Neutron star masses and radii from X-ray bursts in low-mass X-ray binaries

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### Neutron star structure



Main problem – inner core Equation of State (EoS)

## Zoo of NS inner core EoS



Solution – M and R from observations!

# X-ray bursting neutron stars

- X-ray bursting NSs LMXBs with thermonuclear explosions at the neutron star surface
- Sometimes close to the Eddington limit during the burst (photospheric radius expansion (PRE) bursts)
- Burst duration ~10 1000 sec

Ideal sources for NS masses and radii investigations (important for EOS!!!)



Low Mass X-ray Binary (artist veiw)



4U 1724-307 in Terzan 2

Figure from Molkov et al (2000)

### Plane parallel model of the bursting layer

Emergent radiation



#### How emergent spectrum forms?



Due to Compton scattering the emergent spectrum close to the diluted blackbody.

$$F_{v} = \frac{1}{f_{c}^{4}} B_{v}(f_{c}T_{eff}), \quad f_{c} \approx 1.4 - 1.9$$

The apparent size of emitting area depends on the color correction factor  $f_c$ 

$$R_{\infty} = R_{BB} f_c^2$$

Photons which we observe are emitted at the depth

$$\tau_{eff} = \sqrt{\tau_{ff} \tau_T} \approx 1$$
 - thermalization depth

At this depth, electron scattering optical depth  $\tau_{\tau} >> 1$ 

s 
$$k_{ff}(E) \propto E^{-3}$$
,  $k_{ff} << \sigma_T$  at  $E > 0.1 - 1 kT_e$ 

Atmosphere models of X-ray bursts accounting for Compton scattering

- Using Kompaneets equation: London et al. 1984, 1986; Lapidus et al. 1986; Ebisuzaki 1987; Pavlov et al. 1991, Suleimanov et al. 2006, 2011
- Using approximate Compton redistribution function (Guilbert 1981): Madej 1991; Madej et al. 2004; Majczyna et al. 2005
- Using exact relativistic Compton redistribution function (Suleimanov et al. 2012)

## **Basic equations**

Hydrostatic equilibrium

$$\frac{1}{\rho} \frac{dP_{gas}}{dr} = -\frac{GM_{NS}}{R_{NS}^2 (1 - R_g / R_{NS})^{1/2}} + \frac{4\pi}{c} \int H_v (k_{ff} + \sigma_e) \, dv$$

Radiation transfer

Kompaneets operator

$$\frac{\partial^2 (f_v J_v)}{\partial \tau_v^2} = \frac{k_{ff}}{k_{ff} + \sigma_e} (J_v - B_v) \left[ -\frac{\sigma_e}{k_{ff} + \sigma_e} \frac{kT}{m_e c^2} x \frac{\partial}{\partial x} (\frac{\partial J_v}{\partial x} - 3J_v + \frac{T_{eff}}{T} x J_v (1 + C \frac{J_v}{x^3})) \right]$$

$$x = \frac{hv}{kT_{eff}} \quad C = c^2 h^2 / 2(kT_{eff})^3$$
Compton scattering

Radiative equilibrium

$$\int k_{ff} (J_{\nu} - B_{\nu}) dx \left[ -\sigma_e \frac{kT}{m_e c^2} \int (4J_{\nu} - \frac{T_{eff}}{T} x J_{\nu} (1 + \frac{CJ_{\nu}}{x^3})) dx \right] = 0$$

 $k_{ff}$  - true absorption opacity (mainly free-free transitions)

 $\sigma_{e}$  - Thomson electron scattering opacity

#### Accurate treatment using exact relativistic redistribution function for Compton scattering

Radiation transfer equation (RTE)

$$\mu \frac{dI(x,\mu)}{d\tau_x} = I(x,\mu) - S(x,\mu), \qquad d\tau_x = -(\sigma(x,\mu) + k(x))\rho(z) dz$$

