Lattice QCD at finite temperature and density

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Introduction to lattice QCD

lattice QCD, 1974

non-pertubative, first-principles, numerical approach; all systematic errors can be quantified

Particle Data Book on quark masses, 2001

"Eventually, lattice gauge theory methods will be accurate enough to determine the light quark masses."

Particle Data Book on quark masses, 2012

"The determination of quark masses using lattice simulations is well established. With improved algorithms and access to more powerful computing resources, the precision of the results has improved immensely in recent years."

$$\begin{aligned} m_s &= 95.5(1.1)(5.5)\ \text{MeV} \\ m_{ud} &= 3.469(47)(48)\ \text{MeV} \end{aligned}$$

Users guide to lattice QCD results

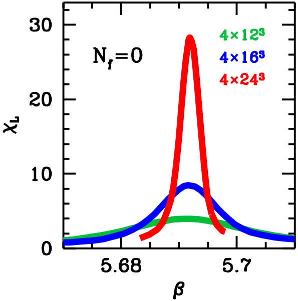
evaluation criteria

	🔴	🟠	🟢
continuum limit	one a	two a's	extrapolation using three or more a's
physical mass	one non-phys. mass	more non-phys. masses	phys. mass
fermion type	quenched	staggered	Wilson
finite V analysis	one V	two V's	extrapolation using three or more V's

(🔴) is not neccesarily bad, but uncertanity is not estimated

Order of the transition

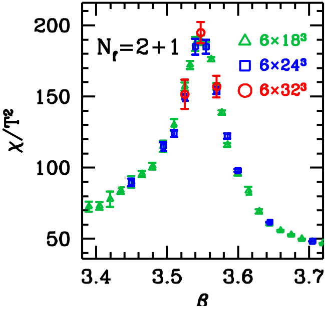
phase transition occurs only in infinite volume $\Rightarrow$ finite size scaling
look at the susceptibility of the Polyakov-line $\frac{1}{V}\left(\langle L^2\rangle - \langle L\rangle^2\right)$ in the quenched theory for different volumes



first order transition
peak width $\propto 1/V$, peak height $\propto V$

Order of the transition

look at the chiral susceptibility $\frac{1}{V}\frac{d^2 logZ}{dm^2}$ in the full $(n_f = 2+1)$ theory for aspect ratios 3–6 (an order of magnitude change in the volume)

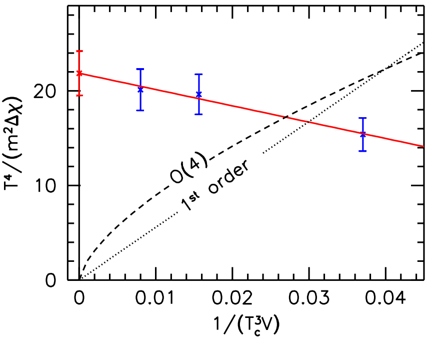


cross-over or analytic transition

peak width ≈ constant, peak height ≈ constant

Order of the transition: crossover ●●●●

renormalized, continuum extrapolated chiral susceptibility



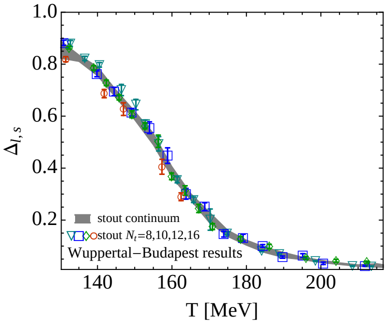
the result is consistent with an approximately constant behavior for a factor of 5 difference within the volume range
finite size scaling analysis (●), continuum result(●), physical quark masses(●) with staggered discretization(●)

Transition temperature

crossover transition

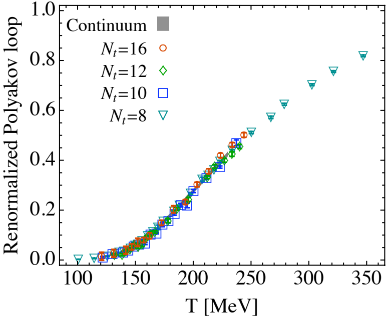
$\Rightarrow$ no single T_c, but 30-40 MeV broad region

