THE EVOLUTION OF THE F-MODE INSTABILITY

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Motivation

- GW driven f-mode instability of relativistic stars
- Time evolution of the instability
- GW signal of the f-mode and its detection prospectives

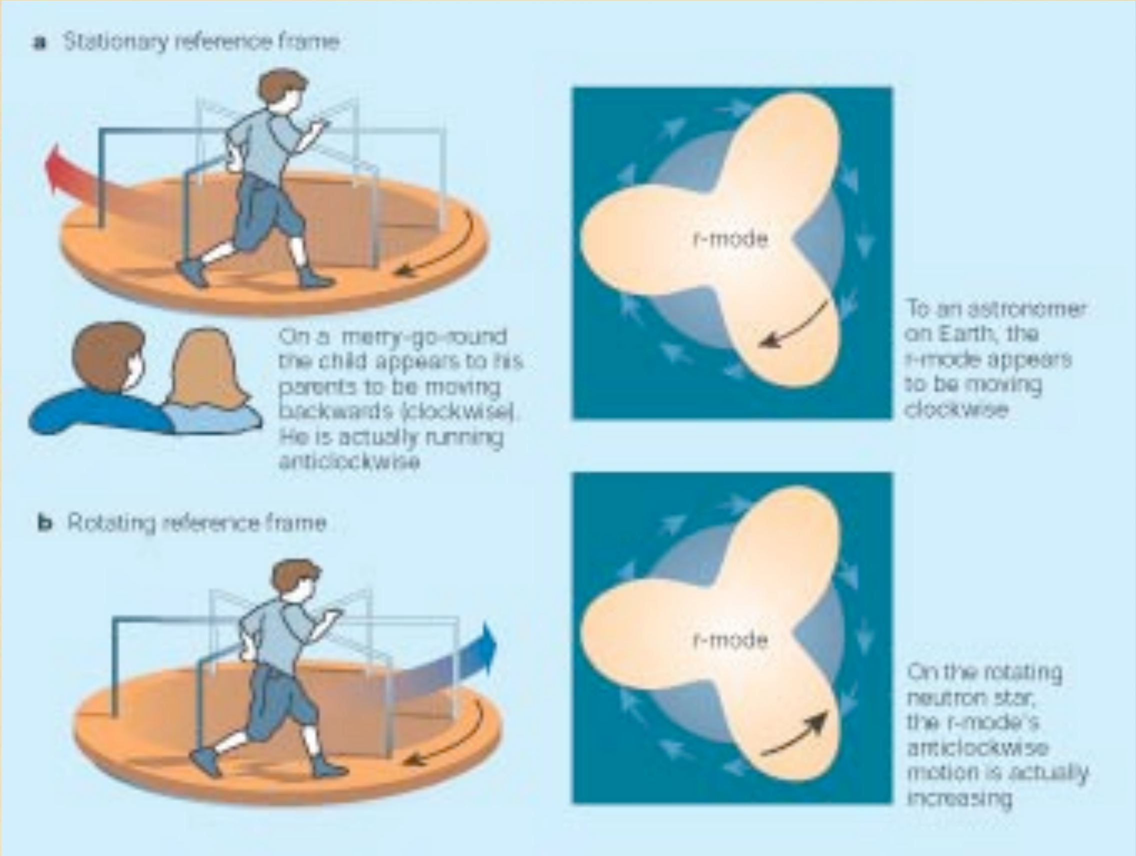
CFS instability

- Rotating NS are prone to CFS gravitational-wave instability

- Radiation drives a mode unstable if

$$\omega_r\left(\omega_r - m\Omega\right) \leq 0 \quad \Longrightarrow \tau_{\rm gw} \leq 0$$

and $$\delta\rho \sim e^{-t/\tau_{\rm gw}}$$



- Viscous mechanisms limit the gravitational-wave instability

$$\delta\rho \sim e^{i\omega t - t/\tau}$$ where $$\frac{1}{\tau} = \frac{1}{\tau_{\rm gw}} + \frac{1}{\tau_{\rm b}} + \frac{1}{\tau_{\rm s}} + \dots$$ and $$\frac{1}{\tau} = \frac{\dot{E}}{2E}$$

<u>*Instability condition:*</u> $$\frac{1}{\tau} \leq 0$$ where $$\tau = \tau\left(\Omega, T\right)$$

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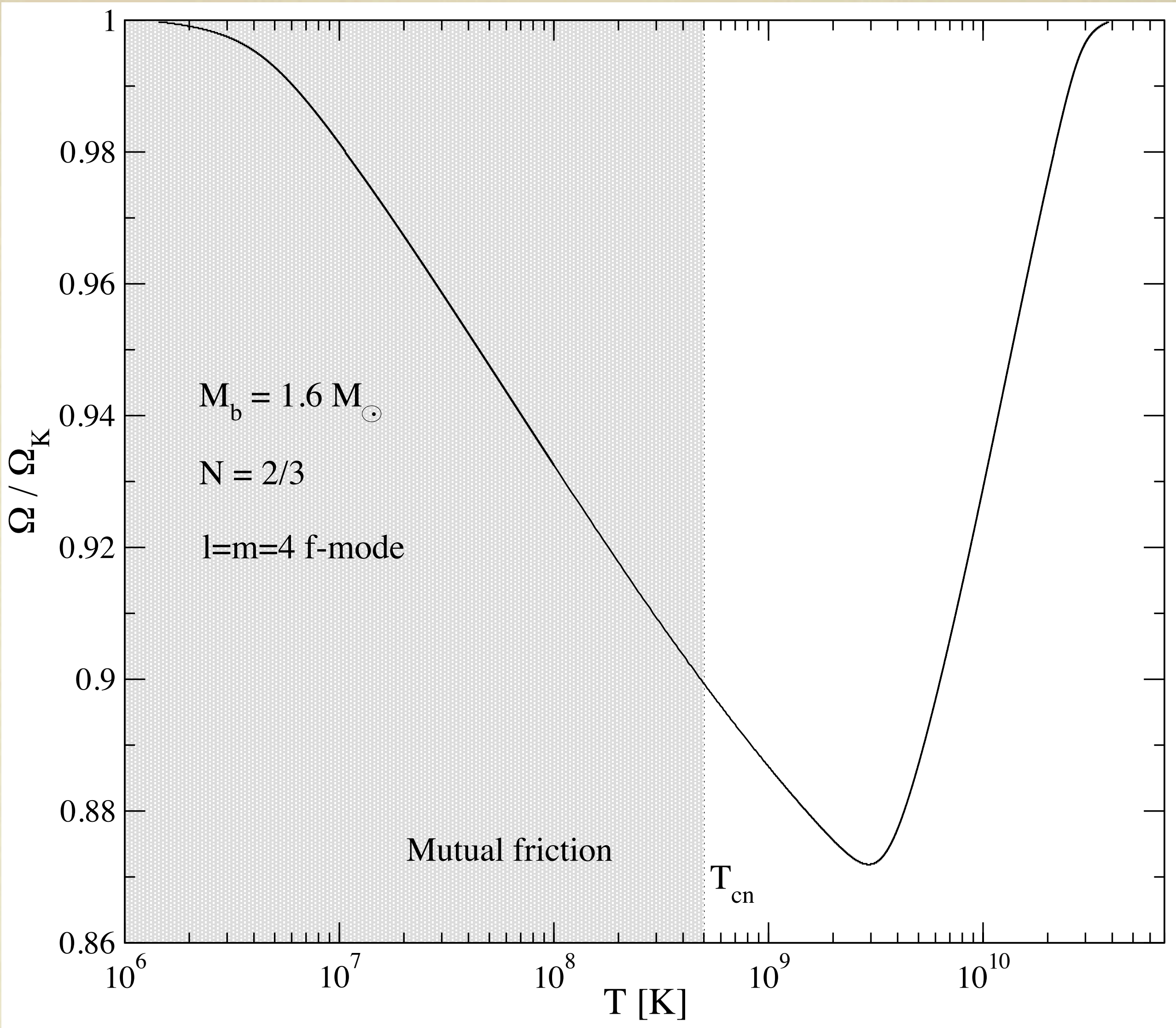


$$\delta\rho \sim e^{i\omega t - t/\tau} \quad \text{where} \quad \frac{1}{\tau} = \frac{1}{\tau_{\rm gw}} + \frac{1}{\tau_{\rm b}} + \frac{1}{\tau_{\rm s}} + \dots \quad \text{and} \quad \frac{1}{\tau} = \frac{\dot{E}}{2E}$$

Instability condition: $$\frac{1}{\tau} \le 0 \quad \text{where} \quad \tau = \tau\left(\Omega, T\right)$$

Equations

Method

- We calculate the mode-frequency and eigenfunctions from time simulations.
- With the energy volume integrals we determine the damping/growth times.
- We study the instability evolution with a set of evolution equations which evolves the mode amplitude, stellar rotation and temperature.

