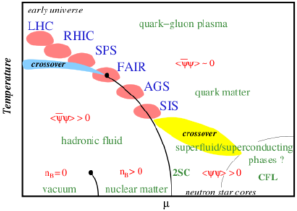
The Role of Fluctuations in the Phase Diagramm of QC_2D Functional Methods

Naseemuddin Khan, Jan M. Pawlowski, Michael Scherer, Fabian Rennecke

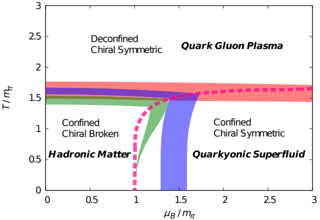
University of Heidelberg

QCD meets Cold Atoms - Episode III
26th August 2012

Motivation

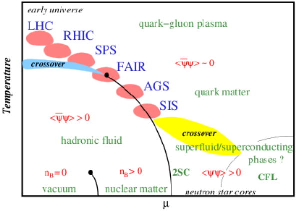


[Schaefer Delta Meeting '10]

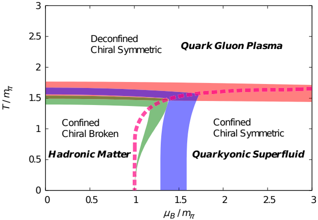


[Brauner, Fukushima & Hidaka '09]

Motivation



[Schaefer Delta Meeting ’10]

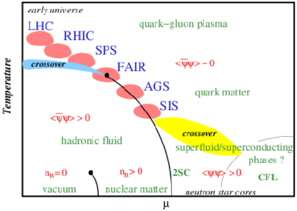


[Brauner, Fukushima & Hidaka ’09]

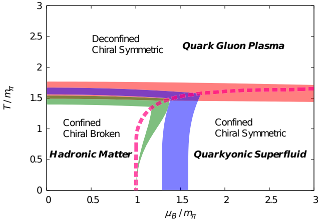
QC_2D

- no sign problem → compare with lattice results

Motivation



[Schaefer Delta Meeting ’10]

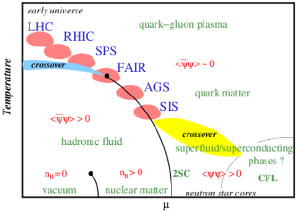


[Brauner, Fukushima & Hidaka ’09]

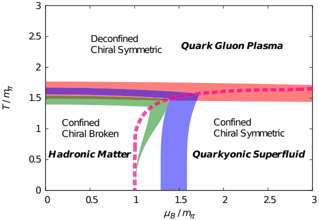
QC_2D

- no sign problem $\to$ compare with lattice results

Motivation



[Schaefer Delta Meeting '10]

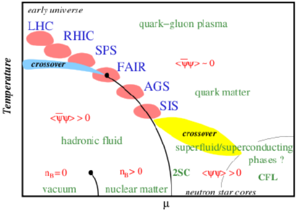


[Brauner, Fukushima & Hidaka '09]

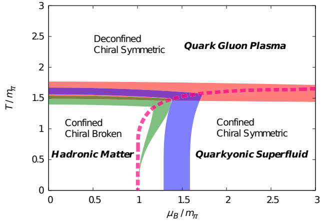
QC_2D

- no sign problem → compare with lattice results
- colour neutral bound states of two quarks → phases of superfluidity easily acessible

Motivation



[Schaefer Delta Meeting '10]

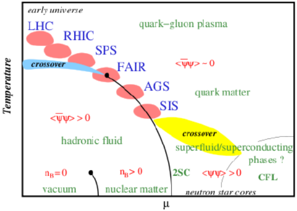


[Brauner, Fukushima & Hidaka '09]

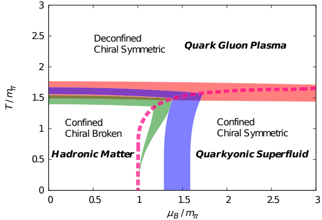
QC$_2$D

- no sign problem → compare with lattice results
- colour neutral bound states of two quarks → phases of superfluidity easily acessible

Motivation



[Schaefer Delta Meeting '10]



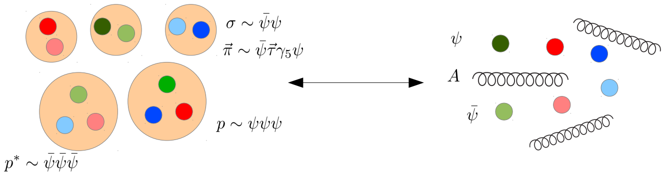
[Brauner, Fukushima & Hidaka '09]

QC_2D

- no sign problem → compare with lattice results
- colour neutral bound states of two quarks → phases of superfluidity easily acessible
- impact of baryons on the phase diagram

