(Nearly) perfect fluidity in cold atomic gases: Recent results

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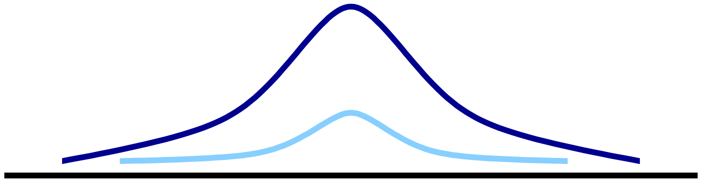
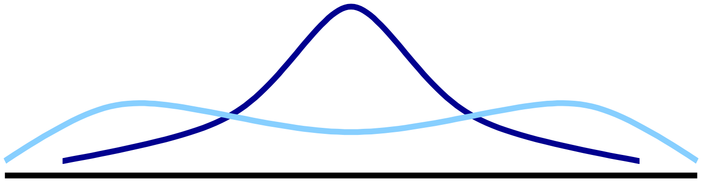
Fluids: Gases, Liquids, Plasmas, . . .

Hydrodynamics: Long-wavelength, low-frequency dynamics of conserved or spontaneously broken symmetry variables.



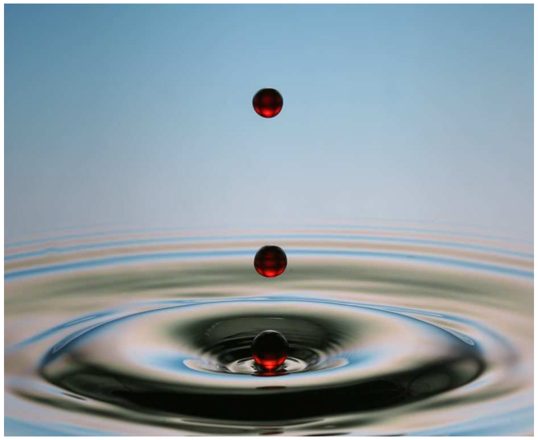
$\tau \sim \tau_{micro}$



$\tau \sim \lambda$

Historically: Water

$(\rho, \epsilon, \vec{\pi})$



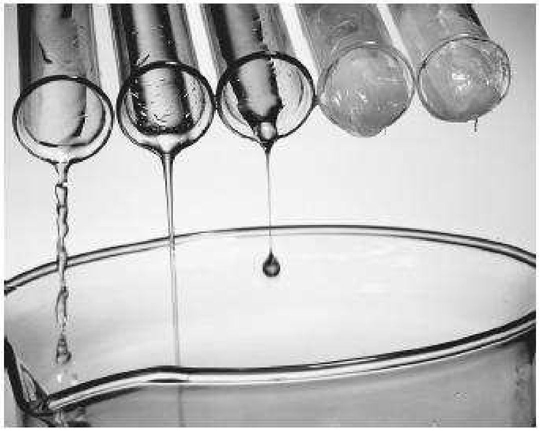
Simple non-relativistic fluid

Simple fluid: Conservation laws for mass, energy, momentum

$$\frac{\partial \rho}{\partial t} + \vec{\nabla}(\rho \vec{v}) = 0$$

$$\frac{\partial \epsilon}{\partial t} + \vec{\nabla} \vec{j}^{\epsilon} = 0$$

$$\frac{\partial}{\partial t}(\rho v_i) + \frac{\partial}{\partial x_j} \Pi_{ij} = 0$$



Constitutive relations: Energy momentum tensor

$$\Pi_{ij} = P\delta_{ij} + \rho v_i v_j + \eta \left(\partial_i v_j + \partial_j v_i - \frac{2}{3}\delta_{ij}\partial_k v_k \right) + O(\partial^2)$$

reactive dissipative 2nd order

Expansion $\Pi^0_{ij} \gg \delta\Pi^1_{ij} \gg \delta\Pi^2_{ij}$

Regime of applicability

Expansion parameter $Re^{-1} = \frac{\eta(\partial v)}{\rho v^2} = \frac{\eta}{\rho L v} \ll 1$

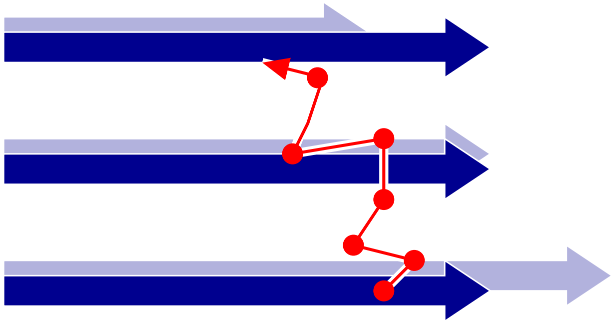
$$\frac{1}{Re} \quad = \quad \underset{\text{fluid property}}{\frac{\eta}{\hbar n}} \quad \times \quad \underset{\text{flow property}}{\frac{\hbar}{m v L}}$$

Consider $mvL \sim \hbar$: Hydrodynamics requires $\eta/(\hbar n) < 1$

Shear viscosity in kinetic theory

Kinetic theory: conserved quantities carried by quasi-particles

$$\frac{\partial f_p}{\partial t} + \vec{v}\cdot\vec{\nabla}_x f_p + \vec{F}\cdot\vec{\nabla}_p f_p = C[f_p]$$



$$\boxed{\eta \sim \frac{1}{3} n\,\bar{p}\,l_{mfp}}$$

Weakly interacting gas: $l_{mfp} \sim 1/(n\sigma)$ $\Rightarrow$ $\eta \sim \frac{1}{3}\frac{\bar{p}}{\sigma}$

$$\boxed{\eta(\sigma \to 0) \to \infty}$$

Strongly interacting gas:

$$\boxed{\frac{\eta}{n} \sim \bar{p} l_{mfp} \geq \hbar}$$

but: kinetic theory not reliable!

Holographic duals: Transport properties

Thermal (conformal) field theory $\equiv AdS_5$ black hole

CFT entropy $\Leftrightarrow$ Hawking-Bekenstein entropy

$\sim$ area of event horizon

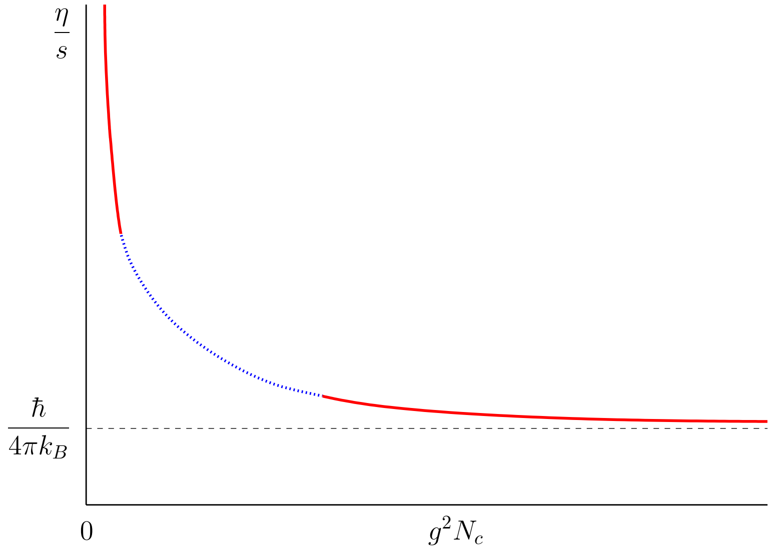
shear viscosity $\Leftrightarrow$ Graviton absorption cross section

$\sim$ area of event horizon

Strong coupling limit

$$\boxed{\frac{\eta}{s} = \frac{\hbar}{4\pi k_B}}$$

Son and Starinets (2001)



Strong coupling limit universal? Provides lower bound for all theories?

Answer appears to be no; e.g. theories with higher derivative gravity duals.