Mode Frequency

- Evolution of the relativistic perturbation equations in Cowling approximation

$$\delta\left(\nabla_\nu T^{\mu\nu}\right) = 0 \qquad \text{where} \qquad \delta g_{\mu\nu} = 0$$

- Standard model

$$N = 1$$

$$M = 1.4 M_\odot$$

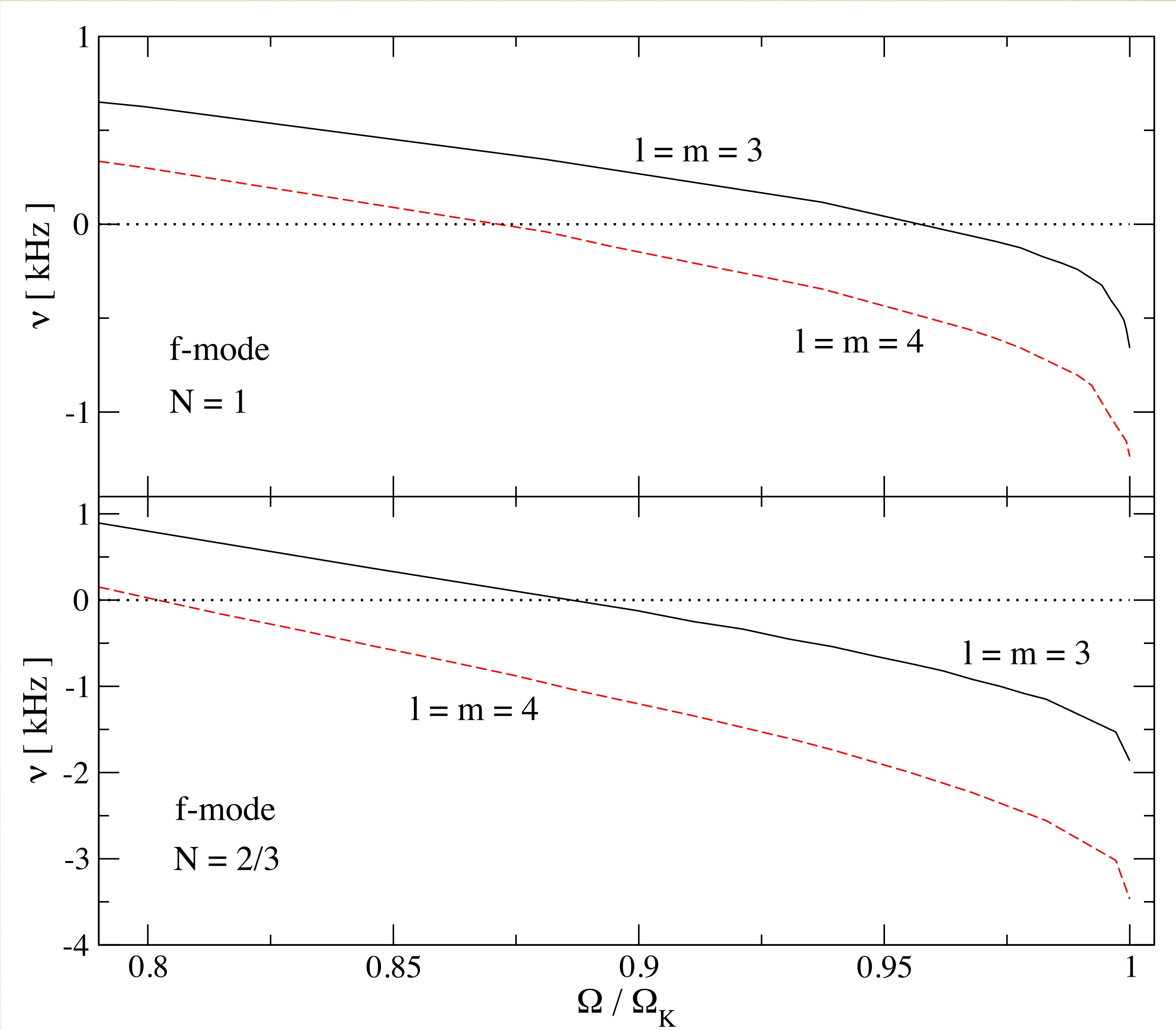
$$\nu_K = 673\,\mathrm{Hz}$$

- Supramassive model

$$N = 2/3$$

$$M = 1.6 M_\odot$$

$$\nu_K = 1783\,\mathrm{Hz}$$



Viscous and GW timescales

- Assumption: dissipative timescales are much longer than the oscillation period

- Bulk and shear viscosity

$$\frac{1}{\tau_{\rm b}} = \frac{1}{2E}\int dV \zeta\, \delta\sigma\delta\sigma^* \qquad \zeta \sim T^6$$

$$\frac{1}{\tau_{\rm s}} = \frac{1}{E}\int dV \eta\, \delta\sigma^{ij}\delta\sigma^*_{ij} \qquad \eta \sim T^{-2}$$

$$\delta\sigma^{ij} = \frac{1}{2}\left(\nabla^i\delta u^j + \nabla^j\delta u^i - \frac{2}{3}g^{ij}\nabla\delta\sigma\right)$$

$$\delta\sigma = \nabla_j\delta u^j$$

- GW radiation reaction

$$\frac{1}{\tau_{\rm gw}} = \frac{\omega}{2E}\sum_{l\geq 2} N_l\left(\omega - m\Omega\right)^{2l+1}\left(\left|\delta D_{lm}\right|^2 + \left|\delta J_{lm}\right|^2\right)$$

$$\omega\left(\omega - m\Omega\right) \leq 0 \quad \Longrightarrow \tau_{\rm gw} \leq 0$$

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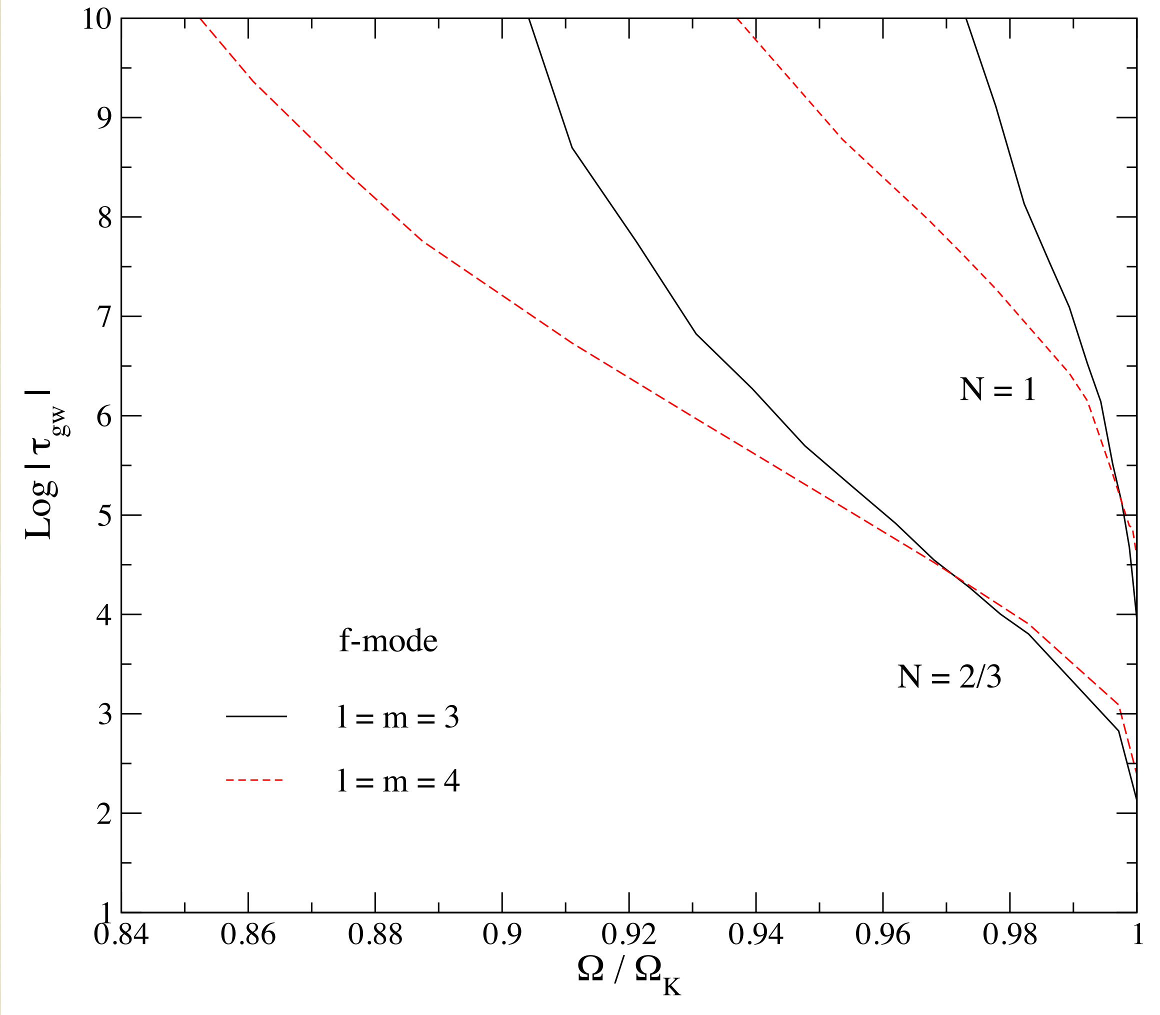

