Pressure Corrections to the Equation of State in Nuclear Mean Field.

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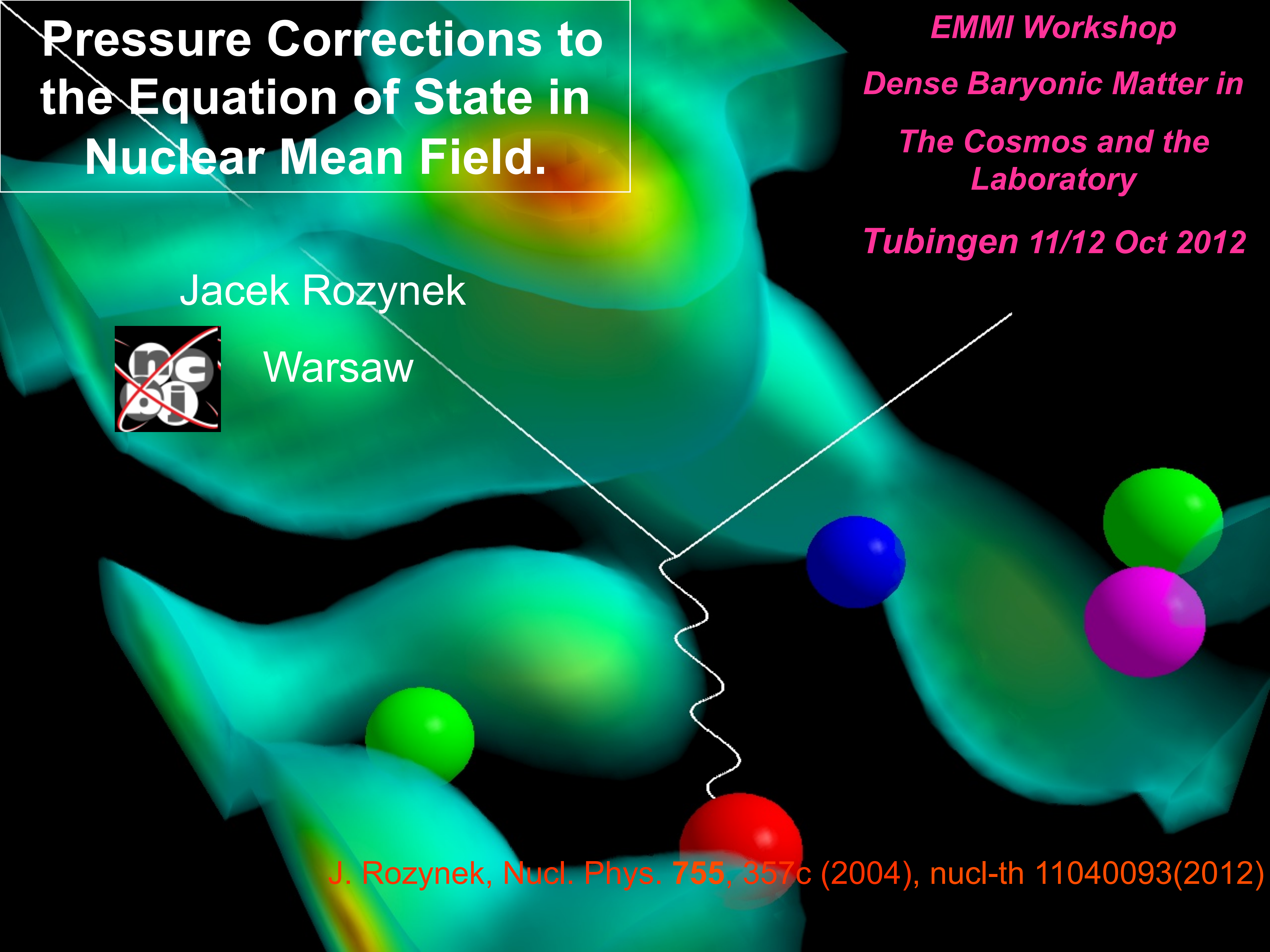
Warsaw

EMMI Workshop

Dense Baryonic Matter in The Cosmos and the Laboratory

Tubingen 11/12 Oct 2012

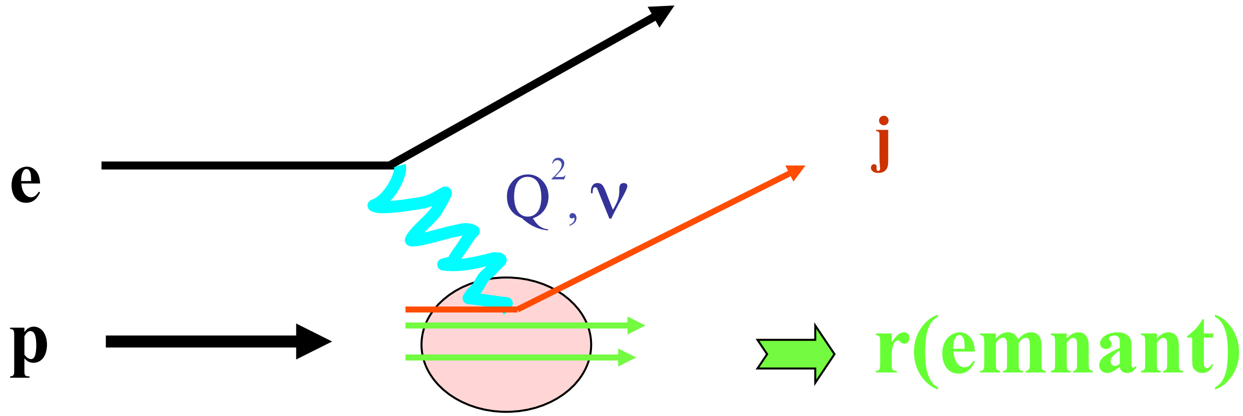


J. Rozynek, Nucl. Phys. **755**, 357c (2004), nucl-th 11040093(2012)

Plan

1. Nuclear structure function in Deep Inelastic Scattering (DIS) - review.
   - EMC effect
   - Hadrons with quark primodial distributions
   - Nuclear Bjorken Limit - $M_N(x)$
2. Finite pressure corrections.
   - Momentum Sum Rule
   - Below and above the saturation density
3. Implication to the EOS for Nuclear matter.
   - New equation of state
   - Heavy Ion and Astrophysical applications
4. Conclusions.

D I S



Hit quark has momentum $\mathbf{k_+ = x\ \ p_+}$

Experimentaly $x = \mathbf{Q^2/2M\nu}$
and is iterpreted as fraction of longitudinal nucleon
momentum carried by parton(quark) for $\nu^2 > Q^2 \to \infty$.

On light cone Bjorken x is defined as

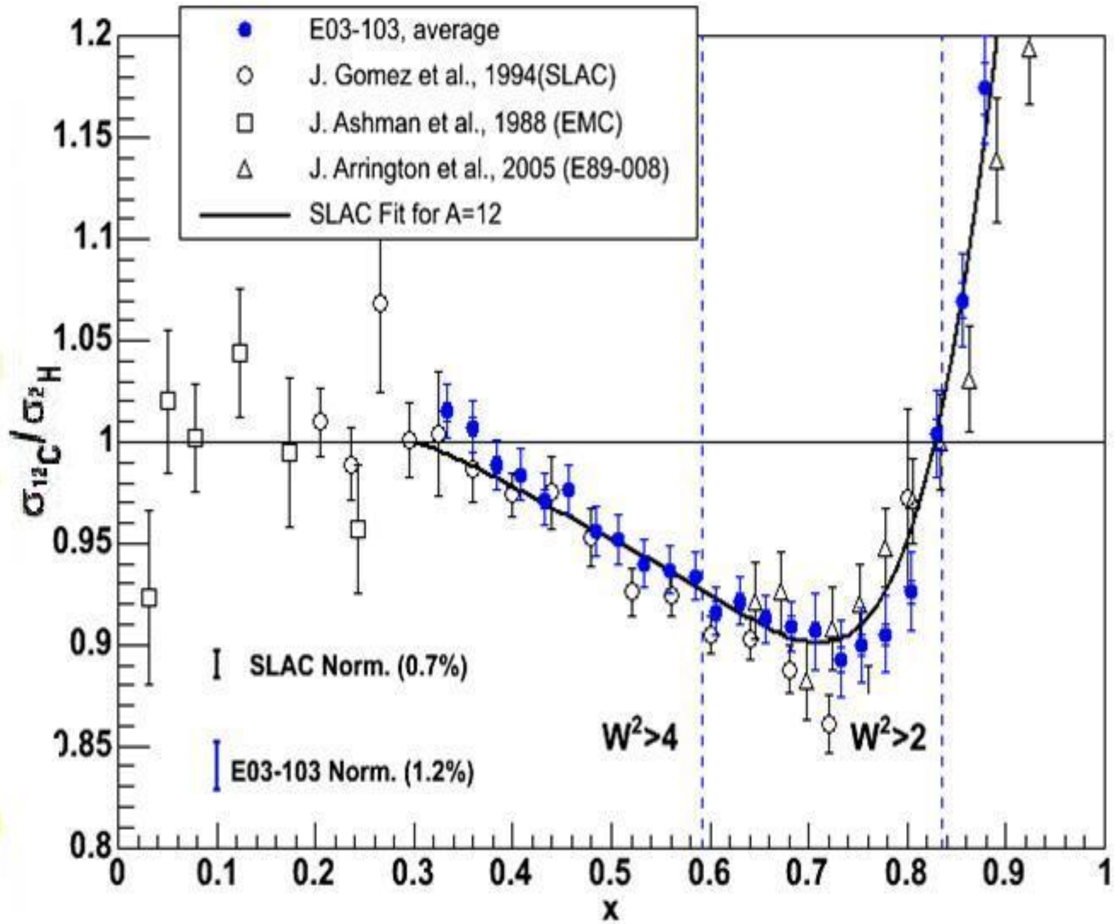
$\mathbf{x = k_+ / p_+}$ where $p_+ = p_0 + p_z$

analogously in nuclei $\mathbf{y = p_+/M}$ where M – nuclear Mass
in the rest frame

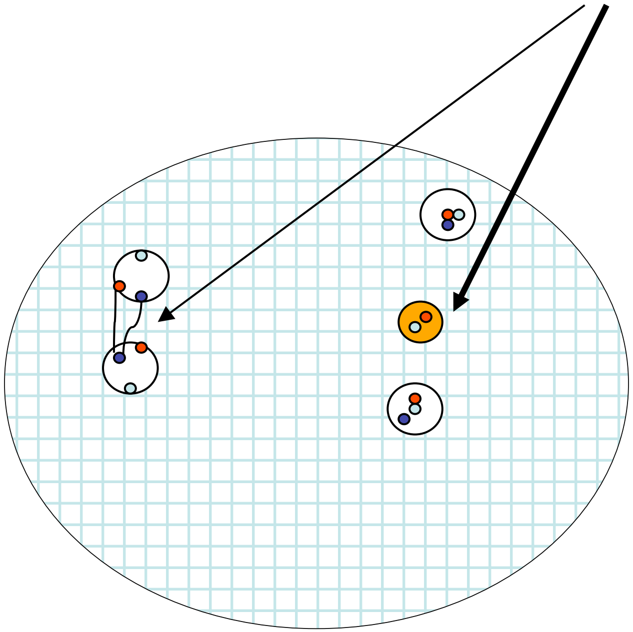
EMC effect

Historically ratio

$$R(x) = F_2^A(x) / AF_2^N(x)$$



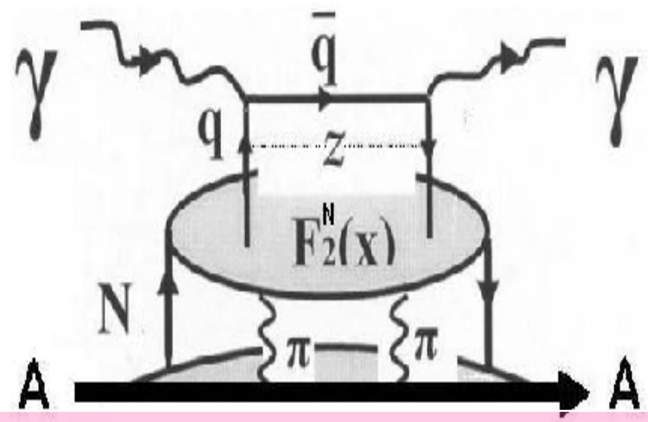
Nucleon in the medium – main differences where the nuclear pions are



