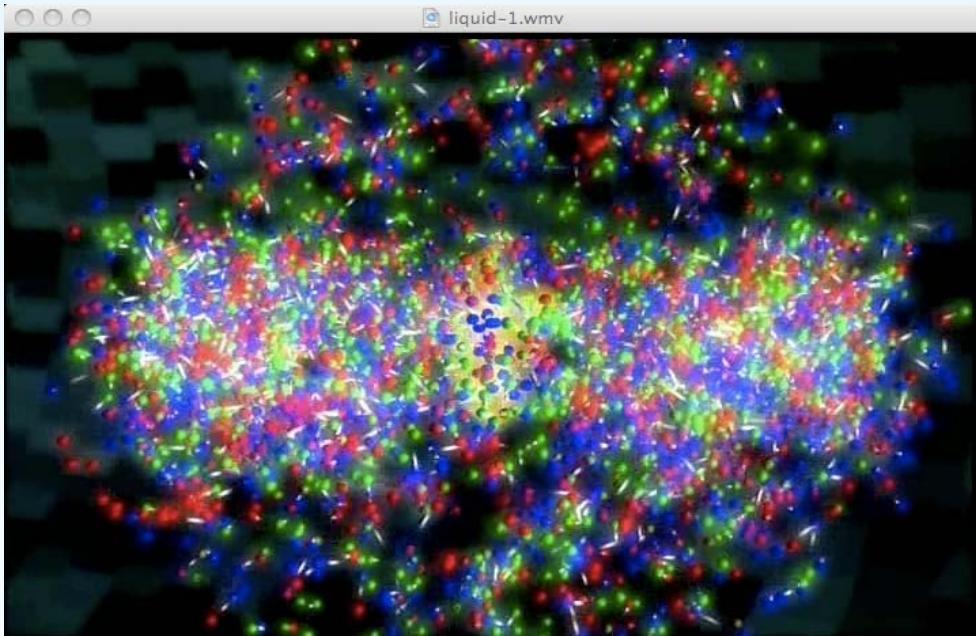


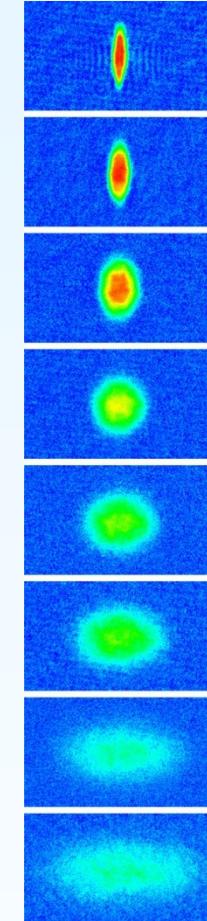
From Strongly-Interacting Fermi Gases to Nuclear Matter

PHYSICS
N
C

John E. Thomas



Quark-gluon plasma $T = 10^{12}$ K BIG BANG
Computer simulation of RHIC collision



Ultracold atomic gas
 $T = 10^{-7}$ K

JETLab Group



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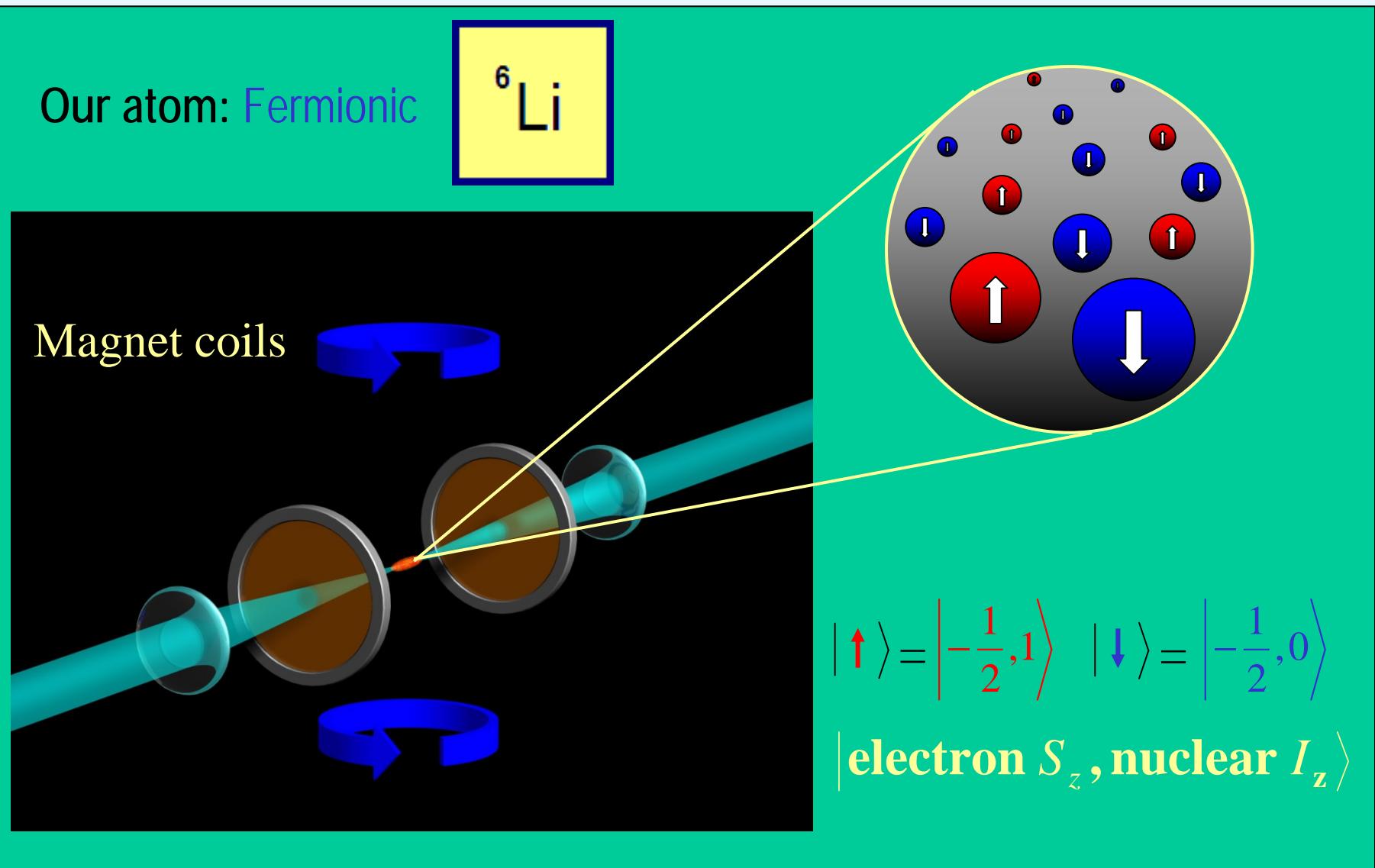
Jessie Petricka*

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Outline

- *Introduction: Optically trapped Fermi gases:*
 - Universal behavior
- *Thermodynamics of strongly-interacting Fermi gases:*
 - Global entropy and energy
 - Temperature calibration
- *Quantum viscosity in strongly-interacting Fermi gases:*
 - Shear forces and heating in collective modes and expanding gases
 - Comparison to the minimum viscosity conjecture
 - Vanishing Bulk viscosity
- *Strongly-interacting Fermi gases in two dimensions*
 - Dimers and polarons in radio-frequency spectra

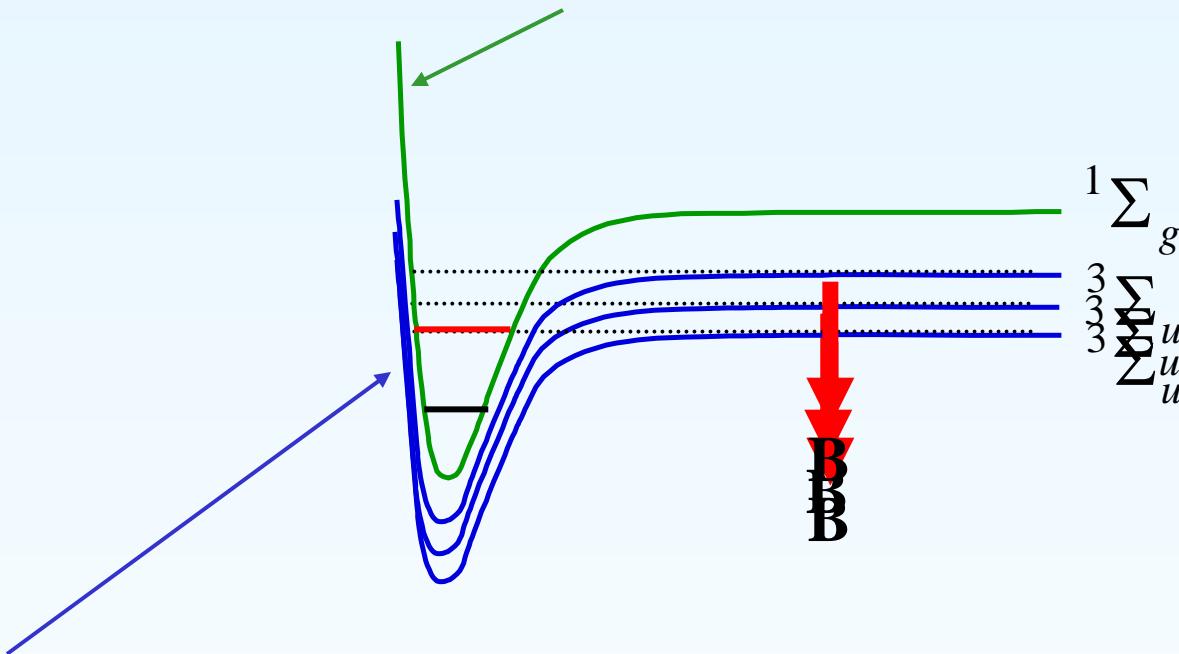
Optically Trapped Fermi Gas



Feshbach Resonance

Resonant Coupling between Colliding Atom Pair – Bound Molecular State

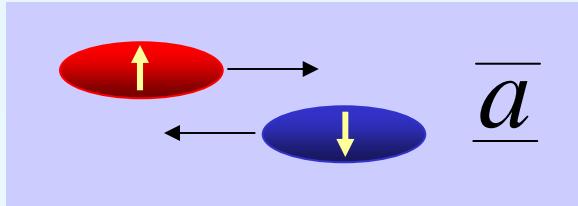
Singlet Diatomic Potential: Electron Spins Anti-Parallel



Triplet Diatomic Potential: Electron Spins Parallel

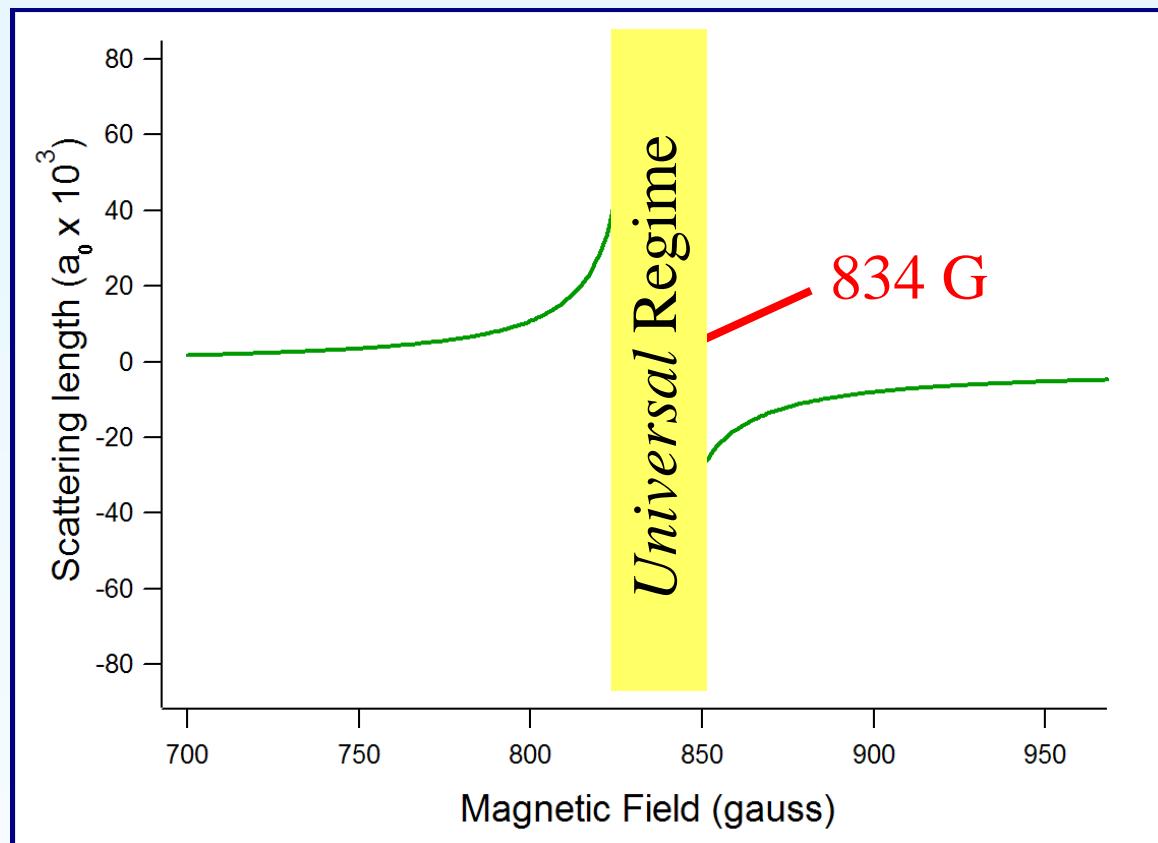
Zero Energy Scattering Length $a_S \rightarrow \pm\infty$

Feshbach Resonance: Tunable Interactions



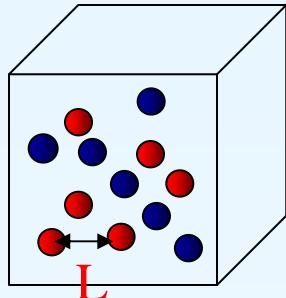
S-wave scattering length

Interparticle Spacing :
 $L \approx 2000 a_0$



*Generated using formula
published in Bartenstein, et al,
PRL **94** 103201 (2005)

The *Universal* Regime: *Natural Units*



T=0: Atom spacing L
is the *only* length scale.

Consequences of the *Heisenberg Uncertainty Principle*:

- Energy and Temperature have *Natural Units* determined by L
- Viscosity—stickiness?

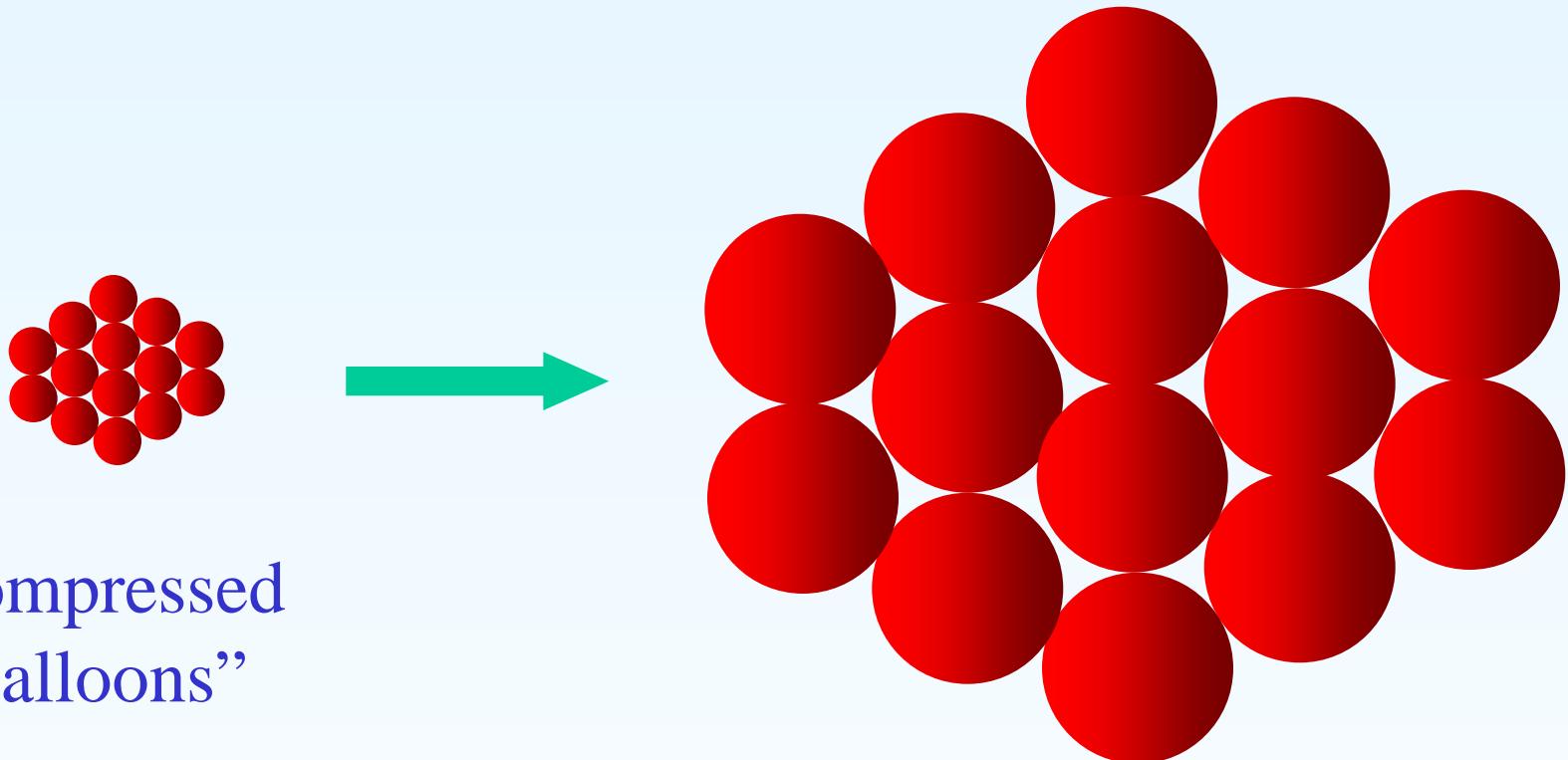
$$\frac{\hbar^2}{2mL^2}$$

Viscosity: Momentum/Area



$$\frac{\hbar}{L^3}$$

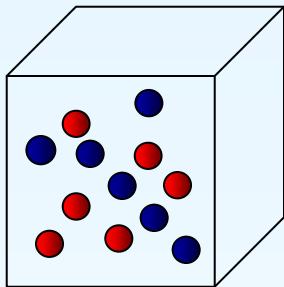
Magic of a Universal Strongly Interacting Fermi Gas



