Effects of baryon number fluctuation around QCD critical point

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working with
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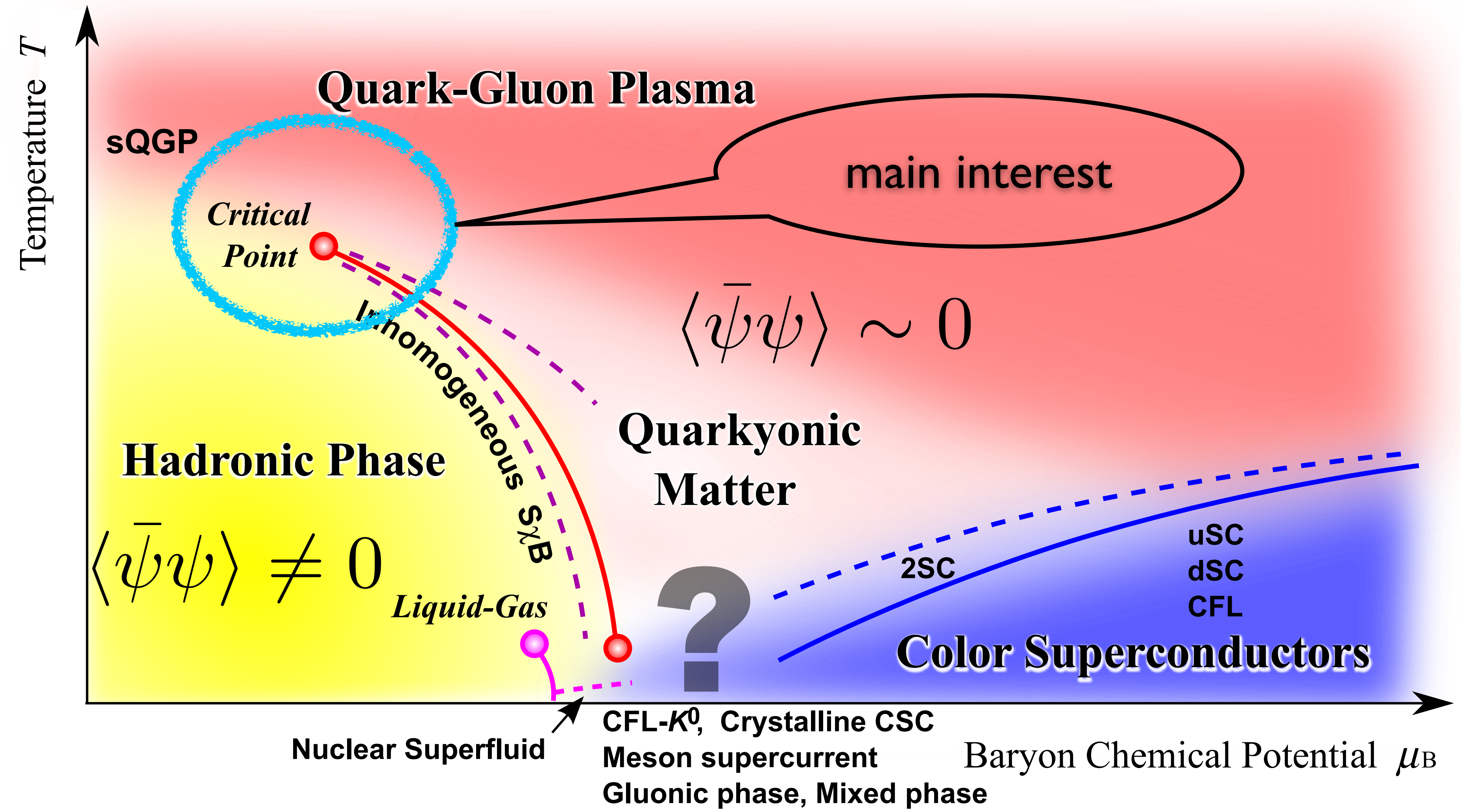
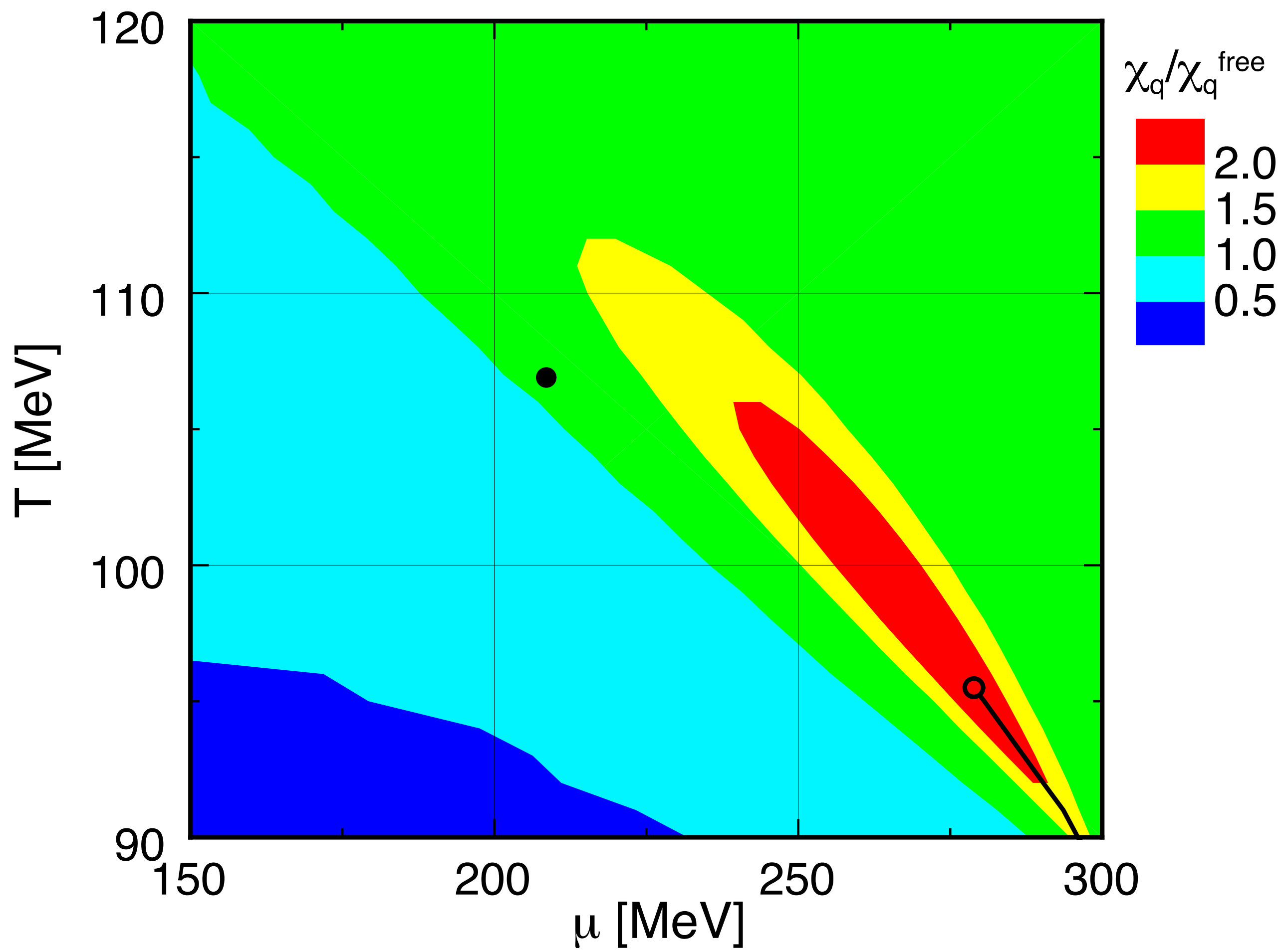
QCD phase diagram



