Thermalization, Bose Condensation and the Glasma

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Hirschegg, Austria

Outline:

The sQGP Paradigm and a little Heavy Ion Lore

Saturation and Coherent Fields in the Hadron Wavefunction

The Formation of the Glasma

Early Time Evolution of the Glasma

Thermalization and Transport

Implications

RHIC and LHC Data on heavy Ions

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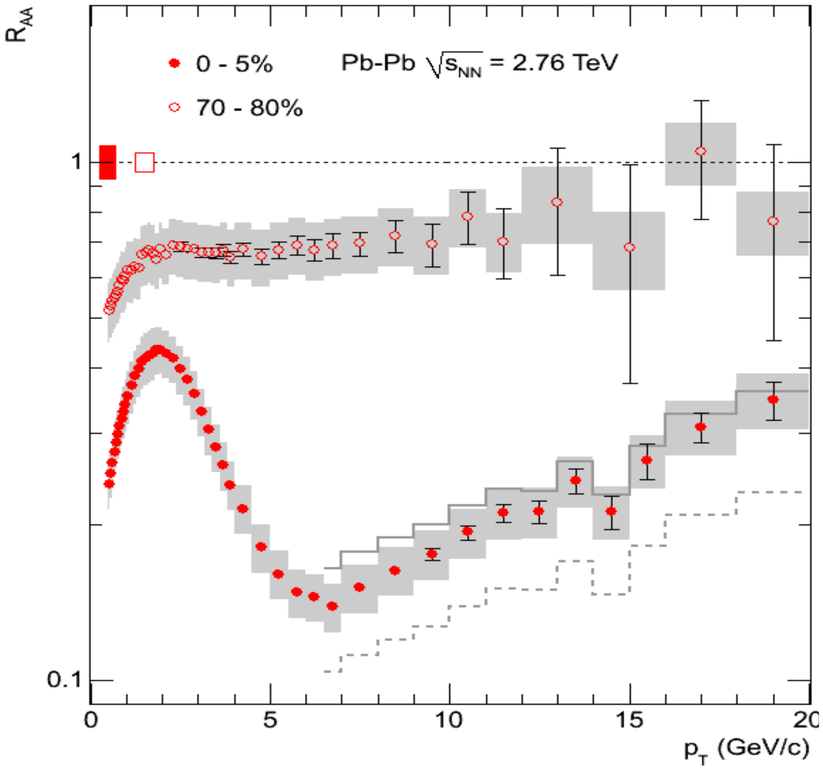
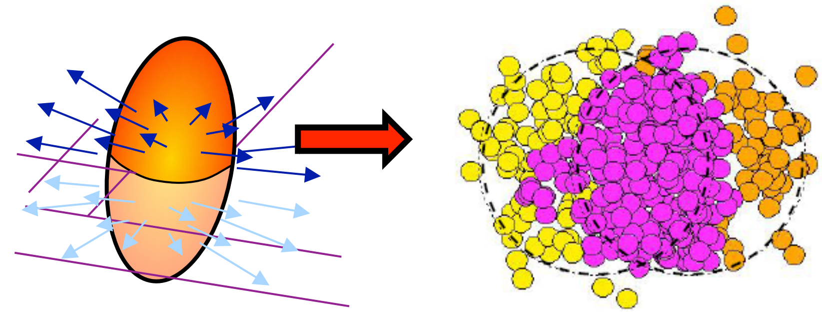
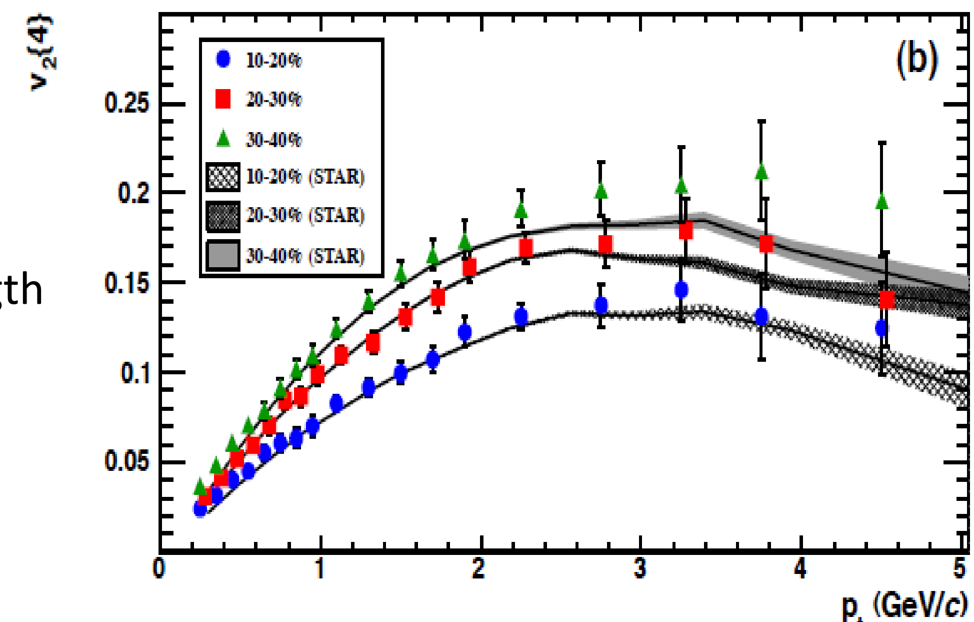