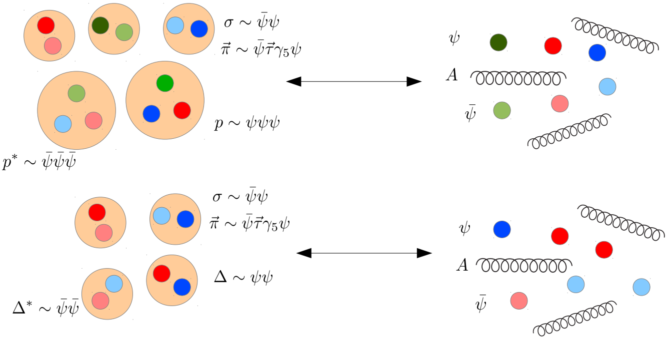
Motivation

degrees of freedom



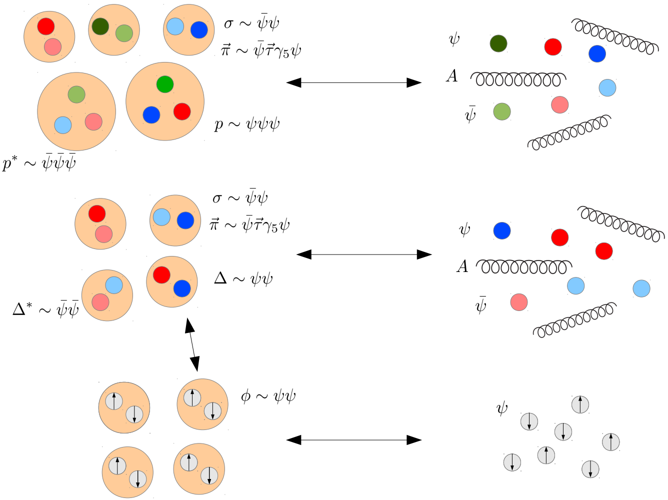
Motivation

degrees of freedom



Motivation

degrees of freedom



Features of QC_2D

Features of QC_2D

pseudoreality of $SU(2)_c$ gauge group generators: $t_a^* = t_a^T = -t_2 t_a t_2$

Features of QC_2D

pseudoreality of $SU(2)_c$ gauge group generators: $t_a^* = t_a^T = -t_2 t_a t_2$

- antiunitarity of Dirac operator: $D^* = -t_2 C\gamma_5 D\gamma_5 C t_2$

Features of QC_2D

pseudoreality of $SU(2)_c$ gauge group generators: $t_a^* = t_a^T = -t_2 t_a t_2$

- antiunitarity of Dirac operator: $D^* = -t_2 C\gamma_5 D\gamma_5 C t_2$
- extended flavour symmetry:

$$SU(N_f)_L \times SU(N_f)_R \times U(1)_B \to SU(2N_f)$$

Features of QC_2D

pseudoreality of $SU(2)_c$ gauge group generators: $t_a^* = t_a^T = -t_2 t_a t_2$

- antiunitarity of Dirac operator: $D^* = -t_2 C \gamma_5 D \gamma_5 C t_2$
- extended flavour symmetry:

$$SU(N_f)_L \times SU(N_f)_R \times U(1)_B \to SU(2N_f)$$

Features of QC_2D

pseudoreality of $SU(2)_c$ gauge group generators: $t_a^* = t_a^T = -t_2 t_a t_2$

- antiunitarity of Dirac operator: $D^* = -t_2 C\gamma_5 D \gamma_5 C t_2$
- extended flavour symmetry:

$$SU(N_f)_L \times SU(N_f)_R \times U(1)_B \to SU(2N_f)$$

$$\mathcal{L}_{\mathrm{QC_2D}} = \psi_L^\dagger i\sigma_\mu D_\mu \psi_L + \psi_R^\dagger i\sigma_\mu^\dagger D_\mu \psi_R$$

Features of QC_2D

pseudoreality of $SU(2)_c$ gauge group generators: $t_a^* = t_a^T = -t_2 t_a t_2$

- antiunitarity of Dirac operator: $D^* = -t_2 C\gamma_5 D\gamma_5 C t_2$
- extended flavour symmetry:

$$SU(N_f)_L \times SU(N_f)_R \times U(1)_B \to SU(2N_f)$$

$$\mathcal{L}_{\mathrm{QC_2D}} = \psi_L^\dagger i\sigma_\mu D_\mu \psi_L + \psi_R^\dagger i\sigma_\mu^\dagger D_\mu \psi_R = \Psi^\dagger i\sigma_\mu D_\mu \Psi$$

Features of QC_2D

pseudoreality of $SU(2)_c$ gauge group generators: $t_a^* = t_a^T = -t_2 t_a t_2$

- antiunitarity of Dirac operator: $D^* = -t_2 C\gamma_5 D\gamma_5 C t_2$
- extended flavour symmetry:

$$SU(N_f)_L \times SU(N_f)_R \times U(1)_B \to SU(2N_f)$$

$$\mathcal{L}_{\mathrm{QC_2D}} = \psi_L^\dagger i\sigma_\mu D_\mu \psi_L + \psi_R^\dagger i\sigma_\mu^\dagger D_\mu \psi_R = \Psi^\dagger i\sigma_\mu D_\mu \Psi$$

$$\Psi = \begin{pmatrix} \psi_L \\ \tilde{\psi}_R \end{pmatrix} = \begin{pmatrix} u_L \\ d_L \\ \tilde{u}_R \\ \tilde{d}_R \end{pmatrix}, \qquad \tilde{\psi}_R = \sigma_2 t_2 \psi_R^*$$

Features of QC_2D

pseudoreality of $SU(2)_c$ gauge group generators: $t_a^* = t_a^T = -t_2 t_a t_2$

- antiunitarity of Dirac operator: $D^* = -t_2 C\gamma_5 D\gamma_5 C t_2$
- extended flavour symmetry:

$$SU(N_f)_L \times SU(N_f)_R \times U(1)_B \to SU(2N_f)$$

$$\mathcal{L}_{\mathrm{QC_2D}} = \psi_L^\dagger i\sigma_\mu D_\mu \psi_L + \psi_R^\dagger i\sigma_\mu^\dagger D_\mu \psi_R = \Psi^\dagger i\sigma_\mu D_\mu \Psi$$