Expanded “Balloons”

Universal Ground State Energy

Ideal Fermi Gas

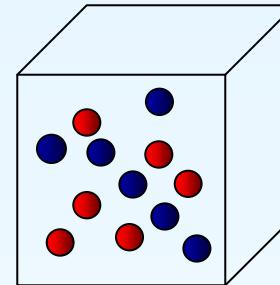


$$B = 527.5 \text{ G}$$

$$a = 0$$

E_{ideal} = Fermi energy

Strongly Interacting Fermi Gas



$$B = 834 \text{ G}$$

$$a \gg L$$

$$E_{\text{gnd}} = (1 + \beta) E_{\text{ideal}}$$

Bertsch 1998, Baker 1999, Heiselberg 2001

Theory: Carlson (2008) $\beta = -0.60(1)$

Experiment: Duke (2008) $\beta = -0.61(2)$

Experiment: MIT (2011) $\beta = -0.624(5)$

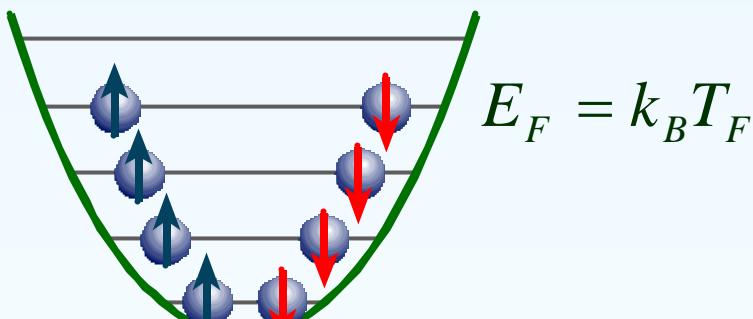
Quantum Degeneracy in Fermi Gases

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$$|\uparrow\rangle = \left|-\frac{1}{2}, 1\right\rangle \quad |\downarrow\rangle = \left|-\frac{1}{2}, 0\right\rangle$$

Harmonic Potential:

$$\varepsilon = (n_x + n_y + n_z)h\nu$$



Zero Temperature

Trap Fermi Temperature Scale:

$$E_F = k_B T_F = h\nu(3N)^{1/3}$$

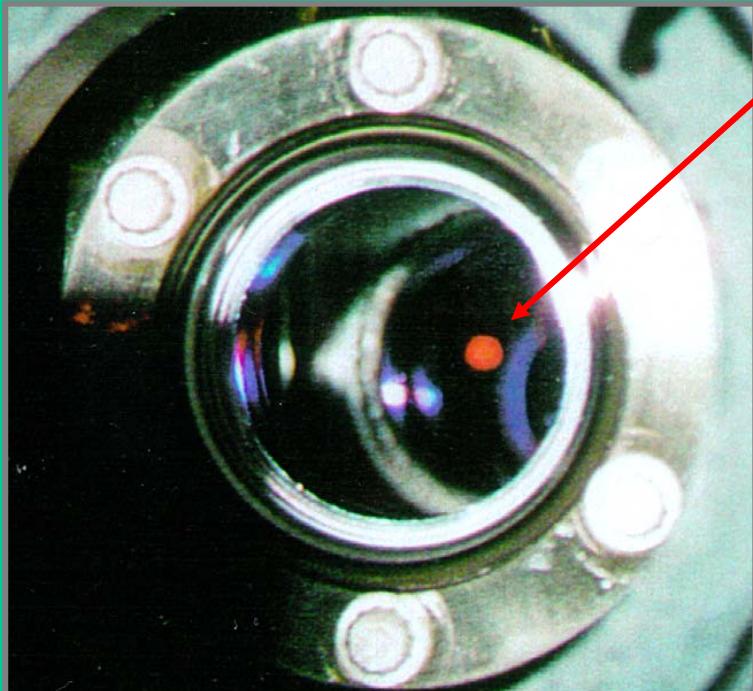
Optical Trap Parameters:

$$\nu = (\nu_x \nu_y \nu_z)^{1/3} = 600 \text{ Hz}$$
$$N = 2 \times 10^5$$

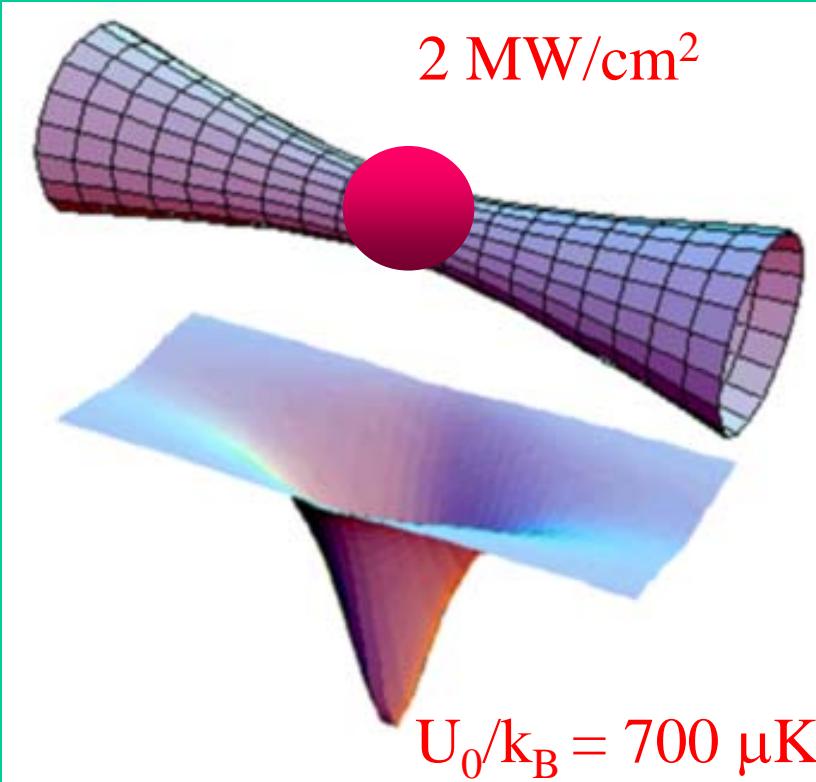
$$T_F = 2.4 \text{ } \mu\text{K}$$

Preparation of Degenerate ${}^6\text{Li}$ gas

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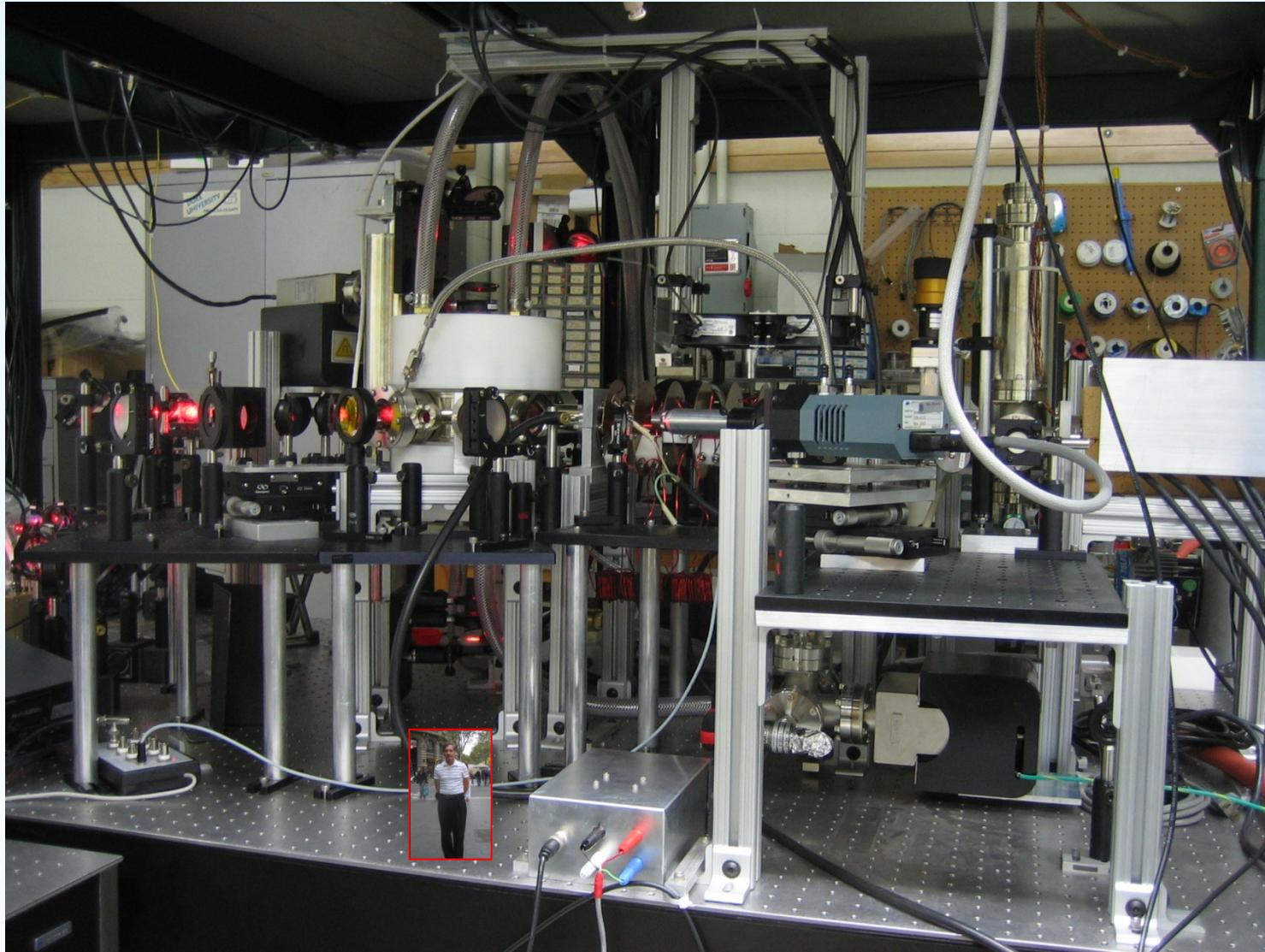


Atoms precooled
in a magneto-optical trap
to $150 \mu\text{K}$



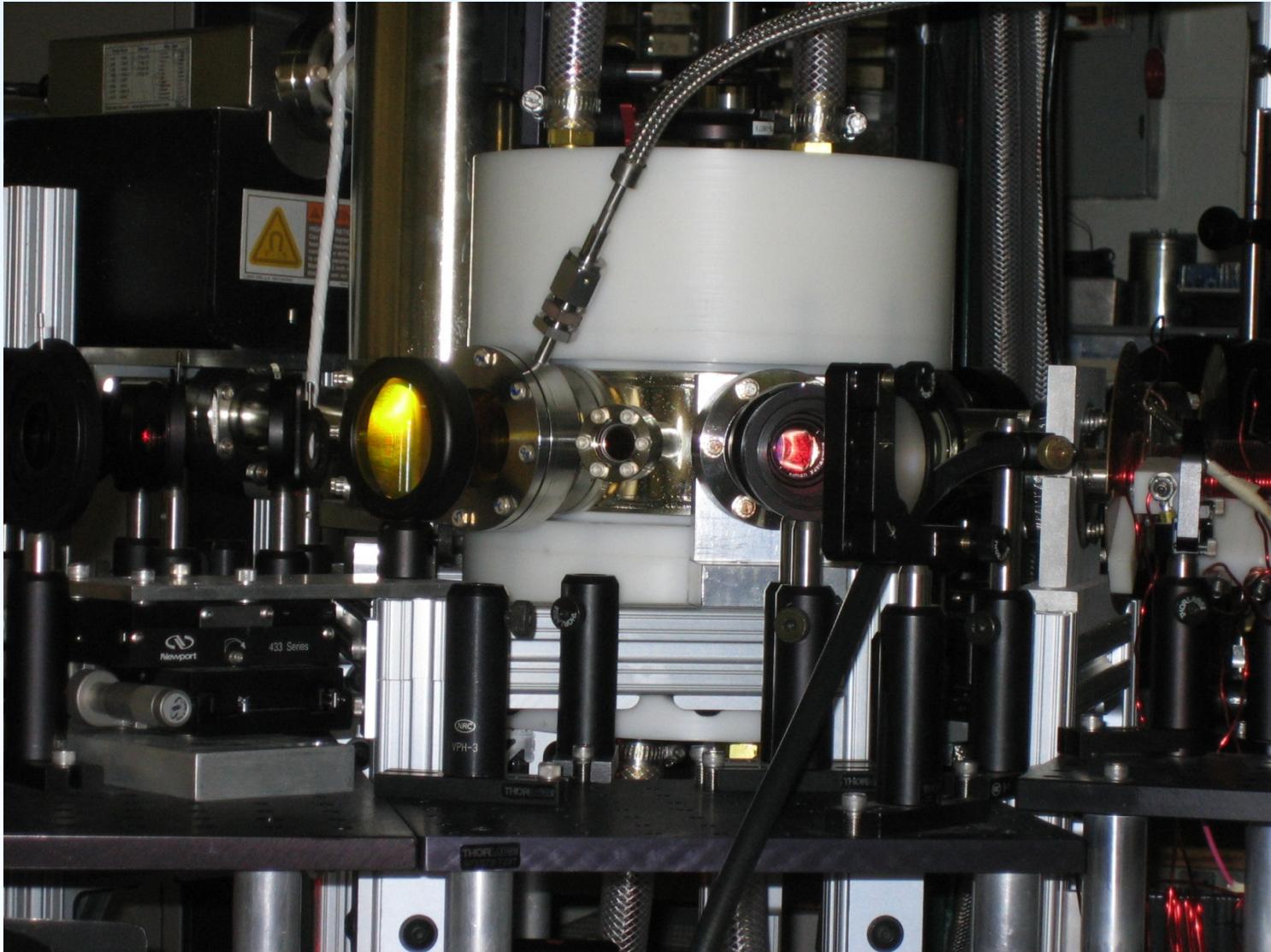
Experimental Apparatus

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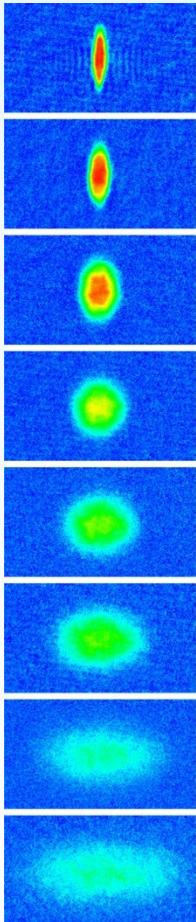
Experimental Apparatus

PHYSICS



Strongly Interacting Fermi Systems in Nature

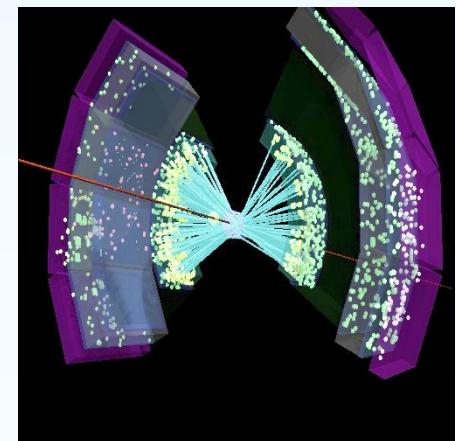
- ❖ Ultracold Atomic Fermi Gases
- ❖ Quark-Gluon Plasma
- ❖ Neutron Matter
- ❖ High-Temperature Superconductors
- ❖ Black Holes in String Theory



Strongly Interacting Degenerate
 ^6Li gas $T = 10^{-7}$ K

Duke, Science (2002) O'Hara et al.

