# Phase Diagram of Two-Color QCD in a Dyson-Schwinger Approach

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Quark Gluon Plasma meets Cold Atoms - Episode III
Hirschegg, Austria

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# (Selected) Features of Two-Color QCD($QC_2D$)

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$\rightarrow$ cf. talk by Lorenz v. Smekal (16:30 today)

## Dirac operator obeys antiunitary symmetry

- no sign problem in fermion determinant at $\mu \neq 0$
- results from lattice calculations available

## Pauli-Gürsey symmetry (at $\mu = m_q = 0$)

- extended flavor symmetry ($SU(N_f) \times SU(N_f) \times U(1)_B \rightarrow SU(2N_f)$)
- pions and diquarks degenerate

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# Dyson-Schwinger Equation for Quarks (qDSE)

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$$S^{-1}(p) = Z_2 S_0^{-1}(p) + Z_{1F} g^2 \int \frac{d^4 q}{(2\pi)^4} \gamma_\mu \frac{\tau^a}{2} S(q) \Gamma^b_\nu(p,q) D^{ab}_{\mu\nu}(k = p - q)$$

# Dyson-Schwinger Equation for Quarks (qDSE) ...and its Truncation

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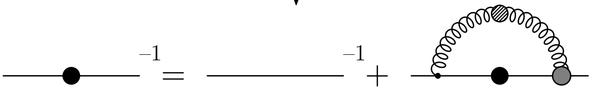
$$S^{-1}(p) = Z_2 S_0^{-1}(p) + Z_{1F} g^2 \int \frac{d^4 q}{(2\pi)^4} \gamma_\mu \frac{\tau^a}{2} S(q) \Gamma_\nu^b(p,q) D_{\mu\nu}^{ab}(k = p - q)$$

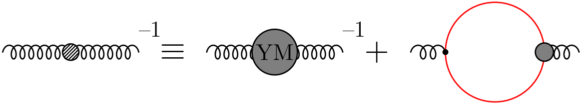
$$\Downarrow$$



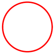

: ansatz for the vertex

$$\Gamma_\mu^a(k) = \tilde{Z}_3 \tau^a \gamma_\mu \left( \frac{d_1}{d_2 + k^2} + \frac{k^2}{\Lambda^2 + k^2} \left( \frac{\beta_0 \alpha(\nu) \mathrm{Ln}\left[\frac{k^2}{\Lambda^2} + 1\right]}{4\pi} \right)^{2\delta} \right)$$

# Dyson-Schwinger Equation for Gluons (gDSE) ...and its Truncation




$$D_{\mu\nu}(k) = \frac{Z_T(k)}{k^2 + G(k)} P^T_{\mu\nu}(k) + \frac{Z_L(k)}{k^2 + F(k)} P^L_{\mu\nu}(k)$$

 : quark loop in Hard-Dense/Hard-Thermal Loop Approximation

$$G(k_4, |\vec{k}|) = m^2_{g,T}(T, \mu, k) \ldots , \qquad F(k_4, |\vec{k}|) = 2m^2_{g,L}(T, \mu, k) \ldots$$

with

$$m^2_{g,T/L}(T, \mu, k) \sim N_f \left( \frac{\pi}{3} T^2 + \frac{1}{\pi} \mu^2 \right) Z_{T/L}(k) \Gamma(k)$$

# Dyson-Schwinger Equation for Gluons (gDSE) ...and its Truncation

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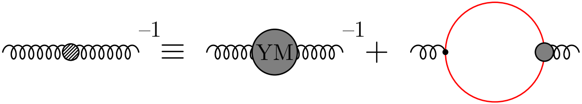
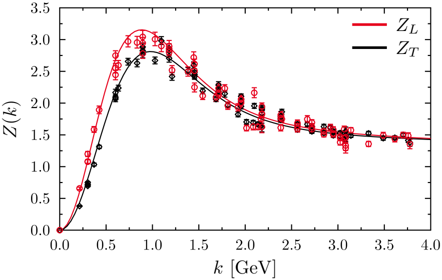



$$D_{\mu\nu}(k) = \frac{Z_T(k)}{k^2 + G(k)} P^T_{\mu\nu}(k) + \frac{Z_L(k)}{k^2 + F(k)} P^L_{\mu\nu}(k)$$

:

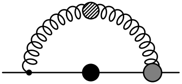
use input from Lattice (C. S. Fischer et al., EPJ C68 (2010))

right: $T = 109\,\text{MeV}$



# Dyson-Schwinger Equation for Quarks (qDSE) ...in the Nambu-Gorkov Formalism


$$= \begin{pmatrix} S^+ & T^- \\ T^+ & S^- \end{pmatrix}$$


$$= \begin{pmatrix} \Sigma^+ & \Phi^- \\ \Phi^+ & \Sigma^- \end{pmatrix}$$


$$= D_{\mu\nu}(k)$$


$$= \begin{pmatrix} \tau^a & \\ & -\tau^{aT} \end{pmatrix} \gamma_\mu \Gamma(p,q)$$

