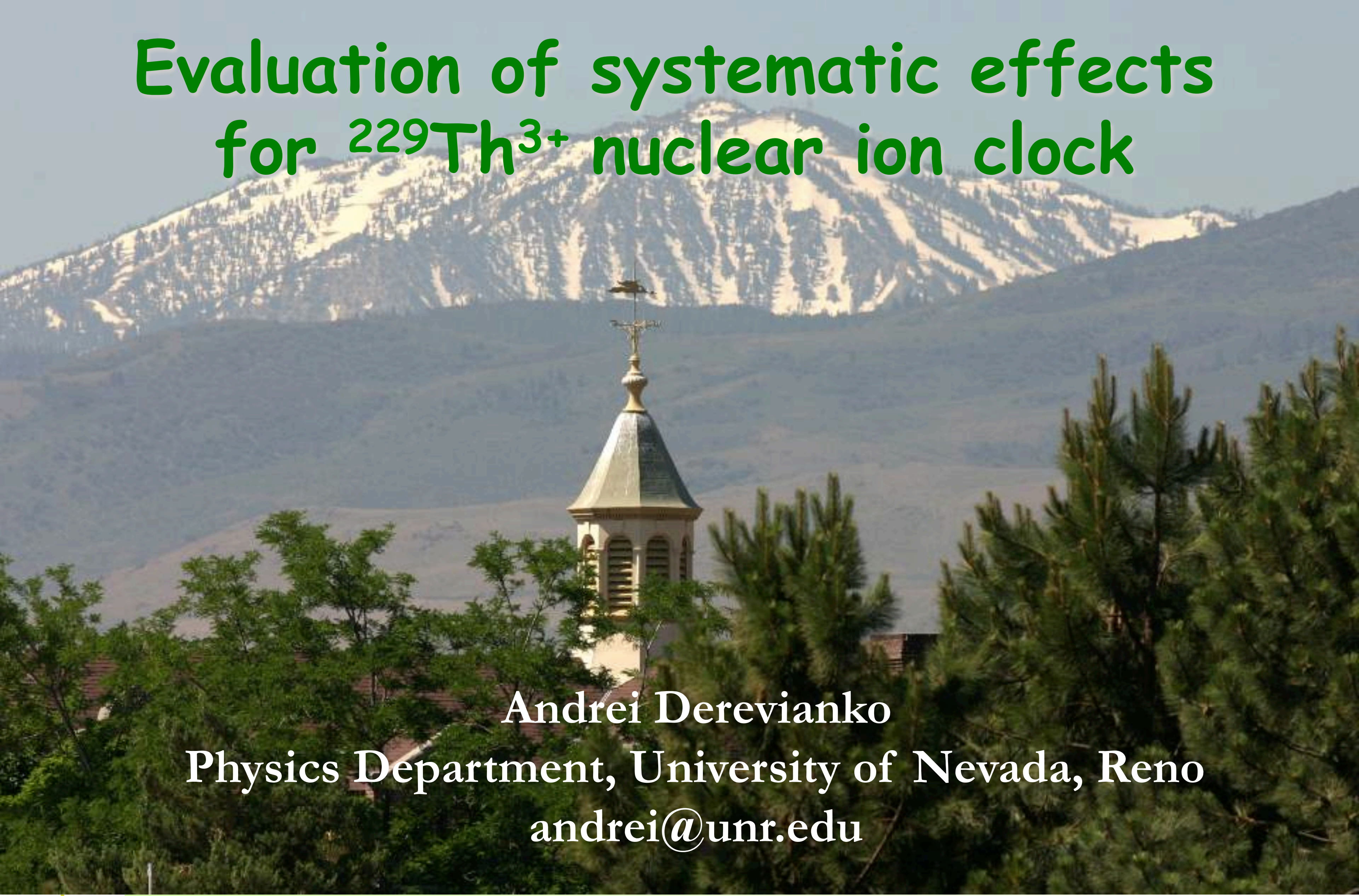
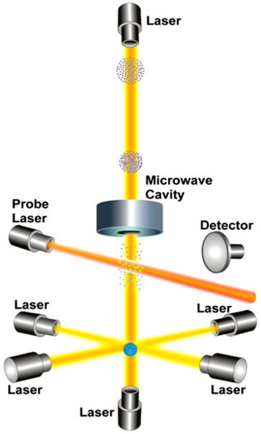
Evaluation of systematic effects for ^{229}Th^{3+} nuclear ion clock



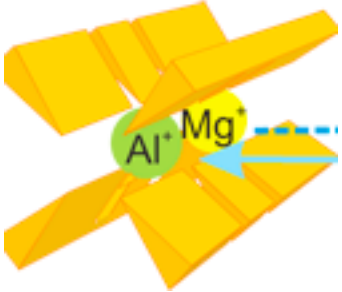
Andrei Derevianko
Physics Department, University of Nevada, Reno
andrei@unr.edu

EMMI workshop “The ^{229}Th Nuclear Isomer Clock”, GSI, Sept. 27, 2012

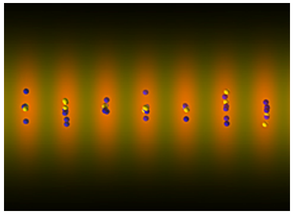
Modern atomic clocks



Microwave (Cs, Rb)
^{133}Cs: defines the SI units of length and time



Singly-charged ion clocks (Al+,Hg+,Cd+,…)
Accuracy record-holders



Optical lattice clocks (Sr, Hg, Yb,…)
Ultrastable

The accuracy frontier

Clock	Fractional inaccuracy
Primary cesium standard	3×10^{-16}
Ion clocks (Al^+/Mg^+)	9×10^{-18}
Optical lattice clocks (Sr)	1.5×10^{-16}

