QCD-like theories at finite density

EMMI Workshop

Quark-Gluon Plasma meets Cold Atoms - Episode III

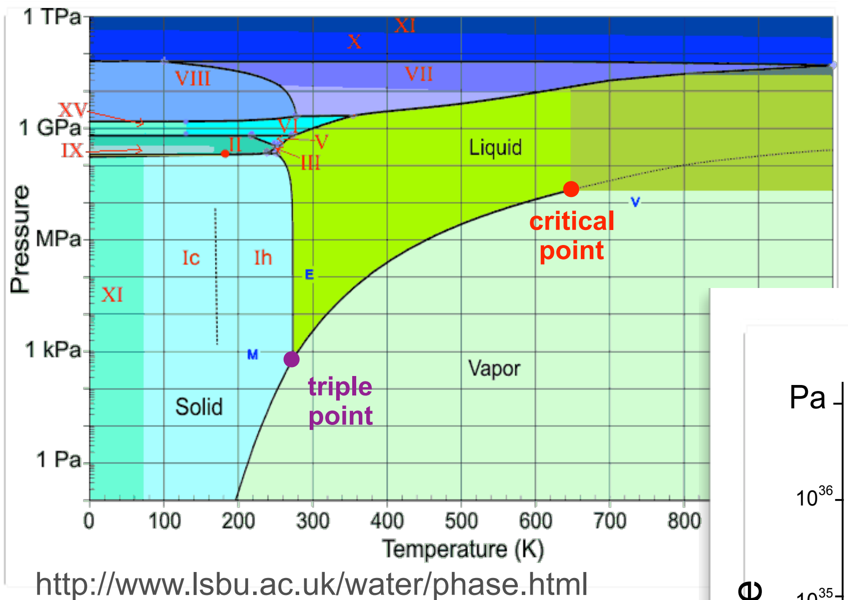
Hirschegg, 25 August 2012

Lorenz von Smekal

Contents

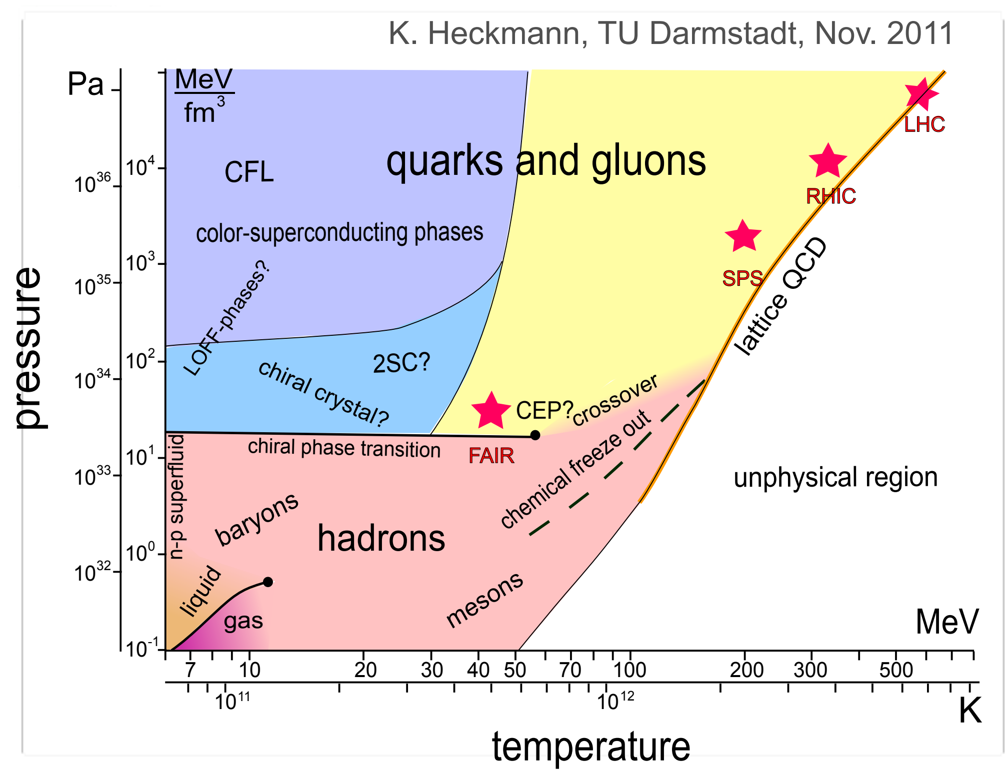
See also: L.v.S. in "Physics at all scales: The Renormalization Group," the 49th Schladming Winter School on Theoretical Physics, Nucl. Phys. B (PS) 228 (2012) pp. 179 - 220 [arXiv:1205.4205]

Phase Diagram



Water

QCD



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2-Color QCD

SU(2) is pseudo-real:

Dirac operator $\mathcal{D}$ has antiunitary symmetry T,

with $T^2 = 1$ (also at $\mu \neq 0$).

- **no sign problem**

χPT: Kogut, Stephanov, Toublan, Verbaarschot & Zhitnitsky, Nucl. Phys. B 582 (2000) 477

Lattice: Hands, Montvay, Scorzato & Skullerud, Eur. Phys. J. C 22 (2001) 451

Hands, Kenny, Kim & Skullerud, Eur. Phys. J. A 47 (2011) 60

NJL: Ratti & Weise, Phys. Rev. D 70 (2004) 054013

He, Phys. Rev. D 82 (2010) 096003.

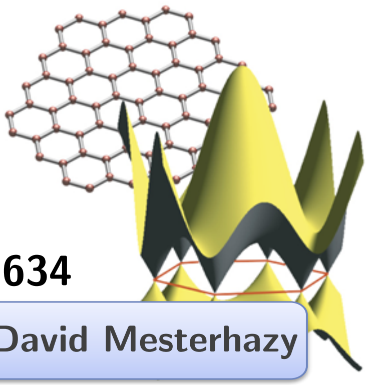
PNJL: Brauner, Fukushima & Hidaka, Phys. Rev. D 80 (2009) 074035

- **extended flavor symmetry (Pauli-Gürsey), at μ = 0**

$SU(N_f) \times SU(N_f) \times U(1)$ becomes $SU(2N_f)$

$N_f = 2$: connects pions and σ-meson with scalar (anti)diquarks.

Compare: even number of fermions in (2+1) dimensions:
QED$_3$ (semimetal-insulator transition, $N_f < 4$),
electronic properties of Graphene (half-filling, $N_f = 2$) – **SFB 634**
but also QCD$_3$ or generally SU(N_c) with $N_c \geq 3$.



→ David Mesterhazy

Quark-Meson-Diquark model for QC_2D

- **color-singlet diquarks (bosonic baryons)**
- ***N*$_f$=2: Explicit (spontaneous) breaking pattern by Dirac mass (quark condensate),**

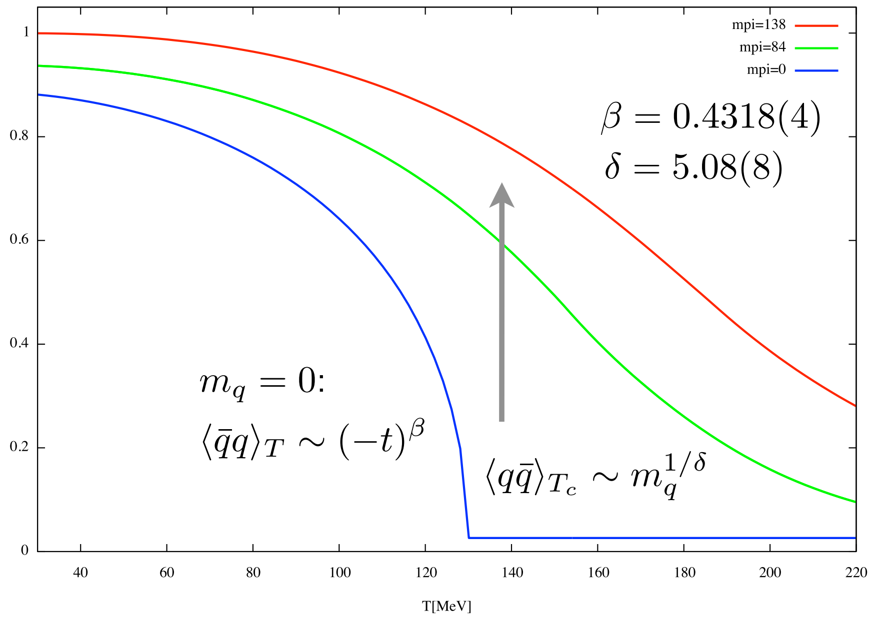
$$SU(4) \to Sp(2)$$

or $$SO(6) \to SO(5)$$

Parameter space: 15 dimensional → 10 dimensional

Coset: S^5

5 Goldstone bosons: pions and scalar (anti)diquarks



- ***O*(6) universality class:**

FRG, LPA: $\beta = 0.432$, $\delta = 5$

Litim, NPB 631 (2002) 128

Lattice: $\beta = 0.425(2)$, $\delta = 4.77(4)$

Holtmann, Schulze, PRE 68 (2003) 036111

FRG beyond LPA in progress

→ **Naseem Kahn**

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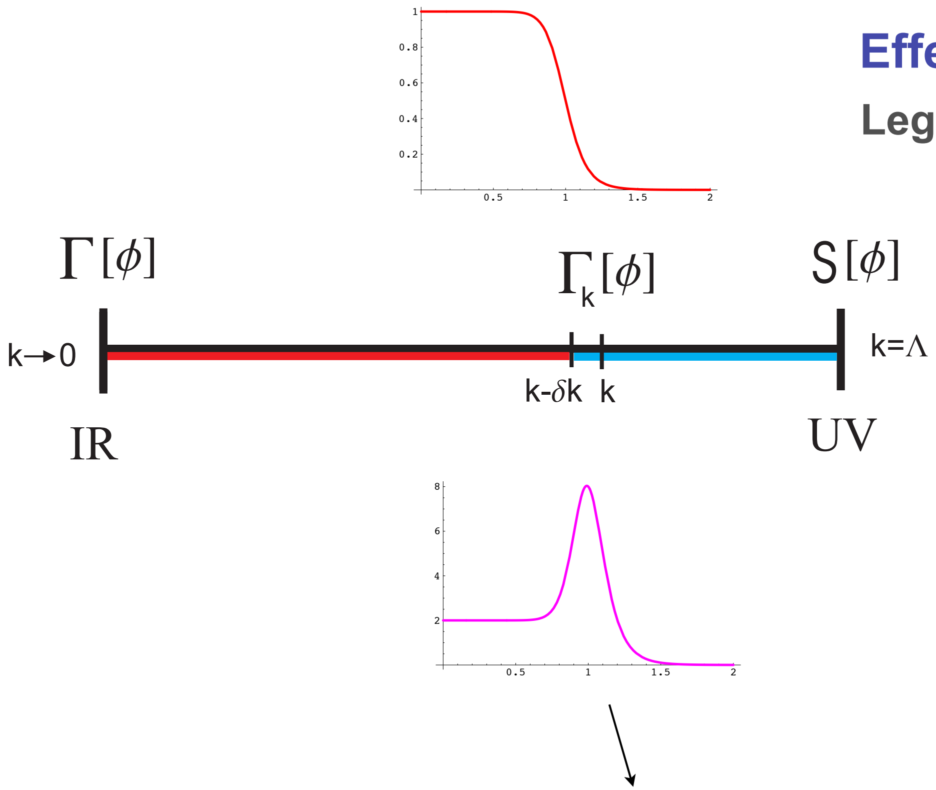
Functional RG (Flow) Equations



