Ultracold fermions in two and three dimensions

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Hirschegg, 27.8. 2012

Outline of the talk

- Introduction:

  The many-body problem in ultracold atoms

  BCS-BEC crossover and Unitary Fermi gas

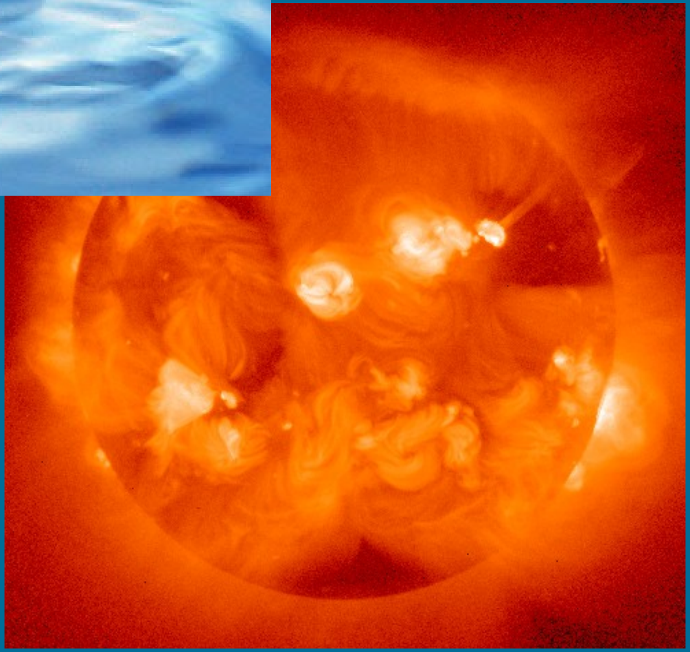
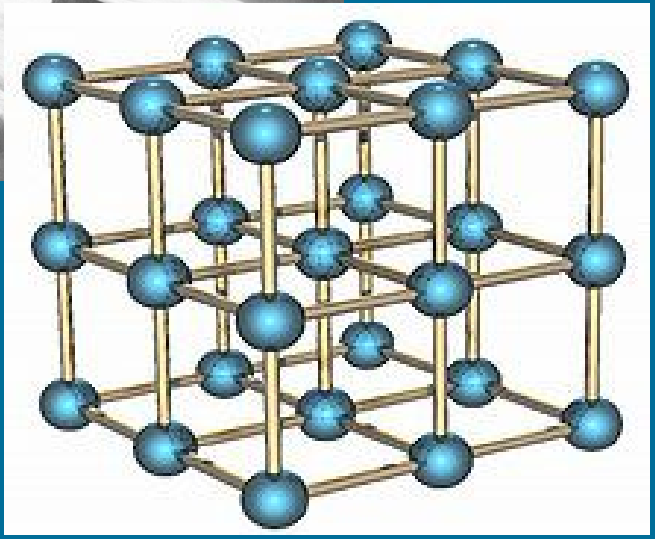
- Functional Renormalization Group study:

  Contact in the Unitary Fermi gas

  The two-dimensional BCS-BEC crossover

The many-body problem

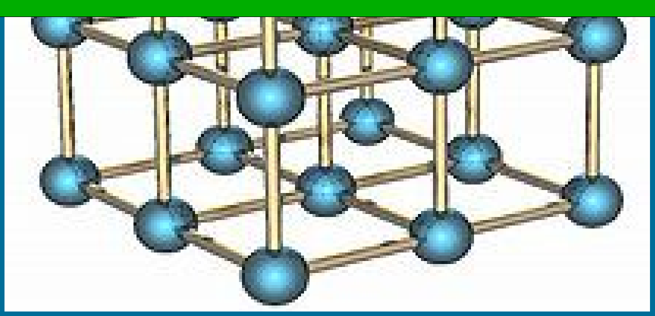


The many-body problem

possibility of a statistical description

collective degrees of freedom



The many-body problem

1st step: Find the right Hamiltonian H

2nd step: Determine the partition function Z

$$Z(\mu, T) = \mathrm{Tr}\left(e^{-\beta(H-\mu N)}\right)$$

The many-body problem

1^{st} step: ~~Find the right Hamiltonian H~~

H is known for cold atoms and QCD!

2^{nd} step: Determine the partition function Z

$$Z(\mu, T) = \mathrm{Tr}\left(e^{-\beta(H-\mu N)}\right)$$

The many-body problem

1st step: ~~Find the right Hamiltonian H~~

H is known for cold atoms and QCD!

2nd step: Determine the partition function Z

$$Z(\mu, T) = \mathrm{Tr}\left(e^{-\beta(H-\mu N)}\right) = \int \mathrm{D}\phi e^{-S[\phi]}$$

path integral

Euclidean quantum field theory

Shopping list

What are the generic features of
quantum many-body systems?

What are reliable theoretical methods
to describe such systems?

What observables reveal advancements
and short-comings of theory?

Shopping list

cold atoms

neutron stars

What are the generic features of
quantum many-body systems?

high-Tc superconductors

early universe

What are reliable theoretical methods
to describe such systems?

What observables reveal advancements
and short-comings of theory?

heavy ion collisions

nuclear matter

quark gluon plasma

Shopping list

Theory

Phase diagram and Equation of state

$$P(\mu, T) = \frac{k_B T}{V} \log Z(\mu, T)$$

Density distribution

Transport coefficients

$$\eta(\mu, T)$$

Experiments with cold atoms

Density images

Collective mode frequencies and damping constants

Expansion after release from trap

Response functions

...

Shopping list

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Response functions

...

The equation of state

Classical ideal gas: $P(n, T) = nk_B T$

Virial expansion for interacting gas:

$$P(n, T) = nk_B T(1 + B_2(T)n + \dots)$$

Van-der-Waals equation of state:

$$P(n, T) = \frac{nk_B T}{1 - bn} - an^2 \simeq nk_B T\left(1 + (b - \frac{a}{k_B T})n + \dots\right)$$

Pressure P(μ,T)

Bose gas

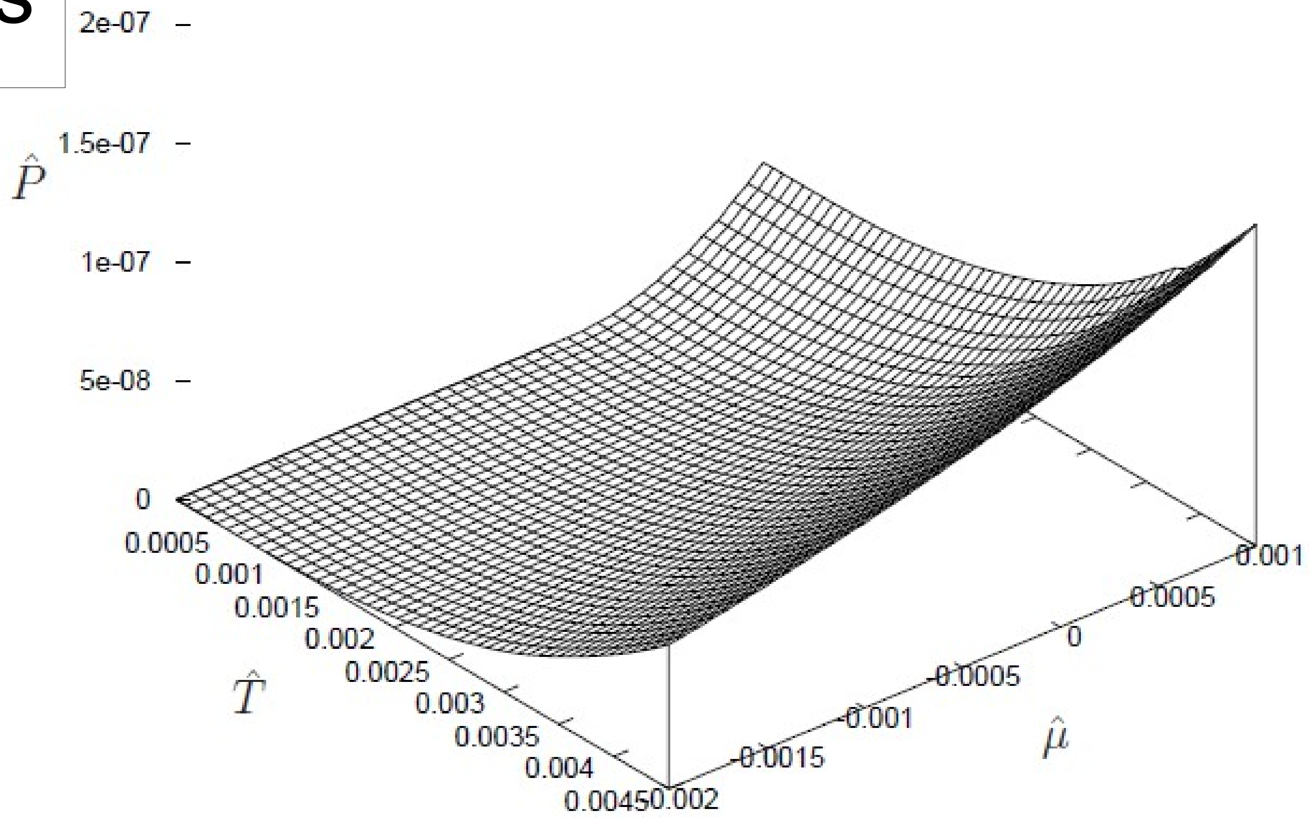


Figure 2.1: Pressure $\hat{P} = Pa^5 m/\hbar^2$ as a function of $\hat{T}$ and $\hat{\mu}$