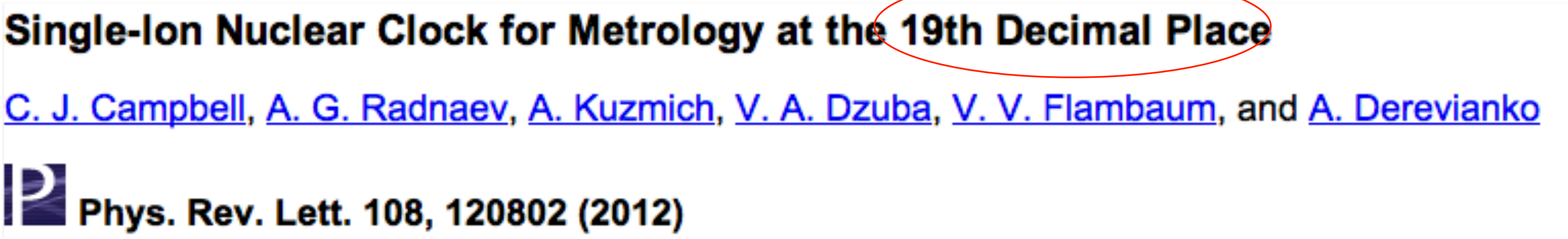
Projected "end-of-the-road" for ion and lattice clocks : 10^{-18}

Can we do better?

10^{-19}

☐ Nuclear clock
☐ Highly-charged ions and optical clockwork

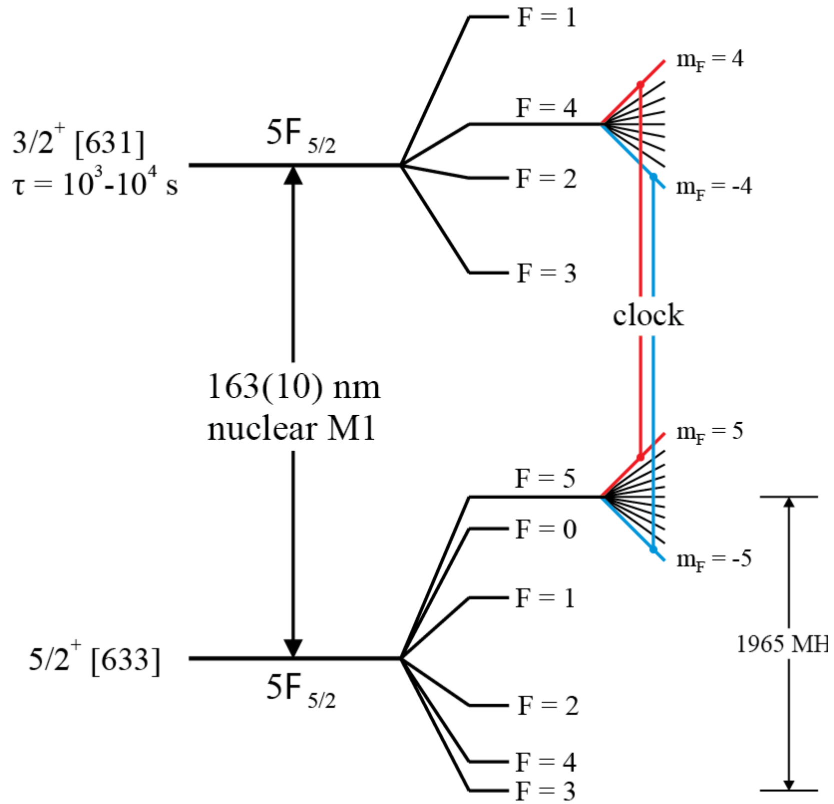
The ultimate clock?



Single-Ion Nuclear Clock for Metrology at the 19th Decimal Place

C. J. Campbell, A. G. Radnaev, A. Kuzmich, V. A. Dzuba, V. V. Flambaum, and A. Derevianko

Phys. Rev. Lett. 108, 120802 (2012)



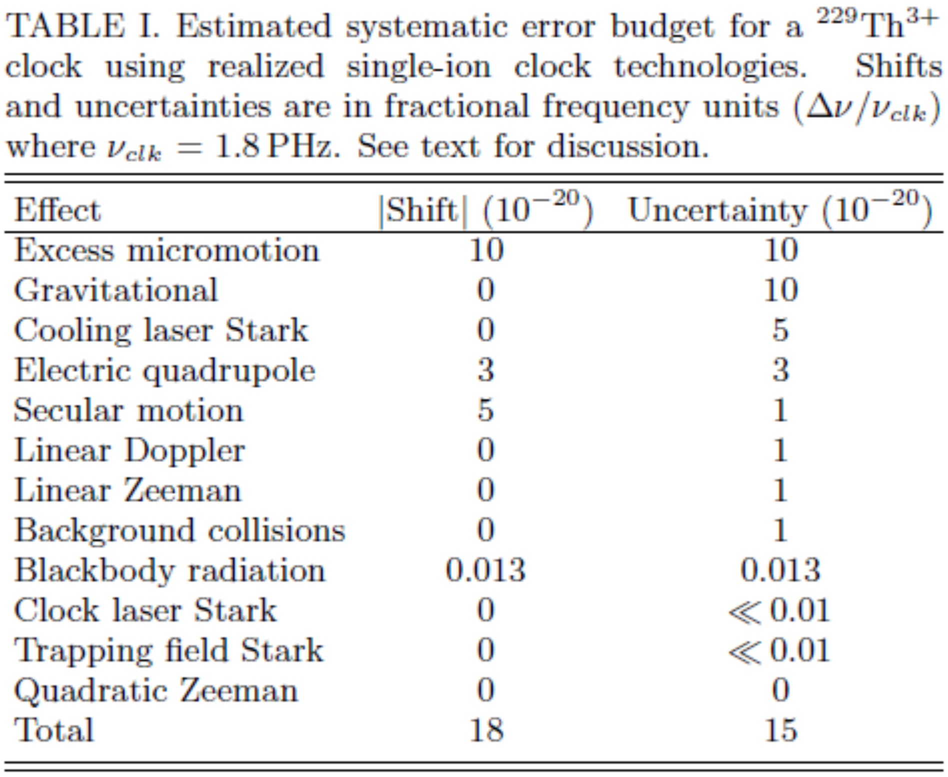
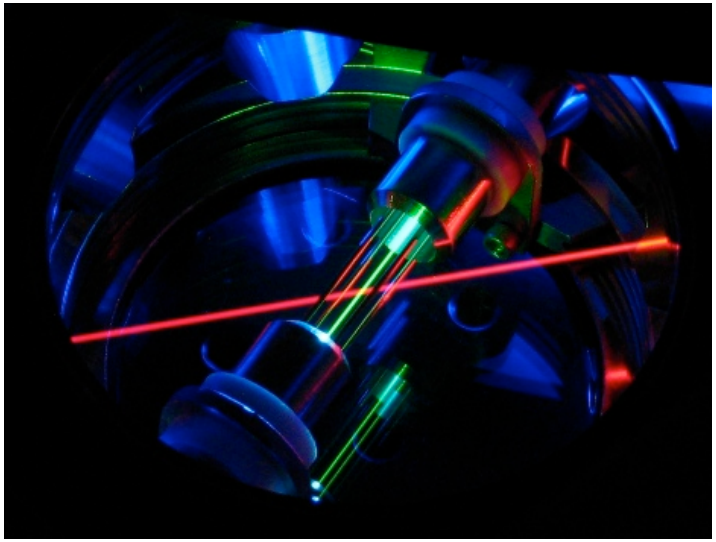