Fukushima and Hatsuda(2010)

- Chiral phase transition
- Order parameter $\langle\bar{\psi}\psi\rangle$
- 1st order boundary and its end point (2nd order)

Critical region



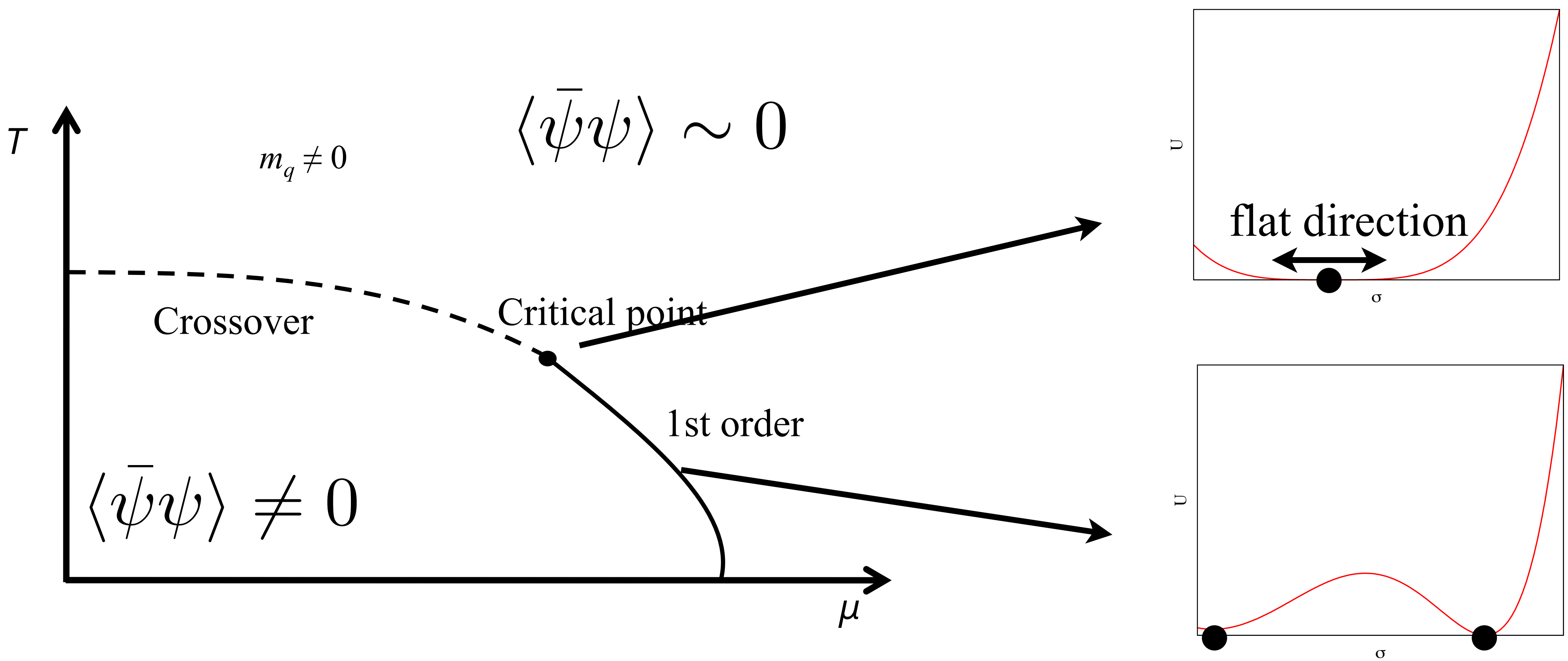
$$\chi_q = \frac{\partial n_b}{\partial \mu} = \frac{\partial^2 P}{\partial \mu^2}, \quad R_q = \chi_q/\chi_q^{\text{free}}$$

Hatta and Ikeda (2003)

- Critical point is not point like.
- Susceptibility is enhanced near the critical point.
- We need to evaluate the size of the critical region.

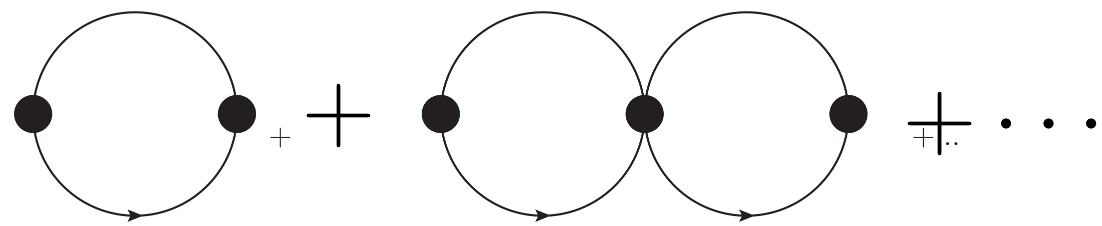
Effective potential

$$U(\sigma) = a(T,\mu)\sigma^2 + b(T,\mu)\sigma^4 + c(T,\mu)\sigma^6 - d(T,\mu)\sigma + \cdots$$