“Jet quenching” => Large Energy Loss /Length

Very Strong Collective Flow Patterns
Consistent with Perfect Fluid
Hydrodynamics
Small Viscosity to Entropy

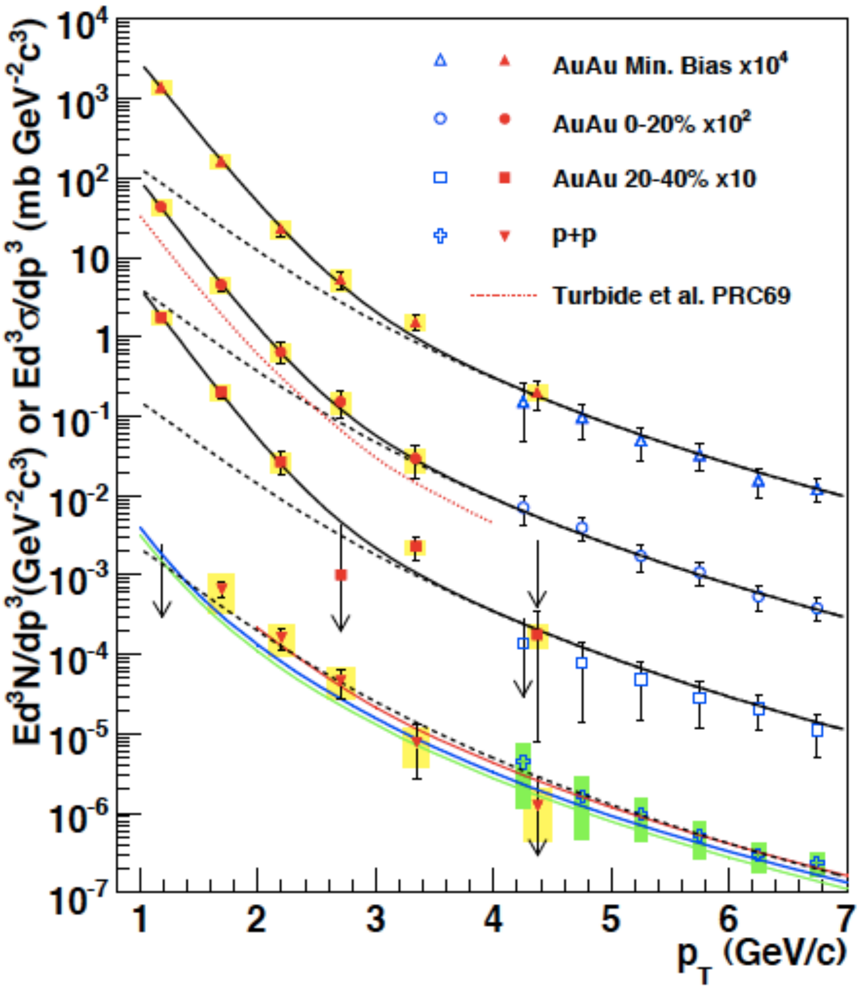




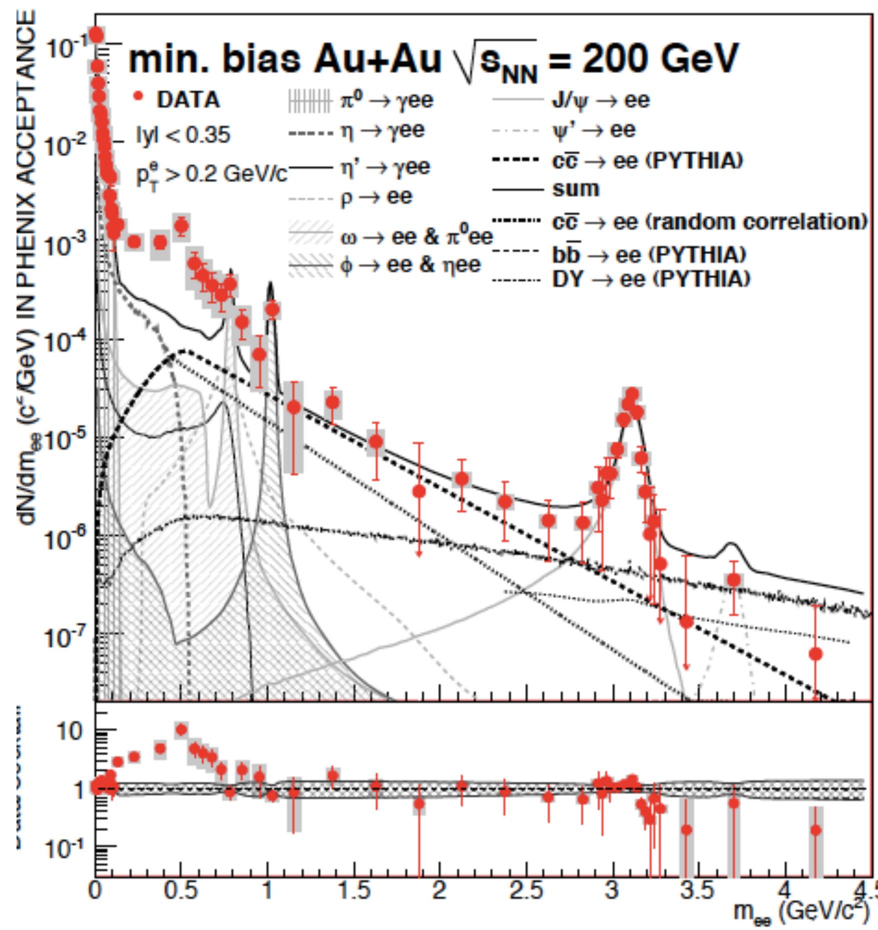
Strongly Interacting QGP Paradigm:
Usual Claim:
A well thermalized Quark Gluon Plasma?