Effective theories for fluids (Unitary Fermi Gas, $T > T_F$)

$$\mathcal{L} = \psi^\dagger \left(i\partial_0 + \frac{\nabla^2}{2M} \right) \psi - \frac{C_0}{2} (\psi^\dagger \psi)^2$$

$$\frac{\partial f_p}{\partial t} + \vec{v} \cdot \vec{\nabla}_x f_p = C[f_p] \qquad \omega < T$$

$$\frac{\partial}{\partial t}(\rho v_i) + \frac{\partial}{\partial x_j} \Pi_{ij} = 0 \quad \omega < T \frac{n}{\eta}$$

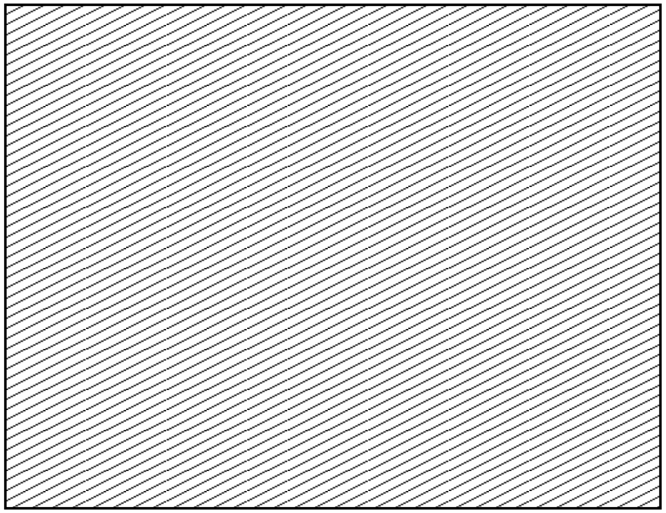
Effective theories (Strong coupling)

$$\mathcal{L} = \bar{\lambda}(i\sigma \cdot D)\lambda - \frac{1}{4}G^a_{\mu\nu}G^a_{\mu\nu} + \ldots \Leftrightarrow S = \frac{1}{2\kappa_5^2}\int d^5x\sqrt{-g}\mathcal{R} + \ldots$$

$$SO(d+2,2) \rightarrow Schr(d) \qquad AdS_{d+3} \rightarrow \mathcal{X}_{d+3}$$

$$\frac{\partial}{\partial t}(\rho v_i) + \frac{\partial}{\partial x_j}\Pi_{ij} = 0 \quad (\omega < T)$$

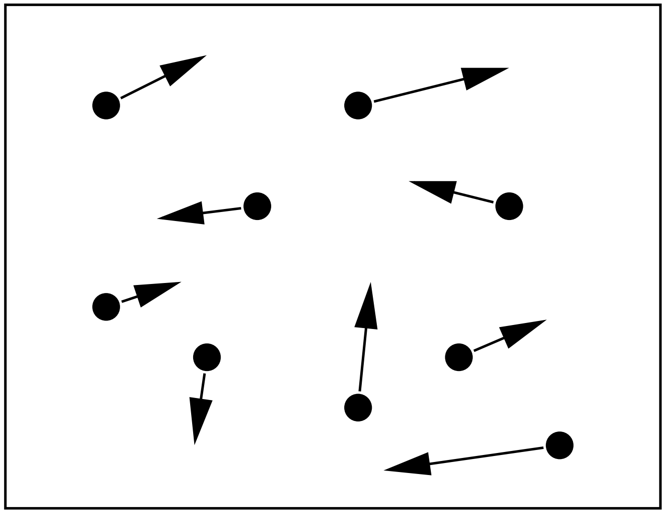
Kinetics vs No-Kinetics



AdS/CFT low viscosity goo

gravitational dual

$\eta/s \simeq 1/(4\pi)$



kinetic liquid

quasi-particles

$\eta/s \gtrsim 1$

Outline

I. Conformal second order hydrodynamics

II. Fluctuations

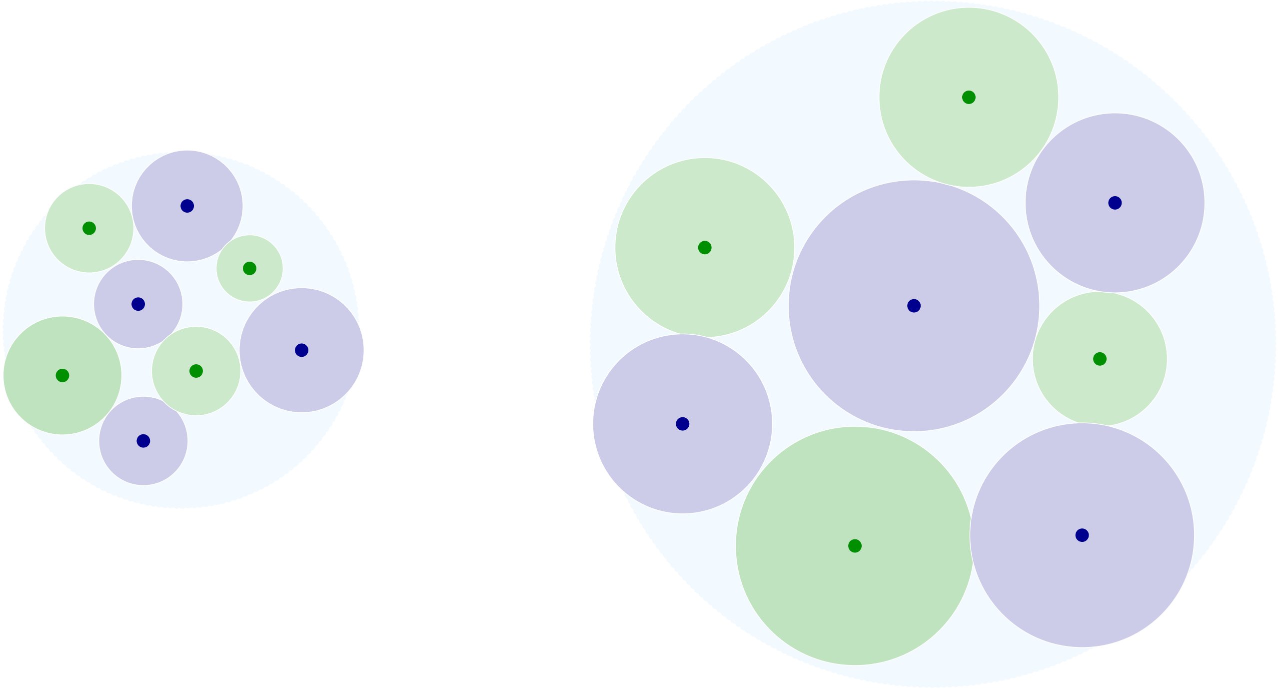
III. Kinetic theory

IV. Experiment

V. Outlook: QGP vs Cold Atoms

I. Scale invariant fluid dynamics

Many body system: Effective cross section $\sigma_{tr} \sim n^{-2/3}$ (or $\sigma_{tr} \sim \lambda^2$)



Systems remains hydrodynamic despite expansion

Scale and conformal symmetry

Gallilean boosts	$\vec{x}' = \vec{x} + \vec{v}t$	$t' = t$
scale trafo	$\vec{x}' = e^s \vec{x}$	$t' = e^{2s} t$
conformal trafo	$\vec{x}' = \vec{x}/(1+ct)$	$1/t' = 1/t + c$

Ideal fluid dynamics

$$\Pi^0_{ij} = P\delta_{ij} + \rho v_i v_j , \qquad \boxed{P = \frac{2}{3}\mathcal{E}}$$

First order viscous hydrodynamics

$$\delta^{(1)}\Pi_{ij} = -\eta\sigma_{ij} , \quad \sigma_{ij} = \left(\nabla_i v_j + \nabla_j v_i - \frac{2}{3}\delta_{ij}(\nabla \cdot v) \right) , \qquad \boxed{\zeta = 0}$$

