Inhomogeneous condensates in the parity doublet model

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Outline

Why?

most works dealing with inhomogeneous condensation do not consider nucleons or vacuum phenomenology

model build with mesonic and baryonic degrees of freedom:

extended linear sigma model and parity doublet model

- includes scalar-, pseudoscalar-, vector- and axialvector-mesons, nucleons and their chiral partners
- decay rates and scattering lengths
- nuclear matter saturation can be achieved

first step towards inhomogeneous condensation $\rightarrow$ chiral density wave (CDW)

The meson fields

globally $U(3)_R \times U(3)_L$ invariant Lagrangian:

scalar and pseudoscalar fields

$$\Phi = \sum_{a=0}^{3} \phi_a t_a = (\sigma + \imath\eta_N)\, t^0 + (\mathbf{a}_0 + \imath\boldsymbol{\pi}) \cdot \mathbf{t} \ , \quad \Phi \to U_L \Phi U_R^\dagger$$

$$\Phi^\dagger = \sum_{a=0}^{3} \phi_a t_a = (\sigma - \imath\eta_N)\, t^0 + (\mathbf{a}_0 - \imath\boldsymbol{\pi}) \cdot \mathbf{t} \ , \quad \Phi^\dagger \to U_R \Phi^\dagger U_L^\dagger$$

vector and axial-vector fields

$$V^\mu = \sum_{a=0}^{3} V_a^\mu t_a = \omega^\mu t^0 + \boldsymbol{\rho}^\mu \cdot \mathbf{t} \ , \quad A^\mu = \sum_{a=0}^{3} A_a^\mu t_a = f_1^\mu t^0 + \mathbf{a}_1^\mu \cdot \mathbf{t}$$

$R^\mu \equiv V^\mu - A^\mu$ with $R^\mu \to U_R R^\mu U_R^\dagger$ and $L^\mu \equiv V^\mu + A^\mu$ with $L^\mu \to U_L L^\mu U_L^\dagger$

The Lagrangian for the mesons

$$\mathscr{L}_M = \mathrm{Tr}\left[(D_\mu\Phi)^\dagger (D^\mu\Phi) - m^2\Phi^\dagger\Phi - \lambda_2\left(\Phi^\dagger\Phi\right)^2\right] - \lambda_1\left[\mathrm{Tr}\left(\Phi^\dagger\Phi\right)\right]^2$$
$$+\mathrm{Tr}\left[H\left(\Phi^\dagger+\Phi\right)\right] + c\left(\det\Phi^\dagger - \det\Phi\right)^2$$
$$-\frac{1}{4}\mathrm{Tr}\left(L_{\mu\nu}L^{\mu\nu} + R_{\mu\nu}R^{\mu\nu}\right) + \mathrm{Tr}\left[\left(\frac{1}{2}m_1^2+\Delta\right)\left(L_\mu L^\mu + R_\mu R^\mu\right)\right]$$
$$+\frac{1}{2}h_1\mathrm{Tr}\left(\Phi^\dagger\Phi\right)\mathrm{Tr}\left(L_\mu L^\mu + R_\mu R^\mu\right) + h_2\mathrm{Tr}\left(\Phi^\dagger L^\mu L_\mu\Phi + \Phi R^\mu R_\mu\Phi^\dagger\right) + 2h_3\mathrm{Tr}\left(\Phi R_\mu\Phi^\dagger L^\mu\right)$$
$$+2h_3\mathrm{Tr}\left(\Phi R_\mu\Phi^\dagger L^\mu\right) - \imath\frac{g_2}{2}\left(\mathrm{Tr}\{L_{\mu\nu}[L^\mu,L^\nu]\} + \mathrm{Tr}\{R_{\mu\nu}[R^\mu,R^\nu]\}\right) + \ldots$$

spontaneous symmetry breaking, explicit symmetry breaking, trace anomaly.

covariant derivative and field strength tensors :