→ Similar “Elliptic” Flow ←



Quark-gluon plasma
 $T = 10^{12}$ K

The Minimum Viscosity Conjecture—String Theory

Viscosity—Hydrodynamics



$$\frac{\eta}{s} \geq \frac{1}{4\pi} \frac{\hbar}{k_B}$$

Kovtun et al.,
PRL 2005

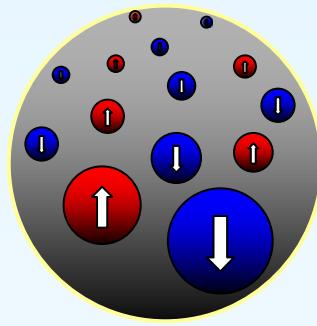


Entropy density—Thermodynamics

Minimum defines a *Perfect* normal fluid

In a ${}^6\text{Li}$ gas we can *measure* η and s .

Thermodynamics of Strongly Interacting Fermi gases



- Ground State Energy
- Finite temperature: Energy and Entropy
- *Temperature calibration*

“Universal” – independent of the microscopic interactions

Global energy E measurement

Universal Gas obeys the **Virial Theorem**

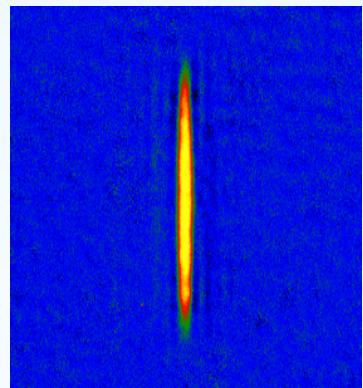
In a HO potential: $E = 2\langle U \rangle$

Thomas (2005)
Castin (2004)
Werner and Castin (2006)
Son (2007)



Energy per particle

$$E = 3m\omega_z^2 \langle z^2 \rangle$$



For a *universal* quantum gas,
the energy E is determined
by the *cloud size*

Measuring the Energy E versus Entropy S by Adiabatic Sweep of Magnetic Field B



Start 834 G

End 1200 G

Strongly interacting at 834 G:
 E nergy E_S known from cloud size
— Universal Fermi gas

Weakly interacting at 1200 G:
 E ntropy S_W known from cloud size
— Weakly Interacting Fermi gas

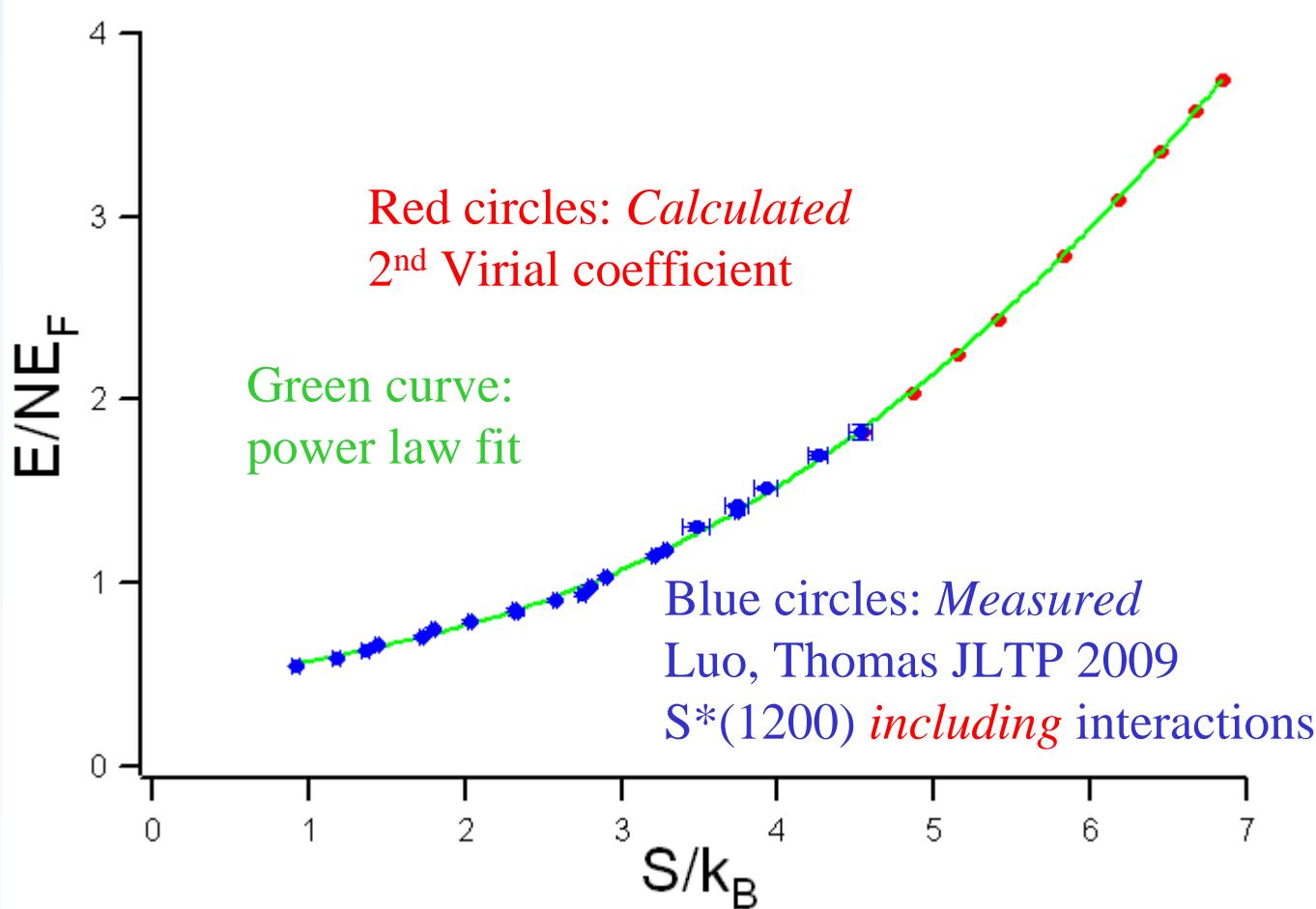
Energy Measurement:

$$E_S = 3m\omega_z^2 \langle z^2 \rangle_{834G}$$

Adiabatic:

$$S_S = S_W$$

Energy per particle versus Entropy per Particle



Temperature Calibration



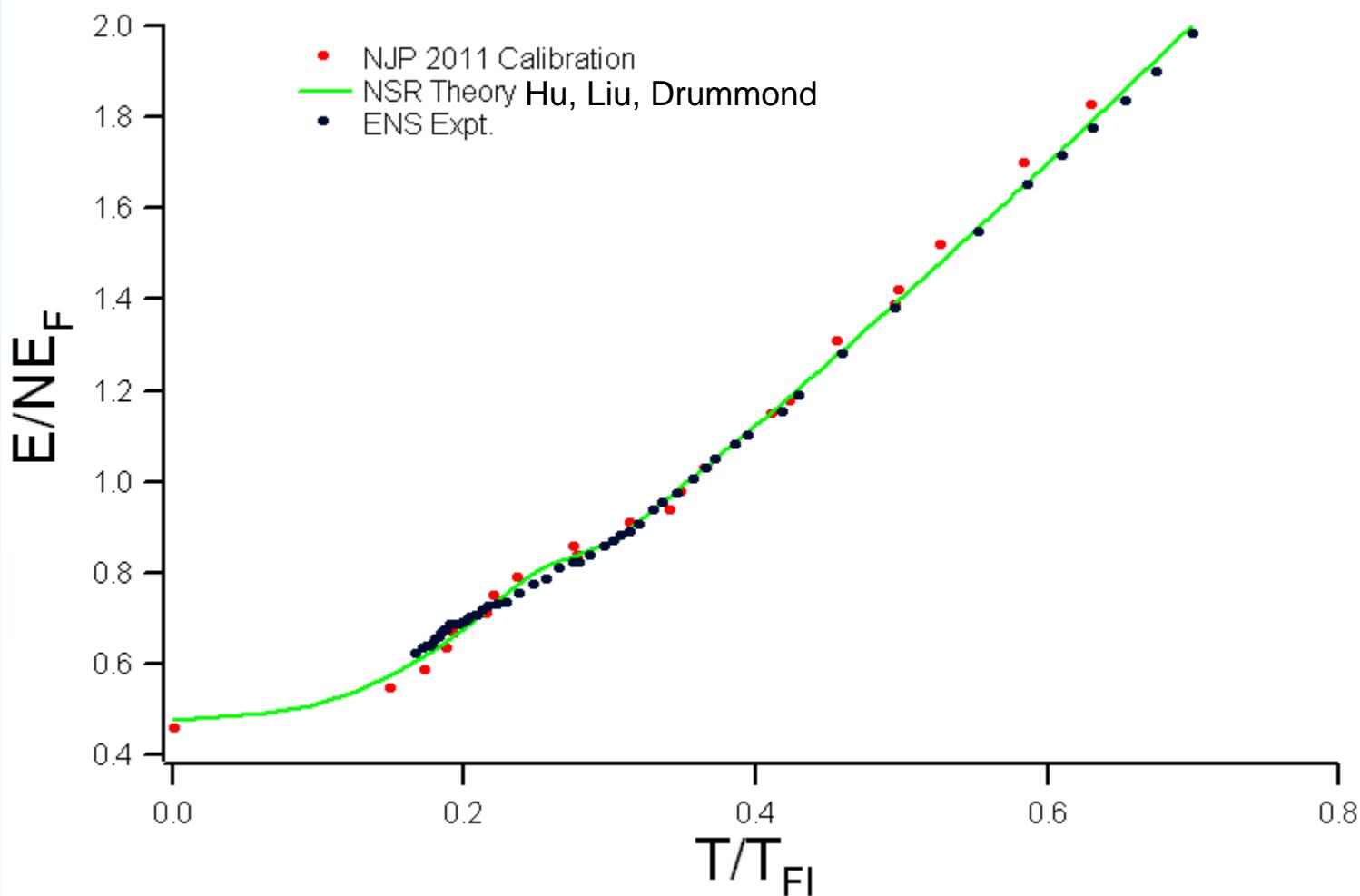
Power law fit to global E versus S data:

$$E_<(S) = E_0 + aS^b; \quad 0 \leq S \leq S_c$$

$$E_>(S) = E_1 + cS^d; \quad S \geq S_c$$

Temperature from: $T = \frac{\partial E}{\partial S}$

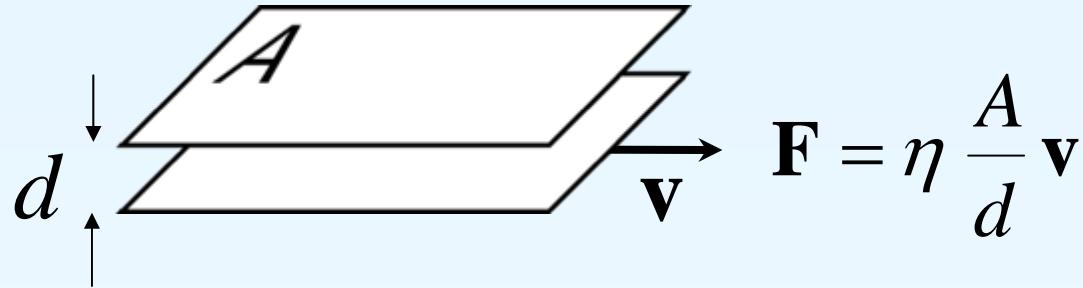
Energy versus Temperature



“Quantum” Viscosity Hydrodynamics

Quantum Viscosity

Shear forces



Viscosity scale:

$$\eta = \frac{p}{\sigma} \quad p = \hbar k \quad \sigma = \frac{4\pi}{k^2}$$

$$\eta \propto \hbar k^3$$

Quantum scale—requires Planck's constant!

Quantum Viscosity at Low and High Temperature



$$\eta \propto \hbar k^3$$

Low Temperature

$$T \leq T_F$$

$$k \approx k_F \approx 1/L$$

$$\eta \approx \hbar n$$

High Temperature

$$T \geq T_F$$

$$k \approx k_{\text{Thermal}} \approx \sqrt{2m k_B T} / \hbar$$

$$\eta \propto T^{3/2} / \hbar^2$$

Entropy density scale: $s \approx n k_B$

Low temperature: $\eta / s \approx \hbar / k_B$ String theory limit

Universal Shear Viscosity

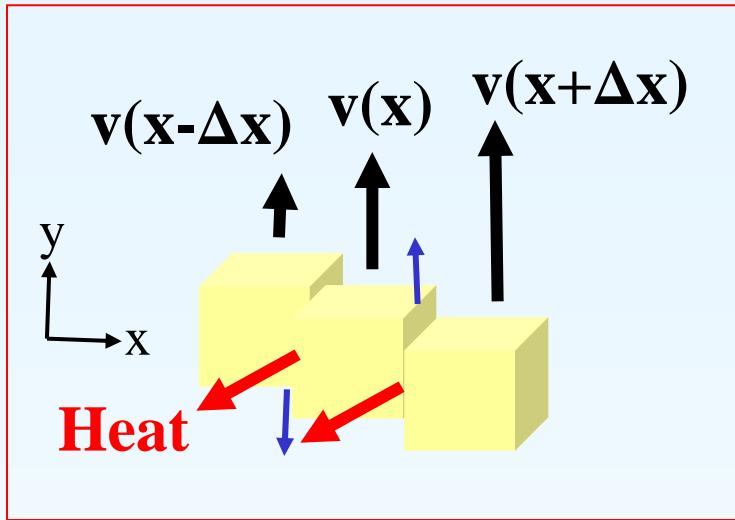


$$\eta(\mathbf{x}, t) = \alpha(\theta) \hbar n(\mathbf{x}, t) \quad \theta = \frac{T}{T_F(n)}$$

Measuring Universal Shear Viscosity
at Low and at High Temperature:

Breathing Mode and Elliptic Flow

Viscous Hydrodynamics



- Shear force at *each* surface $\eta \frac{\partial v_y}{\partial x}$
- *Net* shear force on *volume element* $\frac{\partial}{\partial x} \left(\eta \frac{\partial v_y}{\partial x} \right)$
- Friction *heating* at each surface $\dot{q} = \frac{\eta}{2} \left(\frac{\partial v_y}{\partial x} \right)^2$

Pressure Forces with Heating

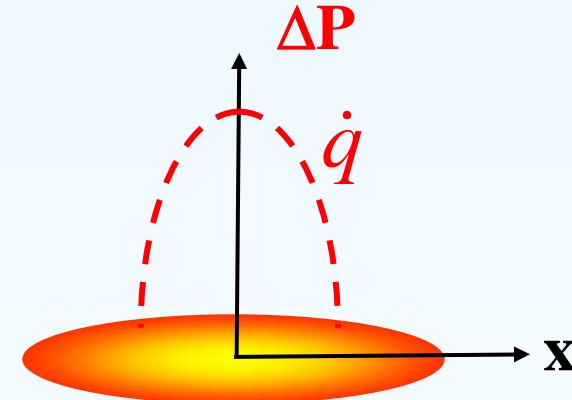
$$P(x) \rightarrow \text{Yellow Block} \leftarrow P(x + \Delta x)$$

Scalar pressure gradient: *Outward* force—expands after release.

Friction force: Inward—*slows* the flow

Friction *Heating*:

$$\dot{q} = \frac{\eta}{2} \left(\frac{\partial v_y}{\partial x} \right)^2$$



*The viscosity must vanish
at the cloud edges*

Heating gradient: *Outward* pressure force that *speeds* the flow!

