QCD at finite density with Dyson-Schwinger equations

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Quark Gluon Plasma meets Cold Atoms
Episode III

Outline

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Motivation

Dyson-Schwinger equations

Color superconductivity

Results

Inhomogeneous phases

Summary and outlook

Motivation

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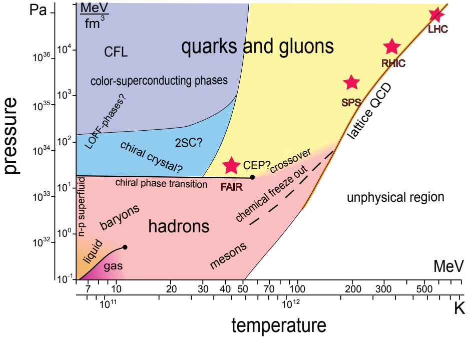

K. Heckmann (2011)

What happens at high densities?

- color superconducting phases?
  $\rightarrow$ weak coupling, effective models
  $\rightarrow$ Dyson-Schwinger equations at $T = 0$ (Nickel, Wambach, Alkofer (2006))
- inhomogeneous phases?
  $\rightarrow$ S. Carignanos talk
- our aim: investigate phases with Dyson-Schwinger equations at finite T and μ

Outline

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Dyson-Schwinger equations (DSEs)

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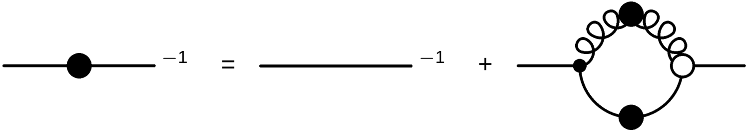
Quark DSE



$$S^{-1}(p) = Z_2 \left(S_0^{-1}(p) + \Sigma(p) \right)$$

Dyson-Schwinger equations (DSEs)

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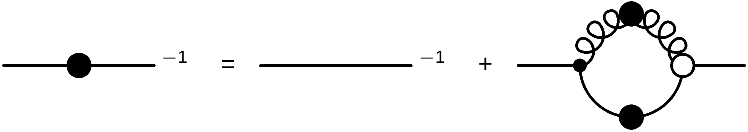
Quark DSE



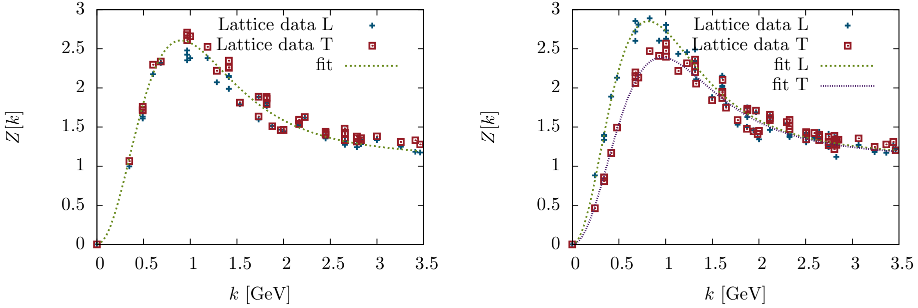
$$S^{-1}(p) \quad = \quad Z_2 \left(S_0^{-1}(p) \quad + \quad \Sigma(p) \right)$$

- exact QCD equation
- need: gluon propagator and dressed quark gluon vertex
  $\rightarrow$ gluon DSE, vertex DSE
- infinite tower of equations $\rightarrow$ truncation

Gluon Truncation

Gluon propagator (Landau gauge) (data and fit from Fischer, Maas, Müller (2010)):

$$D^{ab}_{\mu\nu}(k) = \delta^{ab}\left(\frac{Z_T(k^2)}{k^2}P^T_{\mu\nu}(k) + \frac{Z_L(k^2)}{k^2}P^L_{\mu\nu}(k)\right)$$



T = 0 MeV　　　　T = 125 MeV

Gluon Polarization 1

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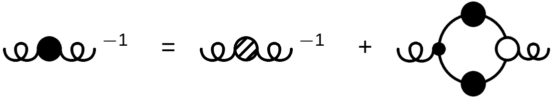
Gluon DSE (truncated)

Effects of quark on the gluon propagator:



$$D^{-1,ab}_{\mu\nu}(k) = D^{-1,ab}_{\mu\nu,YM}(k) + \Pi^{ab}_{\mu\nu}(k)$$

$$D^{ab}_{\mu\nu}(k) = \frac{Z_T(k^2)}{k^2 + G^{ab}(k)} P^T_{\mu\nu}(k) + \frac{Z_L(k^2)}{k^2 + F^{ab}(k)} P^L_{\mu\nu}(k)$$