$\hat{T} = Ta^2 m k_B/\hbar^2$

$\hat{\mu} = \mu a^2 m/\hbar^2$

Density $n=(\partial P/\partial \mu)_T$

Bose gas

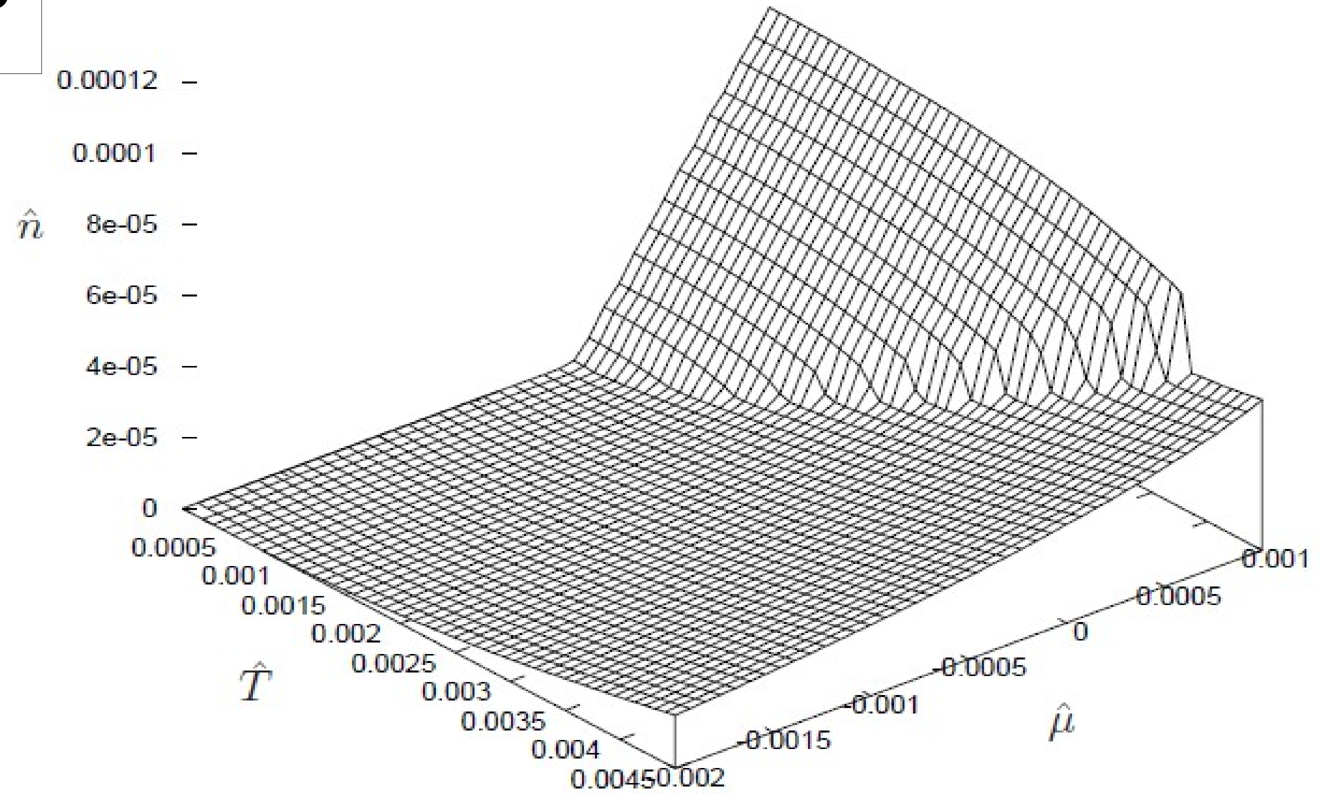


Figure 2.3: Density $\hat{n} = na^3$ as a function of $\hat{T}$ and $\hat{\mu}$

$\hat{T} = Ta^2 m k_B/\hbar^2$

$\hat{\mu} = \mu a^2 m/\hbar^2$

Isothermal compressibility $(\partial^2 P/\partial \mu^2)_T$

Bose gas

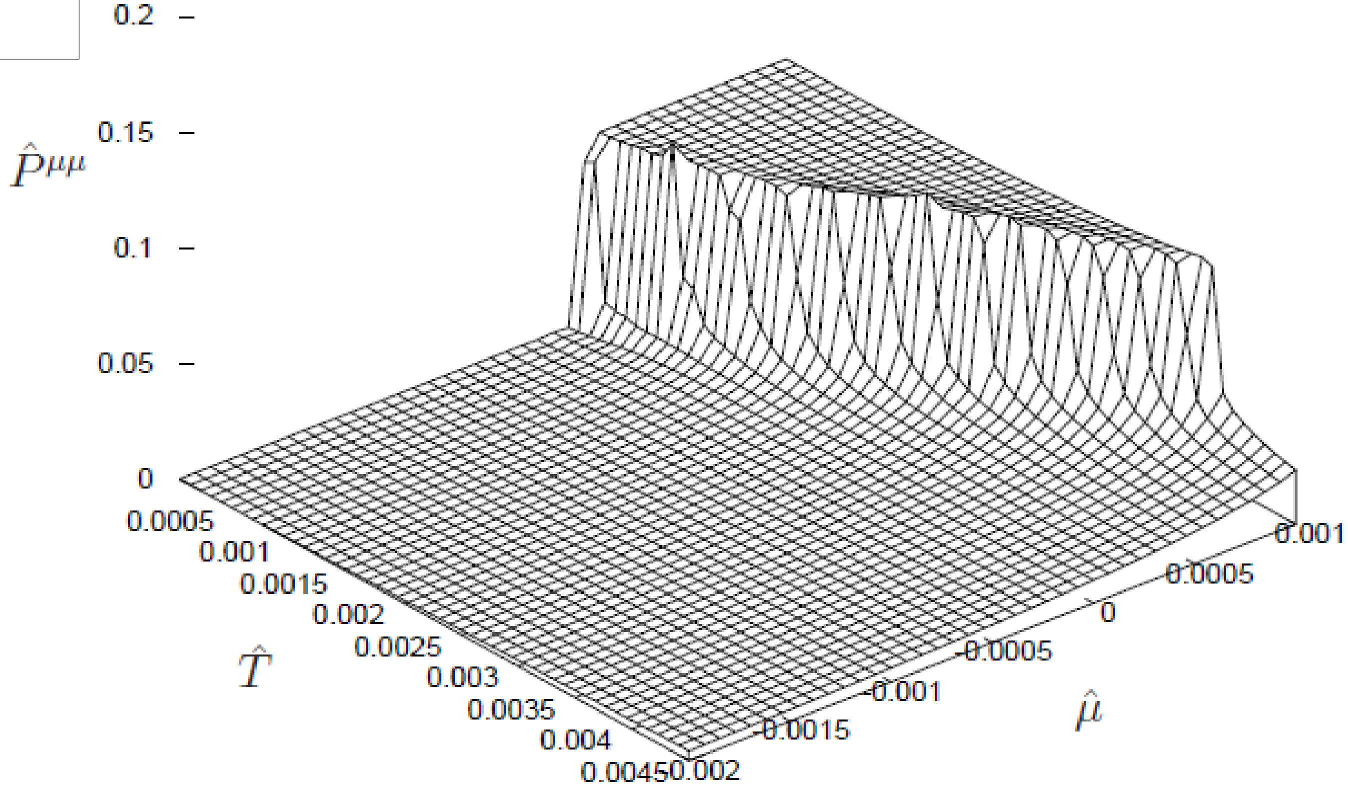


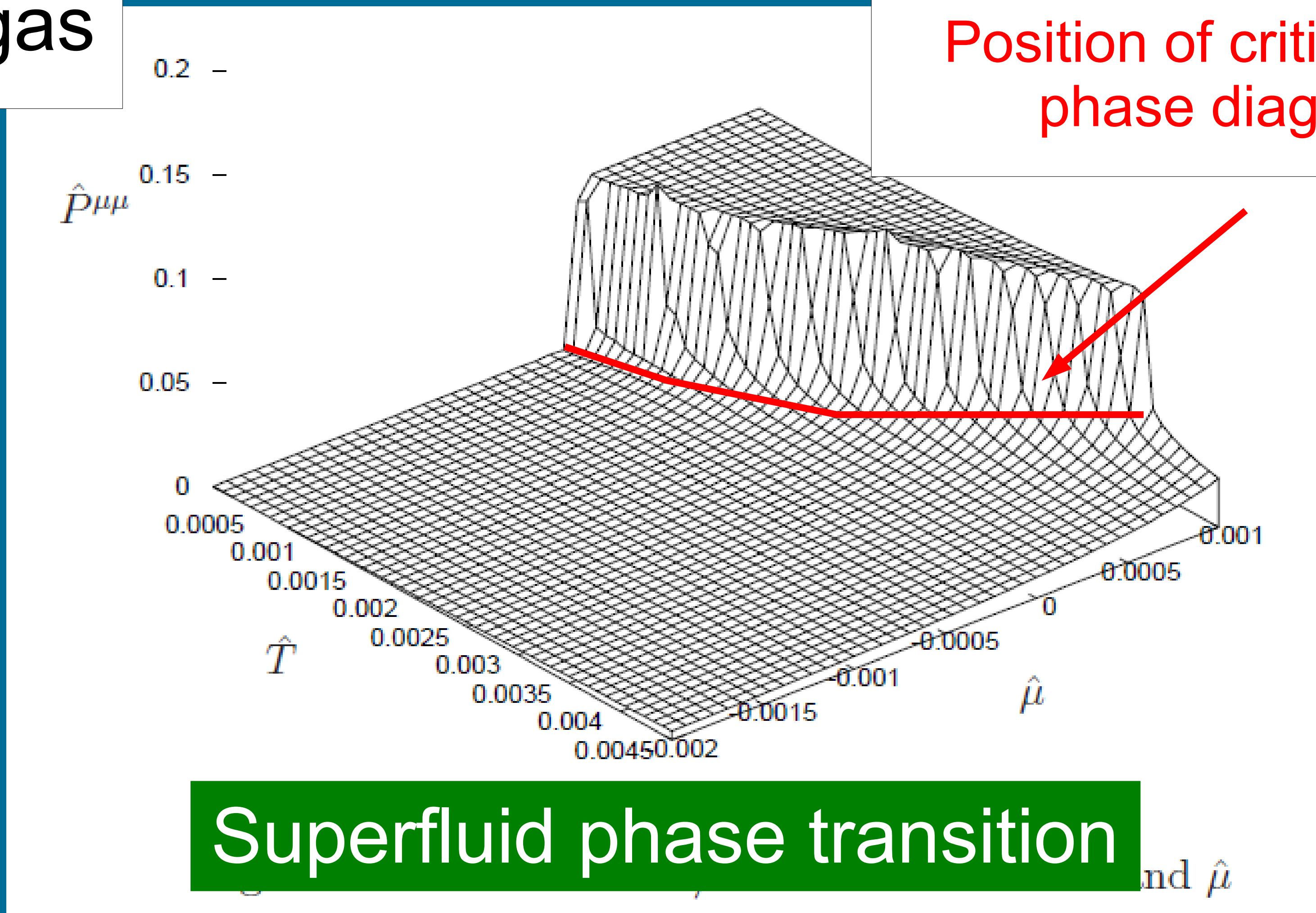
Figure 2.5: $\hat{P}^{\mu\mu} = P^{\mu\mu} a\hbar^2/m$ as a function of $\hat{T}$ and $\hat{\mu}$

$\hat{T} = Ta^2 mk_B/\hbar^2$

$\hat{\mu} = \mu a^2 m/\hbar^2$

Isothermal compressibility $(\partial^2 P/\partial\mu^2)_T$

Bose gas



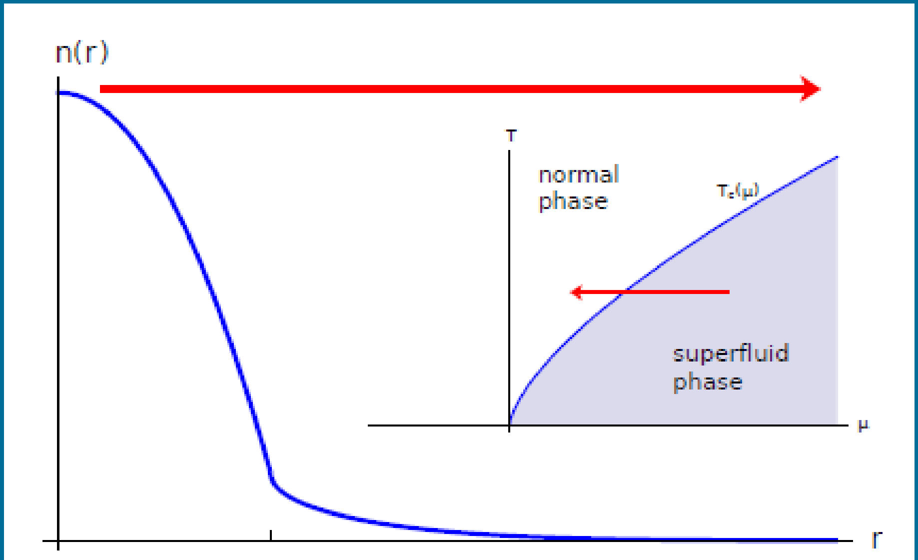
Position of critical line: phase diagram