$\Rightarrow$ different variables give different pseudocritical T_c's



chiral condensate

$T_c \sim 150\text{MeV}$

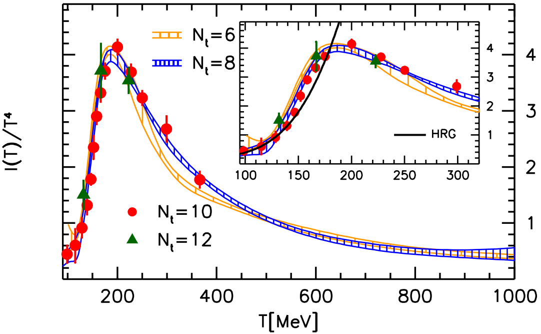


Polyakov-loop

$T_c \sim 170\text{MeV}$

Equation of state

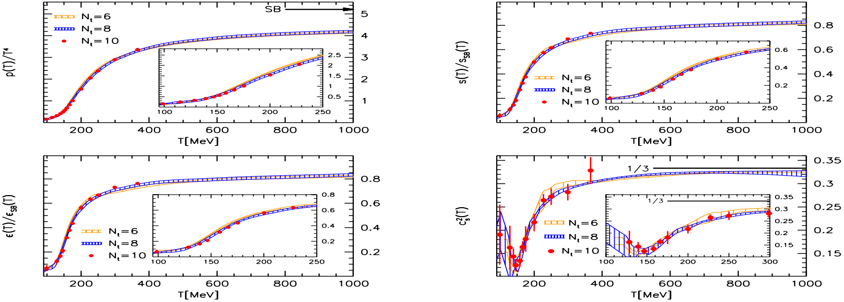
on the lattice we measure the dimensionless trace anomaly $\frac{\epsilon - 3p}{T^4}$



different lattice spacings are in good agreement, but continuum limit is not yet done (●); physical mass (●); staggered quarks (●)

Equation of state

pressure, energy density, entropy, speed of sound are derived



"smaller than error" parametrization for $T = 0...1000$ MeV:

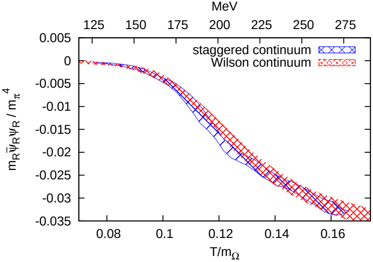
$$\frac{\epsilon - 3p}{T^4} = \exp(-h_1/t - h_2/t^2) \cdot \left(h_0 + \frac{f_0 \cdot [1 + \tanh(f_1 + f_2 t)]}{1 + g_1 t + g_2 t^2} \right) \Bigg|_{t = T/200\mathbf{MeV}}$$

h_0	h_1	h_2	f_0	f_1	f_2	g_1	g_2
0.1396	-0.1800	0.0350	2.76	6.79	-5.29	-0.47	1.04

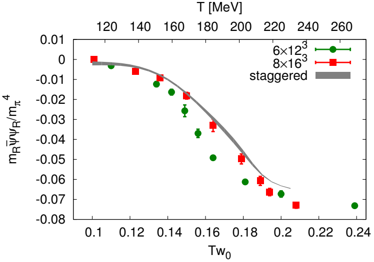
Transition with Wilson ●●●, overlap ●●●

staggered discretization is theoretically unclean
non-staggered discretizations are very expensive

Wilson vs staggered: continuum limit ●, $m_\pi = 545$ MeV ●



overlap vs staggered: two lattice spacings ●, m_π=350 MeV ●



all numerical evidence until now supports the correctness of staggered, continuum limit with smaller masses will follow