Hydrodynamic Forces



- Net Force with Friction:

$$m(\partial_t + \mathbf{v} \cdot \nabla) \mathbf{v}_i = f_i - \partial_i U_{trap} + \sum_j \frac{\partial_j (\eta \sigma_{ij} + \zeta \sigma'_{ij})}{n}$$

Force arising from scalar pressure: $f_i = -\frac{\partial_i P}{n}$

Shear viscosity: $\eta \quad \sigma_{ij} = \partial_j v_i + \partial_i v_j - \frac{2}{3} \delta_{ij} \nabla \cdot \mathbf{v}$

Bulk viscosity: $\zeta \quad \sigma'_{ij} = \delta_{ij} \nabla \cdot \mathbf{v}$

- Initial Condition: $f_i(t=0) = \partial_i U_{Trap}(\mathbf{x}) = m \omega_i^2 x_i$

Universal Pressure with Heating



- Friction **Heating rate** per unit volume

$$\dot{q} = \frac{1}{2} \eta \sum_{ij} \sigma_{ij}^2 + \zeta (\nabla \cdot \mathbf{v})^2$$

- Energy conservation: $\left(\partial_t + \mathbf{v} \cdot \nabla + \frac{5}{3} \nabla \cdot \mathbf{v} \right) \mathcal{E} = \dot{q}$
- Universal Pressure: $P = \frac{2}{3} \mathcal{E}$ Ho, PRL 2004

$$\left(\partial_t + \mathbf{v} \cdot \nabla + \frac{5}{3} \nabla \cdot \mathbf{v} \right) P = \frac{2}{3} \dot{q}$$

Cao, Elliot, Wu, Joseph, Petricka, Schaefer, and Thomas
Science **331**, 58 (2011)

Universal Viscous Hydrodynamics



Equation for

$$f_i = -\frac{\partial_i P}{n}$$

$$\left(\partial_t + \mathbf{v} \cdot \nabla + \frac{2}{3} \nabla \cdot \mathbf{v}\right) f_i + \sum_j (\partial_i v_j) f_j - \frac{5}{3} (\partial_i \nabla \cdot \mathbf{v}) \frac{P}{n} = -\frac{2}{3} \frac{\partial_i \dot{q}}{n}$$



$$\text{Scale transformation: } n(x, y, z, t) = \frac{n\left(\frac{x}{b_x}, \frac{y}{b_y}, \frac{z}{b_z}\right)}{b_x(t)b_y(t)b_z(t)}$$

$$\mathbf{v}_i = x_i \frac{\dot{b}_i}{b_i} \quad f_i = a_i(t)m\omega_i^2 x_i$$

$$b_i(0) = 1; \quad \dot{b}_i(0) = 0; \quad a_i(0) = 1$$

Extracting the *Shear* Viscosity

$$\eta(\mathbf{x}, t) = \alpha(\theta) \hbar n(\mathbf{x}, t)$$

$$\theta = \frac{T}{T_F(n)}$$

$$\dot{a}_i + 2 \frac{\dot{b}_i}{b_i} a_i + \frac{2}{3} \sum_j \frac{\dot{b}_j}{b_j} a_i = \frac{\hbar \bar{\alpha}}{3m \omega_i^2 \langle x_i^2 \rangle_0 b_i^2(t)} \sum_{ij} \sigma_{ij}^2$$

$$\frac{\ddot{b}_i}{b_i} = (a_i - 1_{trap}) \omega_i^2 - \frac{\hbar \bar{\alpha}}{m \langle x_i^2 \rangle_0 b_i^2(t)} \sigma_{ii} \quad \partial_j v_i = \delta_{ij} \frac{\dot{b}_i}{b_i}$$

$$b_i(0) = 1; \quad \dot{b}_i(0) = 0; \quad a_i(0) = 1$$

Trap-averaged
Viscosity coefficient

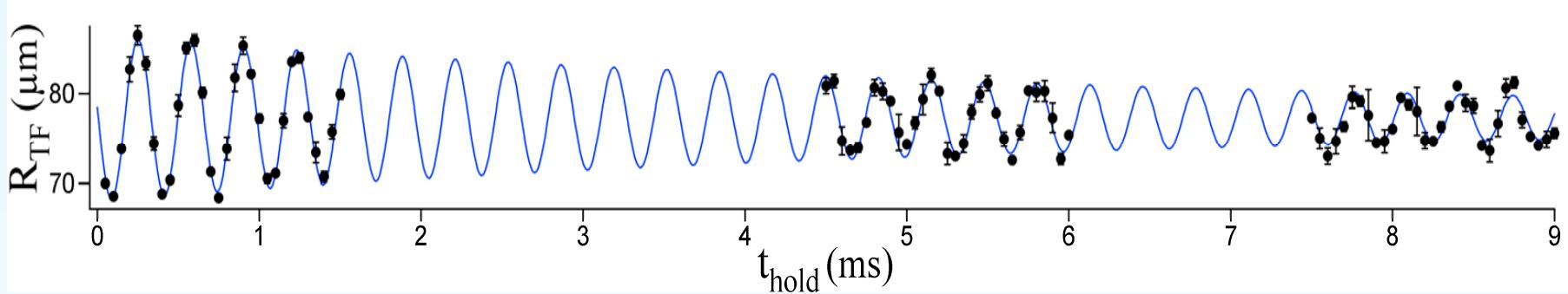
$$\bar{\alpha} \equiv \frac{1}{N\hbar} \int d^3 \mathbf{x} \eta(\mathbf{x}, t) = \frac{1}{N} \int d^3 \mathbf{x} n(\mathbf{x}, t) \alpha(\theta)$$

Adiabatic invariant: Time independent!

Precision Measurement of Viscosity at *Low* Temperature: Breathing Mode

Quantum Viscosity

- Low Temperature: Damping of the Breathing Mode



For viscous damping:

$$\eta = \alpha \hbar n$$

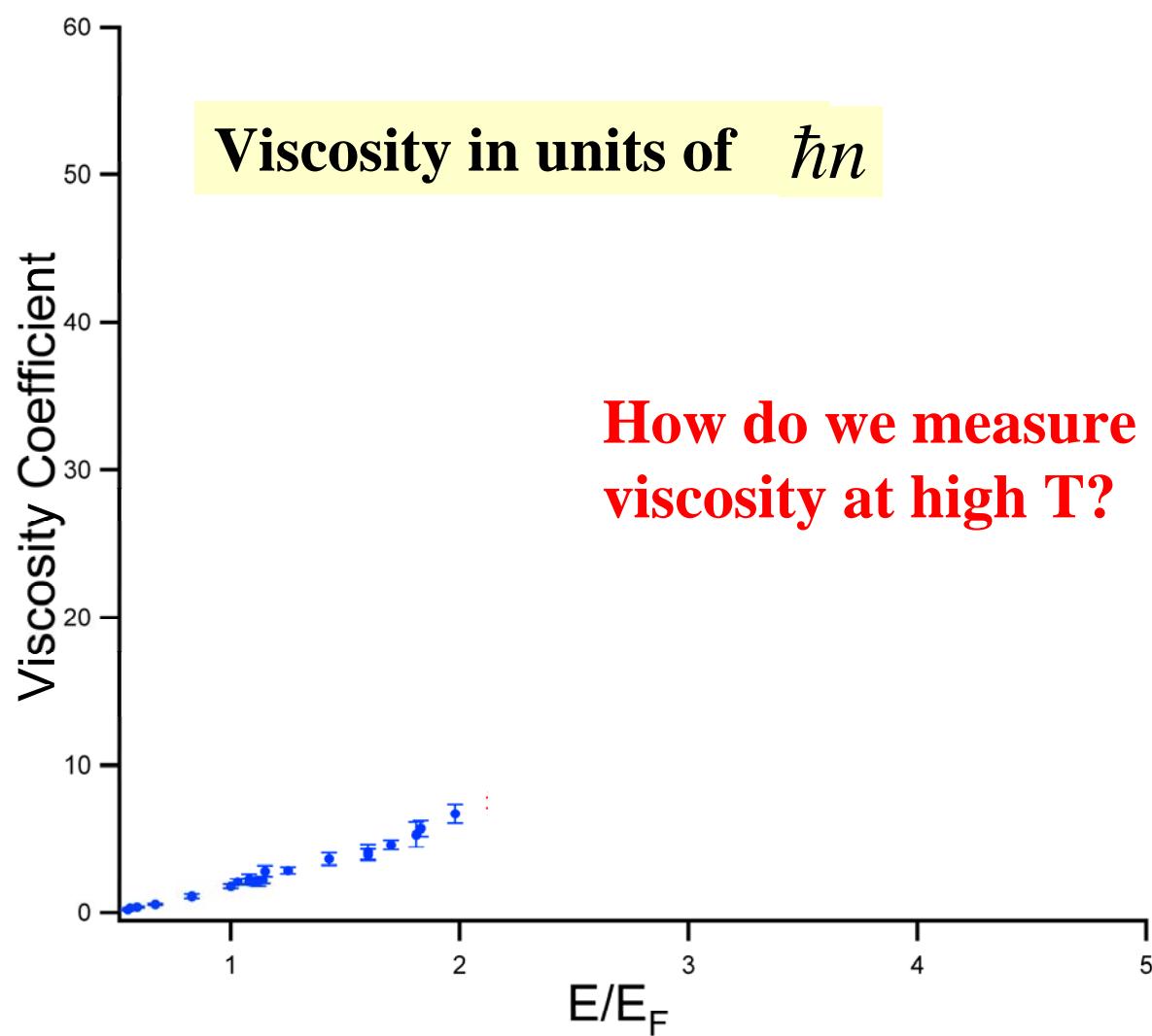
Damping rate:

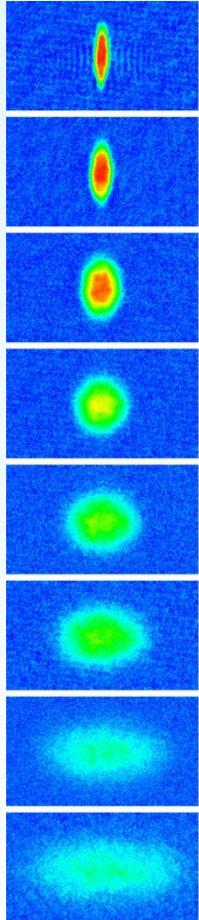
$$\frac{1}{\tau} = \frac{\hbar \bar{\alpha}}{3m \langle x^2 \rangle_0}$$

- Heating is negligible

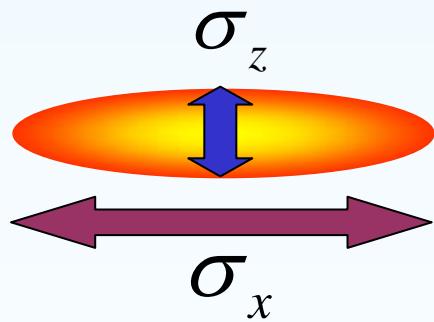
- Measure trap-averaged *viscosity coefficient* $\bar{\alpha}$

Viscosity Coefficient: Low Temperature





High Temperature Quantum Viscosity in Elliptic Flow

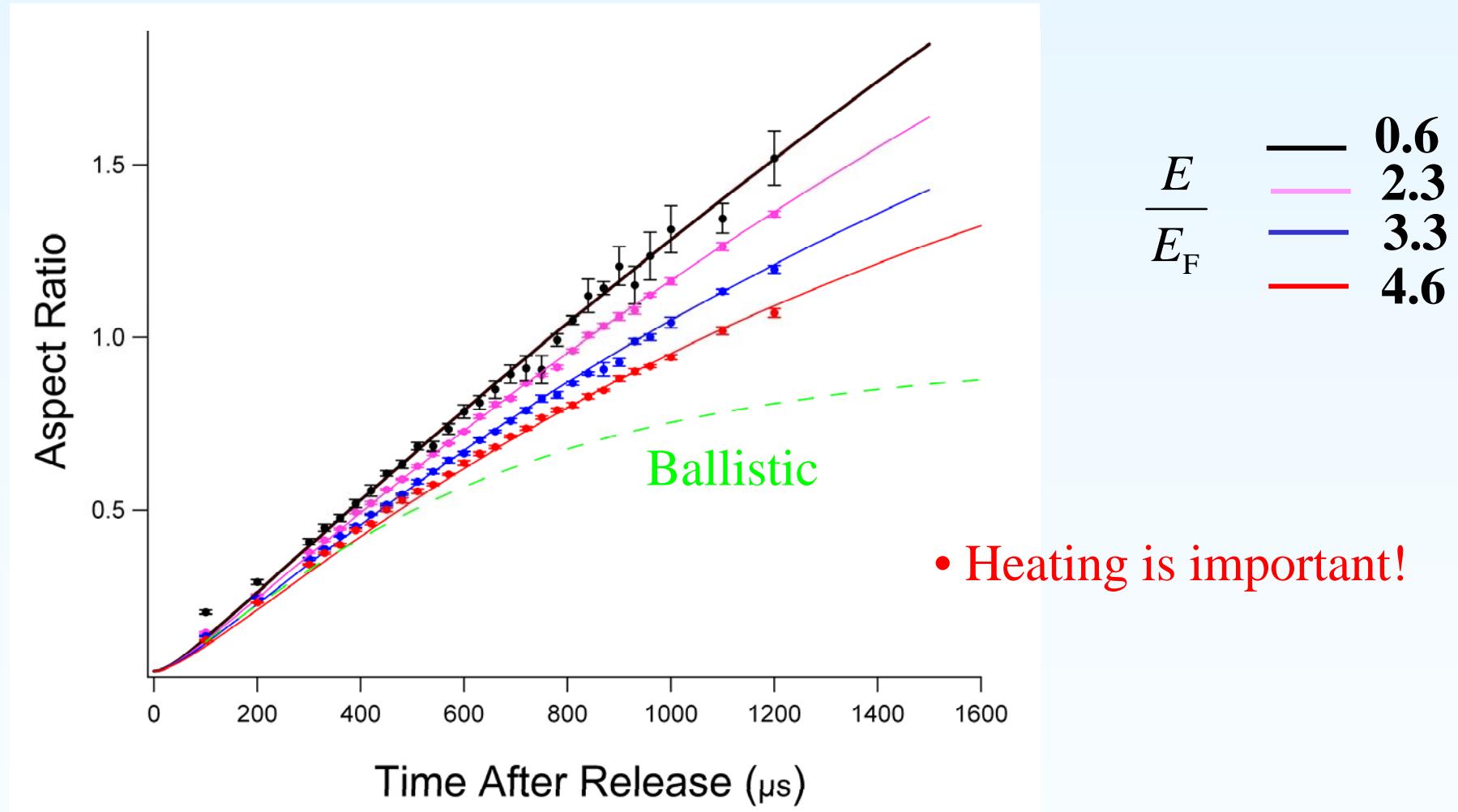


- Measure
Aspect Ratio:

$$\frac{\sigma_x}{\sigma_z}$$

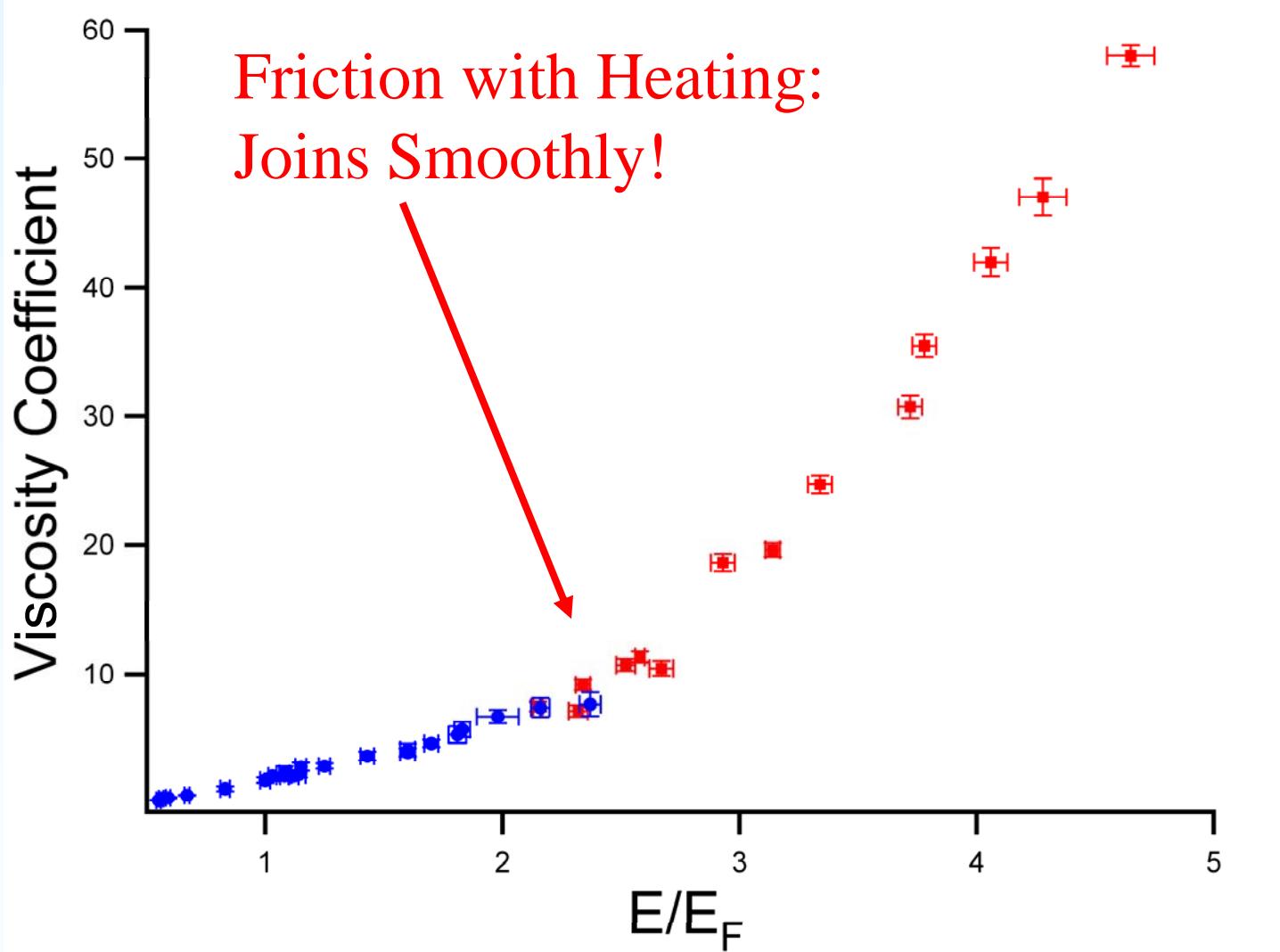
Quantum Viscosity

- High Temperature: Expansion Dynamics—Elliptic Flow

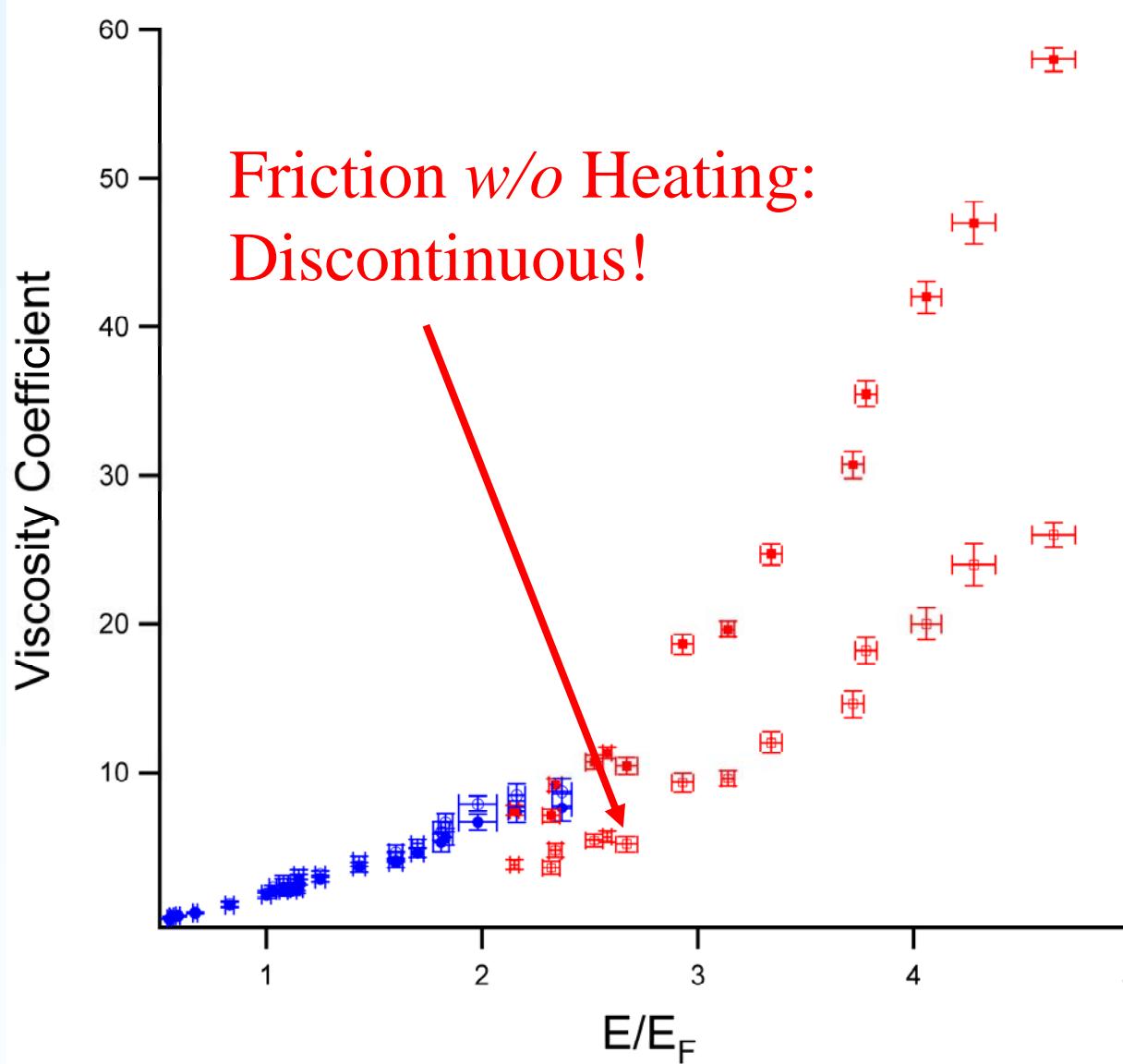


High and Low Temperature Data

Viscosity in units of $\hbar n$

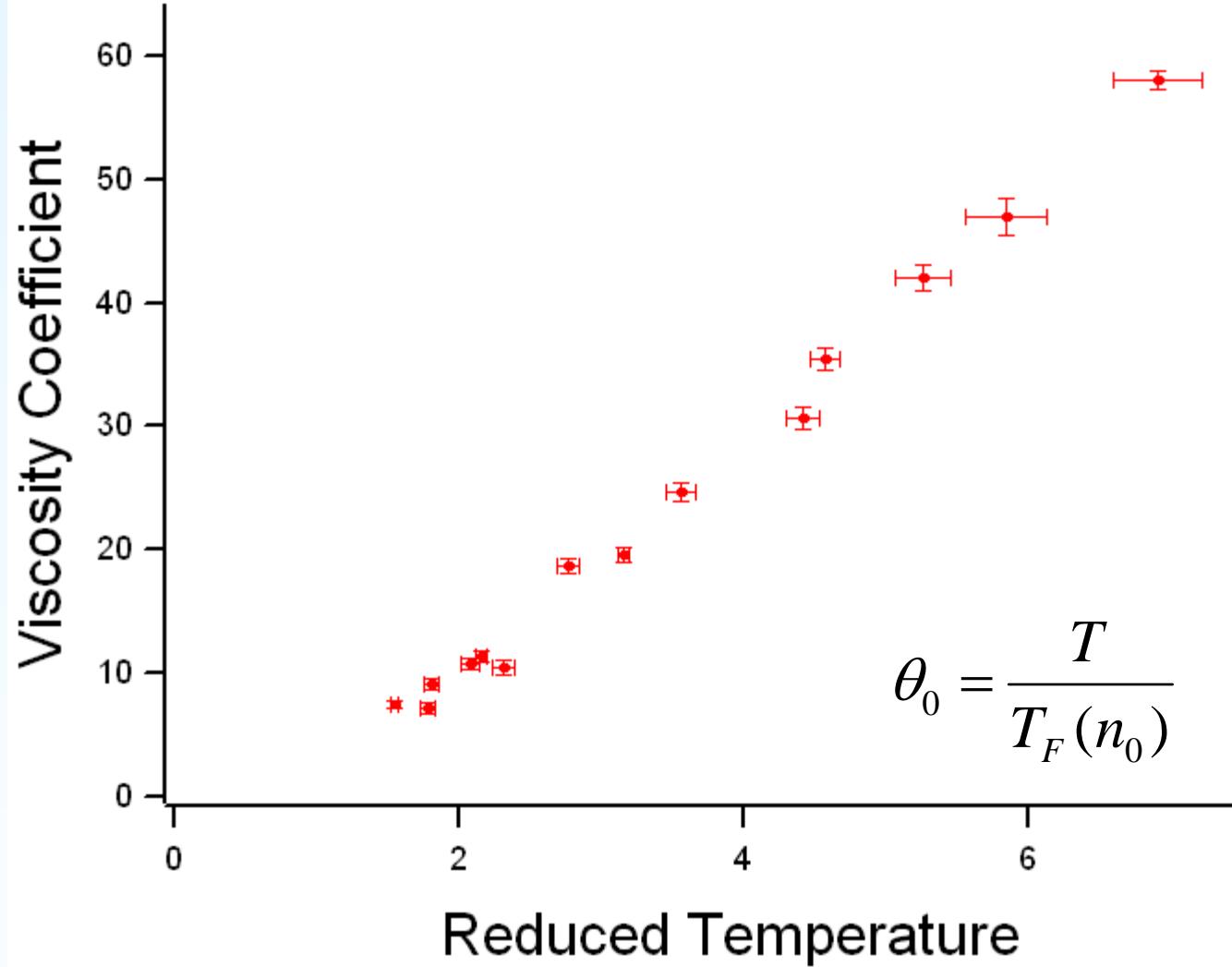


Effect of the *Heating* Rate



Universal High Temperature Scaling

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Ratio of the Shear Viscosity to the Entropy Density

$$\frac{\eta}{s} = \frac{\alpha \hbar n}{s} = \frac{\alpha \hbar}{\frac{s}{n}}$$

$$\frac{\eta}{s} = \frac{\hbar}{k_B} \frac{\langle \alpha \rangle}{S/k_B}$$

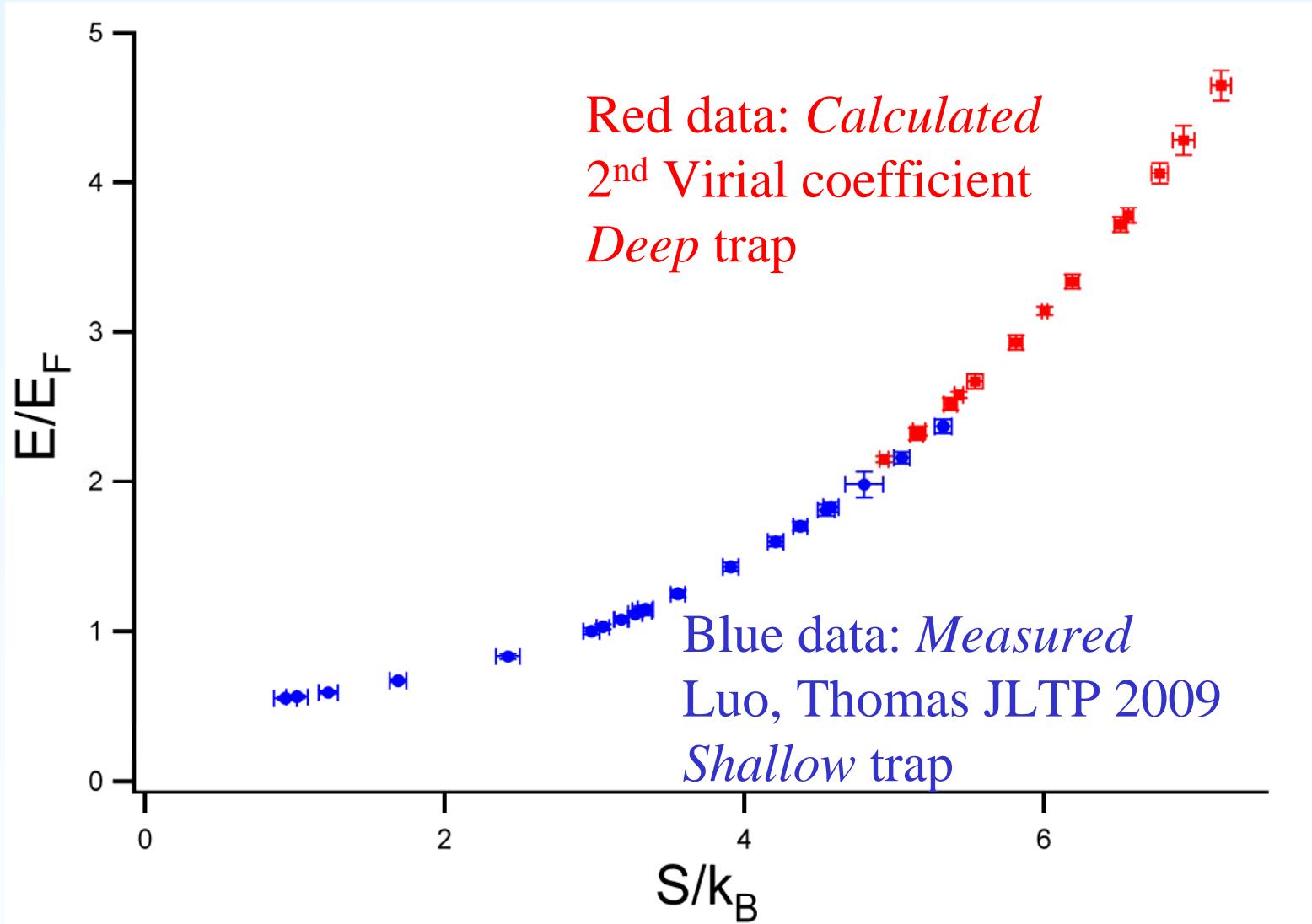
Trap averaged viscosity coefficient

Average entropy per particle

Schaefer, PRA **76**, 063618 (2007)

Turlapov, et al., JLTP **150**, 567 (2008)

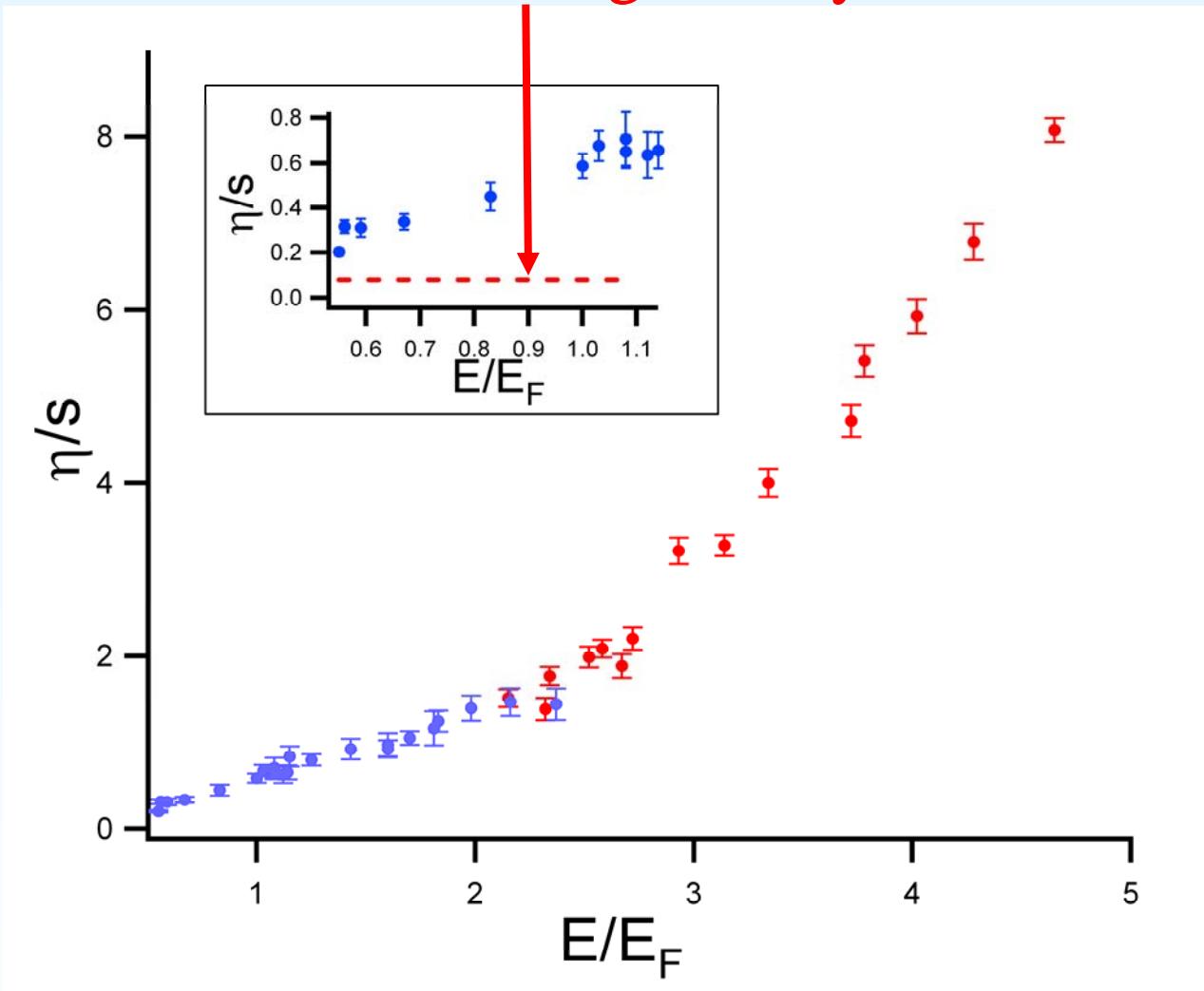
Energy per particle versus Entropy per Particle