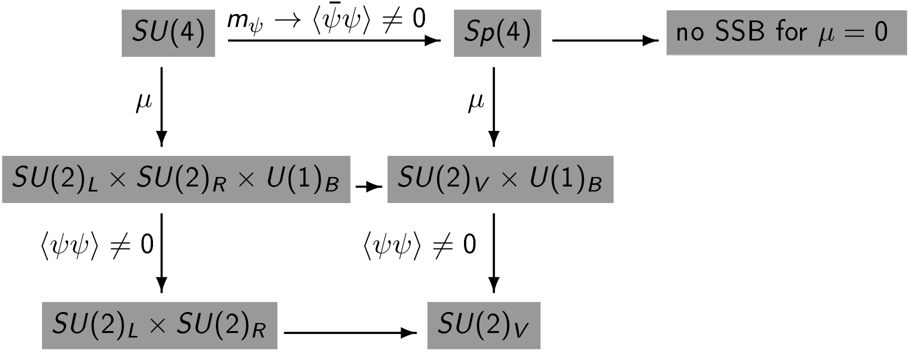
$$\Psi = \begin{pmatrix} \psi_L \\ \tilde{\psi}_R \end{pmatrix} = \begin{pmatrix} u_L \\ d_L \\ \tilde{u}_R \\ \tilde{d}_R \end{pmatrix}, \qquad \tilde{\psi}_R = \sigma_2 t_2 \psi_R^*$$

allows us to rotate $\psi_L \to \tilde{\psi}_R$, similarly $\langle\bar{\psi}\psi\rangle \to \langle\psi\psi\rangle$

Features of QC_2D

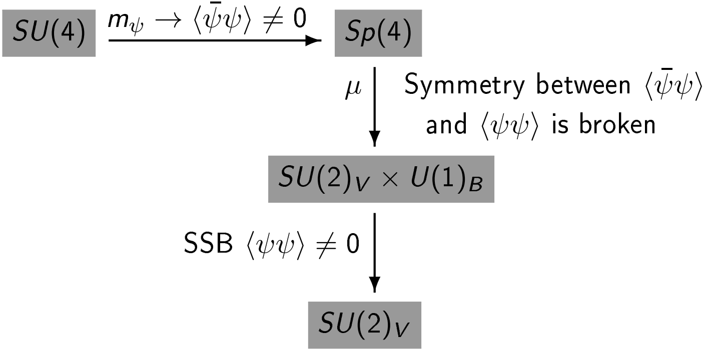
Symmetry Breaking Pattern $N_f = 2$ [Kogut *et* al '99]

$SU(4)$


$SU(4)$
$m_\psi \to \langle\bar{\psi}\psi\rangle \neq 0$
$Sp(4)$
no SSB for $\mu = 0$
μ
μ
$SU(2)_L \times SU(2)_R \times U(1)_B$
$SU(2)_V \times U(1)_B$
$\langle\psi\psi\rangle \neq 0$
$\langle\psi\psi\rangle \neq 0$
$SU(2)_L \times SU(2)_R$
$SU(2)_V$

Features of QC_2D

Symmetry Breaking Pattern $N_f = 2$ [Kogut *et* al '99]



Functional RG

Exact RG Flow Equation

$$\partial_k \Gamma_k[\Phi] = \frac{1}{2}\,\mathrm{STr}\,\frac{1}{\Gamma_k^{(2)}[\Phi] + R_k}\partial_k R_k$$

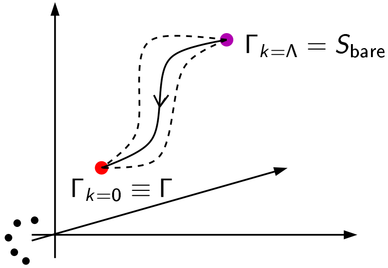
[Wetterich '91]

Functional RG

Exact RG Flow Equation

$$\partial_k \Gamma_k[\Phi] = \frac{1}{2}\,\mathrm{STr}\,\frac{1}{\Gamma_k^{(2)}[\Phi] + R_k}\partial_k R_k$$ [Wetterich '91]

- Γ_k interpolates between mircoscopic action S and full quantum effective action Γ

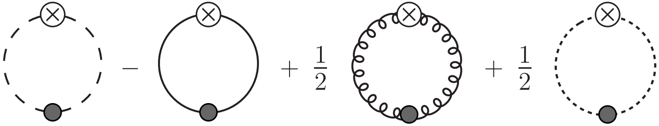


[Gies '06]

Functional RG
Exact RG Flow Equation

- integrate out fluctuations, $\Phi = (\varphi, \psi, \bar{\psi}, A, c, \bar{c})$

$$\partial_k \Gamma_k[\Phi] = \frac{1}{2}$$

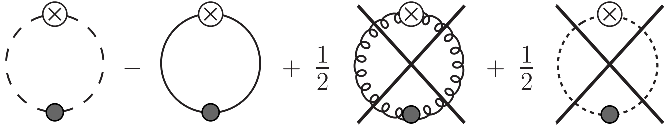


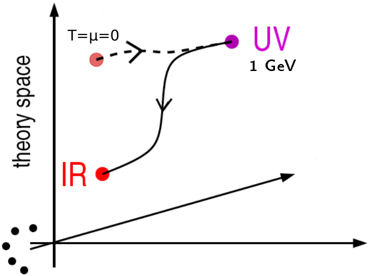
Functional RG

Exact RG Flow Equation

- integrate out fluctuations, $\Phi = (\varphi, \psi, \bar{\psi})$

$$\partial_k \Gamma_k[\Phi] = \frac{1}{2}\ \bigcirc_{\text{dashed}} - \bigcirc + \frac{1}{2}\ \bigcirc_{\text{gluon}} + \frac{1}{2}\ \bigcirc_{\text{dotted}}$$