Effective action: $\Gamma[\phi_j] = (j, \phi_j) - \ln Z[j]$

Legendre transform

→ **1PI vertex functions**

$\Gamma^{(n)}(x_1, \ldots x_n)$

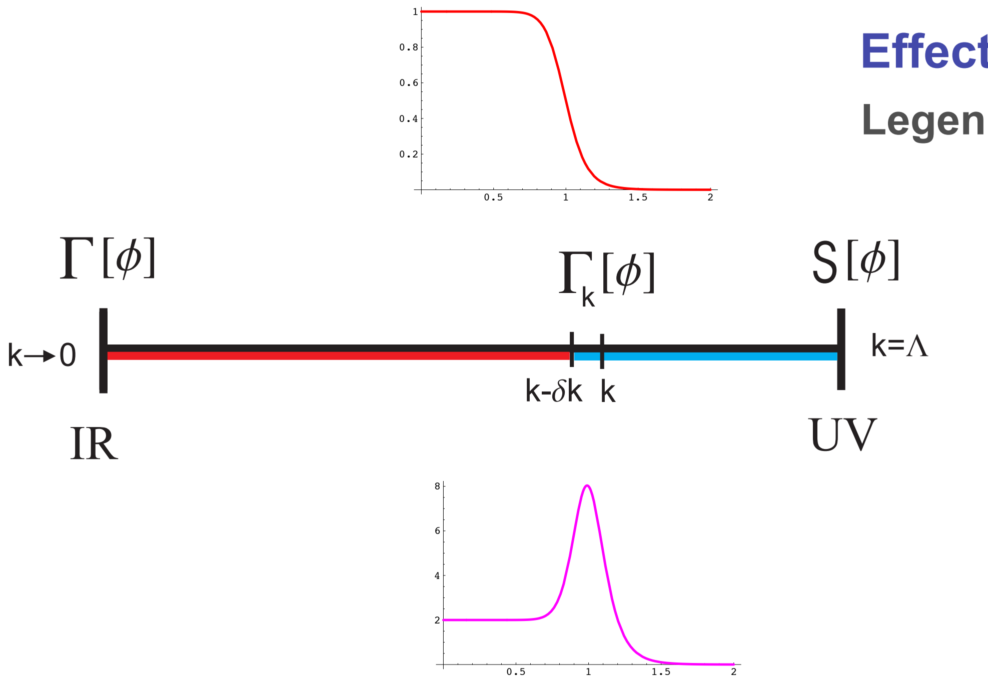
→ **free energy with**

$\phi_j = \langle \phi \rangle_j$

$$k\partial_k \Gamma_k[\Phi] = \frac{1}{2} \mathrm{Tr} \left\{ \partial_t R_k \, (\Gamma_k^{(2)} + R_k)^{-1} \right\}$$

Wetterich, Phys. Lett. B 301 (1993) 90

Functional RG (Flow) Equations



Effective action: $\Gamma[\phi_j] = (j, \phi_j) - \ln Z[j]$

Legendre transform

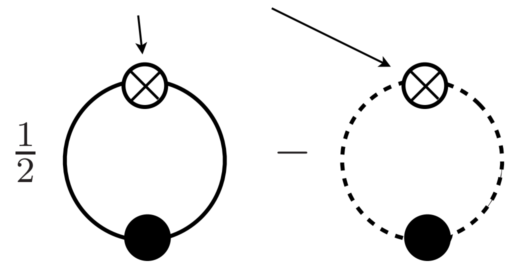
→ **1PI vertex functions**

$\Gamma^{(n)}(x_1, \ldots x_n)$

→ **free energy with**

$\phi_j = \langle \phi \rangle_j$

$$k\partial_k \Gamma_k[\phi] = \frac{1}{2}$$

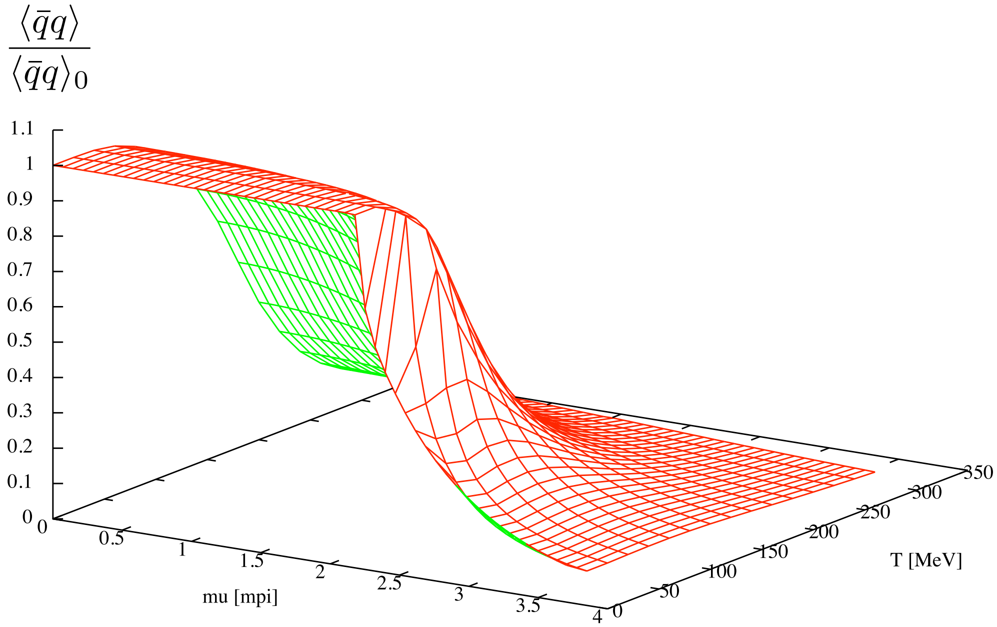


Wetterich, Phys. Lett. B 301 (1993) 90

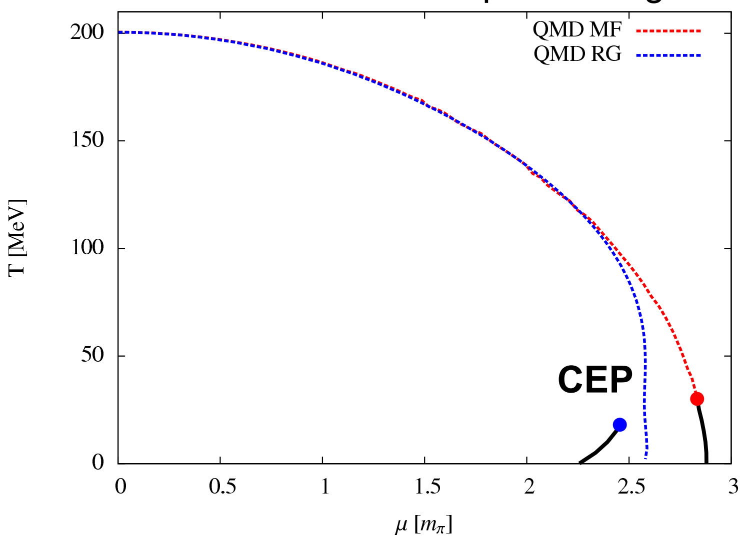
Phase Diagram - SU(4) Symmetric

- **no diquark condensation, flow equation for 1 dim field variable, O(6) symmetric potential**

$$U = U(\phi^2) \text{ where } \vec{\phi} = (\sigma, \vec{\pi}, \mathrm{Re}\Delta, \mathrm{Im}\Delta)$$



phase diagram



- 1st order chiral transition and CEP at $\mu \approx 2.5\, m_\pi$

- **but wait! need to properly include dynamics of our bosonic baryons....**

2-Color QCD at Finite Density

- **finite chemical potential μ:**

$N_f = 2,\ m_q = \mu = 0:$ $SU(4) \simeq SO(6)$

$m_q \neq 0:$ $SO(5)$ — **$\mu_B > m_\pi$: BEC $\to$ BCS** — $\mu \neq 0:$ $SO(4) \times SO(2)$

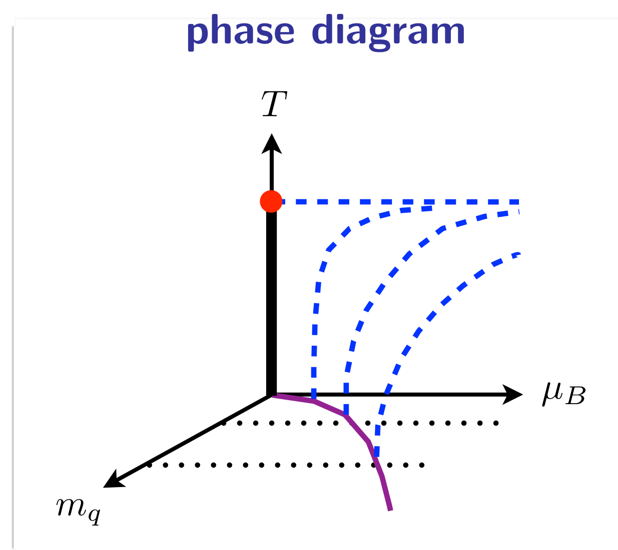
$m_q,\ \mu \neq 0:$ $SO(3) \times SO(2)$

$U(1)_B$ breaks sponta-neously at $\mu_B = m_\pi$

- **χPT: vacuum alignment,**

$2\mu = \mu_B < m_\pi$: $\langle \bar{q}q \rangle \neq 0$, $\langle qq \rangle = 0$, $\langle \bar{q}q \rangle$-like,

$2\mu = \mu_B > m_\pi$: $\langle \bar{q}q \rangle \propto \left(\frac{m_\pi}{\mu_B}\right)^2$, $\langle qq \rangle \propto \sqrt{1 - \left(\frac{m_\pi}{\mu_B}\right)^4}$, turns $\langle qq \rangle$-like.