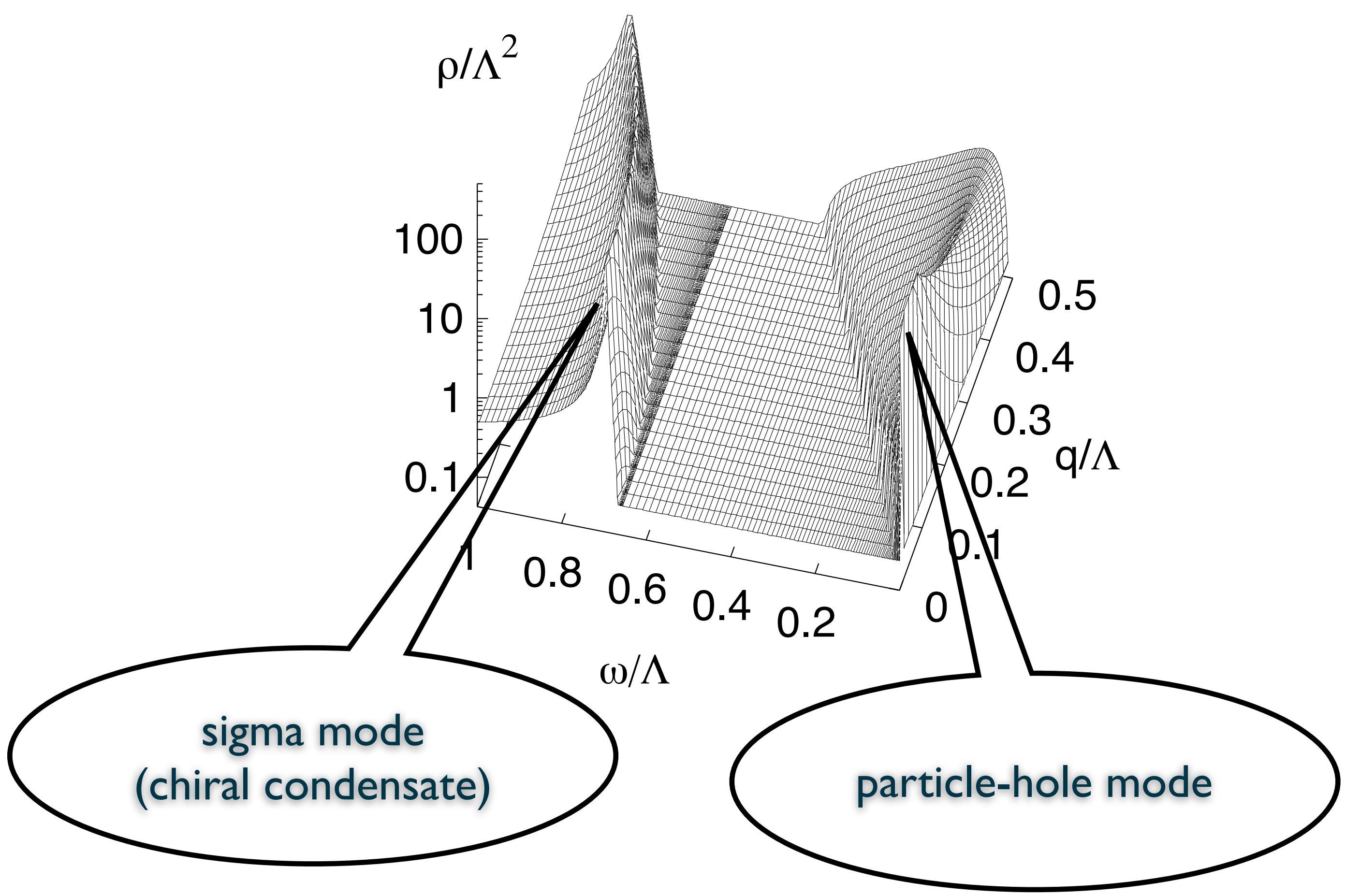


- Effective potential as a function of order parameter $\sigma \sim \bar{\psi}\psi$
- A Soft mode accompanying the CP is sigma mode?

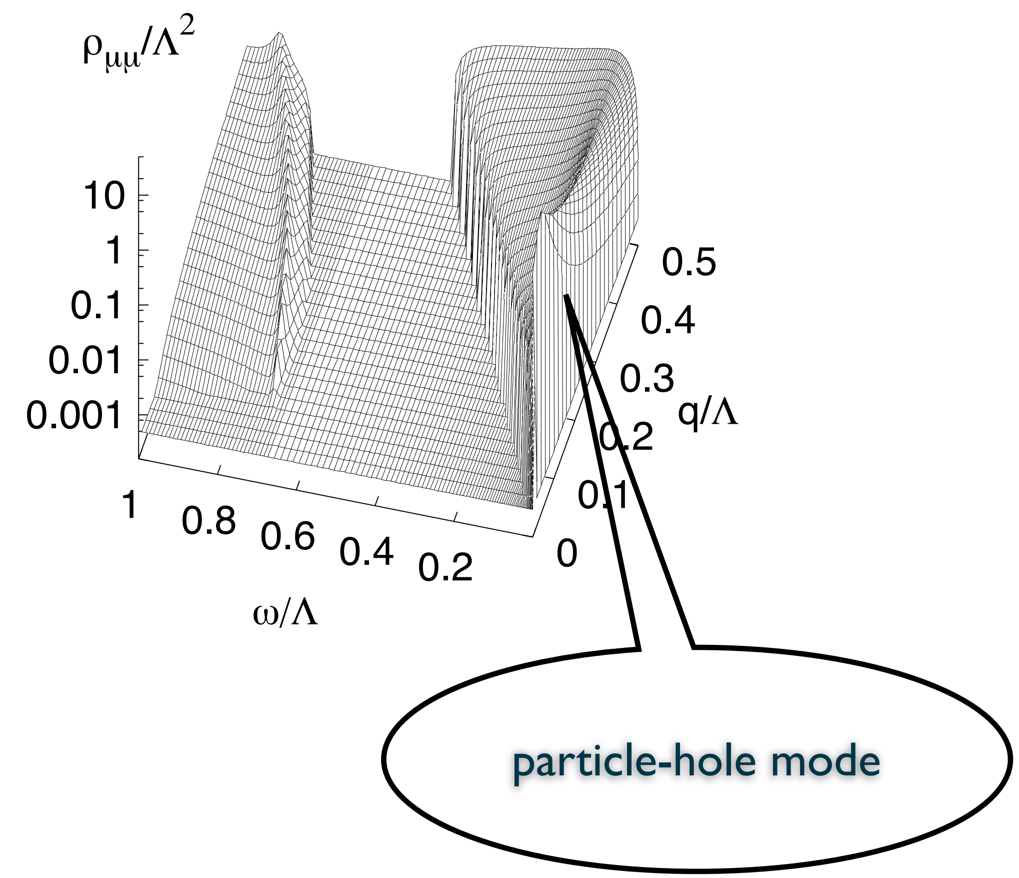
Spectral functions near CP

$\langle(\bar{\psi}O\psi)(\bar{\psi}O\psi)\rangle \sim$  (RPA)

$\Big\langle(\bar{\psi}\psi)(\bar{\psi}\psi)\Big\rangle$



$\Big\langle(\bar{\psi}\gamma_0\psi)(\bar{\psi}\gamma_0\psi)\Big\rangle$

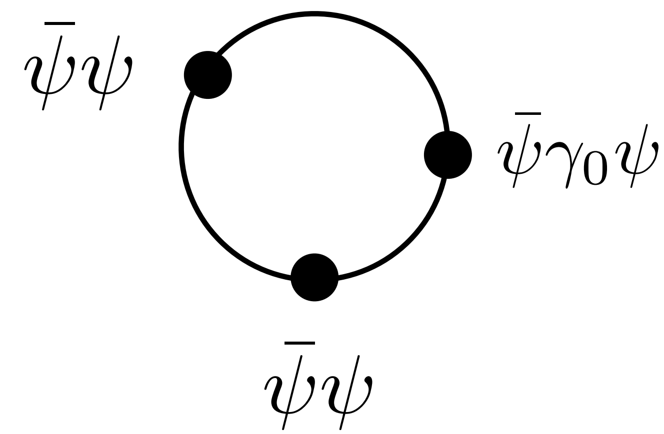


H. Fujii, M. Ohtani Phys.Rev. D70 (2004) 014016

- The soft mode is a linear combination of sigma and baryon-number density (particle-hole mode).

Coupling with baryon density

$$\mu \neq 0$$



$$U = a\sigma^2 + b\sigma^4 + c\sigma^6 - d\sigma + \boxed{n_b\sigma^2}$$

H. Fujii, M. Ohtani Phys.Rev. D70 (2004) 014016

D. Son and M. Stephanov, Phys. Rev. D 70, 056001 (2004).

- A coupling between chiral condensate and quark-number density is essential.
- We have to evaluate the thermodynamic potential with the mixing.