Summary of results

Zero density				
transition is crossover	🟢	🟢	🟠	🟢
transition temperature(s)	🟢	🟢	🟠	-
equation of state	🟠	🟢	🟠	-
transition with Wilson fermions	🟢	🔴	🟢	-
transition with overlap fermions	🟠	🔴	🟢	-
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Finite chemical potential

sign problem

for non-vanishing μ's exponentially large number of equally important field configurations with oscillating contribution, numerical algorithms (importance sampling) do not work

until 2001 lattice QCD could not say anything for $\mu > 0$

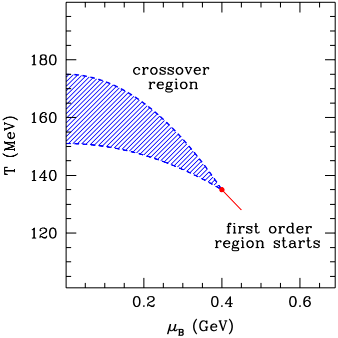
techniques

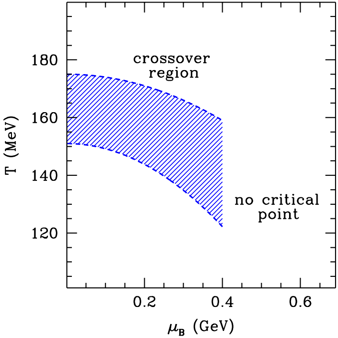
- 2001: multiparameter reweighting [Fodor-Katz]
- 2002: Taylor expanding in μ [Bielefeld-Swansee]
- 2002: imaginary μ [de Forcrand-Philipsen, D'Elia-Lombardo]
- 2005: canonical partition functions [de Forcrand-Kratochvila]

they do not solve the sign problem, but make the small-moderate μ region accesible
for small μ's the Taylor-expansion is the least expensive

Curvature of the transition line

curvature is the leading order behaviour: $\kappa = -T_c \left. \frac{dT_c(\mu)}{d\mu^2} \right|_{\mu=0}$



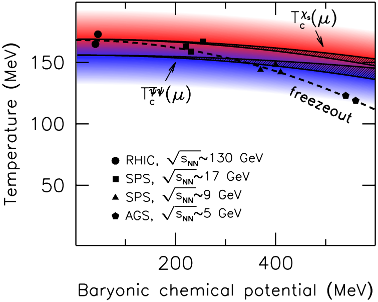


Does the crossover region shrink or expand?

Is there a critical endpoint?

Curvature of the transition line

leading order $O(\mu^2)$ analysis for chiral condensate ($T_c \sim 150$ MeV) and strange number susceptibility ($T_c \sim 170$ MeV)



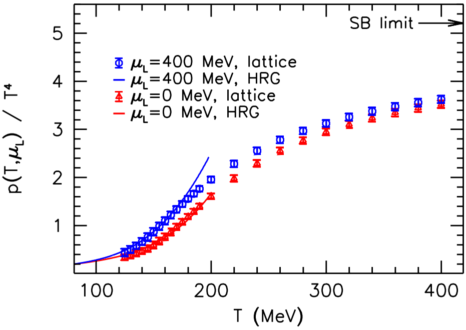
the two curvatures are the same ($\kappa = 0.008(2)$), the widths do not change with μ

transition does not get weaker nor stronger $\Rightarrow$ critical point?

Equation of state for small μ

leading order expansion of pressure:

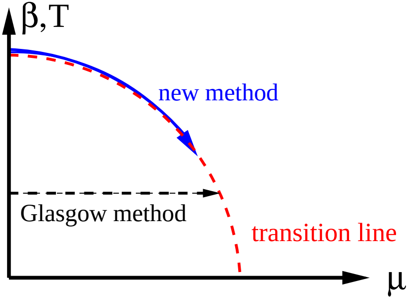
$$\frac{p(T,\mu)}{T^4} = \frac{p(T)}{T^4} + c_2(T)\frac{\mu^2}{T^2} + \ldots$$



Critical endpoint: multiparameter reweighting [Fodor-Katz,'01]

has to go beyond leading order in μ

reweighting: generate data at zero μ and add correction factor (weight), which takes into account all orders in μ