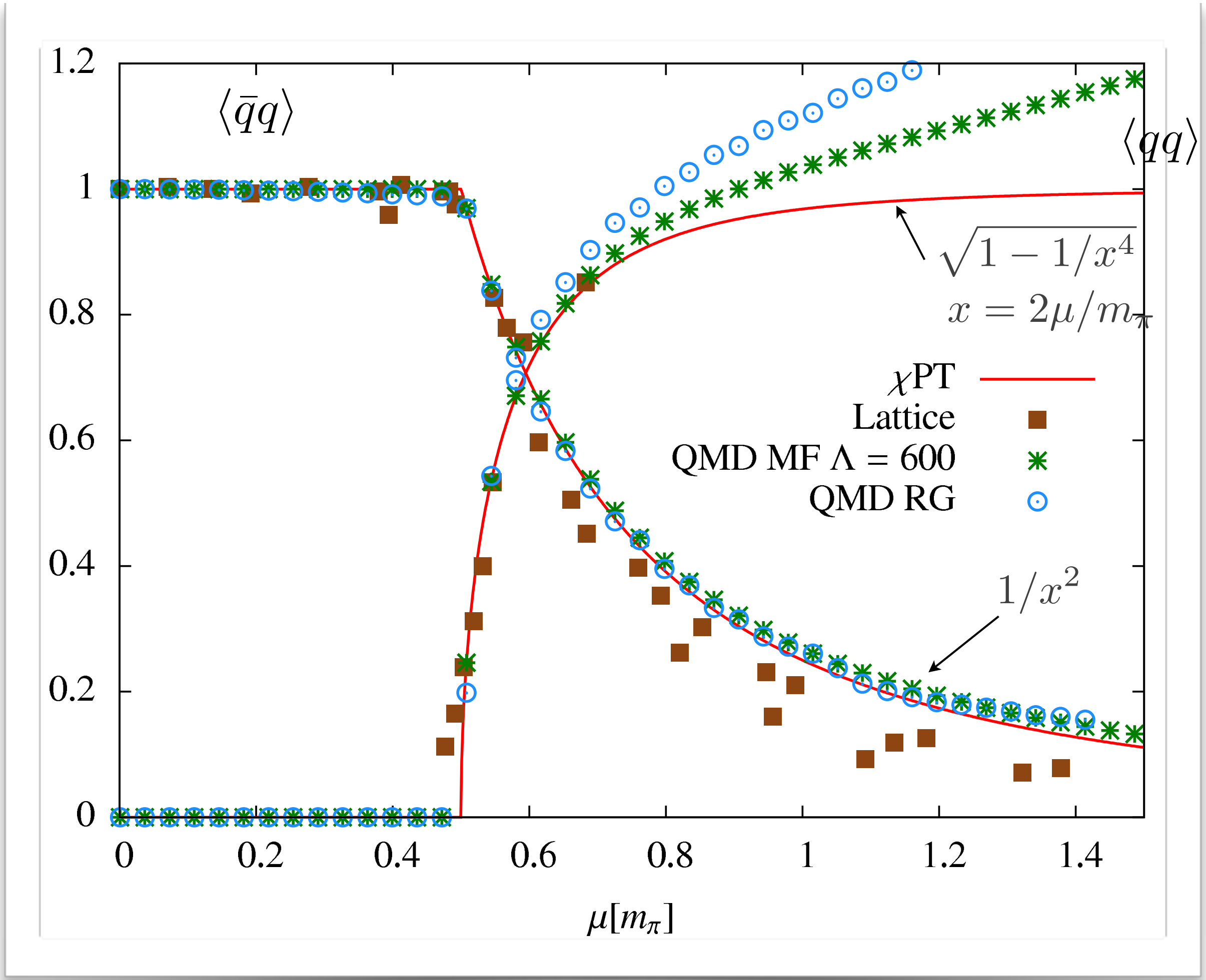


- **need 2 field variables in effective potential**

$U = U(\rho^2, d^2)$ where $\vec{\rho} = (\sigma, \vec{\pi})$ and $d^2 = |\Delta|^2$

Vacuum Alignment, *T*=0

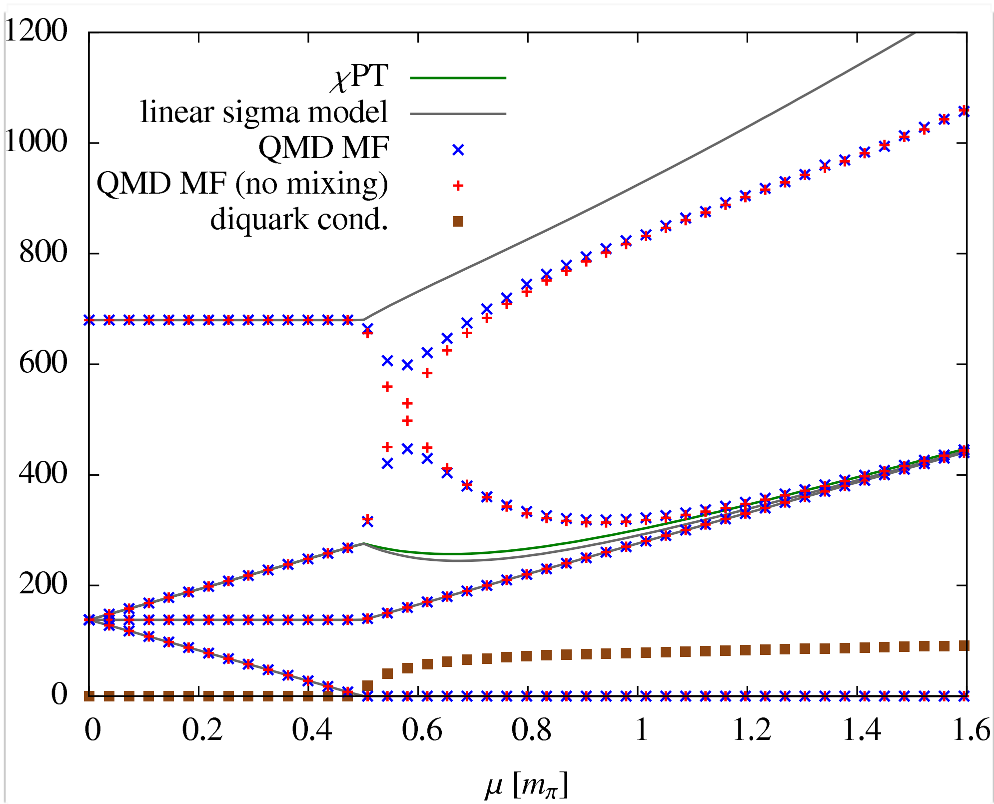
- **Diquark condensation at $2\mu = \mu_B = m_\pi$**



From DSEs
→ Pascal Buescher

Vacuum Alignment, *T*=0

- **RPA pole masses, QMD model:**



- **PNJL model:**

  Brauner, Fukushima & Hidaka, Phys. Rev. D 80 (2009) 074035

- **NJL with isospin chemical potential:**

  He, Jin & Zhuang, Phys. Rev. D 71 (2005) 116001

  Xiong, Jin & Li, J. Phys. G 36 (2009) 125005

- **Functional RG:**

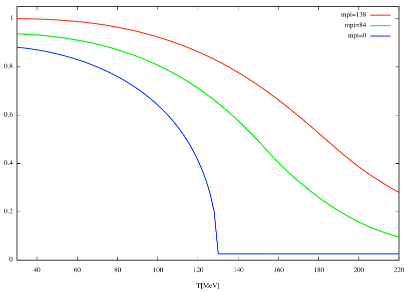
  Strodthoff, Schaefer & LvS, Phys. Rev. D 85 (2012) 074007

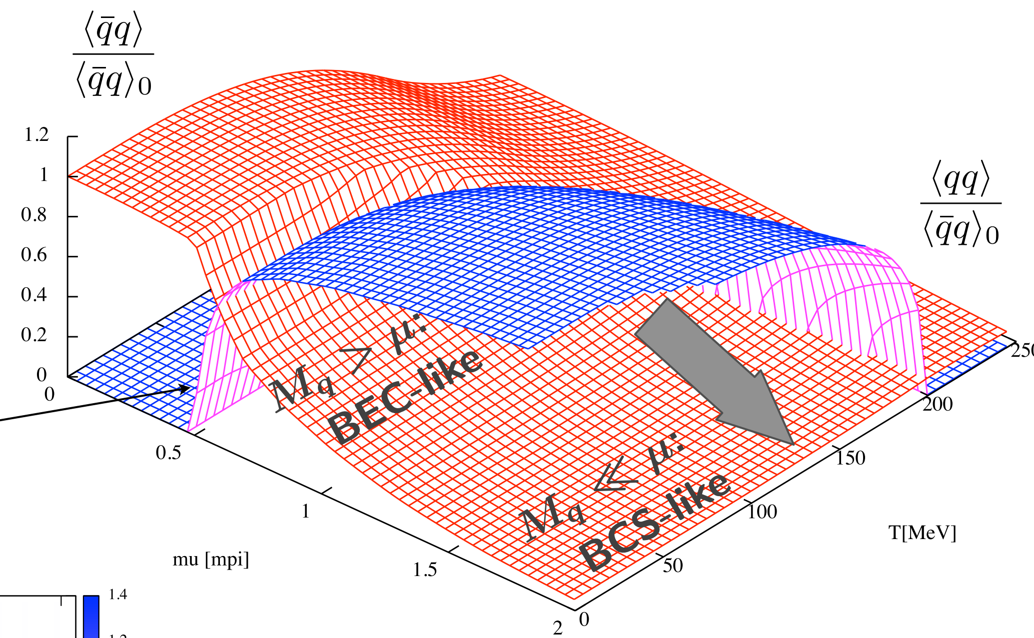
  Kamikado, Strodthoff, LvS & Wambach, arXiv:1207.0400