[Diehl *et* al '10]

- $SU(4) \simeq SO(6) \to O(6)$-order parameter potential + explicit breaking terms

$$U_k = V_k(\vec{\pi}^2 + \sigma^2 + \Delta_1^2 + \Delta_2^2) - c\sigma - \mu^2|\Delta|^2$$

- $SU(4) \simeq SO(6) \to O(6)$-order parameter potential + explicit breaking terms

$$U_k = V_k(\vec{\pi}^2 + \sigma^2 + \Delta_1^2 + \Delta_2^2) - c\sigma - \mu^2|\Delta|^2$$

- Global minimum determines the condensates $\langle\sigma\rangle$, $\langle|\Delta|\rangle$:

$$\begin{array}{ll} \text{Normal phase:} & m^2 - \mu^2 > 0\,, \quad \langle\sigma\rangle = \frac{c}{m^2}\,, \quad \langle|\Delta|\rangle = 0 \\ \text{Superfluid phase:} & m^2 - \mu^2 < 0\,, \quad \langle\sigma\rangle = \frac{c}{\mu^2}\,, \quad \langle|\Delta|\rangle \neq 0 \end{array}$$

- $SU(4) \simeq SO(6) \to O(6)$-order parameter potential + explicit breaking terms

$$U_k = V_k(\vec{\pi}^2 + \sigma^2 + \Delta_1^2 + \Delta_2^2) - c\sigma - \mu^2|\Delta|^2$$

- Global minimum determines the condensates $\langle\sigma\rangle$, $\langle|\Delta|\rangle$:

$$\begin{array}{ll} \text{Normal phase:} & m^2 - \mu^2 > 0\,, \quad \langle\sigma\rangle = \frac{c}{m^2}\,, \quad \langle|\Delta|\rangle = 0 \\ \text{Superfluid phase:} & m^2 - \mu^2 < 0\,, \quad \langle\sigma\rangle = \frac{c}{\mu^2}\,, \quad \langle|\Delta|\rangle \neq 0 \end{array}$$

Functional RG

Effective Potential

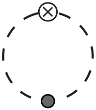
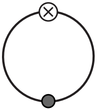
- $SU(4) \simeq SO(6) \to O(6)$-order parameter potential + explicit breaking terms

$$U_k = V_k(\vec{\pi}^2 + \sigma^2 + \Delta_1^2 + \Delta_2^2) - c\sigma - \mu^2|\Delta|^2$$

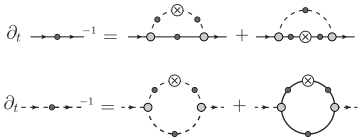
- Global minimum determines the condensates $\langle\sigma\rangle$, $\langle|\Delta|\rangle$:

$$\begin{array}{rlll} \text{Normal phase:} & m^2 - \mu^2 > 0\,, & \langle\sigma\rangle = \frac{c}{m^2}\,, & \langle|\Delta|\rangle = 0 \\ \text{Superfluid phase:} & m^2 - \mu^2 < 0\,, & \langle\sigma\rangle = \frac{c}{\mu^2}\,, & \langle|\Delta|\rangle \neq 0 \end{array}$$

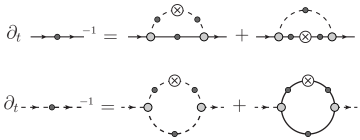
- Flow

$\partial_t U_k = \frac{1}{2}$  $-$ 

- Fluctuations of the propagators → wave function renormalizations $Z_{\Delta,k}$, $Z_{\phi,k}$, $Z_{\psi,k}$

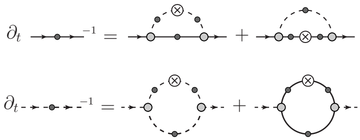


- Fluctuations of the propagators → wave function renormalizations $Z_{\Delta,k}$, $Z_{\phi,k}$, $Z_{\psi,k}$



- Running Yukawa coupling, $m_{Quark} = h_k \langle \bar{\psi}\psi \rangle = h_k \langle \sigma \rangle$

- Fluctuations of the propagators → wave function renormalizations $Z_{\Delta,k}$, $Z_{\phi,k}$, $Z_{\psi,k}$

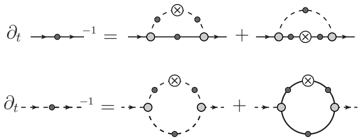


- Running Yukawa coupling, $m_{Quark} = h_k \langle \bar{\psi}\psi \rangle = h_k \langle \sigma \rangle$

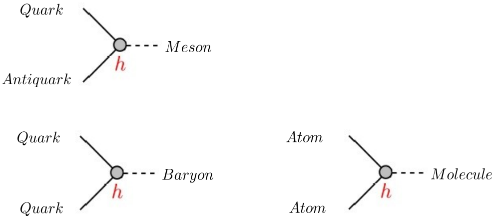
Functional RG

Improving the truncation

- Fluctuations of the propagators → wave function renormalizations $Z_{\Delta,k}$, $Z_{\phi,k}$, $Z_{\psi,k}$

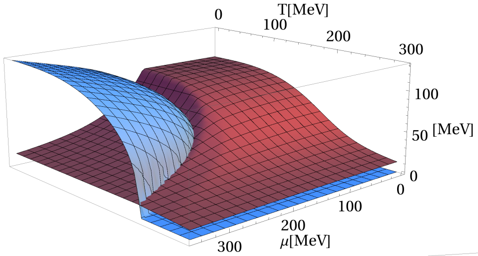


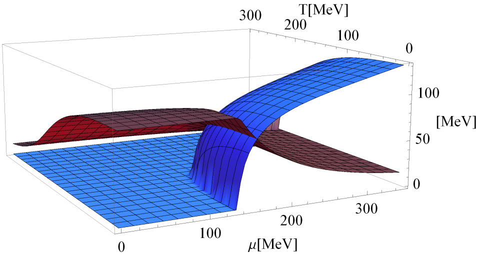
- Running Yukawa coupling, $m_{Quark} = h_k \langle \bar{\psi}\psi \rangle = h_k \langle \sigma \rangle$