- M Birse PLB 299(1985), JR IJMP(2000), G Miller J Smith PR (2001)
- GE Brown, M Buballa, Li, Wambach , Bertsch, Frankfurt, Strikman

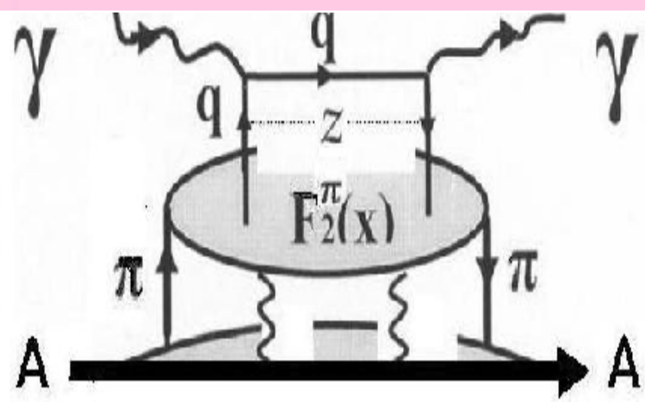
The graphical representation of the convolution model for the deep inelastic electron nucleus scattering with: (a) the active nucleon *N* (with hit quark *q*) including exchange final state interaction with nucleon spectators and (b) virtual pion

(a)



$$F_2^A(x) = \int dy \rho_N^A(y) F_2^N(x/y) + \int dy \rho_\pi^A(y) F_2^\pi(x/y);$$

(b)



J. Rozynek, G. Wilk, Phys.Lett. B**473**, 167 (2000)

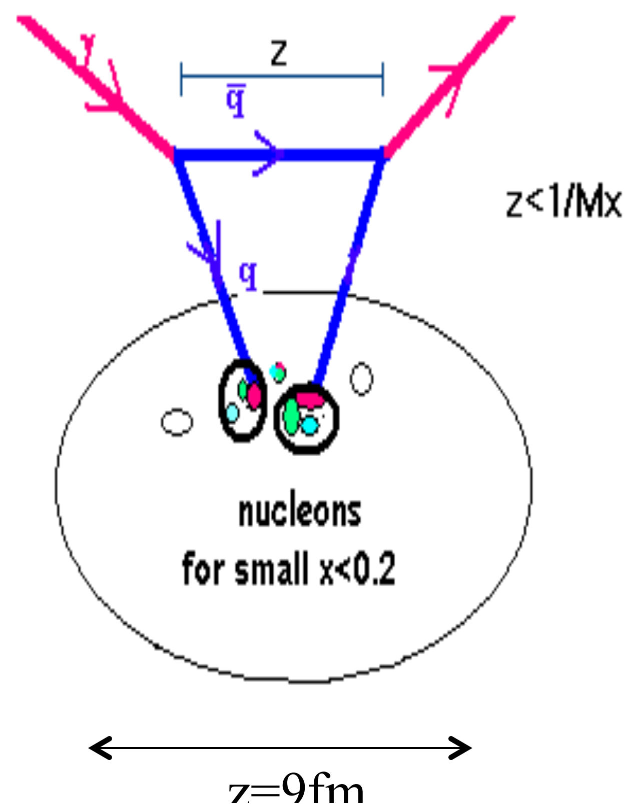
Ttwo resolutions scales in deep inelastic scattering

1 $1/Q^2$ → connected with virtuality of γ probe .
(A-P evolution equation - **well known)**

○

1/Mx = z →distance how far can propagate the quark in the medium. (Final state quark interaction - **not known**)

z distance where is shadowing for that pions which carry the nucleon-nucleon interaction



For x=0.05 z=4 fm

Hard core radius	for x=0.6	z=0.4fm
Inter nucleon distance	for x=0.2	z=1.2fm

Nucleon mass in saturation

$$\frac{1}{A}\sum_{i=1}^{nA} k_{Ai}^{+} = M_N + \epsilon_A = \varepsilon_N$$

$$M_N + \epsilon_A = \frac{4}{(2\pi)^3\rho}\int_{|p|>p_F} d^3p\sqrt{(M_B+U_s)^2+\vec{p}^2} + U_v \quad + E_{meson}$$

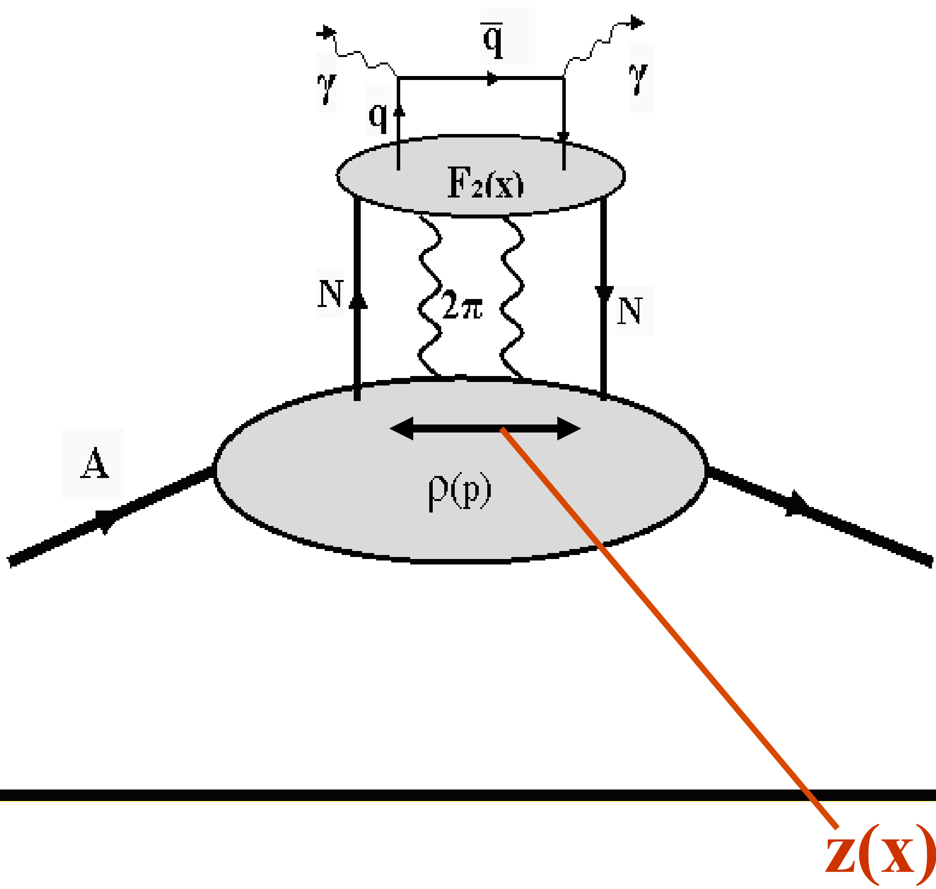
$$M_B \cong \frac{M_A}{A} - E_K \simeq M_N + U_N \qquad for\ x \geq x_C \equiv 0.6$$

$$M_B = M_N \qquad for\ x < x_N \sim 0.2$$

Nuclear final state interaction

r_N - av. NN distance

r_C - nucleon radius

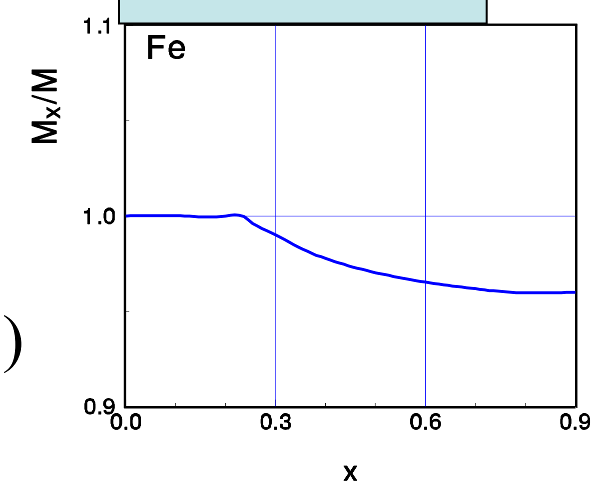


- if $z(x) > r_N$

  $M(x) = M_N$

if $z(x) < r_C$

$M(x) = M_B$



Effective nucleon Mass $M(x)=M(\, z(x)\, ,\, r_C\, ,r_N\,)$

J.R. Nucl.Phys.A

Relativistic Mean Field

RMF models for neutrons stars see i.e. Glendenning book

In standard RMF electrons will be scattered on nucleons in average scalar and vector potential:

$[\alpha \mathbf{p} + \beta(M+U_S) - (e - U_V)]\psi=0$

where $U_S=-g_S/m_S\rho_S$ $U_V=-g_V/m_V\rho$

$U_S = -400\text{MeV}\rho/\rho_0$

$U_V = 300\text{MeV}\rho/\rho_0$

Gives the nuclear distribution of longitudinal nucleon momenta $p_+=y_A M_A$

$$\rho^A(y\ ) = \frac{4}{\rho}\int \frac{d^4p}{(2\pi)^4} S_N(p^0,\mathrm{p})\left[1+\frac{p_3}{E(p)}\right]\delta\left[y-\frac{(p_0+p_3)}{\mu}\right]$$

$S_N()$ - spectral fun. μ - nucleon chemical pot.

Momentum Sum Rule - continuation

$$\frac{1}{A}\int F_2^A(x)dx = \frac{1}{A}\int dy(y)\rho^A(y)\int F_2^N(x)\,dx$$

$$\rho^A(y) = \frac{4}{\rho}\int_{|\mathrm{p}|>p_F}\frac{d^3p}{(2\pi)^3}(1+\frac{p_3}{E_p^*})\delta(y-(E^*(p)+U_V+p^3)/$$

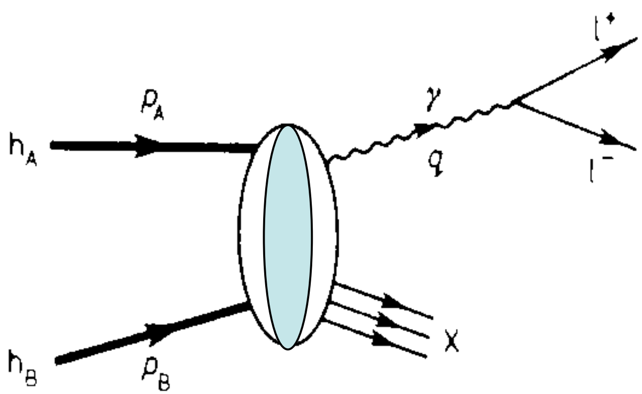
$$= \tfrac{3}{4}\left(\frac{\varepsilon_A}{k_F}\right)^2\left[\left(\frac{p_F}{\varepsilon_A}\right)^2-\left(y-\frac{E_F}{\varepsilon_A}\right)^2\right],$$