Effective model near CP

$$\mathcal{L} = \bar{\psi}[i\partial\!\!\!/ - g_s(\sigma + i\gamma_5\vec{\tau}\cdot\vec{\pi}) + g_d\varphi\gamma_0]\psi + \frac{1}{2}(\partial_\mu\sigma)^2 + \frac{1}{2}(\partial_\mu\vec{\pi})^2 + \frac{1}{2}(\partial_\mu\varphi)^2$$
$$- a(\sigma^2 + \vec{\pi}^2) - b(\sigma^2 + \vec{\pi}^2)^2 - \frac{m_\varphi}{2}\varphi^2 + c\sigma$$

- We introduce new filed φ (baryon-number density with appropriate normalization).
- g_d is density coupling which is familiar in Walecka model (σ-ω model).
- quark-quark interaction is attractive.

Parameters

$$\mathcal{L} = \bar{\psi}[i\partial\!\!\!/ - g_s(\sigma + i\gamma_5\vec{\tau}\cdot\vec{\pi}) + g_d\varphi\gamma_0]\psi + \frac{1}{2}(\partial_\mu\sigma)^2 + \frac{1}{2}(\partial_\mu\vec{\pi})^2 + \frac{1}{2}(\partial_\mu\varphi)^2$$
$$- a(\sigma^2 + \vec{\pi}^2) - b(\sigma^2 + \vec{\pi}^2)^2 - \frac{m_\varphi}{2}\varphi^2 + c\sigma$$

- a, b,c and g_s are fixed by vacuum physical value such as m_π, f_π, Mσ~600 [MeV] and Mq~300[MeV].
- We fix M_φ and vary g_d.
- A ratio (g_d/M_φ) controls a strength of the mixing.

Functional-RG

$$k\partial_k \Gamma_k[\varphi] = \frac{1}{2}\mathrm{Tr}\left[\frac{k\partial_k R_{kB}}{R_{kB}+\Gamma_k^{(0,2)}[\varphi]}\right] - \mathrm{Tr}\left[\frac{k\partial_k R_{kF}}{R_{kF}+\Gamma_k^{(2,0)}[\varphi]}\right]$$

C. Wetterich, Phys. Lett. B301, 90 (1993)

$\Gamma_k[\phi]$: effective potential at scale k $\qquad S[\phi] + \frac{1}{2}R_k\phi^2$

$$R_k(p) \sim k^2 \text{ for } \mathrm{p}^2 << \mathrm{k}^2$$

$$R_k(p) \sim 0 \text{ for } \mathrm{p}^2 >> \mathrm{k}^2$$

- R_k prevents the propagation of the mode q < k.

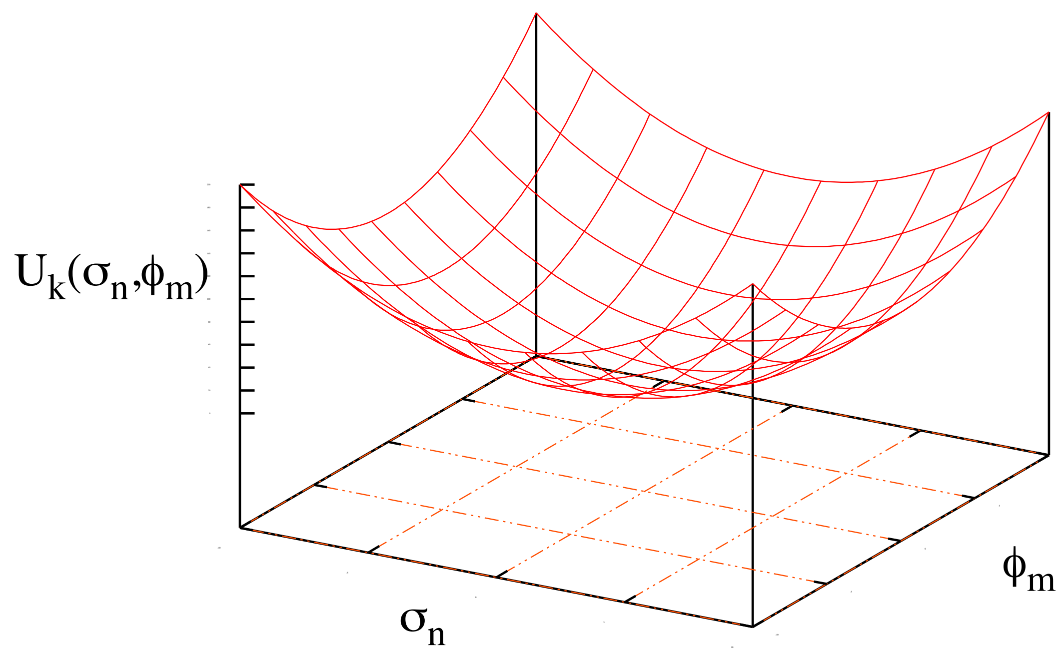
$$\Gamma_{k=\Lambda}[\phi] = S[\phi] \qquad \text{classical}$$

