Collective modes in trapped Fermi gases from the hydrodynamic to the collisionless limit

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Outline

1. Introduction
2. Boltzmann equation with in-medium interaction
3. Approximate solution: Method of moments
4. Numerical solution: Test particles
5. Role of 4th-order moments
6. Summary and conclusions

References:

S. Chiacchiera, T. Lepers, D. Davesne, and M. Urban, PRA 79, 033613 (2009),
T. Lepers, D. Davesne, S. Chiacchiera, and M. Urban, PRA 82, 023609 (2010),
S. Chiacchiera, T. Lepers, D. Davesne, and M. Urban, PRA 84, 043634 (2011).

Introduction

Already discussed at this workshop:

- Quark-gluon plasma at RHIC or LHC: elliptic flow

  hydrodynamic behaviour with shear viscosity η close to the lower bound

  $$\eta_{min} = \frac{\hbar s}{4\pi k_B} \quad (s = \text{entropy density})$$

- Fermi gas in the unitary limit: expansion, collective modes

  hydrodynamic behaviour, η again of the order of η_{min}

But what does "hydrodynamic" mean?

Hydrodynamic behaviour

- Starting point: distribution function $f(\vec{r}, \vec{p}, t)$
- Example: uniform ideal Fermi gas in equilibrium:

$$f_{eq}(\vec{p}; \mu, T) = \frac{1}{e^{(p^2/2m-\mu)/T} + 1}$$

- A system behaves hydrodynamically if it stays locally in equilibrium:

$$f(\vec{r}, \vec{p}, t) \approx f_{eq}(\vec{p} - m\vec{v}(\vec{r}, t); \mu(\vec{r}, t), T(\vec{r}, t))$$

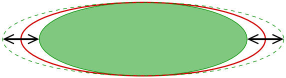
$\rightarrow$ it is enough to know $\vec{v}(\vec{r}, t)$, $\mu(\vec{r}, t)$ and $T(\vec{r}, t)$
which depend only on $\vec{r}$ and t but not on $\vec{p}$.

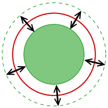
- Local equilibrium is restored by collisions on a time scale τ (relaxation time $\sim$ mean time between collisions)

$\rightarrow$ The same system may behave hydrodynamically or not depending on whether one is interested in slow or fast processes.

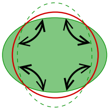
Collective modes in trapped Fermi gases

- Experiments done at Duke, Innsbruck, ENS, ...
- Typical situation: strongly anisotropic trap $V_{trap}(\vec{r}) = \frac{m}{2} \sum_{i=x,y,z} \omega_i^2 r_i^2$

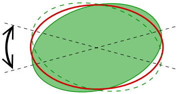
  with $\omega_z \ll \omega_x, \omega_y \quad \rightarrow \quad$ strongly elongated cloud
- Collective modes: small oscillations of the cloud size or shape
- Axial breathing mode:  slow $\rightarrow$ hydrodynamic
- Radial modes (in the xy plane): much faster $\rightarrow$ not always hydrodynamic!



radial breathing mode



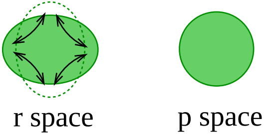
radial quadrupole mode



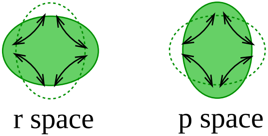
scissors mode

Example: Quadrupole mode

- Hydrodynamic regime ($\omega_\perp \tau \ll 1$):
  $\omega_Q = \sqrt{2}\omega_\perp$ independent of the interaction (no compression)