Photons and Dileptons

QGP & Hydro:
Magnitude and shape
Flow not explained



QGP & Hydro
Magnitude and shape not explained
Dilepton have p_T slope of 100 MeV
No agreement on data



Saturation and Coherent Fields:

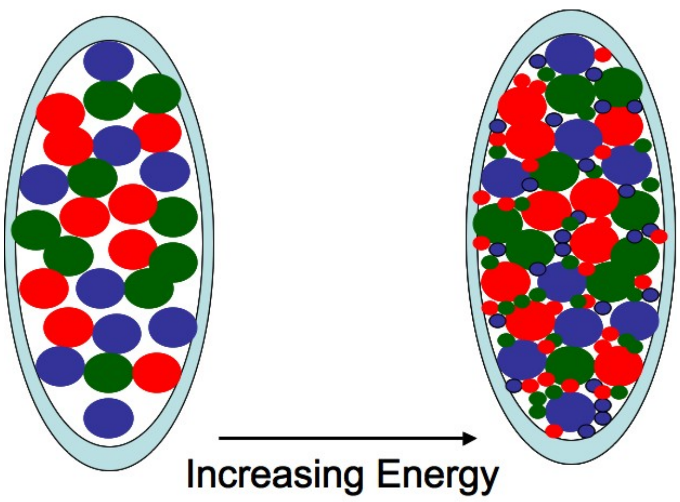
Color Glass Condensate:

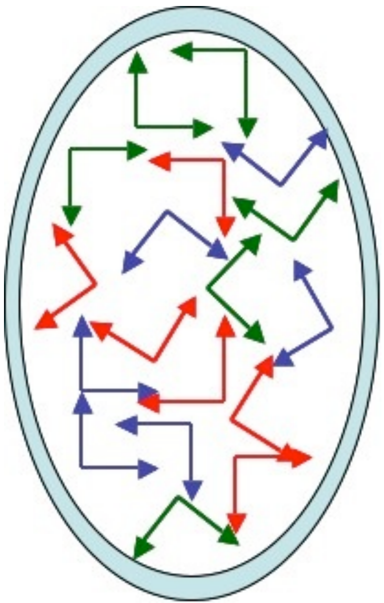
High density gluon fields:

$$\frac{dN}{dy\,d^2r_T\,d^2p_T} \sim \frac{1}{\alpha_s}$$

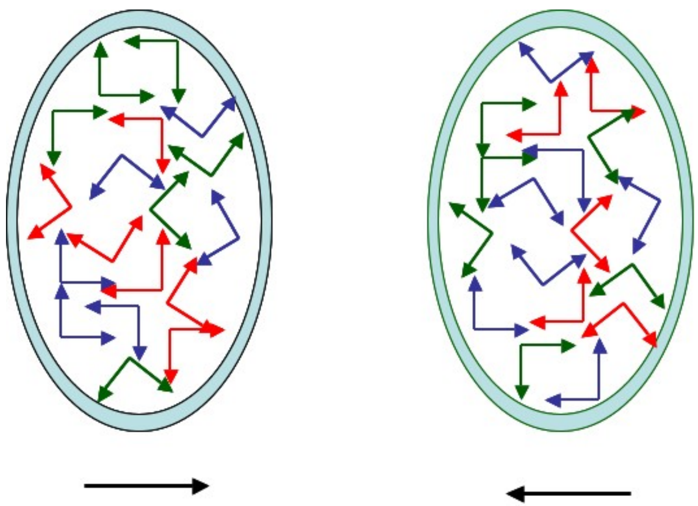
Parton distributions replaced by ensemble of coherent classical fields
Renormalization group equations for sources of these fields

$$Q^2_{sat} >> \Lambda^2_{QCD}$$

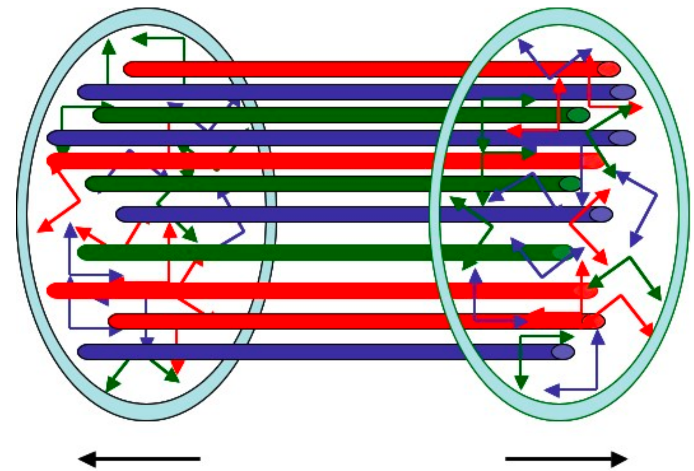




Collisions of two sheets of colored glass



Sheets get dusted with color electric
and color magnetic fields



The initial conditions for a Glasma evolve classically and the classical fields radiate into gluons
Longitudinal momentum is red shifted to zero by longitudinal expansion

But the classical equations are chaotic:

Small deviations grow exponentially in time

Chaos and Turbulence:

CGC field is rapidity independent => occupies restricted range of phase space
Wiggling strings have much bigger classical phase space
A small perturbation that has longitudinal noise grows exponentially

$$A_{classical} \sim 1/g$$

$$A_{quantum} \sim 1$$

After a time

$$t \sim \frac{ln^p(1/g)}{Q_{sat}}$$

system isotropizes,

But it has not thermalized!

Thermalization naively occurs when scattering times are small compared to expansion times. Scattering is characterized by a small interaction strength.

How can the system possibly thermalize, or even strongly interact with itself?

Initial distribution:

$$\frac{dN}{d^3x d^3p} \sim \frac{Q_{sat}}{\alpha_s E} \; F(E/Q_{sat})$$

A thermal distribution would be:

$$\frac{dN}{d^3x d^3p} \sim \frac{1}{e^{E/T} - 1} \sim T/E$$

Only the low momentum parts of the Bose-Einstein distribution remain

$$E \sim Q_{sat}$$

$$\text{“}T \sim Q_{sat}/\alpha_s\text{”}$$

As dynamics migrates to UV, how do we maintain isotropy driven by infrared modes with a scale of the saturation momentum?

Phase space is initially over-occupied

$$f_{thermal} = \frac{1}{e^{(E-\mu)/T} - 1}$$

Chemical potential is at maximum the particle mass

$$\rho_{max} \sim T^3 \qquad \epsilon_{max} \sim T^4$$

$$\rho_{max}/\epsilon_{max}^{3/4} \leq C$$

But for isotropic Glasma distribution

$$\rho_{max}/\epsilon_{max}^{3/4} \leq 1/\alpha_S^{1/4}$$

Where do the particle gluons go?