Superfluid phase transition

$\hat{T} = Ta^2 m k_B/\hbar^2$

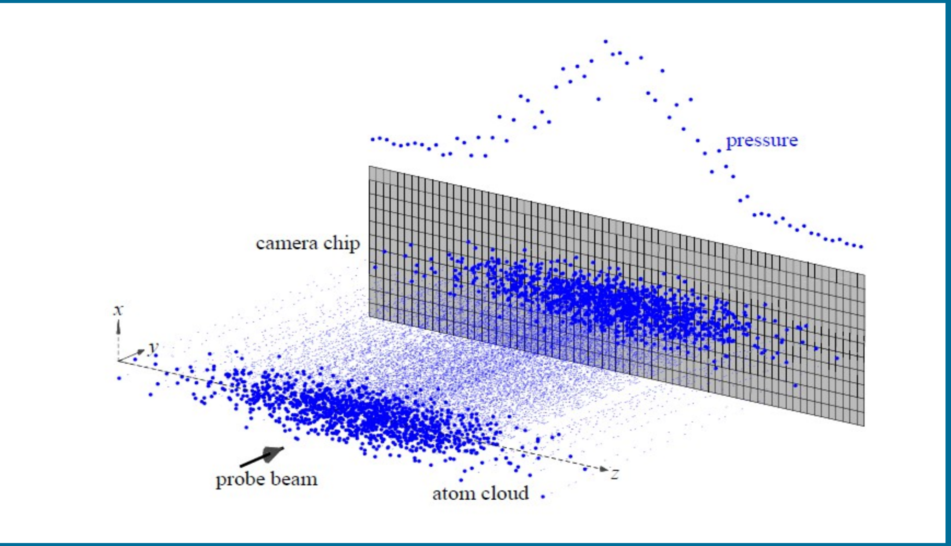
$\hat{\mu} = \mu a^2 m/\hbar^2$

Thermodynamics from density profiles



$$P(\mu, T) \rightarrow P(\mu - V_{\mathrm{ext}}(\vec{x}), T)$$

local density approximation

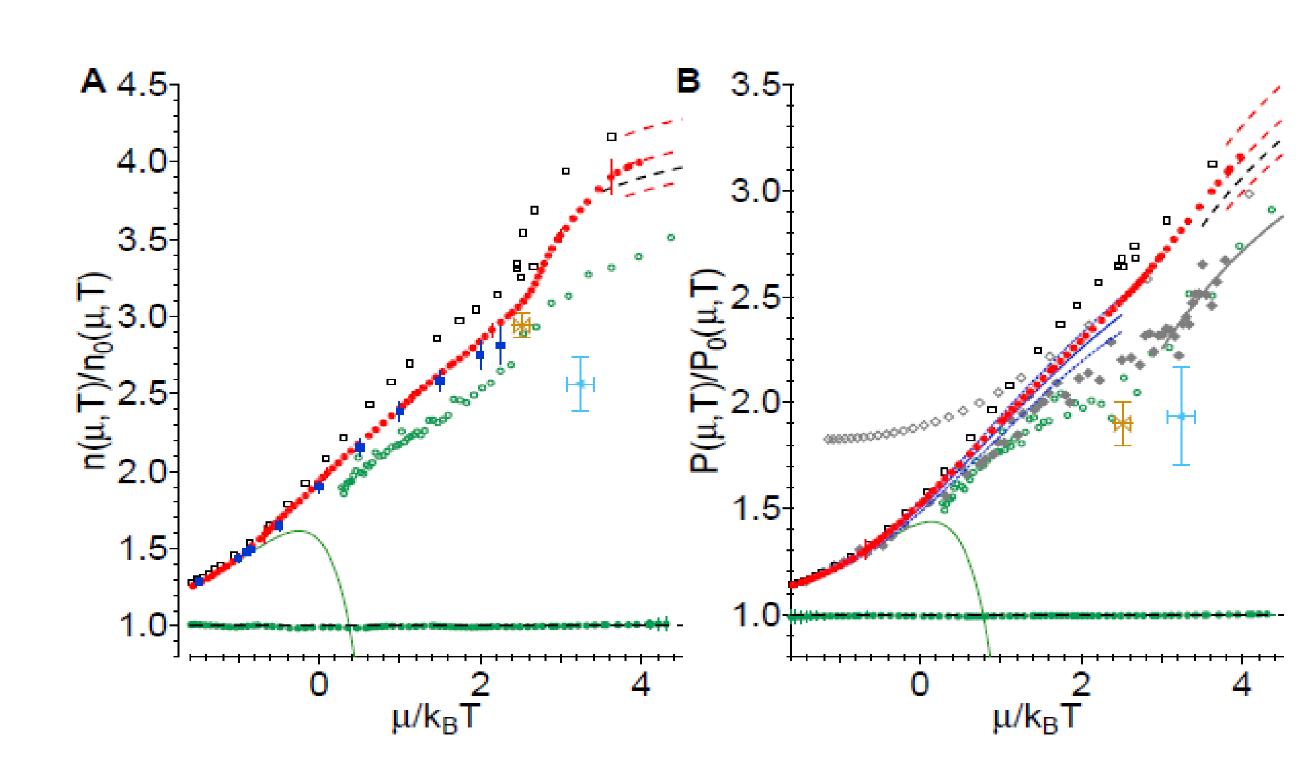


$$P(\mu_z, T) = \frac{m\omega_r^2}{2\pi}\overline{n}(z),$$

T.-L. Ho, Q. Zhou,
Nature Physics **6**, 131 (2010)

Figure: S. Nascimbène et al.,
New Journal of Physics 12 (2010) 103026

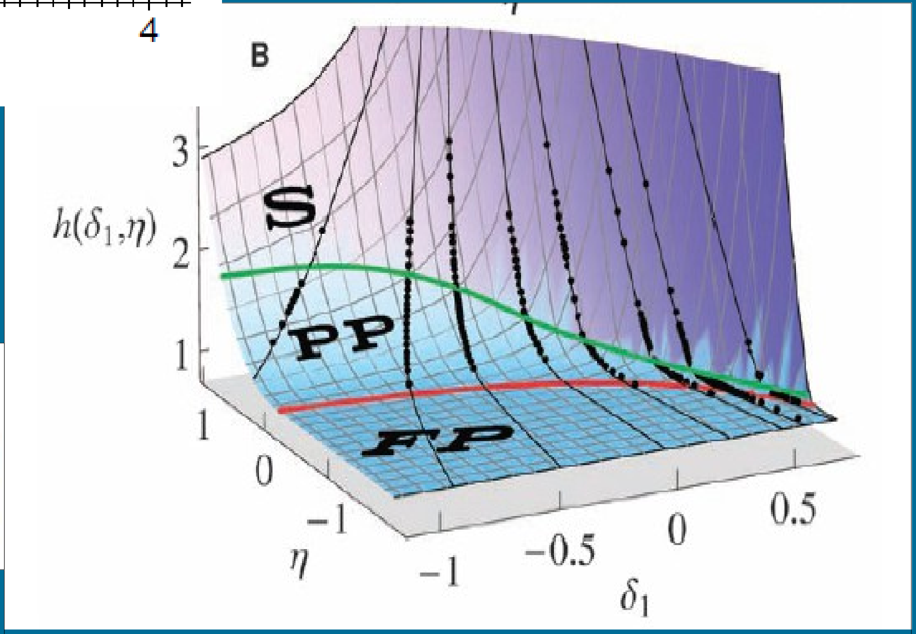
Thermodynamics from density profiles



M. J. H. Ku et al., Science **335**, 563-567 (2012)

imbalanced two-component Fermi gas at T=0:

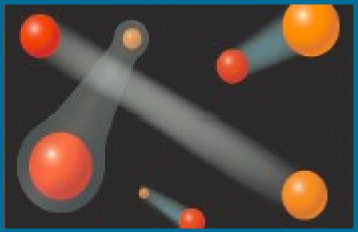
$$P(\mu_1,\mu_2,a) = P_0(\mu_1)h\left(\delta_1 \equiv \frac{\hbar}{\sqrt{2m\mu_1 a}}, \eta \equiv \frac{\mu_2}{\mu_1}\right)$$



N. Navon et al., Science **328**, 729 (2010)

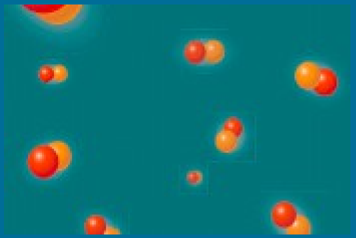
The BCS-BEC Crossover

Two cornerstones of quantum condensation:



BCS

Cooper pairing
of weakly attractive
fermions



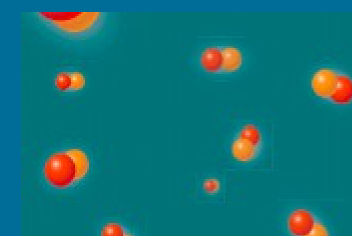
BEC

Bose condensation
of weakly repulsive
bosons

The BCS-BEC Crossover

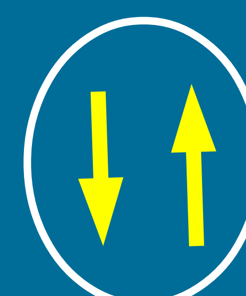
Two cornerstones of quantum condensation:

BCS



BEC

$(k_{\mathrm{F}} a)^{-1}$



$\sigma = 4\pi a^2$

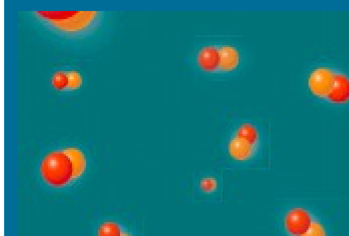
The BCS-BEC Crossover

Two cornerstones of quantum condensation:

BCS

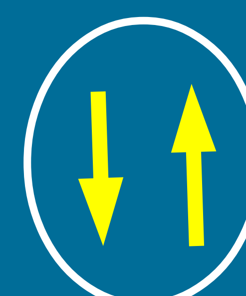
Unitary Fermi gas

$$(k_{\mathrm{F}} a)^{-1} = 0, \;\; \sigma = \frac{4\pi}{p^2}$$



BEC

$$(k_{\mathrm{F}} a)^{-1}$$



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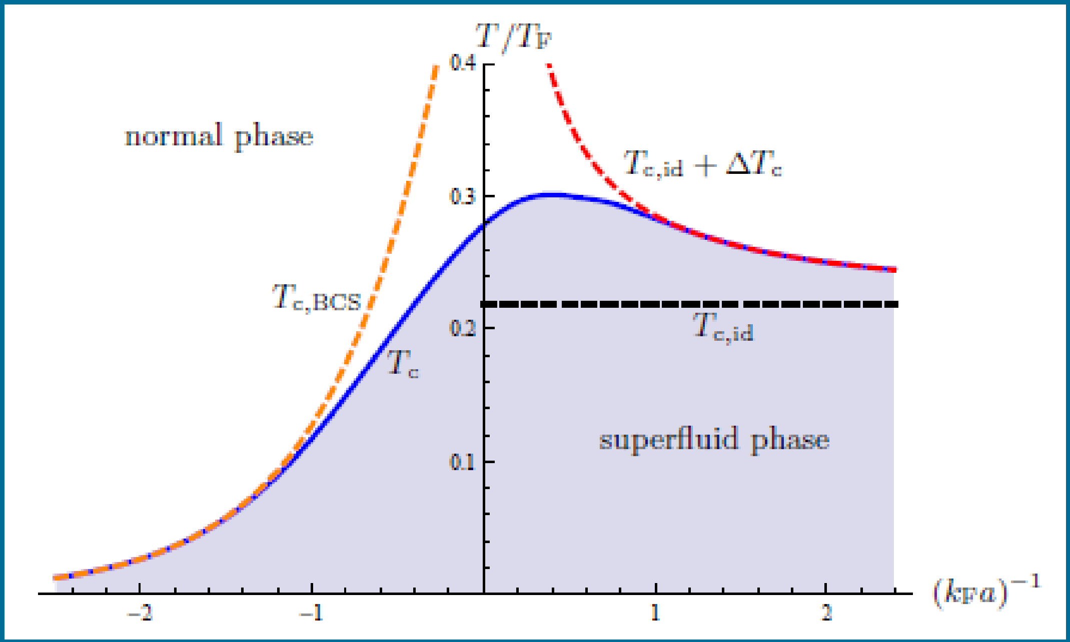
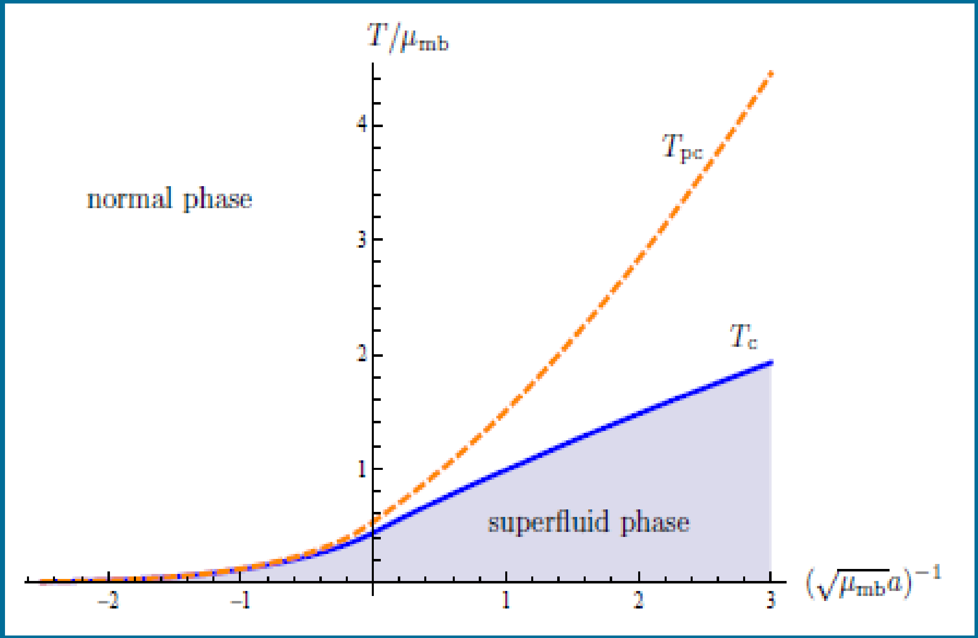
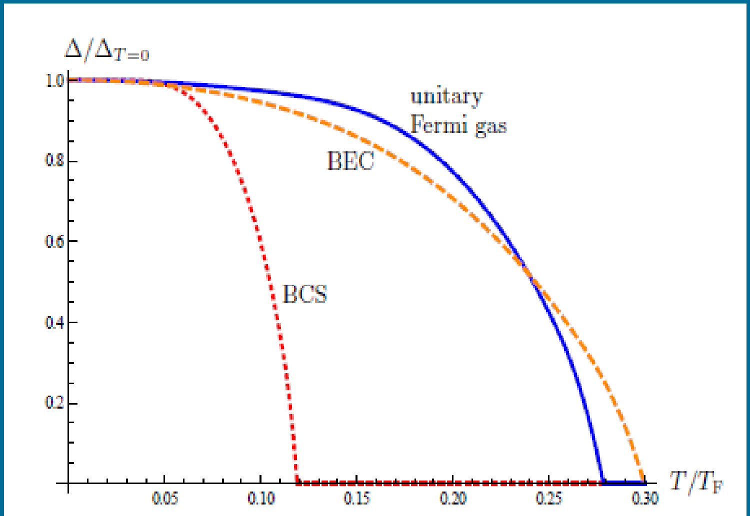
The BCS-BEC Crossover