Second order conformal hydrodynamics

Relaxation of shear stress is a second order hydro term. Complete list

$$\begin{aligned}
\delta^{(2)}\Pi^{ij} \;=\; & \eta\tau_\pi\left[{}^{\langle}D\sigma^{ij\rangle} + \frac{2}{3}\sigma^{ij}(\nabla\cdot v)\right] \\
& + \lambda_1\sigma^{\langle i}{}_k\sigma^{j\rangle k} + \lambda_2\sigma^{\langle i}{}_k\Omega^{j\rangle k} + \lambda_3\Omega^{\langle i}{}_k\Omega^{j\rangle k} \\
& + \gamma_1\nabla^{\langle i}T\nabla^{j\rangle}T + \gamma_2\nabla^{\langle i}P\nabla^{j\rangle}P + \gamma_3\nabla^{\langle i}\nabla^{j\rangle}T + \ldots
\end{aligned}$$

$$D = \partial_0 + v\cdot\nabla \quad A^{\langle ij\rangle} = \tfrac{1}{2}\left(A_{ij} + A_{ji} - \tfrac{2}{3}g_{ij}A^k{}_k\right) \quad \Omega^{ij} = \left(\nabla_i v_j - \nabla_j v_i\right)$$

New transport coefficients $\tau_\pi, \lambda_i, \gamma_i$

Can be written as a relaxation equation for $\pi^{ij} \equiv \delta\Pi^{ij}$

$$\pi^{ij} = -\eta\sigma^{ij} - \tau_\pi\left[{}^{\langle}D\pi^{ij\rangle} + \frac{5}{3}(\nabla\cdot v)\pi^{ij}\right] + \ldots$$

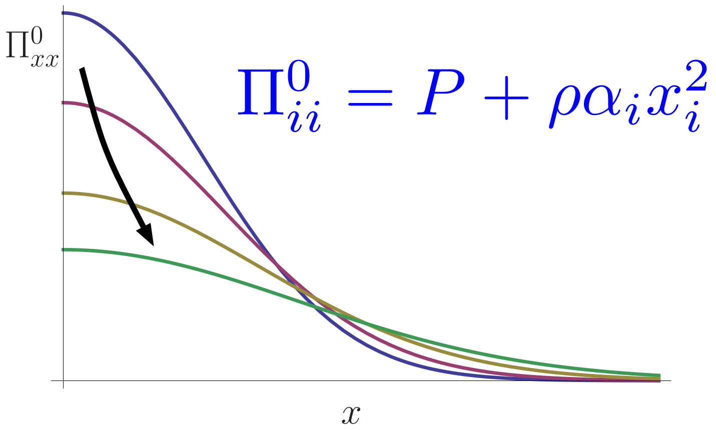
Chao, Schaefer (2011)

Why second order fluid dynamics?

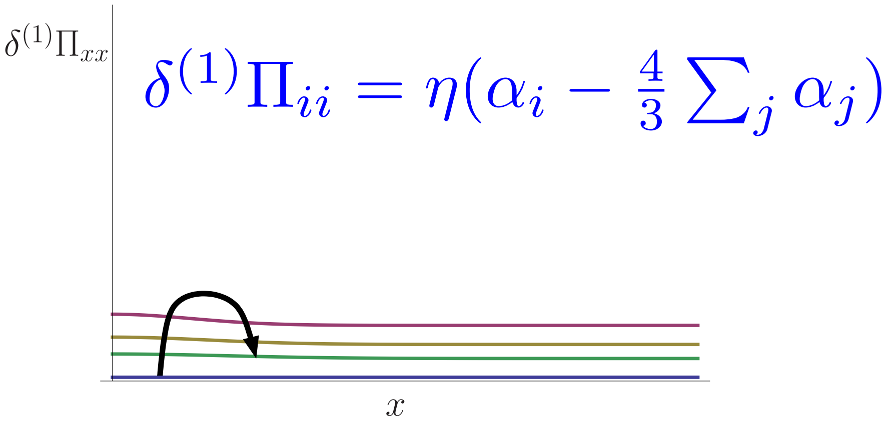
Scaling ("Hubble") expansion

$$\rho(x_i,t) = \rho_0(b_i(t)x_i), \qquad v_i(x_j,t) = \alpha_i(t)x_i, \qquad \alpha_i(t) = \dot{b}_i(t)/b_i(t)$$

Compare ideal and dissipative stresses



$$\Pi^0_{ii} = P + \rho\alpha_i x_i^2$$



$$\delta^{(1)}\Pi_{ii} = \eta(\alpha_i - \tfrac{4}{3}\textstyle\sum_j \alpha_j)$$

Ideal stresses propagate with speed $\sim c_s$, dissipative stresses propagate with infinite speed. Hydro always breaks down in the dilute corona.

Solved by relaxation time $\tau_\pi \sim \frac{\eta}{P}$.

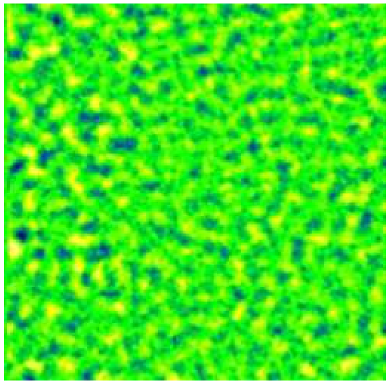
II. Fluctuations

If hydrodynamics is an effective (field?) theory

then where are the loop corrections?

Thermal fluctuations

Hydrodynamic variables fluctuate



$$\langle \delta v_i(x,t)\delta v_j(x',t)\rangle = \frac{T}{\rho}\delta_{ij}\delta(x-x')$$

Linearized hydrodynamics propagates fluctuations as shear or sound

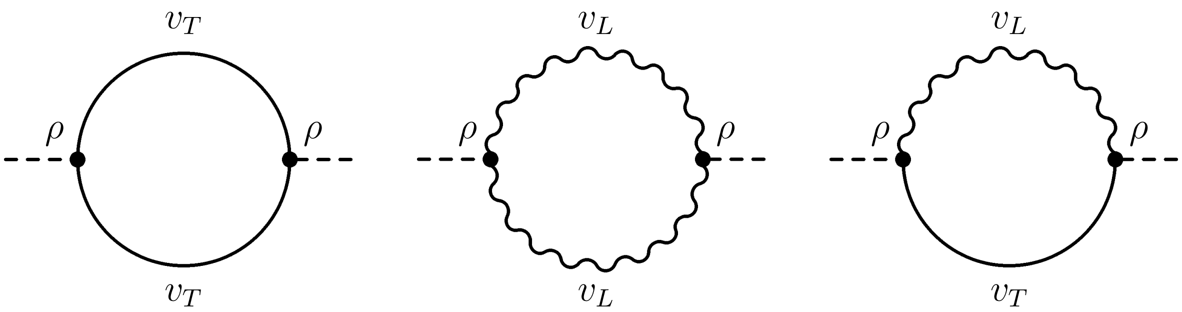
$$\langle \delta v_i^T \delta v_j^T \rangle_{\omega,k} = \frac{2T}{\rho}(\delta_{ij} - \hat{k}_i\hat{k}_j)\frac{\nu k^2}{\omega^2 + (\nu k^2)^2} \qquad \textit{shear}$$

$$\langle \delta v_i^L \delta v_j^L \rangle_{\omega,k} = \frac{2T}{\rho}\hat{k}_i\hat{k}_j\frac{\omega k^2 \Gamma}{(\omega^2 - c_s^2 k^2)^2 + (\omega k^2 \Gamma)^2} \qquad \textit{sound}$$

$v = v_T + v_L$: $\quad \nabla \cdot v_T = 0,\ \nabla \times v_L = 0$ $\qquad \nu = \eta/\rho, \quad \Gamma = \frac{4}{3}\nu + \ldots$