Quark-Meson-Diquark Model

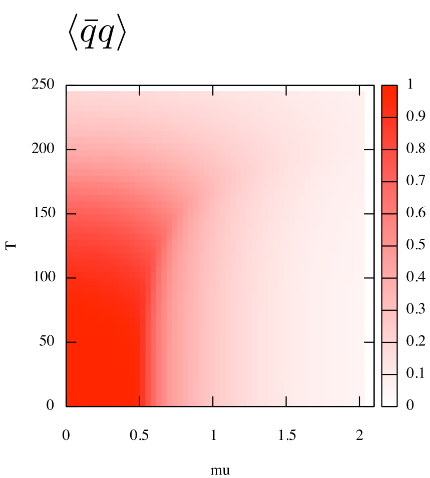
- **Finite T and μ:**

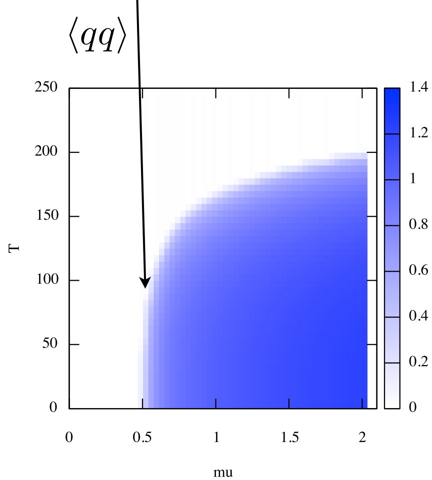
along $\mu = 0$ axis as before





baryon density rises abruptly

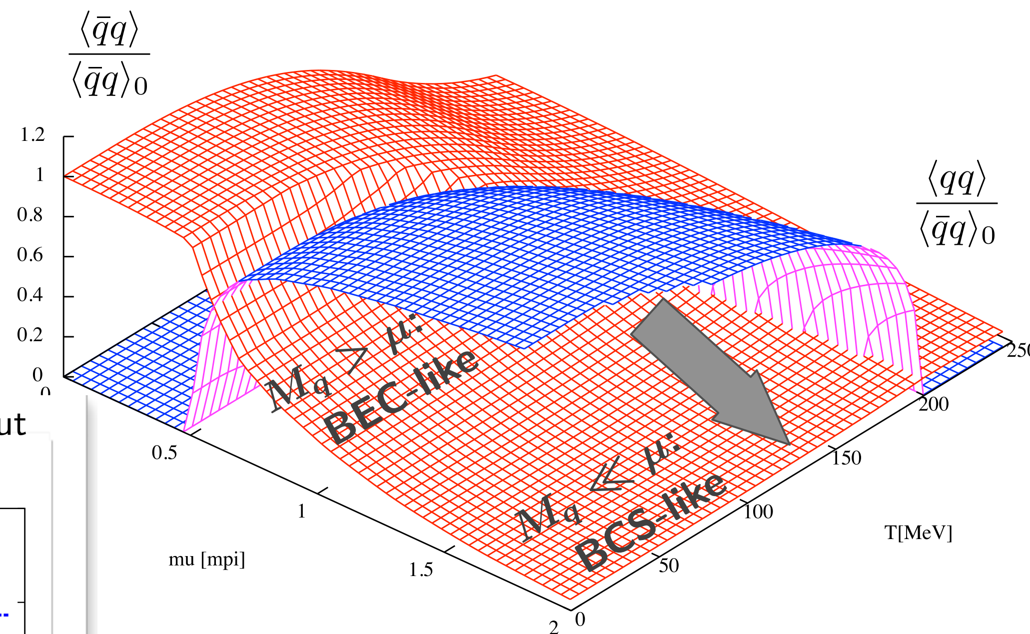




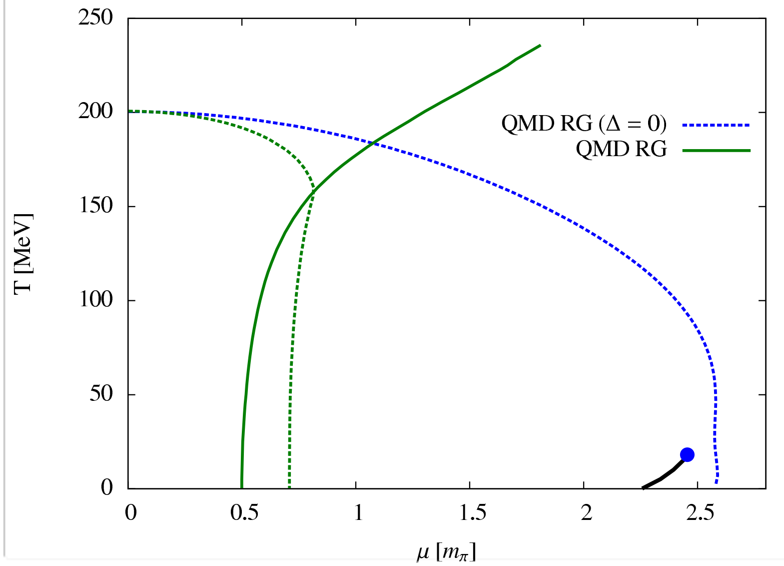
baryon density:
order parameter for $N_c = N_f = 2$
as it is for $N_c = N_f \to \infty$

Quark-Meson-Diquark Model

- **Finite T and μ:**



compare phase diagram with and without collective baryonic fluctuations

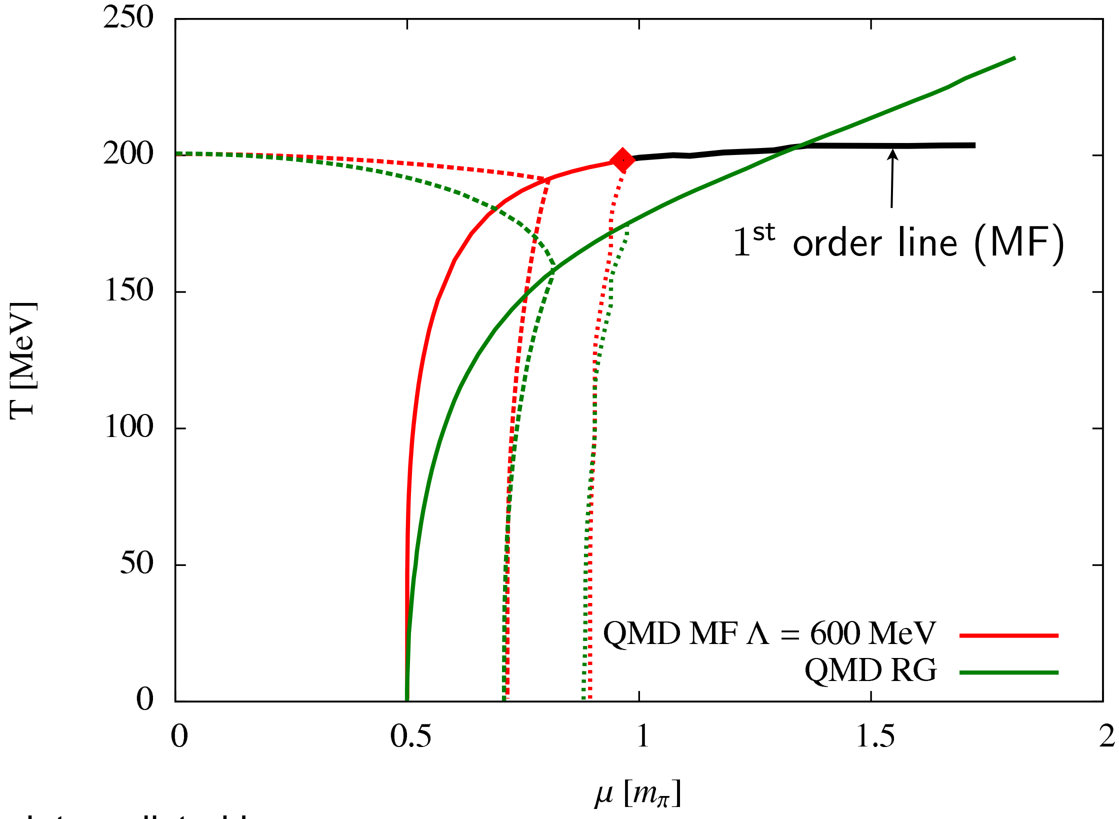


no low-T $1^{\rm st}$ order transition,
no CEP at $\mu \sim 2.5\ m_\pi$!

PhD projects: Nils Strodthoff (FRG), David Scheffler (Lattice), with C. Schmidt & D. Smith

Quark-Meson-Diquark Model

- **Functional RG vs mean field:**



- Tricritical point predicted in:

  Splittorff, Toublan & Verbaarschot, Nucl. Phys. B 620 (2002) 290

QM Model for QCD – Isospin Chemical Potential

- **N_f = 2 quarks & mesons with Yukawa coupling:**

$$\mathcal{L} = \bar{\psi}(\partial\!\!\!/ + g(\sigma + i\gamma^5\vec{\pi}\vec{\tau}) - \mu\gamma^0 - \mu_I\tau_3\gamma^0)\psi$$
$$+ \frac{1}{2}(\partial_\mu\sigma)^2 + \frac{1}{2}(\partial_\mu\pi_0)^2 + U(\rho^2, d^2) - c\sigma$$
$$+ \frac{1}{2}\left((\partial_\mu + 2\mu_I\delta^0_\mu)\pi_+(\partial_\mu - 2\mu_I\delta^0_\mu)\pi_-\right)$$

Son & Stephanov, Phys. Rev. Lett. 86 (2001) 592
Cohen, Phys. Rev. Lett. 91 (2003) 222001
Kogut & Sinclair Phys. Rev. D 70 (2004) 094501; PoS LAT2006 147
de Forcrand, Stephanov & Wenger PoS LAT2007 237
Detmold, Orginos & Shi, arXiv:1205.4224
He, Jin & Zhuang, Phys. Rev. D 71 (2005) 116001
Mu, He & Liu, Phys. Rev. D 82 (2010) 056006

- **chemical potentials:**

$$\mu_u = \mu + \mu_I \qquad \mu_d = \mu - \mu_I$$

$\mu \gg \mu_I$: $\mu_I \rightsquigarrow$ imbalance between up and down

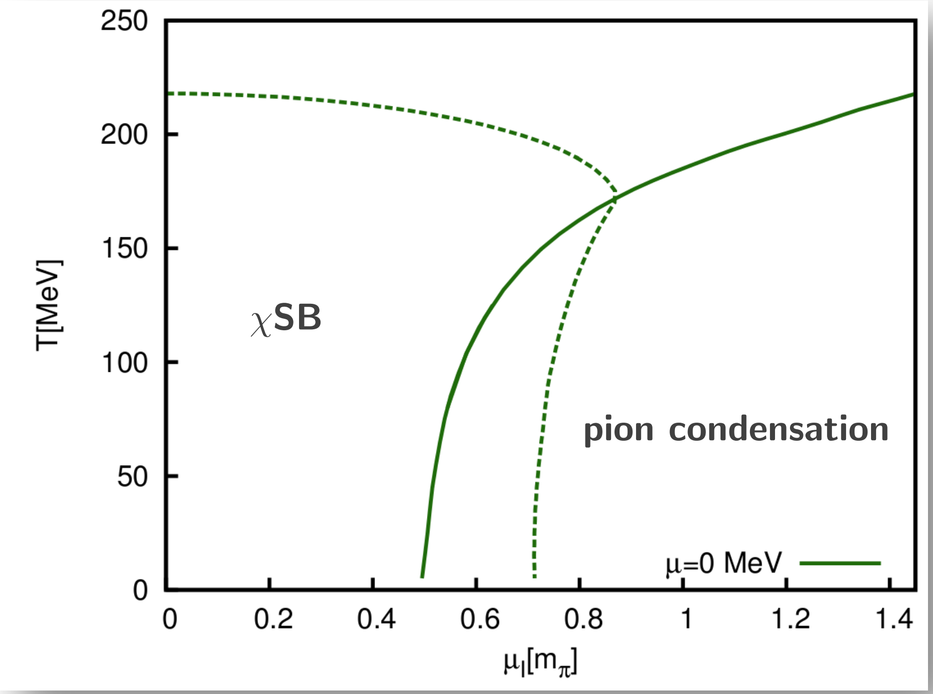
$\mu_I \gg \mu$: $\mu \rightsquigarrow$ imbalance between up and anti-down

