Gravitational waves from colour-magnetic ‘mountains’ in neutron stars

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Based on Glampedakis, DIJ & Samuelsson
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Context: Gravitational wave emission form ‘mountains’

- A neutron star rotating steadily with spin $f_{\rm spin}$ at distance r radiates GWs:

$$h = 3 \times 10^{-29} \left(\frac{\epsilon}{10^{-7}} \right) \left(\frac{f_{\rm spin}}{10\,{\rm Hz}} \right)^2 \left(\frac{1\,{\rm kpc}}{r} \right),$$

  where the ellipticity $\epsilon = (I_{yy} - I_{xx})/I_{zz}$ may be non-zero due to:

  1. Strains in solid crust and possibly core, or
  2. Magnetic forces (this talk).

Direct upper limits

- Direct upper limits already obtained, from non-detection of GWs by LIGO/Virgo, e.g.

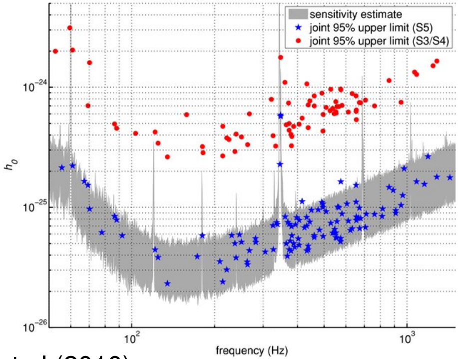
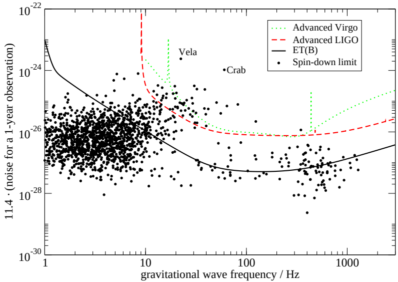


Figure: Abbott et al (2010).

Spin-down upper limit

- Can place a theoretical upper limit on GW emission by assuming 100% conversion of spin-down energy into GWs:



- Need theoretical modelling of mountains to say when upper limits start to get interesting.

Magnetic mountains: normal matter

- Magnetic field lines have an effective tension, and deform star (Chandrasekhar & Fermi 1953).
- Rough estimate:

$$\epsilon \sim \frac{\int B^2 \, dV}{GM^2/R} \sim 10^{-12} \left(\frac{B}{10^{12}\,\mathrm{G}} \right)^2 .$$

- For normal neutron star matter, ellipticities are small, GWs undetectable.

Protonic superconductivity: flux tubes

- A magnetic field threads a superconductor by forming flux tubes.
- Each tube carries a flux $\phi_0 = hc/2e$.
- A field B then has an area density of flux tubes:

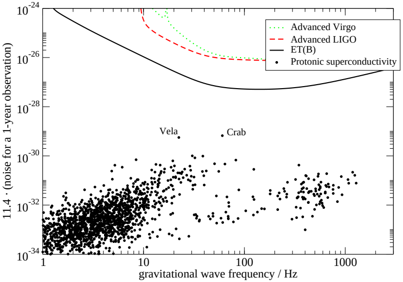
$$N = \frac{B}{\phi_0} \sim 5 \times 10^{18} \frac{B}{10^{12}\,\mathrm{G}}\,\mathrm{cm}^{-2}.$$

- For a typical field strength, this implies $\sim 10^{31}$ flux tubes!

Protonic superconductivity: flux tubes

- The flux tube energy per unit length (tension) is dominated by the kinetic energy of the proton circulation around the flux tube.
- Effect of this is to increase flux tube tension by a factor of H_c/B, where $H_c \sim 10^{15}$ G, increasing ellipticity:

$$\epsilon \sim 10^{-9} \frac{B}{10^{12}\,\mathrm{G}}.$$



Clearly, GWs not detectable.

Colour-magnetic flux tubes

- If CFL or 2SC phases occur in neutron star cores, can get *colour-magnetic flux tubes* (Iida & Baym 2002, Iida 2005, Alford & Sedrakian 2010).
- Relevant vector potential field A_μ^{X} is a mixture of electromagnetic A_μ and gluonic A_μ^8 parts:

$$A_\mu^{\mathrm{X}} = A_\mu \sin\chi + A_\mu^8 \cos\chi,$$

  where mixing angle is

$$\chi \sim \frac{e}{g} \ll 1.$$

- This leads to flux tube tension $\sim 10^3$ larger than in protonic superconductivity case.

