Effect of short-range interactions on the quantum critical behavior of spinless fermions on the honeycomb lattice

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DFG Deutsche Forschungsgemeinschaft

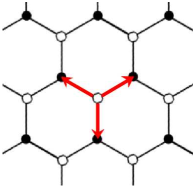
Introduction

Graphene – single layer of carbon atoms on a bipartite honeycomb lattice

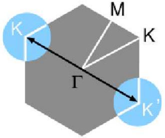
Tight-binding model:

$$H_0 = -t \sum_{\langle i,j \rangle,\sigma} \left(u_\sigma^\dagger(\mathbf{r}_i) v_\sigma(\mathbf{r}_j) + \text{H.c.} \right)$$

$$E(\mathbf{k}) = \pm t \left| \sum_{i=1,2,3} e^{i\mathbf{k}\cdot\boldsymbol{\delta}_i} \right|$$



Wallace, Phys. Rev. **71**, 622 (1947)



Two inequivalent Dirac points at corners of first Brillouin zone (BZ) with massless linear dispersion

$$E(k) = \pm v_F k \ , \quad k/\Lambda \ll 1$$

Relativistic wave equation for massless spin-$\frac{1}{2}$ particles

Kotov et al., arXiv:1012.3484

Low-energy excitations and symmetry properties

$N_f = 2$ flavors of massless Dirac fermions ($v_F = 1$)

$$\mathcal{L} = \bar{\Psi}^a i\gamma_\mu \partial_\mu \Psi^a$$

Symmetry properties:

1) **Lorentz symmetry** (C_{3v} lattice symmetry)

2) $SU(2)$ **chiral symmetry** with generators γ_3, γ_5, and $\gamma_{35} = \frac{i}{2}[\gamma_3, \gamma_5]$

3) **Time reversal symmetry**, orthogonal $\mathcal{T} = i\gamma_2 K$ (exact), and symplectic $\mathcal{S} = -i\gamma_0\gamma_1 K$

Herbut, Juričić, and Roy, Phys. Rev. B **79**, 085116 (2009)
Beenakker, Rev. Mod. Phys. **80**, 1337 (2008)

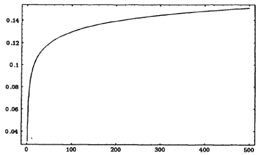
$\Rightarrow$ extended $U(2N_f)$ flavor symmetry

Electron-electron interactions

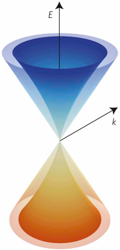
Dielectric medium plays a significant role for the interactions ⇒ screening effects

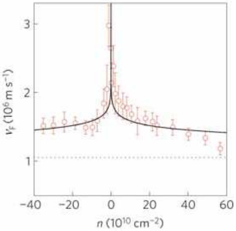
Suspended graphene at charge neutral point – massless Dirac fermions interacting via Coulomb interactions

Renormalization of Fermi velocity due to Coulomb interactions:



González et al.,
Nucl. Phys. B**424**, 595 (1994)





Elias et al., Nature Phys. **7**, 701 (2007)

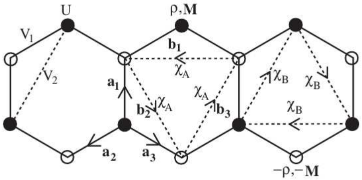
Possible many-body nature of $n = 0, \pm 1$ (anomalous) integer quantum Hall states

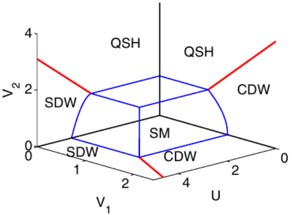
Abanin et al., Phys. Rev. Lett. **98**, 196806 (2007)
Jiang et al., Phys. Rev. Lett. **99**, 106802 (2007)

Short-range repulsive interactions

Are the quantum critical properties can be controlled by short-range interactions?

$$H = H_0 + U \sum_i n_{i\uparrow} n_{i\downarrow} + V_1 \sum_{\langle i,j \rangle} (n_i - 1)(n_j - 1) + V_2 \sum_{\langle\langle i,j \rangle\rangle} (n_i - 1)(n_j - 1)$$





Raghu et al., Phys. Rev. Lett. **100**, 156401 (2008)
Herbut, Phys. Rev. Lett. **97**, 146401 (2006)

Several types of instabilities discussed in the literature, e. g. chirally broken phases, AF phase, superconducting long-range order etc.