$$\downarrow$$

$$\Gamma_{k=0}[\phi] = \Gamma[\phi] \qquad \text{quantum}$$

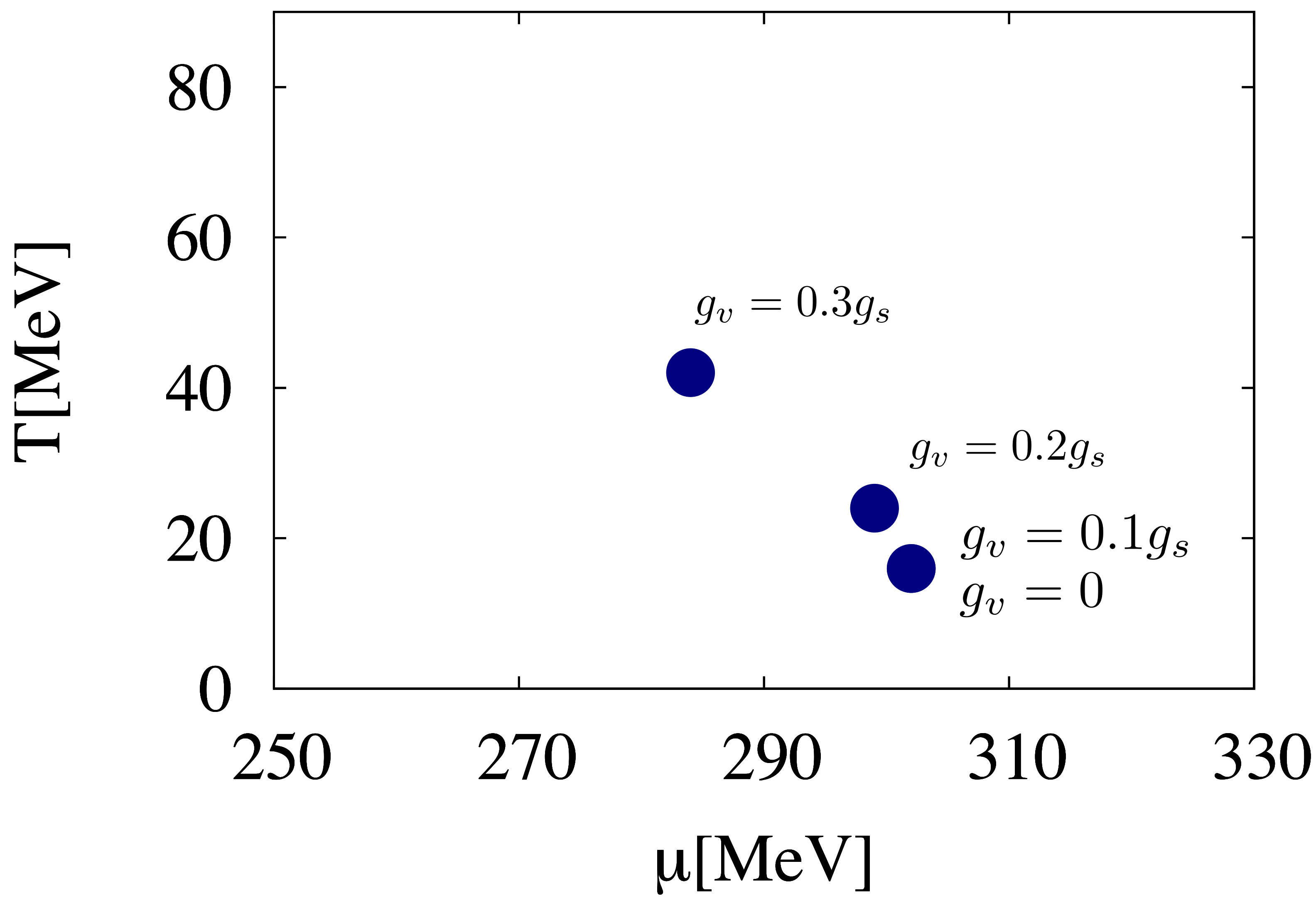
How to solve it

$$\Gamma_k^{LPA} = \text{Kinetic part} + \mathrm{U_k}(\sigma^2+\pi^2,\varphi) - \mathrm{c}\sigma$$

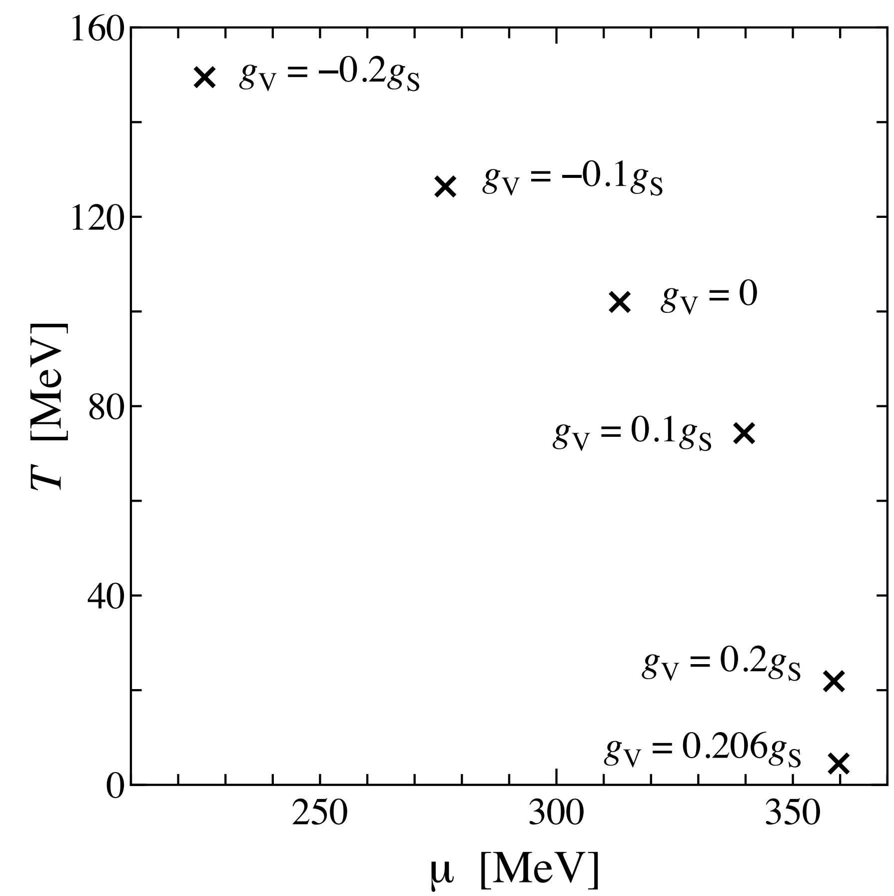


- Assume a functional form of the effective action (Local potential approximation[LPA])
- Solve coupled ordinary differential equations for $U_k(\sigma_n,\varphi_m)$: (grid method)

Phase Structure



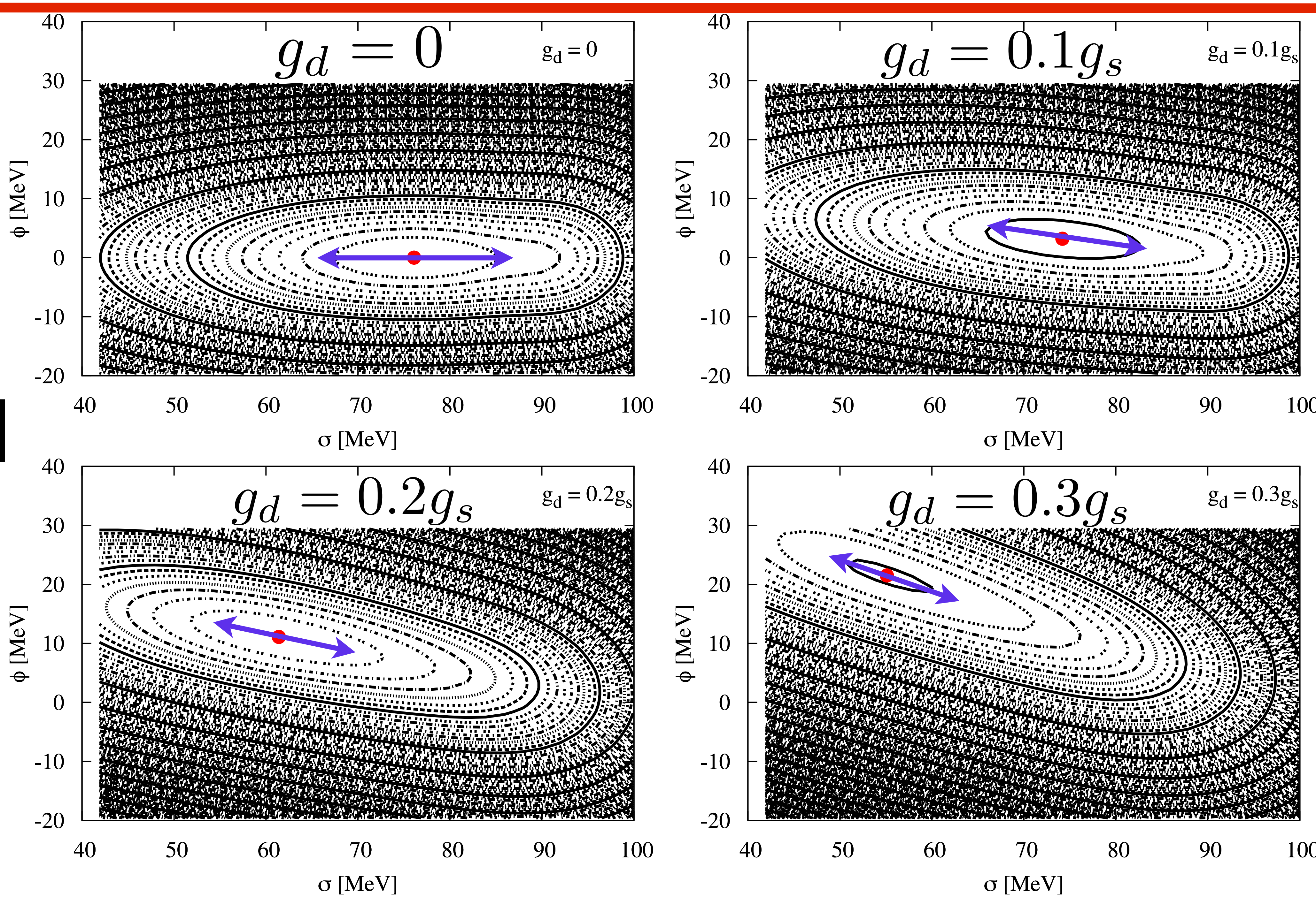
cf. NJL with vector



K. Fukushima (2008)

- QCD CP exists for finite g_d.
- The position of CP slightly moves to higher T and lower μ direction.