Colour-magnetic flux tubes

- Estimate ellipticity:

$$\epsilon_{\rm CFL} \sim 10^{-7} \left(\frac{f_{\rm vol}}{1/2}\right) \left(\frac{B_{\rm int}}{10^{12}\,{\rm G}}\right) \left(\frac{\mu_{\rm q}}{400\,{\rm MeV}}\right)^2,$$

  where

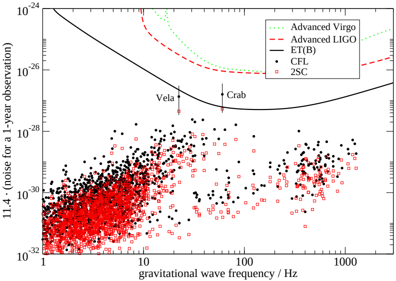
  - $f_{\rm vol}$ = fraction of stellar volume in deconfined state,
  - $B_{\rm int}$ = *internal* magnetic field strength,
  - $\mu_{\rm q}$ = quark chemical potential.

- Can allow for internal field to be some multiple of external field:

$$B_{\rm int} = \alpha B_{\rm ext}.$$

Colour-magnetic flux tubes

- For given stellar parameters $f_{\rm vol}$, α and $\mu_{\rm q}$ can then balance observed spin-down of pulsars against combined GW & EM torque to estimate $B_{\rm int}$ and hence h.
- GW amplitudes scale as $h \sim f_{\rm vol}\alpha\mu_{\rm q}^2$; for sensible values ($f_{\rm vol} = 0.5$, $\alpha = 2$, $\mu_{\rm q} = 400$ MeV) obtain:

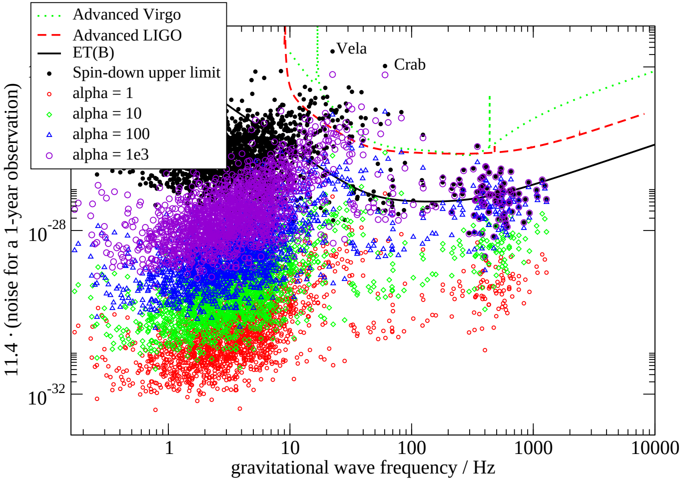


Clearly of interest for Crab and Vela pulsars.

Varying the internal field strength

- The internal magnetic field strength may significantly exceed the external one.
- For some of the young pulsars, measurements of the braking index
$$n = \frac{\ddot{\Omega}\Omega}{\dot{\Omega}^2} < 3$$
can be interpreted as evidence for a buried magnetic field emerging from the core.
- For the (old) millisecond pulsars, accretion earlier in the star's life may have buried a canonical $\sim 10^{12}$ G field in core.
- So, there exists motivation for exploring $\alpha \gg 1$.

Varying the internal field strength



Summary

- Colour-magnetic flux tubes may occur in neutron stars.
- Flux tube tension is $\sim$ 1000 times larger than for protonic superconductivity.
- GW emission may be detectable by third generation detector for Crab and Vela.
- If internal field strength $\gtrsim 10^2$ times external field, several tens of other pulsar may be detectable.
- Obvious caveats:
  - Microphysics uncertain
  - Strength and arrangement of internal field not known
- *Proximity of pulsars to detectability curves motivates more work in addressing key issues.*