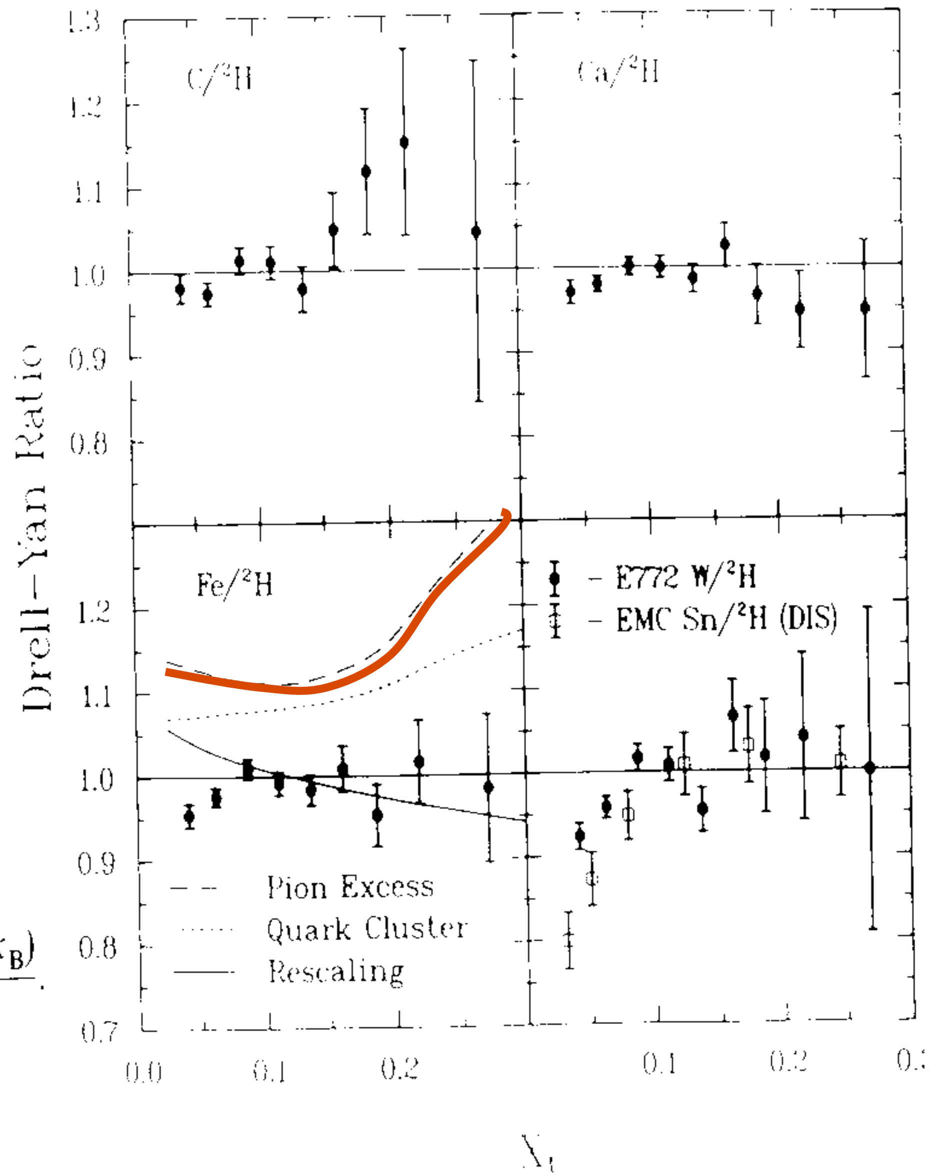
Here the nucleon spectral function was taken in the impulse approximation: $S_N = n(p)\delta(p^o-(E^*(p)+U_V))$ and $E^*(p)=\sqrt{M_N^2+p^2}$. and y takes the values given by inequality $(E_F-p_F)/\varepsilon_A < (E_F+p_F)/\varepsilon_A$ where $\varepsilon_A = M_N+\epsilon$.

The second sum rule is sensitive to Fermi energy E_F as can be seen from the integral:

$$\int dy\, y\rho^A(y) = \frac{E_F}{\varepsilon_A}$$

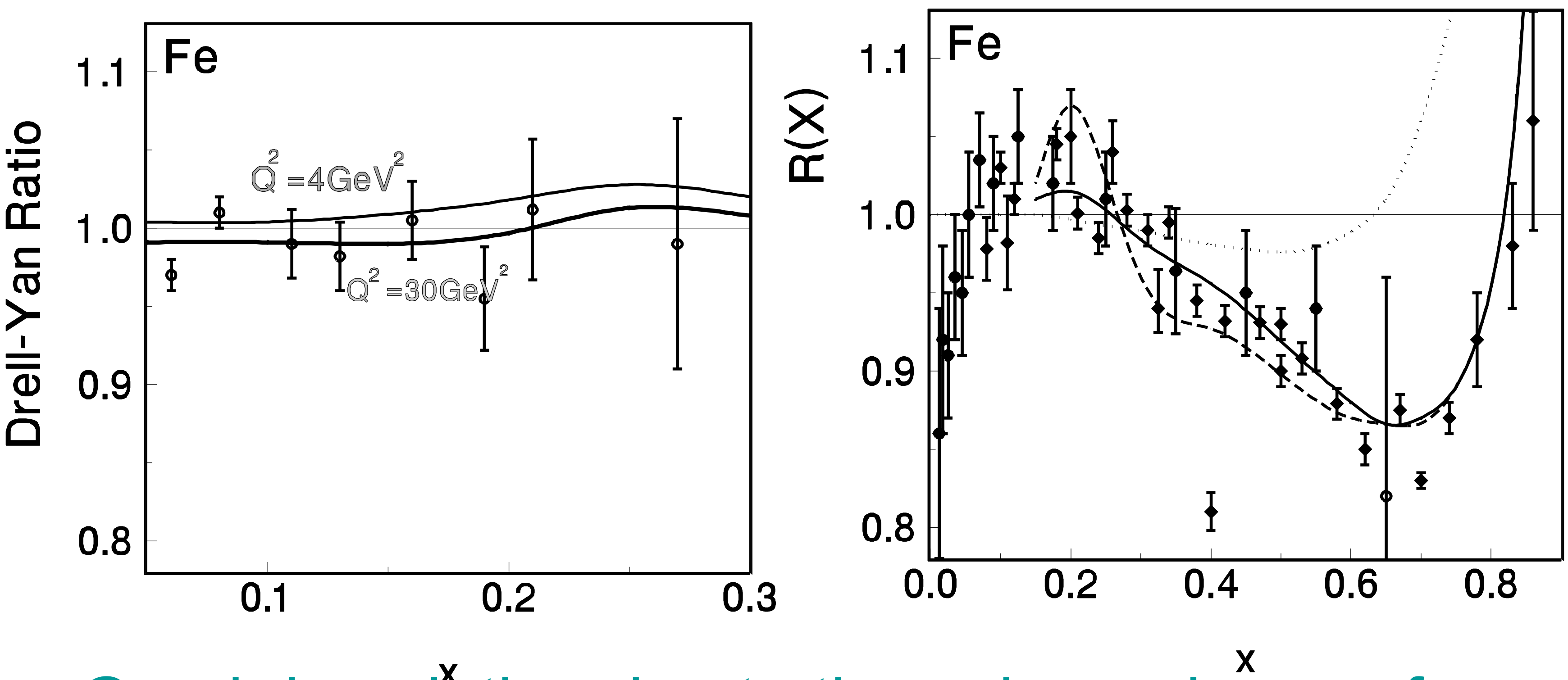
Drell Yan





$$\frac{d^2\sigma}{dq^2 d\xi} = \frac{4\pi\alpha^2}{9q^2 s} \sum Q_j^2 \frac{q_j^A(x_A)\bar{q}_j^B(x_B)+\bar{q}_j^A(x_A)q_j^B(x_B)}{x_A+x_B}.$$

Drell Yan Calculations & EMC effect



Good description due to the x dependence of nucleon mass.

J. Rozynek, G.Wilk, Phys. Rev. C **71**, 068202 (2005)

Conclusions

- Good fit to data for Bjorken x>0.1 by modfying the nucleon mass in the medium (~24 MeV depletion) will correct the EOS for NM. Although such subtle changes of nucleons mass is difficult to measure inside nuclear medium due to final state interaction this reduction of nucleon mass is compatible with recent observation of similar reduction in Delta invariant mass in the decay spectrum to (N+Pion)

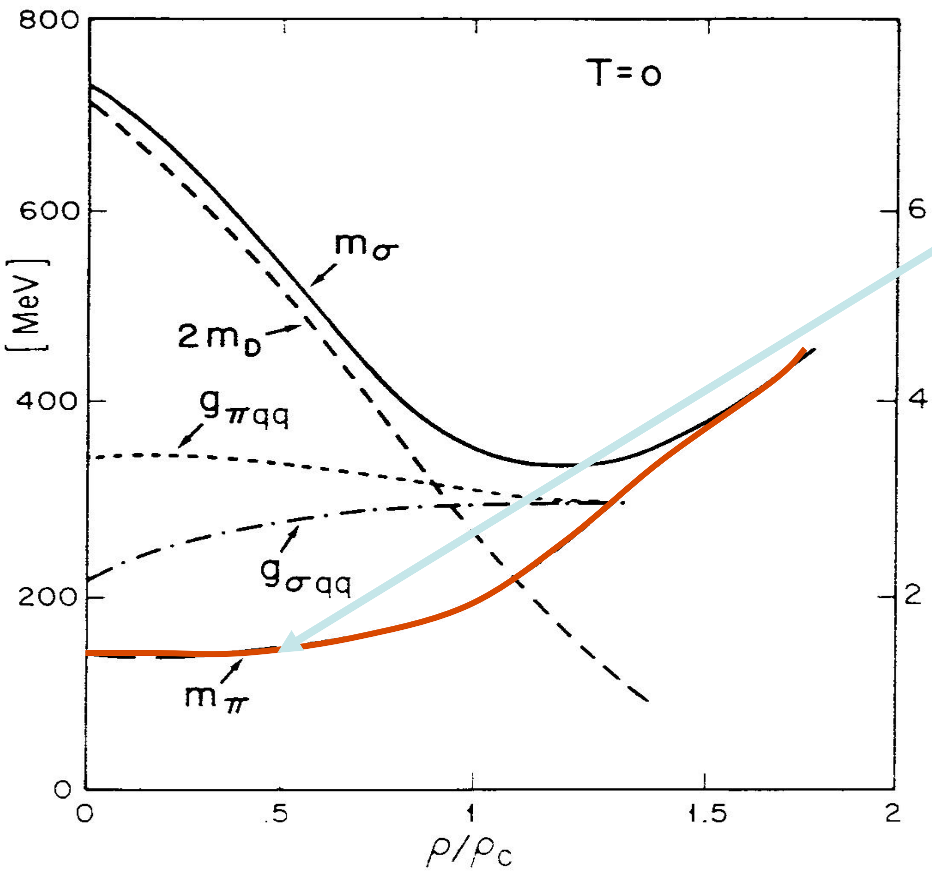
  T.Matulewicz Eur. Phys. J A9 (2000)

- (~ 1% only) of nuclear momentum is carried by sea quarks nuclear pions) due to x dependent effective nucleon mass supported by Drell-Yan nuclear experiments for higher densities increase for soft EOS towards chiral phase transition.

- x – dependent correction to the $< k_T^2 >$ distribution

EOS in NJL

EMC effect



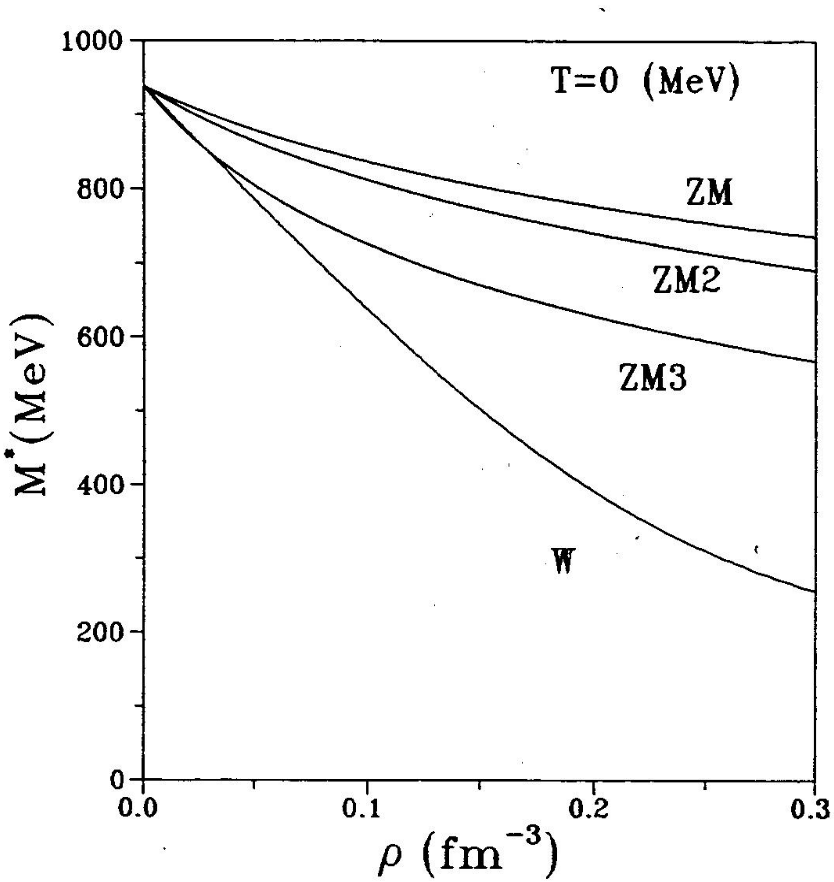
Bernard,Meissner,Zahed PRC (1987)

- pion mass in the medium in chiral symmetry restoration
- Nucleon mass in the medium ?