Results

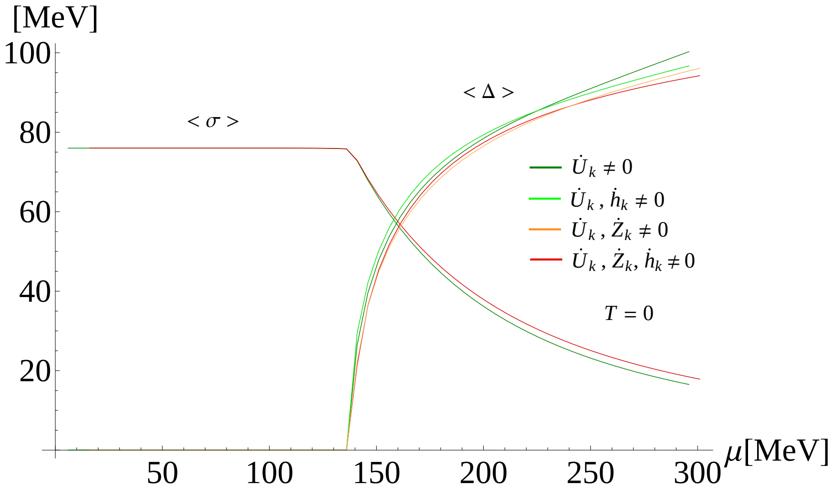
The QC_2D Phase Diagram





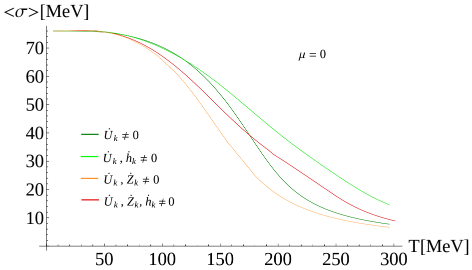
Results (preliminary)

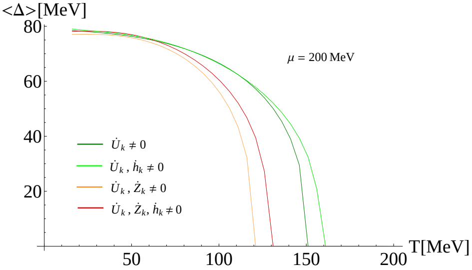
μ Dependence

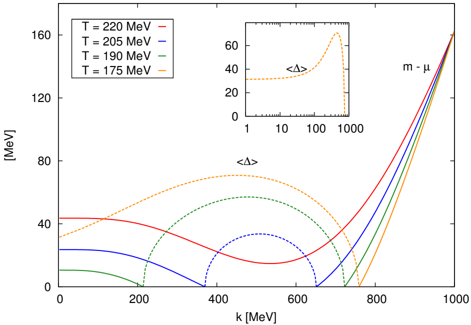


Results (preliminary)

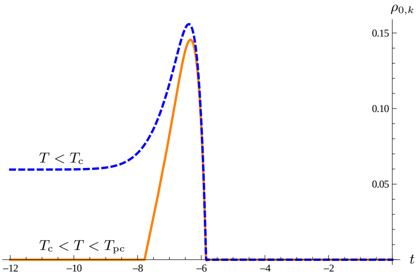
Temperature Dependence







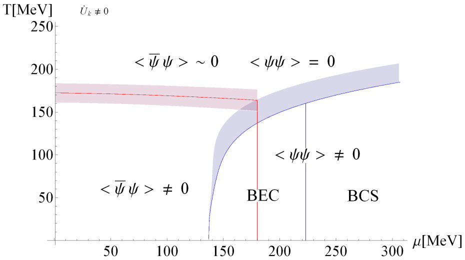
← $\mu = 230$ MeV

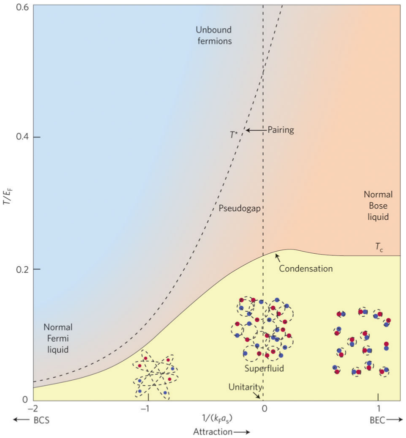


[Boettcher *et* al 2012]

Results (preliminary)

The QC_2D Phase Diagram

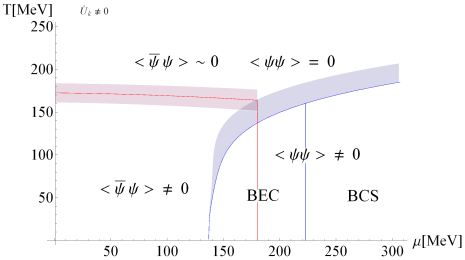


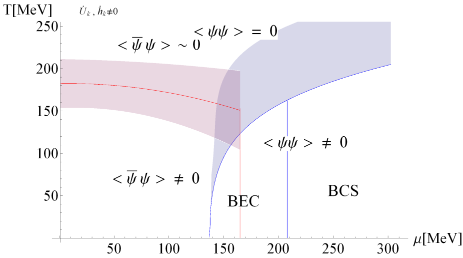


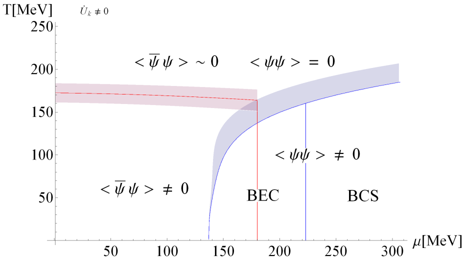
[Randeria, Nature Physics 6 (2010)]