- **$\mu = 0$, map to QMD model for QC_2D:**

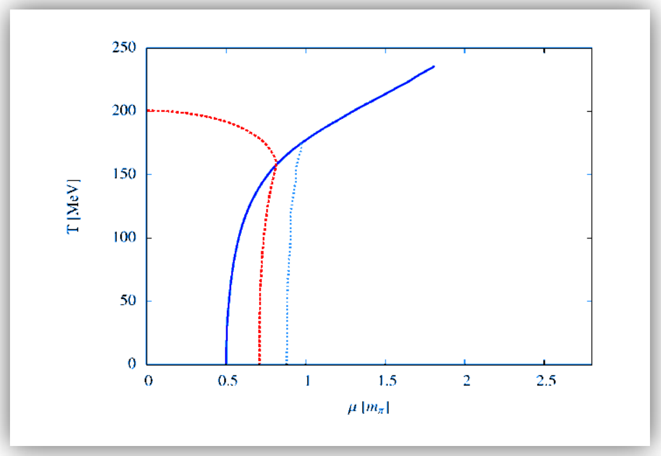
$$N_c\colon\ 3 \to 2 \qquad (\psi_u, \psi_d) \to (\psi_r, \tau_2 C\bar{\psi}_g) \qquad \mu_I \to \mu$$
$$\pi_+, \pi_- \to \Delta, \Delta^* \qquad \pi_0 \to \vec{\pi}$$

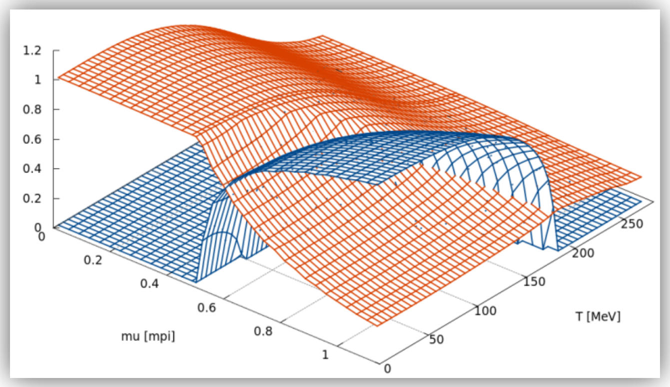
QM Model for QCD – Isospin Chemical Potential

- **Isospin density**



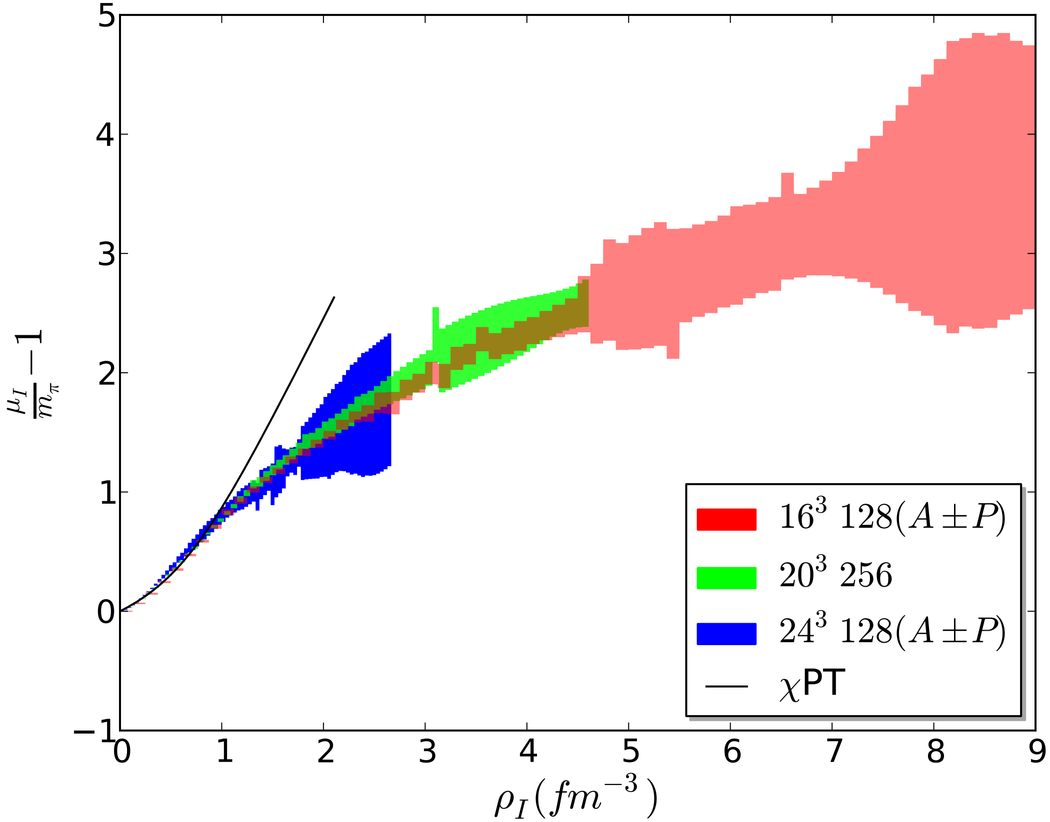
2-color QCD:





QM Model for QCD – Isospin Chemical Potential

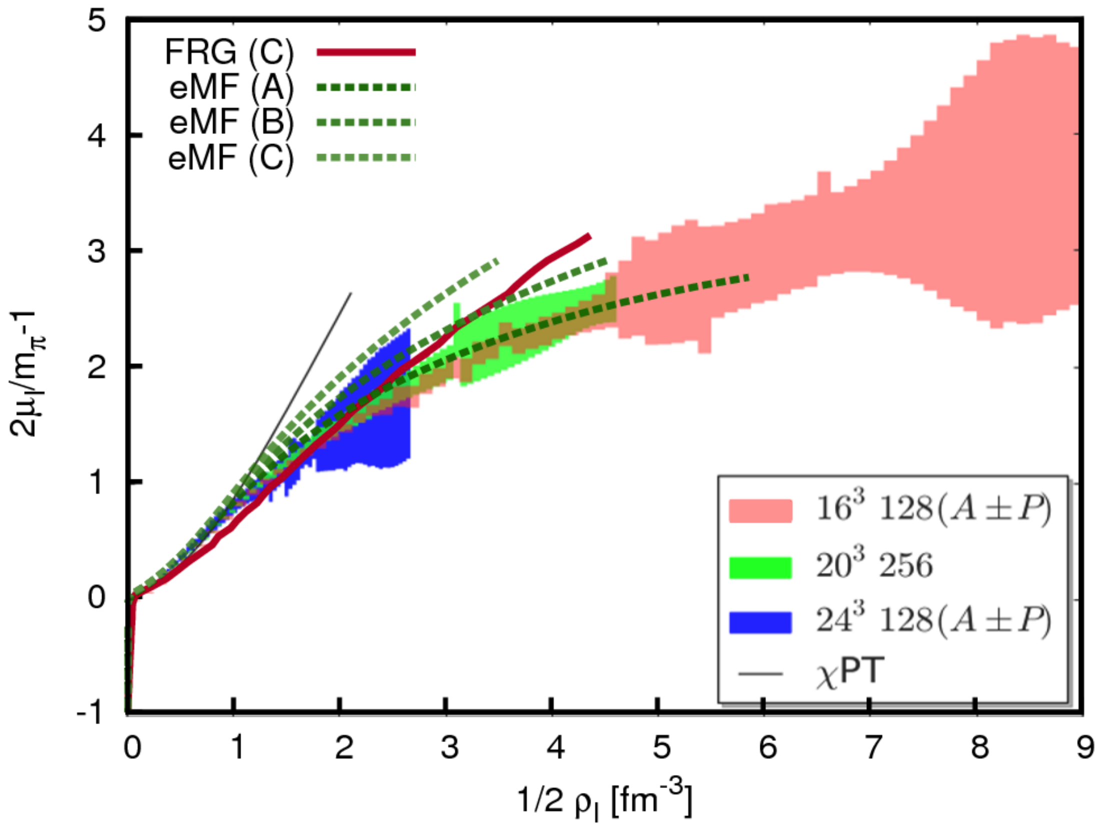
- *T* = 0 isospin density - lattice QCD:



Detmold, Orginos & Shi, arXiv:1205.4224 [hep-lat]

QM Model for QCD – Isospin Chemical Potential

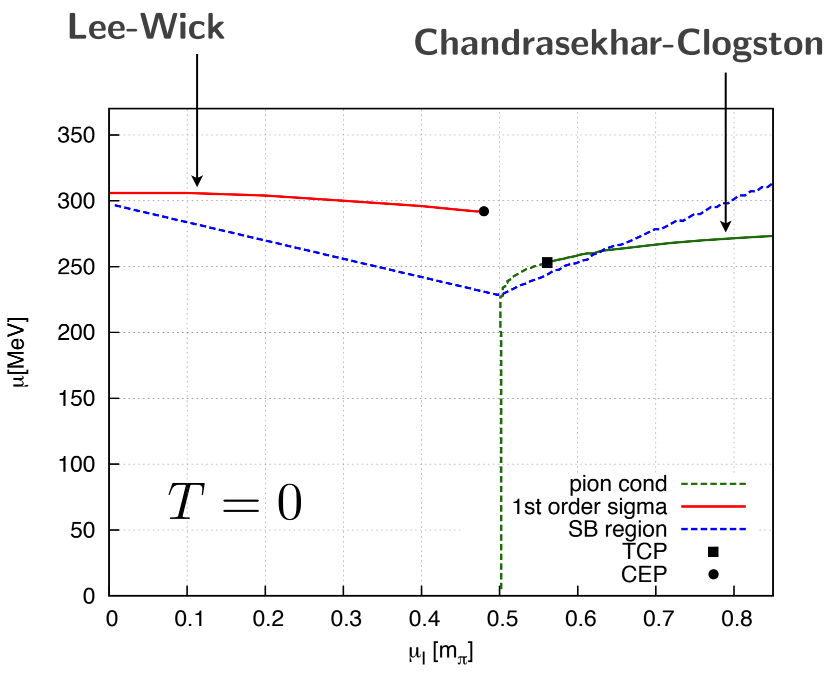
- $T = 0$ isospin density - lattice QCD:

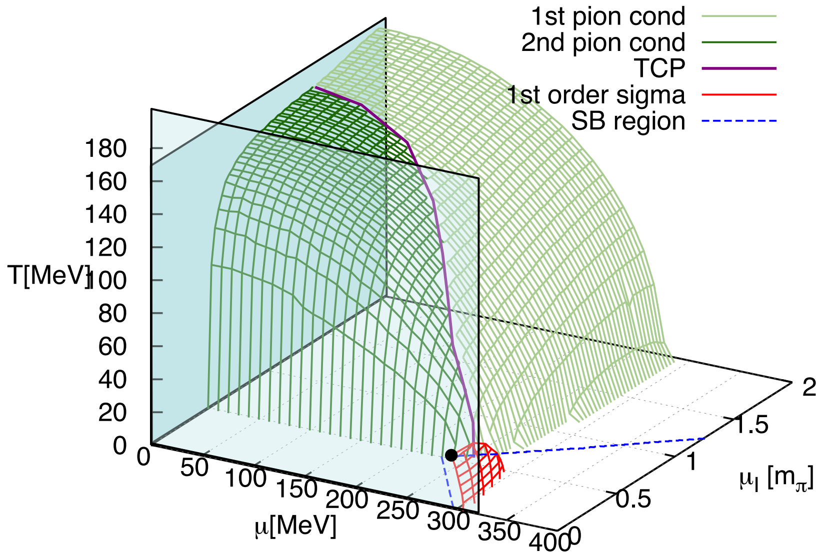


Detmold, Orginos & Shi, arXiv:1205.4224 [hep-lat]

QM Model for QCD – Isospin Chemical Potential

- **Fermionic flow (extended mean-field):**



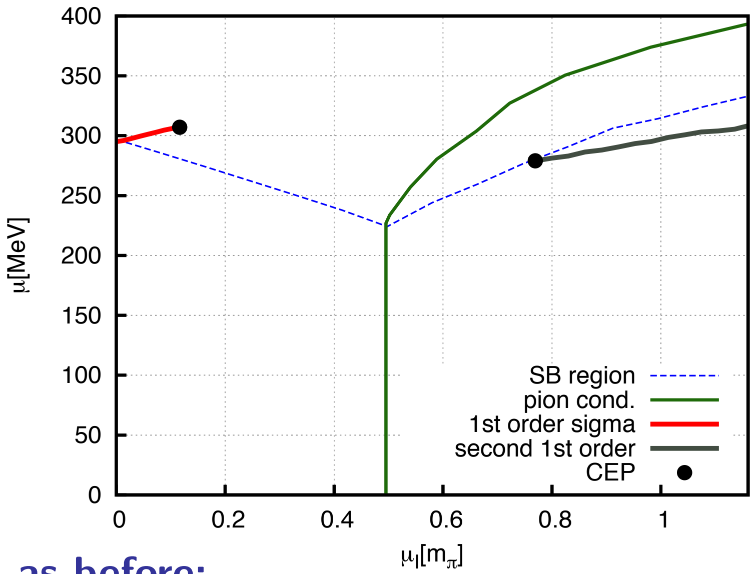


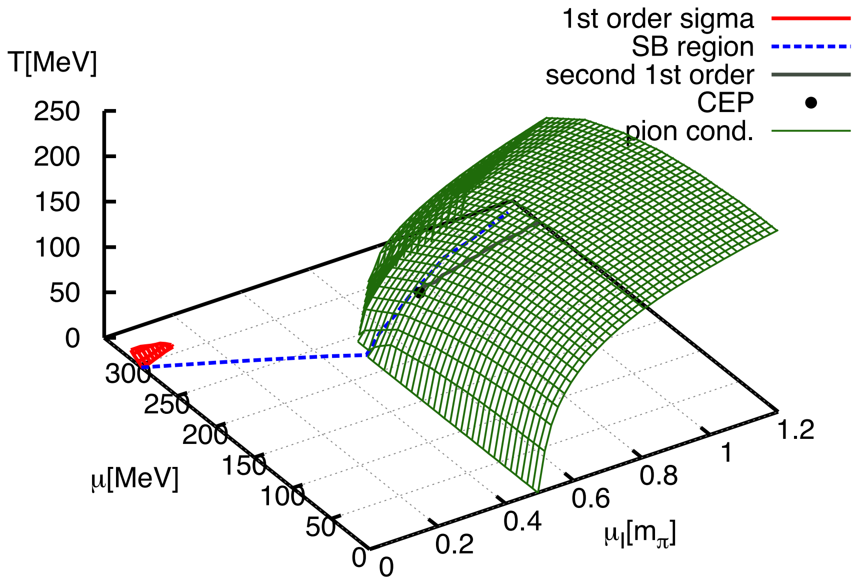
Kamikado, Strodthoff, LvS & Wambach, arXiv:1207.0400

QM Model for QCD – Isospin Chemical Potential

- **Full mesonic flow (2 dimensional):**

$T = 0$:





- **as before:**

$U = U(\rho^2, d^2)$, but replace $\rho^2 = \sigma^2 + \vec{\pi}^2$ and $d^2 = |\Delta|^2$
by $\rho^2 = \sigma^2 + \pi_0^2$ and $d^2 = \pi_1^2 + \pi_2^2 = \pi_+ \pi_-$

Kamikado, Strodthoff, LvS & Wambach, arXiv:1207.0400

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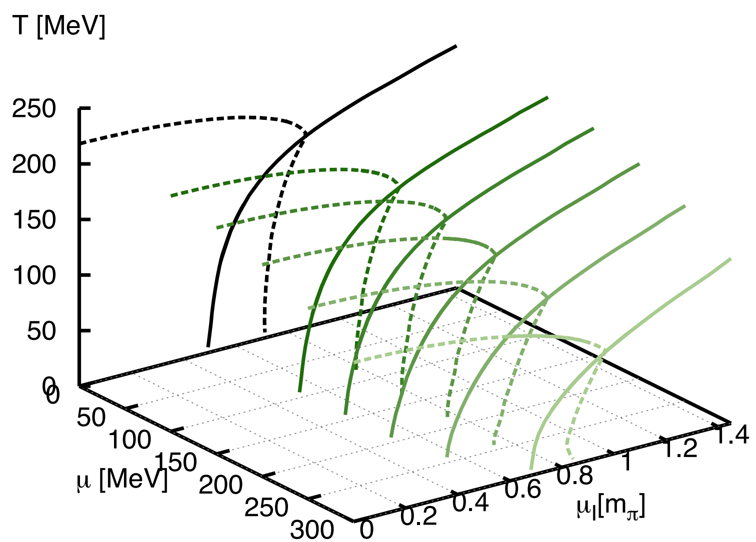
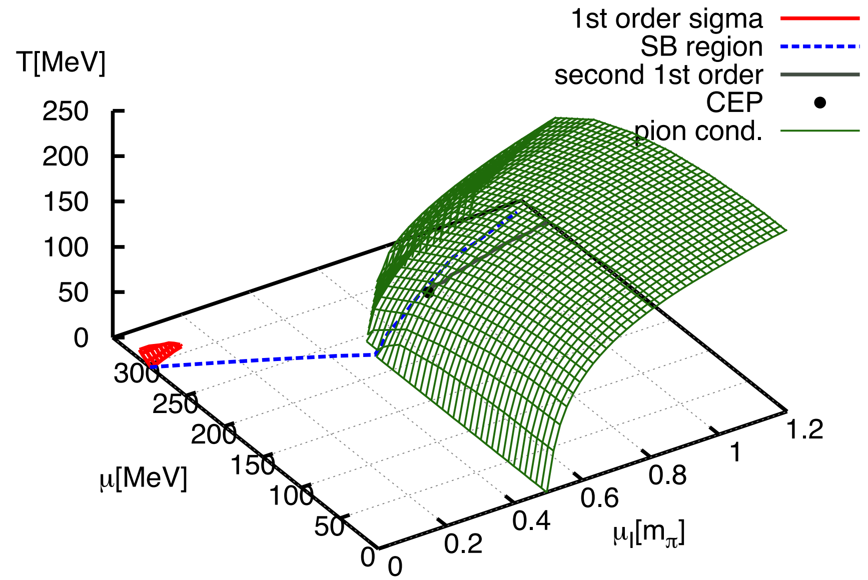
QM Model for QCD – Isospin Chemical Potential

- **Full mesonic flow (2 dimensional):**





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QM Model for QCD – Isospin Chemical Potential

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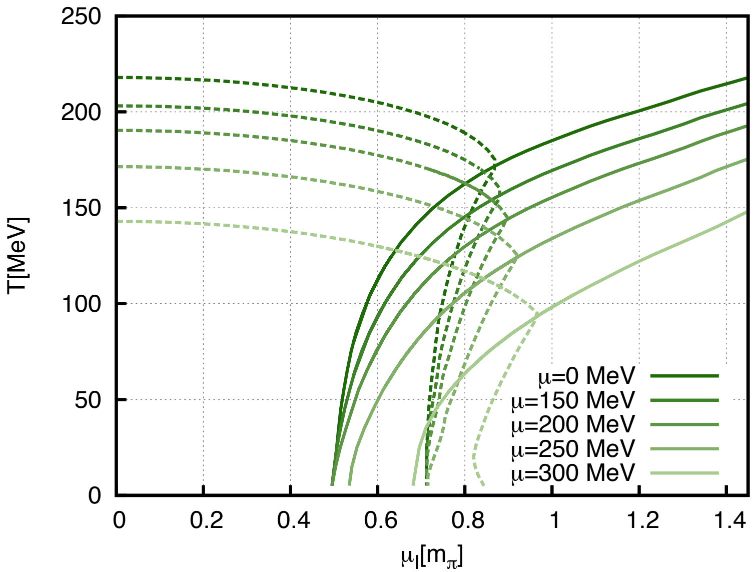
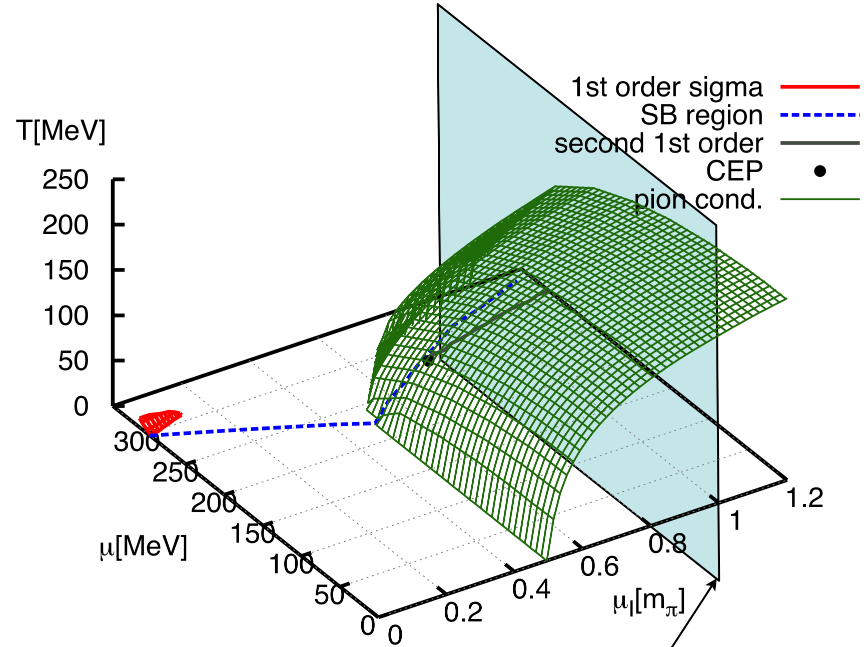