Electron scattering opacity

$$\sigma(x,\mu) = \frac{\sigma_{\rm e}}{x} \int_0^\infty x_1 \, dx_1 \, \int_{-1}^1 d\mu_1 \, R(x,\mu,x_1,\mu_1) \, \exp\left(-\frac{x_1-x}{\Theta(z)}\right) \left(1 + \frac{C \, I(x_1,\mu_1)}{x_1^3}\right)$$

$$\sigma_{\rm e} = \sigma_{\rm T} \frac{n_{\rm e}}{\rho}, \quad C = \frac{h^2}{2m_{\rm e}^3 c^4} \qquad \sigma_{\rm T} = 6.65 \cdot 10^{-25} \,{\rm cm}^2$$

Source function

$$\tau_{e} = \sigma_{T} \frac{n_{e}}{\rho}, \quad C = \frac{h^{2}}{2m_{e}^{3}c^{4}} \quad \sigma_{T} = 6.65 \cdot 10^{-25} \text{ cm}^{2} \qquad x_{1} = \frac{hv_{1}}{m_{e}c^{2}}, \quad x = \frac{hv}{m_{e}c^{2}},$$
Source function
$$S(x,\mu) = \frac{k(x)}{\sigma(x,\mu) + k(x)}B(x) + \qquad \Theta(z) = \frac{kT(z)}{m_{e}c^{2}}$$

$$\frac{x^{2}}{\sigma(x,\mu) + k(x)} \left(1 + \frac{CI(x,\mu)}{x^{3}}\right) \int_{0}^{\infty} \frac{dx_{1}}{x_{1}^{2}} \int_{-1}^{1} d\mu_{1}R(x,\mu,x_{1},\mu_{1})I(x_{1},\mu_{1}),$$

Redistribution function (RF)

$$R(x, x_1, \mu, \mu_1) = \int_0^{2\pi} R(x, x_1, \eta) \, d\varphi, \qquad \eta = \mu \mu_1 + \sqrt{1 - \mu^2} \sqrt{1 - \mu_1^2} \cos \varphi$$

### Model atmosphere calculations

(Suleimanov, Poutanen, Werner 2011, A&A 527, A139 / 2012, A&A, 545, A120)

- 6 chemical compositions: H, He, solar H/He with  $Z = 1, 0.3, 0.1, 0.01 Z_{sun}$ 

- **3** surface gravities: *log g* = 14.0, 14.3 and 14.6

- 28 relative luminosities  $I = L / L_{edd}$ from 0.001 to 1.1 (super-Eddington luminosities for Thomson cross-section)

Dashed lines – Kompaneets approximation

Solid lines – exact Compton scattering kernel



# New set of atmosphere models. Radiative acceleration.



Dashed curves – Paczynski's (1983) approximation for averaged opacity

$$\sigma_e(T) \approx \sigma_e \left( 1 + \left( \frac{T}{4.5 \times 10^8 K} \right)^{0.86} \right)^{-1}$$

Slightly improved approximation gives better result

$$\sigma_e(T) \approx \sigma_e \left( 1 + \left( \frac{T}{4.5 \times 10^8 K} \right)^{0.98} \right)^{-1}$$

Used approximation

$$g_{rad} = \sigma_e(T) \, \frac{\sigma_{SB} T_{eff}^4}{c}$$

### Color correction $f_c$ calculations

Calculated spectra are redshifted and fitted by diluted blackbody (oneand two-parameters functions) (assuming  $M = 1.4 M_{sun}$ ) in the *PCA/RXTE* energy band (3-20) keV  $F_E = wB_E(f_c T_{eff})$ 



Minimizing deviations in photon number flux

$$\sum_{n=1}^{N} \frac{(F_{E_n} - w_2 B_{E_n}(f_{c,2}T_{\text{eff}}))^2}{E_n^2}$$