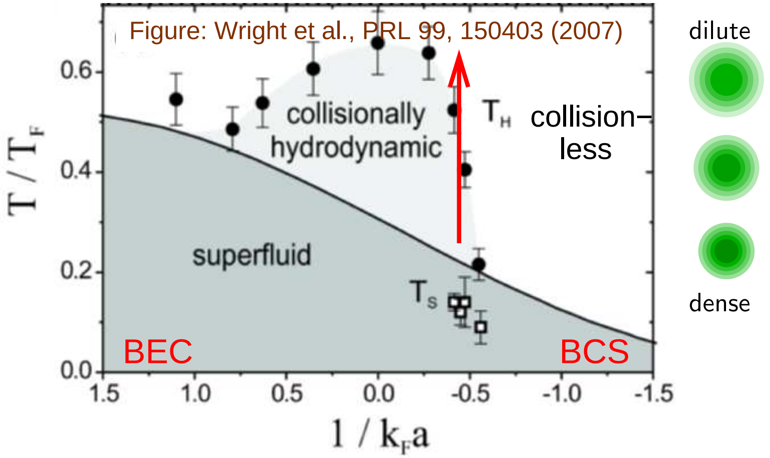
- Collisionless regime ($\omega_\perp \tau \gg 1$):
  Purely ballistic motion in the trap potential $\longrightarrow \omega_Q = 2\omega_\perp$
  Shape of momentum distribution oscillates, too.



  Mean field potential U leads to (small) corrections to ω_Q
- Intermediate cases ($\omega_\perp \tau \sim 1$): Strong damping!

Dynamical regimes

- This talk: transition from collisional hydrodynamics to collisionless regime



(data points: temperatures where scissors-mode damping is maximum)

Theoretical framework

- (Viscous) hydrodynamics insufficient $\longrightarrow$ kinetic theory
- Boltzmann equation:

$$\dot{f} + \frac{\vec{p}}{m} \cdot \vec{\nabla}_r f - \vec{\nabla}_r V \cdot \vec{\nabla}_p f = -I[f]$$

$V = V_{trap} + U =$ potential (trap + mean field)

- Collision term:

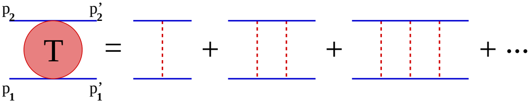
$$I[f] = \int \frac{d^3 p_1}{(2\pi)^3} \int d\Omega \frac{d\sigma}{d\Omega} |\vec{v} - \vec{v}_1| [f f_1 (1 - f')(1 - f_1') - f' f_1' (1 - f)(1 - f_1)]$$

$\frac{d\sigma}{d\Omega} =$ cross section

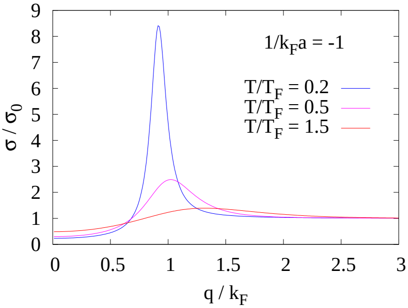
$(1 - f')$ etc. = Pauli blocking factors (suppress collisions at low T)

In-medium cross section

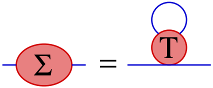
- Scattering cross section in free space: $\sigma_0 = \dfrac{4\pi a^2}{1 + (qa)^2}$
- Scattering amplitude in the gas is modified by Pauli blocking of intermediate states
- Calculate in-medium T matrix in ladder approximation



- At low temperature ($T \to T_c$), the in-medium cross section σ gets strongly enhanced (precursor of the pole in the T matrix at $T = T_c$) [Bruun and Smith, PRA 76, 045602 (2007); in nuclear physics: Alm et al., PRC 50, 31 (1994)]



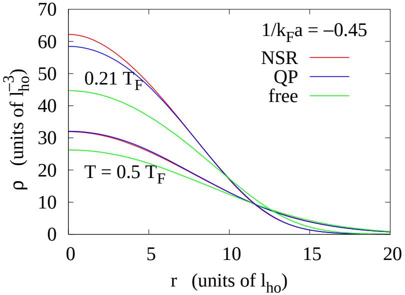
Mean field



- Calculate self-energy $\Sigma(\omega, k)$ with in-medium T matrix
- Quasiparticle (QP) approximation:

$$U = \Sigma(0, k_\mu)$$

where $k_\mu = \sqrt{2m\,\max(\mu, 0)}$
[Perali et al., PRB 66, 024510 (2002)]