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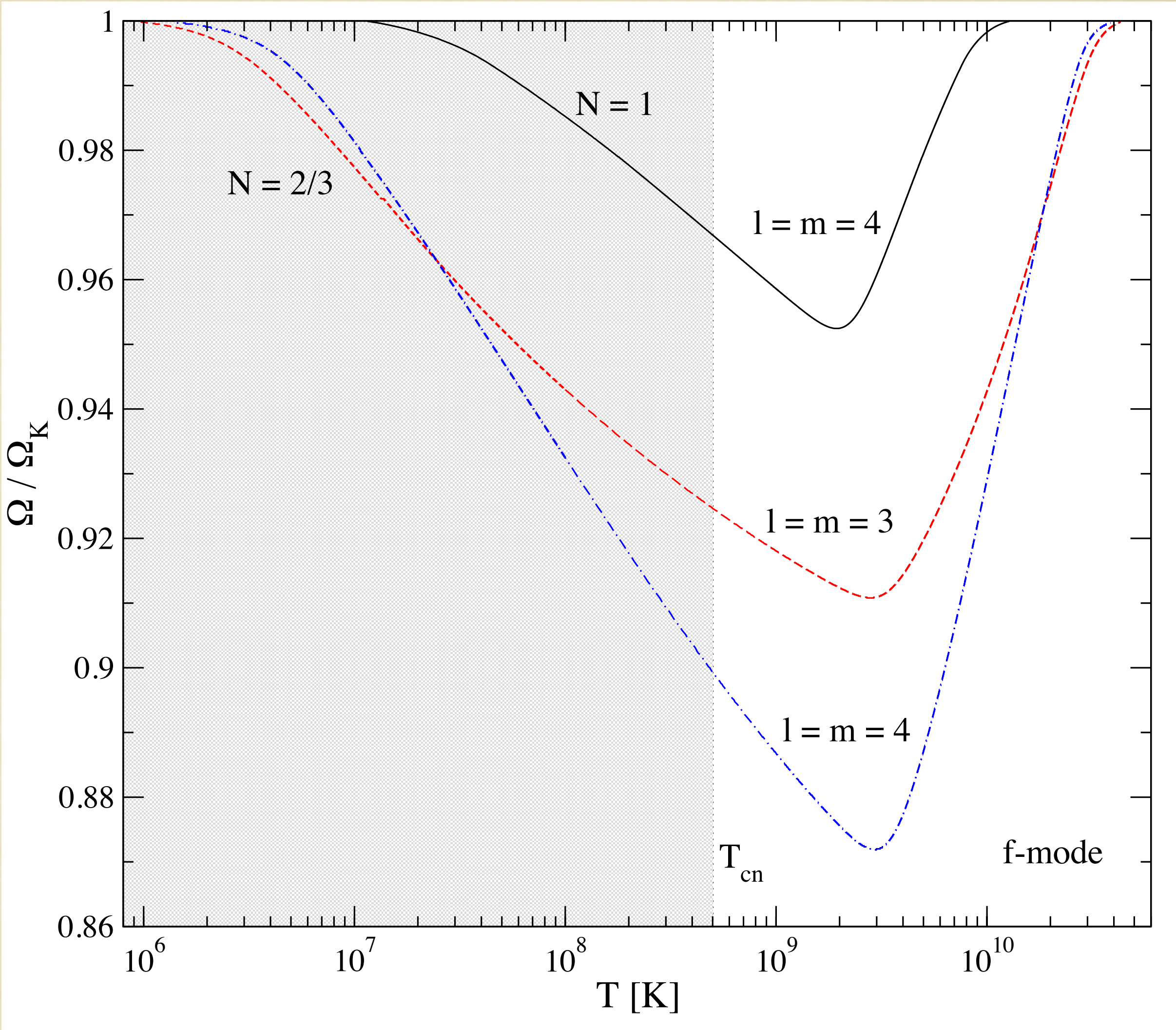

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$$\omega\,(\omega - m\Omega) \leq 0 \implies \tau_{\rm gw} \leq 0$$

Instability Evolution

- Basic equations

$$\frac{dE}{dt} = -\frac{2E}{\tau} \qquad E = \alpha \tilde{E}(\Omega)$$

$$\frac{dJ}{dt} = \frac{dJ_{gw}}{dt} + \frac{dJ_{mag}}{dt} \qquad J = J_s + \alpha \tilde{J}_c(\Omega)$$

$$C_v \frac{dT}{dt} = -L_\nu + H_s \qquad H_s = \frac{2E}{\tau_s}$$

- Amplitude Normalization

$$E = \alpha E_{\rm rot} \qquad \alpha = 1 \Longrightarrow E \simeq 10^{-2} M_\odot c^2$$

Note: $\delta\rho \sim \alpha^{1/2}$

- Mode growth

$$\frac{d\alpha}{dt} = -\frac{2\alpha}{\tau_{gw}} - \frac{2\alpha}{\tau_v}\frac{1+\alpha Q}{D} + \frac{2P}{D}\frac{\alpha}{\tau_{mag}},$$

$$\frac{d\Omega}{dt} = \frac{2F}{D}\left(\frac{\alpha}{\tau_v} - \frac{1}{\tau_{mag}}\right),$$

- Non-linear saturation

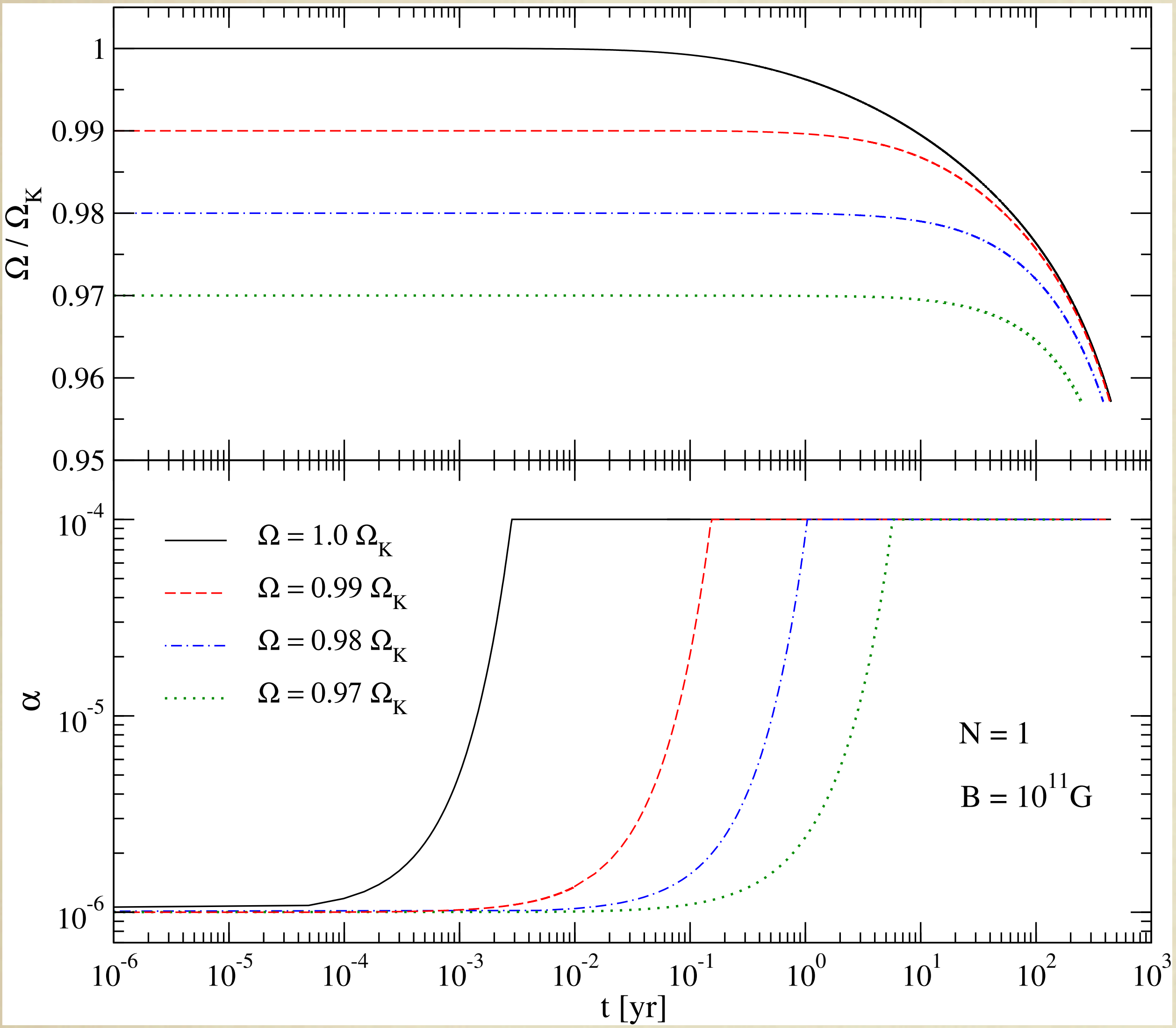
$$\frac{d\alpha}{dt} = 0$$

$$\frac{d\Omega}{dt} = -\frac{2F}{1+\alpha Q}\left(\frac{\alpha}{\tau_{gw}} + \frac{1}{\tau_{mag}}\right)$$