Hydro Loops: “Breakdown” of second order hydro

Response function $G_R^{xyxy} = \langle \theta(t)[\Pi^{xy}, \Pi^{xy}] \rangle_{\omega,k}$ $\Pi_{xy} = \rho v_x v_y$



$$G_R^{xyxy} = P + \delta P + i\omega[\eta + \delta\eta] + \omega^2 \left[\eta\tau_\pi + \delta(\eta\tau_\pi)\right]$$

$$\delta\eta \sim T \left(\frac{\rho}{\eta}\right)^2 \left(\frac{P}{\rho}\right)^{1/2} \qquad \delta(\eta\tau_\pi) \sim \frac{1}{\sqrt{\omega}} \left(\frac{\rho}{\eta}\right)^{3/2}$$

Hydro Loops: “Breakdown” of second order hydro

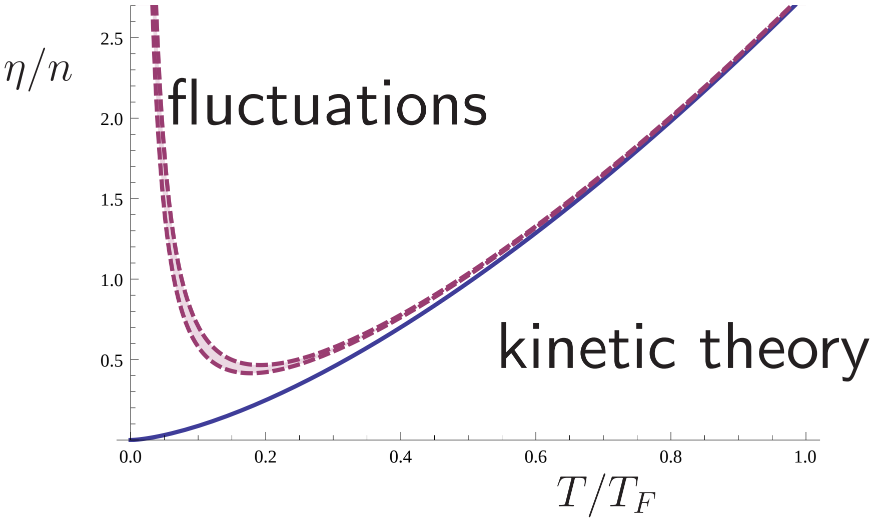
$$\delta\eta \sim T \left(\frac{\rho}{\eta}\right)^2 \left(\frac{P}{\rho}\right)^{1/2} \qquad \delta(\eta\tau_\pi) \sim \frac{1}{\sqrt{\omega}} \left(\frac{\rho}{\eta}\right)^{3/2}$$

Small shear viscosity enhances fluctuation corrections.

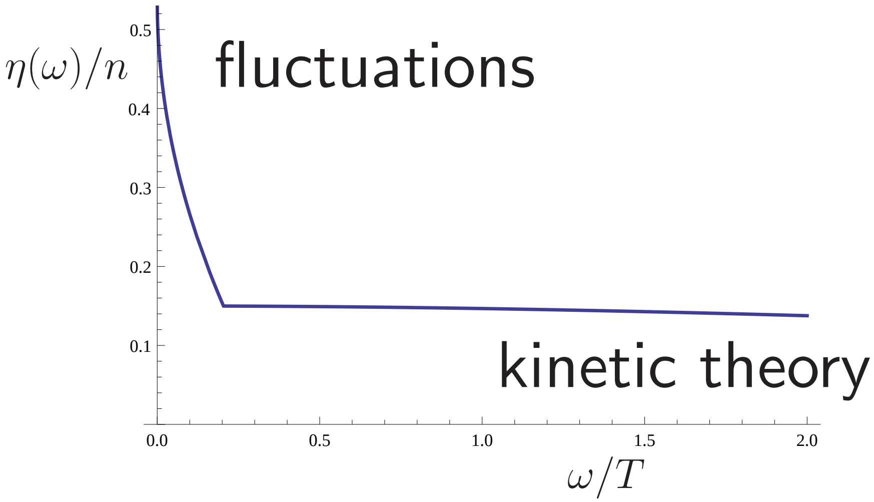
Small η leads to large $\delta\eta$: There must be a bound on η/n.

Relaxation time diverges: 2nd order hydro without fluctuations inconsistent.

Fluctuation induced bound on η/n



$$(\eta/n)_{min} \simeq 0.3$$



spectral function

non-analytic $\sqrt{\omega}$ term

see also Kovtun, Moore, Romatschke (2011)

III. Linear response and kinetic theory

Consider background metric $g_{ij}(t,\mathbf{x}) = \delta_{ij} + h_{ij}(t,\mathbf{x})$. Linear response

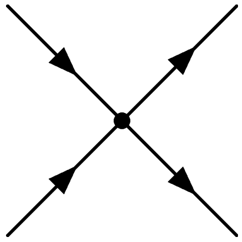
$$\delta\Pi^{ij} = -\frac{1}{2} G_R^{ijkl} h_{kl}$$

Kubo relation: $\eta(\omega) = \frac{1}{\omega} \mathrm{Im} G_R^{xyxy}(\omega, 0)$

Kinetic theory: Boltzmann equation

$$\left(\frac{\partial}{\partial t} + \frac{p^i}{m} \frac{\partial}{\partial x^i} - \left(g^{il} \dot{g}_{lj} p^j + \Gamma^i_{jk} \frac{p^j p^k}{m} \right) \frac{\partial}{\partial p^i} \right) f(t, \mathbf{x}, \mathbf{p}) = C[f]$$

$C[f] =$

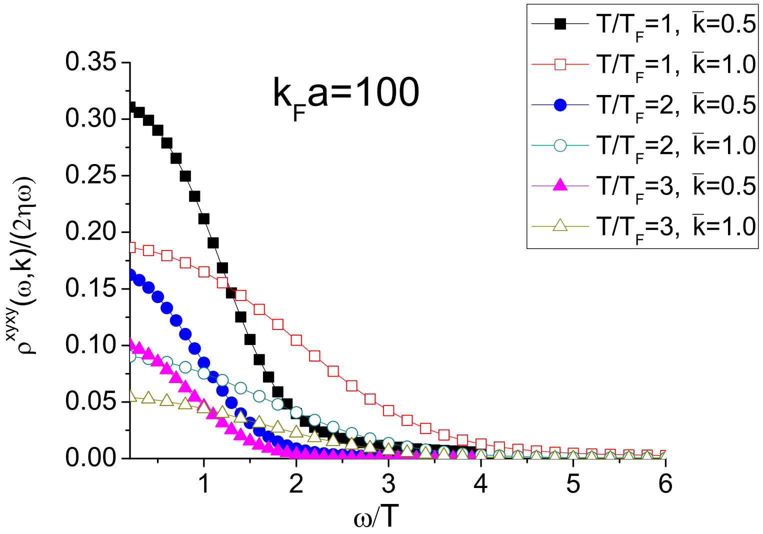


Kinetic theory

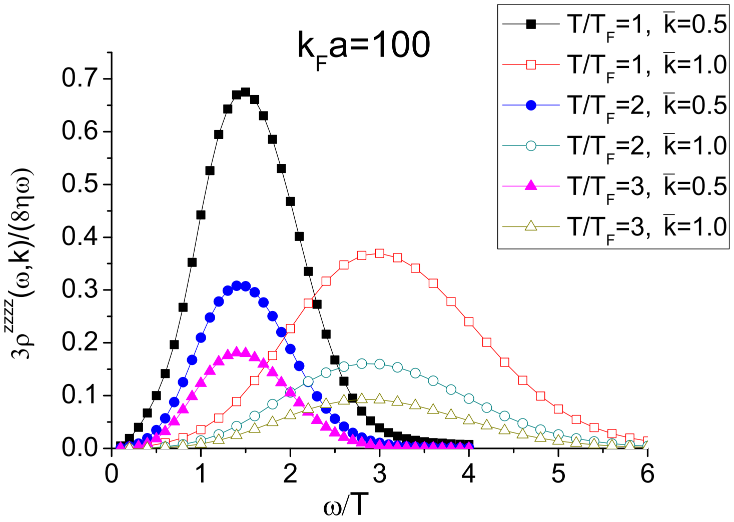
linearize $f = f_0 + \delta f$, solve for δf, $\hookrightarrow \delta\Pi_{ij}$, $\hookrightarrow G_R$, $\hookrightarrow \eta(\omega)$

$$\eta(\omega) = \frac{\eta}{1+\omega^2\tau_\pi^2} \qquad \eta = \frac{15}{32\sqrt{\pi}}(mT)^{3/2} \qquad \tau_\pi = \frac{\eta}{nT}$$

shear channel



sound channel



Braby, Chao, T.S. (2010)