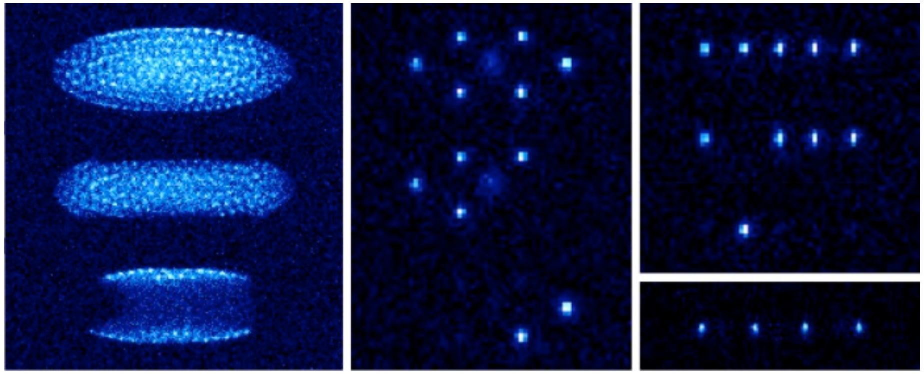
TABLE I. Estimated systematic error budget for a ^{229}Th^{3+} clock using realized single-ion clock technologies. Shifts and uncertainties are in fractional frequency units ($\Delta\nu/\nu_{clk}$) where $\nu_{clk} = 1.8$ PHz. See text for discussion.

Effect	$\lvert$Shift$\rvert$ (10^{-20})	Uncertainty (10^{-20})
Excess micromotion	10	10
Gravitational	0	10
Cooling laser Stark	0	5
Electric quadrupole	3	3
Secular motion	5	1
Linear Doppler	0	1
Linear Zeeman	0	1
Background collisions	0	1
Blackbody radiation	0.013	0.013
Clock laser Stark	0	$\ll 0.01$
Trapping field Stark	0	$\ll 0.01$
Quadratic Zeeman	0	0
Total	18	15

Trapping $^{229}Th^{3+}$



GATech linear Paul trap



Wigner crystals of laser-cooled Th^{3+} ions (T~mK)

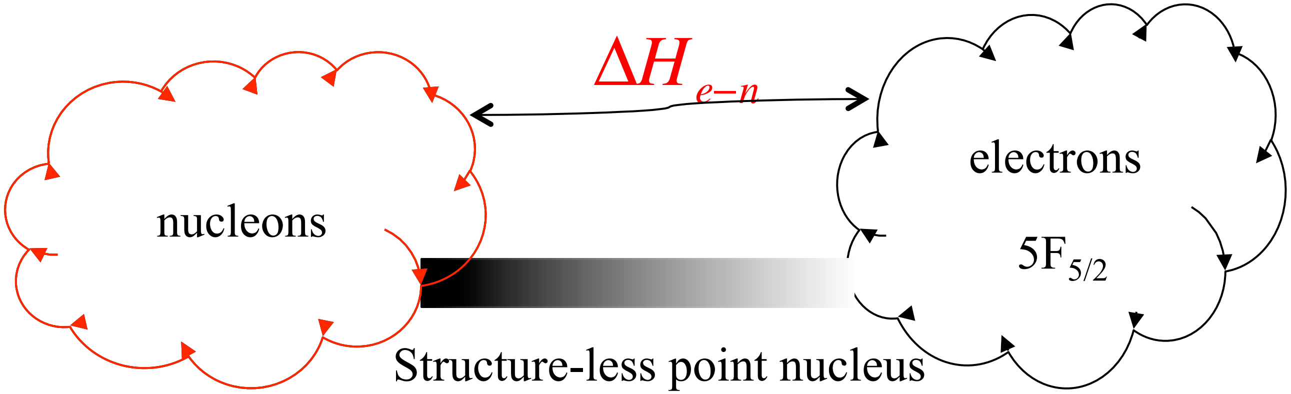
C. Campbell, A. Radnaev, A. Kuzmich, PRL 106, 223001 (2011)

Systematic effects

Clock frequency is affected by:

- ✓ Stark shifts: cooling/probe lasers, trapping filed
- ✓ Zeeman shifts
- ✓ Electric quadrupole (gradients of trapping field)
- ✓ Blackbody radiation
- ✓ Doppler shifts
- ✓ Gravity

Compound system



ΔH_{e-n}

nucleons

electrons

$5F_{5/2}$

Structure-less point nucleus

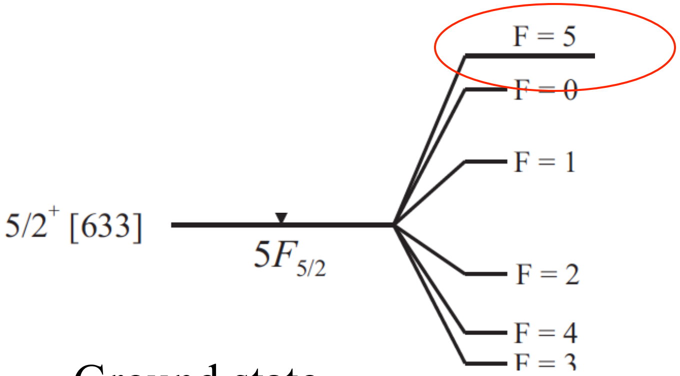
$$H = H_{\text{nuclear}} + H_{\text{electrons}} + H_{\text{Couloumb-point-nucleus}} + \Delta H_{e-n}$$

Nucleus = source of EM potentials =
= E0 (Coulomb) + magnetic-dipole (mu) + electric-quadrupole (Q)+…

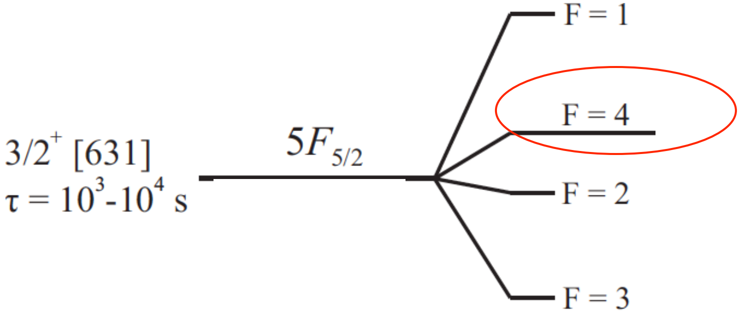
Nuclear transition => jump in rho, mu, Q

Keep the electrons in the ground state (Th^{3+} $5F_{5/2}$)

The virtues of stretching



Ground state



Excited state

Stretched states (decoupling electrons and nucleons)

$$|F = J + I, M_F = F> = |5F_{5/2}, M_J = J> |I, M_I = I>$$

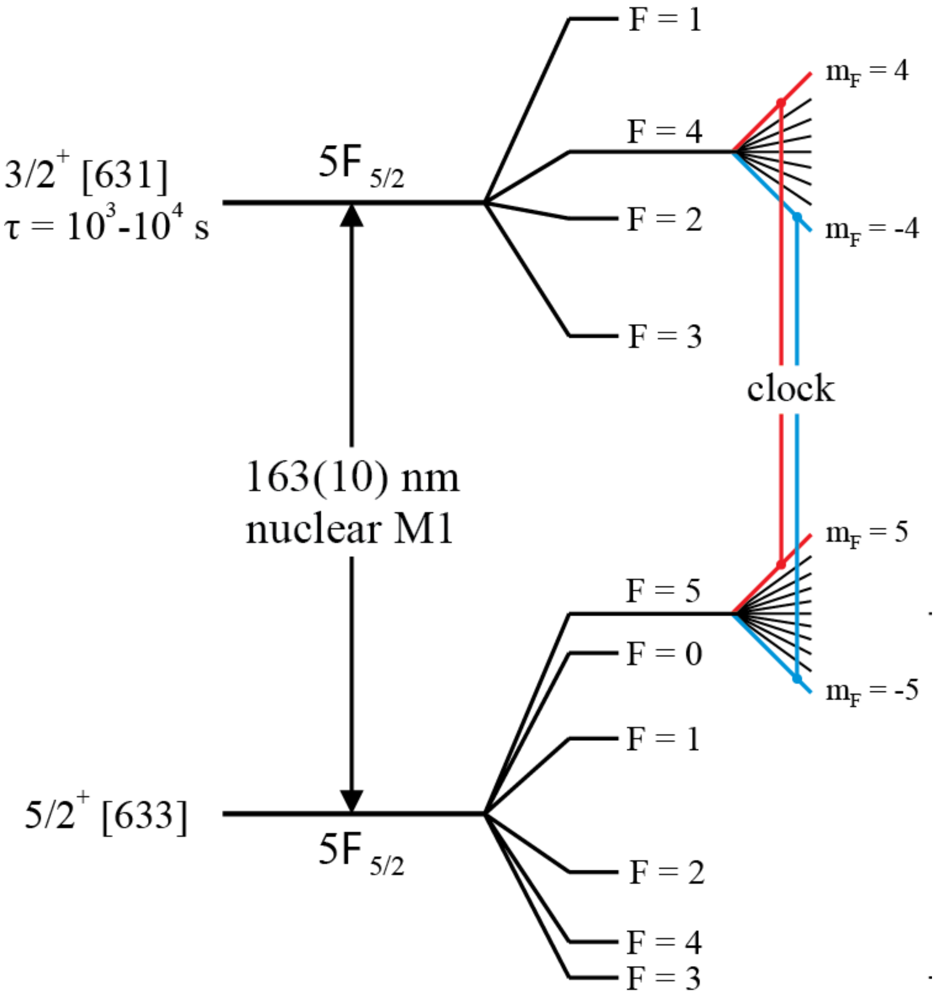
Perturbation

$$\widehat{X} = \widehat{X}_{\text{nuc}} + \widehat{X}_{\text{el}}$$

Clock shift

$$h\delta\nu_{\text{clock}} = \langle X \rangle_e - \langle X \rangle_g = \langle X_{\text{nuc}} \rangle_e - \langle X_{\text{nuc}} \rangle_g + \Delta X_{\text{e-n}}$$!