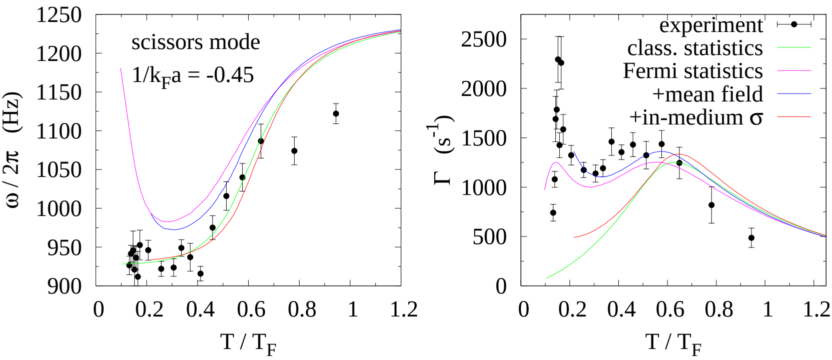
- Local-density approximation (LDA): replace $\mu \to \mu - V$
- U strongly affects the density profiles
- Out of equilibrium: assume $\delta U(\vec{r}, t) = \left(\frac{\partial U}{\partial \rho}\right)_{eq} \delta\rho(\vec{r}, t)$

Approximate solution of the Boltzmann equation: Method of moments

- Linearize Boltzmann equation for small deviations from equilibrium
- Write distribution function as $f(\vec{r},\vec{p},t) = f_{eq}(\vec{r},\vec{p}) + \frac{df_{eq}}{d\mu}\Phi(\vec{r},\vec{p},t)$
- Ansatz for Φ: Polynomial in $\vec{r}$ and $\vec{p}$ with time-dependent coefficients
- Determine time-dependence by taking moments of the Boltzmann equation
- Usually [Bruun et al., Riedl. et al., Chiacchiera et al., ...]:
  Include only polynomials of second order in $\vec{r}$ and $\vec{p}$
  (equivalent to "generalised scaling" [Pedri et al.])

Example: Scissors mode ($1/k_F a = -0.45$)

Experiment Wright et al., PRL 99, 150403 (2007)



- Mean field improves agreement with the data.
- In-medium σ is too strong and compensates Pauli blocking effect [Bruun and Smith, PRA 75, 04612 (2007); Riedl et al., PRA 78, 053609 (2008)]

What's wrong?

- With the in-medium σ, the obtained relaxation time τ is too short.
- Is this a defect of the underlying theory or just of the approximate solution of the Boltzmann equation (method of moments)?
- Example: Within the ansatz for the quadrupole mode

$$\Phi = c_1(x^2 - y^2) + c_2(p_x^2 - p_y^2) + c_3(xp_x - yp_y)$$

  the Fermi sphere deformation is the same everywhere in the trap,
  but it should be smallest near the center where collisions are most frequent.

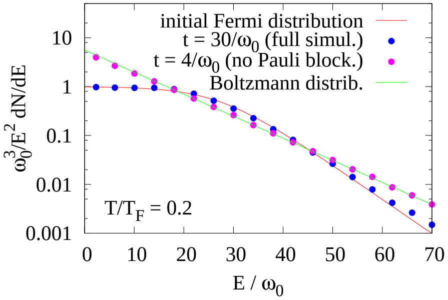
- Possible solutions:
  (a) Numerical solution of the Boltzmann equation (test particles)
  (b) Include higher-order moments into the method of moments (e.g. 4th order, including a term $\propto r^2(p_x^2 - p_y^2)$)
  (c) Relaxation-time approximation with $\tau = \tau(\vec{r})$ [Wu and Zhang (2012)]

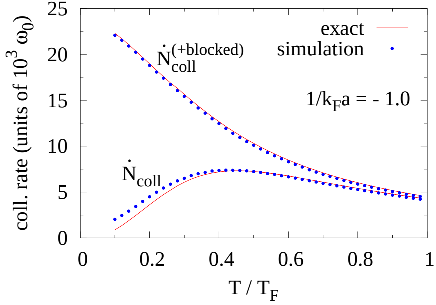
Numerical solution of the Boltzmann equation

- Use the method of test particles
  (often employed in the simulation of heavy-ion collisions)
- Simplifications:
  - Spherical trap: $V = \frac{1}{2} m\omega_0^2 r^2$
  - Neglect medium effects: $U = 0$, $\sigma = \sigma_0$