$\tau$: SU(2) generators in color space

$$\begin{aligned} \Sigma^+(p) &\propto \int d^4q \quad \tau^a \gamma^\mu S^+(q) \tau^a \gamma^\nu \quad \Gamma(p,q) D_{\mu\nu}(p-q) \\ \Phi^+(p) &\propto \int d^4q \quad -\tau^{aT} \gamma^\mu T^+(q) \tau^a \gamma^\nu \quad \Gamma(p,q) D_{\mu\nu}(p-q) \end{aligned}$$

# Pauli-Gürsey Symmetry
# ...in the Formalism

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Consider color-flavor product:

$$\Sigma^{+}(p) \propto \int \mathrm{d}^4 q \quad \tau^a \gamma^\mu S^{+}(q) \tau^a \gamma^\nu \quad \Gamma_\mu(p,q) D_{\mu\nu}(p-q)$$
$$\Phi^{+}(p) \propto \int \mathrm{d}^4 q \quad -\tau^{aT} \gamma^\mu T^{+}(q) \tau^a \gamma^\nu \quad \Gamma_\mu(p,q) D_{\mu\nu}(p-q)$$

color-flavor components of the propagators:

$$S^{+}(p) = \quad \mathbf{1} \quad s^{+}(p)$$
$$T^{+}(p) = \quad \tau^2 \sigma^2 \quad t^{+}(p)$$

$\sigma$: SU(2) generators in flavor space

$$\Sigma^{+}(p) \propto \int \mathrm{d}^4 q \quad 3\gamma^\mu S^{+}(q) \gamma^\nu \quad \Gamma(p,q) D_{\mu\nu}(p-q)$$
$$\Phi^{+}(p) \propto \int \mathrm{d}^4 q \quad 3\gamma^\mu T^{+}(q) \gamma^\nu \quad \Gamma(p,q) D_{\mu\nu}(p-q)$$

**$\Rightarrow$ same coupling for $\bar{q}q$ and $qq$!**

# Calculating Condensates

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## Chiral Condensate

$$\langle \bar{q}q \rangle \propto \int \mathrm{d}^4 q \mathrm{Tr} S^+(q)$$

- integral UV-divergent: quadratic$\sim m$, logarithmic$\sim \langle \bar{q}q \rangle$
- regularize with condensate at high temperature $\langle \bar{q}q \rangle' \equiv \langle \bar{q}q \rangle - \langle \bar{q}q \rangle|_{\mu=0}^{T=600\text{ MeV}}$

## Diquark Condensate

$$\langle qq \rangle \equiv \left\langle q^T C \tau^2 \sigma^2 \gamma_5 q \right\rangle \propto \int \mathrm{d}^4 q \mathrm{Tr} \left[ \tau^2 \sigma^2 \gamma_5 T^+(q) \right]$$

- integral converges

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# Pauli-Gürsey Symmetry
# ...in Numerical Calculations

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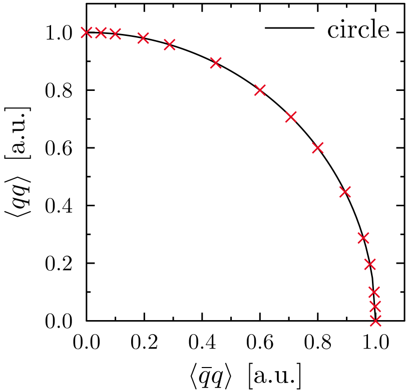
$\mu = m = 0$

$$\langle \bar{q}q \rangle \propto \int d^4q \mathrm{Tr} S^+(q)$$

$$\langle qq \rangle \equiv \langle q^T C \tau^2 \sigma^2 \gamma_5 q \rangle \propto \int d^4q \mathrm{Tr} \left[ \tau^2 \sigma^2 \gamma_5 T^+(q) \right]$$



# Silver Blaze Property (at finite quark mass)

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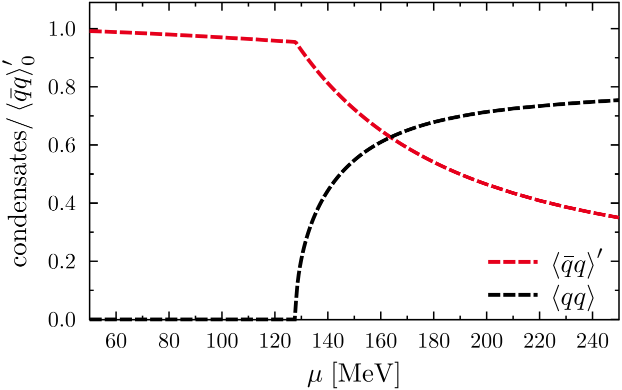
- for $T = 0$, thermodynamic observables independent of $\mu$ unless $\mu$ exceeds lowest excitation threshold
- for $QC_2D$: observables unchanged for $\mu < m_\pi/2$