R. Rapp and J.Wambach, Adv. Nucl. Phys. 25, 1 (2000)

Brown- Rho scaling

Effective Mass in RMF



ZM - Zimanyi Moszkowski

- W - Nucleon bare mass in the Walecka mean field approach
- BS – J.Boguta, (H.Stocker)
- Phys.Lett.106B,1981, (120B, 1983) constructed by changing of covariant derivative in W model. Langrangian describes the motion of baryons with effective mass and the density dependent scalar (vector) coupling constant.

But in the medium we have correction to the Hugenholtz-van Hove theorem:

$$E_F = \frac{d}{dA}(E)_\Omega = \frac{d}{d\varrho}\left(\frac{E}{\Omega}\right)$$

$$\boxed{E_F = \varepsilon_N + \varrho \frac{d\varepsilon_N}{d\varrho}}$$

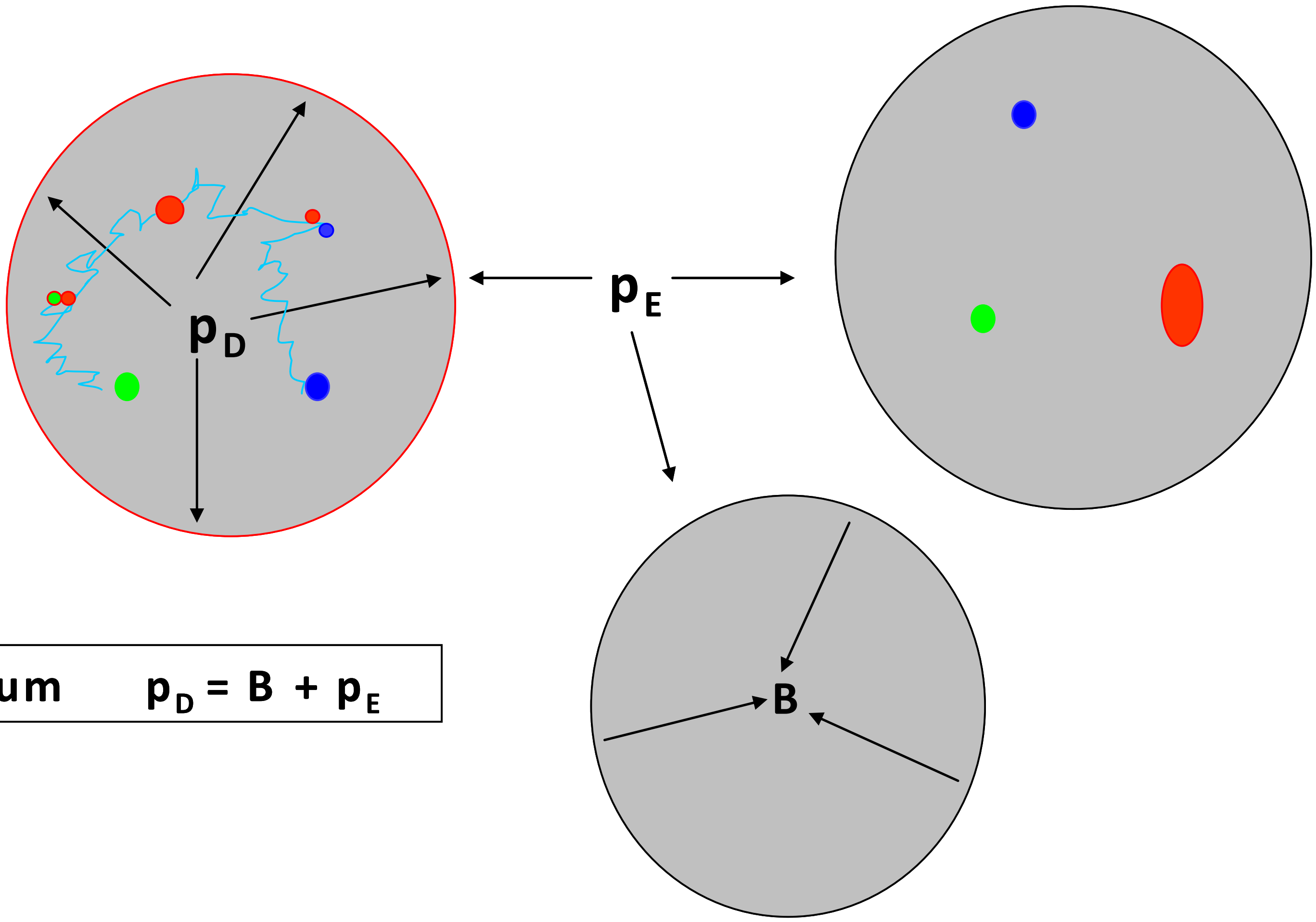
$$p = -\left(\frac{\partial E}{\partial \Omega}\right)_A = \varrho^2 \frac{d}{d\varrho}(\varepsilon_N)$$

For positive p $E_F/M > (E/A)/M$

Longitudinal momentum Sum Rule

For p<0 $\int dy\, y \rho^A(y) = \frac{E_F}{\varepsilon_A} < 1.$ ***But for p>0*** $\int dy\, y \rho^A(y) = \frac{E_F}{\varepsilon_A} > 1.$

TO SWELL OR NOT TO SWELL ? (Nucleon)



equilibrium $p_D = B + p_E$

Above the saturation

- For higher density, the average distances between nucleons are smaller, therefore the room for nuclear pions given by x dependent nucleon mass M_{med} will vanish gradually.

- Also the pion effective cross section is strongly reduced at high nuclear densities above the threshold in N+N=N+N+ π reaction calculated in Dirac-Brueckner approach (also with RPA insertions to self energy of N and Δ .

  B. ter Haar and R. Malfliet, Phys. Rev. C **36**, 1611 (1987), Phys. Rep. **149**, 287 (1987).
  E. Oset, L.L. Salcedo, Nucl. Phys. **468**, 631 (1987), "The Nuclear Methods and the Nuclear Equation of State", ed. M. Baldo, World Scientific 1999.

- Therefore for positive pressure the nuclear pions carry much less then 1% of the nuclear longitudinal momentum.

- Dealing with a non-equilibrium correction to the nuclear distribution we will restrict considerations to the nucleon part without additional virtual pions between them.

Positive Pressure in NM

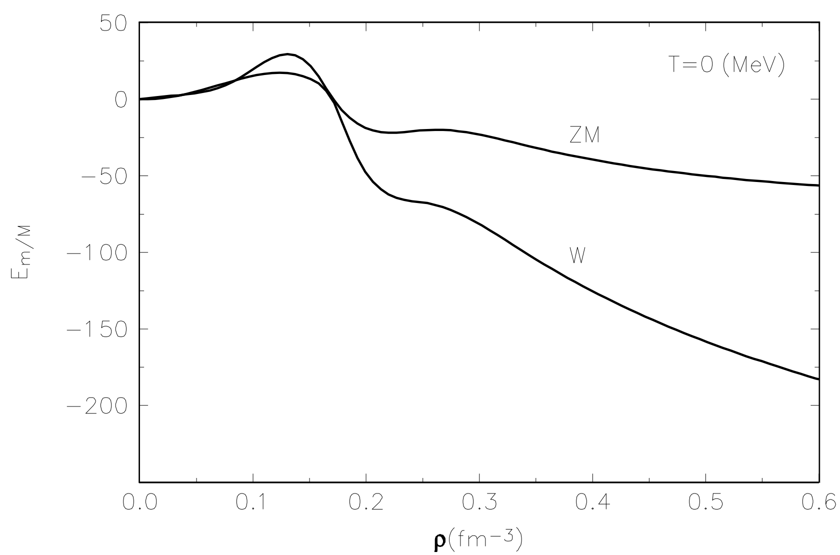
Calculation of the nuclear SF for positive pressure have to include the changes in Fermi energy along with the finite pressure. It increases E_F with density much stronger. The second sum rule is sensitive and we see that second the sum rule is broken by the factor E_F/ε_A To restore the MSR we have to change the quark SF. Scaling the Björken x by E_F/ε will give finally in MSR the nuclear mass M_A:

$$\int F_2^A(x)dx = \int dy y \rho^A(y) \int F_2^N\left(\frac{E_F}{\varepsilon_A}x\right)dx =$$
$$A\int F_2^N(x)dx = M_A$$

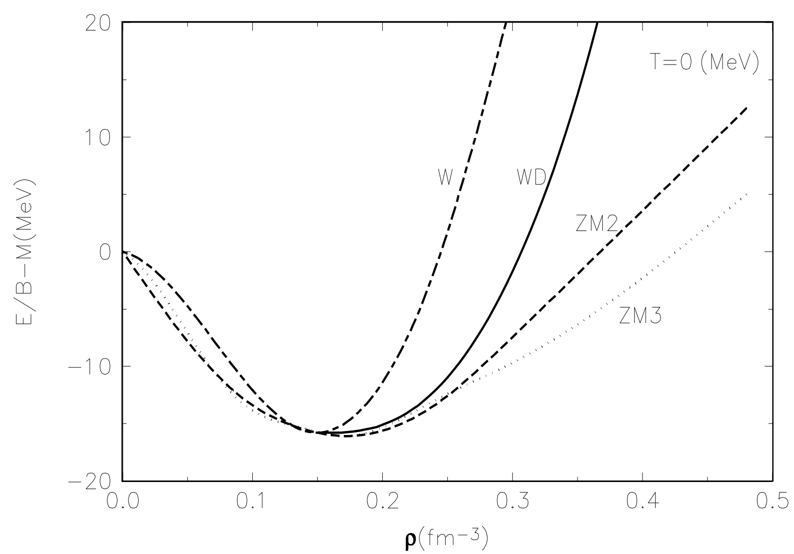
Now the new nucleon nass will dependent on the nucleon density and pressure but will remain constant below saturation point.

$$M_{med}=M/\left(1+(\delta\varepsilon/\delta\rho)(\rho/\varepsilon)\right)$$

and we have new equation for the relativistic (Walecka type) effective mass which now include the pressure correction.