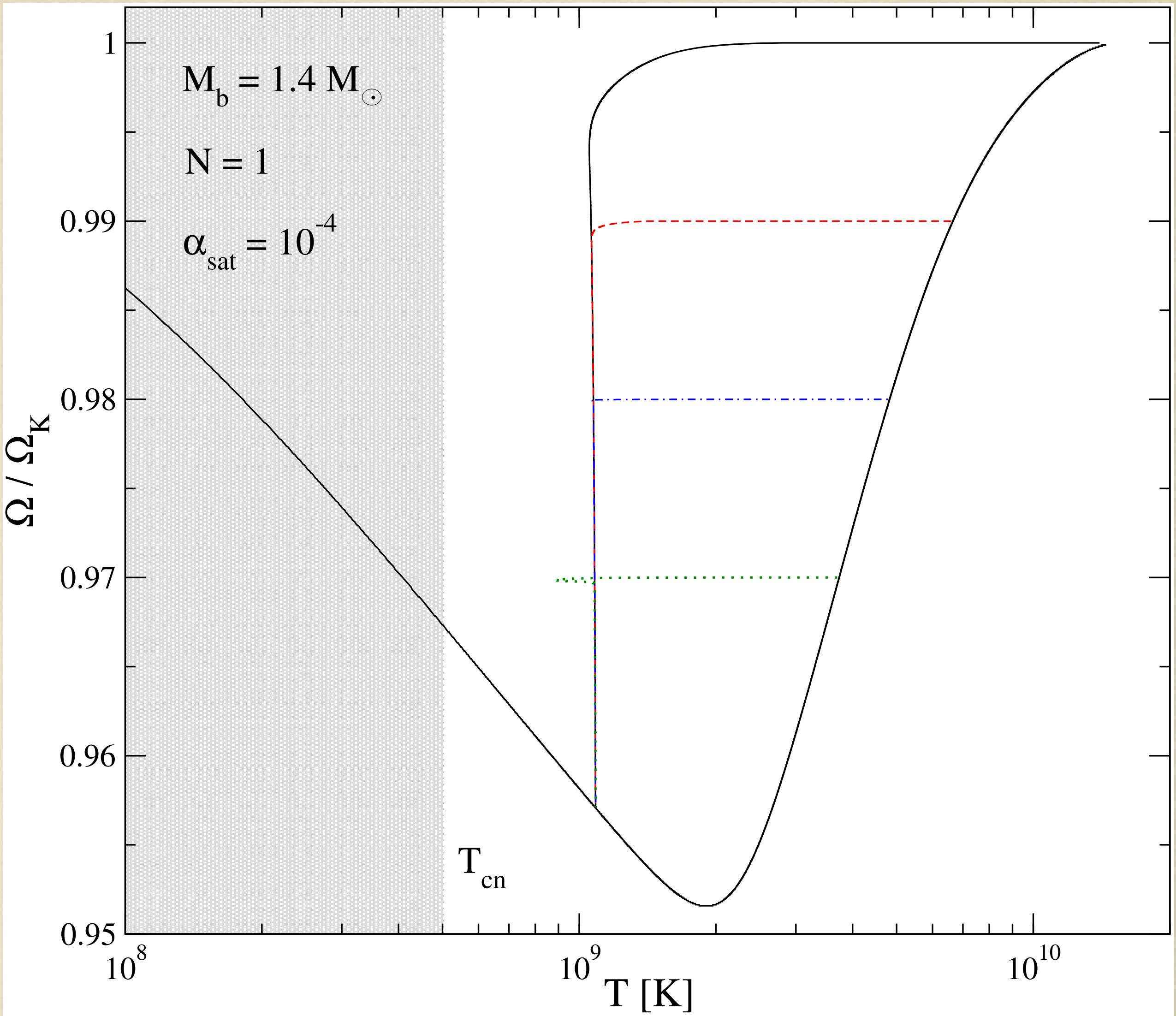
Results

l=m=4 f-mode Evolution

N = 1 polytrope



Instability trajectory



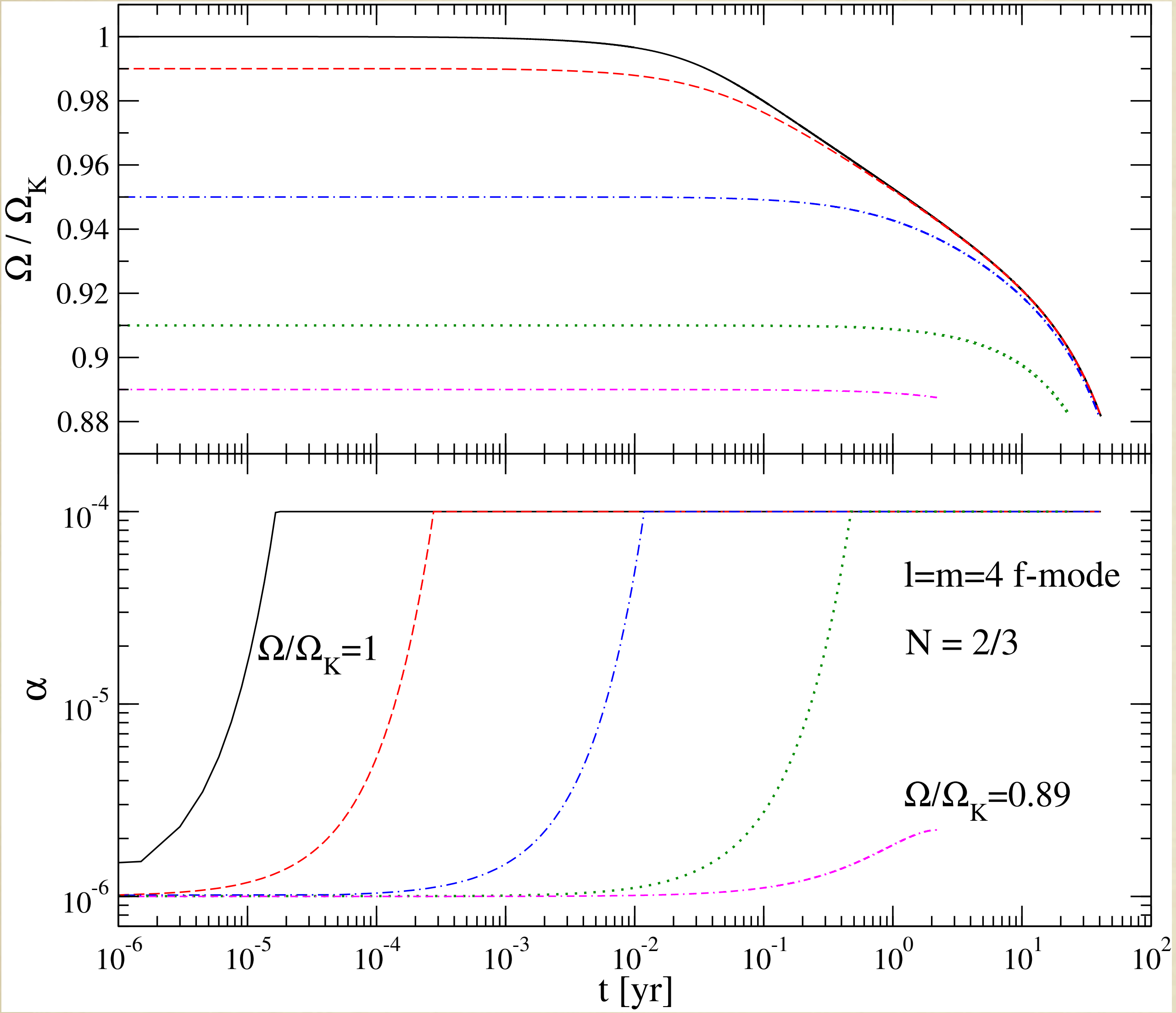
- Amplitude: $\alpha_0 = 10^{-6}$ and $\alpha_{sat} = 10^{-4} \Longrightarrow E \approx 10^{-6} M_\odot c^2$