$D^\mu\Phi = \partial^\mu\Phi - \imath g_1(\Phi R^\mu - L^\mu\Phi)$

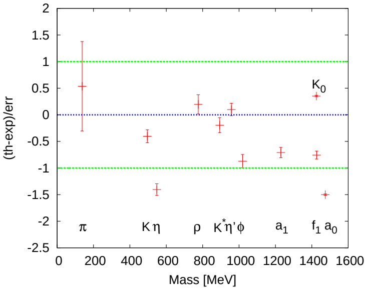
$L^{\mu\nu} = \partial^\mu L^\nu - \imath e A^\mu\left[T_3, L^\nu\right] - \{\partial^\nu L^\mu - \imath e A^\nu\left[T_3, L^\mu\right]\}$

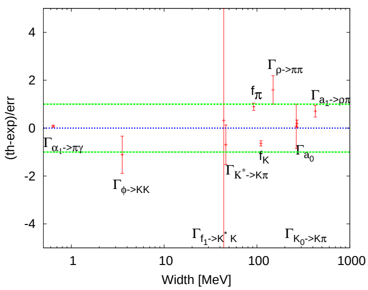
$R^{\mu\nu} = \partial^\mu R^\nu - \imath e A^\mu\left[T_3, R^\nu\right] - \{\partial^\nu R^\mu - \imath e A^\nu\left[T_3, R^\mu\right]\}$

model vs. reality - meson sector

spontaneous chiral-symmetry breaking requires shift of σ_N and σ_S by their expectation values $\rightarrow$ leads to axial-vector and pseudoscalar mixing

11 parameters $\rightarrow \chi^2 = 12.33$ e.g. $\chi^2/(21\text{ observables} - 11\text{ parameters}) = 1.23$





D. Parganlija, P. Kovacs, G. Wolf, F. Giacosa and D. H. Rischke, arXiv:1208.2054 [hep-ph].

The mirror assignment

mirror assignment

$$\begin{aligned} \psi_{1,R} &\to U_R\ \psi_{1,R}\ , \qquad & \psi_{1,L} &\to U_L\ \psi_{1,L}, \\ \psi_{2,R} &\to U_L\ \psi_{2,R}\ , \qquad & \psi_{2,L} &\to U_R\ \psi_{2,L} \end{aligned}$$

baryon Lagrangian

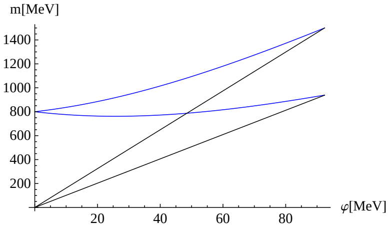
$$\begin{aligned} \mathscr{L}_B = {} & \bar{\psi}_{1,L} \imath \not{D}_{1,L} \psi_{1,L} + \bar{\psi}_{1,R} \imath \not{D}_{1,R} \psi_{1,R} + \bar{\psi}_{2,L} \imath \not{D}_{2,L} \psi_{2,L} + \bar{\psi}_{2,R} \imath \not{D}_{2,R} \psi_{2,R} \\ & - \hat{g}_1 \left(\bar{\psi}_{1,L} \Phi \psi_{1,R} + \bar{\psi}_{1,R} \Phi^\dagger \psi_{1,L} \right) - \hat{g}_2 \left(\bar{\psi}_{2,L} \Phi^\dagger \psi_{2,R} + \bar{\psi}_{2,R} \Phi \psi_{2,L} \right) \\ & + m_0 \left(\bar{\psi}_{2,L} \psi_{1,R} - \bar{\psi}_{2,R} \psi_{1,L} - \bar{\psi}_{1,L} \psi_{2,R} + \bar{\psi}_{1,R} \psi_{2,L} \right) \end{aligned}$$

$D^\mu_{1,R} = \partial^\mu - \imath c_1 R^\mu$, $D^\mu_{1,L} = \partial^\mu - \imath c_1 L^\mu$, $D^\mu_{2,R} = \partial^\mu - \imath c_2 R^\mu$, and $D^\mu_{2,L} = \partial^\mu - \imath c_2 L^\mu$.