# Silver Blaze Property (at finite quark mass)

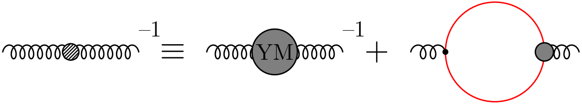
for $T = 0$ and $\mu < m_\pi/2$

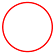
- observables should be independent of $\mu$




- $m \neq 0$
- $T = 20$ MeV
- $\langle \bar{q}q \rangle' \equiv \langle \bar{q}q \rangle - \langle \bar{q}q \rangle\big|_{\mu=0}^{T=600\text{ MeV}}$
- $\langle \bar{q}q \rangle'_0 \equiv \langle \bar{q}q \rangle'\big|_{\mu=0}^{T=20\text{ MeV}}$

# Hard-Thermal/Hard-Dense Loop




 $\sim N_f \left( \frac{\pi}{3} T^2 + \frac{1}{\pi} \mu^2 \right)$

**⇒ explicit dependence on $\mu$ violates Silver Blaze property**

# Silver Blaze Property (at finite quark mass)

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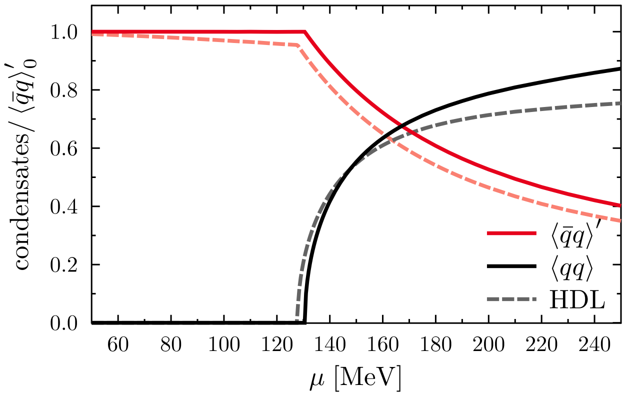
**without HDL**:




- $m \neq 0$
- $T = 20\,\text{MeV}$
- $\langle \bar{q}q \rangle' \equiv \langle \bar{q}q \rangle - \langle \bar{q}q \rangle \big|_{\mu=0}^{T=600\,\text{MeV}}$
- $\langle \bar{q}q \rangle'_0 \equiv \langle \bar{q}q \rangle' \big|_{\mu=0}^{T=20\,\text{MeV}}$

# Silver Blaze Property
# (at finite quark mass)

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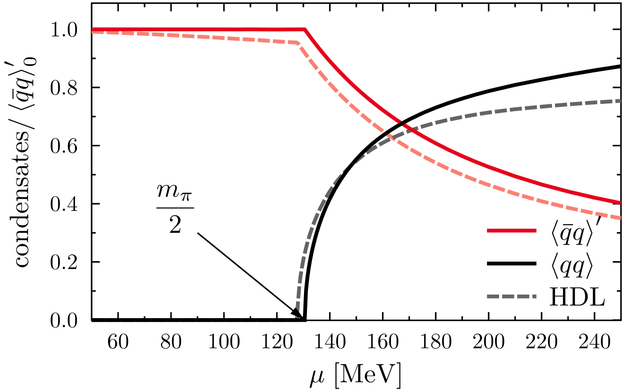
**without HDL**:




- $m \neq 0$
- $T = 20$ MeV
- $\langle\bar{q}q\rangle' \equiv \langle\bar{q}q\rangle - \langle\bar{q}q\rangle\big|_{\mu=0}^{T=600\,\text{MeV}}$
- $\langle\bar{q}q\rangle'_0 \equiv \langle\bar{q}q\rangle'\big|_{\mu=0}^{T=20\,\text{MeV}}$