Second order hydrodynamics from kinetic theory

Boltzmann equation (BGK approximation)

$$\begin{aligned}\delta^{(2)}\Pi^{ij} \;&=\; \frac{\eta^2}{P}\left[{}^{\langle}D\sigma^{ij\rangle} + \frac{2}{3}\sigma^{ij}(\nabla\cdot v)\right] \\ &\quad + \frac{\eta^2}{P}\left[\sigma^{\langle i}{}_k\sigma^{j\rangle k} + \sigma^{\langle i}{}_k\Omega^{j\rangle k}\right] + O(\kappa\eta\nabla^i\nabla^j T)\end{aligned}$$

relaxation time $\tau_\pi = \frac{\eta}{P} \simeq \frac{\eta}{nT}$

Shear & bulk viscosity: Sum rules

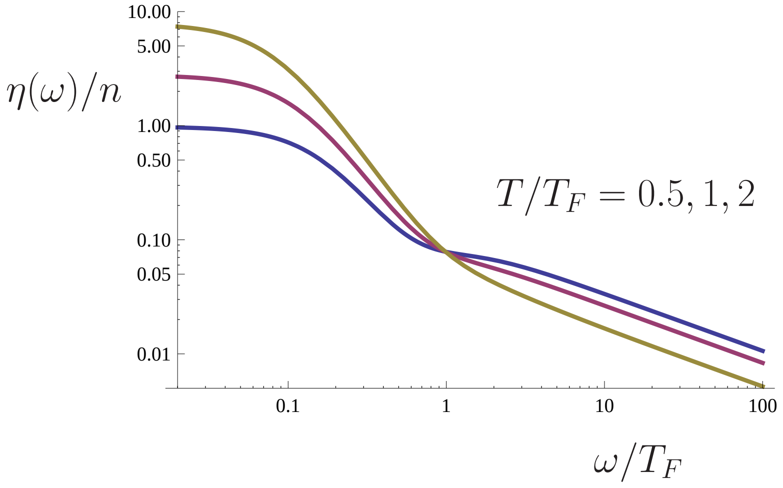
Randeria & Taylor proved the sum rules (corrected by Enss & Zwerger)

$$\frac{1}{\pi}\int dw\left[\eta(\omega)-\frac{C}{15\pi\sqrt{m\omega}}\right]=\frac{\mathcal{E}}{3}-\frac{C}{10\pi ma}$$

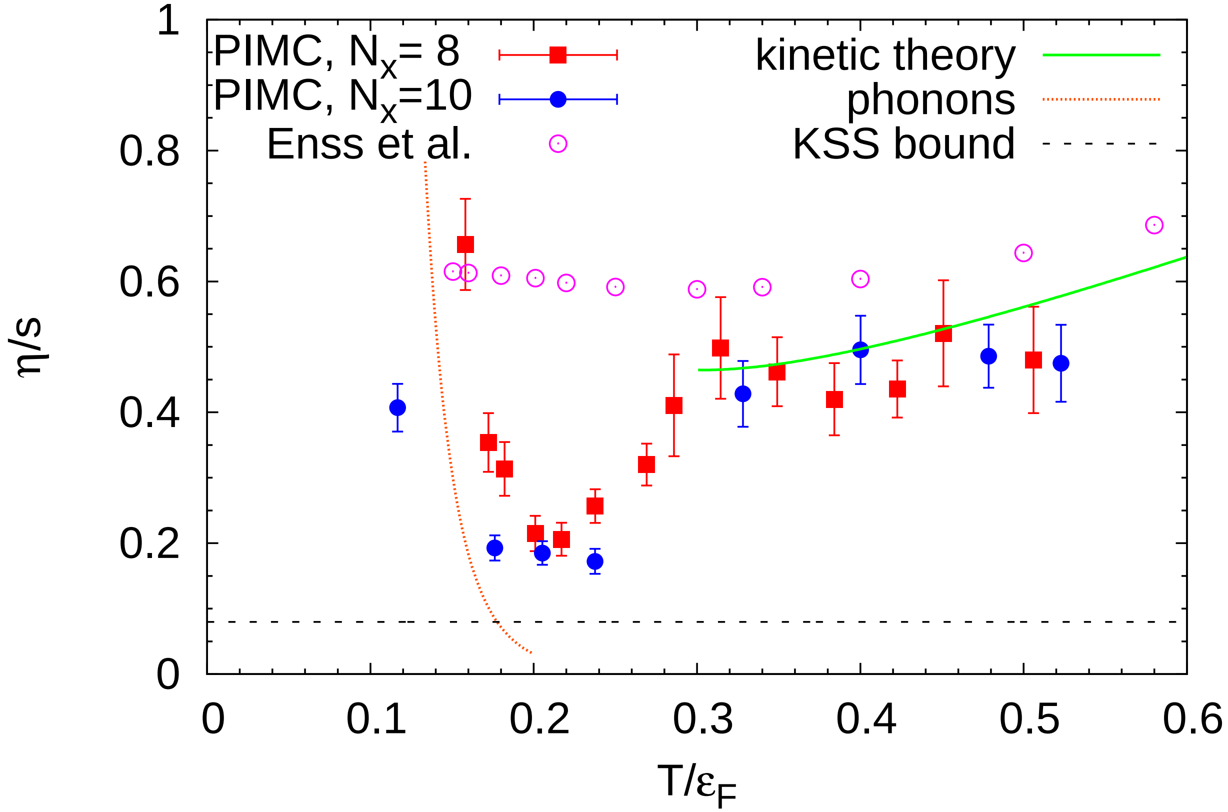
$$\frac{1}{\pi}\int dw\;\zeta(\omega)=\frac{1}{72\pi ma^2}\left(\frac{\partial C}{\partial a^{-1}}\right)$$

where C is Tan's contact, $n_k \sim C/k^4$.

Model spectral function: Kinetic theory for $\omega < T$, OPE for $\omega > T$.