The density dependent energy carried by meson field



Nuclear energy per nucleon for Walecka abd nonlinear models

Basic Equations

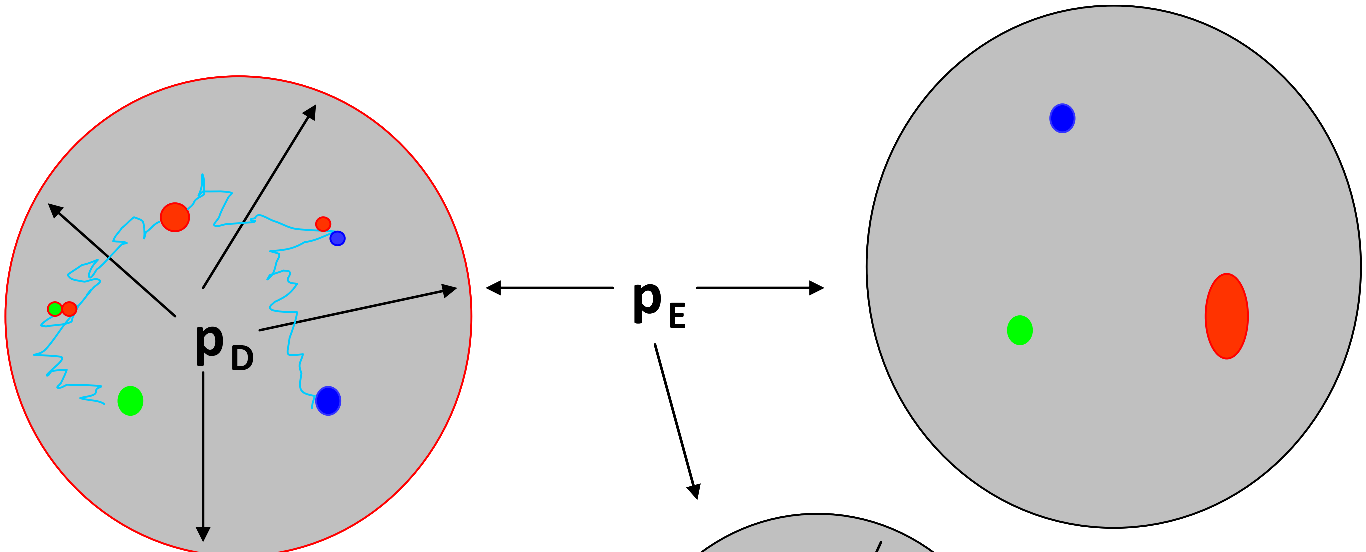
$$M_{med} = \frac{\varepsilon_A}{E_F^A} M_N = M_N / \left(1 + \frac{\varrho \frac{d}{d\varrho}(\varepsilon_A)}{\varepsilon_A}\right) \simeq M_N \left(1 - \frac{p_E}{\varrho \varepsilon_A}\right)$$

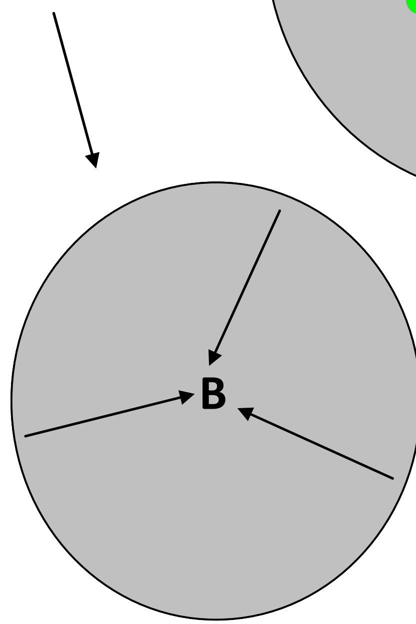
$$\varepsilon_N = C_1^2 \rho + \frac{C_2^2}{\rho}(M_{med} - M^*)^2 + \frac{\gamma}{(2\pi)^3 \rho} \int d^3p \sqrt{(p^2 + M^{*2})}$$

$$M^* = M_{med} - \frac{\gamma}{2C_2^2 (2\pi)^3} \int d^3p \frac{M^*}{\sqrt{(p^2 + M^{*2})}}$$

$$p = \frac{g_v^2}{2m_v^2} \rho_B^2 - \frac{m_s^2}{2g_s^2} (M - M^*)^2 + \frac{1}{3} \frac{\gamma}{(2\pi)^3} \int_0^{k_F} d^3k \frac{\tilde{k}^2}{(\tilde{k}^2 + M^{*2})^{1/2}}$$

TO SWELL OR NOT TO SWELL ? (Nucleon)





equilibrium $p_D = B + p_E$

$$E_{Bag} = \frac{n\omega_0 - Z_0}{R} + \frac{4\pi}{3} B R^3 \qquad \textbf{(1)}$$

This formula was obtained by studying the free quark flow described by the stress tensor $T_D^{\mu\nu}$ is given on bag surface along the vertical n^μ to this surface by:

$$n_\mu T_D^{\mu\nu} = \frac{1}{2} \frac{\partial}{\partial x_\nu} \sum \overline{q}_a(x) q_a(x) \qquad \textbf{(2)}$$

where $q_a(x)$ denotes the quark field with given flavor and color. Because the $\overline{q}_a(x) q_a(x) = 0$ on the surface its derivative must lie along the normal and define the intrinsic pressure p_D.

$$n_\mu T_D^{\mu\nu} = n_\nu p_D \qquad \textbf{(3)}$$

The pressure condition for p_D can be obtained from a usual criterium :

$$(\partial E_{Bag} / \partial \Omega_{Bag})_n = 0 \qquad \textbf{(4)}$$

However in a compressed medium the pressure p_D generated by free quarks inside the bag is balanced at the bag surface[27] not only by a intrinsic confining force represented by the bag constant B but additionaly by external pressure p_E generated by elastic collisions with other hadrons[18, 19, 28]. In this way the residual strong interaction with all other quarks localized outside the given bag is taken into account:

$$p_D = B + p_E \qquad \textbf{(5)}$$

Finally using the equation (4) and resulting spherical solution for the bag radius we obtain:

$$\frac{3\omega_0 - Z_0}{R^2} = 4\pi (B + p_E) R^2$$

$$R = \left[\frac{3\omega_0 - Z_0}{4\pi (B + p_E)} \right]^{1/4} \qquad \textbf{(6)}$$

At the saturation $p_E = 0$ and the bag constant B can be determined by the value of the nucleon radius $R \simeq 0.6 fm$. Above the saturation point the R increases[19, 20]. The main reason: the external pressure was not taken into account and the value of B wrongly determines the increasing bag radius R (nucleon swelling).

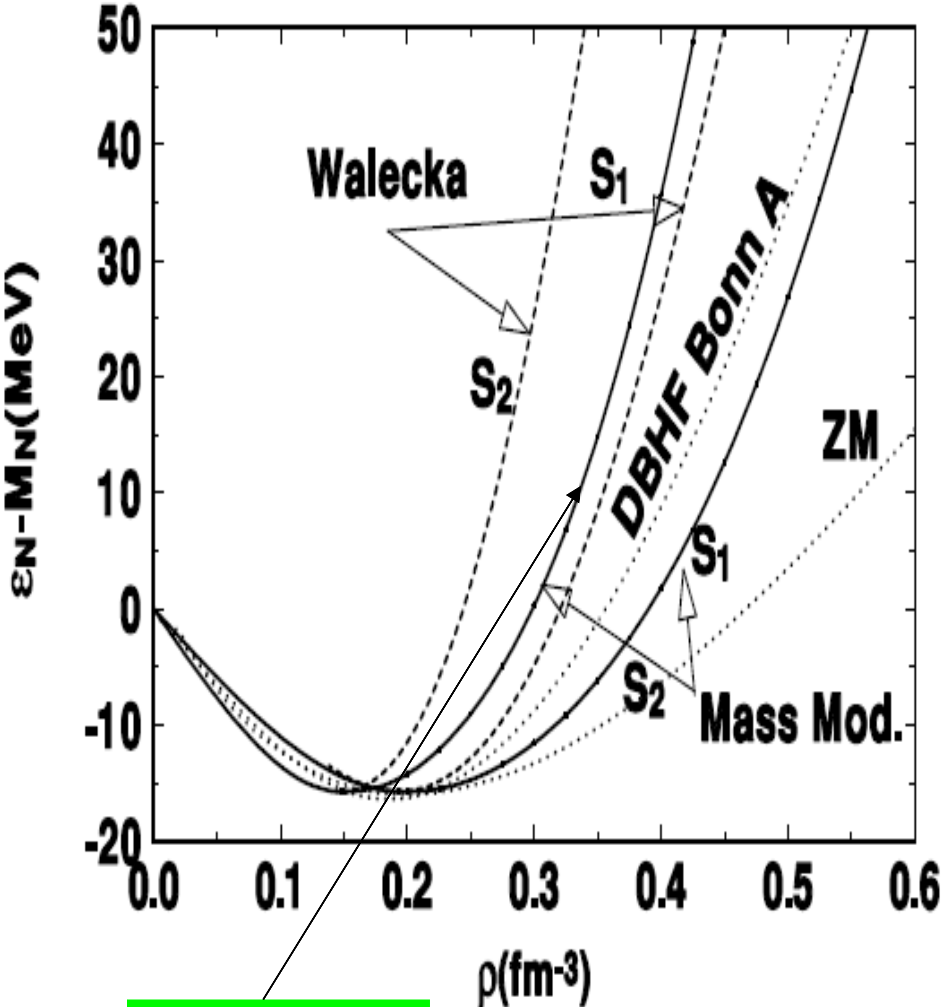
EOS

Now the nucleon energy in the rest frame is given by:

$$E_{Bag} = 4\pi R^3 \left[\frac{4}{3}(B + p_E) - \frac{p_E}{3}\right] \simeq E_{Bag}^{p_E=0}[1 - \frac{p_E}{\varrho E_{Bag}^{p_E=0}}]$$

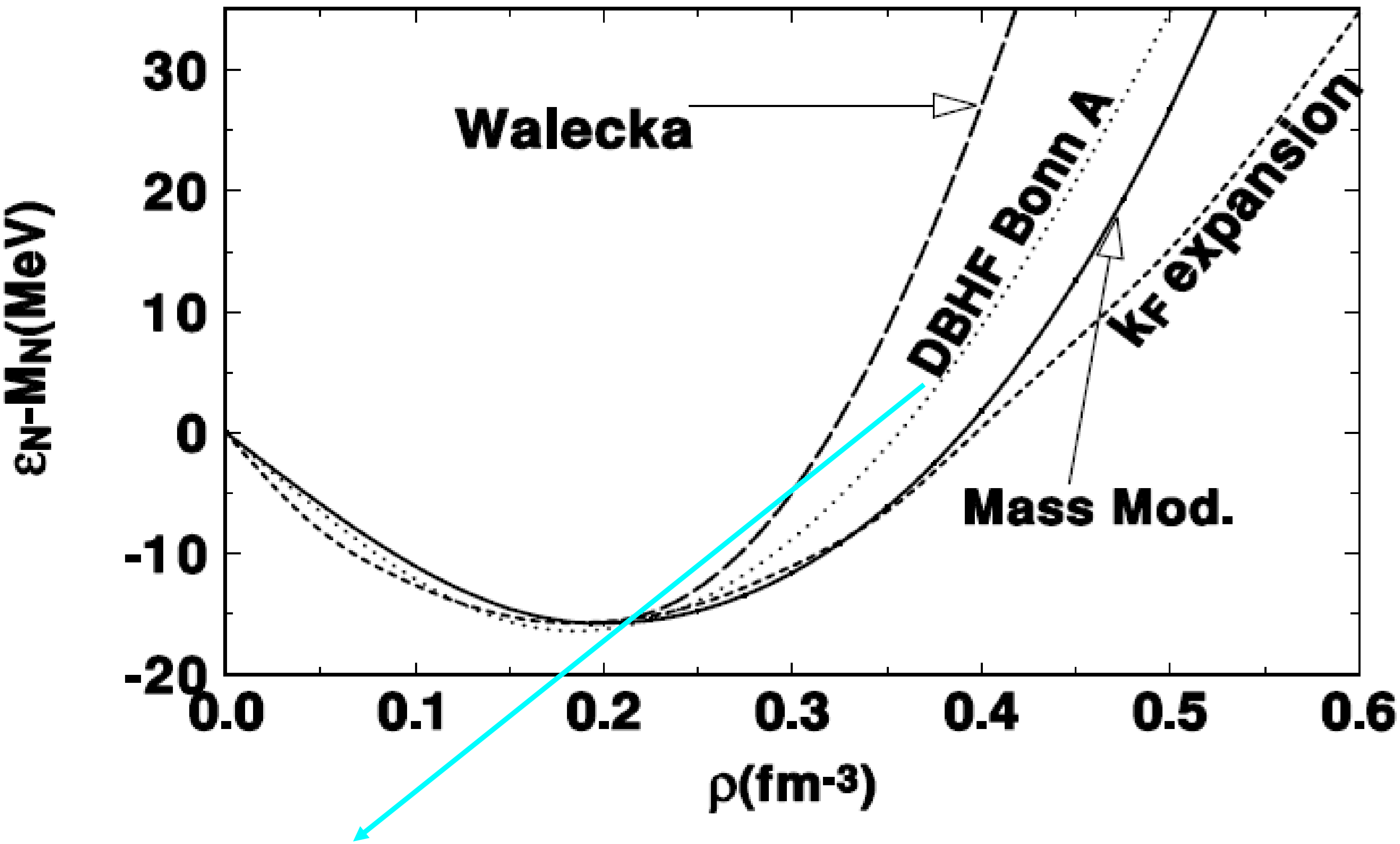
with $\varrho \simeq 1/\Omega_{Bag} > \varrho_0$ where the volume of a space between nucleons above the saturation density ϱ_0 was neglected. E_{Bag} differs from the the nucleon mass by the c.m. correction [20] to the nucleon parton model.