Gluon Polarization 2

$$D^{ab}_{\mu\nu}(k) = \frac{Z_T(k^2)}{k^2 + G^{ab}(k)} P^T_{\mu\nu}(k) + \frac{Z_L(k^2)}{k^2 + F^{ab}(k)} P^L_{\mu\nu}(k)$$

- calculation with fully dressed quarks or
- HDL / HTL - like approximation:
  - bare quark propagators
  - large Temperatures or chemical potentials $T, \mu \gg k$
  - (constant) vacuum parts absorbed in renormalization

$$F(k_4, |\vec{k}|) = 2m_g^2(k) \cdot \ldots, \quad G(k_4, |\vec{k}|) = \frac{\pi}{2} m_g^2(k) \frac{k_4}{|\vec{k}|} \cdot \ldots$$

$$m_g^2 = \alpha_s(k^2) \left(\frac{N_f \mu^2}{\pi} + \frac{N_f T^2 \pi}{3} \right)$$

Vertex Truncation

- exact vertex: vertex DSE $\rightarrow$ 4 point functions, non-abelian terms,... (complicated)

Simplifications

- abelian vertex construction:

$$\Gamma^a_\mu(p,q;k) \rightarrow \Gamma(p,q)\gamma_\mu \frac{\lambda^a}{2}$$

- Ansatz for the dressing function $\Gamma(p,q)$
- perturbative QCD UV behaviour + phenomenological infrared strength

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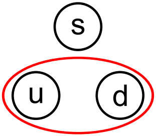
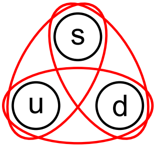
Color Superconductivity: pairing patterns

Cooper instability

- fermionic system + attractive force $\rightarrow$ Cooper pairs
- in QCD: diquarks $\langle q^T C \mathcal{O} q \rangle$
- most important phases:



2SC phase
high m_s / low μ



CFL (-like) phase
low m_s / high μ

Color superconductivity in the Dyson-Schwinger framework

Nambu Gor'kov formalism

- define bispinors $\Psi = \begin{pmatrix} \psi \\ C\bar{\psi}^T \end{pmatrix}$, $\bar{\Psi} = \begin{pmatrix} \bar{\psi} & \psi^T C \end{pmatrix}$
- $\mathcal{S}_0 = \begin{pmatrix} S_0^+ & 0 \\ 0 & S_0^- \end{pmatrix}$, $\mathcal{S} = \begin{pmatrix} S^+ & T^- \\ T^+ & S^- \end{pmatrix}$, $\Sigma = \begin{pmatrix} \Sigma^+ & \Phi^- \\ \Phi^+ & \Sigma^- \end{pmatrix}$, $\Gamma = \begin{pmatrix} \Gamma^+ & \Delta^- \\ \Delta^+ & \Gamma^- \end{pmatrix}$
- T, Φ: anomalous propagators / self energies, representing color superconducting phases

Gluon polarization with color superconductivity

Properties of the gluon polarization

- Debye and Meissner masses:

$$
\begin{aligned}
m^2_{D,ab} &= \lim_{\vec{p}\to 0} \Pi^{ab}_{TL}(\omega_m = 0, \vec{p}) \\
m^2_{M,ab} &= \lim_{\vec{p}\to 0} \Pi^{ab}_{TT}(\omega_m = 0, \vec{p})
\end{aligned}
$$

Gluon polarization with color superconductivity

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Properties of the gluon polarization

- Debye and Meissner masses:

$$m^2_{D,ab} = \lim_{\vec{p}\to 0} \Pi^{ab}_{TL}(\omega_m = 0, \vec{p})$$

$$m^2_{M,ab} = \lim_{\vec{p}\to 0} \Pi^{ab}_{TT}(\omega_m = 0, \vec{p})$$

- transversality: $k_\mu k_\nu D^{ab}_{\mu\nu}(k) = 0$
- ensured by regularization and truncation
- requires off-diagonal (in Nambu Gor'kov space) contributions to the quark-gluon vertex

Outline

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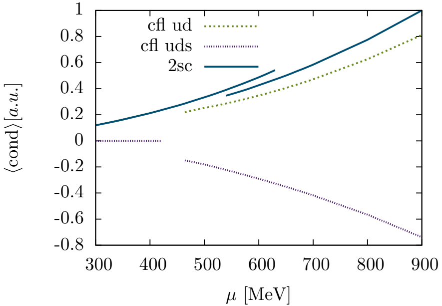
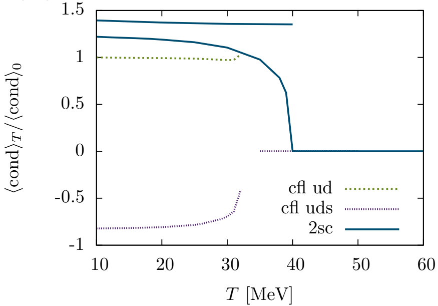
Condensates

$$\langle q^T C\mathcal{O}q\rangle = -Z_2 \int \frac{d^4p}{(2\pi)^4} \mathrm{Tr}\,[\mathcal{O}T^+]$$