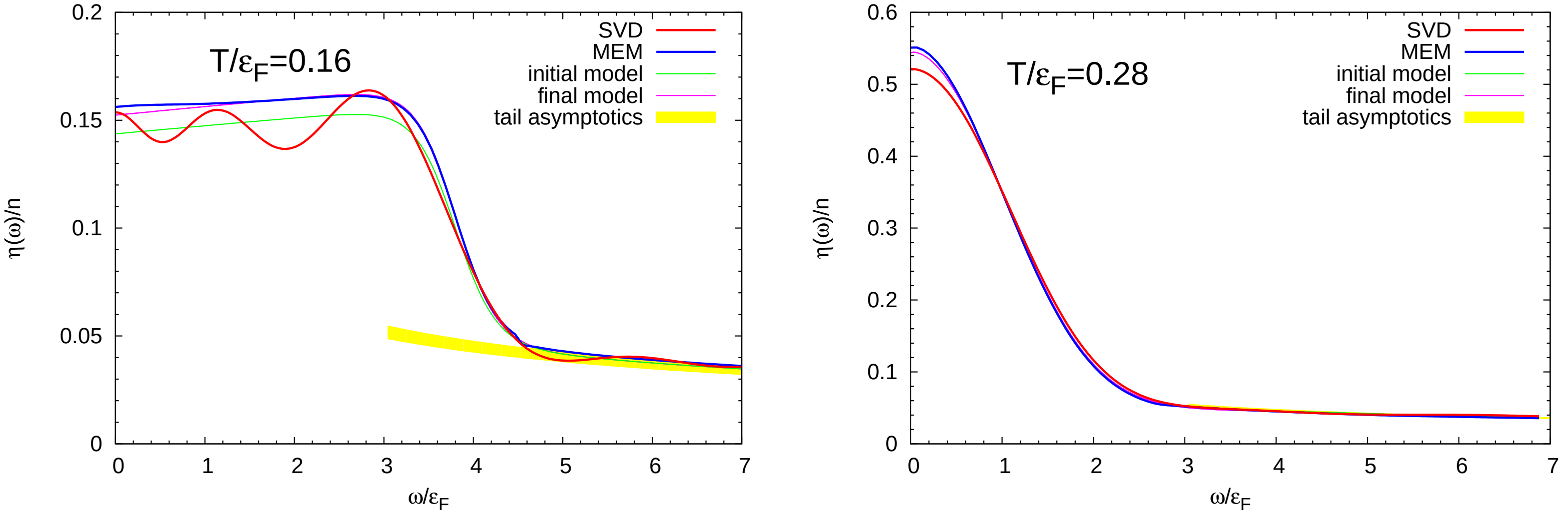


Lattice data: η/s



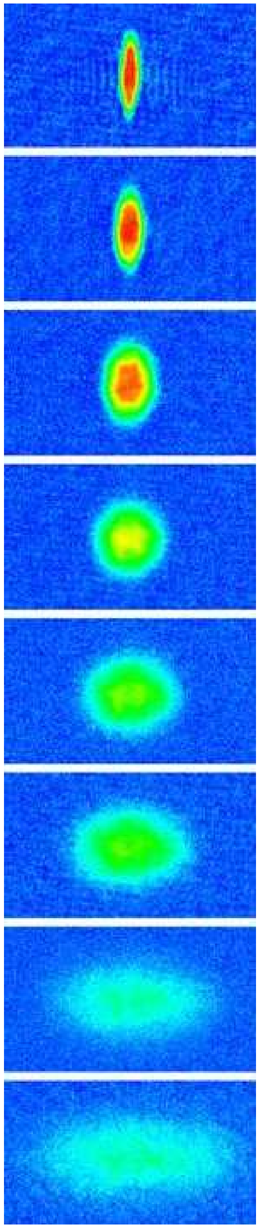
Wlazlowski, Magierski & Drut, arXiv:1204.0270

Lattice data: Spectral functions

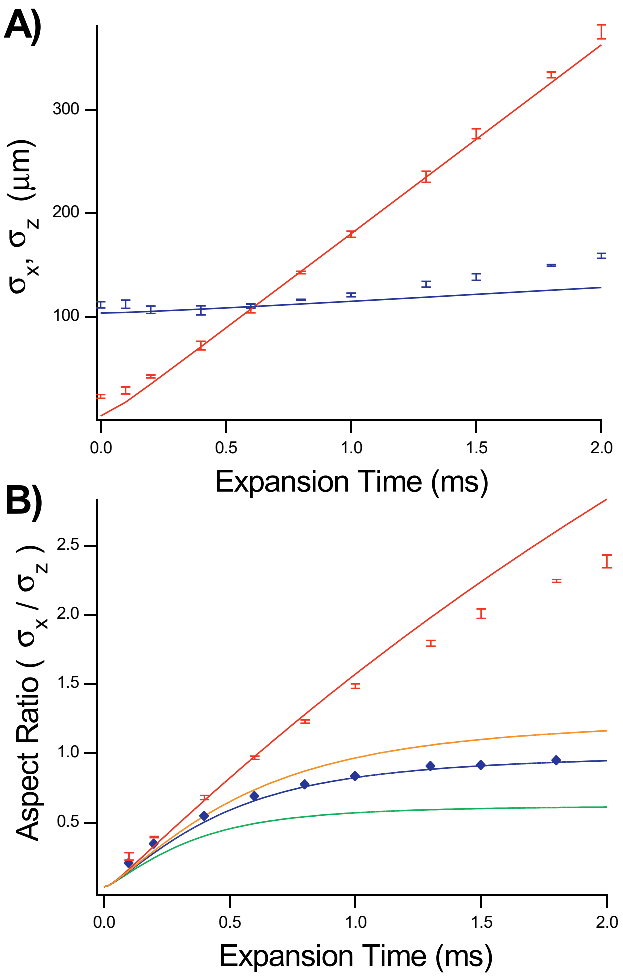


Wlazlowski, Magierski & Drut, arXiv:1204.0270

IV. Experiments: Flow and Collective Modes



O'Hara et al. (2002)