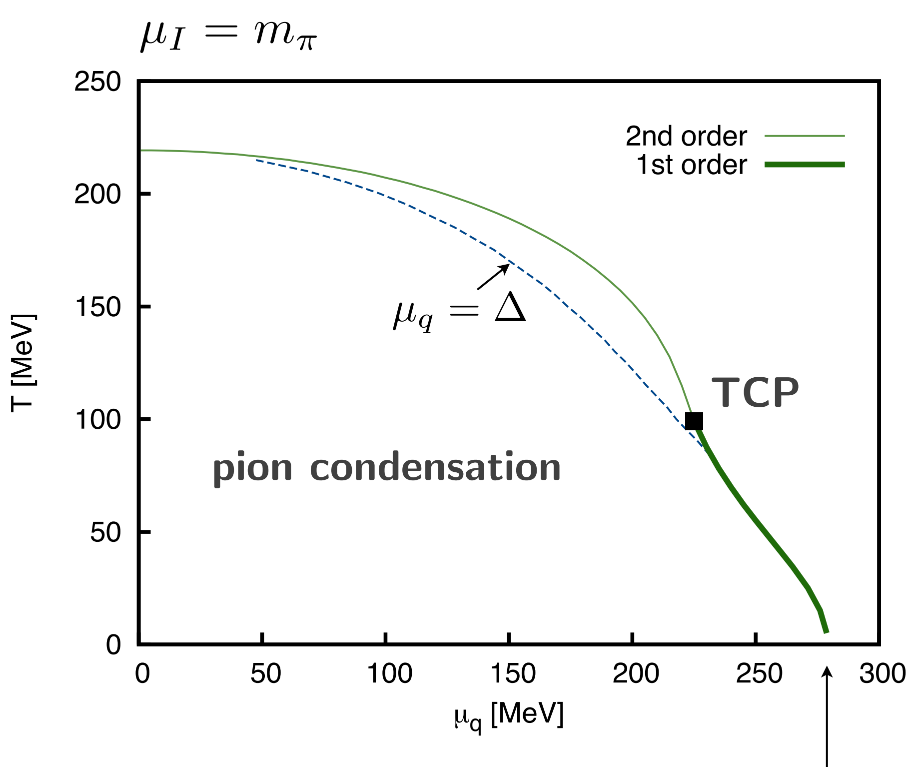


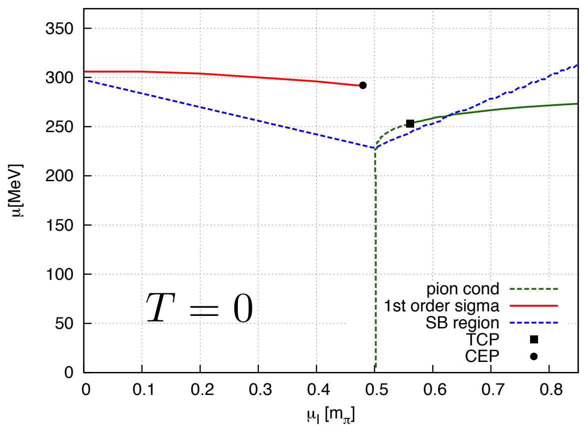
fixed $\mu_I = m_\pi$

μ **for (up/anti-down) imbalance**

QM Model for QCD – Isospin Chemical Potential

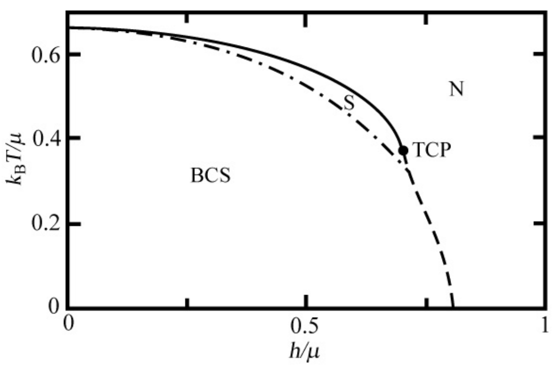
- **Fermionic flow (extended mean-field):**





- **compare:**

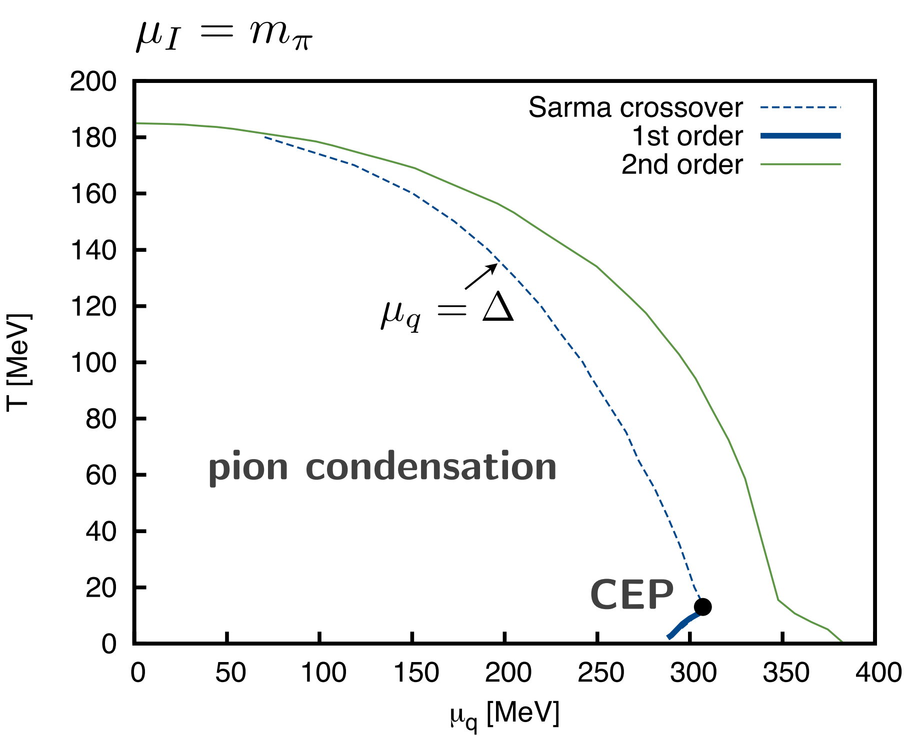
Imbalanced Fermi Gases

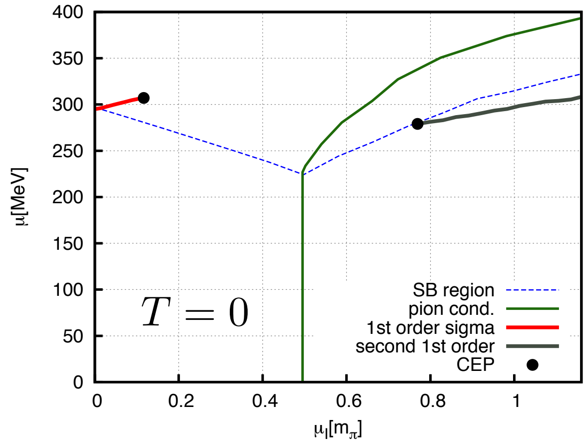


Gubbels, Stoof, arXiv:1205.0568

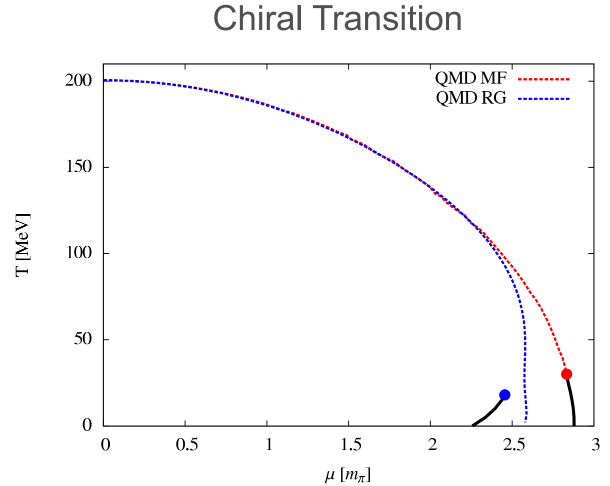
QM Model for QCD – Isospin Chemical Potential

- **Full flow with mesonic fluctuations:**





- **compare:**



- **stable Sarma phase down to *T* = 0**

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G_2 Gauge Theory at Finite Density

G_2 is real:

Dirac operator $\mathcal{D}$ has antiunitary symmetry S,

with $S^2 = -1$ (symplectic, $\beta = 4$).

Holland, Minkowski, Pepe & Wiese, Nucl. Phys. B 668 (2003) 207
Wellegehausen, Wipf & Wozar, Phys. Rev. D 83 (2011) 114502
Maas, LvS, Wellegehausen & Wipf, arXiv:1203.5653

- **no sign problem**

  real and positive for single flavor: $SU(2) \to U_B(1)$

  2 Goldstone bosons: scalar (anti)diquarks

- **O(3) symmetric effective potential**

  $U = U(\phi^2)$ where $\vec{\phi} = (\sigma, \mathrm{Re}\Delta, \mathrm{Im}\Delta)$