The pressure corrections essentially given by $E_{press} = (p/\varrho)$ are similar especially when the nucleon radius and $(B + p_E)$ remains constant[29] in the bag (23) model. Such a solution of $R(\varrho)$ which is constant with ϱ testifies the properly soft EoS in ZM model showed in Fig ------------------------------------→



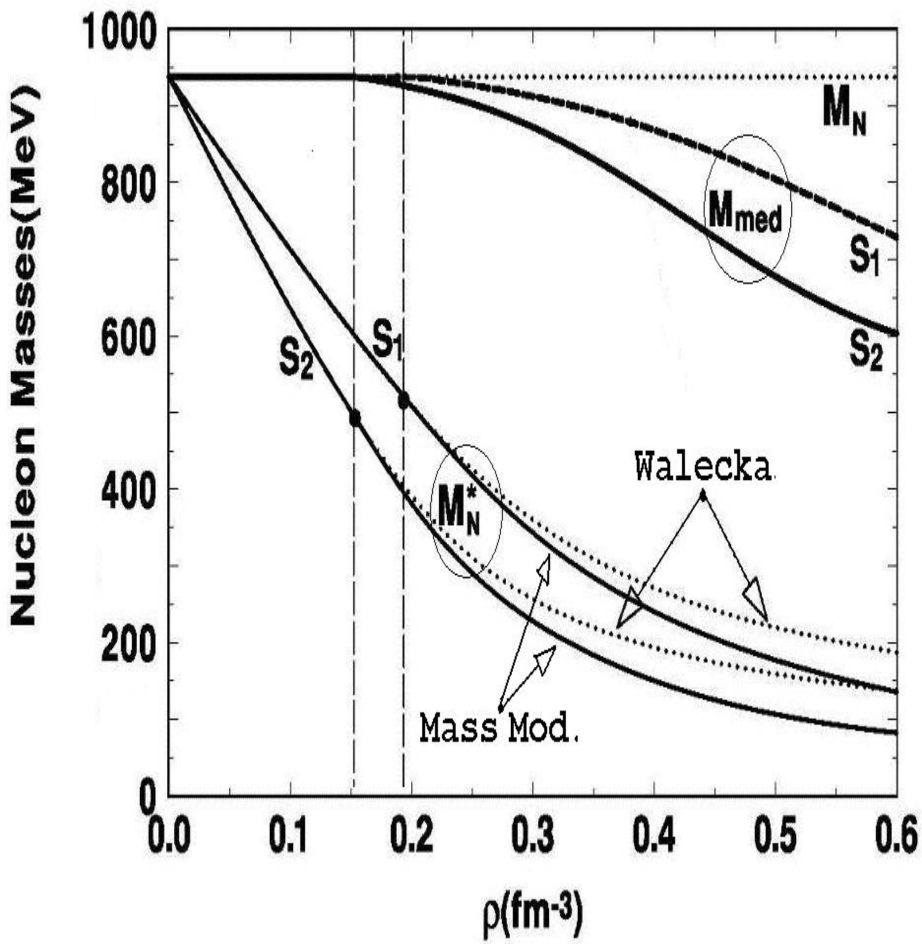
(GB,F,F)

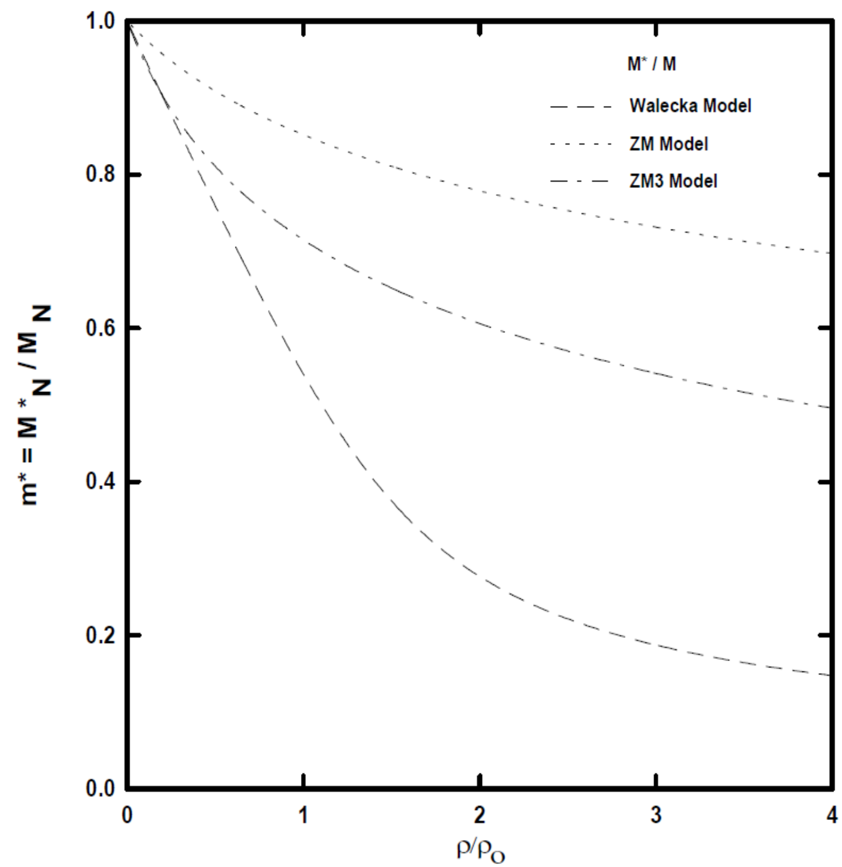
EOS



T. Gross-Boelting, C. Fuchs, A. Faessler, Nuclear Physics A **648**, 105 (1999); E. N. E. van Dalen, C.Fuchs, A. Faessler, Phys. Rev. Lett. **95**, 022302 (2005); Fuchs J. Phys. G **35**, 014049 (2008).

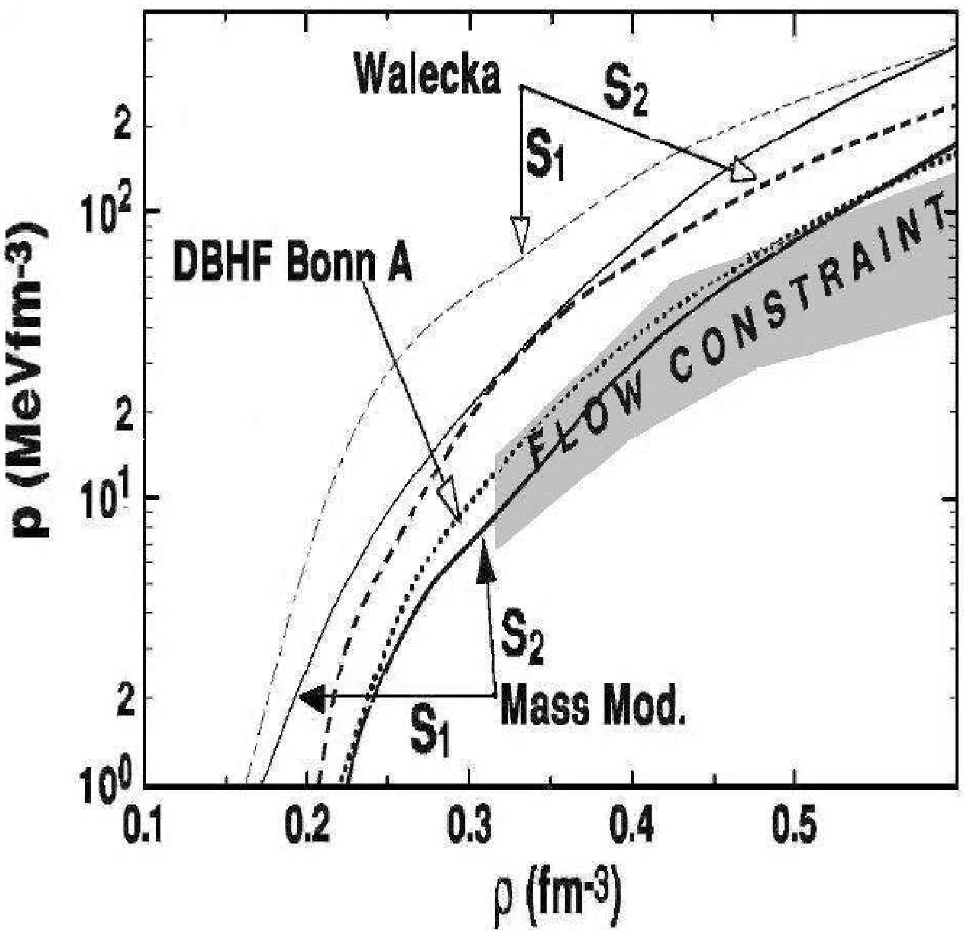
Masses - solution of modified RMF equation with finite pressure corrections