Virtual clock transitions



Two STRETCHED transitions with
opposite shifts in B-field:
Averaging =>
Linear Zeeman goes away

Differential template

$$\Delta X = \gamma_X \Delta X^{\mathrm{nuc}} + \Delta X^{\mathrm{iso}} + \Delta X^{\mathrm{hfs}}$$

Shielding factor for Th^{4+}

Isomer shift (E0)

HFS-mediated shift (M1+E2)

Relativistic many-body calculations of atomic structure – typical accuracy 10%.
Dirac-Hartree-Fock + Bruckner core-polarization + random-phase-approximation

Isomer contribution

$$\Delta X^{\mathrm{iso}} = \left(\frac{dX}{dR_{\mathrm{rms}}} \right) \Delta R_{\mathrm{rms}}$$

difference in the RMS radii of the nuclear ground- and isomer-state charge distributions

---

Example: Quadrupole moment (in a.u.) of the ground electronic state

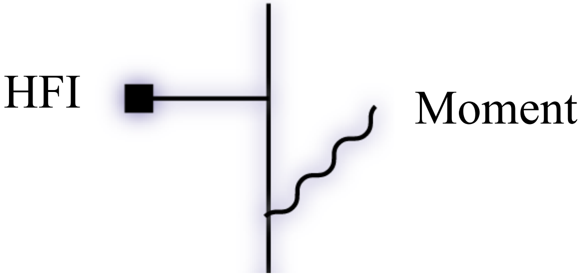
C_{fermi}, fm	DHF	DHF+RPA	DHF+BO
6.5	-1.832544	-1.382214	-1.639564
7.0	-1.832442		
7.5	-1.832336	-1.381968	-1.639402
$\Delta Q/\Delta C$,au/fm	+0.000208	+0.000246	+0.000162

$$dQ/dC_{Fermi} \approx 2 \times 10^{-4} \;\; |e|\, a_0^2/fm$$

hyperfine-mediated contribution

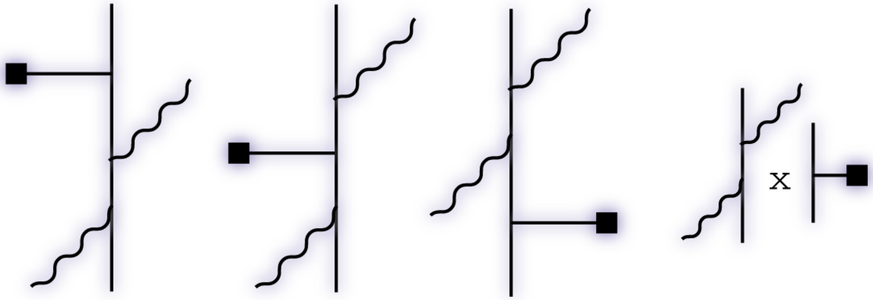
$$\Delta X^{\mathrm{hfs}} = \left(\frac{\mu_m - \mu_g}{\mu_N} \right) \bar{X}_{\mu}^{\mathrm{hfs}} + \left(\frac{Q_m - Q_g}{|e| b} \right) \bar{X}_{Q}^{\mathrm{hfs}}$$

Various moments =



$$\frac{\bar{Q}_Q^{hfs}}{|e| \ fm^2} = 2 \sum_{n_k J_k} \left(\begin{array}{ccc} J_g & 2 & J_k \\ -J_g & 0 & J_g \end{array} \right)^2 \frac{\langle n_g J_g || Q || \, n_k J_k \rangle \langle n_k J_k || \mathcal{T}^{(2)} || \, n_g J_g \rangle}{E_g - E_k}.$$