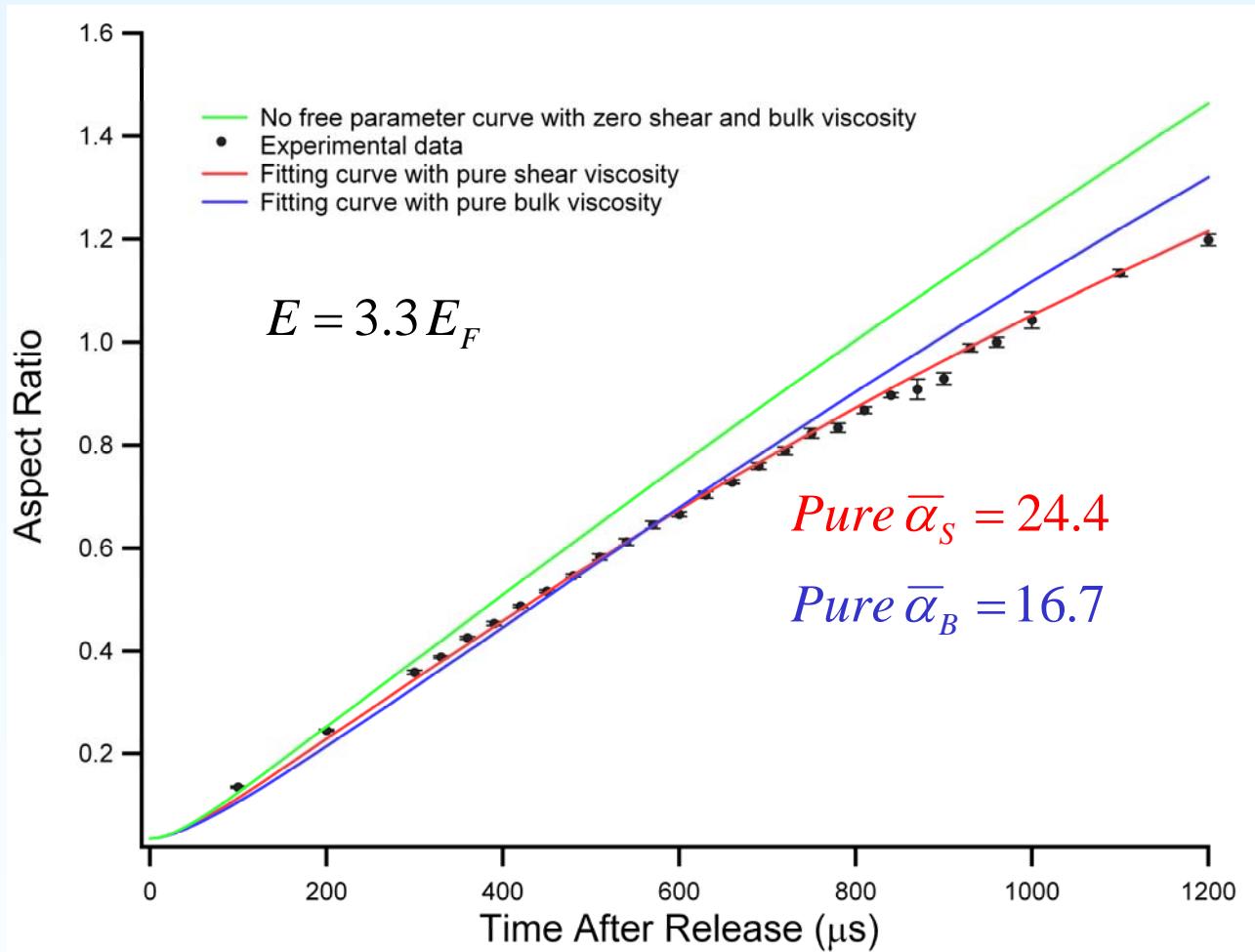
Ratio of the Viscosity to the Entropy

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String Theory Limit

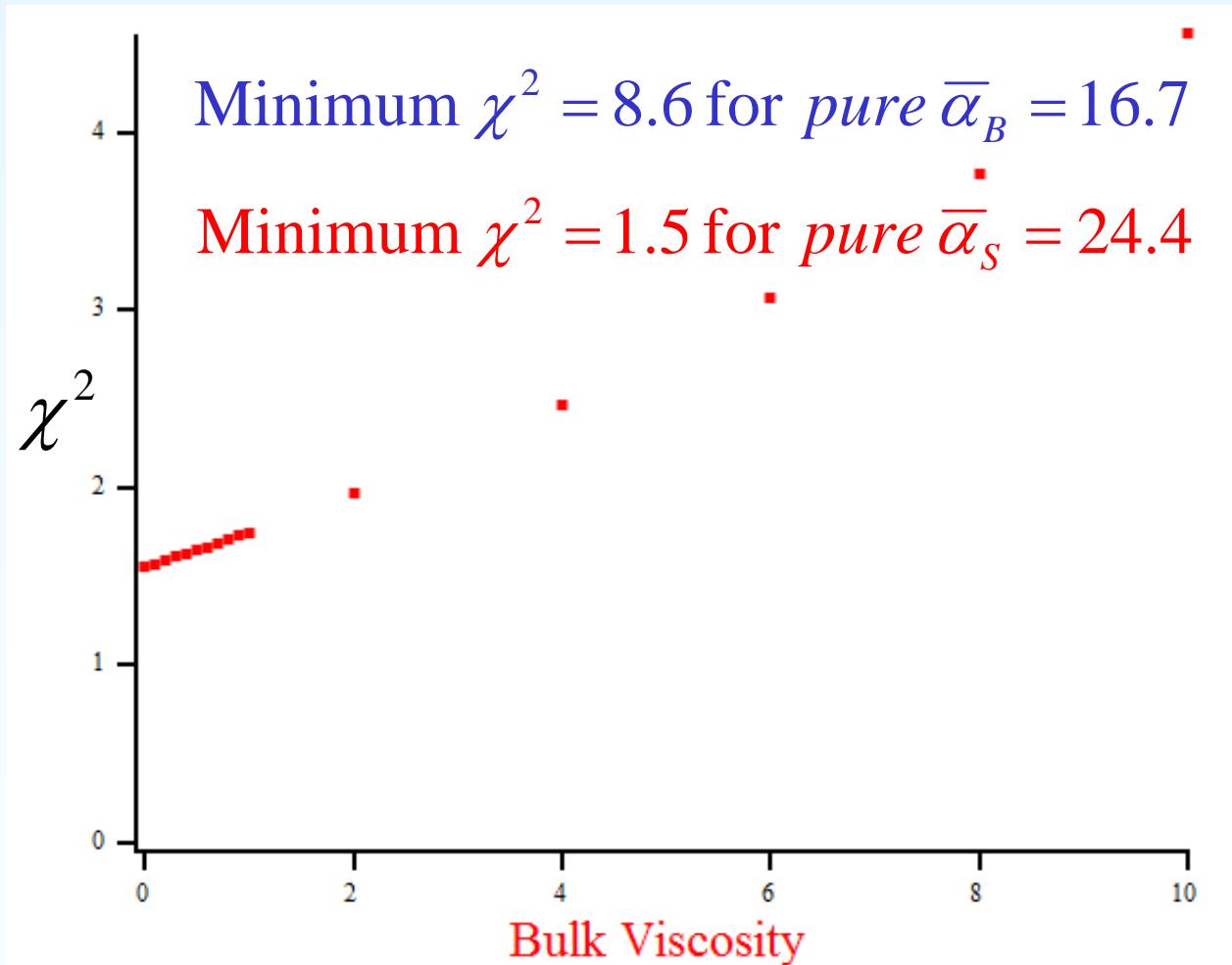


What about *Bulk* Viscosity?



Vanishing Bulk Viscosity

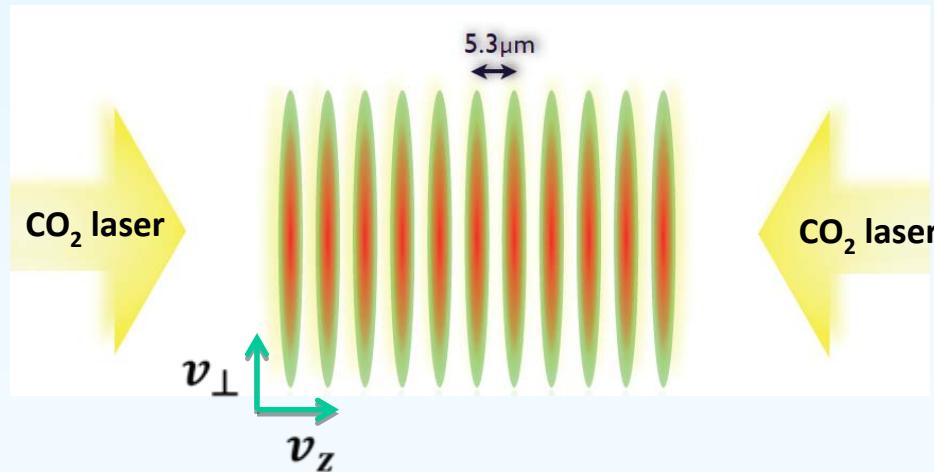
- Two parameter fit, optimum shear viscosity for each bulk viscosity



Radio-frequency spectra of a quasi-two-dimensional Fermi gas:

Quasi-Two-Dimensional Fermi gas

- We study confinement-induced dimers and many-body effects in a quasi-2D system.

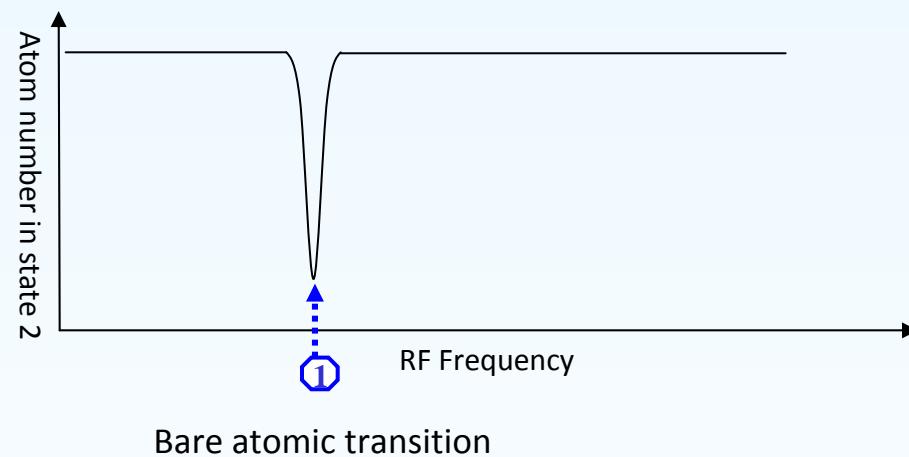
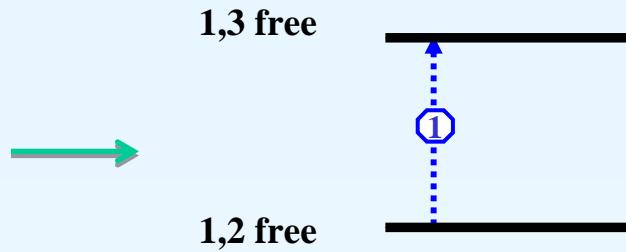
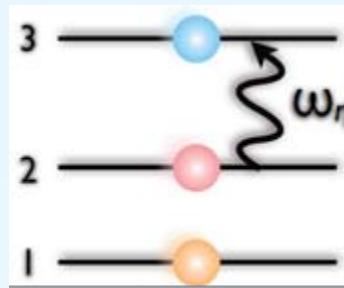


Along CO₂ laser direction

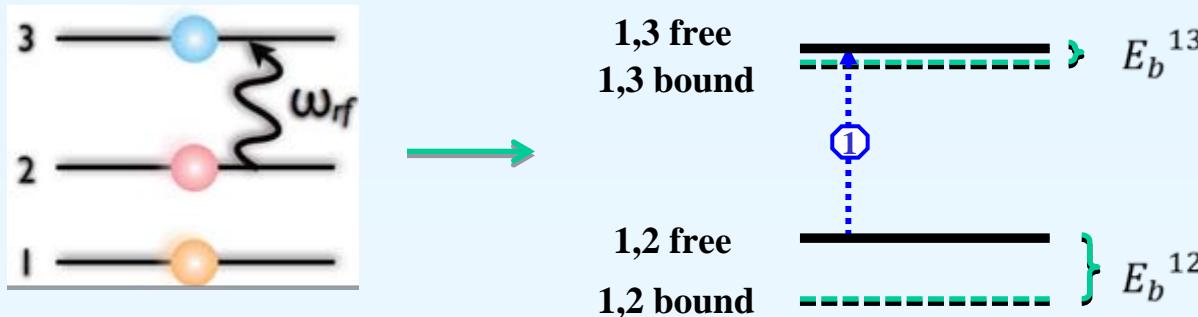


- 50-50 mixture of ⁶Li |1⟩ and |2⟩
- Atom number per pancake ~ 1000
- $v_{\perp}/v_z = 1/25$
- $E_{F\perp} \sim 1.5 \hbar\nu_z$ (v_z : 25kHz-180kHz; $E_{F\perp}$: 2μK-12μK)

RF transitions: Bare atom



RF transitions: dimer-to-dimer



Confinement-induced dimer binding energy for 3D harmonic trap: $E_b \equiv \varepsilon_b \hbar \nu_z$

Magnetic field dependent
S-wave scattering length
with pseudopotential:
 $a[B]$

$$V(\mathbf{r}) = \frac{4\pi\hbar^2 a}{m} \delta(\mathbf{r}) \partial_r [r \dots]$$

$$a \rightarrow \infty$$

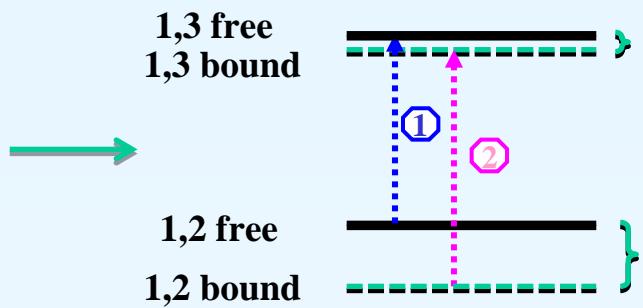
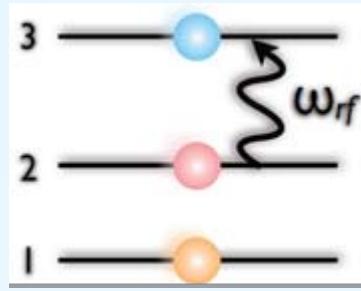
$$\frac{l_z}{a} = \int_0^\infty \frac{du}{\sqrt{4\pi u^3}} \left\{ 1 - \prod_j \left(\frac{2\beta_j u}{1 - e^{-2\beta_j u}} \right)^{1/2} e^{-\varepsilon_b u} \right\}$$

$$l_z \equiv \sqrt{\hbar / (m \omega_z)} \quad \beta_j \equiv \nu_j / \nu_z$$

$$E_b \equiv 0.245 \hbar \nu_z \quad E_b \equiv 0.290 \hbar \nu_z$$

$$\beta_\perp \equiv \nu_\perp / \nu_z = 0 \quad \beta_\perp \equiv \nu_\perp / \nu_z = 1/25$$

RF transitions: dimer-to-dimer



$$\psi_{E_b}(z, \rho) \cong \phi_0(z) \frac{\kappa}{\sqrt{\pi}} K_0(\kappa \rho)$$

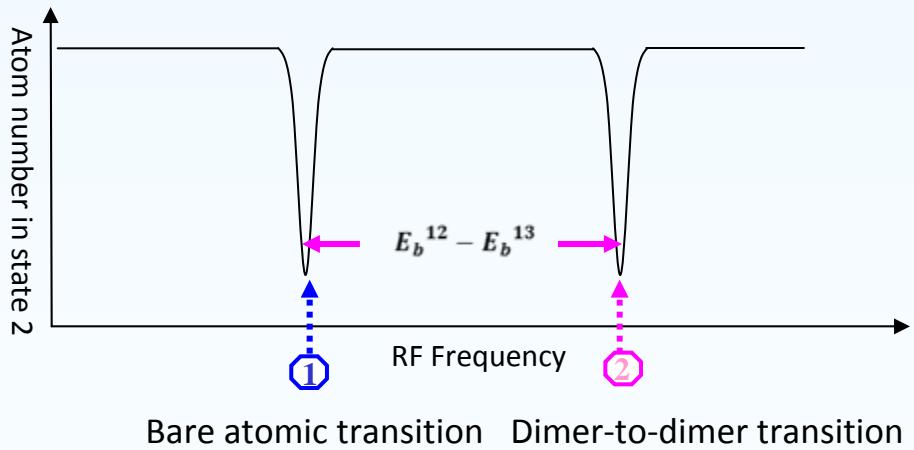
$$E_b = \frac{\hbar^2 \kappa^2}{m}$$

② Bound-to-bound transition spectrum:

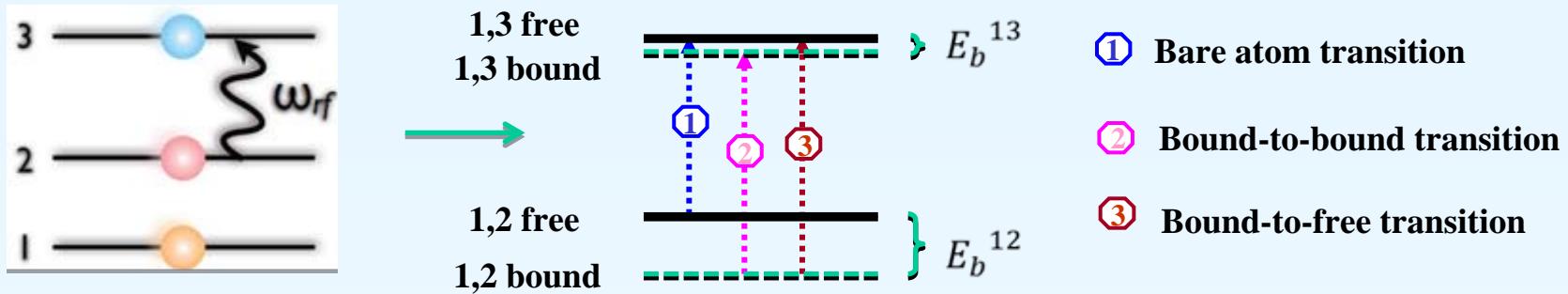
$$\int d\nu I_{bb}(\nu) \equiv \varepsilon_{bb}(q) = \frac{q^2}{4 \sinh^2(q/2)}$$

$$q \equiv \ln(E_b^{13} / E_b^{12})$$

$$I_{bb}(\nu) = \varepsilon_{bb}(q) \delta \left[\nu - \frac{E_b^{12} - E_b^{13}}{h} \right]$$