Results (preliminary)

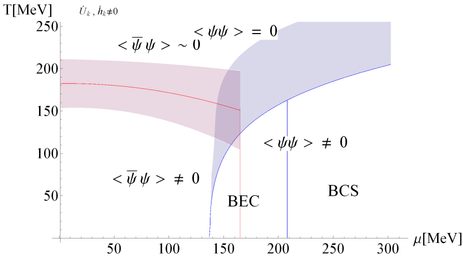
The QC_2D Phase Diagram



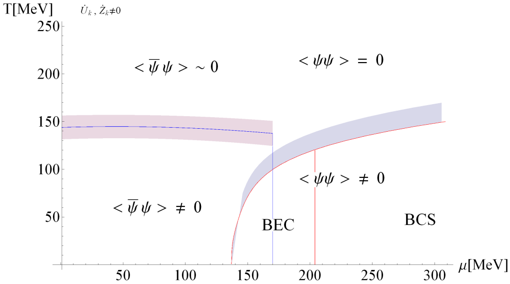




T[MeV]
$\dot{U}_k \neq 0$
250
200
150
100
50
$< \bar{\psi}\psi > \sim 0$
$< \psi\psi > = 0$
$< \psi\psi > \neq 0$
$< \bar{\psi}\psi > \neq 0$
BEC
BCS
50
100
150
200
250
300
μ[MeV]



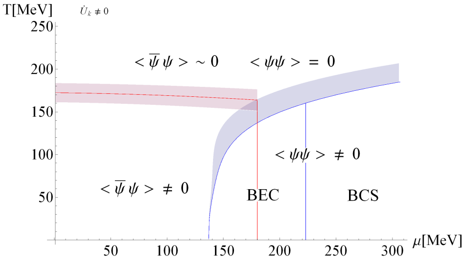
T[MeV]
$\dot{U}_k, \dot{h}_k \neq 0$
250
200
150
100
50
$< \psi\psi > = 0$
$< \bar{\psi}\psi > \sim 0$
$< \psi\psi > \neq 0$
$< \bar{\psi}\psi > \neq 0$
BEC
BCS
50
100
150
200
250
300
μ[MeV]

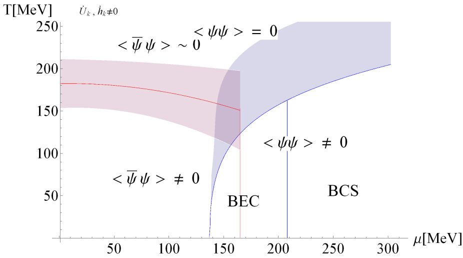


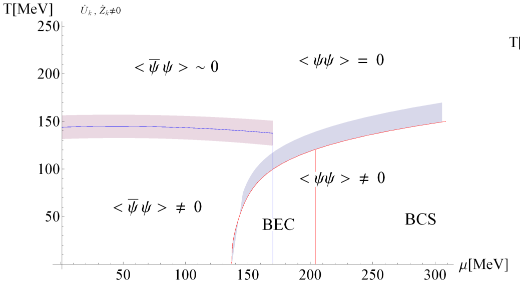
T[MeV]
$\dot{U}_k, \dot{Z}_k \neq 0$
250
200
150
100
50
$< \bar{\psi}\psi > \sim 0$
$< \psi\psi > = 0$
$< \psi\psi > \neq 0$
$< \bar{\psi}\psi > \neq 0$
BEC
BCS
50
100
150
200
250
300
μ[MeV]

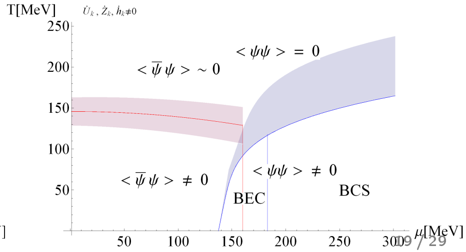
Results (preliminary)

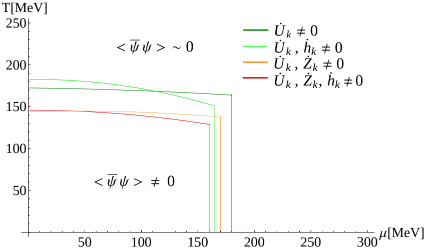
The QC_2D Phase Diagram



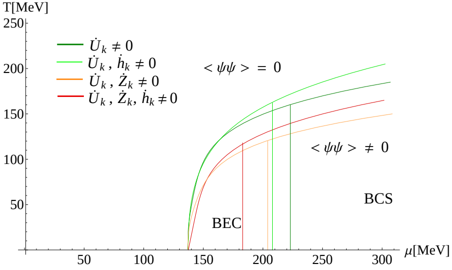








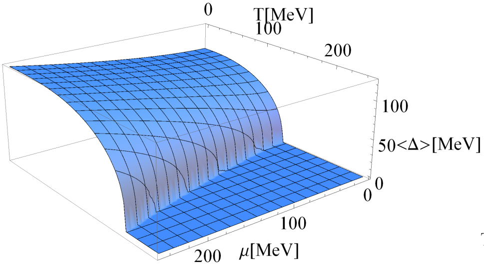
T[MeV]
250
200
150
100
50
50
100
150
200
250
300
μ[MeV]
< ψ̄ψ > ~ 0
< ψ̄ψ > ≠ 0
U̇_k ≠ 0
U̇_k , ḣ_k ≠ 0
U̇_k , Ż_k ≠ 0
U̇_k , Ż_k, ḣ_k ≠ 0