arXiv:hep-ph/9805306v1

mirror assignment

not just nucleon N but also its chiral partner N^*
chiral eigenstates are not equal to mass eigenstates
$\Rightarrow$ mass eigenstates emerge after diagonalization



naive assignment ($m_0 = 0$):

$$m_N = m_{\Psi_1} \propto \varphi$$
$$m_{N^*} = m_{\Psi_2} \propto \varphi$$

mirror assignment ($m_0 \neq 0$):

$$m_N = \frac{1}{2}\sqrt{(\hat{g}_1 + \hat{g}_2)^2\,\varphi^2 + 4m_0^2} + \frac{1}{4}(\hat{g}_1 - \hat{g}_2)\,\varphi$$
$$m_{N^*} = \frac{1}{2}\sqrt{(\hat{g}_1 + \hat{g}_2)^2\,\varphi^2 + 4m_0^2} - \frac{1}{4}(\hat{g}_1 - \hat{g}_2)\,\varphi$$

Further extensions and achievements

- origin of m_0 term:
  glueball condensate G_0 or tetraquark condensate χ_0 with
  $m_0 = aG_0 + b\chi_0$
- πN scattering lengths and decay widths of N^*
- at finite density (mean-field) nuclear matter saturation can be achieved

arXiv:1003.4934, arXiv:1105.5003v1, arXiv:0907.5084, arXiv:1103.3238

CDW and implementation

Model is simplified to very basic level:

- mesons other than $\sigma, \vec{\pi}$, ω^μ are ignored
- higher-order interactions of vector mesons are ignored
- m_0 treated as a constant
- for the moment do not aim to describe vacuum phenomenology
- mean-field approximation

minimal baryonic Lagrangian

$$
\begin{aligned}
\mathscr{L} = & \bar{\psi}_1 \imath \partial\!\!\!/ \psi_1 + \bar{\psi}_2 \imath \partial\!\!\!/ \psi_2 - g_\omega^{(1)} \bar{\psi}_1 \imath \gamma_\mu \omega^\mu \psi_1 - g_\omega^{(2)} \bar{\psi}_2 \imath \gamma_\mu \omega^\mu \psi_2 \\
& - \frac{1}{2} \hat{g}_1 \bar{\psi}_1 \left(\sigma + \imath \gamma_5 \vec{\tau} \cdot \vec{\pi}\right) \psi_1 - \frac{1}{2} \hat{g}_2 \bar{\psi}_2 \left(\sigma - \imath \gamma_5 \vec{\tau} \cdot \vec{\pi}\right) \psi_2 \\
& + m_0 \left(\bar{\psi}_2 \gamma_5 \psi_1 - \bar{\psi}_1 \gamma_5 \psi_2\right) + \mathscr{L}_M
\end{aligned}
$$

arXiv:nucl-th/0608044v2

The chiral density wave

Ansatz for chiral density wave:

$$\langle\sigma\rangle \sim \varphi \cos(2\ f\ x)\ , \qquad \langle\pi_0\rangle \sim \varphi \sin(2\ f\ x)$$

$$\begin{aligned}\mathscr{L}_B =& \bar\psi_1 \imath\partial\!\!\!/ \psi_1 + \bar\psi_2 \imath\partial\!\!\!/ \psi_2 + m_0 \left(\bar\psi_2\gamma_5\psi_1 - \bar\psi_1\gamma_5\psi_2\right) \\ & - \frac{1}{2}\hat g_1 \varphi \bar\psi_1 \left[\cos(2fx) + \imath\gamma_5\tau_3 \sin(2fx)\right]\psi_1 - \frac{1}{2}\hat g_2 \bar\psi_2 \left[\cos(2fx) - \imath\gamma_5\tau_3 \sin(2fx)\right]\psi_2 \\ & + \dots \\ =& \bar\psi_1 \imath\partial\!\!\!/ \psi_1 + \bar\psi_2 \imath\partial\!\!\!/ \psi_2 + m_0 \left(\bar\psi_2\gamma_5\psi_1 - \bar\psi_1\gamma_5\psi_2\right) \\ & - \frac{1}{2}\hat g_1 \varphi \bar\psi_1 \exp\left(+\imath 2\gamma_5\tau_3 fx\right)\psi_1 - \frac{1}{2}\hat g_2 \varphi \bar\psi_2 \exp\left(-\imath 2\gamma_5\tau_3 fx\right)\psi_2 \\ & + \dots\end{aligned}$$

recall:

$\exp(\imath a\tau_3) = \cos(a) + \imath\tau_3 \sin(a)$ and $\exp(\imath a\gamma_5\tau_3) = \cos(a) + \imath\gamma_5\tau_3 \sin(a)$

arXiv:1102.4049v1

Towards the grand canonical potential

transformation of the fermion fields

$$\bar{\psi}_1 \to \bar{\psi}_1 \exp[-\imath\gamma_5\tau_3 fx], \quad \psi_1 \to \exp[-\imath\gamma_5\tau_3 fx]\psi_1$$
$$\bar{\psi}_2 \to \bar{\psi}_2 \exp[+\imath\gamma_5\tau_3 fx], \quad \psi_2 \to \exp[+\imath\gamma_5\tau_3 fx]\psi_2$$

- $\bar{\psi}_1 \exp[+\imath\gamma_5\tau_3 2fx]\psi_1 \to \bar{\psi}_1\psi_1, \quad \bar{\psi}_2 \exp[-\imath\gamma_5\tau_3 2fx]\psi_2 \to \bar{\psi}_2\psi_2$
- $\bar{\psi}_1\gamma_\mu\psi_1 \to \bar{\psi}_1\gamma_\mu\psi_1, \quad \bar{\psi}_2\gamma_\mu\psi_2 \to \bar{\psi}_2\gamma_\mu\psi_2$
- $\bar{\psi}_2\gamma_5\psi_1 \to \bar{\psi}_2\gamma_5\psi_1, \quad \bar{\psi}_1\gamma_5\psi_2 \to \bar{\psi}_1\gamma_5\psi_2$
- $\bar{\psi}_1\imath\partial\!\!\!/\psi_1 \to \bar{\psi}_1\imath\partial\!\!\!/\psi_1 + \bar{\psi}_1\gamma_1\gamma_5\tau_3 f\psi_1, \quad \bar{\psi}_2\imath\partial\!\!\!/\psi_2 \to \bar{\psi}_2\imath\partial\!\!\!/\psi_2 - \bar{\psi}_2\gamma_1\gamma_5\tau_3 f\psi_2$

$\Rightarrow$ spacial dependence transformed to an additional momentum dependence

The mesonic Lagrangian

$$\mathscr{L}_M = \frac{1}{2}\partial_\mu\sigma\partial^\mu\sigma + \frac{1}{2}\partial_\mu\vec{\pi}\partial^\mu\vec{\pi} - \frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \frac{1}{2}m^2(\sigma^2+\vec{\pi}^2) - \frac{\lambda}{4}(\sigma^2+\vec{\pi}^2)^2 + h_0\sigma + \frac{1}{2}m_\omega^2\omega_\mu\omega^\mu$$

$F_{\mu\nu} = \partial_\mu\omega_\nu - \partial_\nu\omega_\mu$
within mean-field approximation:

$$F_{\mu\nu}F^{\mu\nu} \to 0$$
$$\omega_\mu\omega^\mu \to \bar{\omega}_0^2$$
$$\sigma^2 + \vec{\pi}^2 \to \varphi^2$$
$$\partial_\mu\sigma\partial^\mu\sigma + \partial_\mu\vec{\pi}\partial^\mu\vec{\pi} \to 4f^2\varphi^2$$

$$V_M = 2f^2\varphi^2 + \frac{1}{4}\lambda\varphi^4 - \frac{1}{2}m^2\varphi^2 - h_0\varphi - \frac{1}{2}m_\omega^2\bar{\omega}_0^2$$

Grand canonical potential

$$\frac{\Omega}{V} = 2f^2\varphi^2 + \frac{1}{4}\lambda\varphi^4 - \frac{1}{2}m^2\varphi^2 - \epsilon\varphi - \frac{1}{2}m_\omega^2\bar{\omega}_0^2$$
$$+ \sum_{k=1}^{4} \frac{2}{(2\pi)^2} \int d^3p \left(\sqrt{\vec{p}^2 + \bar{m}_k(p_1)^2} - \mu^*\right) \Theta \left(\mu^* - \sqrt{\vec{p}^2 + \bar{m}_k(p_1)^2}\right)$$