Order parameters and symmetries

1) **Staggered sublattice potential (CDW)**

$$\Delta_3 \sim \langle \bar{\Psi}^a \gamma_3 \Psi^a \rangle$$

flavor singlet, breaks chiral $SU(2) \to U(1)$

Semenoff, Phys. Rev. Lett. **53**, 2449 (1984)

2) **Topological insulator**

$$\Delta_{35} \sim i\langle \bar{\Psi}^a \gamma_{35} \Psi^a \rangle$$

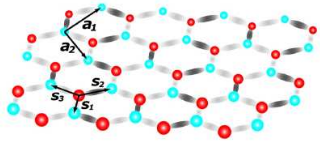
breaks TRS, invariant under extended flavor symmetry $U(2N_f)$

Haldane, Phys. Rev. Lett. **61**, 2015 (1988)

3) **Kekulé hopping texture**

$$\Delta \sim i\langle \bar{\Psi}^a \left(\cos\alpha + \gamma_5 \sin\alpha\right) \Psi^a \rangle$$

TRS invariant, but breaks chiral $SU(2) \to U(1)$



Hou et al., Phys. Rev. Lett. **98**, 186809 (2007)

Discrete symmetries

	$\mathcal{P}$	$\mathcal{C}$	$\mathcal{T}$
$i\,\bar{\Psi}\Psi$	$+$	$+$	$+$
$i\,\bar{\Psi}\gamma_{35}\Psi$	$+$	$+$	$-$
$i\,\bar{\Psi}\gamma_\mu\Psi$	$i\,\bar{\Psi}\hat{\gamma}_\mu\Psi$	$-i\,\bar{\Psi}\gamma_\mu\Psi$	$i\,\bar{\Psi}\gamma_\mu\Psi$

Transformation properties of fermionic bilinears, $\hat{\gamma}_\mu = (\gamma_0, -\gamma_1, -\gamma_2)$

	$\mathcal{T} = i\gamma_2 K$	$\mathcal{S} = -i\gamma_0\gamma_1 K$	$\mathcal{TS} = i\gamma_{35}$
$i\,\bar{\Psi}\Psi$	$+$	$+$	$+$
$\bar{\Psi}\gamma_3\Psi$	$+$	$-$	$-$
$\bar{\Psi}\gamma_5\Psi$	$+$	$-$	$-$
$i\,\bar{\Psi}\gamma_{35}\Psi$	$-$	$-$	$+$
$i\,\bar{\Psi}\gamma_\mu\Psi$	$+$	$+$	$+$

Transformation properties of fermionic bilinears under antiunitary symmetries

Functional renormalization group

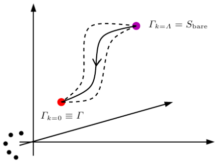
Exact renormalization group equation

$$\partial_t \Gamma_k = \frac{1}{2}\,\mathrm{STr}\left\{\partial_t R_k \left(\Gamma_k^{(1,1)} + R_k\right)^{-1}\right\}\ , \quad \partial_t \equiv k\frac{\partial}{\partial k}$$

Wetterich, Phys. Lett. B **301**, 90 (1993)

Include fluctuations successively via the regulator function R_k

Starting from some microscopic model $\Gamma_{k=\Lambda} \simeq S$ we solve the flow equation to obtain scale-dependent effective average action Γ_k