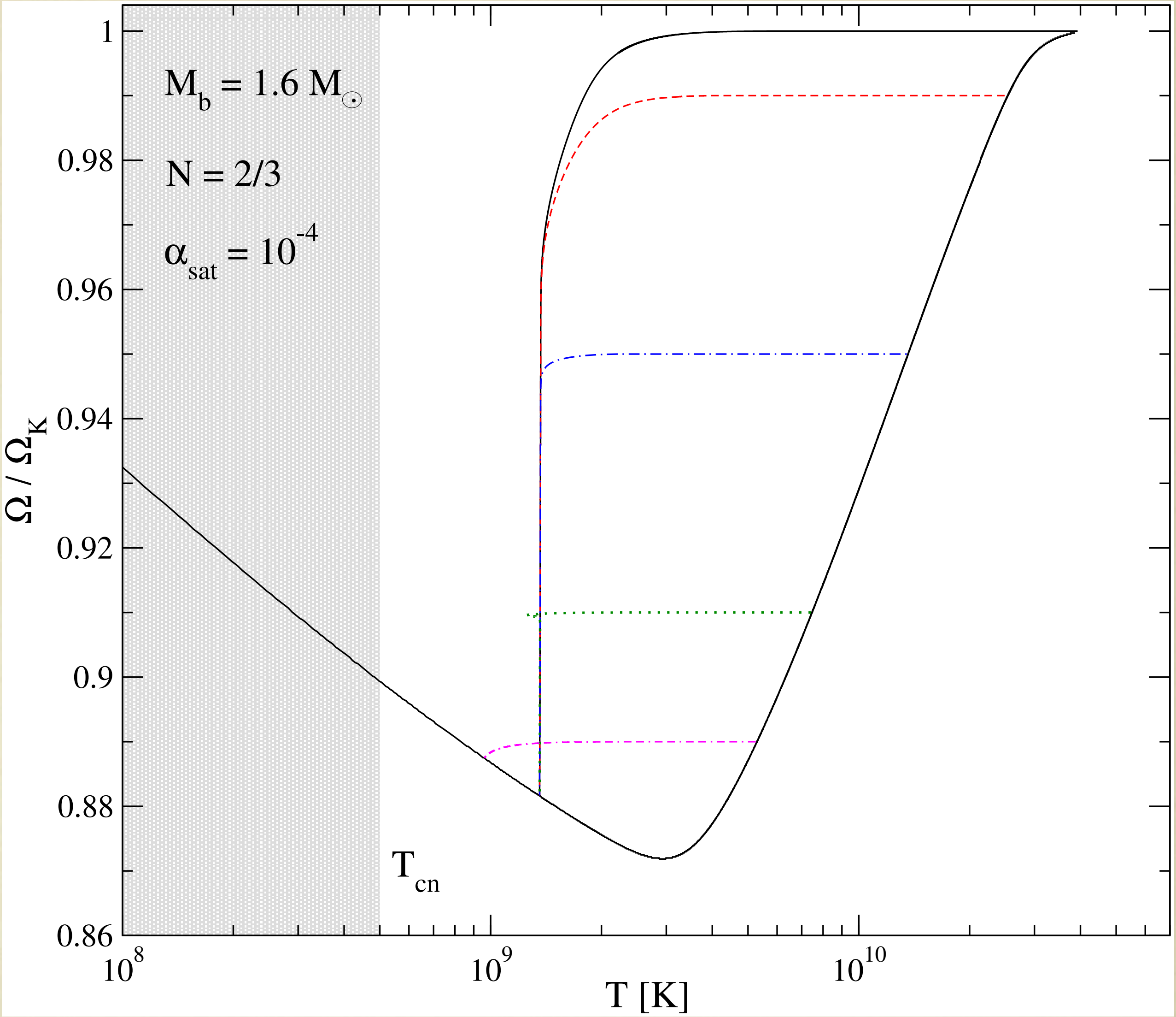
- At $\Omega = \Omega_{\rm K}$ the growth time is $\tau_{\rm gw} \sim 10^4 \rm s$

l=m=4 f-mode Evolution

N = 2/3 polytrope

Trajectory



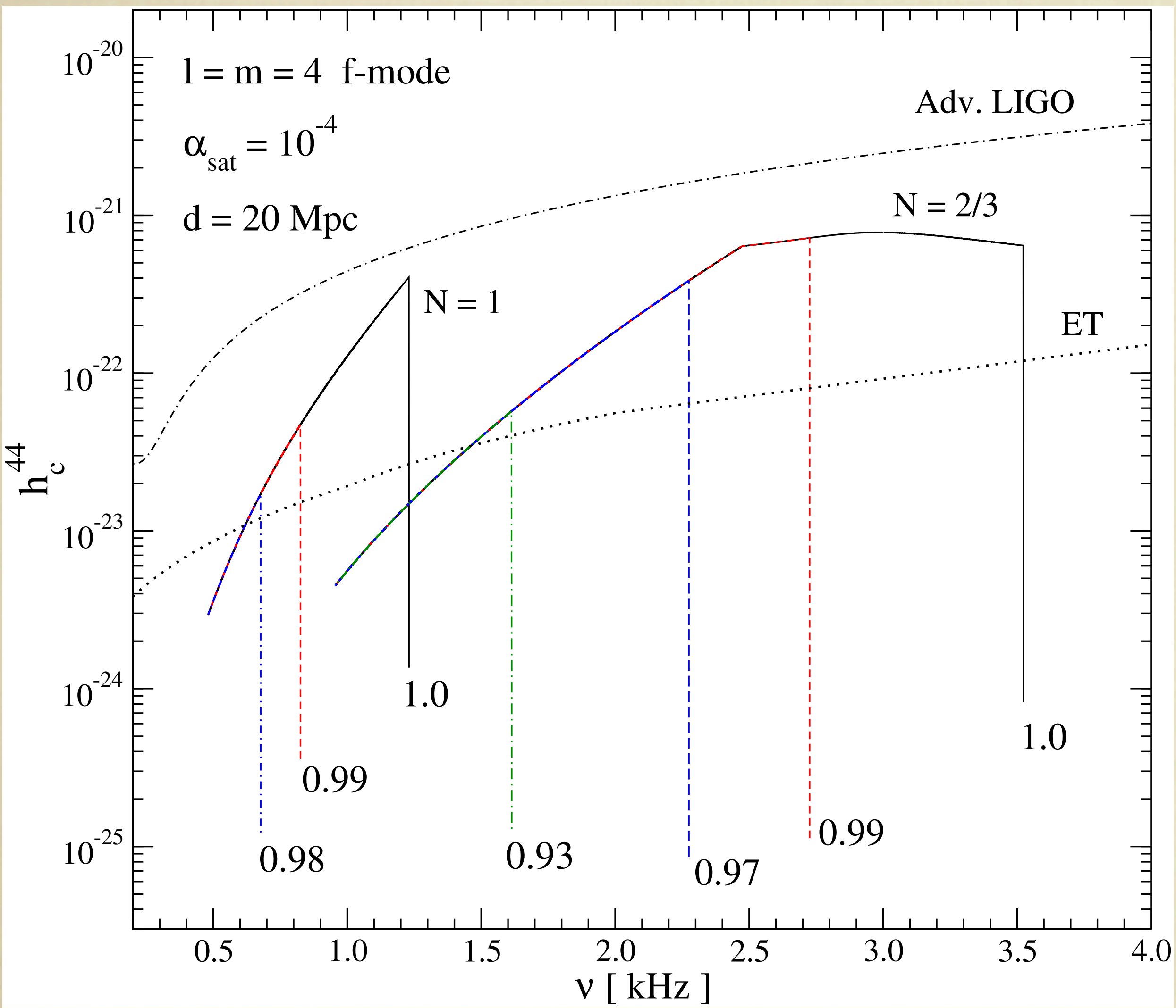


- At $\Omega = \Omega_K$ the growth time is $\tau_{gw} \sim 10^2 s$

GW signal

- Characteristic strain $$h_c = h\sqrt{\nu^2\left|\frac{dt}{d\nu}\right|}$$ $t_{obs} \leq 1\text{yr}$