Hydrodynamic

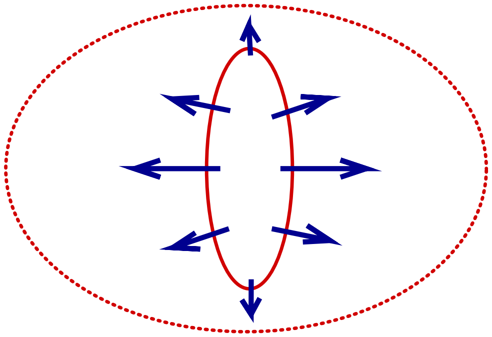
expansion converts

coordinate space

anisotropy

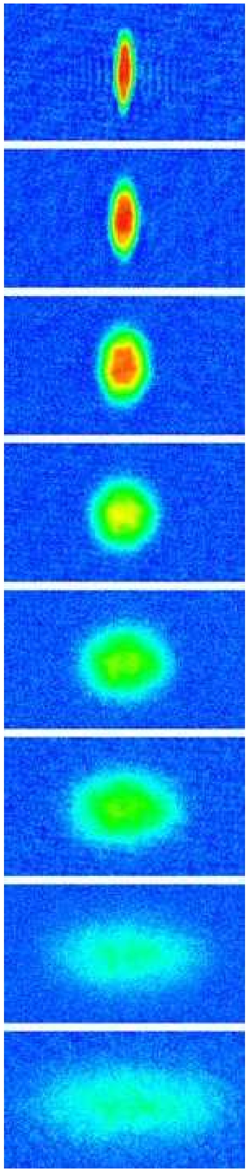
to momentum space

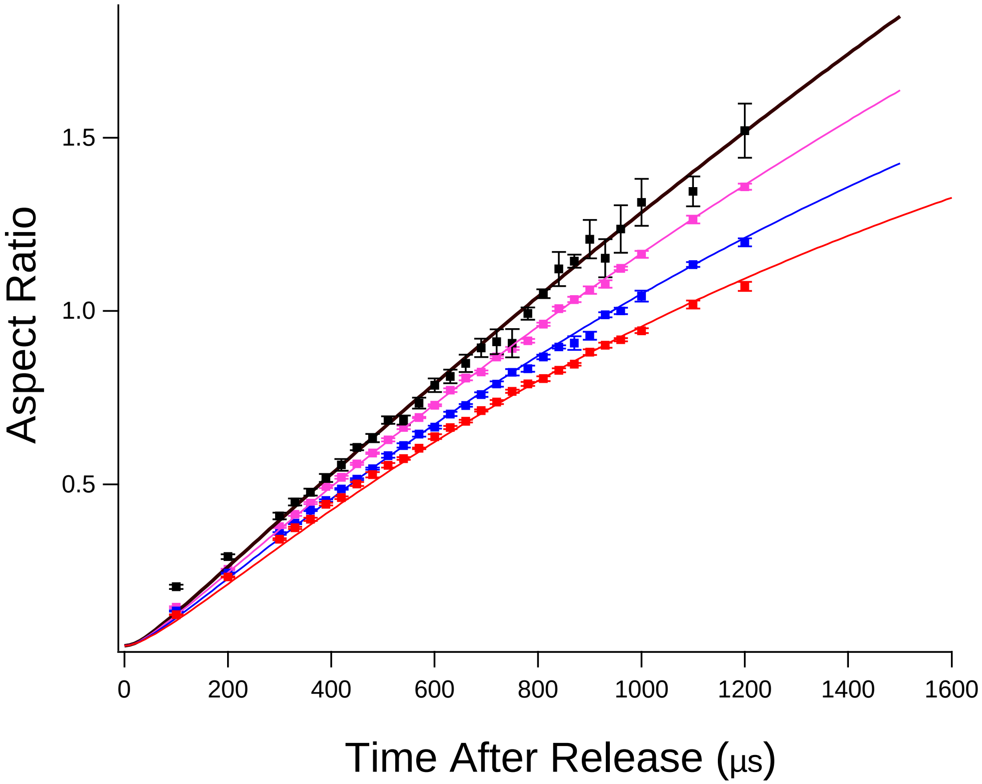
anisotropy



Elliptic flow: High T limit

Quantum viscosity $\eta = \eta_0 \dfrac{(mT)^{3/2}}{\hbar^2}$





$$\eta = \eta_0 (mT)^{3/2}$$

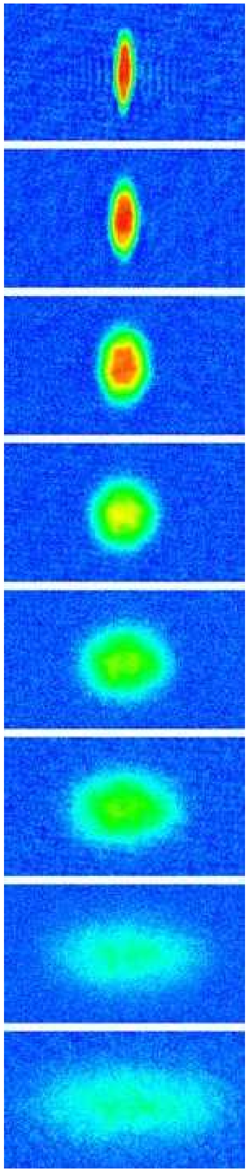
$$\tau_\pi = \eta / P$$

Cao et al., Science (2010)

fit: $\eta_0 = 0.33 \pm 0.04$

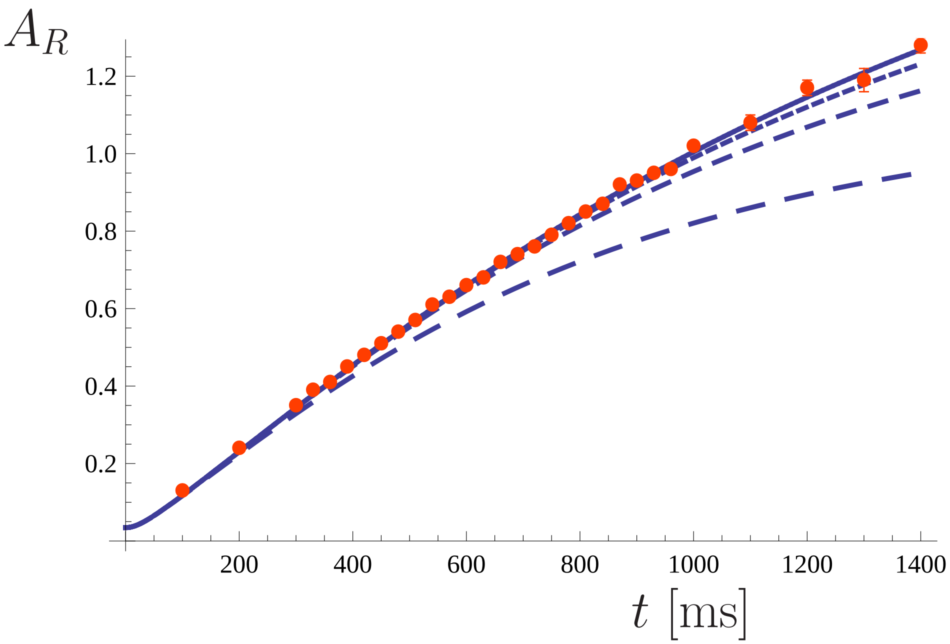
theory: $\eta_0 = \dfrac{15}{32\sqrt{\pi}} = 0.26$

Elliptic flow: Freezeout?



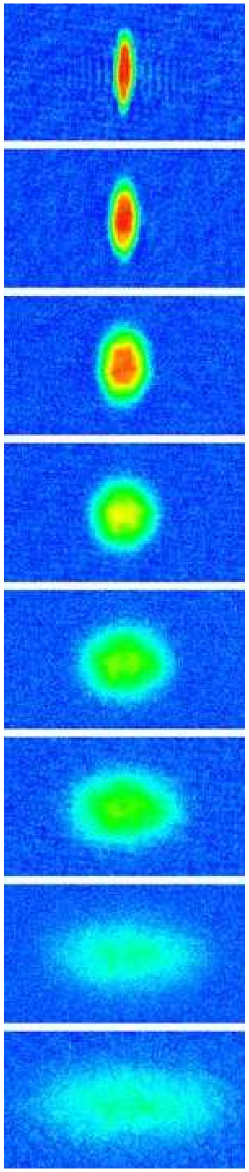
switch from hydro to (weakly collisional) kinetics