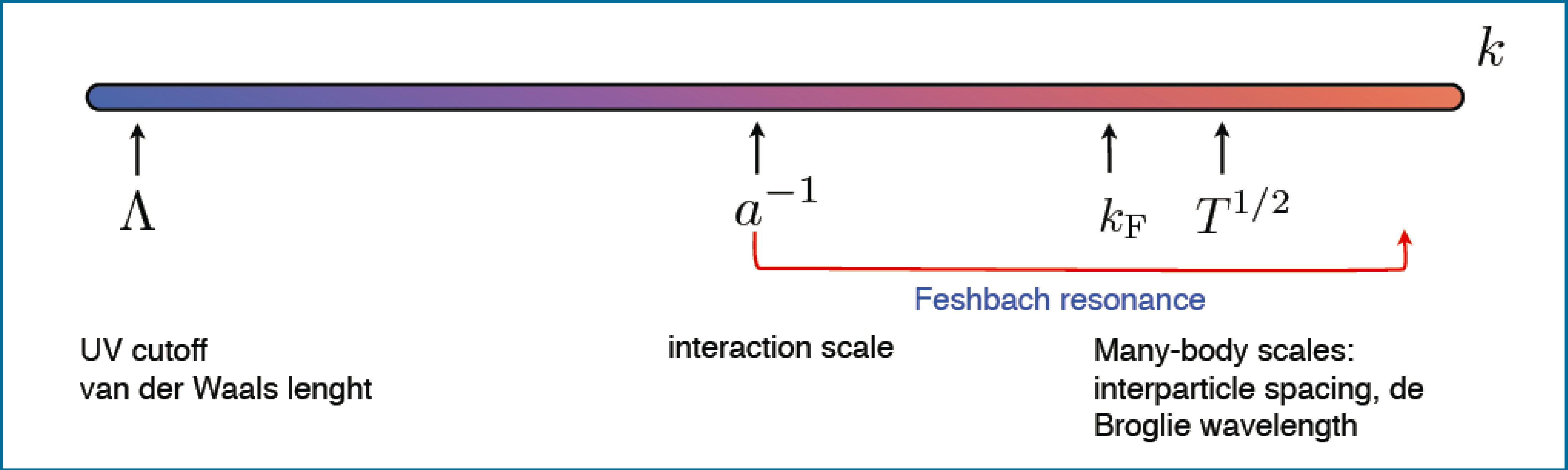
3D BCS-BEC crossover

(results from Functional Renormalization Group)

Microscopic Model

Many-body Hamiltonian

$$\hat{H} = \int d^3x \left(\sum_{\sigma=1,2} \hat{\psi}_\sigma^\dagger (-\nabla^2) \hat{\psi}_\sigma + \lambda_{\psi,\Lambda} \hat{\psi}_1^\dagger \hat{\psi}_2^\dagger \hat{\psi}_2 \hat{\psi}_1 \right)$$



Microscopic Model

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Microscopic action

$$S[\varphi, \psi] = \int_x \left(\sum_{\sigma=1,2} \psi^*_\sigma (\partial_\tau - \nabla^2 - \mu) \psi_\sigma + m^2_{\varphi,\Lambda} \varphi^* \varphi \right.$$

$$\left. - h_\varphi (\varphi^* \psi_1 \psi_2 - \varphi \psi^*_1 \psi^*_2) \right)$$

Macroscopic physics

How to compute the partition function?

$$Z(\mu, T) = \int \mathrm{D}\varphi \mathrm{D}\psi \, \mathrm{e}^{-S[\varphi,\psi]}$$

Integration

Macroscopic physics

How to compute the partition function?

$$Z_k(\mu, T) = \int \mathrm{D}\varphi \mathrm{D}\psi \, e^{-S[\varphi,\psi] + \Delta S_k}$$

scale dependent partition function

Macroscopic physics

How to compute the partition function?

$$Z_k(\mu, T) = \int \mathrm{D}\varphi \mathrm{D}\psi \, e^{-S[\varphi,\psi]+\Delta S_k}$$

scale dependent partition function

$$\partial_k Z_k(\mu, T) = \ldots$$

Solve flow equation

Wetterich equation

$\Gamma[\Phi] = J \cdot \Phi - \log Z[J]$ effective action

Wetterich equation

$\Gamma[\Phi] = J \cdot \Phi - \log Z[J]$ effective action

$$\partial_k \Gamma_k = \frac{1}{2} \mathrm{STr} \left(\frac{1}{\Gamma_k^{(2)} + R_k} \partial_k R_k \right)$$

fluctuations

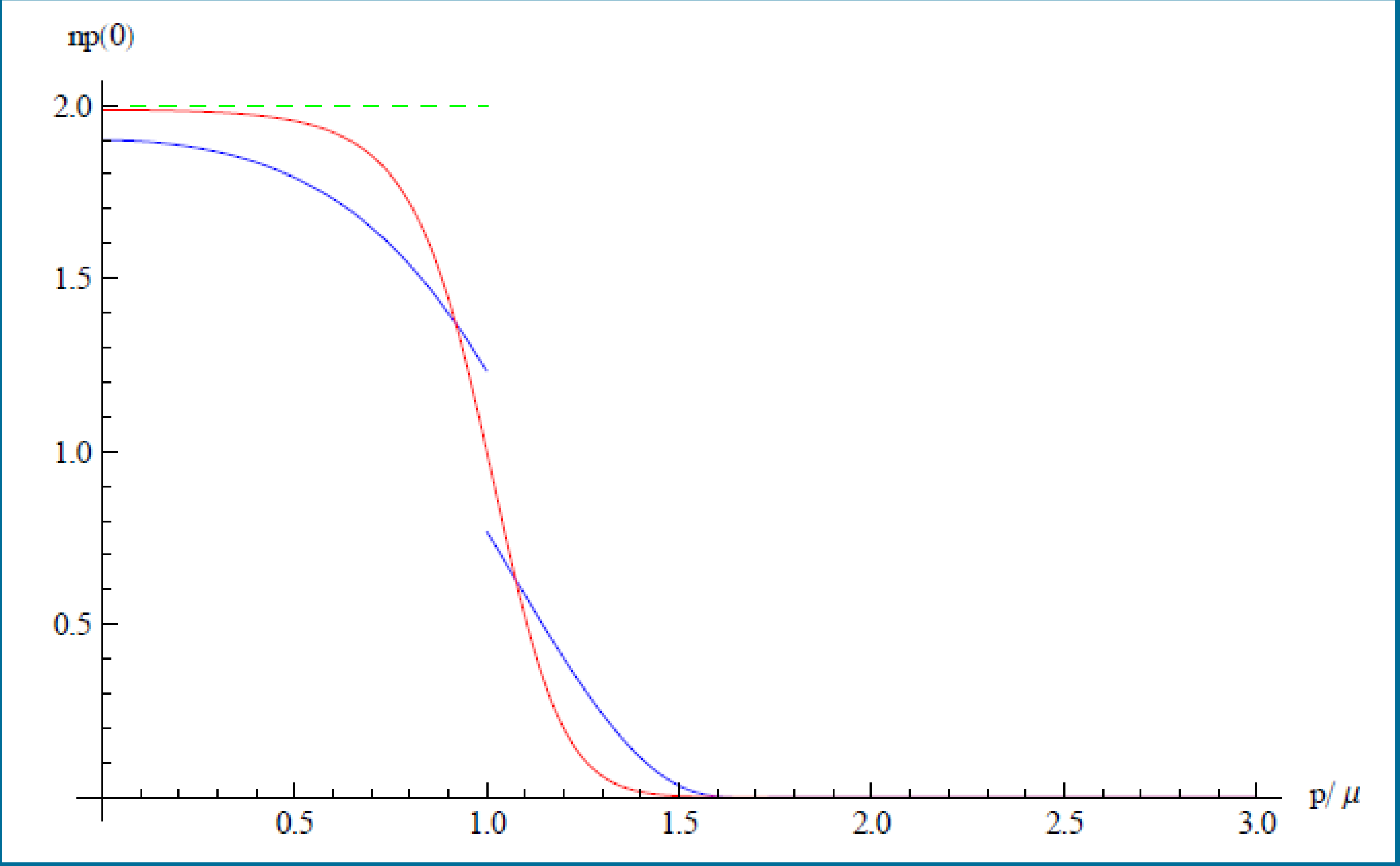
$\Gamma_{k=\Lambda} = S$ → $\Gamma_{k=0} = \Gamma$

Microphysics Macrophysics

Contact in the BCS-BEC Crossover

Momentum distribution

Ideal Fermi gas: Fermi-Dirac distribution



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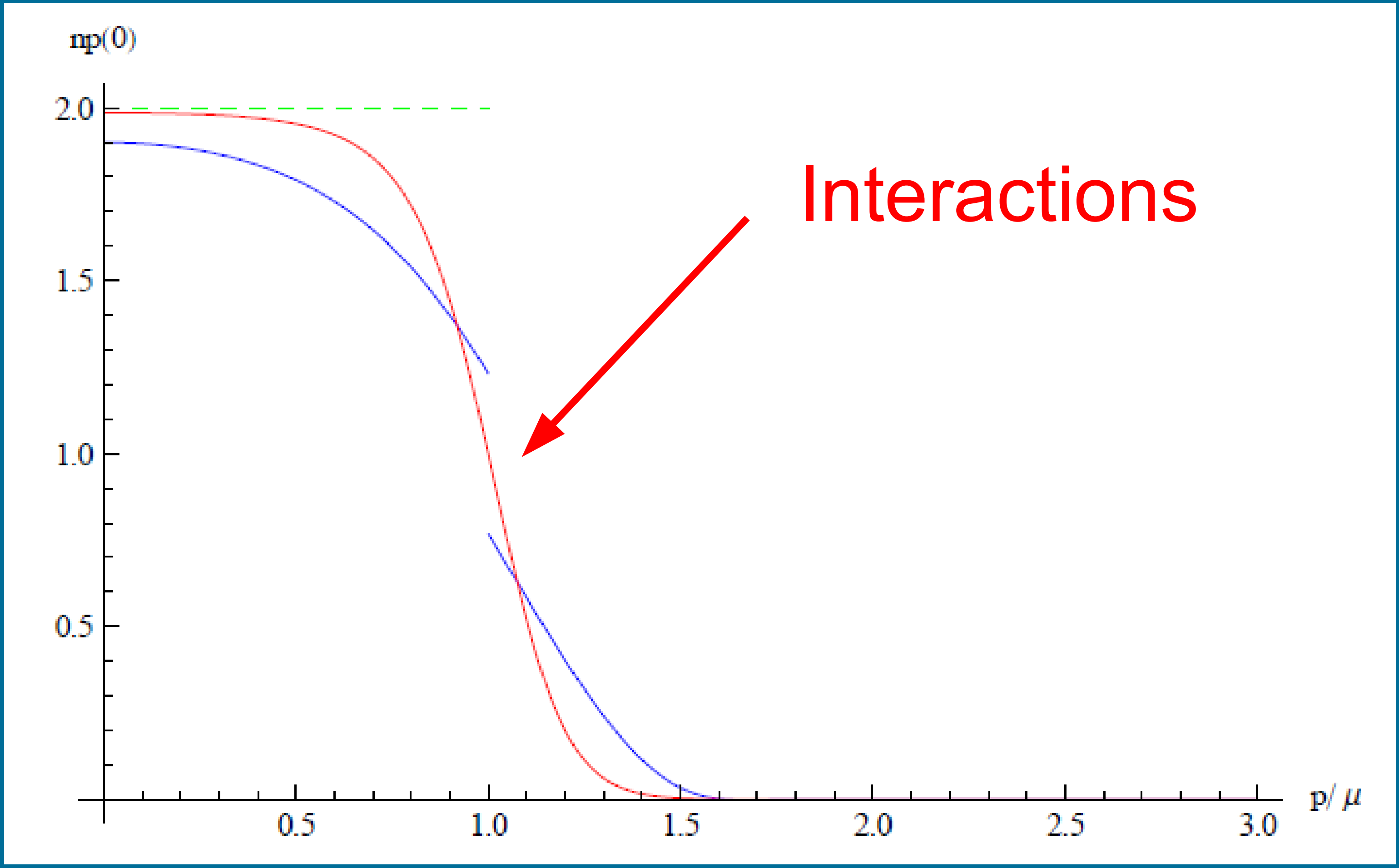