RF transitions: dimer-to-unbound (“free”)



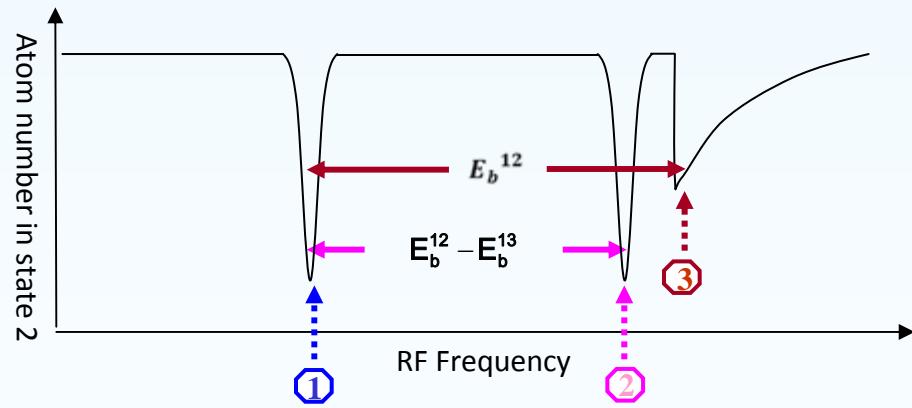
③ “Free” wavefunction: $\psi_{E_\perp}(z, \rho) \cong \phi_0(z) \frac{1}{\sqrt{A}} \left\{ J_0(k_\perp \rho) - \frac{\pi i}{\ln(E_b/E_\perp) + \pi i} H_0(k_\perp \rho) \right\}; E_\perp = \frac{\hbar^2 k_\perp^2}{m}$

Bound-to-free transition spectrum:

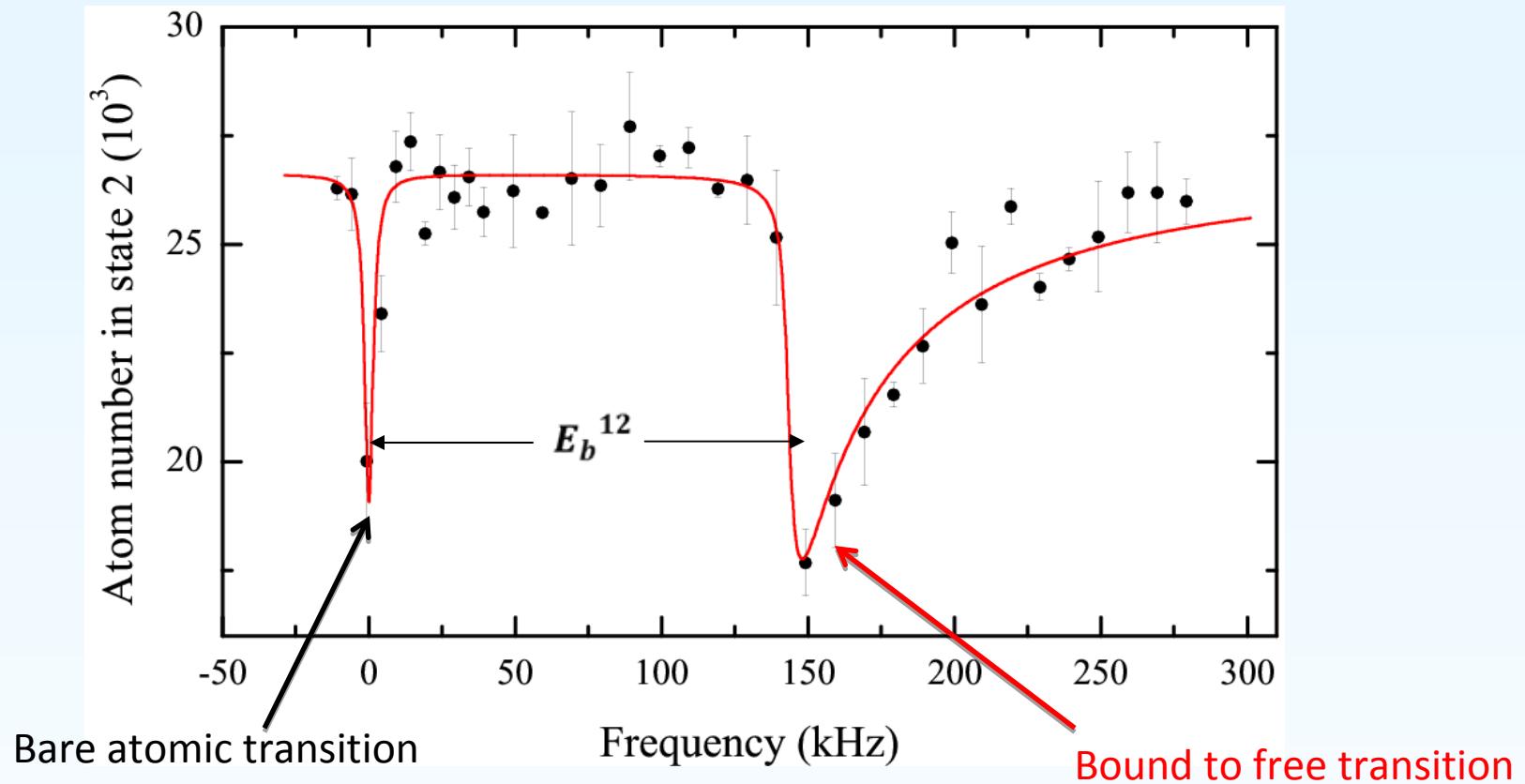
$$I_{bf}(\nu) = \frac{E_b^{12}}{h\nu^2} \theta\left(\nu - \frac{E_b^{12}}{h}\right) \frac{q^2}{\left[q - \ln\left(\frac{h\nu}{E_b^{12}} - 1\right)\right]^2 + \pi^2}$$

$$q \equiv \ln\left(E_b^{13}/E_b^{12}\right)$$

$$\int d\nu I_{bf}(\nu) = 1 - \varepsilon_{bb}(q)$$



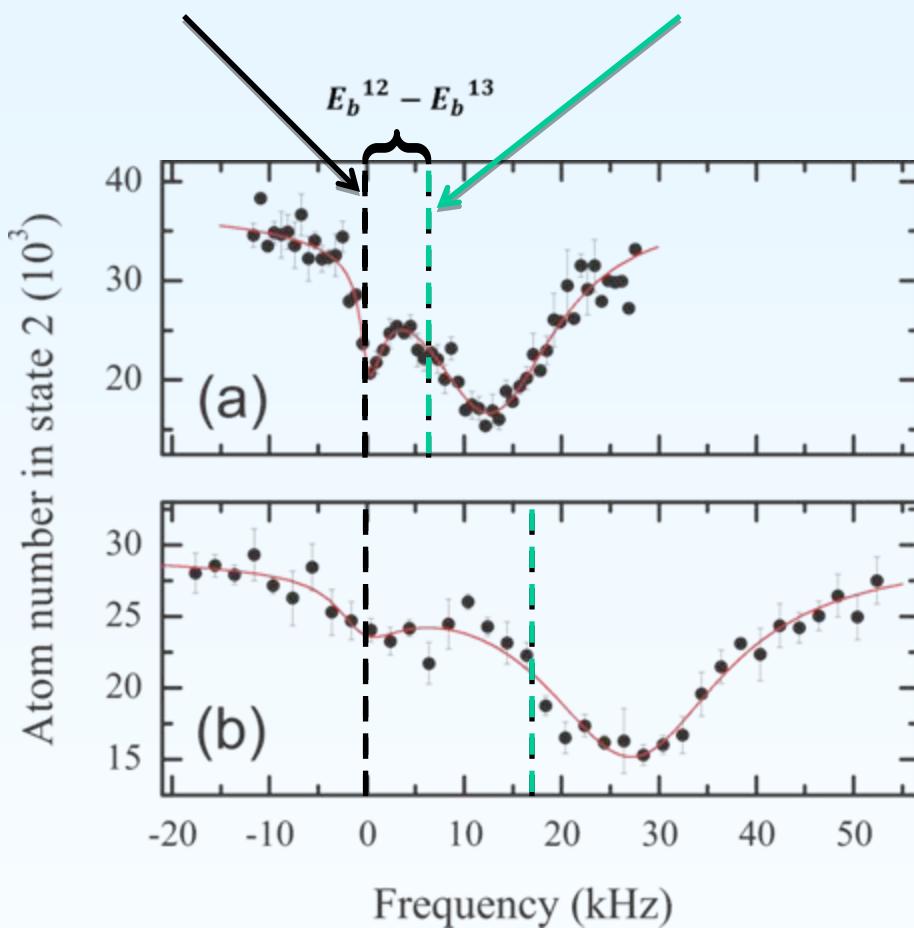
RF 12-to-13 spectrum at 720 G



$$\epsilon_{bb} = 0.27, E_b^{12} = 145 \text{ kHz}, E_b^{13} = 2.9 \text{ kHz}$$

RF 12-to-13 spectrum at 832G

Bare atomic transition



Bound to bound transition

$$\nu_z = 24.5 \text{ kHz}$$

$$\epsilon_{bb} = 0.66, \\ E_b^{12} = 7.25 \text{ kHz}, E_b^{13} = 0.81 \text{ kHz}$$

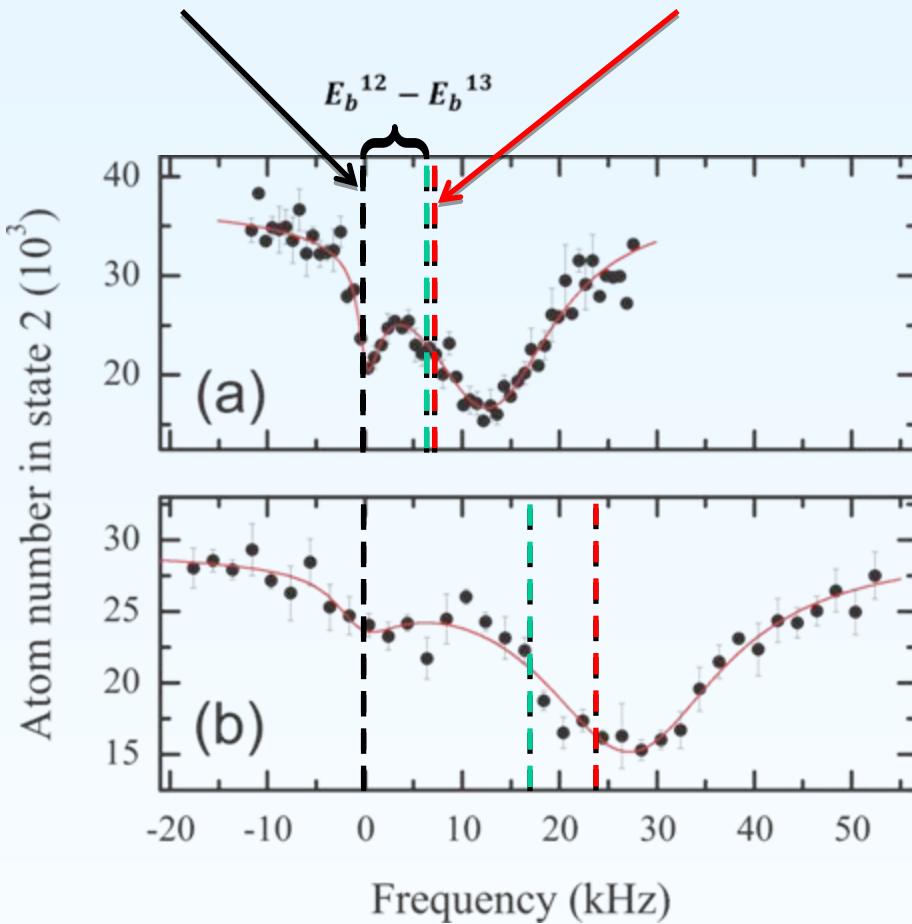
$$\nu_z = 82.0 \text{ kHz}$$

$$\epsilon_{bb} = 0.83, \\ E_b^{12} = 23.9 \text{ kHz}, E_b^{13} = 5.79 \text{ kHz}$$

RF spectrum at 832G

Bare atomic transition

Bound to free transition

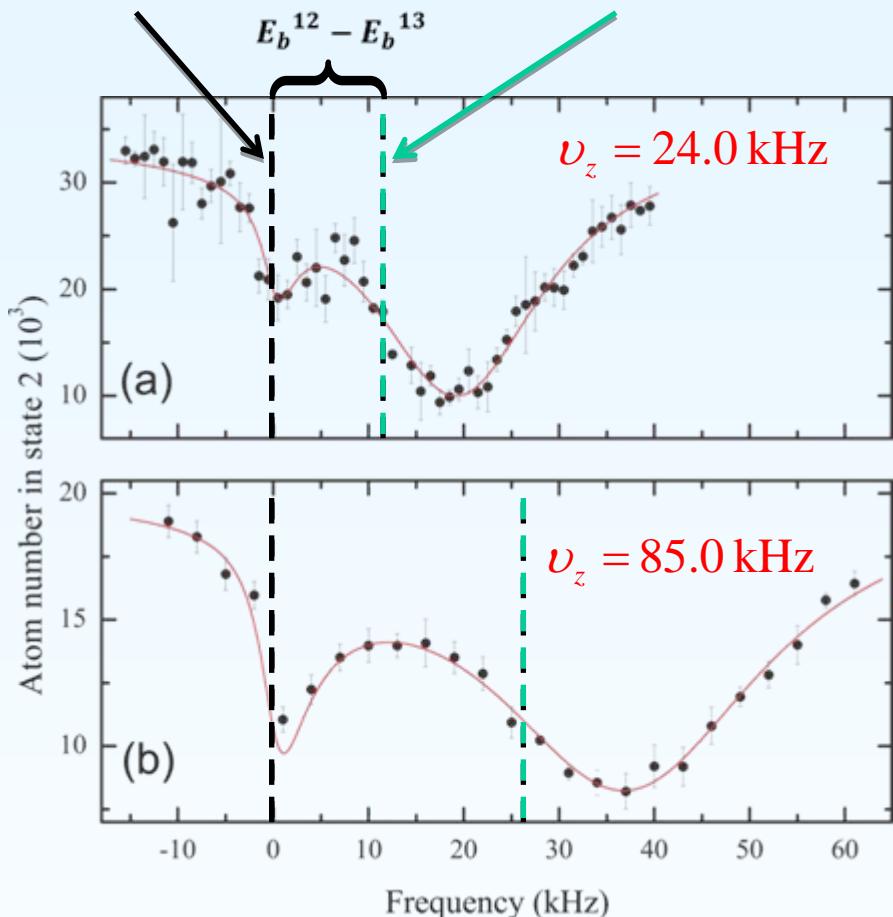


$$\epsilon_{bb} = 0.66, \\ E_b^{12} = 7.25 \text{ kHz}, E_b^{13} = 0.81 \text{ kHz}$$

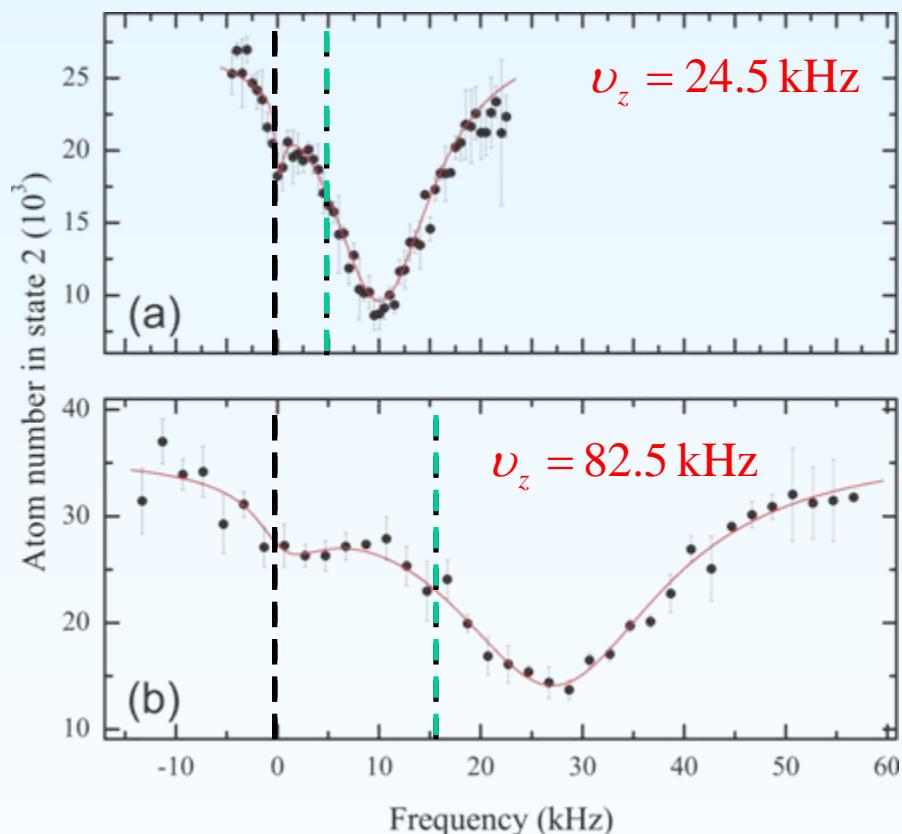
$$\epsilon_{bb} = 0.83, \\ E_b^{12} = 23.9 \text{ kHz}, E_b^{13} = 5.79 \text{ kHz}$$