Potential on the CP



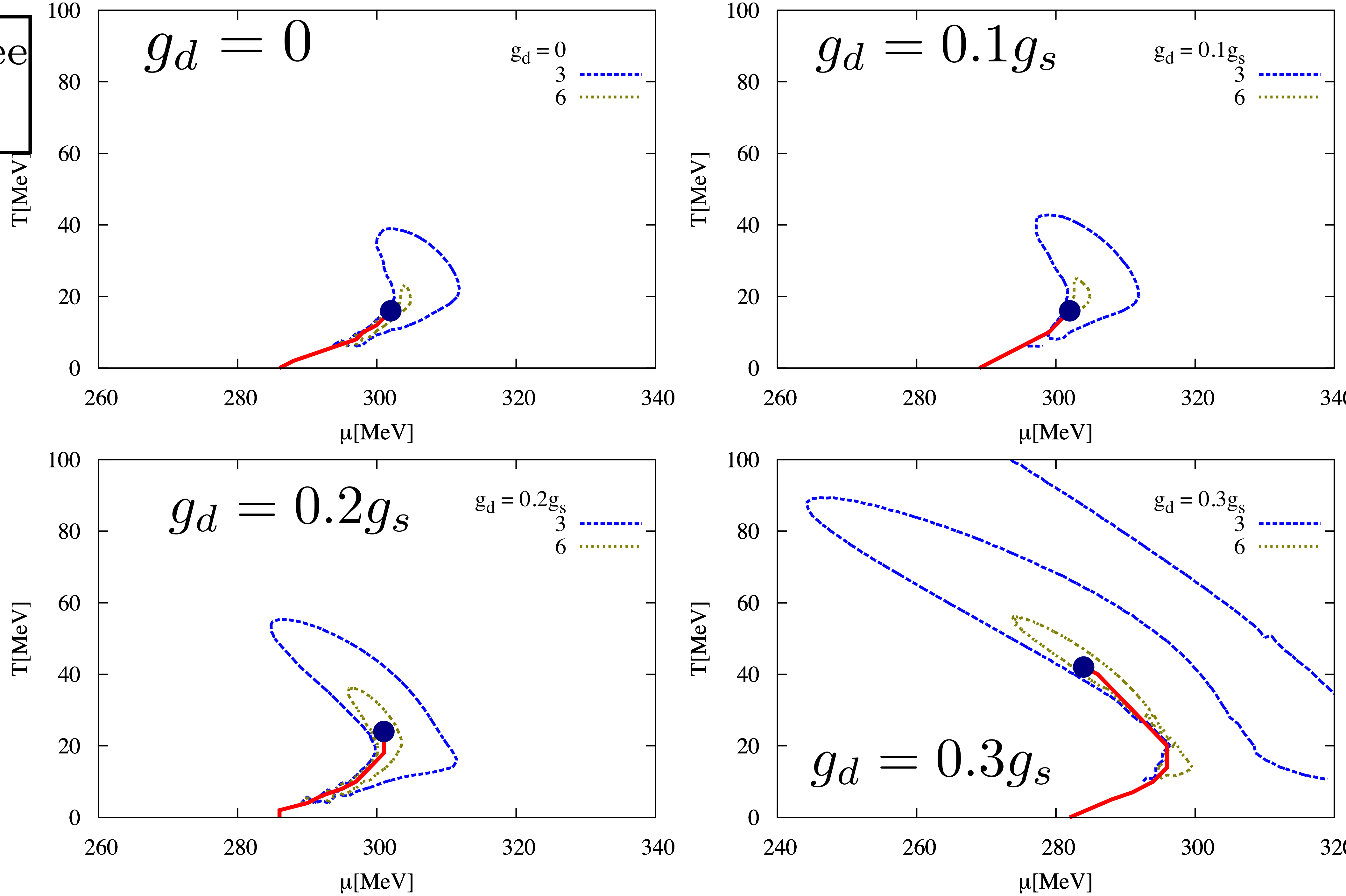
φ[MeV]

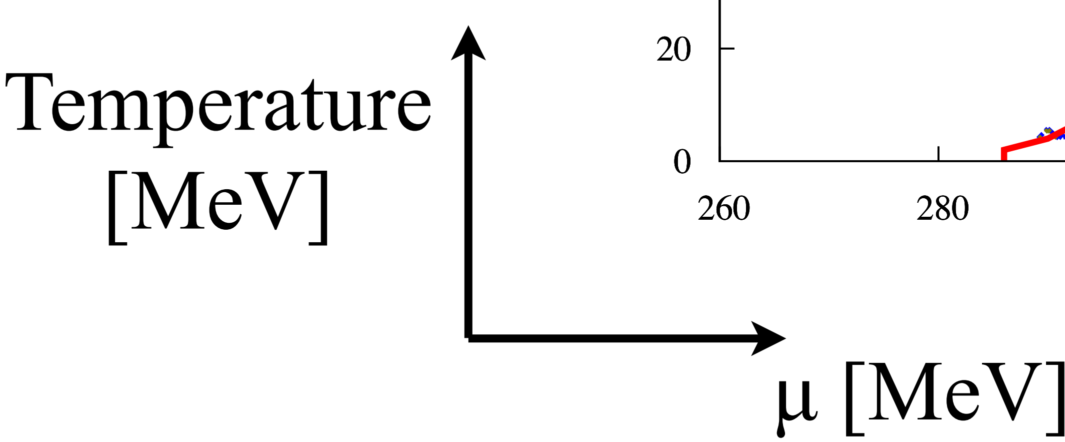
σ[MeV]

- The flat direction changes to the φ direction with g_d increasing.