If inelastic collisions were unimportant, then as the system thermalized, the ratio of the energy density and number density are conserved

$$f_{thermal} = \rho_{cond}\delta^3(p) + \frac{1}{e^{(E-m)/T} - 1}$$

One would form a Bose-Einstein Condensate

Over-occupied phase space => Field coherence in important
Interactions can be much stronger that g^2

$$N_{coh} g^2$$

Might this be at the heart of the large amount of jet quenching, and strong flow patterns seen at RHIC?

Problem we try to solve:

How does the system evolve from an early time over-occupied distribution to a thermalized distribution

We argue that the system stays strongly interacting with itself during this time due to coherence

First: Kinetic Evolution Dominated by Elastic Collisions in a Non-Expanding Glasma

$$\partial_t f(p,X) = C_p[f]$$

Blaizot, Gelis, Liao, LM, Venugopalan

$$f(p,X) = \frac{\Lambda_s(t)}{\alpha_s p} g(p/\Lambda(t))$$

$$\Lambda_s(t_i) \sim \Lambda(t_i) \sim Q_{sat}$$

Small angle approximation for transport equation:

$$\left.\frac{\partial f}{\partial t}\right|_{\mathrm{coll}} \sim \frac{\Lambda_s^2 \Lambda}{p^2} \partial_p \left\{ p^2 \left[\frac{df}{dp} + \frac{\alpha_s}{\Lambda_s} f(p)(1+f(p)) \right] \right\}$$

$$\frac{\Lambda\Lambda_s}{\alpha_s} \equiv -\int_0^\infty dp\, p^2 \frac{df}{dp}$$

$$\frac{\Lambda\Lambda_s^2}{\alpha_s^2} \equiv \int_0^\infty dp\, p^2 f(1+f)$$

Due to coherence, the collision equation is independent of coupling strength! Equation describes a strongly interacting but weakly coupled QGP

There is a fixed point of this equation corresponding to thermal equilibrium when

$$T \sim \Lambda \sim \Lambda_s/\alpha_s$$

Estimates of various quantities
(Momentum integrations are all dominated by the hard scale)

$$n_{\rm g} \sim \frac{1}{\alpha_{\rm s}} \Lambda^2 \Lambda_s \qquad \epsilon_{\rm g} \sim \frac{1}{\alpha_{\rm s}} \Lambda_{\rm s} \Lambda^3 \qquad \frac{\epsilon_g}{n_g} \sim \Lambda$$

$$n = n_{\rm c} + n_{\rm g} \qquad \epsilon_{\rm c} \sim n_{\rm c}\, m \sim n_c \sqrt{\Lambda \Lambda_s}$$

$$m^2 \sim \alpha_{\rm s} \int dp\, p^2 \frac{df(p)}{d\omega_p} \sim \Lambda \Lambda_{\rm s}$$

The collision time follows from the structure of the transport equation and is

$$t_{scat} = \frac{\Lambda}{\Lambda_s^2}$$

The scattering time is independent of the interaction strength

Thermalization in a non-expanding box

$$\epsilon = \Lambda_s \Lambda^3 \sim constant$$

$$t_{scat} = \frac{\Lambda}{\Lambda_s^2} \sim t$$

So that

$$\Lambda_s \sim Q_s \left(\frac{t_0}{t}\right)^{\frac{3}{7}} \qquad \Lambda \sim Q_s \left(\frac{t}{t_0}\right)^{\frac{1}{7}}$$

$$n_g \sim n_0 \left(\frac{t_o}{t}\right)^{1/7} \qquad m \sim Q_s (t_0/t)^{1/7} \qquad \frac{\epsilon_c}{\epsilon_g} \sim \left(\frac{t_0}{t}\right)^{1/7}$$

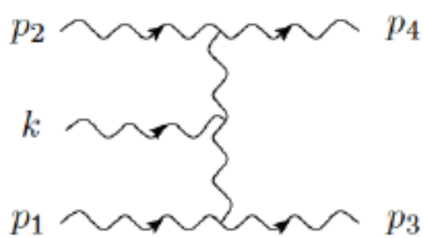
$$s \sim \Lambda^3 \sim Q_s^3 (t/t_0)^{3/7}$$

At thermalization $\Lambda_s = \alpha_s \Lambda$

$$t_{th} \sim \frac{1}{Q_s} \left(\frac{1}{\alpha_s}\right)^{\frac{7}{4}} \qquad s \sim Q_s^3 / \alpha_s^{3/4} \sim T^3$$

How do inelastic processes change this?