l=m=4 f-mode



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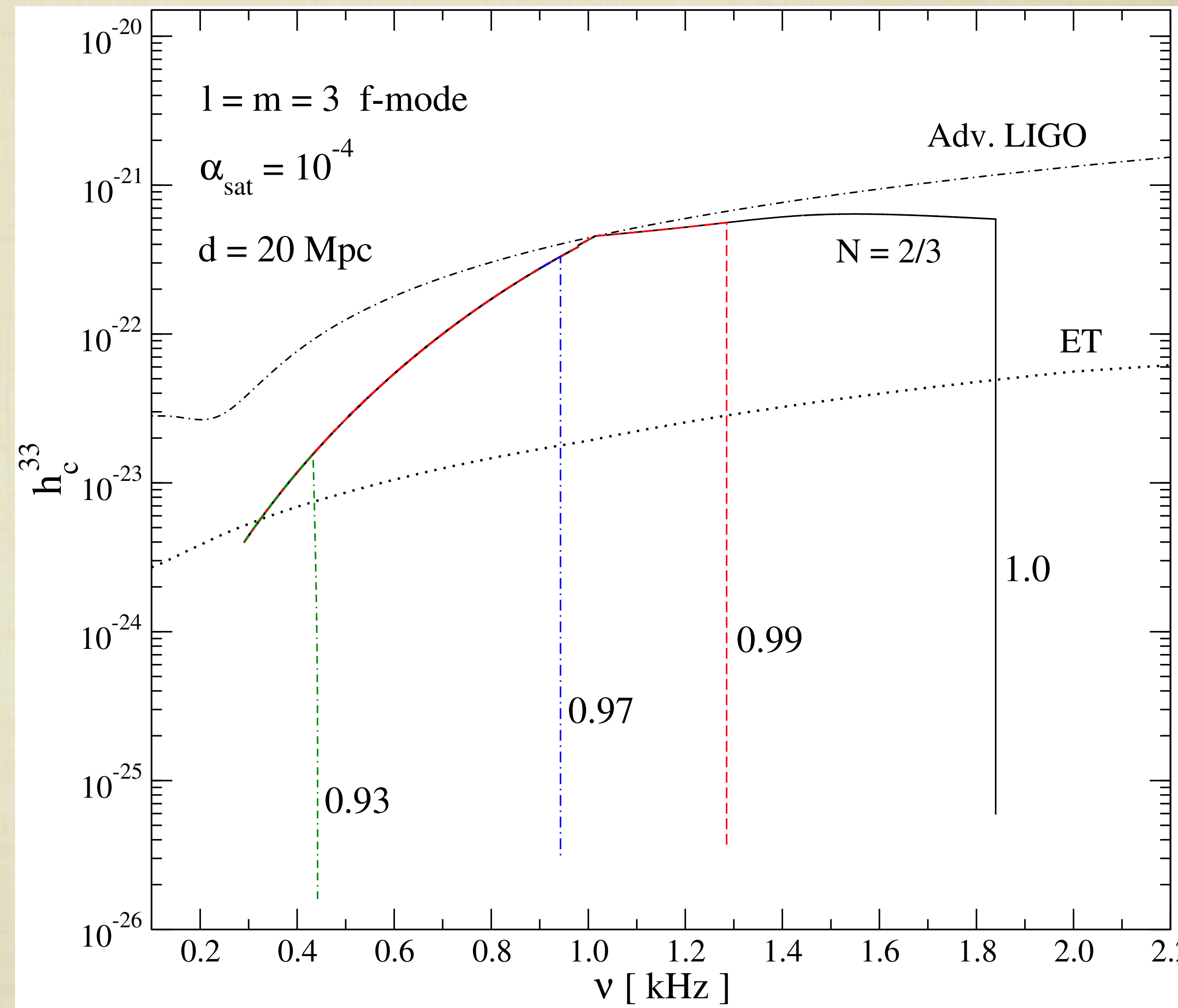


- event rate: ~30-60 SNs per year within Virgo Cluster.

Magnetic Torque

N = 2/3 polytrope

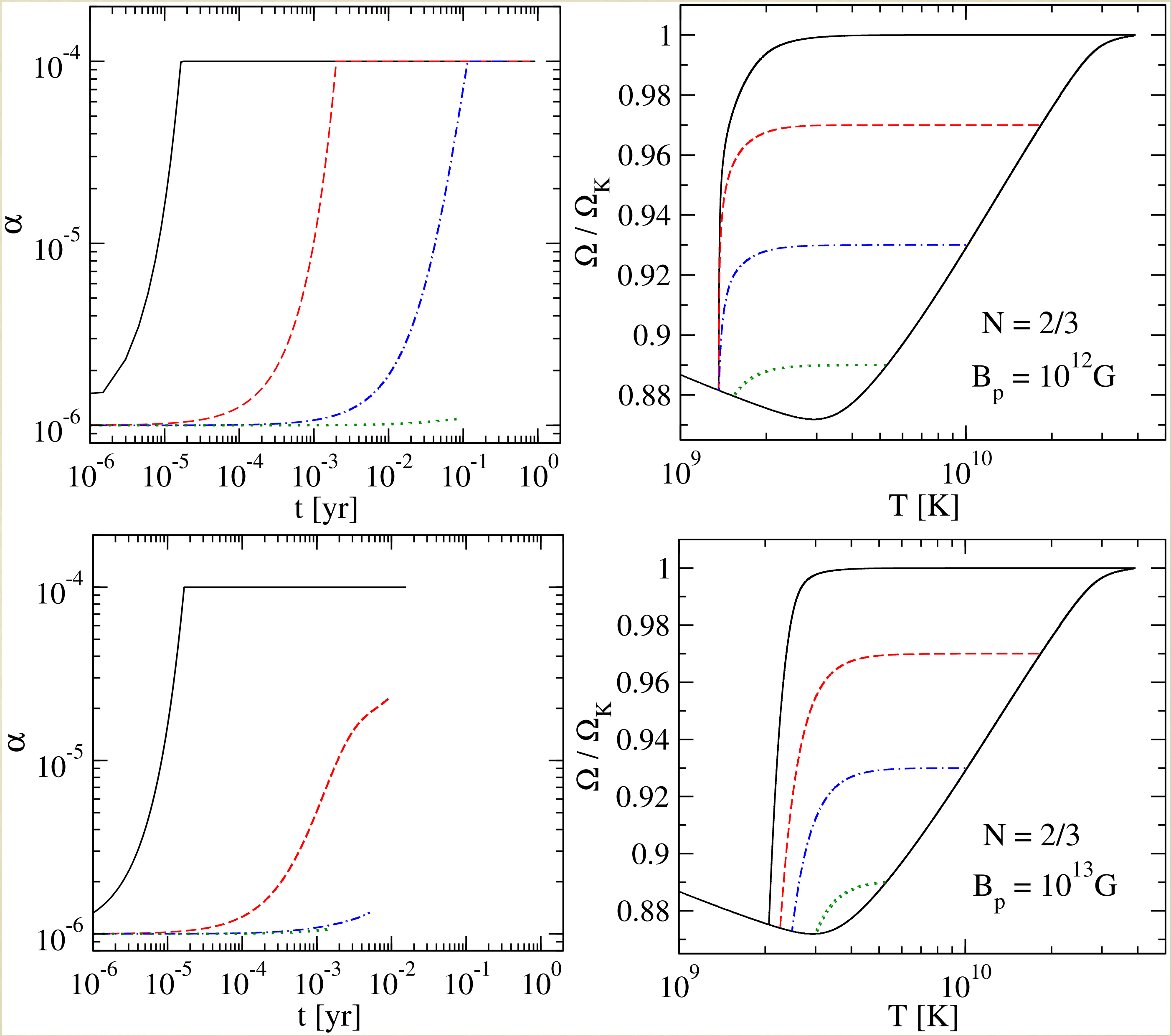
- Magnetic field accelerates the transition through the instability window