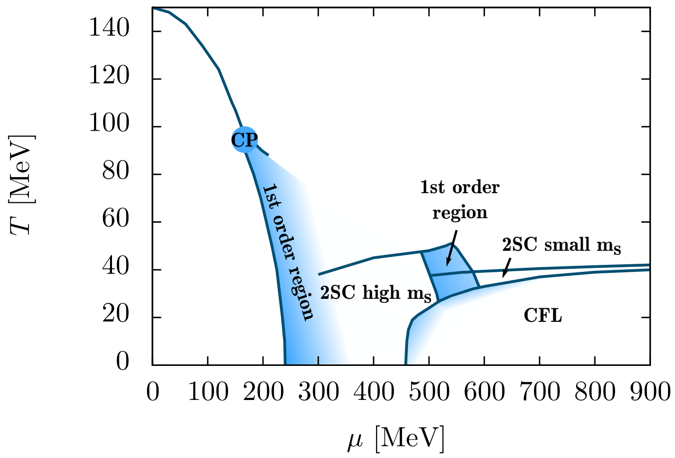


dependence of csc condensates on chemical potential (T = 10 MeV)



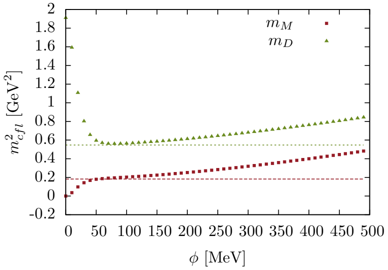
dependence of csc condensates on temperature (μ = 580 MeV)

Phase diagrams

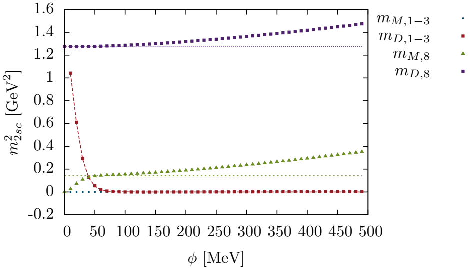


Gluon masses in the weak coupling limit

Self energy ansatz: $\Phi^{+}(p) = \phi_i \gamma_5 M_{2SC/CFL,i}$



Debye and Meissner masses in the CFL phase (T = 10 MeV, μ = 1 GeV)



Debye and Meissner masses in the 2SC phase (T = 10 MeV, μ = 1 GeV)

(weak coupling results from Rischke (2000))

Gluon masses - full calculation

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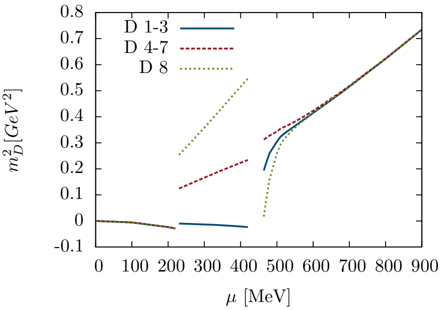
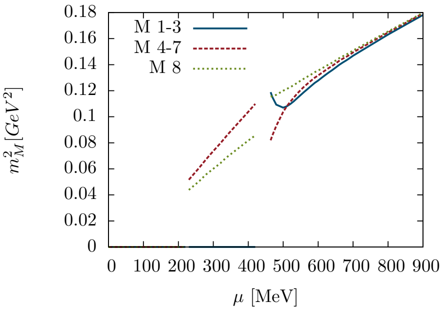


Debye masses (T = 10 MeV)



Meissner masses (T = 10 MeV)

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Inhomogeneous phases

General remarks

- till now: spatially homogeneous matter
- chiral 1st order transition possibly covered by inhomogeneous condensates $\rightarrow$ see S. Carignanos talk (NJL model)
- $\langle \bar{q}q \rangle = \langle \bar{q}q \rangle(x)$

Dyson-Schwinger equations - approximations

- need some simplifications:
- HDL / HTL truncation
- 1-dimensional modulations
- chiral density wave (chiral spiral): $\langle \bar{q}q \rangle(x) = \langle \bar{q}q \rangle e^{iQx}$

Chiral spiral

- non-diagonal structure in momentum space
- Dirac decomposition requires 10 components