- **diquark condensation as before**

- **but have fermionic baryons also**

  as QCD with adjoint quarks

- **breaks down to QCD**

  $$G_2 \xrightarrow{\text{Higgs}} SU(3)$$

  coset:

  $$G_2/SU(3) \sim SO(7)/SO(6) \sim S^6$$

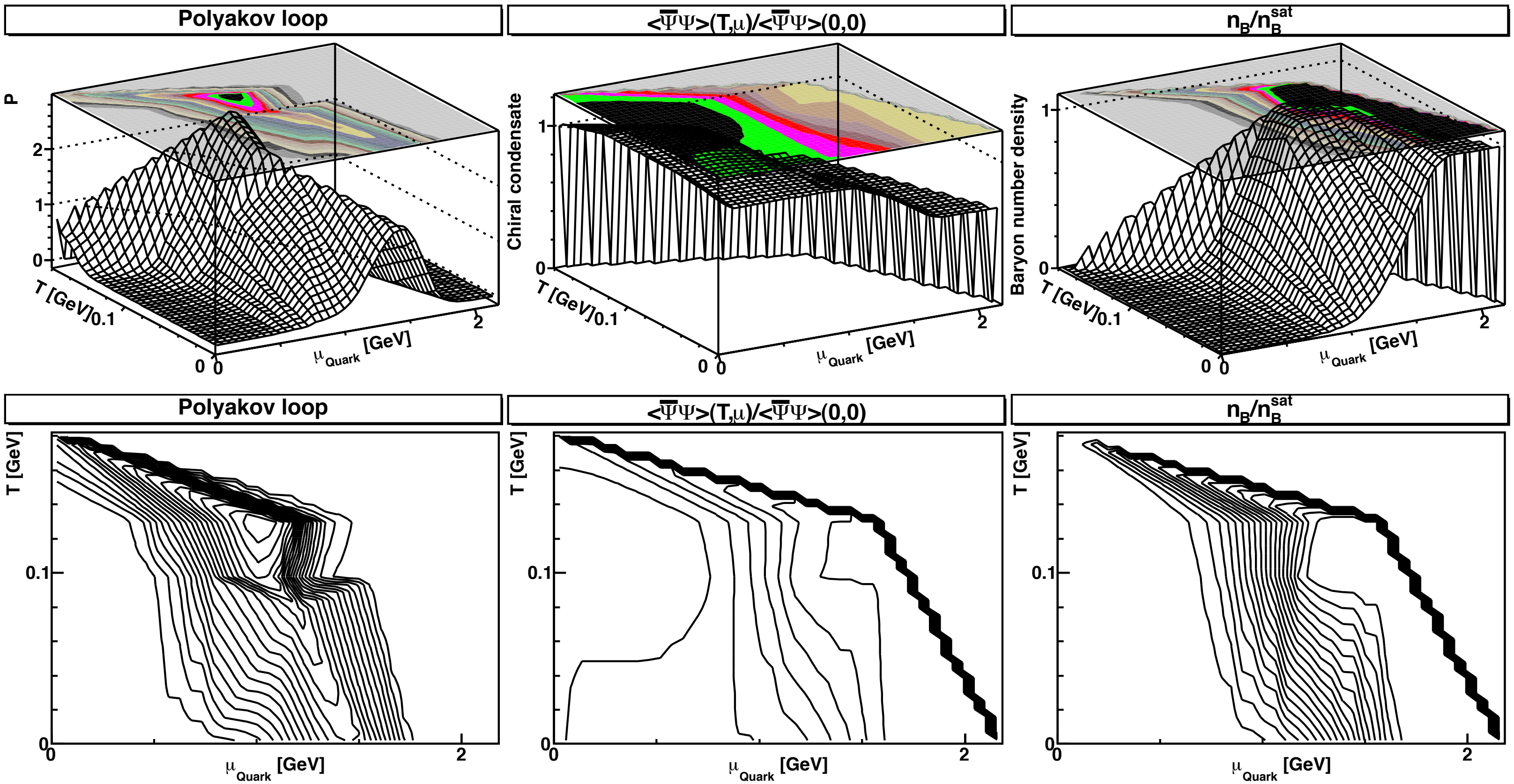
  $$(7) \to (3) \oplus (\bar{3}) \oplus (1) \quad \leftarrow \text{massive Higgs}$$

  $$(14) \to (3) \oplus (\bar{3}) \oplus (8) \quad \leftarrow \text{gluons}$$

  heavy gauge: $(3) \oplus (\bar{3})$

G$_2$ Gauge Theory at Finite Density

- phase diagram with 1 flavor dynamcial Wilson fermion



Maas, LvS, Wellegehausen & Wipf, arXiv:1203.5653.

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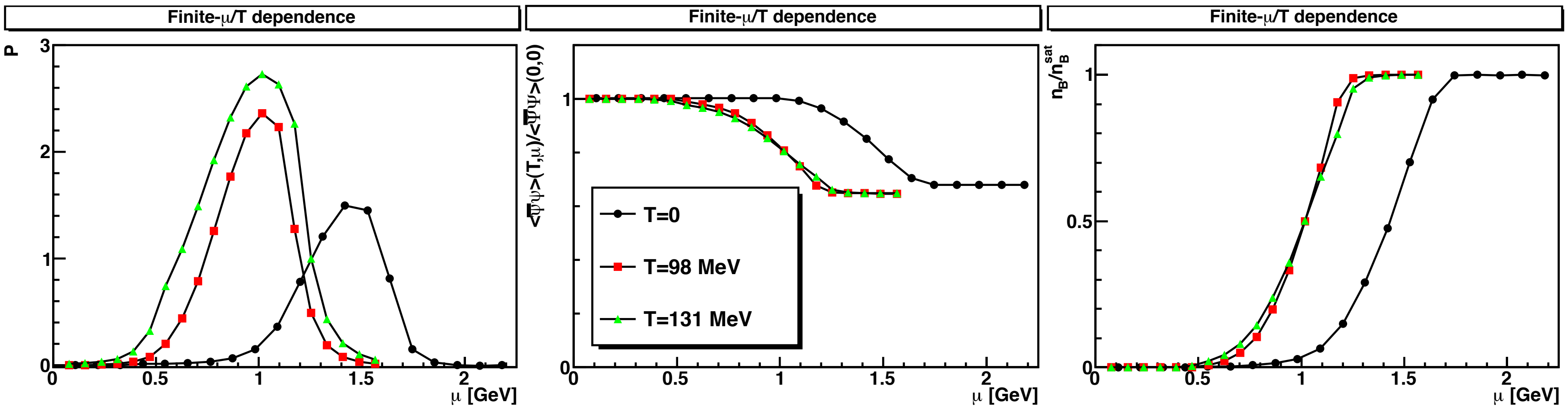
G_2 Gauge Theory at Finite Density

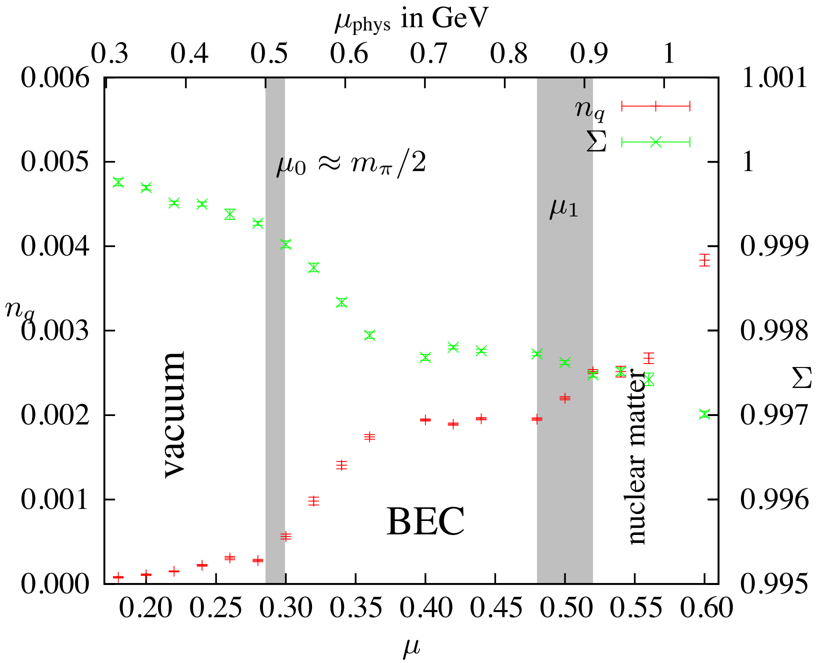
- **finite baryon density (bosonic and fermionic)**

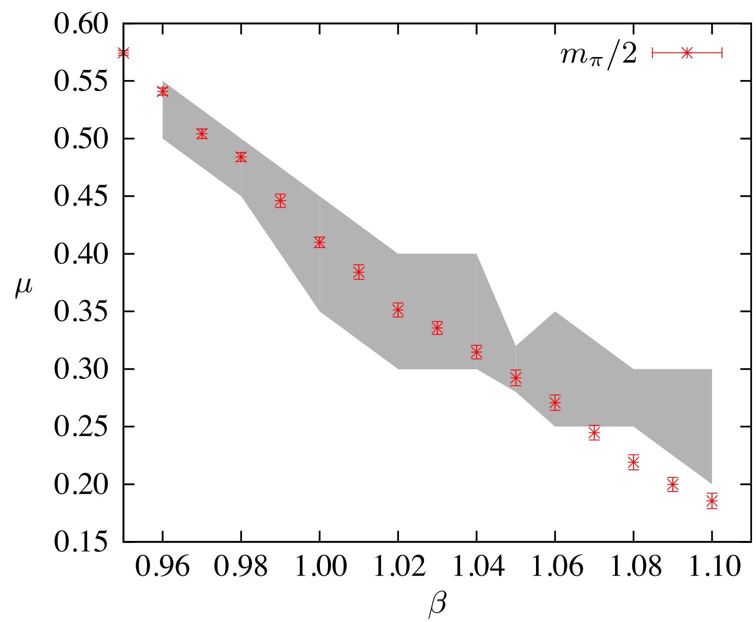


Maas, LvS, Wellegehausen & Wipf, arXiv:1203.5653.

G_2 Gauge Theory at Finite Density

- **onset of diquark condensation:**

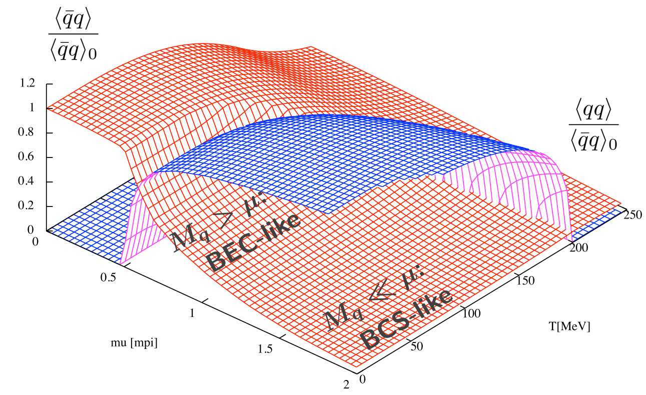




Bjoern Wellegehausen, PhD thesis, Jena 2012.

Summary & Outlook

- **Phase Diagram of Two-Color QCD**
  - need to include baryonic fluctuations
  - functional methods and lattice MC



- **QCD with isospin chemical potential**
  - equivalent problem, population imbalance
- **Phase Diagram of G_2 Gauge Theory**
  - no sign problem − fermionic baryons
- **Fermi**
  - quan ...

Thank You for Your Attention!

- **QCD Phase Diagram**
  - refined functional methods & models, baryonic dofs, finite volume...

DFG Deutsche Forschungsgemeinschaft

European Commission

EMMI ExtreMe Matter Institute
Helmholtz Alliance 'Cosmic Matter in the Laboratory'

HIC for FAIR
Helmholtz International Center

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