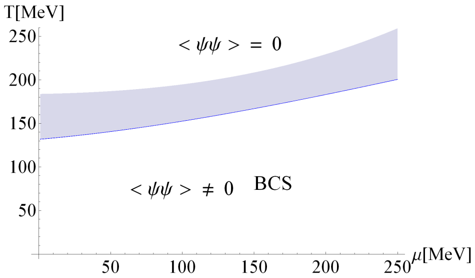


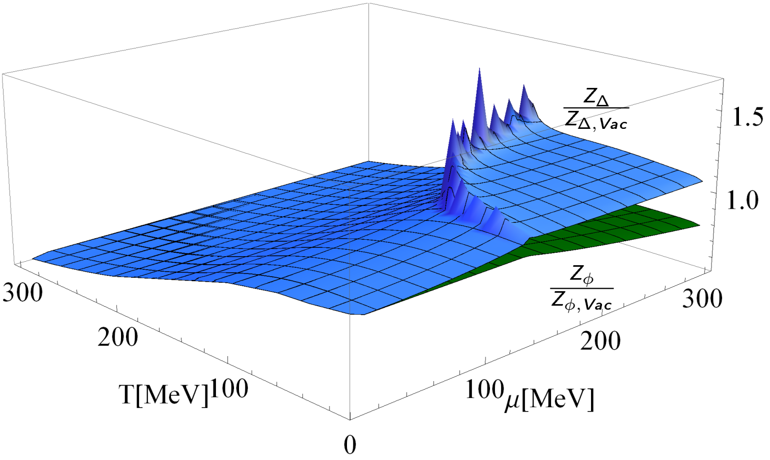
T[MeV]
250
200
150
100
50
50
100
150
200
250
300
μ[MeV]
U̇_k ≠ 0
U̇_k , ḣ_k ≠ 0
U̇_k , Ż_k ≠ 0
U̇_k , Ż_k, ḣ_k ≠ 0
< ψψ > = 0
< ψψ > ≠ 0
BCS
BEC

Results

Chiral Limit $c = 0$, $m_\pi = 0$



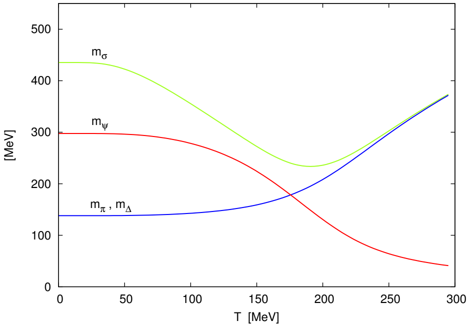




$\frac{Z_\Delta}{Z_{\Delta,Vac}}$
1.5
1.0
$\frac{Z_\phi}{Z_{\phi,Vac}}$
300
200
T[MeV]100
0
100μ[MeV]
200
300

Results

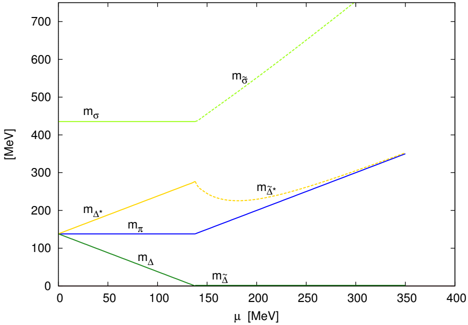
Mass Spectrum



$\leftarrow \quad \mu = 0$

$\det G_{Boson}(p_0, \vec{p} = 0) = 0$

$T = 0 \;\rightarrow$



Summary/Outlook

- Summary

Summary/Outlook

- Summary
  - phase diagram of 2-colour 2-flavour QCD, baryonic degrees of freedom,

Summary/Outlook

- Summary
  - phase diagram of 2-colour 2-flavour QCD, baryonic degrees of freedom,
  - FRG treatment in yields a pre-condensation phase

Summary/Outlook

- Summary
  - phase diagram of 2-colour 2-flavour QCD, baryonic degrees of freedom,
  - FRG treatment in yields a pre-condensation phase
  - impact of running h_k and $Z_{\Delta,k}$, $Z_{\phi,k}$, $Z_{\psi,k}$

Summary/Outlook

- Summary
  - phase diagram of 2-colour 2-flavour QCD, baryonic degrees of freedom,
  - FRG treatment in yields a pre-condensation phase
  - impact of running h_k and $Z_{\Delta,k}$, $Z_{\phi,k}$, $Z_{\psi,k}$
- Outlook

Summary/Outlook

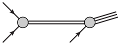
- Summary
  - phase diagram of 2-colour 2-flavour QCD, baryonic degrees of freedom,
  - FRG treatment in yields a pre-condensation phase
  - impact of running h_k and $Z_{\Delta,k}$, $Z_{\phi,k}$, $Z_{\psi,k}$
- Outlook
  - LPA beyond ϕ^4