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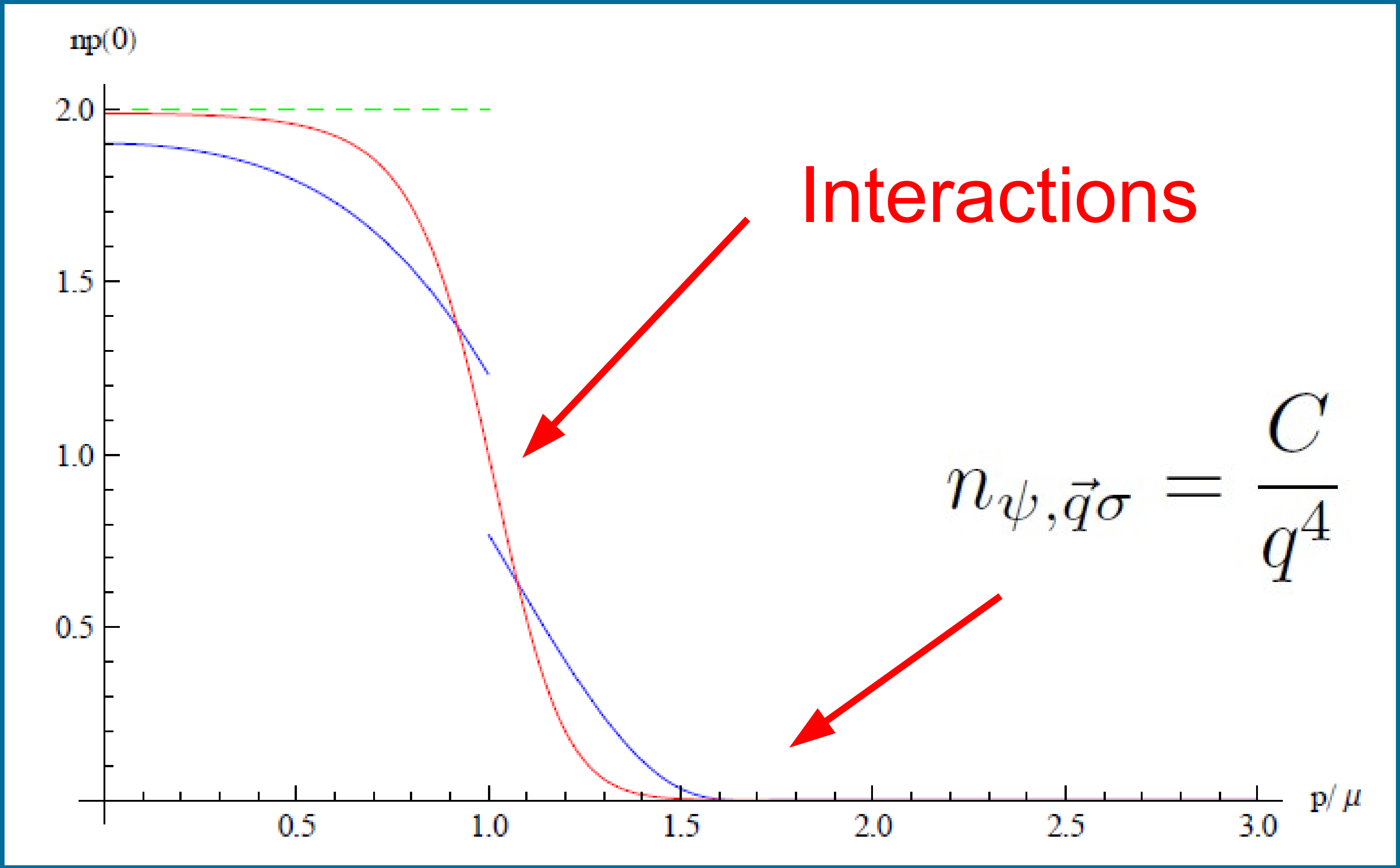

$$n_{\psi,\vec{q}\sigma} = \frac{C}{q^4}$$

Momentum distribution

$$n_{\vec{p}\sigma} \simeq \frac{C}{p^4}$$

Tan contact C

Several exact relations, e.g.:

$$\frac{1}{V}\frac{\mathrm{d}E}{\mathrm{d}(-1/a)} = \frac{C}{4\pi M}$$

$$E = \frac{C}{4\pi M a} + \sum_{\sigma=1,2}\int \frac{\mathrm{d}^3 p}{(2\pi)^3}\frac{p^2}{2M}\left(n_{\vec{p}\sigma} - \frac{C}{p^4}\right)$$

Contact from the FRG

$$n_{\vec{p}\sigma} = -\int_{p_0} G_{\psi\sigma}(p_0, \vec{p})$$

full macroscopic propagator

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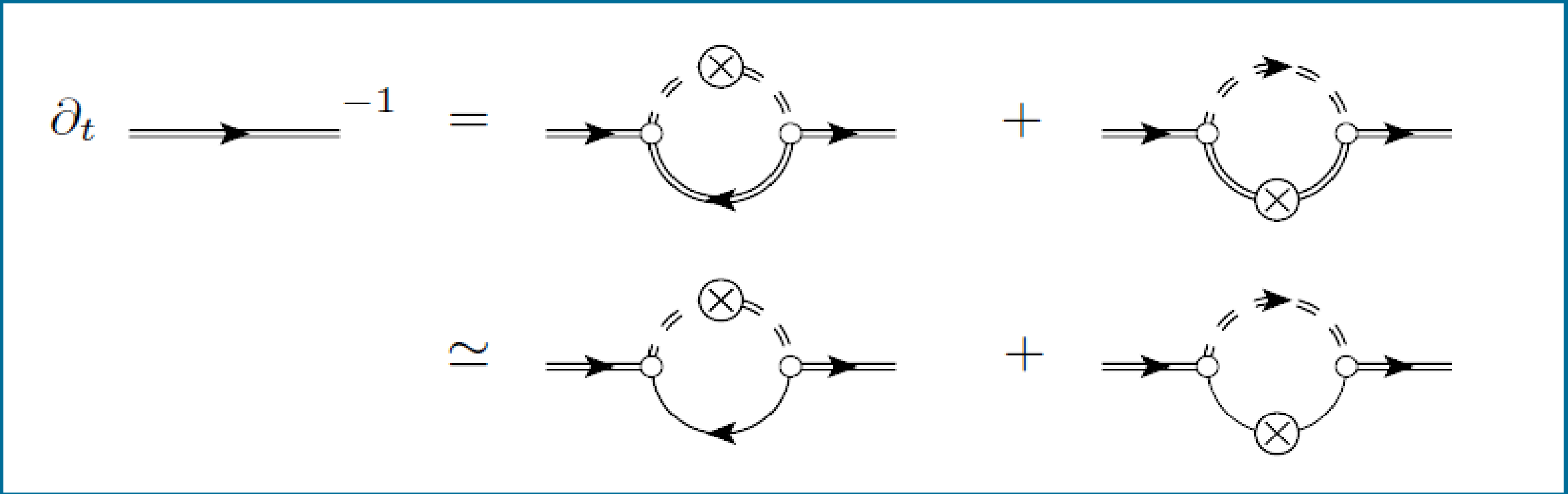
full macroscopic propagator



Contact from the FRG

Factorization of the RG flow for large p:

$$\partial_k G_{\psi,k}^{-1}(P) \simeq \frac{4}{-\mathrm{i}p_0 + p^2 - \mu} \partial_k C_k$$

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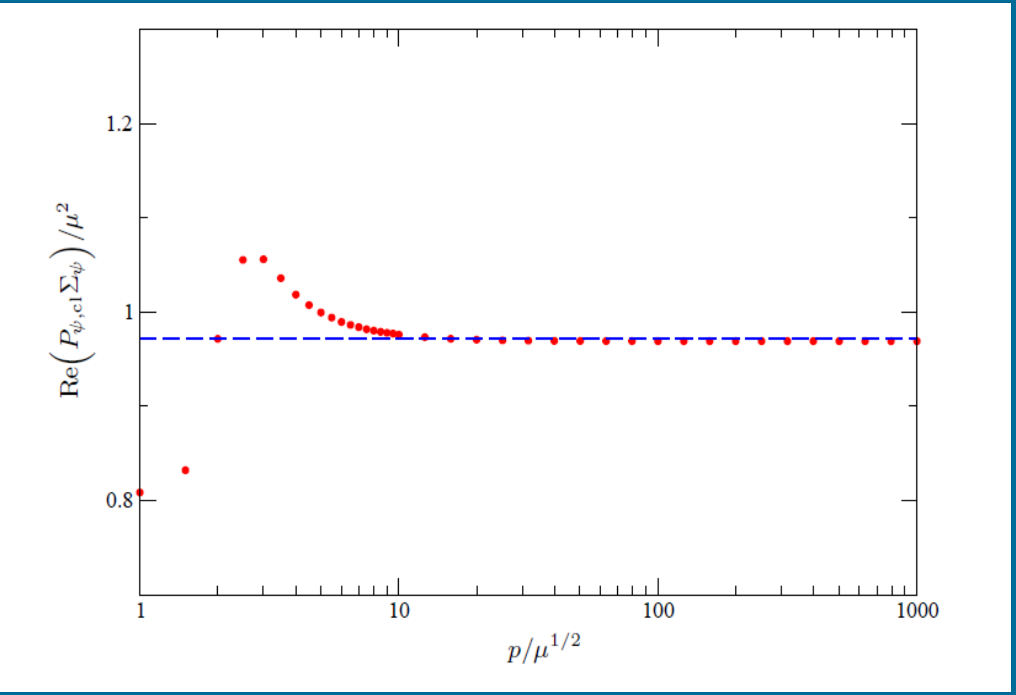

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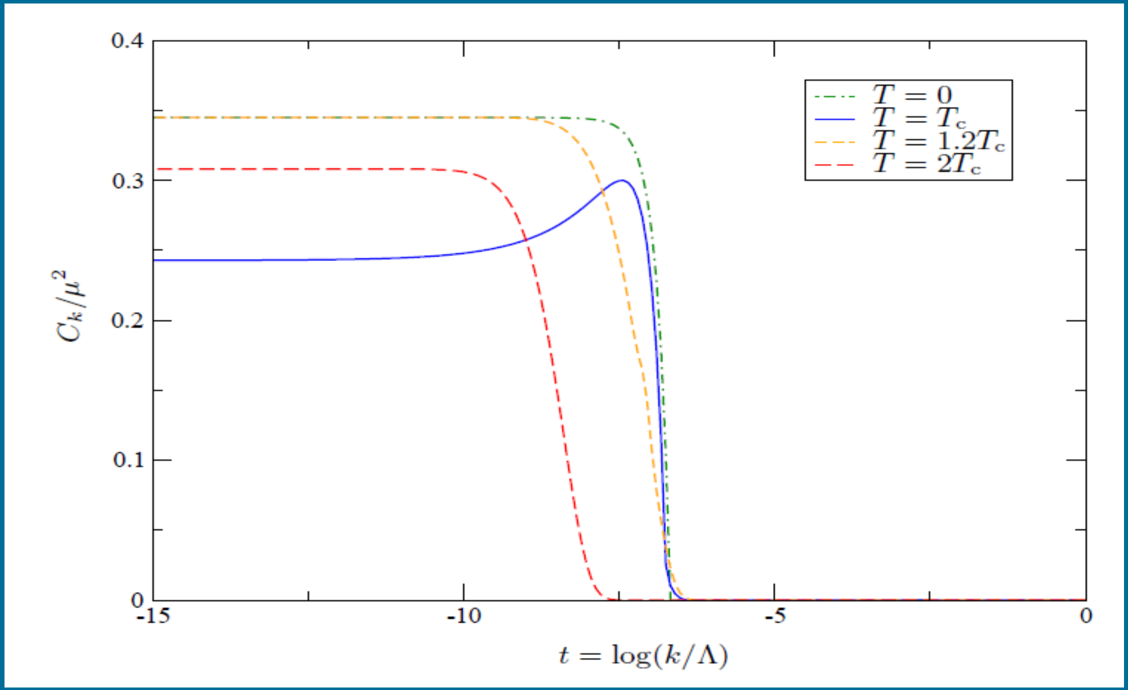
Flowing contact