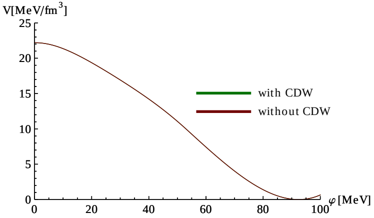
short notation $\mu^* = \mu - g_\omega\bar{\omega}_0$

mean meson fields are obtained by minimizing Ω

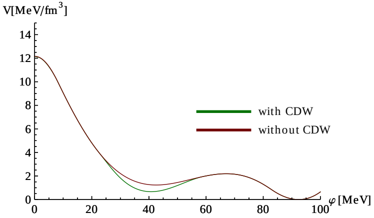
$$0 \stackrel{!}{=} \frac{\partial(\Omega/V)}{\partial\varphi},\ 0 \stackrel{!}{=} \frac{\partial(\Omega/V)}{\partial\bar{\omega}_0},\ 0 \stackrel{!}{=} \frac{\partial(\Omega/V)}{\partial f}$$

Potential in the chiral limit

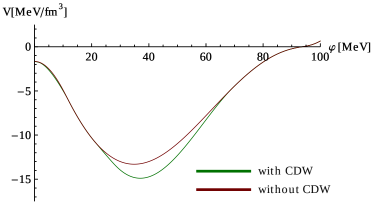
$\mu_B = 800$ MeV, vacuum



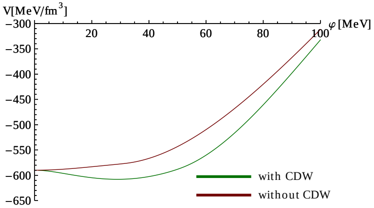
$\mu_B = 900$ MeV, vacuum



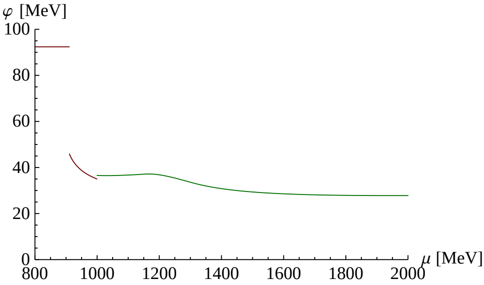
$\mu_B = 950$ MeV



$\mu_B = 1500$ MeV



Condensate φ at finite μ_B and finite m_π

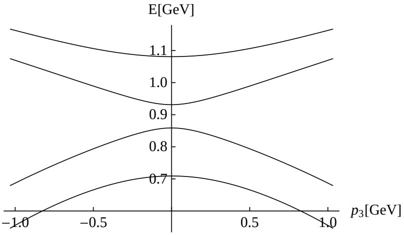


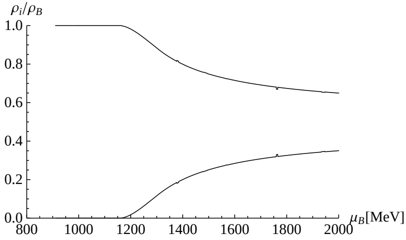
red line: homogeneous condensation

green line: inhomogeneous condensation

- homogeneous nuclear matter ground state is possible
- for moderate μ_B crystalline phase is realized
- chiral symmetry is not restored, indeed value increases with μ_B
- increase of g_ω: inhomogeneous phase is realized for higher μ_B

Dispersion relation and relative density





$\mu_B = 1000$ MeV, $\varphi = 36.6$ MeV, $\bar{\omega}_0 = 30.9$ MeV,

$f = 183.7$ MeV, and $p_1 = p_2 = 0$

- $E_k = \sqrt{\vec{p}^2 + \bar{m}_k(p_3)^2}$, $k = 1 \ldots 4$
- shape remains similar even for high μ_B

Summary and Outlook

- parity doublet model favors inhomogeneous condensation
- crystalline phase has a strong parameter dependence
- chiral symmetry will not be restored for asymptotically large μ_B

- extend to more realistic setup
- calculations beyond mean-field
- test further inhomogeneous realizations beside CDW
- ...