at scale factor $b_\perp^{fr} = 1, 5, 10, 20$

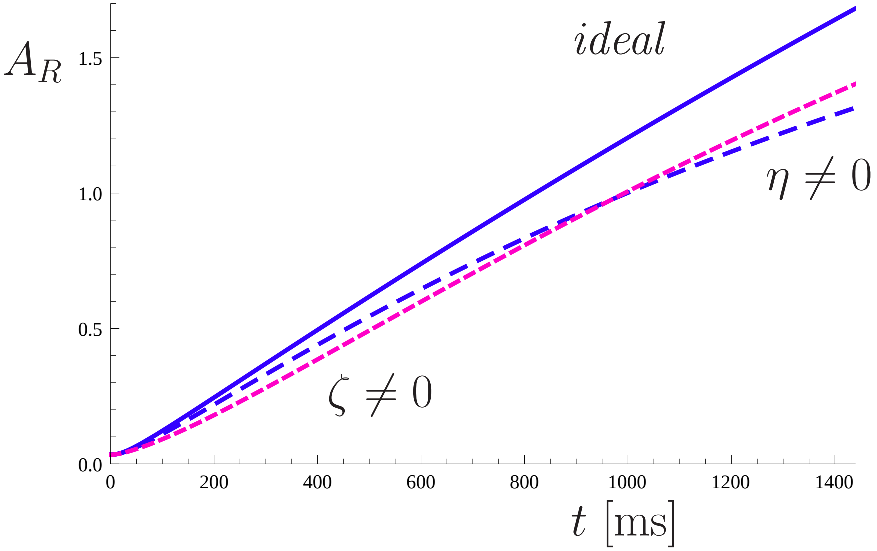


no freezeout seen in the data

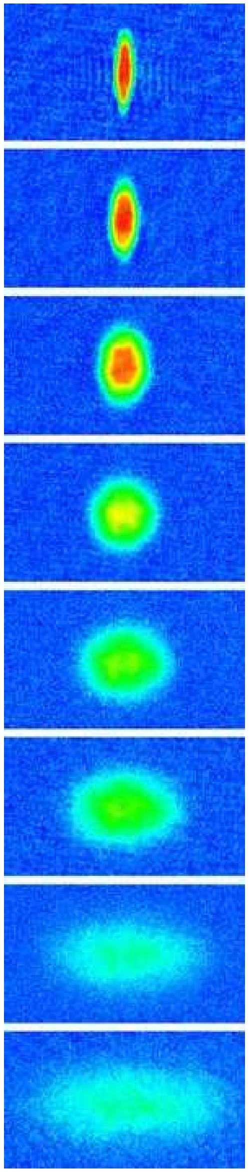
Elliptic flow: Shear vs bulk viscosity



Dissipative hydro with both η, ζ

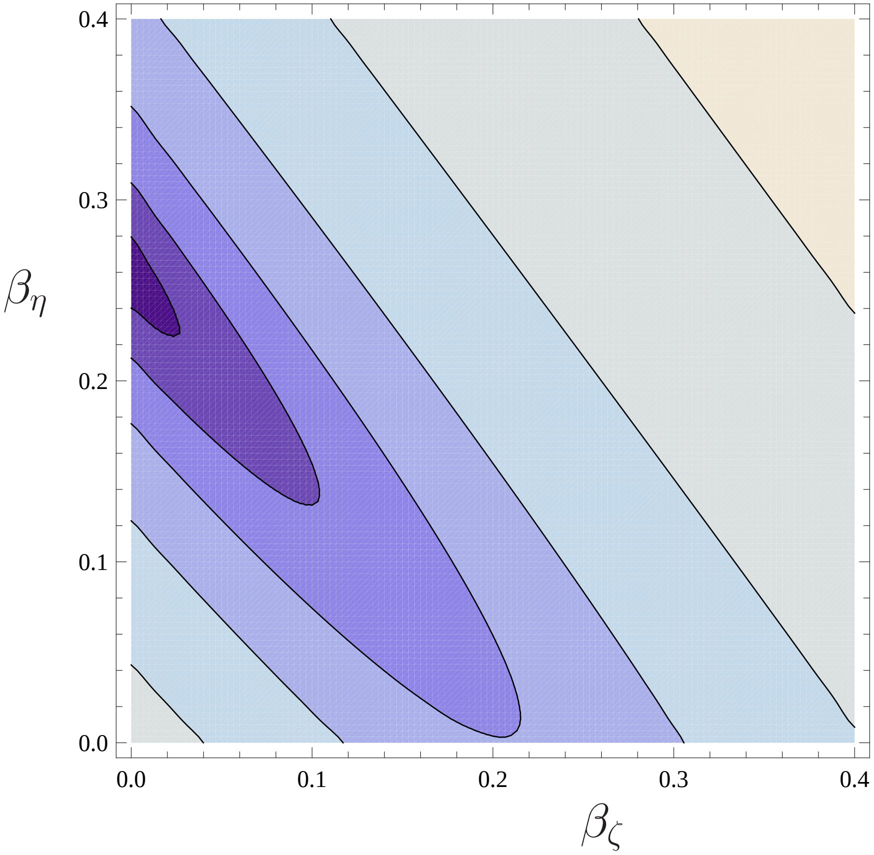


Elliptic flow: Shear vs bulk viscosity



Dissipative hydro with both η, ζ

$$\beta_{\eta,\zeta} = \frac{[\eta,\zeta]}{n}\frac{E_F}{E}\frac{1}{(3\lambda N)^{1/3}}$$



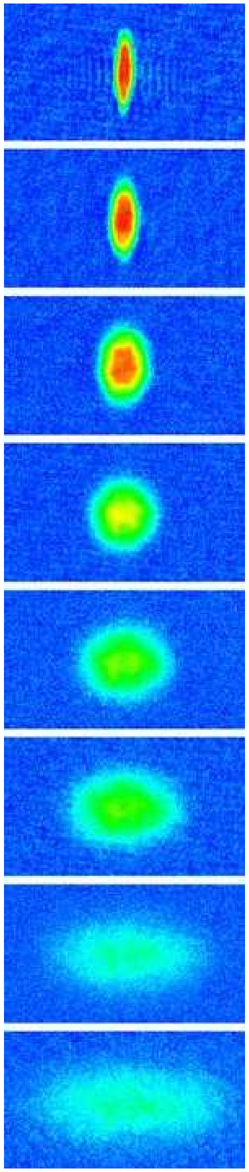
$\eta \gg \zeta$

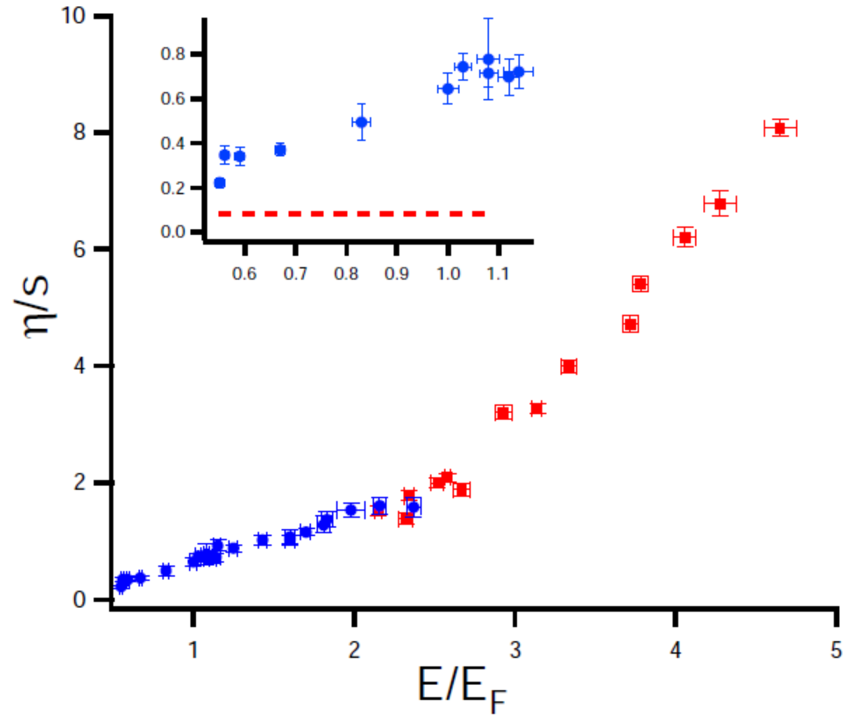
Dusling, Schaefer (2010)

Viscosity to entropy density ratio

consider both collective modes (low T)

and elliptic flow (high T)





Cao et al., Science (2010)

$$\eta/s \leq 0.4$$

V. Outlook: Temperature/density dependence

QGP: η/s typically assumed to be constant. Probably o.k. (in QGP phase, at $\sim 30\%$ level), but corrections difficult to study.

CAG: η/n has significant dependence on $T^{3/2}n^{-1}$. Difficult to unfold. Local s/n known to high accuracy ($\sim 2\%$).

Initial state

QGP: Biggest source of uncertainty. Opportunity: Initial state fluctuations generate odd harmonics.

CAG: Well characterized. Some ability to control initial state, but this opportiunity has not been exploited.

Final state

QGP: Kinetic afterburners standard. Flow does not saturate, some puzzling data about energy dependence (v_2 growth due to mean p_T increase?) and high p_T (non-hydro) flow.

CAG: No need for freezeout, flow saturates. But: need kinetics for corona (currently: relaxation time approach interpolates to kinetic limit).

Higher harmonics

QGP: Important constraint on viscosity, but need initial state models.

CAG: Some data on higher multipole collective modes. Need data from single experiment.

Bulk viscosity

QGP: Hard to measure from flow. Important effects on p_T spectra and hadro-chemistry?

CAG: Zero at unitarity. Can be measured from either flow or collective mode damping. Leading behavior away from unitarity not known.

Fluctuations

QGP: Dominated by initial state fluctuations? Possible effects near critical points. Some attempts at stochastic hydro.

CAG: Dominated by thermal fluctuations. Possible effects near T_c and in two dimensions.

Outlook: New ideas (?)

QGP: Phenomenology: What is the best observable to constrain η/s? Integrated v_2 in ultra-central collisions?

Theory: Anomalous hydrodynamics, new ideas about thermalization (hydro before isotropization and thermalization).

CAG: Experiment: Hydrodynamic engineering; can we design flow profiles? Muiple modes, artificial gauge fields (?)

Theory: How to do hydro/kinetic interface: 2nd order hydro, lattice Boltzmann, BGK kinetics? Holographic $Schr(d)$: Still no dual of unitary fermions, $d = 2$ easier?