Structure in $p - p'$ space

$$S^{-1} = \left(\begin{array}{cc|cc|cc} 0 & \square & \lozenge & 0 & 0 & 0 \\ \square & 0 & 0 & 0 & 0 & 0 \\ \hline 0 & 0 & 0 & \square & \lozenge & 0 \\ 0 & \lozenge & \square & 0 & 0 & 0 \\ \hline 0 & 0 & 0 & 0 & 0 & \square \\ 0 & 0 & 0 & \lozenge & \square & 0 \end{array}\right)$$

Gap equations

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Effective action (HDL / HTL truncations)

$$\Gamma = \mathrm{Tr}\ \ln S^{-1} - \mathrm{Tr}\,(1 - S_0^{-1} S) + \frac{1}{2} \mathrm{Tr}\ S(p) \Gamma^a_{\mu,0} D^{ab}_{\mu\nu}(p-q) S(q) \Gamma^b_\nu$$

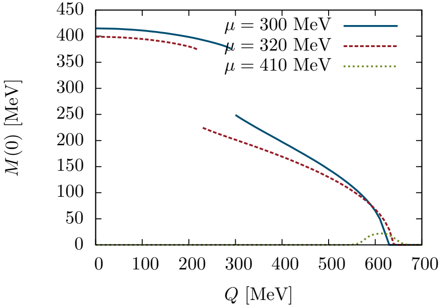
Gap equations

$$\frac{\partial \Gamma}{\partial S(p)} = 0 \rightarrow S^{-1}(p) = S_0^{-1}(p) + \Sigma(p)$$

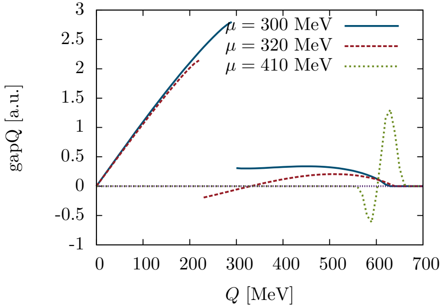
$$\frac{d\Gamma}{dQ} = 0$$

Solve both equations simultaneously!

Mass and gap equation



solution for the mass function for given Q at T = 10 MeV



Gap equation for Q for given Q at T = 10 MeV

M and Q

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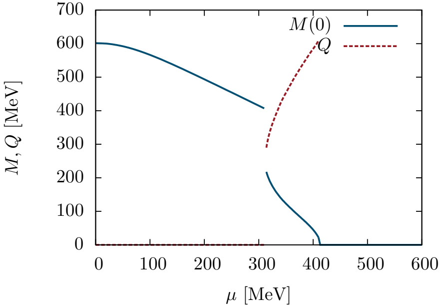
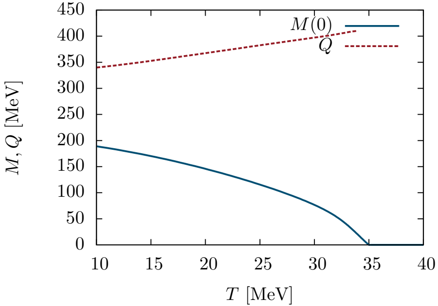
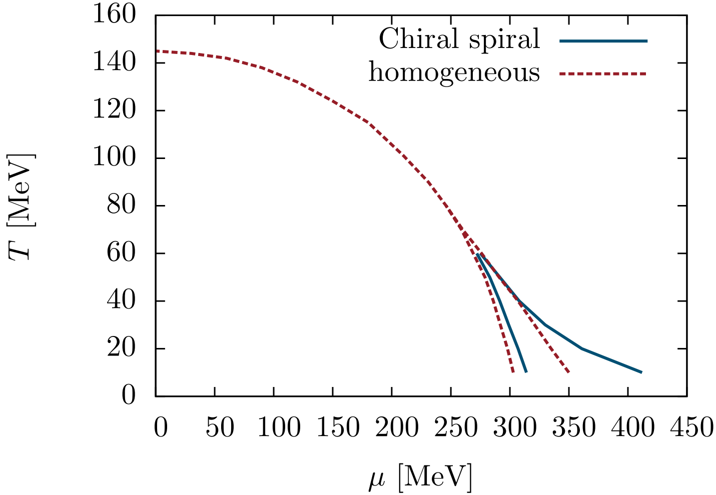


dependence of the mass and wave vector on chemical potential (T = 10 MeV)



dependence of the mass and wave vector on temperature (μ = 320 MeV)

Phase diagram



Summary and Outlook

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Summary

- Color superconductivity with Dyson-Schwinger equations
- CFL-phase for $\mu > 500$ MeV
- 2SC phase at low densities and at finite T
- strange quark phase transition visible in 2SC condensates
- inhomogeneous phases: chiral spiral covers 1st order area

Outlook

- improvement of the vertex
- inhomogeneous color superconducting phases
- ...

Summary and Outlook