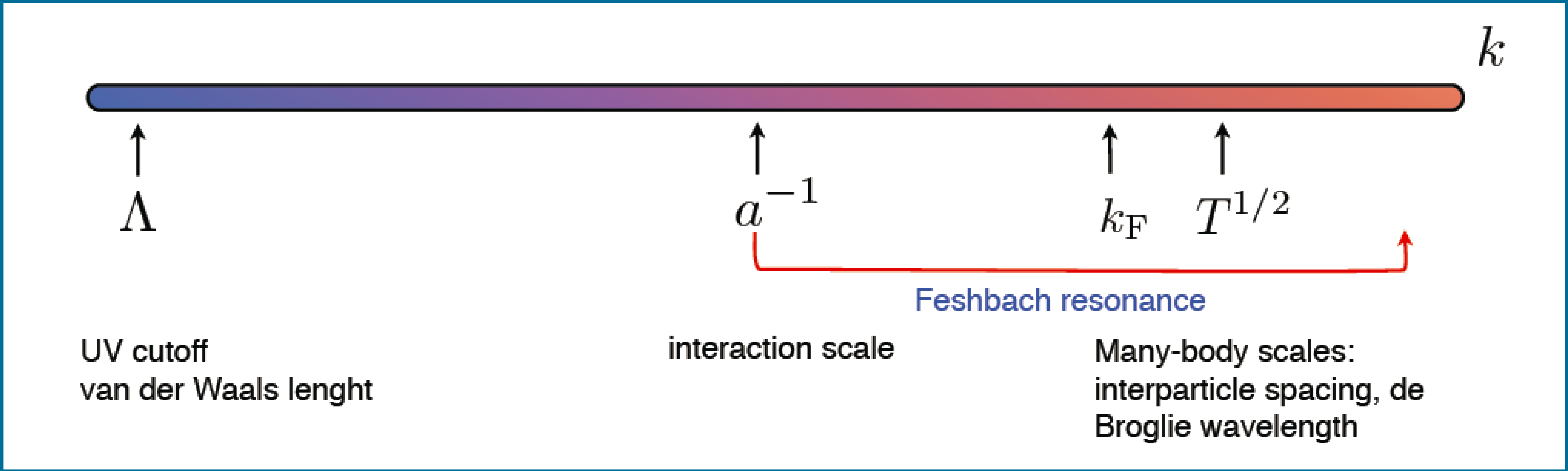
$$\partial_k C_k = \ldots$$



Contact from the FRG

Universal regime is enhanced for the Unitary Fermi gas

$$\Sigma_\psi(P) \simeq \frac{4C}{-\mathrm{i}p_0 + p^2 - \mu} - \delta\mu$$



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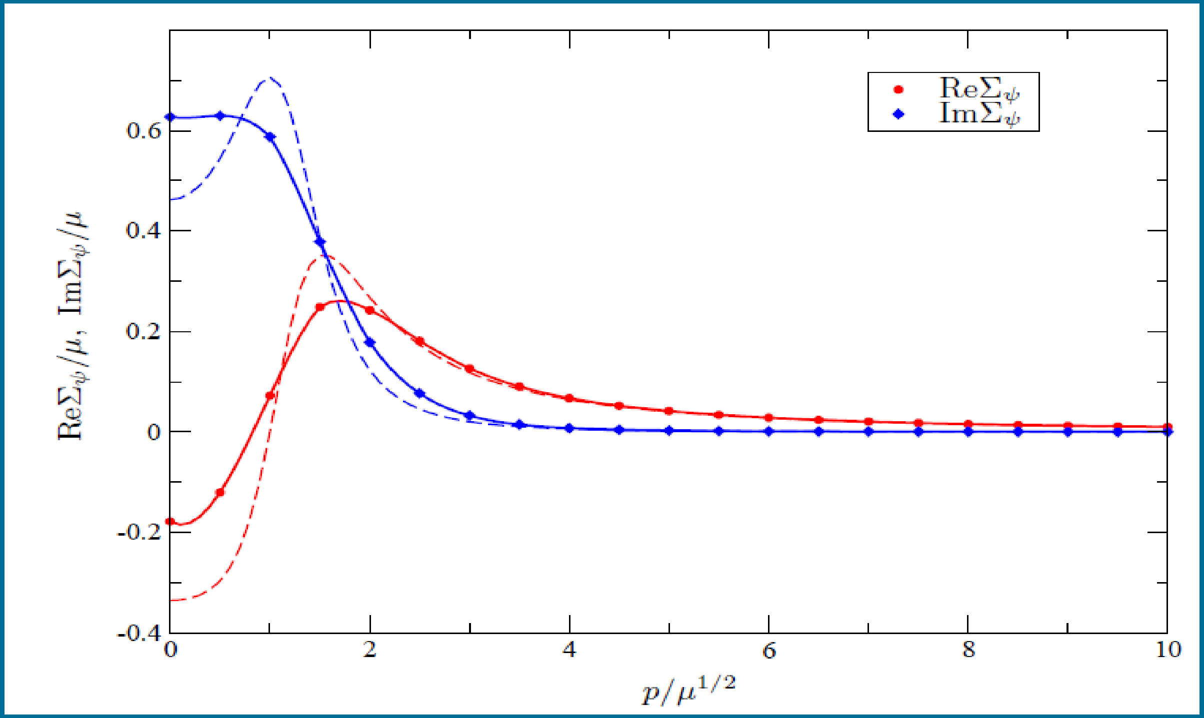
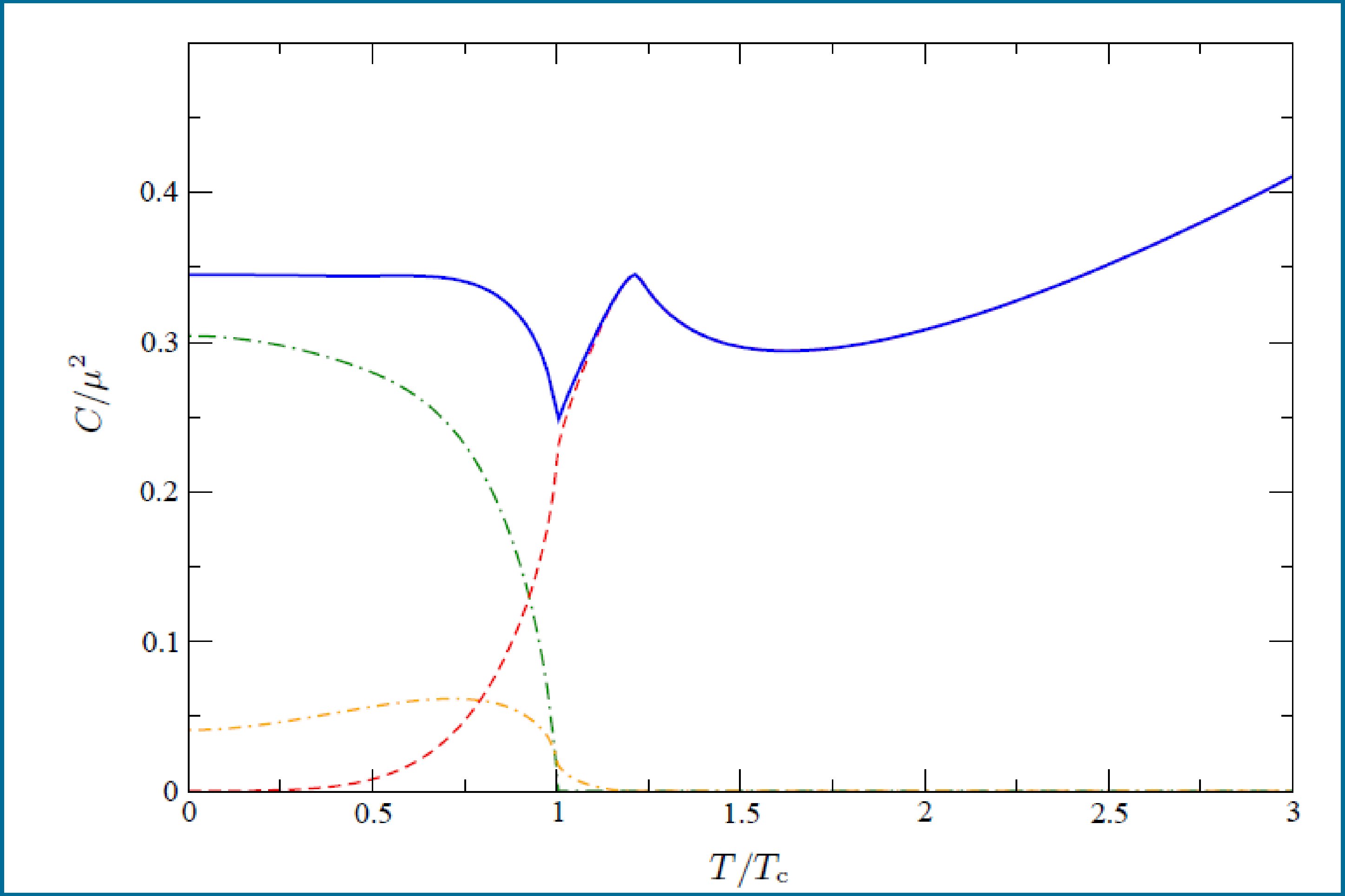