Polarizability =



K. Beloy, U. I. Safronova, and A. Derevianko, PRL 97, 040801 (2006)

Nuclear parameters

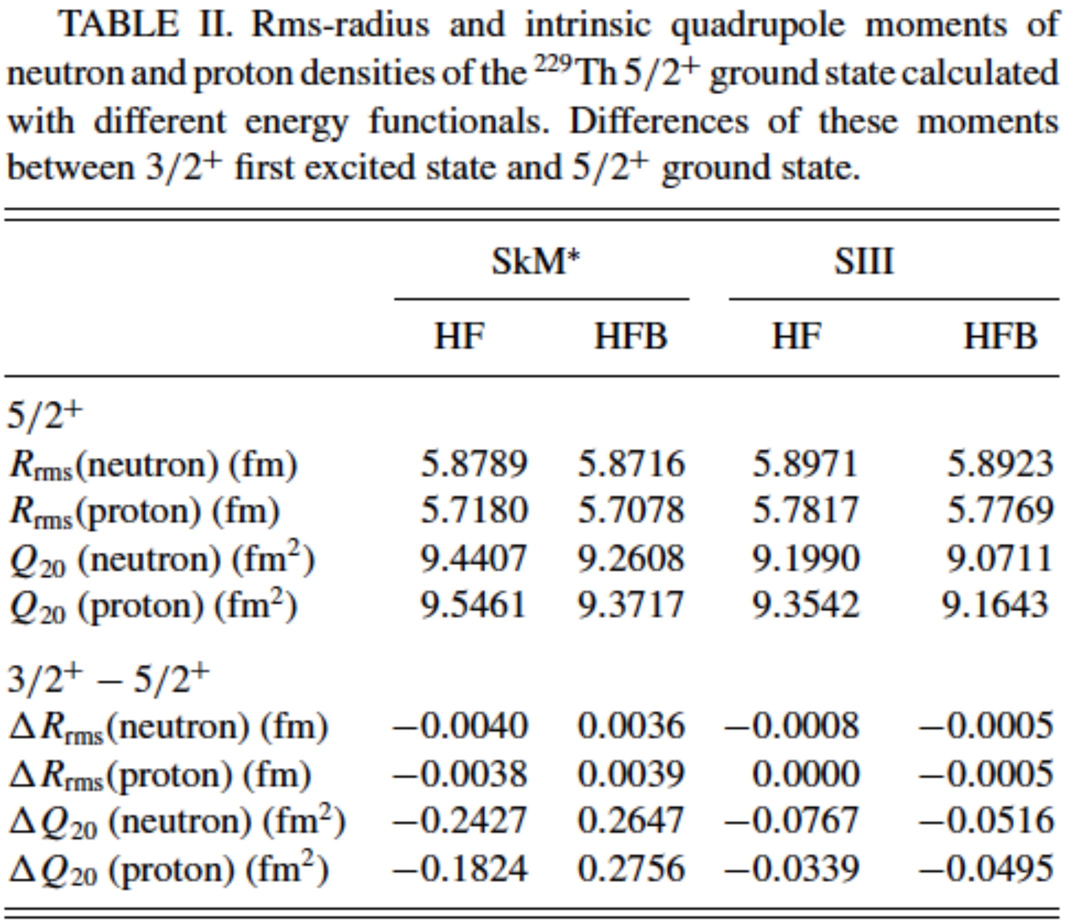


TABLE II. Rms-radius and intrinsic quadrupole moments of neutron and proton densities of the ^{229}Th $5/2^+$ ground state calculated with different energy functionals. Differences of these moments between $3/2^+$ first excited state and $5/2^+$ ground state.

	SkM*		SIII	
	HF	HFB	HF	HFB
$5/2^+$				
$R_{\rm rms}$(neutron) (fm)	5.8789	5.8716	5.8971	5.8923
$R_{\rm rms}$(proton) (fm)	5.7180	5.7078	5.7817	5.7769
Q_{20} (neutron) (fm^2)	9.4407	9.2608	9.1990	9.0711
Q_{20} (proton) (fm^2)	9.5461	9.3717	9.3542	9.1643
$3/2^+ - 5/2^+$				
$\Delta R_{\rm rms}$(neutron) (fm)	−0.0040	0.0036	−0.0008	−0.0005
$\Delta R_{\rm rms}$(proton) (fm)	−0.0038	0.0039	0.0000	−0.0005
ΔQ_{20} (neutron) (fm^2)	−0.2427	0.2647	−0.0767	−0.0516
ΔQ_{20} (proton) (fm^2)	−0.1824	0.2756	−0.0339	−0.0495