Tests of the numerics

Stability of the equilibrium distribution and total collision rate:





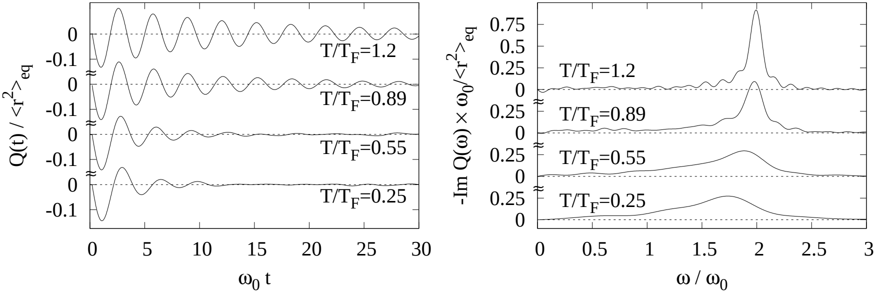
Numerical simulation of the quadrupole mode

- Excitation: $V_1(\vec{r},t) = c(x^2 - y^2)\delta(t)$ (c small $\rightarrow$ linear response)

  $\rightarrow$ at $t = 0$, all test particles get a kick $\vec{p}_i \rightarrow \vec{p}_i - c\vec{\nabla}(x^2 - y^2)$

- Results for $Q(t) = \langle x^2 - y^2 \rangle(t)$ and its Fourier transform $Q(\omega)$

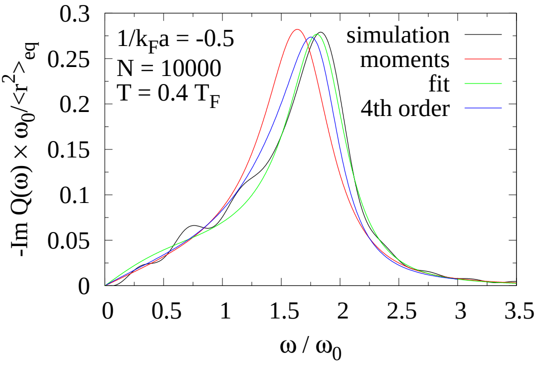
($N = 10000$ atoms, $1/k_F a = -0.5$)



Moments method vs. numerical solution

- Compare response functions obtained from:

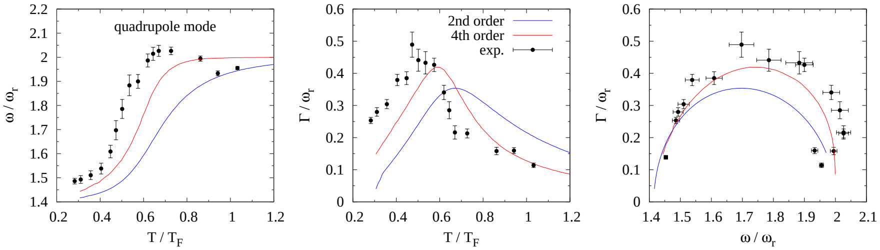
  method of moments (2nd order),
  numerical simulation (test particles),
  extended method of moments (up to 4th-order moments):



- 2nd-order moments result "more hydrodynamic" than numerical solution
- Inclusion of 4th-order moments $\longrightarrow$ considerable improvement

Results of 2nd- and 4th-order moments vs. experiment

- Extended moments method can also be used in the realistic case ($N = 600000$, elongated trap, in-medium σ)
- Radial quadrupole mode [data: Riedl et al., PRA 78, 053609 (2008)]
- Compare 2nd-order and 4th-order results with data



- 4th-order results with in-medium cross section in much better agreement with data than 2nd-order ones

Conclusions

- Collective modes in ultracold trapped Fermi gases can show different behaviour from hydrodynamic to collisionless
- Description in the framework of the Boltzmann equation, approximate solutions obtained with the method of moments
- Comparison with numerical solution indicates that higher-order moments are necessary [similar to using $\tau = \tau(\vec{r})$]
- Frequency and damping of quadrupole mode measured at Innsbruck can be more or less explained

Outlook

- Numerical simulation in the realistic case
- Asymmetric systems ($N_\uparrow \neq N_\downarrow$)
- Superfluid phase