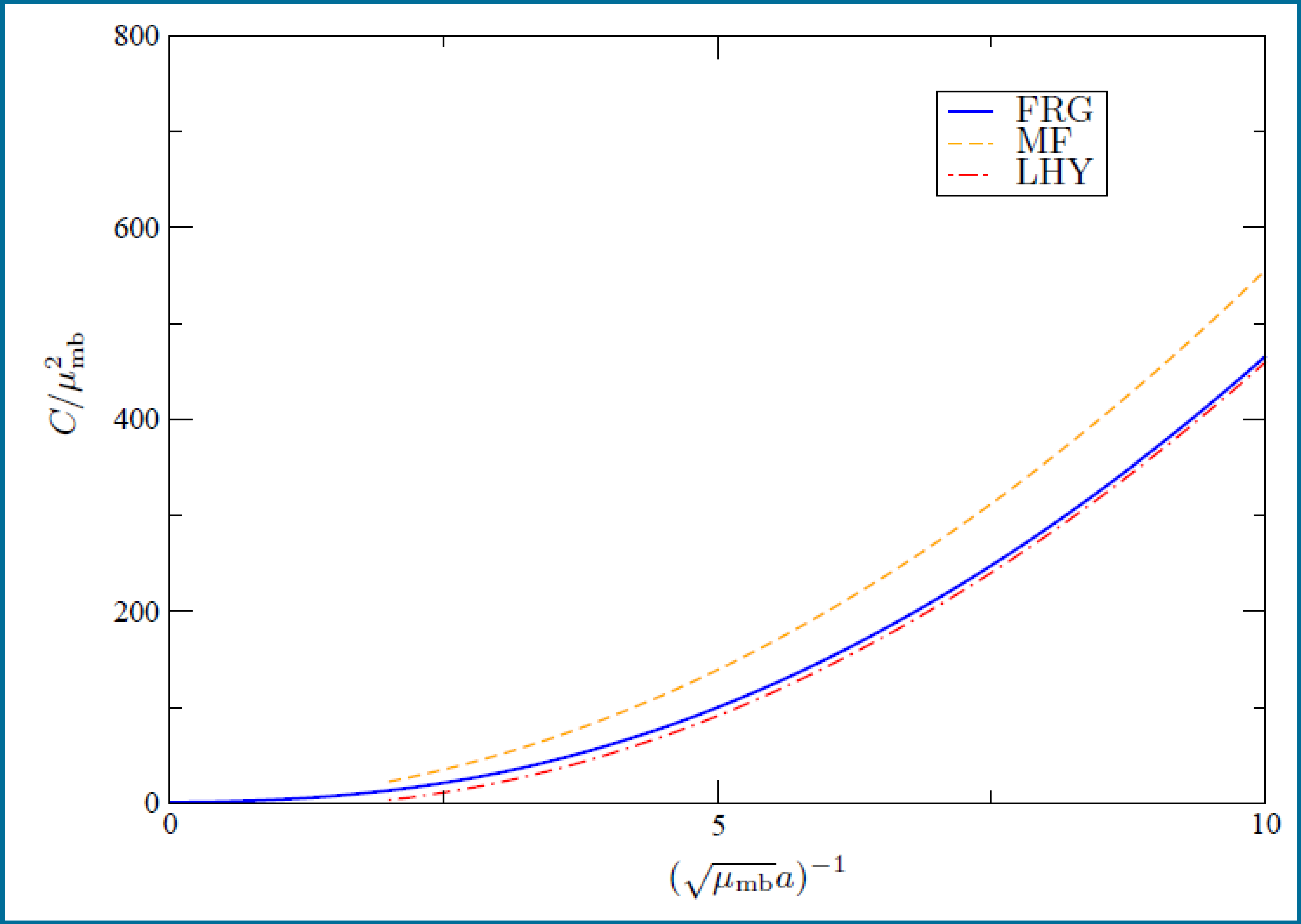
Contact from the FRG

Temperature dependent contact of the Unitary Fermi gas



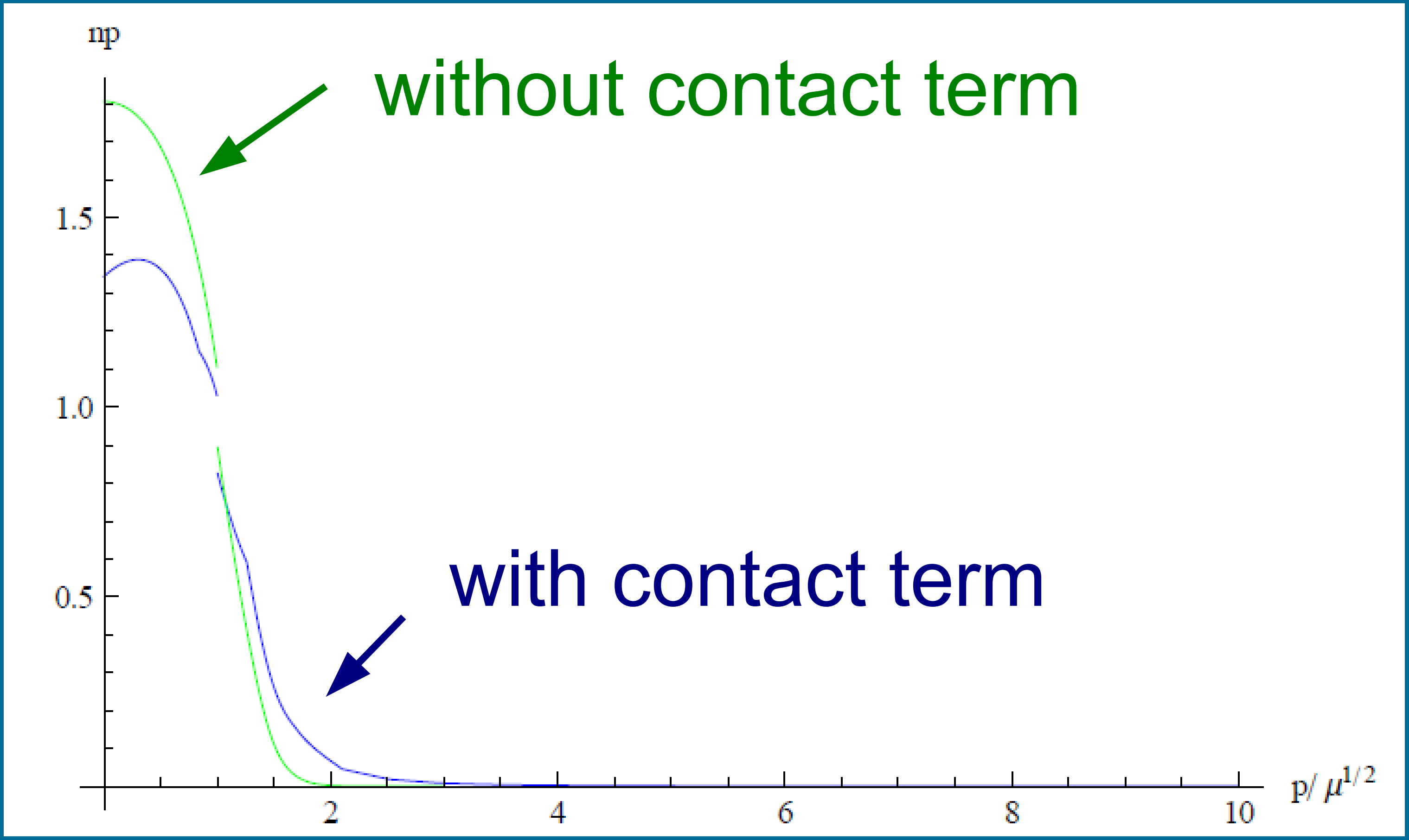
Contact from the FRG

Contact at T=0 in the BCS-BEC crossover



Contact from the FRG

Momentum distribution of the Unitary Fermi Gas at the critical temperature



Increase of density

Contribution from high energetic particles to the density

$$n = 2 \int \frac{\mathrm{d}^3 p}{(2\pi)^3} n_{\vec{p}\sigma}$$

$$\frac{\delta n^{(C)}}{n} = 27.5\%$$ at Tc

Substantial effect on $\frac{T_{\mathrm{c}}}{T_{\mathrm{F}}} \propto \frac{T_{\mathrm{c}}}{n^{2/3}}$

Two-dimensional BCS-BEC Crossover

Two-dimensional BCS-BEC Crossover

Why two dimensions?

Two-dimensional BCS-BEC Crossover

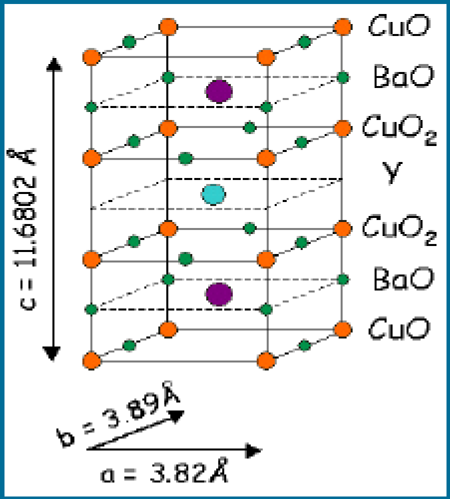
Why two dimensions?

- Enhanced effects of quantum fluctuations
  → test and improve elaborate methods

Two-dimensional BCS-BEC Crossover

Why two dimensions?

- Enhanced effects of quantum fluctuations
  $\rightarrow$ test and improve elaborate methods
- Understand pairing in two dimensions
  $\rightarrow$ high temperature superconductors



Two-dimensional BCS-BEC Crossover

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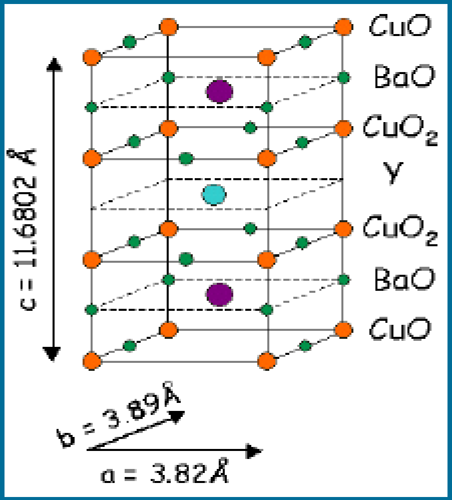
How?



Two-dimensional BCS-BEC Crossover

Why two dimensions?

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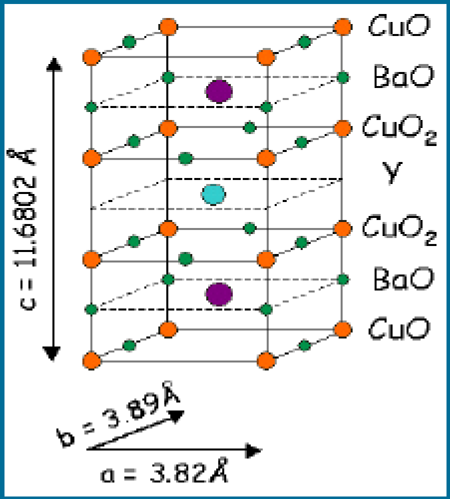
  → test and improve elaborate methods
- Understand pairing in two dimensions
  → high temperature superconductors

How?

Highly anisotropic traps!



What is different?

Scattering physics in two dimensions

$$f_{2\mathrm{d}}(q) \sim \frac{1}{\log(1/q^2 a_{2\mathrm{d}}^2) + \mathrm{i}\pi + \ldots}$$

$$f_{3\mathrm{d}}(q) \sim \frac{1}{-\frac{1}{a} + \frac{1}{2} r_{\mathrm{e}} q^2 - \mathrm{i}q + \ldots}$$

Scattering amplitude

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Scattering amplitude

Crossover parameter $\log(k_{\mathrm{F}} a_{2\mathrm{d}})$

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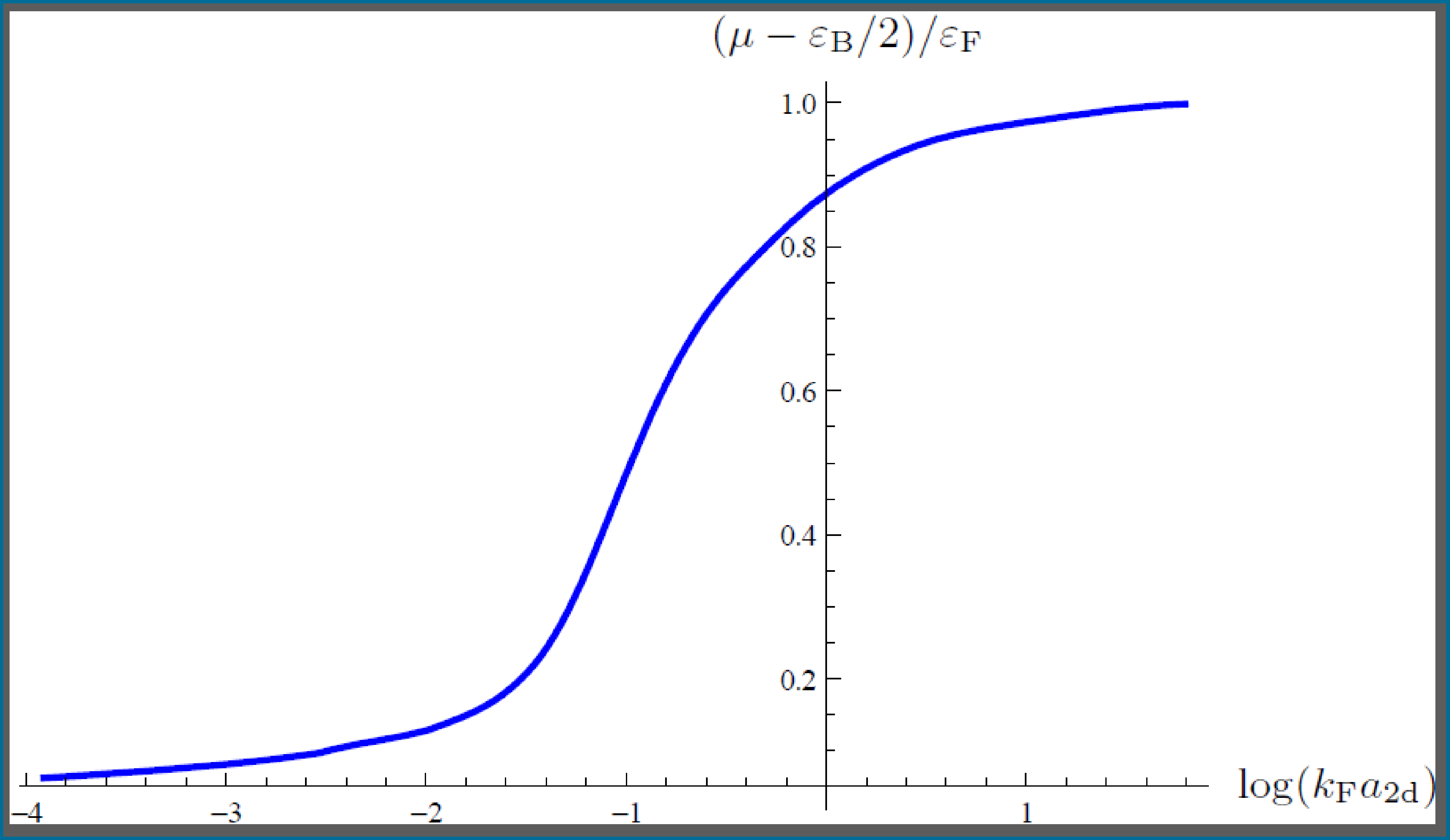
Scattering amplitude