$$|\Delta R_{rms}| < 0.0038\,\mathrm{fm}$$

$$|\Delta Q_{20}| < 0.28|e|\,\mathrm{fm}^2$$

$$\Delta\mu = -0.53\,\mu_n$$

E. Litvinova, H. Feldmeier, J. Dobaczewski, and V. Flambaum, PRC79, 064303 (2009).

Uncertainty budget

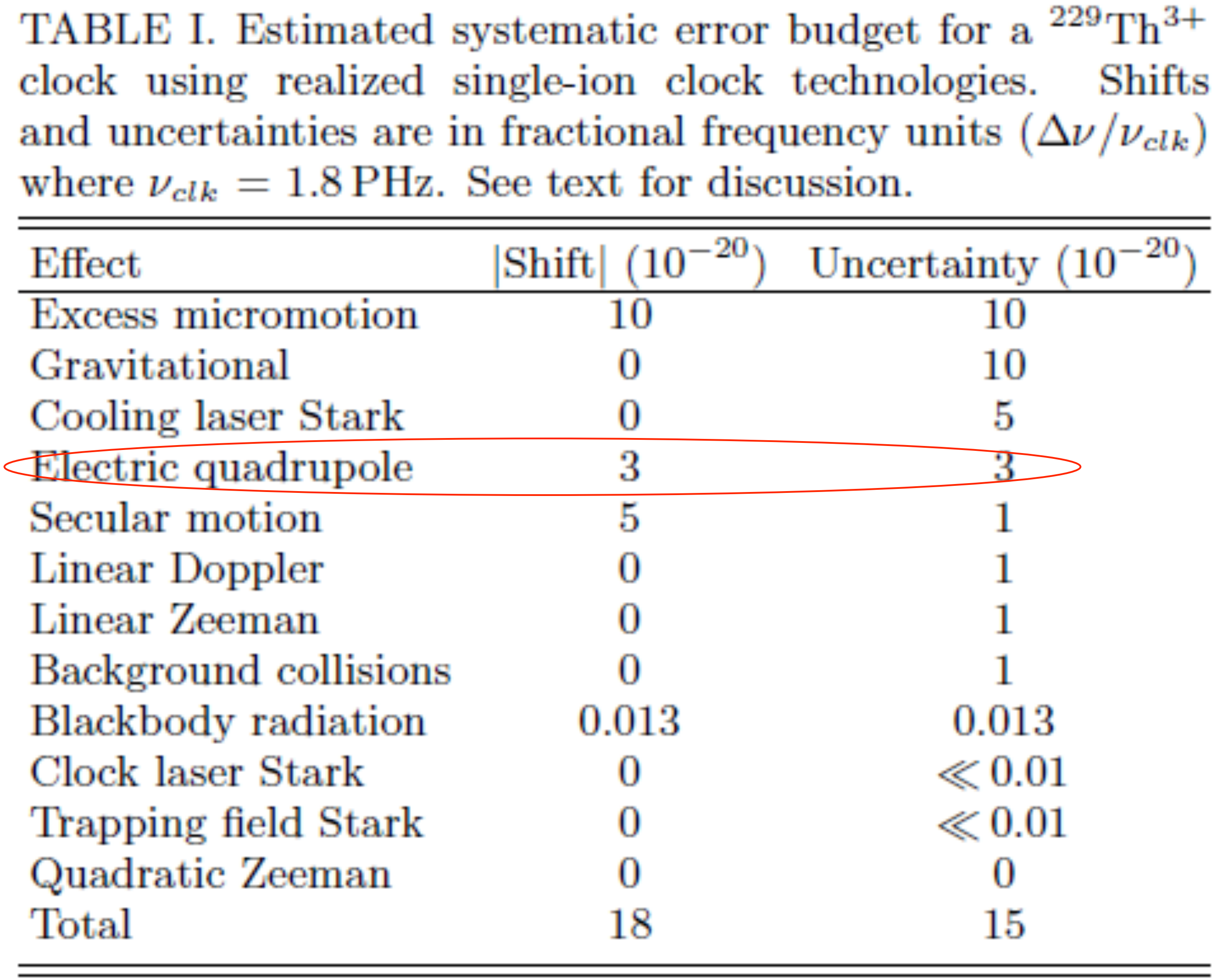


TABLE I. Estimated systematic error budget for a $^{229}\mathrm{Th}^{3+}$ clock using realized single-ion clock technologies. Shifts and uncertainties are in fractional frequency units ($\Delta\nu/\nu_{clk}$) where $\nu_{clk} = 1.8\,\mathrm{PHz}$. See text for discussion.

Effect	|Shift| (10^{-20})	Uncertainty (10^{-20})
Excess micromotion	10	10
Gravitational	0	10
Cooling laser Stark	0	5
Electric quadrupole	3	3
Secular motion	5	1
Linear Doppler	0	1
Linear Zeeman	0	1
Background collisions	0	1
Blackbody radiation	0.013	0.013
Clock laser Stark	0	$\ll 0.01$
Trapping field Stark	0	$\ll 0.01$
Quadratic Zeeman	0	0
Total	18	15

Example: Quadrupole shift

$$\frac{\Delta\nu_{QS}}{\nu_{\mathrm{clock}}} = -\frac{1}{2h\nu_{\mathrm{clock}}}\Delta Q\frac{\partial\mathcal{E}_z}{\partial z}$$

Trapping field gradient $8\times10^{6}\,\mathrm{V/m^2}$

$$\Delta Q = \gamma_Q\Delta Q^{\mathrm{nuc}} + \Delta Q^{\mathrm{iso}} + \Delta Q^{\mathrm{hfs}}$$

$\gamma_Q = -177.5 \quad \Delta Q^{nuc} = -1.37(5)|e|b$

$|\Delta Q^{iso}| < 8\times10^{-7}|e|a_0^2$

$\Delta Q^{hfs} \approx 1.8\times10^{-6}|e|a_0^2$

$\Delta Q \approx 1\times10^{-5}|e|a_0^2$ Dominated by anti-shielded direct nuclear contribution