- Dipole formula

$$\frac{dJ_{mag}}{dt} \sim B_p^2 R^6 \Omega^3$$

- Mag. Torque becomes dominant for

$$B_{\rm p} \geq 10^{12}{\rm G}$$



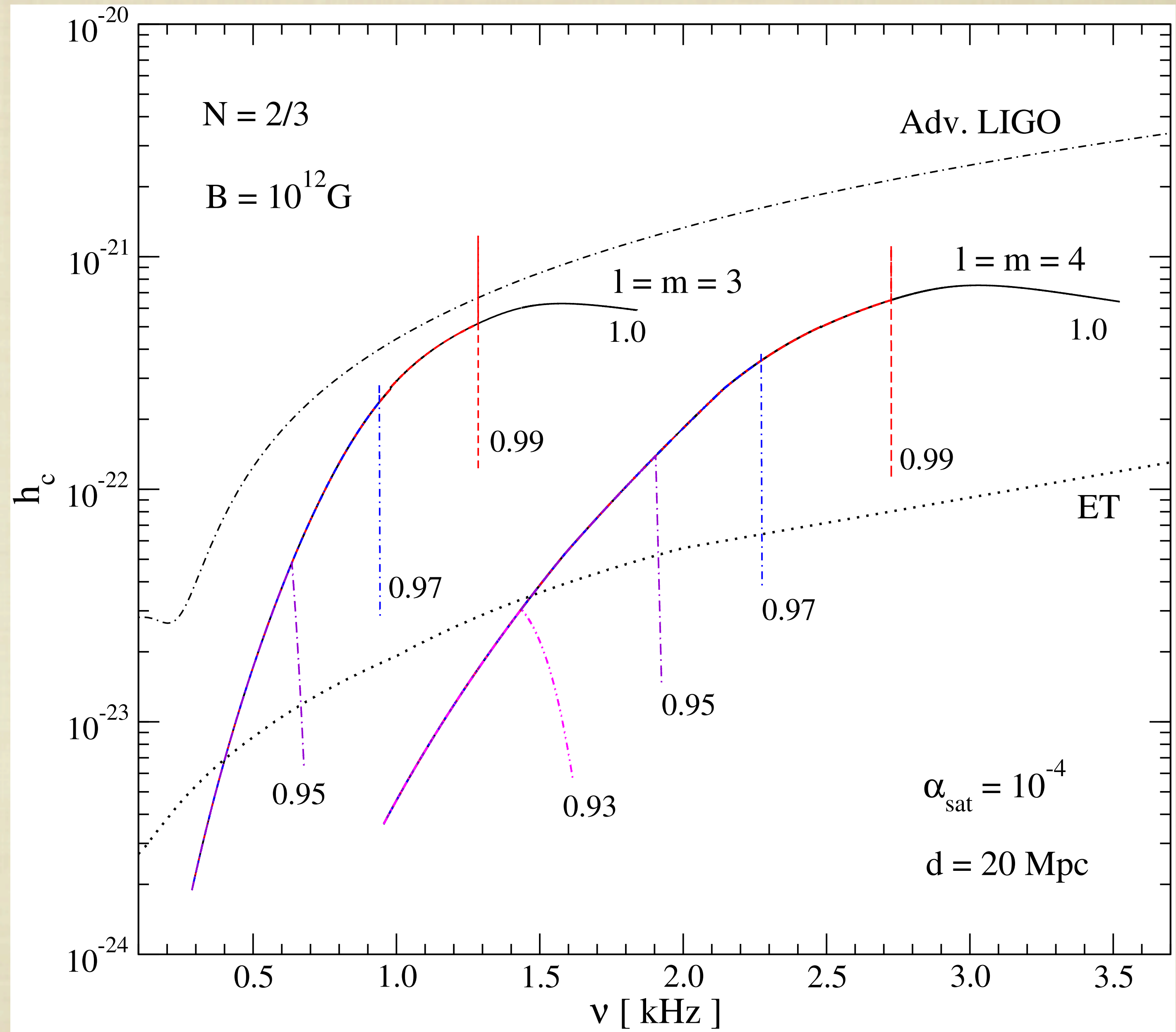
GW signal

- Characteristic strain

  $t_{obs} \leq 1\text{yr}$

- For the more massive model with $B_{\rm p} = 10^{12}\text{G}$ the GW signal may be still detectable by ET.

- Source is at 20 Mpc (Virgo Cluster)



F-mode versus R-mode

N = 2/3 model

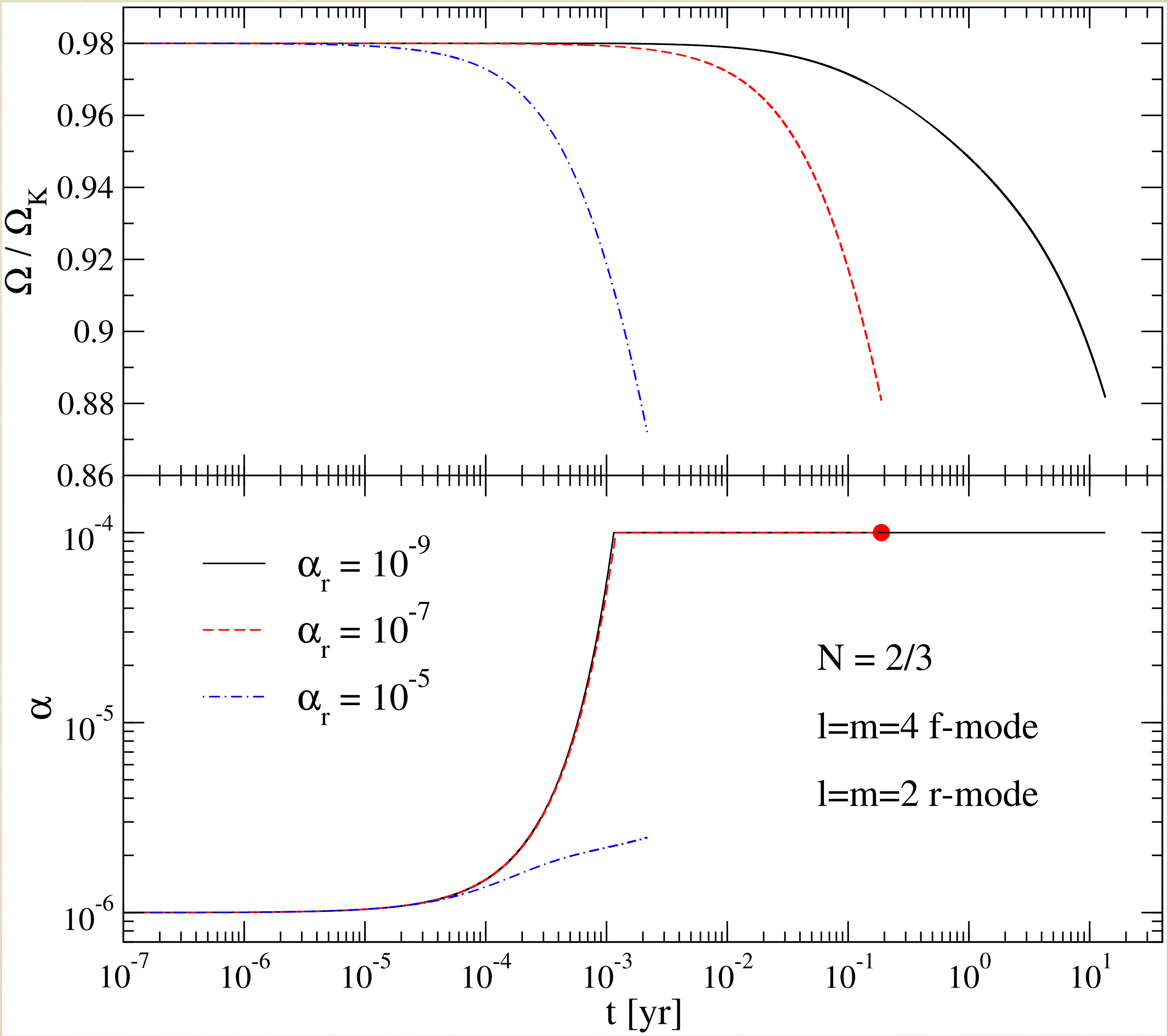
- Non-linear saturation of the f-mode

  $\alpha_{\rm f}^{sat} = 10^{-4}$

  $\Longrightarrow E \approx 10^{-6} M_{\odot} c^2$

- Non-linear mode coupling saturates r-mode

  $\alpha_{\rm r}^{sat} = 10^{-10} - 10^{-6}$



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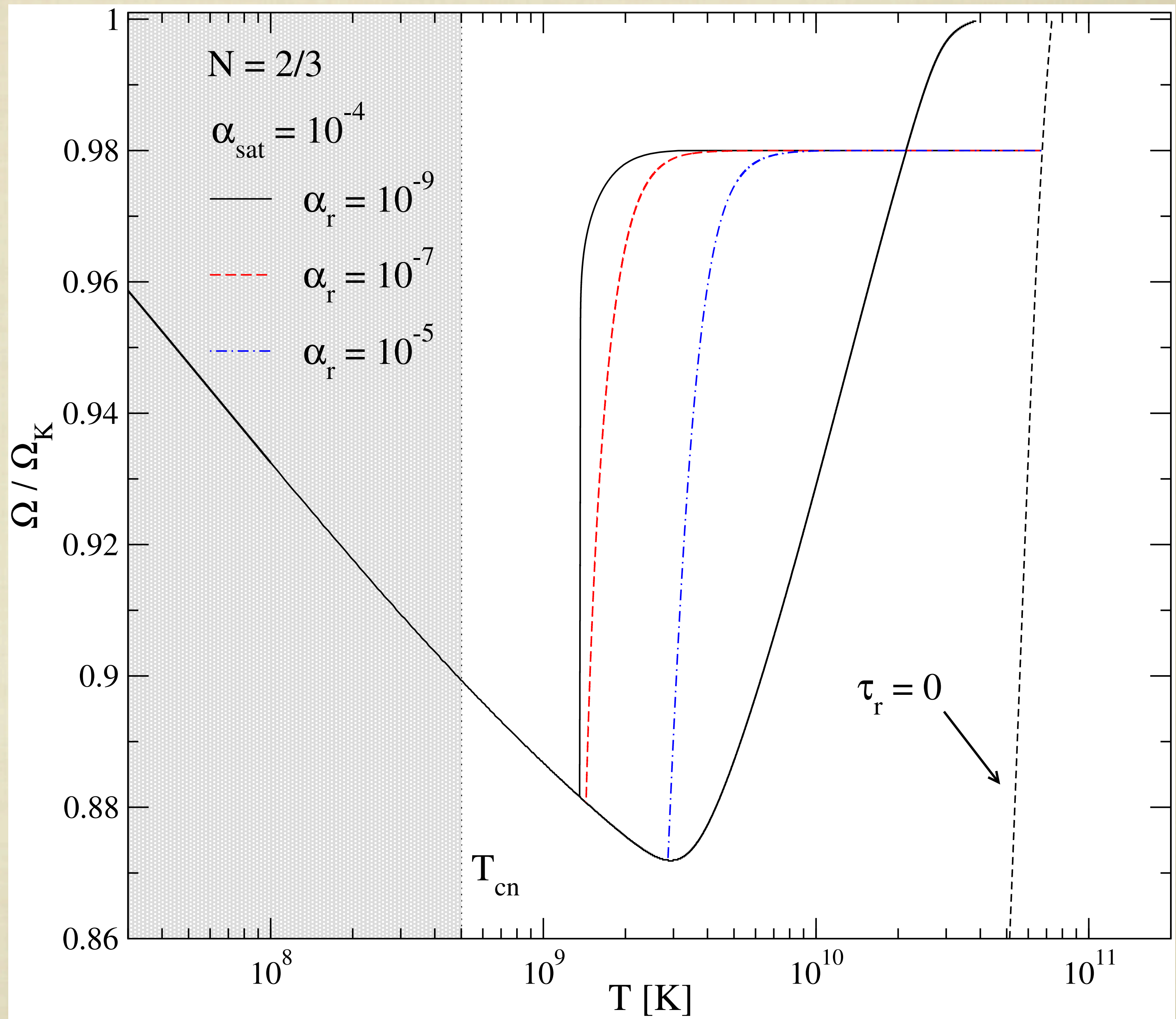