Crossover parameter $\log(k_{\mathrm{F}} a_{2\mathrm{d}})$

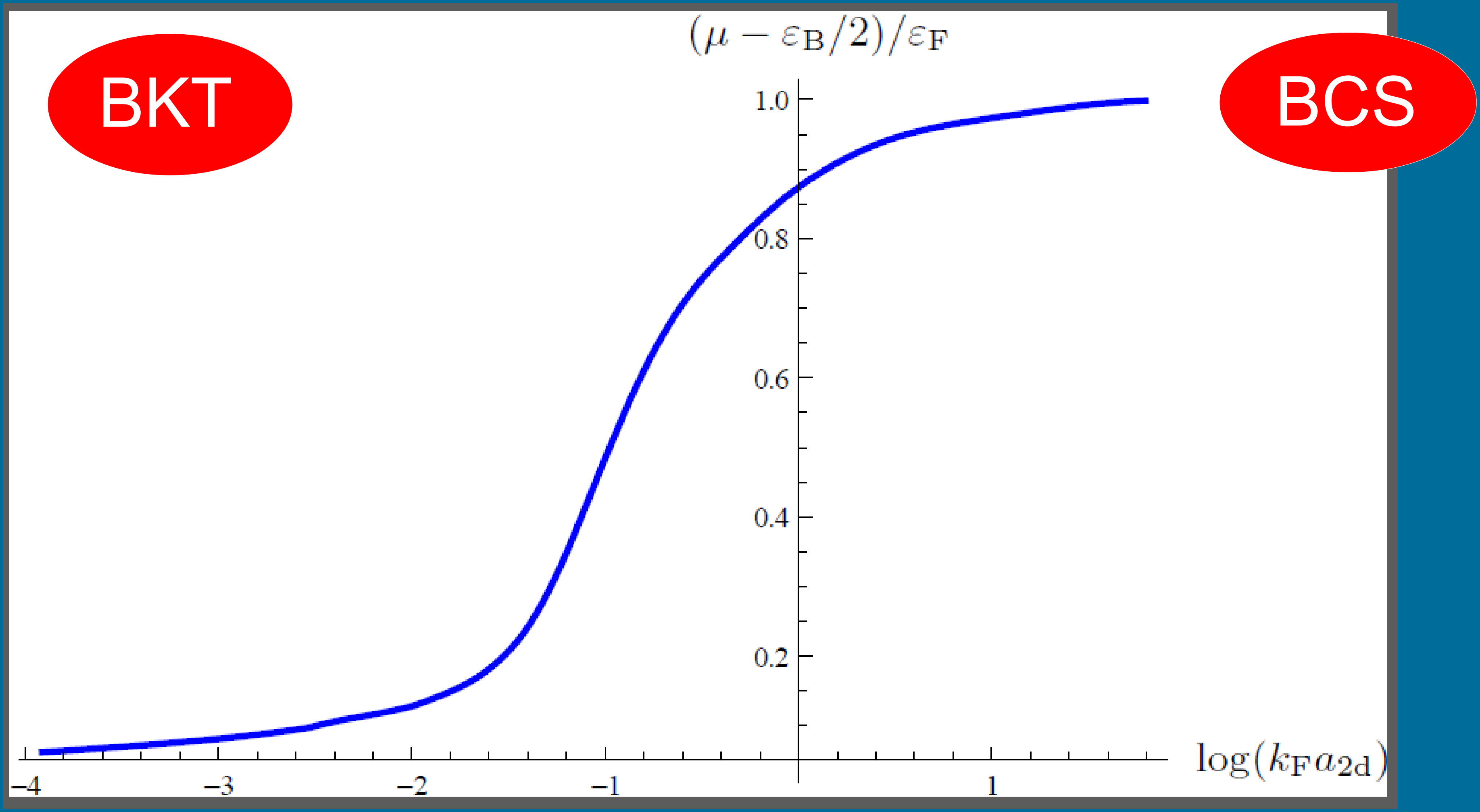
No scale invariance, but strong correlations for $k_{\mathrm{F}} \sim \frac{1}{a_{2\mathrm{d}}}$

Equation of state at T=0



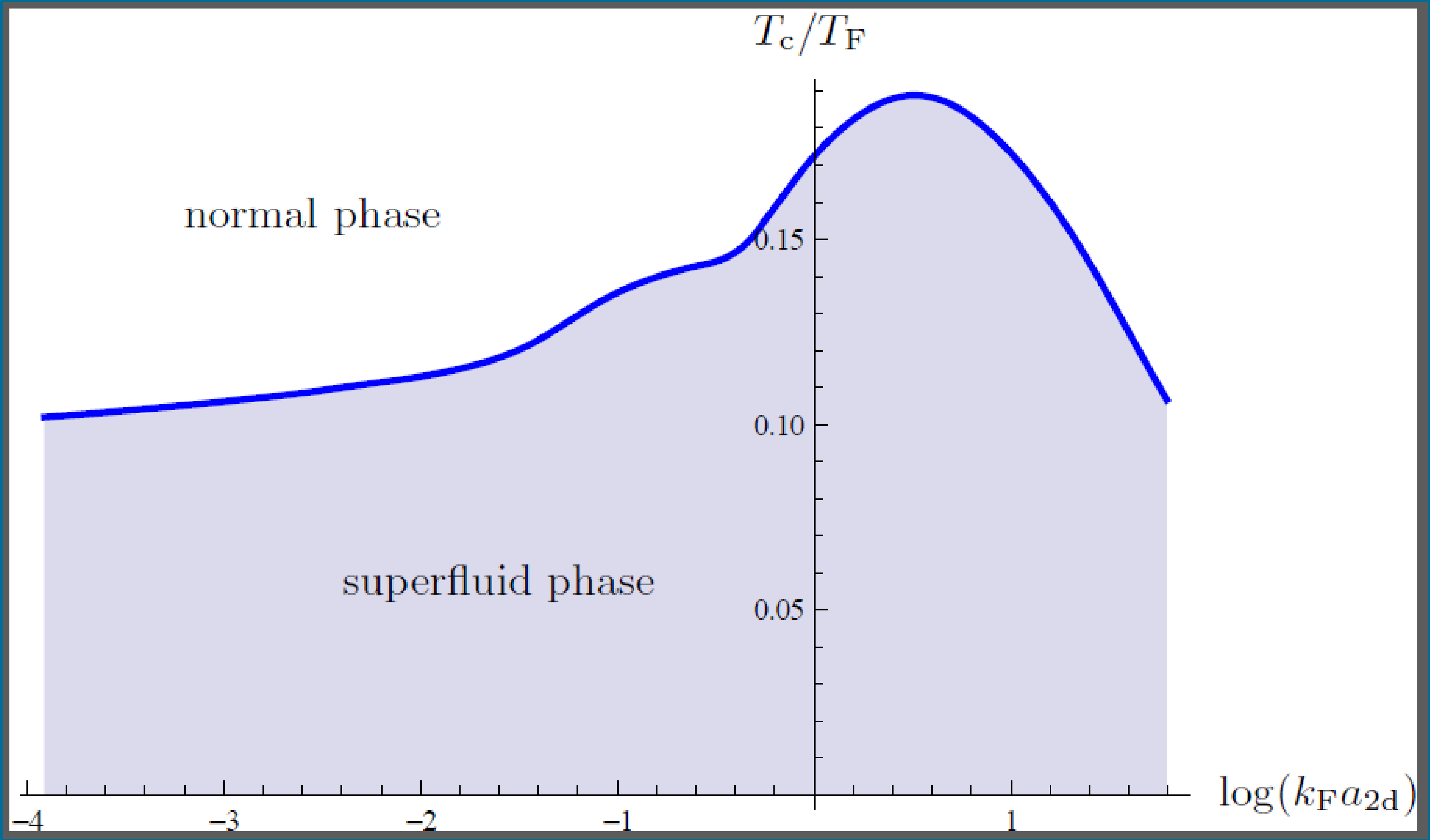
$(\mu - \varepsilon_B/2)/\varepsilon_F = 0.874$ for $\log(k_F a_{2d}) = 0$

Equation of state at T=0



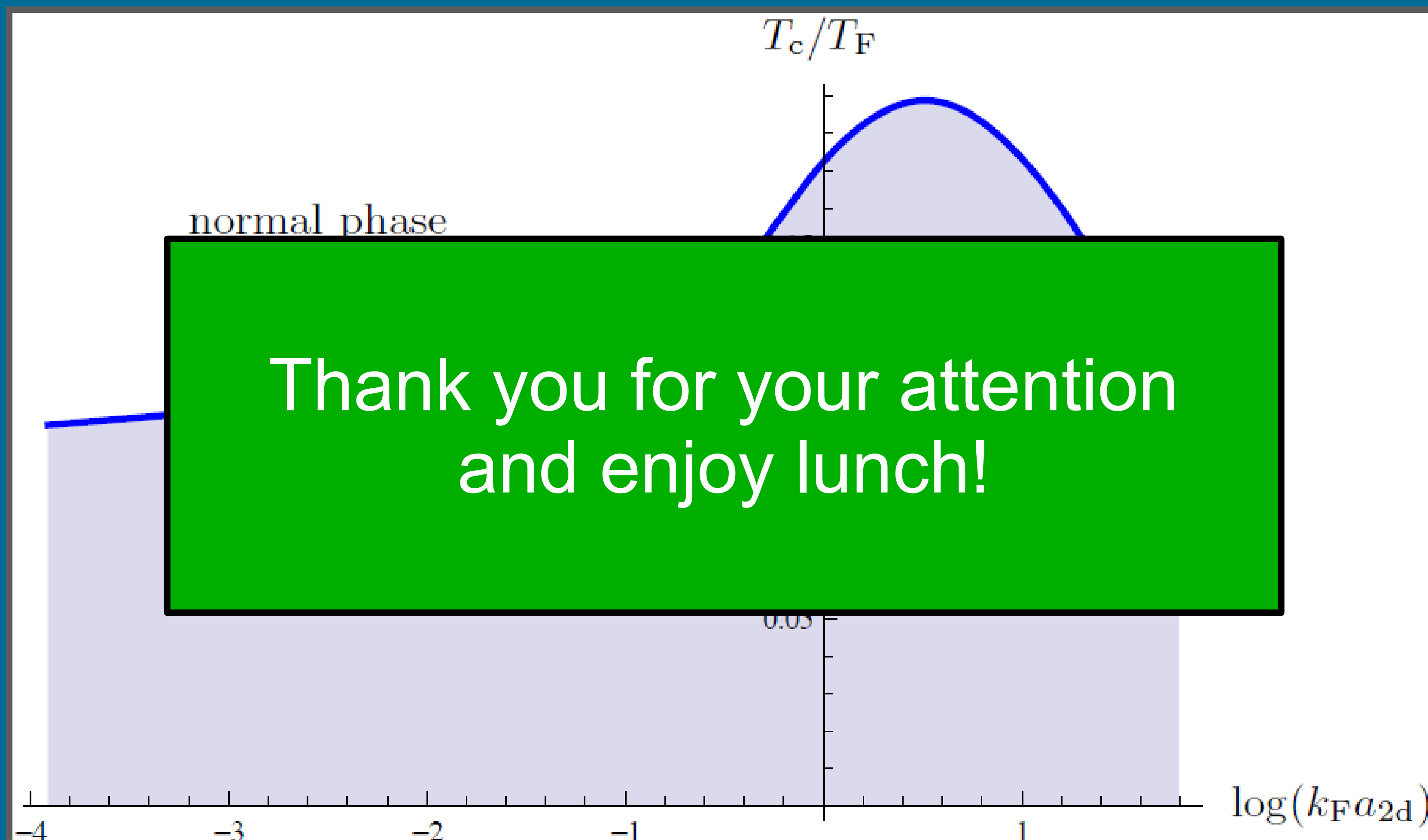
$$(\mu - \varepsilon_B/2)/\varepsilon_F = 0.874 \quad \text{for} \quad \log(k_F a_{2d}) = 0$$

Superfluid phase transition



$T_c/T_F = 0.172$ for $\log(k_F a_{2d}) = 0$

Superfluid phase transition



Thank you for your attention
and enjoy lunch!

$T_c/T_F = 0.172$ for $\log(k_F a_{2d}) = 0$