Summary/Outlook

- Summary
  - phase diagram of 2-colour 2-flavour QCD, baryonic degrees of freedom,
  - FRG treatment in yields a pre-condensation phase
  - impact of running h_k and $Z_{\Delta,k}$, $Z_{\phi,k}$, $Z_{\psi,k}$
- Outlook
  - LPA beyond ϕ^4
  - confinement/deconfinement phase transition

Summary/Outlook

- Summary
  - phase diagram of 2-colour 2-flavour QCD, baryonic degrees of freedom,
  - FRG treatment in yields a pre-condensation phase
  - impact of running h_k and $Z_{\Delta,k}$, $Z_{\phi,k}$, $Z_{\psi,k}$
- Outlook
  - LPA beyond ϕ^4
  - confinement/deconfinement phase transition
  - baryons in 3-colour QCD



Backup Slides

Symmetry group	Generators	Pseudo-/Goldstones
$SU(4)$	15	-

Symmetry Breaking Pattern $N_f = 2$ [Kogut *et* al '99]

Symmetry group	Generators	Pseudo-/Goldstones
$SU(4)$	15	-
$\langle\psi\psi\rangle = \langle\Delta_1\rangle \downarrow$		
$Sp(4)$	10	5 G ($\vec{\pi}$, σ, Δ_2)

Symmetry Breaking Pattern $N_f = 2$ [Kogut *et* al '99]

Symmetry group	Generators	Pseudo-/Goldstones
$SU(4)$	15	-
$\langle\psi\psi\rangle = \langle\Delta_1\rangle \downarrow$		
$Sp(4)$	10	5 G ($\vec{\pi}$, σ, Δ_2)
$\mu \downarrow$		
$SU(2)_L \times SU(2)_R$	6	4 PG ($\vec{\pi}$, σ), 1 G (Δ_2)

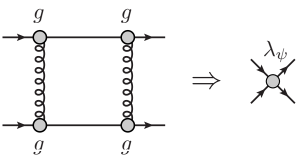
Symmetry Breaking Pattern $N_f = 2$ [Kogut *et* al '99]

Symmetry group	Generators	Pseudo-/Goldstones
$SU(4)$	15	-
$\langle\psi\psi\rangle = \langle\Delta_1\rangle \downarrow$		
$Sp(4)$	10	5 G ($\vec{\pi}$, σ, Δ_2)
$\mu \downarrow$		
$SU(2)_L \times SU(2)_R$	6	4 PG ($\vec{\pi}$, σ), 1 G (Δ_2)
$m_\psi \downarrow$		
$SU(2)_V$	3	3 PPG ($\vec{\pi}$), 1 PG (σ), 1G (Δ_2)

Backup Slides

Hadronization

- NJL model

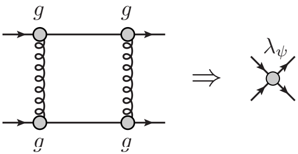


$$\Longrightarrow \quad \lambda_\psi \left[(\bar{\psi}\psi)^2 - (\bar{\psi}\gamma_5\vec{\tau}\psi)^2 - \left|\psi^T \epsilon \psi\right|^2 \right]$$

Backup Slides

Hadronization

- NJL model



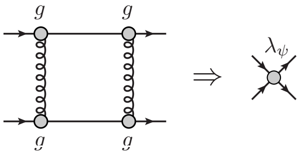
$$\Longrightarrow \quad \lambda_\psi \left[(\bar\psi\psi)^2 - (\bar\psi\gamma_5\vec\tau\psi)^2 - \left|\psi^T\epsilon\psi\right|^2 \right]$$

- Hubbard-Stratonovich transformation

$$\lambda_\psi(\bar\psi\psi)^2 = h\sigma\bar\psi\psi + \frac{1}{2}m^2\sigma^2 \quad \text{with} \quad \lambda_\psi = -\frac{h^2}{2m^2}$$

and EoM(σ) $\rightarrow$ $\sigma = \bar\psi\psi$

- NJL model



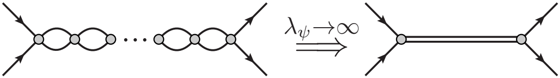
$$\Longrightarrow \quad \lambda_\psi \left[(\bar{\psi}\psi)^2 - (\bar{\psi}\gamma_5\vec{\tau}\psi)^2 - \left|\psi^T \epsilon \psi\right|^2 \right]$$

- Hubbard-Stratonovich transformation

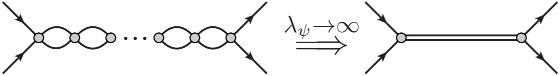
$$\lambda_\psi(\bar{\psi}\psi)^2 = h\sigma\bar{\psi}\psi + \frac{1}{2}m^2\sigma^2 \quad \text{with} \quad \lambda_\psi = -\frac{h^2}{2m^2}$$

and EoM(σ) $\rightarrow$ $\langle\sigma\rangle = \langle\bar{\psi}\psi\rangle \neq 0$ $\rightarrow$ massterm $m_q = h\langle\sigma\rangle$

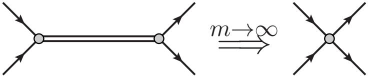
- Large four-fermion coupling limit



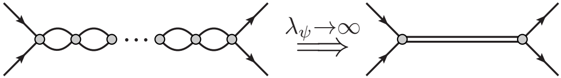
- Large four-fermion coupling limit



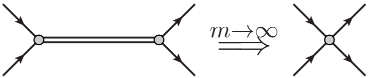
- Large hadron mass limit



- Large four-fermion coupling limit



- Large hadron mass limit



- Dynamical degrees of freedom

Quarks ψ, Gluons A $\Longrightarrow$ ψ, mesons $\phi \sim \bar{\psi}\psi$, baryons $\Delta \sim \psi\psi$, A