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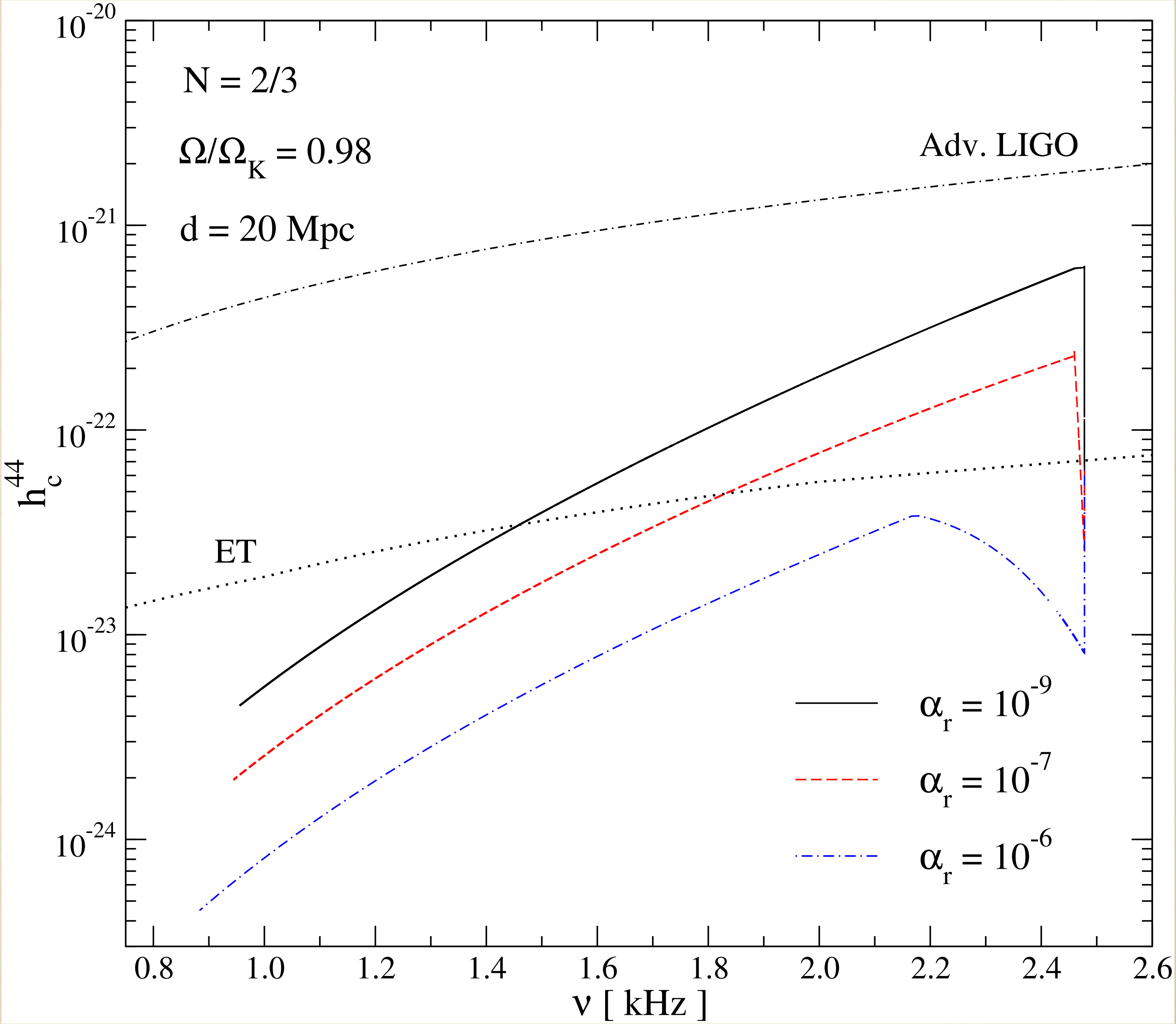


Conclusions

- The GW signal of very compact objects may be detectable from Virgo Cluster by ET.
  - The magnetic torque affects the spin down when $B_{\rm p} \geq 10^{12}{\rm G}$
  - The r-mode may limit the f-mode instability, but we need to know the relative saturation amplitude more accurately.
  - The shear viscosity re-heating may delay the superfluid transition in the core.
- More ingredients in future work.
  - The l=m=2 f-mode may become important if we abandon the Cowling approximation.
  - Study realistic EoS and consider dUrca reactions.
  - Include the Crust and the effects of Ekman layers.

Unstable modes

- F-mode (2≤m≤4)

  CFS unstable in rapidly rotating stars

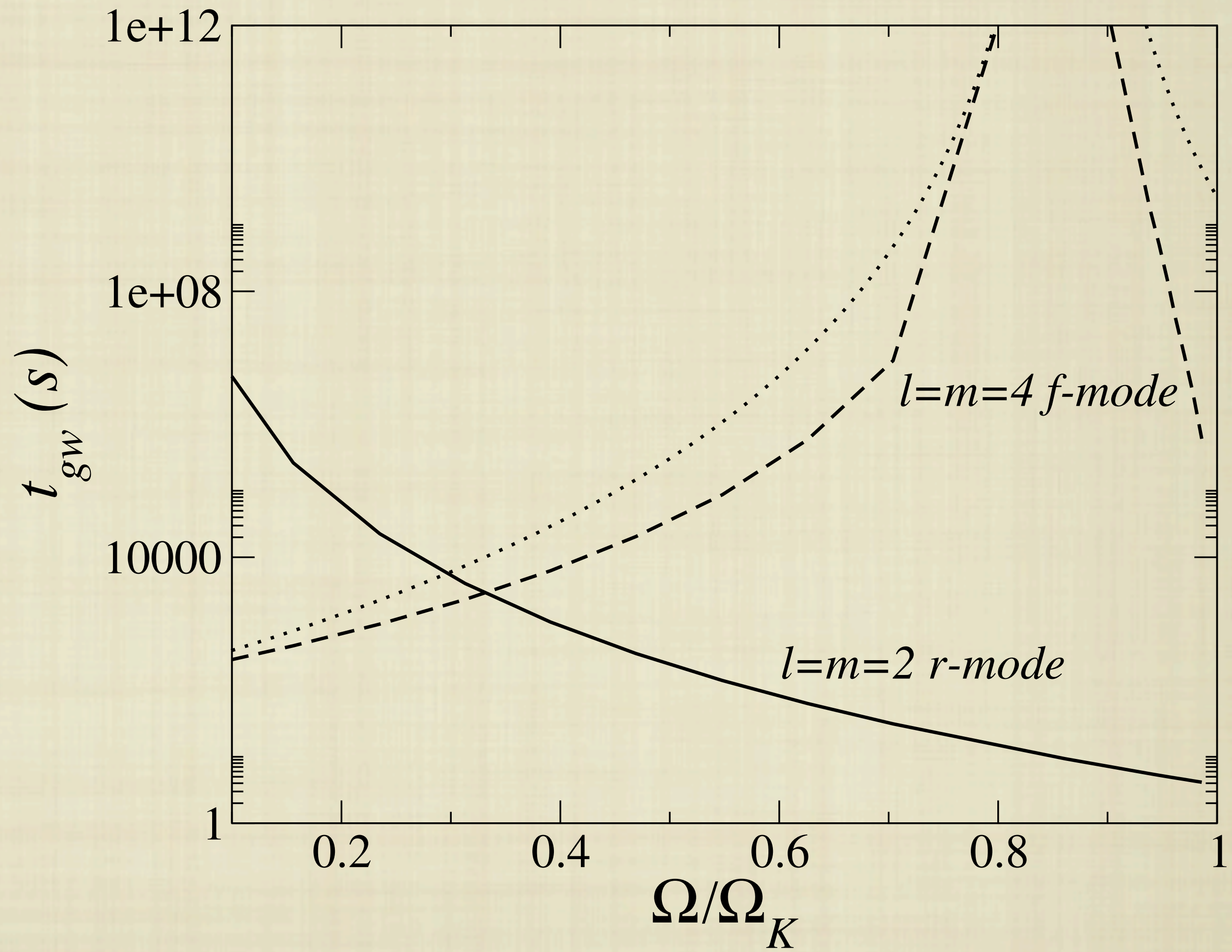
  $\tau_{\rm gw}^{44} \sim 10^2 - 10^4 s$

- R-mode (m=2)

  CFS unstable for any $\boldsymbol{\Omega}$

  $\tau_{\rm gw}^{22} \sim 1 - 10 s$

Newtonian N=1 Star



- General Relativity may strengthen the f-mode instability considerably