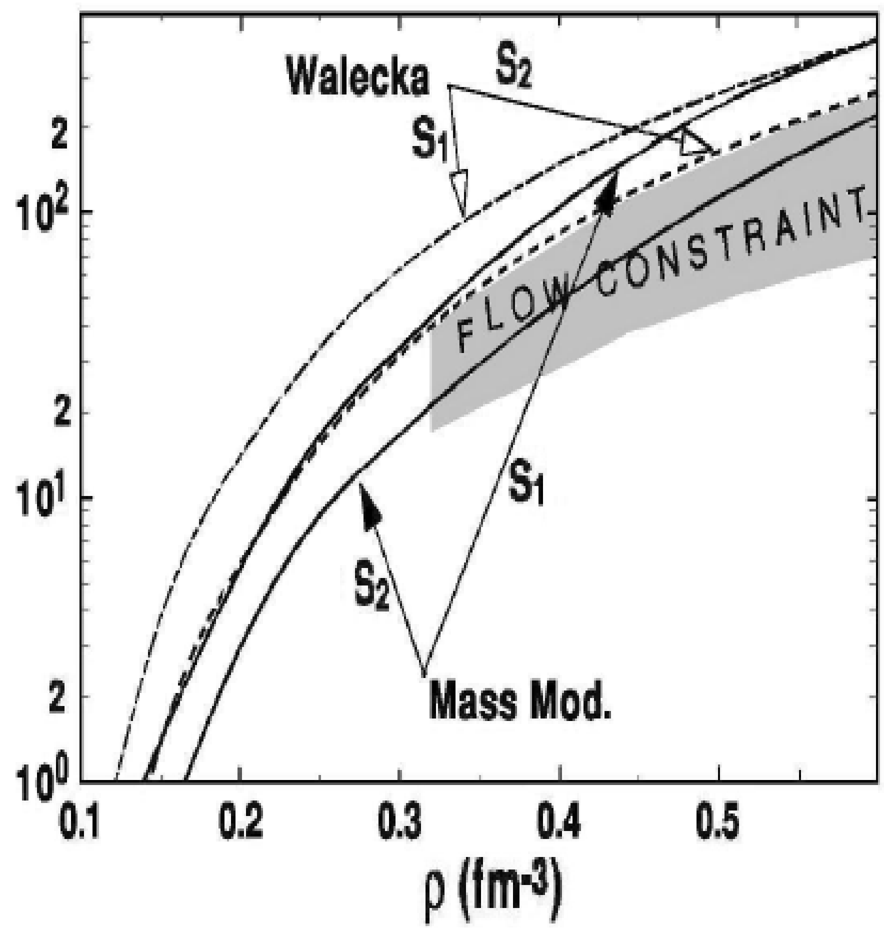




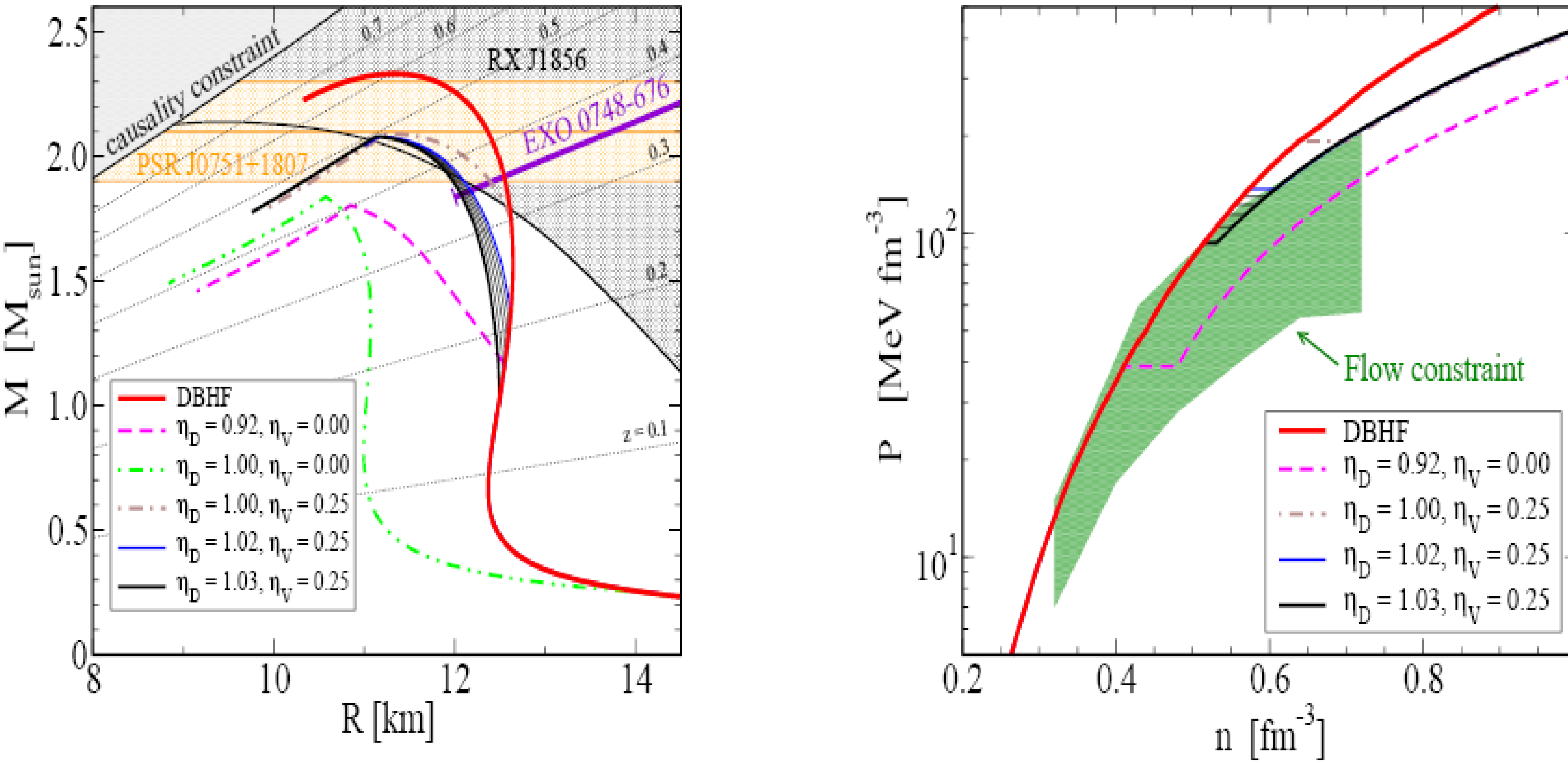
EOS results

P.Danielewicz et al
Science(2002)





Mass-Radius constraint and Flow constraint (II)



- Large Mass ($\sim 2\, M_\odot$) and radius ($R \geq 12$ km) $\Rightarrow$ stiff quark matter EoS;
  Note: DU problem of DBHF removed by deconfinement! and: CFL core Hybrids unstable!
- Flow in Heavy-Ion Collisions $\Rightarrow$ not too stiff EoS !
  Note: Quark matter removes violation by DBHF at high densities

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Klähn, D.B., Sandin, Fuchs, Faessler, Grigorian, Roepke, Truemper, [arxiv:nucl-th/0609067] (2006)

transparency taken from David Blaschke

1.Presented model correspond to the scenario where the part of nuclear momentum carried by meson field and coming from the strongly correlation region, reduce the nucleon mass by corrections proportional to the pressure.

2.The low density limit the spin-orbit splitting of single particle levels remains in agreement with experiments, like in the classical Walecka Relativistic Mean Field Approach, but the equation of state for nuclear matter is softer from the classical scalar-vector Walecka model and now the compressibility $K^{-1}=9(r^2 d^2/dr^2)E/A$ = 230MeV, closed to experimental estimate.

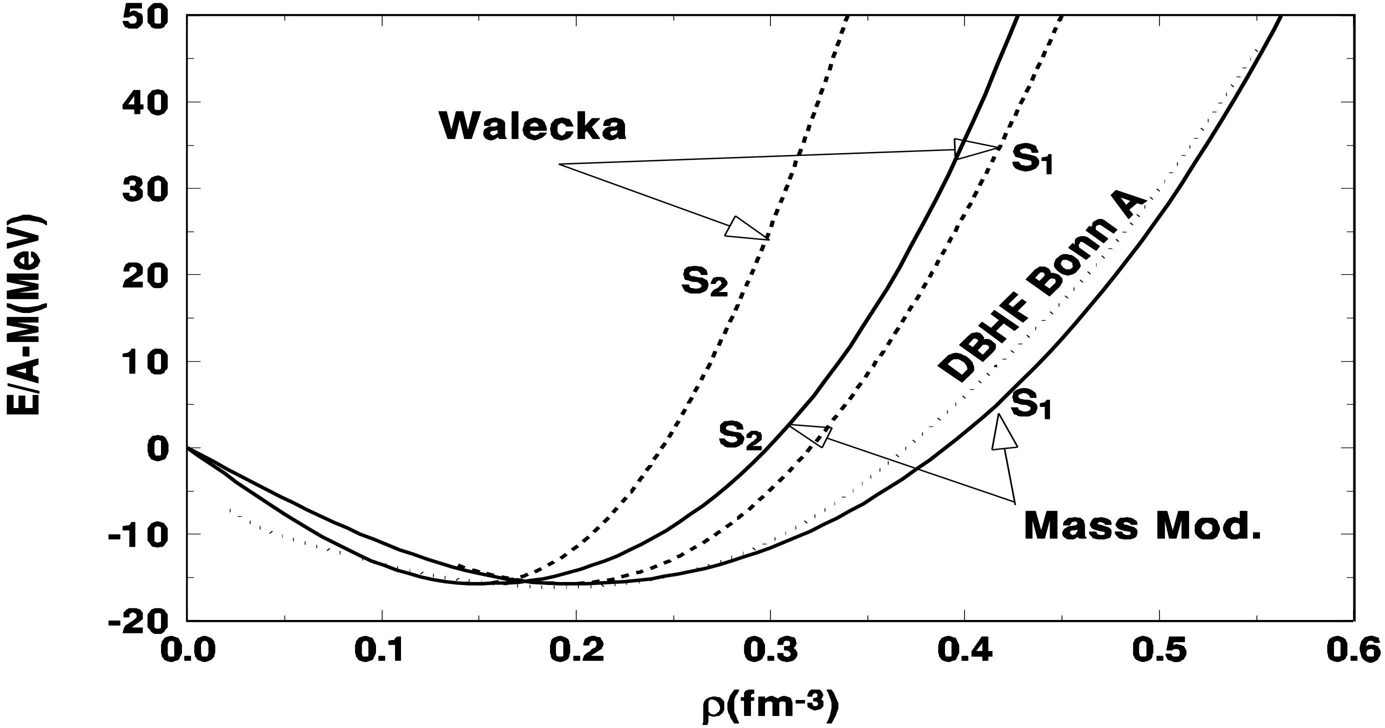
CONCLUSIONS

1.Pressure dependent meson contributions when added to EOS and to the nuclear structure function improve the EOS estimated from HI collisions giving well satisfied Momentum Sum Rule for the parton constituents.

2.New EOS is enough soft(or stiff) to be in agreement with estimates from compact stars for higher densities the strange hiperons will change it slighty.

Finally we conclude that we found the corrections to Relativistic Mean Field Approach from parton structure measured in DIS, which may improve the stiffness properties of the mean field description of nuclear matter from saturation density ρ_0 to $3\rho_0$.

The nucleon energy in function of nuclear matter density for DBHF (GB,F,F) Bonn A and two RMF models - Walecka (dot lines) and Modified Mass Approach (solid). Both RMF models are calculated for S_1 and S_2 parametrization: version[8] S_1 $({C_v}^2 = 195.9, {C_s}^2 = 267.1, \rho_0 = .19\, fm^{-3})$ and version[9] S_2 $({C_v}^2 = 273.8, {C_s}^2 = 357.4, \rho_0 = .16\, fm^{-3})$



Deep inelastic scattering

$$d\sigma = l_{\mu\nu} W^{\mu\nu}$$

$$W_{\mu\nu} = \sum_x \delta(p+q-r) \langle p | J_\mu(0) | X \rangle \langle X | J_\nu(0) | p \rangle$$

$$W_{\mu\nu} \approx \int d^4\xi e^{iq\xi} \langle p | J_\mu(\xi) J_\nu(0) | p \rangle$$

$$W_{\mu\nu} = -(g_{\mu\nu} - q^\mu q^\nu / q^2) W_1(q^2,\nu) + 1/M^2$$

$$(p_\mu - (M\nu/q^2) q_\mu)(p_\nu - (M\nu/q^2) q_\nu) W_2(q^2,\nu)$$

$$(\nu/M) \lim_{\nu\to\infty} W_2(q^2,\nu) = F_2(x_T) \leftarrow Bjorken \quad Scaling$$

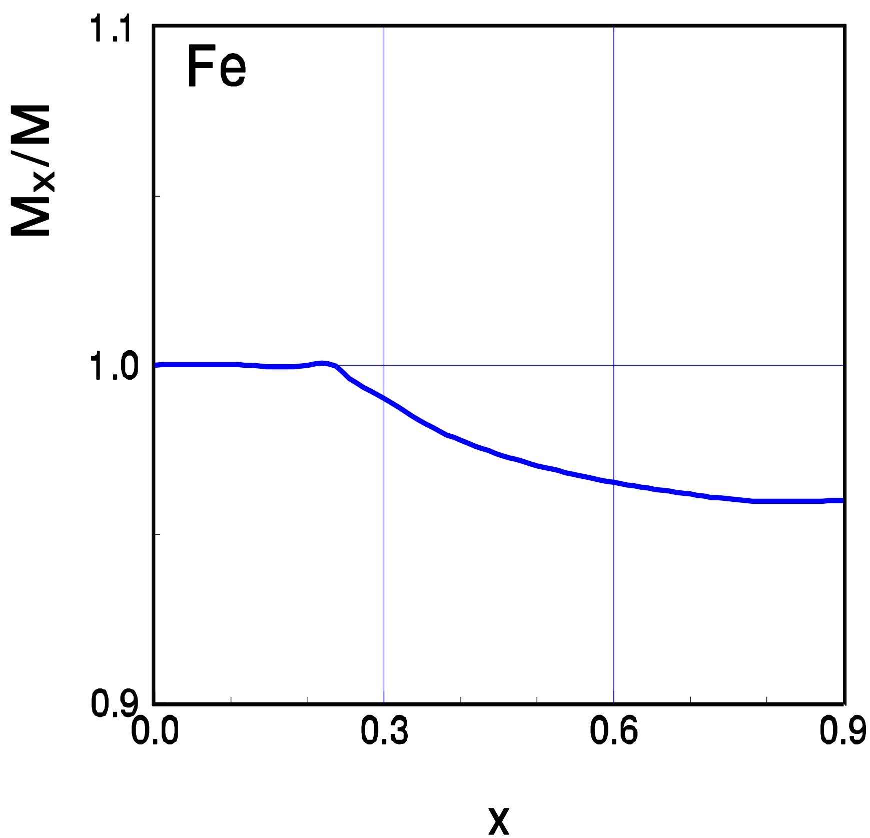
$$q = (\nu,0,0,-\sqrt{\nu^2+Q^2}), \qquad Q^2 = -q^2 \to \infty$$

$$q = (\nu,0,0,-\nu - Mx) \qquad x_T = Q^2/2M\nu \to fixed$$

x dependent nucleon effective mass

- it is possible to show that in DIS $\langle k_T^2 \rangle$ M^2

Bartelski Acta Phys.Pol.B9 (1978)



$$\sqrt{\langle k_T^2 \rangle_{Medium} / \langle k_T^2 \rangle}$$

In the x>0.6 limit

(no NN interaction)