Color correction  $f_{\rm c}$  calculations



Dashed lines – Kompaneets approximation

differences are small at  $L/L_{edd} < 0.8$ 

Solid lines – exact Compton scattering kernel

# **Basic relations**



 $F_{obs}(Edd) = \frac{L_{Edd}}{d^2(1+z)^2}$  observed flux, which corresponds to Eddington luminosity

$$F_{obs} = \sigma T_{BB}^4 \frac{R_{BB}^2}{d^2} = \sigma T_{BB}^4 K$$
 observed flux with fitting parameters

 $F_E \approx \frac{1}{f_c^4} B_E(f_c T_{eff})$  observed spectrum is close to the diluted blackbody color correction (hardness) factor

from 
$$L_{obs} = L_{BB}$$
  $\longrightarrow$   $R_{BB} = \frac{R}{f_c^2}(1+z)$ 

# **Cooling tail method**

•The observed evolution of  $K^{-1/4}$  vs. F should look similar to the theoretical relation  $f_c$  vs.  $F/F_{Edd}$ 

$$K = \left(\frac{R_{bb}}{D_{10}}\right)^2 = \frac{1}{f_c^4} \left(\frac{R_\infty}{D_{10}}\right)^2 \longrightarrow K^{-1/4} = A f_c (F / F_{Edd})$$
$$D_{10} = d/10 \, kpc \qquad \qquad A = (R_\infty [\text{km}]/D_{10})^{-1/2}$$

•From the fits a more reliable estimate of the Eddington flux and apparent radius can be obtained.

and we use now our theoretical dependences

 $f_{\rm c}$  vs.  $F/F_{\rm Edd}$ 

## to find two fitting parameters: A and $F_{\text{Edd}}$

# Cooling tail method



### Three curves on *M-R* plane





# Cooling tails of PRE bursts from 4U 1724-307



- Crosses: Long, >150 sec, PRE burst during hard/low state on Nov 8, 1996.
- Diamonds: two short PRE bursts on Feb 23 and May 22, 2004 during soft state.
- Spectral evolution is spectacularly different!

# M-R relation - 4U 1724-307

•From the best-fit A and  $F_{Edd}$ , we can get constraints on Mand R if we assume some distance distribution (we take flat in 5.3-7.7 kpc with gaussian tails).

- •1. Radius > 13.5 km at 90% confidence for any solar composition for M<2.3 solar.
- •2. Hydrogen-rich atmosphere is preferred.
- •3. Stiff EoS is preferred.



Contours are elongated along  $T_{Edd}$ =const track

$$T_{\rm Edd,\infty} = \left(\frac{gc}{\sigma_{\rm SB}\kappa_{\rm e}}\right)^{1/4} \frac{1}{1+z} = 6.4 \times 10^9 \, A \, F_{\rm Edd}^{1/4} \, {\rm K}$$

# Cooling tails of PRE bursts from 4U 1608-52

Bursts in hard persistent states are taken



Poutanen et al. 2012 (in preparation)

# M-R relation - 4U 1608-52



Results are similar to 4U 1724-307

Poutanen et al. 2012 (in preparation)

# Conclusions

- 1. An extended set of accurate model atmospheres for X-ray bursts covering large range of luminosities, various log g and chemical composition is computed.
- 2. Evolution of blackbody normalization with flux  $K^{-1/4}$  vs. F in "hard state" bursts is well described by the theory. "Soft state" PRE bursts from 4U1724-307 and 4U1608-52 do not show the evolution of  $K^{-1/4}$  vs. F predicted for a passively cooling neutron star, therefore they should not be used for M/R determination.
- 3. Burst properties depend on persistent flux. Optically thick accretion disk blocks nearly 1/2 of the star and possibly affects the short burst (soft state) spectra. In the hard state bursts, accretion is not important (optically thin).
- 4. Neutron star radii are constrained at R>13.5 km favoring stiff equation of state (consistent with existence of the 2M<sub>☉</sub> pulsar).

# Integration over NS surface