Susceptibility

$$R_q = \chi_q / \chi_q^{\text{free}}$$

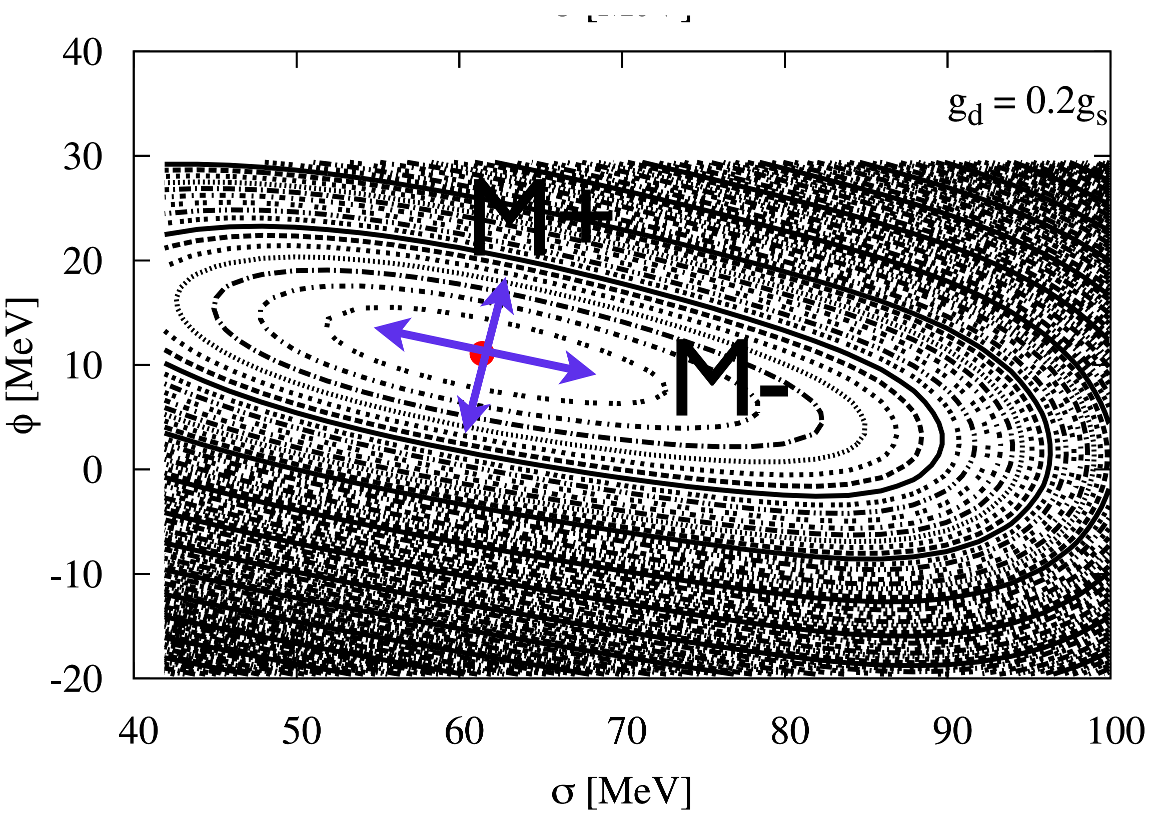




- The critical region is drastically expanded with g_d.