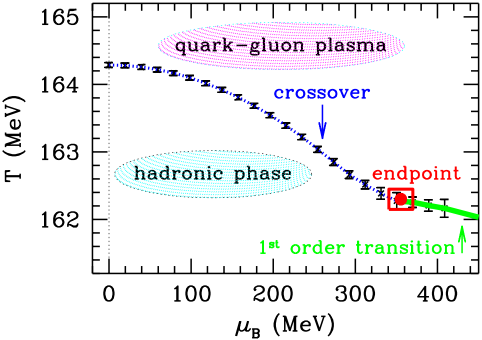


old method (Glasgow): single parameter (μ), purely hadronic $\Rightarrow$ transition

multiparameter (new method): two parameters (μ and T), transition $\Rightarrow$ transition

Endpoint from reweighting [Fodor-Katz,'04]

first lattice QCD result at finite chemical potential



endpoint: $T_E = 162 \pm 2$ MeV, $\mu_E = 360 \pm 40$ MeV

phase transition only in infinite V $\Rightarrow$ finite V analysis (●)

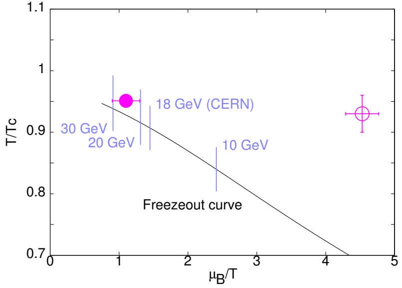
one lattice spacing (●), physical mass (●), staggered (●)

Endpoint from Taylor-expansion [Gavai-Gupta,'05]

expansion of pressure at $\mu = 0$ has a convergence radius, which is smaller than the critical chemical potential

$$\frac{p(T,\mu)}{T^4} = \sum_{n=0}^{\infty} c_n(T) \left(\frac{\mu}{T}\right)^n$$

estimate convergence radius from the first few c_n's



endpoint: $T_E/T_c \leq 0.95$ and $\mu_E/T_c \geq 1.1$

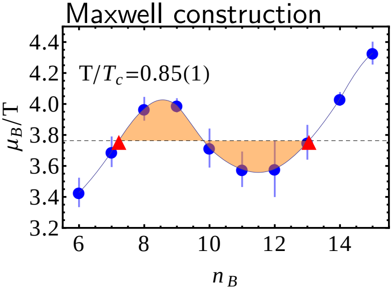
one lattice spacing (●), non-physical mass (●), stagg (●), finite V (●)

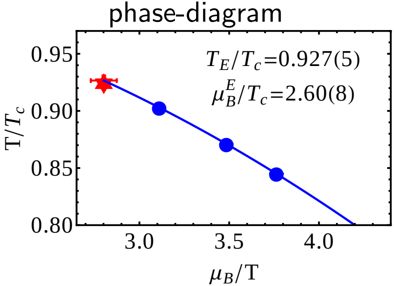
Endpoint from canonical approach [Li-Alexandru-Liu,'11]

use canonical ensemble (C) instead of grand-canonical (GC)

$$Z_{\mathrm{GC}}(\mu, T) = \sum_B Z_{\mathrm{C}}(B, T) \cdot \exp(B\mu/T)$$

set number of baryons(B) and measure chemical potential (μ)





endpoint: $T_E/T_c = 0.925(5)$ and $\mu_E/T_c = 2.60(8)$

two lattice actions (●), non-physical mass (●), Wilson (●), one V (●)

Summary of results

Zero density				
transition is crossover	🟢	🟢	🟠	🟢
transition temperature(s)	🟢	🟢	🟠	-
equation of state	🟠	🟢	🟠	-
transition with Wilson fermions	🟢	🔴	🟢	-
transition with overlap fermions	🟠	🔴	🟢	-
Finite density				
curvature of transition line	🟢	🟢	🟠	-
equation of state to O(μ^2)	🟢	🟢	🟠	-
critical endpoint reweighting	🔴	🟢	🟠	🟢
critical endpoint expansion	🔴	🔴	🟠	🟢
critical endpoint canonical	🟠	🔴	🟢	🔴
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