$\langle k_T^2 \rangle_{\text{Nuclear}} = \langle k_T^2 \rangle_{\text{Nukleon}}$

Nuclear deep inelastic limit revisited

x dependent nucleon „rest” mass in NM

$$F_2^N(x) = f(x)F_2^{2N}(x) + (1 - f(x))F_2^N$$

f(x) - probability that struck quark originated from correlated nucleon

$$M_x = M_N + \frac{1 - f(x)}{2} < V_N >$$

- **Momentum Sum Rule violation**

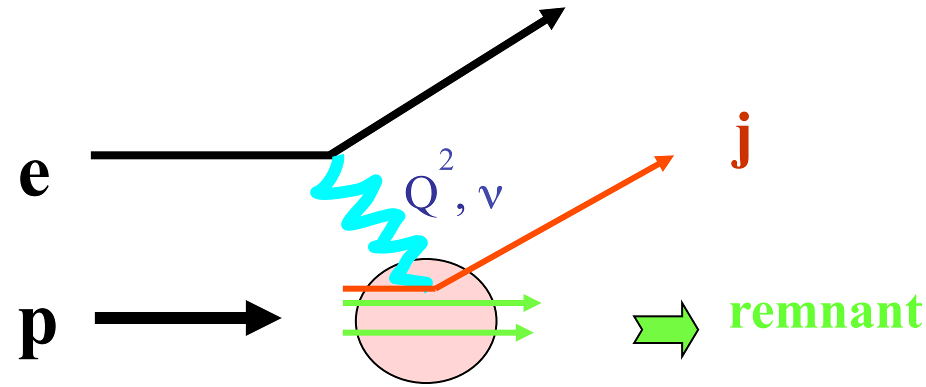
$$\frac{\frac{1}{A}\int F_2^A(x_A)dx_A}{\int F_2^N(x)dx} = \left(1 + C[f]\frac{< V_N >}{M}\right)(1 + \varepsilon)$$

D I S

Hit quark has momentum

$j_+ = x\, p_+$

Experimentaly $x = Q^2/2M\nu$ and is iterpreted as fraction of longitudinal nucleon momentum carried by parton(quark) for $\nu^2 > Q^2 \rightarrow \infty$ (Bjorken lim)



On light cone Bjorken x is defined as $x = j^+/p^+$ where $p^+ = p^0 + p^z$

$d\sigma \sim 1^{\mu\nu} W_{\mu\nu} \quad W_{\mu\nu}(W_1, W_2)$

Bjorken Scaling

$F_2(x) = \lim[(\nu/M)\, W_2(q^2,\nu)]_{Bjo}$

In Nuclear Matter due to final state NN interaction, nucleon mass M(x) depends on x , and consequently from energy ε and density ρ.

for large x (no NN int.) the nucleon mass has limit $M_B \cong M_N + \varepsilon - e_{Fermi}$

Due to renomalization of the nucleon mass in medium we have enhancement of the pion cloud from **momentum sum rule**

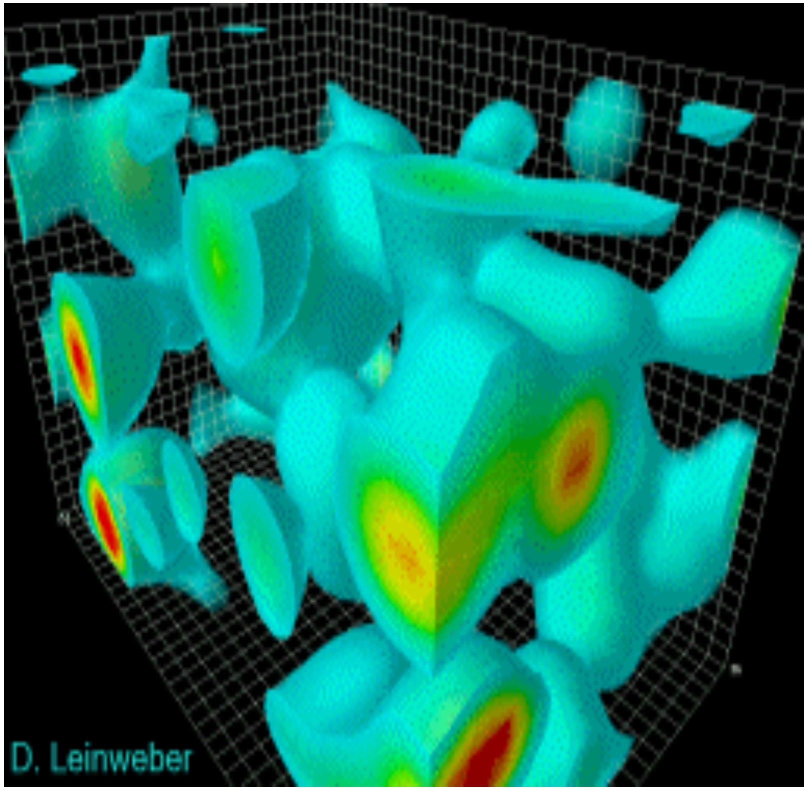
Rescaling inside nucleus

$F_2^A(x) = F_2[xM/M(x)] + F_2^\pi(x)$

In vacuum

$$\langle \bar{q}q \rangle_{\rm vac} \simeq -(225 \pm 25\ {\rm MeV})^3 ,$$

$$\left\langle \frac{\alpha_s}{\pi} G^a_{\mu\nu} G^{a\mu\nu} \right\rangle_{\rm vac} \simeq (360 \pm 20\ {\rm MeV})^4 .$$



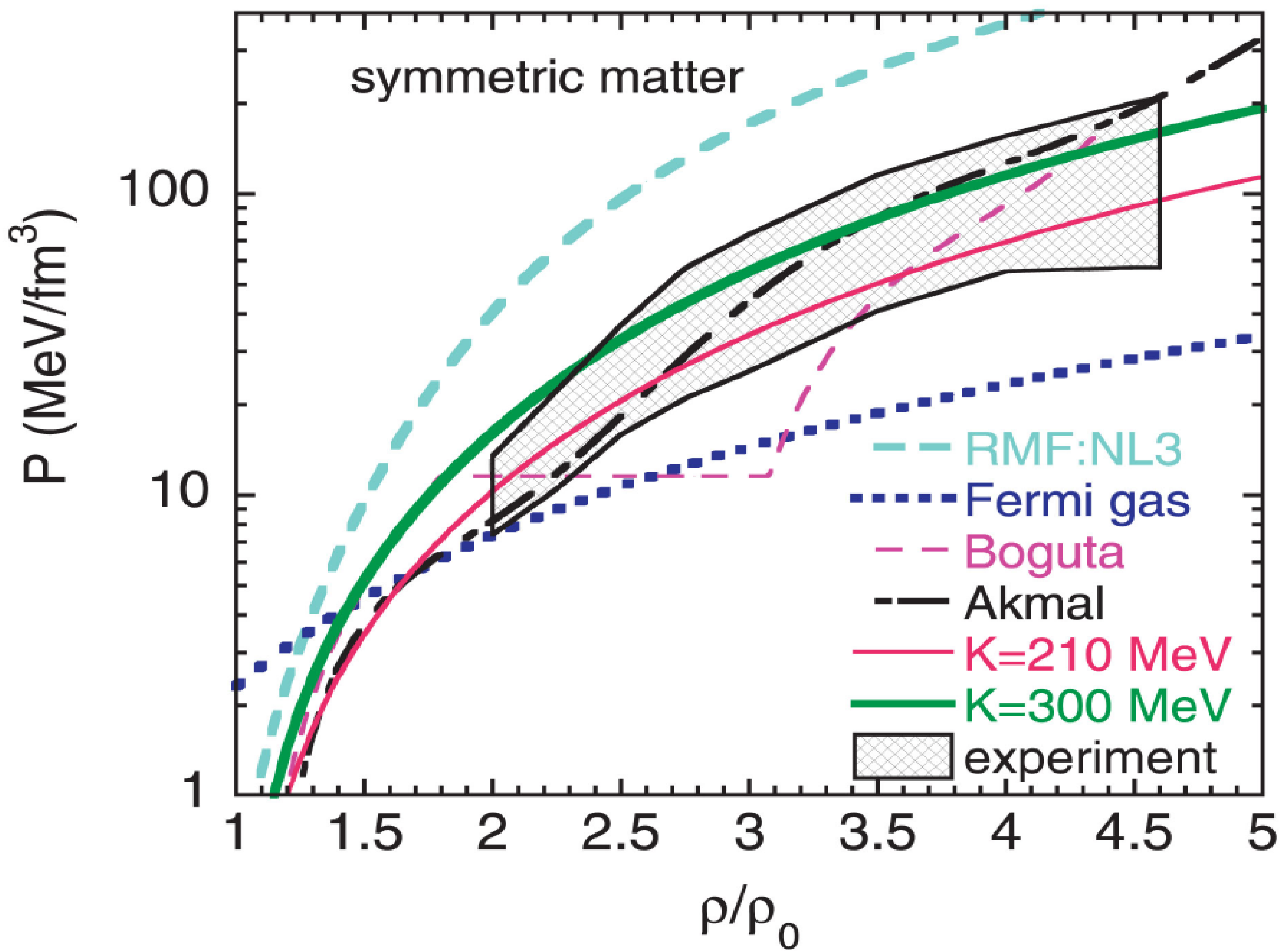
In nuclear medium

$$\frac{\langle \bar{q}q \rangle_{\rho_N}}{\langle \bar{q}q \rangle_{\rm vac}} = 1 - \frac{\rho_N}{\rho_N^\chi} + \cdots ,$$

$$\rho_N^\chi \equiv \frac{m_\pi^2 f_\pi^2}{\sigma_N}$$

$$\sigma_N = 2m_q \int d^3x (\langle N|\bar{q}q|N\rangle - \langle {\rm vac}|\bar{q}q|{\rm vac}\rangle) ,$$

Quark and gluon condensates in nuclear matter
Phys.Rev.C45 1881
Thomas D. Cohen, R. J. Furnstahl,* and David K. Griegel†


symmetric matter
P (MeV/fm^3)
100
10
1
1
1.5
2
2.5
3
3.5
4
4.5
5
ρ/ρ$_0$
RMF:NL3
Fermi gas
Boguta
Akmal
K=210 MeV
K=300 MeV
experiment

NN interaction has the contribution to the nucleon mass in the medium and determine time scale of mass corrections. Therefore for fast measurements (smaller then meson exchange) or equivalently large $x > 0.6$, the potential part of single particle energy is absent which in that moment shift the nucleon mass. The additional contribution from nuclear pions emerges from the MSR satisfied by nuclear SF. Consequently for very fast (in comparison to meson exchange) measurements, the nucleon mass is smaller for x and additional contribution from the pion field is present in the structure function (SF) of nucleus in order to satisfy MSR. In order to satisfy exactly the momentum sum rule (MSR), one has to allow for a nuclear pion excess, which should be small (of the order of $\sim 1\%$) in order to simultaneously