Rates of inelastic and elastic processes are parametrically the same



$$\frac{1}{t_{scat}} \sim \alpha_s^{n+m-2} \left(\frac{\Lambda_s}{\alpha_s}\right)^{n+m-2} \left(\frac{1}{m^2}\right)^{n+m-4} \Lambda^{n+m-5}$$

$$m^2 \sim \Lambda_s \Lambda$$

$$t_{\rm scat} = \frac{\Lambda}{\Lambda_{\rm s}^2},$$

What about the condensate? Difficult to make definite statement.
In relaxation time limit, we would expect:

$$\frac{d}{dt}\rho_{cond} = -\frac{a}{t_{scat}}\rho_{cond} + \frac{b}{t_{scat}} n_{gluons}$$

Either $\rho_{cond} >> n_{gl}$ or $\rho_{cond} = \frac{b}{a} n_{gluons}$

Condensate would rapidly evaporate near thermalization time

Effect of Longitudinal Expansion

$$\partial_t f - \frac{p_z}{t}\partial_{p_z} f = \left.\frac{df}{dt}\right|_{p_z t} = C[f] \qquad \partial_t \epsilon + \frac{\epsilon + P_L}{t} = 0$$

Assume approximate isotropy restored by scattering.
Will check later that this is consistent.

$$P_L = \delta\epsilon \qquad 0 < \delta < 1/3$$

$$\epsilon_g(t) \sim \epsilon(t_0)\left(\frac{t_0}{t}\right)^{1+\delta} \qquad \Lambda_s \sim Q_s\left(\frac{t_0}{t}\right)^{(4+\delta)/7}, \qquad \Lambda \sim Q_s\left(\frac{t_0}{t}\right)^{(1+2\delta)/7}$$

$$\left(\frac{t_{\rm th}}{t_0}\right) \sim \left(\frac{1}{\alpha_s}\right)^{\frac{7}{3-\delta}}$$

The asymmetry parameter:

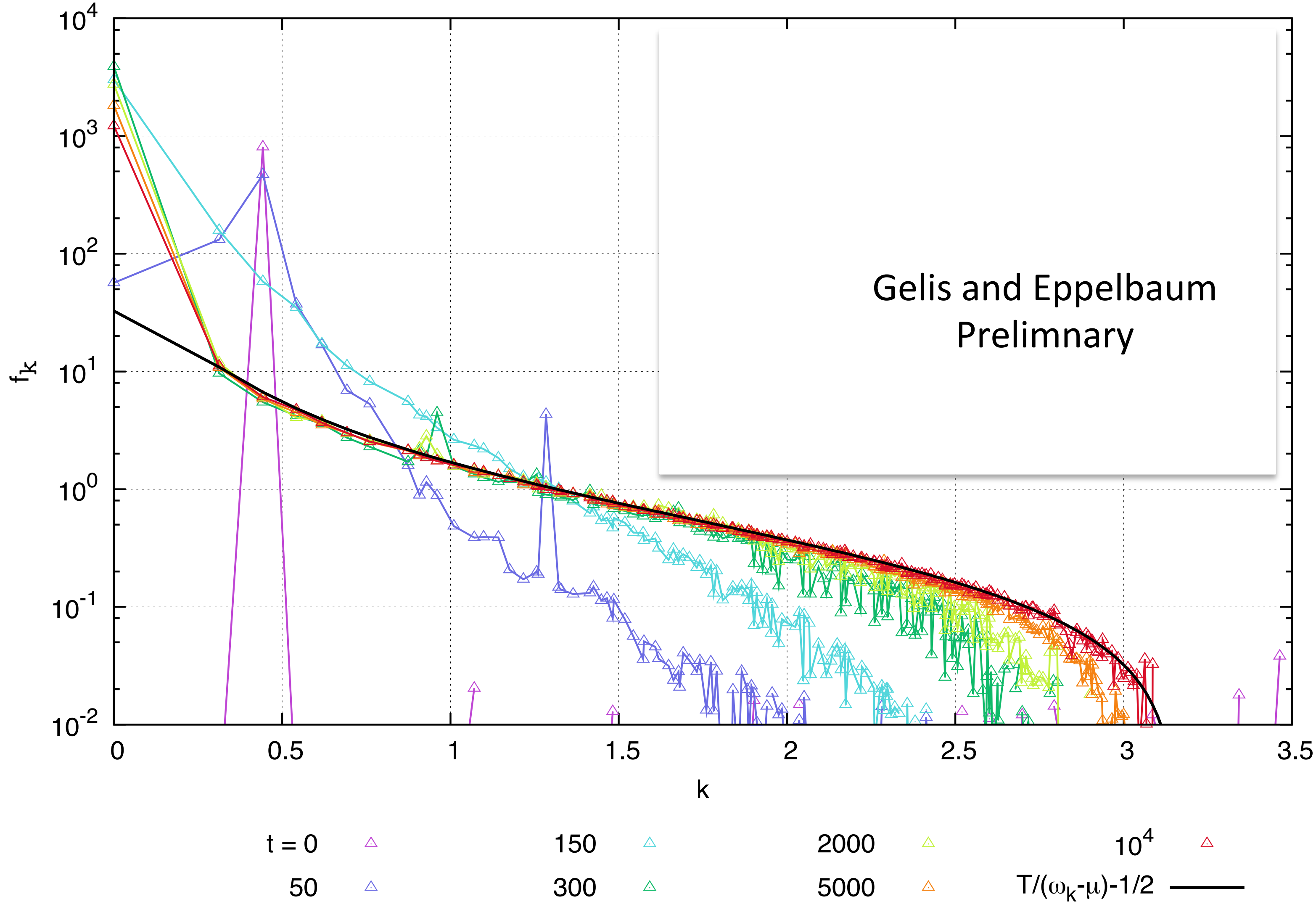
Take moments of transport equation

$$< p_x^2 > \qquad < p_T^2 >$$

Prove that the solutions for these moments reduce to constants times the ultraviolet scale at large times

$$< p_x^2 > / < p_T^2 > \sim constant$$

Need to know solutions to transport equation to determine the value of the anisotropy parameter



10^4
10^3
10^2
10^1
10^0
10^{-1}
10^{-2}
f_k
0
0.5
1
1.5
2
2.5
3
3.5
k
Gelis and Eppelbaum
Prelimnary
t = 0
50
150
300
2000
5000
10^4
$T/(\omega_k-\mu)-1/2$

Numerical Studies of the Transport Equation:

$$\left.\frac{\partial f}{\partial t}\right|_{\mathrm{coll}} \sim \frac{\Lambda_s^2 \Lambda}{p^2} \partial_p \left\{ p^2 \left[\frac{df}{dp} + \frac{\alpha_s}{\Lambda_s} f(p)(1 + f(p)) \right] \right\}$$

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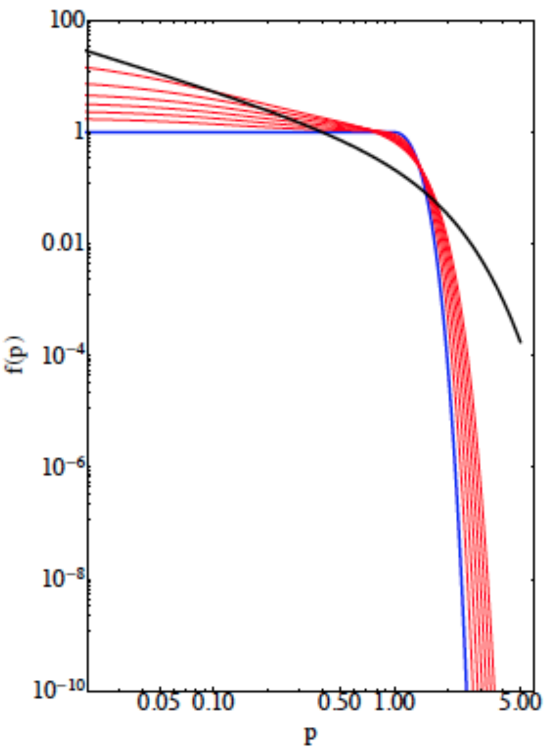
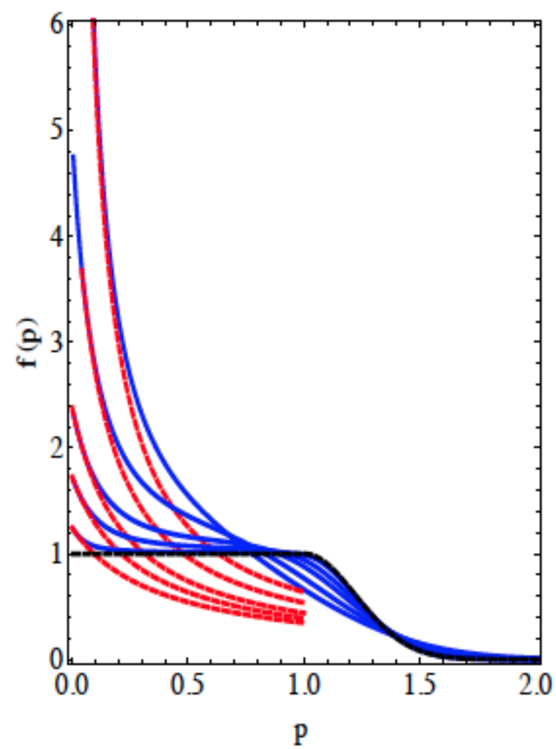
Derived from small angle approximation to Uehling-Uhlenbeck Equation