Gies, arXiv:hep-th/0611146

Truncate Γ_k to arrive at a finite system of coupled differential equations

Systematic derivative expansion of effective average action Γ_k

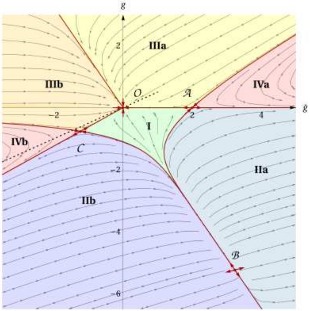
(2+1)-dimensional Thirring model

Classical action with repulsive short-range interactions, invariant under $U(2N_f)$ and discrete symmetries

$U(2N_f)$-complete fermionic model, N_f Dirac fermions

$$\Gamma_k[\bar{\Psi}, \Psi] = \int_x \left\{ \bar{\Psi}^a i\partial\!\!\!/ \Psi^a + \frac{\bar{g}_k}{2N_f} \left(\bar{\Psi}^a \gamma_\mu \Psi^a\right)^2 + \frac{\bar{\bar{g}}_k}{2N_f} \left(\bar{\Psi}^a \gamma_{35} \Psi^a\right)^2 \right\}$$

Janssen & Gies, Phys. Rev. D **82**, 085018 (2010)

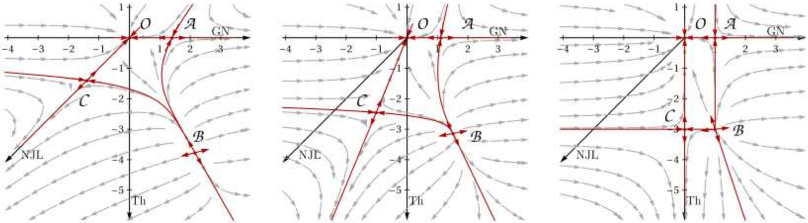


Two types of instabilities that are characterized by:

Nambu-Jona-Lasinio (NJL) and Thirring channel

(2+1)-dimensional Thirring model – continued

Competition between NJL- and Thirring-channel in fermionic RG



Janssen & Gies, arXiv:1208.3327 (2012)

⇒ NJL-type interaction dominates at small N_f

$SU(N)$ matrix Yukawa model

Collective **matrix field** Φ via Hubbard-Stratonovich transformation

Effective average action Γ_k with $U(2N_f)$ symmetry, $N = 2N_f$ species of Weyl spinors

$$\Gamma_k = \int_x \left\{ Z_{F,k}\, \bar{\psi}^a i\partial\!\!\!/ \psi^a + \frac{1}{2} Z_{B,k}\, \mathrm{tr}\, (\partial_\mu \Phi)^2 + i\bar{h}_k\, \bar{\psi}^a \Phi_{ab} \psi^b + V_k(\Phi) \right\}$$

Wavefunction renormalization

$$\partial_t Z_{F,k} \neq 0 \ , \quad \partial_t Z_{B,k} \neq 0$$

$$\eta_F = -\partial_t \ln Z_{F,k} \ , \quad \eta_B = -\partial_t \ln Z_{B,k}$$

$SU(N)$ matrix Yukawa model – continued

Effective average potential $V_k(\Phi)$ function of $U(N)$ invariants $\sim \mathrm{tr}\,\Phi^n$

$$V_k(\Phi) \equiv V_k(\sigma, \rho)$$

$$\sigma = \mathrm{tr}\,\Phi\ , \quad \rho = \frac{1}{2}\,\mathrm{tr}\left(\Phi - \frac{1}{N}\right)^2 \quad (\text{exact for } N = 2)$$

$\Rightarrow N = 2$: **spinless fermions on the honeycomb lattice**

Fluctuations **only** in ρ-channel ($\sigma = 0$) relevant for the $SU(2) \to U(1)$ chiral phase transition