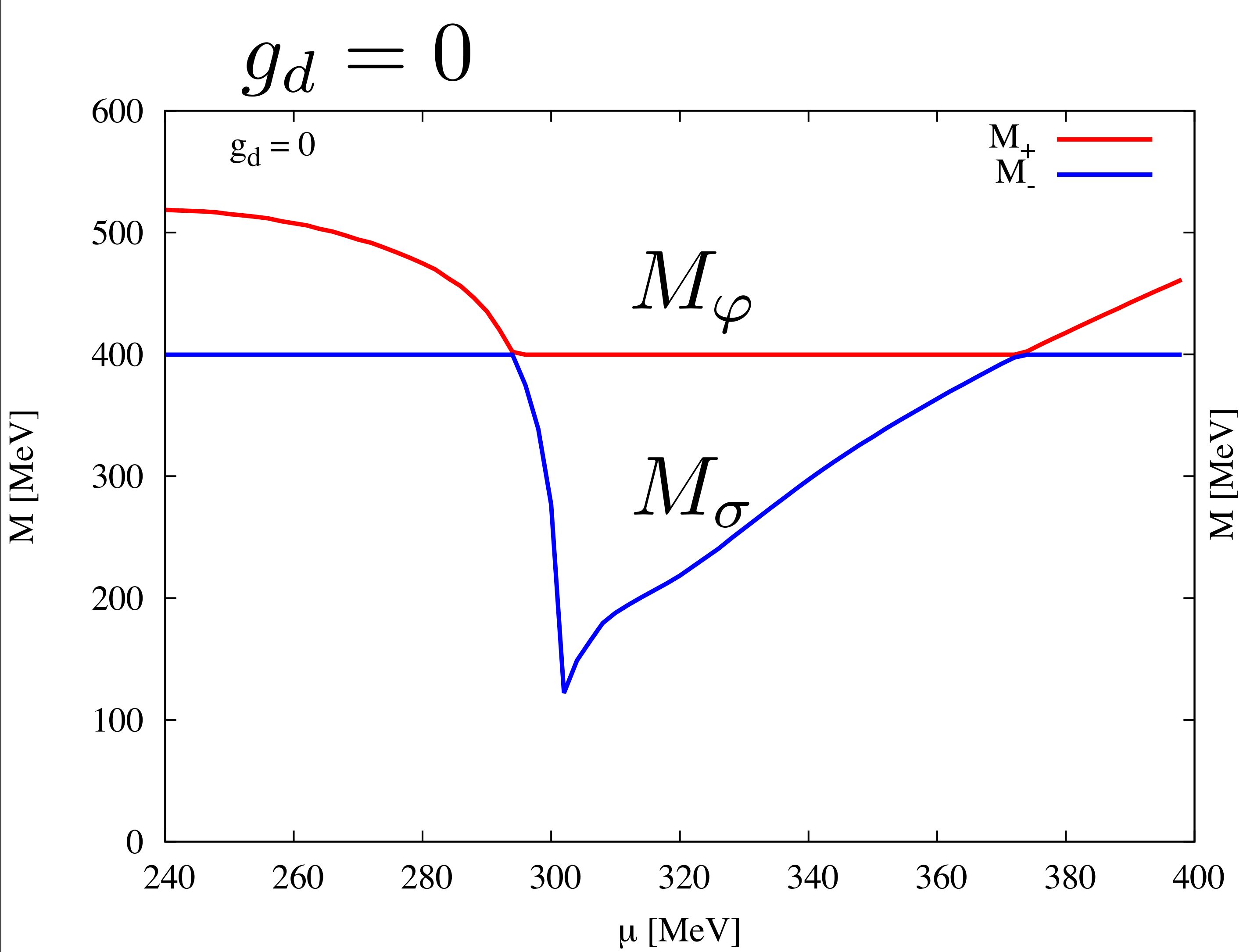
Curvature masses $M_{\pm}$

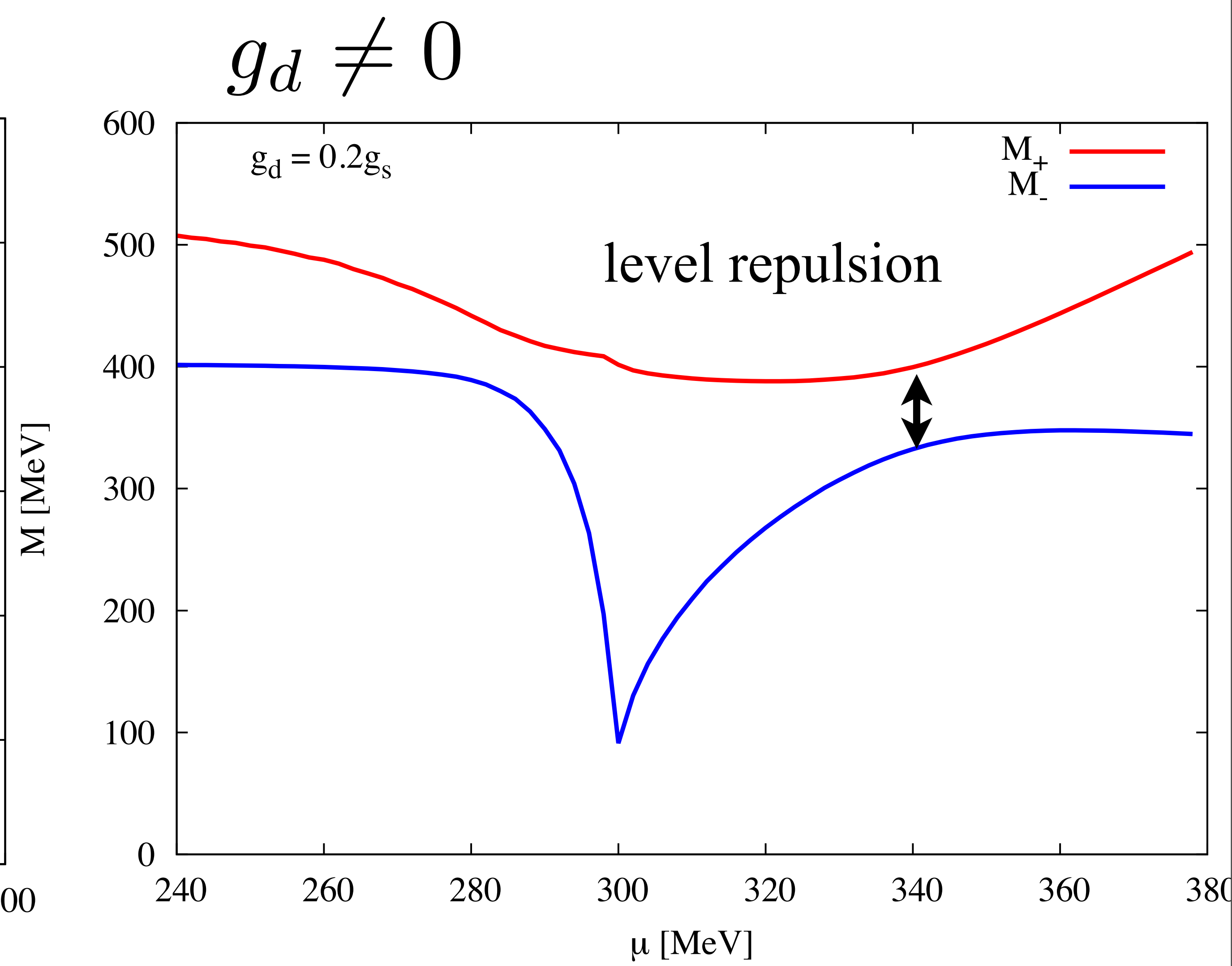
$$M = \left(\begin{matrix} \frac{\partial^2 U}{\partial\sigma\partial\sigma} & \frac{\partial^2 U}{\partial\sigma\partial\varphi} \\ \frac{\partial^2 U}{\partial\varphi\partial\sigma} & \frac{\partial^2 U}{\partial\varphi\partial\varphi} \end{matrix} \right) \Bigg|_{\sigma_0,\varphi_0}$$



- We calculate M+ and M- (eigenvalue of matrix M).
- M- corresponds to curvature of the flat direction.

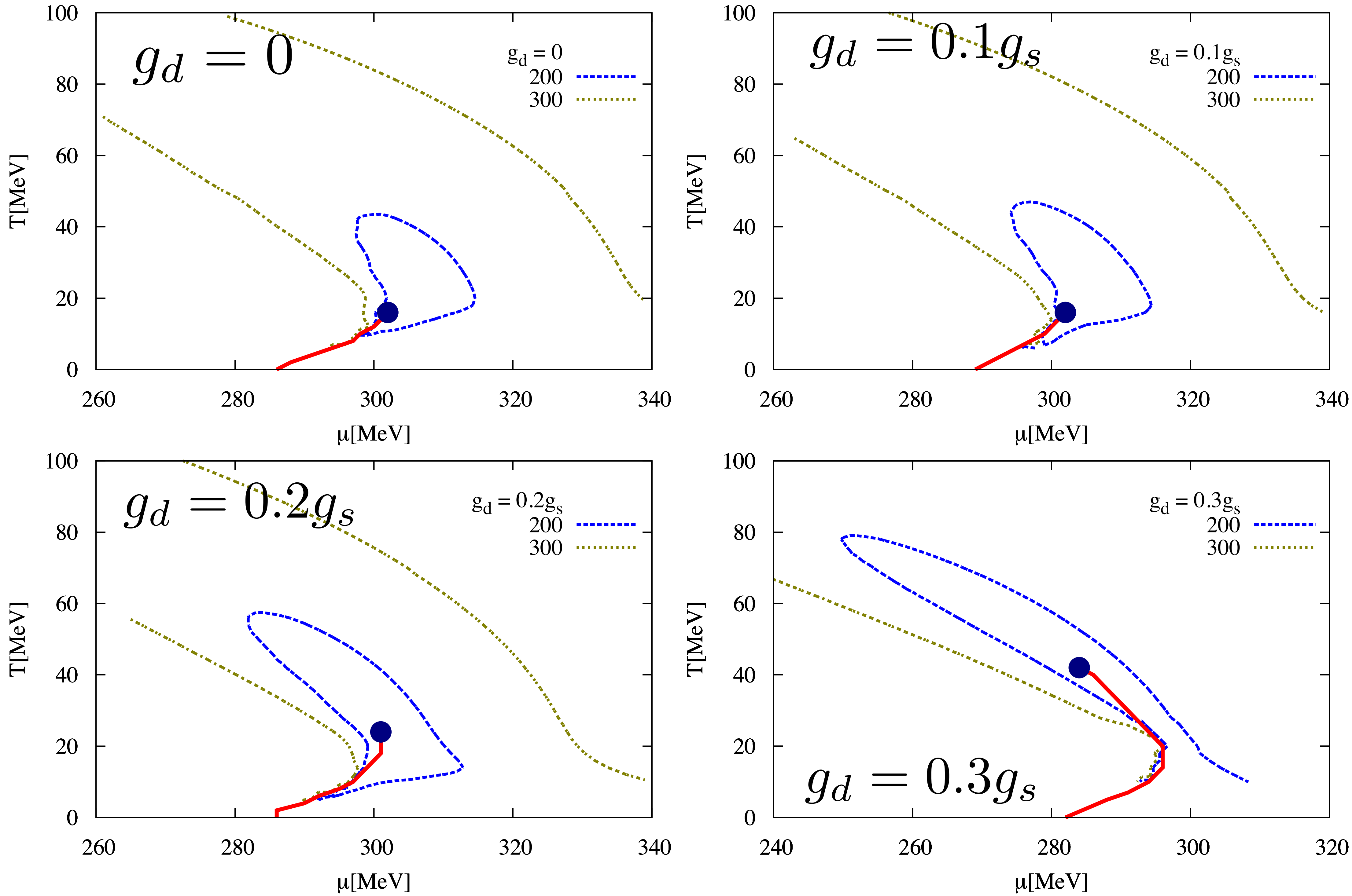
Curvature mass





- We see a kind of level repulsion between M+ and M- ($g_d \neq 0$).
- M- is decreased by the level repulsion.

Curvature mass



- The contour of M-
- Small M- region is also expanded.

Summary and outlook

- We have considered the QCD critical point and its critical region.
- We have considered the model which includes the baryon-density mode as well as chiral modes.
- We have seen the expansion of critical region with the coupling increasing.

- We will calculate the higher moments of the quark or charge number density.

- Thank you for your attendtion