$$\boxed{\frac{\Delta\nu_{QS}}{\nu_{\mathrm{clock}}} \approx 3\times10^{-20}}$$

Uncertainty budget

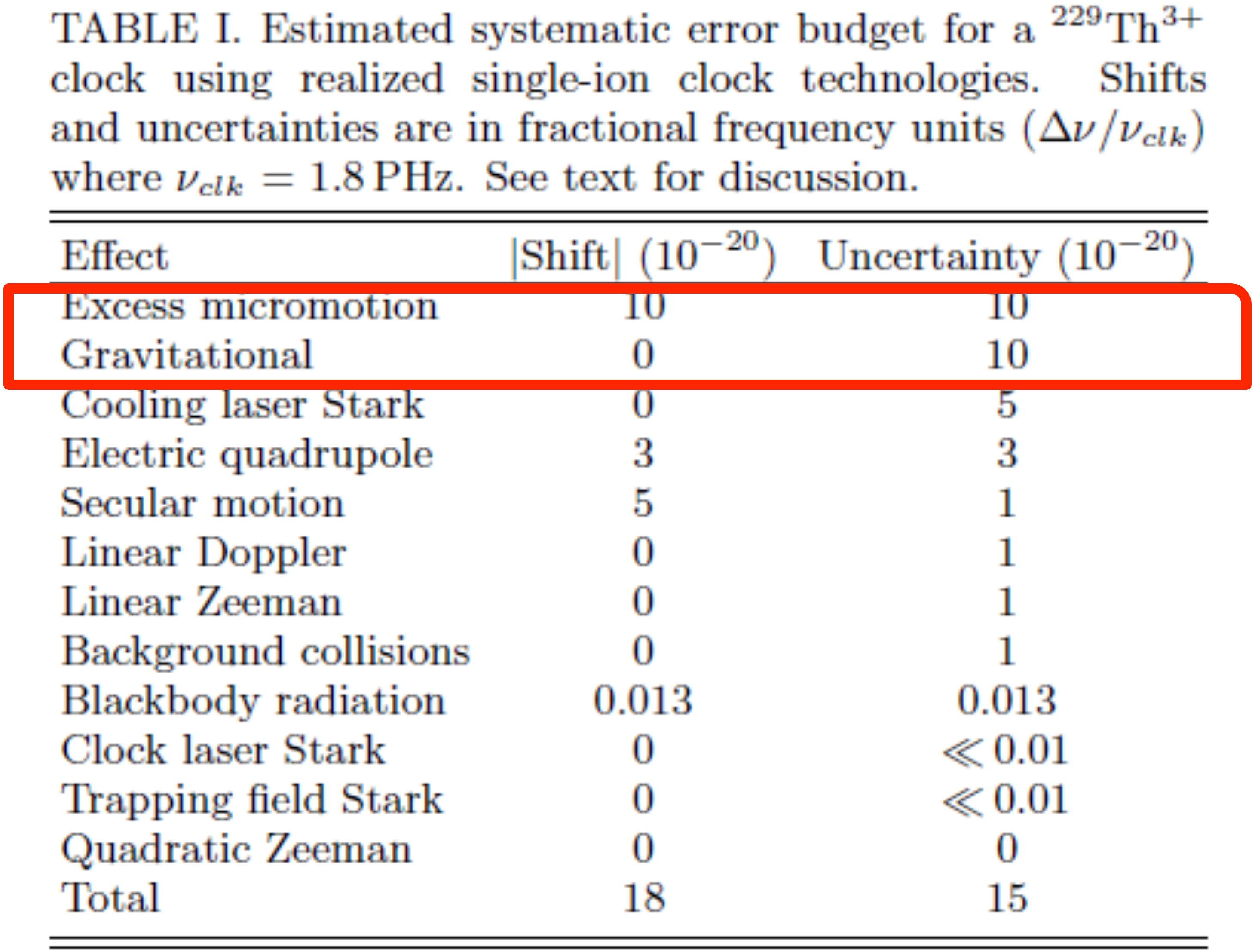


TABLE I. Estimated systematic error budget for a $^{229}\mathrm{Th}^{3+}$ clock using realized single-ion clock technologies. Shifts and uncertainties are in fractional frequency units ($\Delta\nu/\nu_{clk}$) where $\nu_{clk} = 1.8\,\mathrm{PHz}$. See text for discussion.

Effect	|Shift| (10^{-20})	Uncertainty (10^{-20})
Excess micromotion	10	10
Gravitational	0	10
Cooling laser Stark	0	5
Electric quadrupole	3	3
Secular motion	5	1
Linear Doppler	0	1
Linear Zeeman	0	1
Background collisions	0	1
Blackbody radiation	0.013	0.013
Clock laser Stark	0	$\ll 0.01$
Trapping field Stark	0	$\ll 0.01$
Quadratic Zeeman	0	0
Total	18	15

10^{-19} and highly-charged ions

Nuclear clock ^{229}Th (Th^{3+} ion clock):

- Improvement is due to tiny size of the quantum oscillator => suppressed couplings
- *Yet* unobserved optical transition
- Radioactivity

clocks based on highly-charged ions:

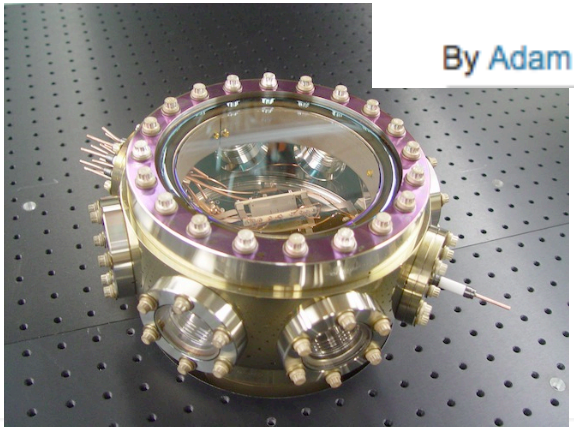
- As the ionic charge increases, the electronic cloud shrinks thereby greatly reducing couplings to detrimental external perturbations
- Highly-forbidden laser-accessible OPTICAL transitions
- HCIs can be trapped and cooled
- The 10^{-19} accuracy mark is feasible

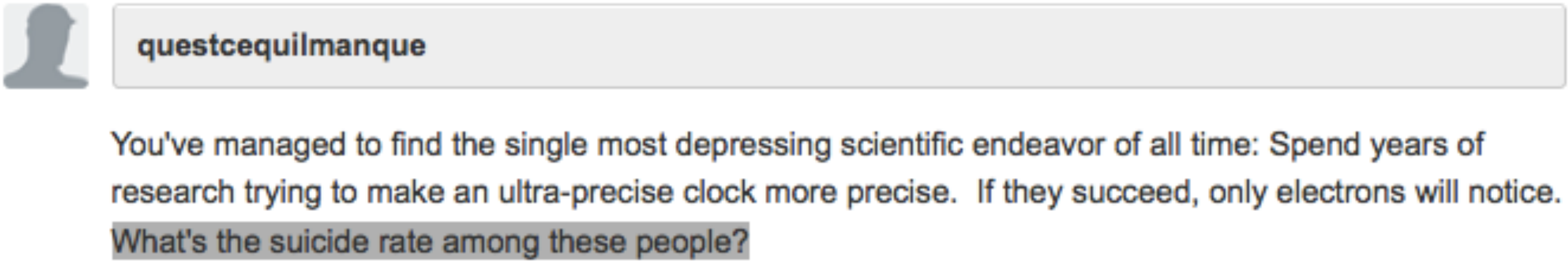
Highly-charged ions as a basis of optical atomic clockwork of exceptional accuracy
A. Derevianko, V. A. Dzuba, V. V. Flambaum, arXiv:1208.3528

WIRED

Laser-Tuned Nuclear Clock Would Be Accurate for Billions of Years

By Adam Mann March 20, 2012 | 5:28 pm | Categories: Physics





questcequilmanque

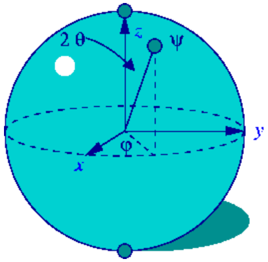
You've managed to find the single most depressing scientific endeavor of all time: Spend years of research trying to make an ultra-precise clock more precise. If they succeed, only electrons will notice. What's the suicide rate among these people?

Why do we need better clocks?

New timepieces will lose only milliseconds over the age of the Universe

GPS on Mars:
Deep-space navigation (DSN network of NASA)

Are constants of nature constant?



Perfect quantum memory (qubits)