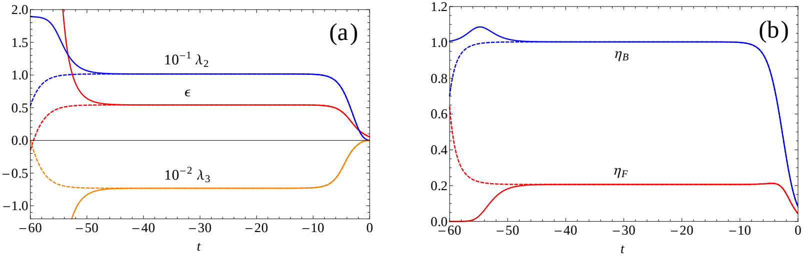
$$V_k(\sigma = 0, \rho) = \bar{m}_k^2\,(\rho - \rho_{0,k}) + \sum_{n=2} \frac{\bar{\lambda}_{n,k}}{n!}\,(\rho - \rho_{0,k})^n$$

1) Symmetric regime: $\rho_{0,k} = 0$

2) χSB regime: k-dependent minimum $\rho_{0,k} \neq 0$ and $\bar{m}_k^2 = 0$

$N = 2$ renormalization group flow

$N = 2$ renormalization group flow in the vicinity of the quantum critical point



(a) Dimensionless renormalized couplings ϵ, λ_2, and λ_3

and

(b) Boson and fermion anomalous dimensions η_B and η_F

$N = 2$ critical couplings

$N = 2$ critical couplings for different orders in the series expansion

	4th order	6th order	8th order	10th order
ϵ_*	0.4842	0.5242	0.5424	0.5288
$\lambda_{2,*}$	10.7678	10.3744	10.1573	10.3210
$\lambda_{3,*}$		-48.5405	-73.0962	-54.6552
$\lambda_{4,*}$			-1956.82	-485.084
$\lambda_{5,*}$				219713
h_*^2	12.8622	12.9203	12.9438	12.9264

$\Rightarrow$ Relavant couplings stable!

$N = 2$ critical exponents

$N = 2$ critical exponents for different orders in the series expansion

	4th order	6th order	8th order	10th order
η_B	0.989	0.999	1.003	1.000
η_F	0.223	0.211	0.207	0.210
ν	1.922	1.936	1.791	1.874
γ	1.942	1.939	1.786	1.875
β	1.911	1.935	1.793	1.874

	Gross-Neveu (FRG)
η_B	0.561
η_F	0.066
ν	0.961
γ	1.384
β	0.745

Rosa et al., Phys. Rev. Lett. **86**, 958 (2001)
Höfling et al., Phys. Rev. B **66**, 205111 (2002)

$\Rightarrow$ Large values for the anomalous dimensions!

Conclusions

$N = 2$ matrix Yukawa model is characterized by large anomalous dimensions

Similar properties have been found in a single Dirac-cone model for the semimetal-superfluid transition
⇒ Second order phase transition $\eta_B \simeq 0.75$ and 1.75, $\eta_F \simeq 0.25$

Obert et al., Ann. Phys. **523**, 621 (2011)

Compact three-dimensional QED $\eta_A = 1$ due to gauge invariance

Herbut and Tesanovic, Phys. Rev. Lett. **76**, 4588 (1996)
Hove and Sudbo, Phys. Rev. Lett. **84**, 3426 (2000)

⇒ Up to now **open question** if these non-trivial properties realized in suspended graphene!

Need simulations of the extended Hubbard model on the honeycomb lattice – **ultracold gases?**

Outlook

⇒ Critical properties of the short-range repulsive interactions on the honeycomb lattice appear to be rather special – *strong fluctuations!*

Is graphene $N = 4$ with unscreened Coulomb interactions controlled by the short-range NJL-type quantum critical point?
⇒ Would be interesting to find some screening effect at the charge neutral point!

Renormalization of Fermi velocity

What about *transport properties?*

Müller et al., Phys. Rev. Lett. **103**, 025301 (2009)