# Silver Blaze Property
## ...comparision with other approaches

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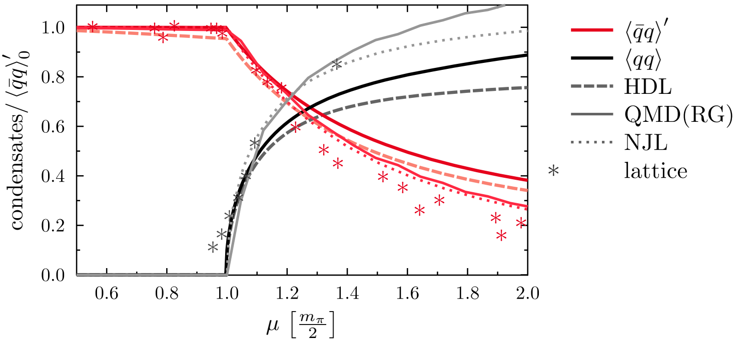


non-DSE plots from N. Strodthoff et al., 1112.5401v2[hep-ph]
lattice data from S. Hands et al., EPJ C17 (2000); EPJ C22 (2001)

# Phase Diagram including $T$ Direction

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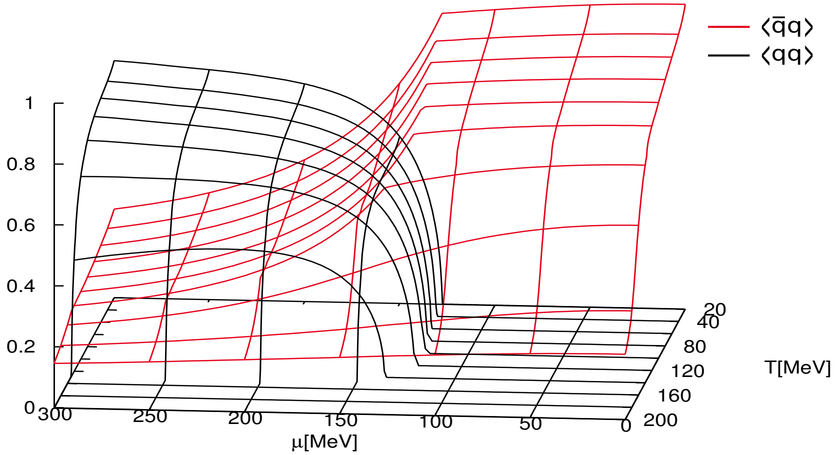

# if diquarks are suppressed

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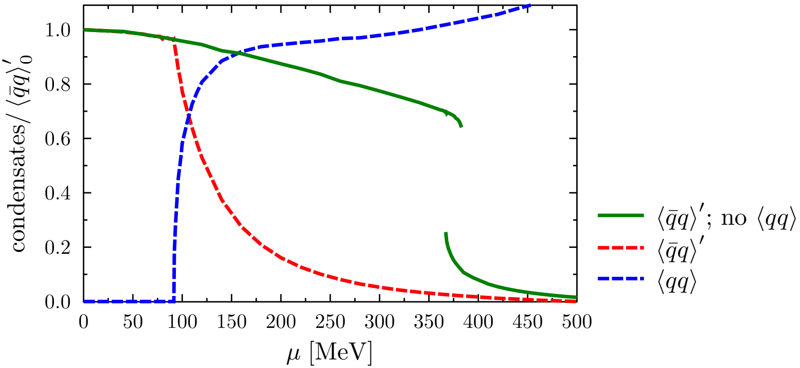


half the current quark mass as before $\langle \bar{q}q \rangle' \equiv \langle \bar{q}q \rangle - \langle \bar{q}q \rangle\big|_{\mu=500\,\text{MeV}}^{T=20\,\text{MeV}}$

# if diquarks are suppressed

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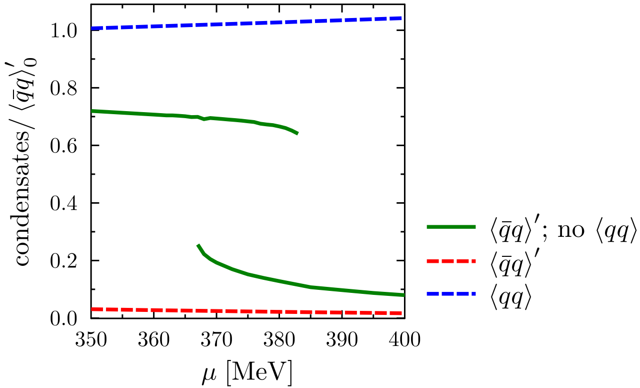

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# Summary & Outlook

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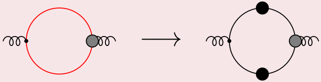
Introduced a Dyson-Schwinger approach to calculate $QC_2D$ which shows

- Pauli-Gürsey symmetry
- Silver Blaze property

## Outlook

- improve truncation (in progress):



- consider other quantities, e.g. the dual condensate
- calculate $m_\pi$ from Bethe-Salpeter equations
  $\Rightarrow$ cross-check with $m_\pi$ from Silver Blaze property

# Thank you!

Also thanks to Daniel Müller and Nils Strodthoff for fruitful discussions.