$$f = \frac{T^*}{p - \mu^*} \quad (\mu^* < 0)$$

Underpopulated:
Infrared rapidly to classical thermal distribution with chemical potential

Overpopulated:
Rapidly develops a delta function singularity

Expanding and nonexpanding boxes

Evolution of the condensate:

Look at contributions to Uehling-Uhlenbeck equation involving condensate:

Modification of regular part of transport equation:

$$D_t f(p) = \xi' ln(M_D) \vec{\nabla} \cdot \left\{ I_a \vec{\nabla} f + \hat{p} I_b f(1+f) \right\}$$
$$+\chi' \frac{n_c}{M_D} \left\{ I_c \partial_p^2 f + ln(M_d)(2f+1)\partial_p f \right\}$$

Explicit equation for the condensate:

$$D_t n_c = \frac{n_c}{M_D} \frac{\chi}{(2\pi)} \left\{ ln(M_D) I_d - I_c I_e \right\} + S$$

S comes from regular scattering terms, and I's are integrals over f's and their derivatives. The sign for growth or decay of condensate depend upon under or over occupation of the distribution

Can in principal see the appearance and growth of condensate within the Uehling-Uhlenbeck equation and study qualitative features

How to think about condensate formation:

Work in time like axial gauge:

$$A^0 = 0$$

$$\delta A^i = A^i(t) - A^i(t=0)$$

Under time independent residual gauge transformations:

$$\delta A_i(x) \rightarrow U(x)\delta A_i(x)U^\dagger(x)$$

$$tr(\delta A_i \delta A_i)$$

Is analogous to Higgs term in electroweak action,
except it is an adjoint field that can condense

In Higgs model, adding a chemical potential for charge modifies the i-epsilon prescription and the static effective potential

$$(M^2 - \mu^2) \mid \phi \mid^2 + \lambda \mid \phi \mid^4$$

For positive mass squared for large enough density, a condensate is generated. In absence of vector fields, a Goldstone boson and a massive scalar (except when mass and chemical potential are equal) are generated. If there is a vector present, it eats the Goldstone boson and all modes become massive.

In non-abelian Higgs model for Higgs in fundamental. Same story as Abelian Higgs model. If adjoint, then there are residual unbroken U(1) symmetries. This results in topological excitation, for SU(2), magnetic monopoles.

?? If there is Bose condensation in QCD, is it associated with formation of color magnetic monopoles as, in analogy with vortex formation in Bose condensates for atomic gas condensate??

Phenomenological Consequences:

Flow is more easily generated since system is anisotropic and also generates flow at earliest times

Jet quenching at early times should be stronger due to coherence of media

Will get more flow for photons than hydrodynamic simulations, but enough?

$$\frac{dN}{dydM^2} \sim \pi R^2 \left(\frac{Q^2_{sat}}{M^2} \right)^p \qquad p \sim 3-4$$

Geometric scaling of photon and di-lepton distributions lead to huge enhancements

The above do not require Bose condensation, only coherence

Contribution to di-lepton spectrum from gluons annihilating in condensate to virtual quark loop. Not suppressed by powers of interaction due to coherence. Di-lepton pair will have very small transverse momentum because condensate gluons have very small transverse momentum.

The last requires a condensate

Issues I would like to understand:

Is the sQGP strongly interacting but weakly coupled or is the intrinsic coupling strength large?

Is the sQGP for some part of its existence a well thermalized QGP, or is it a strongly interacting Glasma?

Can we determine from experiment the degree of anisotropy, P_L vs P_T?

If there is Bose condensation, how do we imagine the degrees of freedom of the condensate? Are their topological excitation (analogs of vortices) associated with such condensation?