RF 12-to-13 spectra at 811G & 842G

Bare atomic transition

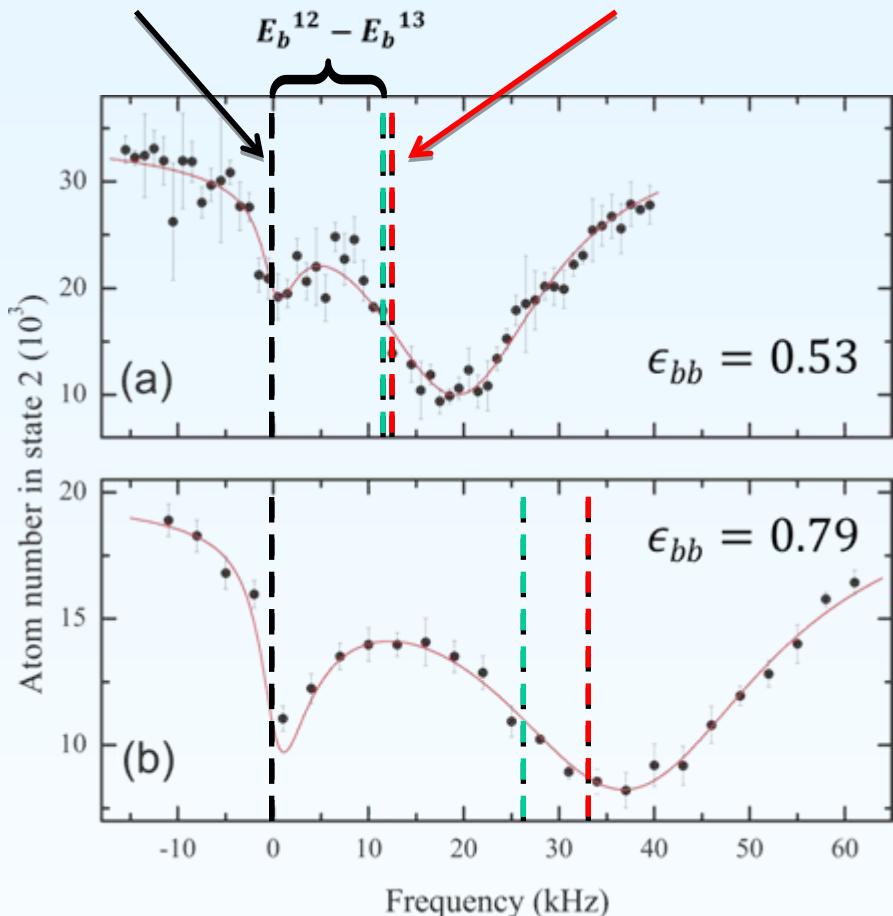


Bound to bound transition

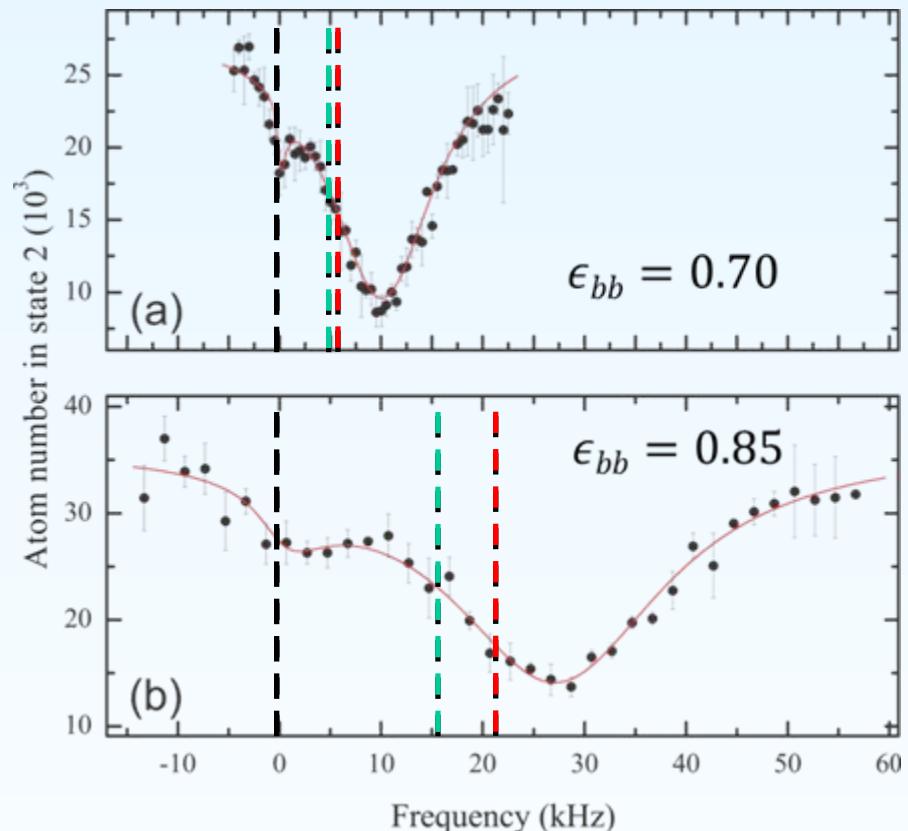


RF spectra at 811G (left) & 842G (right)

Bare atomic transition



Bound to free transition



- Spectra in *disagreement* with dimer theory!

BCS Theory—Many-body effects?

RF spectrum
non-interacting
final state:

$$I_{BCS}(\nu) \propto \frac{1}{\nu^2} \int_0^\infty d^2 \mathbf{x}_\perp |\Delta|^2 \theta[h\nu + \mu - \sqrt{\mu^2 + |\Delta|^2}]$$

Randeria PRL 1989: T = 0 Gap equation in 2D: $E_b = \sqrt{\mu^2 + |\Delta|^2} - \mu$

$$T_{2B}(k_{\perp 0}) = \frac{\hbar^2}{m} \frac{4\pi}{\pi i + \ln[\varepsilon_b / (2\varepsilon_{\mathbf{k}\perp 0})]} \quad \rightarrow \quad h\nu_{\text{threshold}} = E_b$$

Threshold *exactly* at the dimer binding energy!

Observed by Sommer et al, PRL 2012.

Polarons in 2D?

For 2-3 transition, state 3 initially empty:
3-impurity in Fermi sea of atoms in state 1—Polaron?

Chevy's ansatz for the zero-momentum polaron PRA 2006:

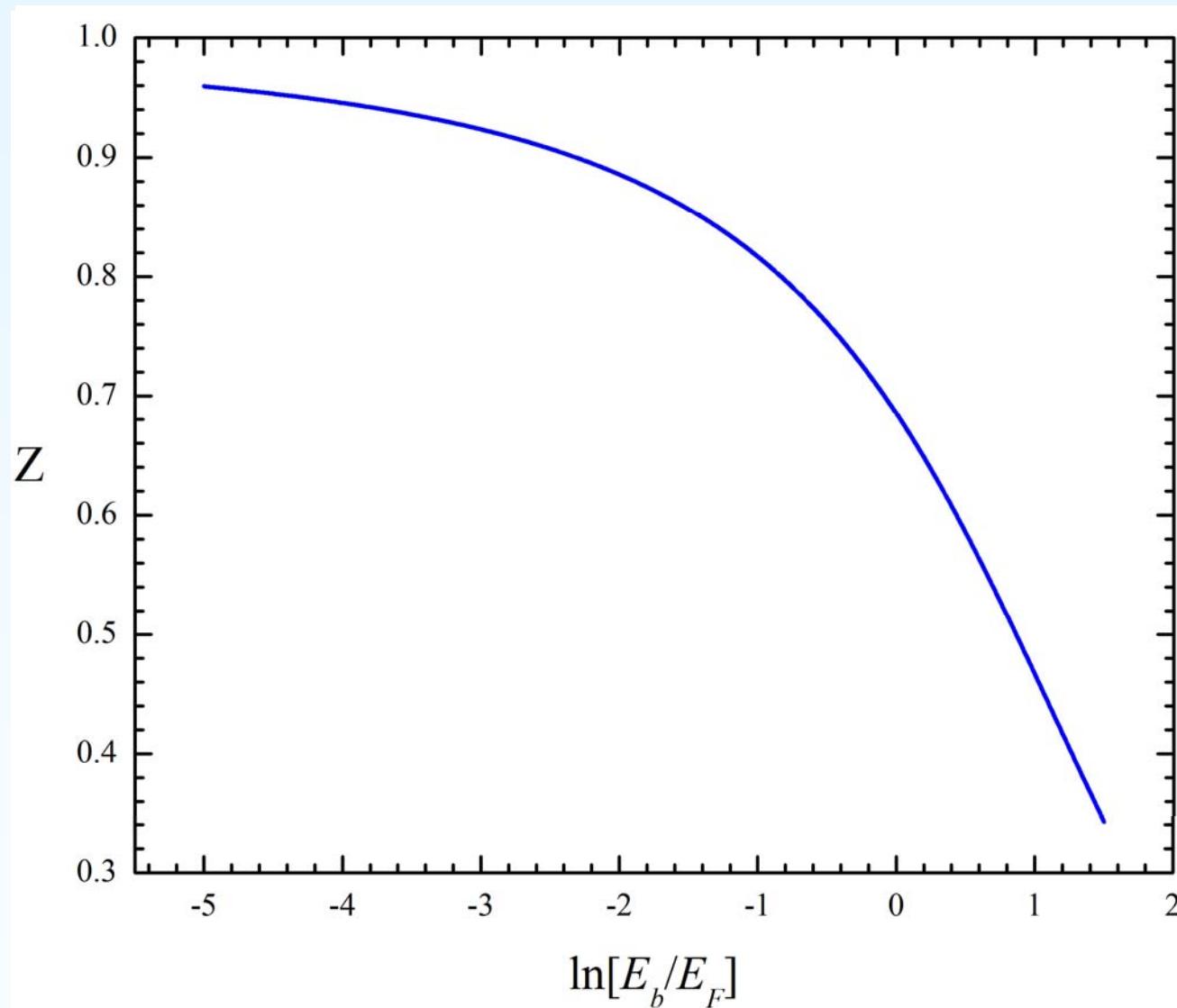
$$|E_{pi}\rangle = \varphi_{0i} |\mathbf{k} = 0\rangle_i |FS\rangle_1 + \sum_{q < k_F < k} \varphi_{\mathbf{k}\mathbf{q}i} |\mathbf{q} \cdot \mathbf{k}\rangle_i c_{\mathbf{k}1}^+ c_{\mathbf{q}1} |FS\rangle_1$$

Follow Shirotzek et al., PRL 2009, but change 3D to 2D:

$$\varepsilon \equiv \frac{E_{pi}}{E_{F\perp}} \quad \boxed{\varepsilon = \Sigma(\varepsilon)}$$

$$\Sigma(\varepsilon) = -2 \int_0^1 \frac{du}{-\ln\left(\frac{E_b}{E_{F\perp}}\right) + \ln\left[\sqrt{(1-\varepsilon/2)^2 - u} + (1-\varepsilon/2 - u/2)\right]}$$

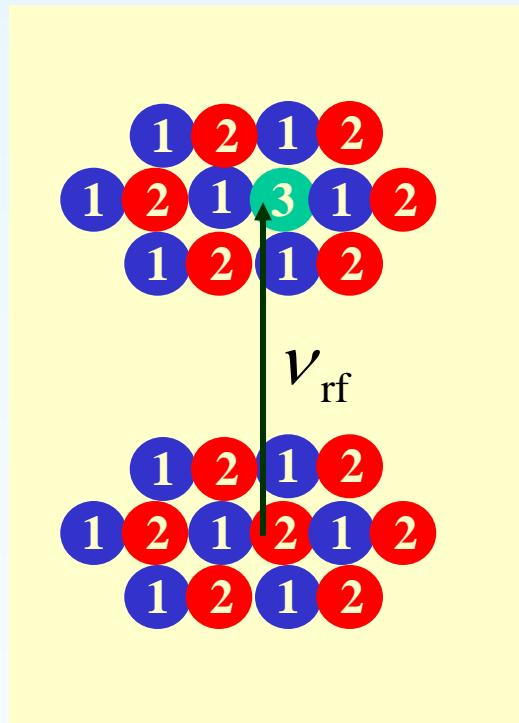
Polaron Energy E_p and Weight Z



2-1 Polaron-to-3-1 Polaron Transition

Problem: 3-1 Polaron energy is HUGE!

*Try 2-1 polaron-to-3-1 polaron transition!



*Following Zwierlein et al., PRL 2003 and Baym et al., PRL 2007, we assume that 2-3 interactions cause *no shifts* for 2-to-3 transitions.

Polaron-to-polaron transitions

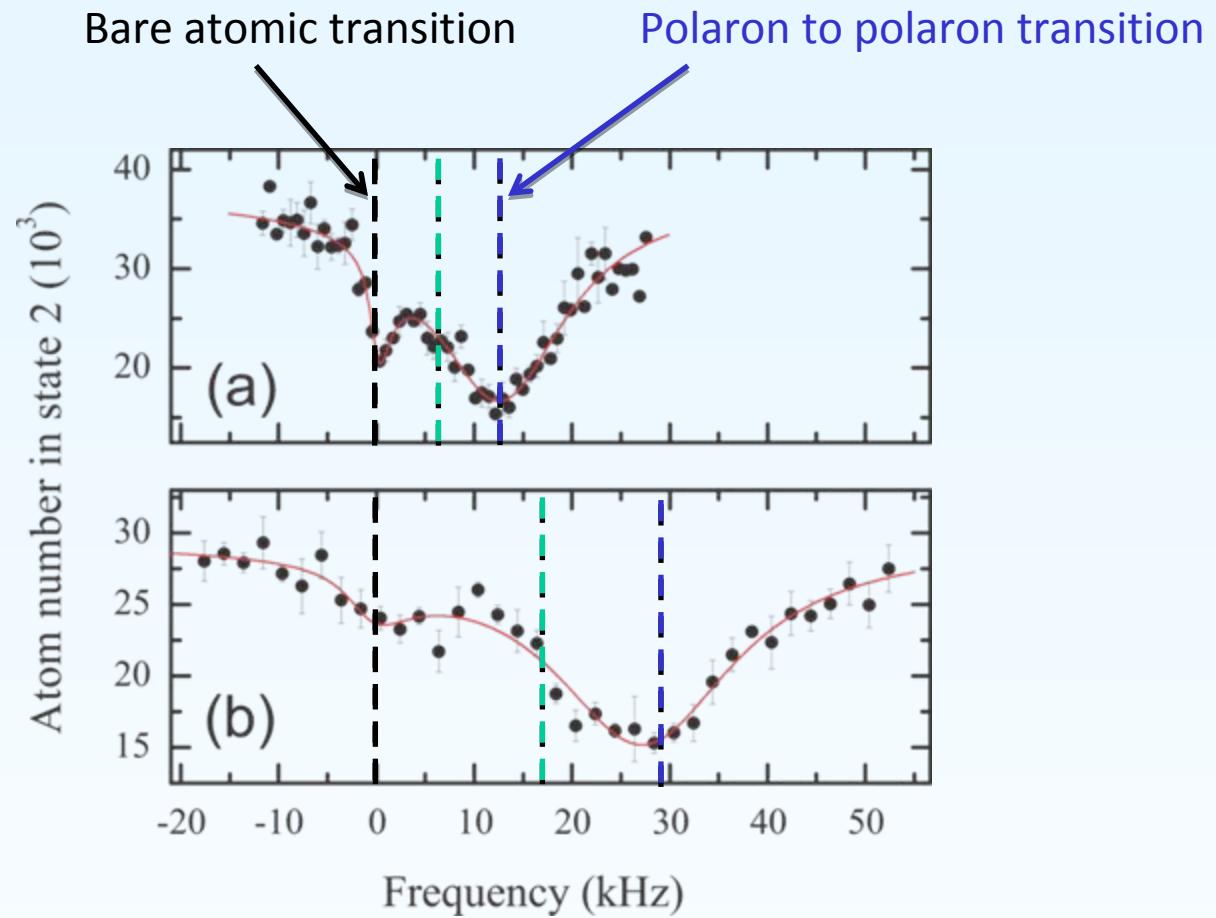
- Polaron: an impurity atom in $|2\rangle$ or $|3\rangle$ immersed in a bath of $|1\rangle$ atoms.

$$h\Delta\nu_{\text{polaron}} = E_b(2,1) - E_b(3,1)$$

$$E_F \text{ local} = \lambda_1 E_{F\perp} \quad \lambda_1 = 0.67$$

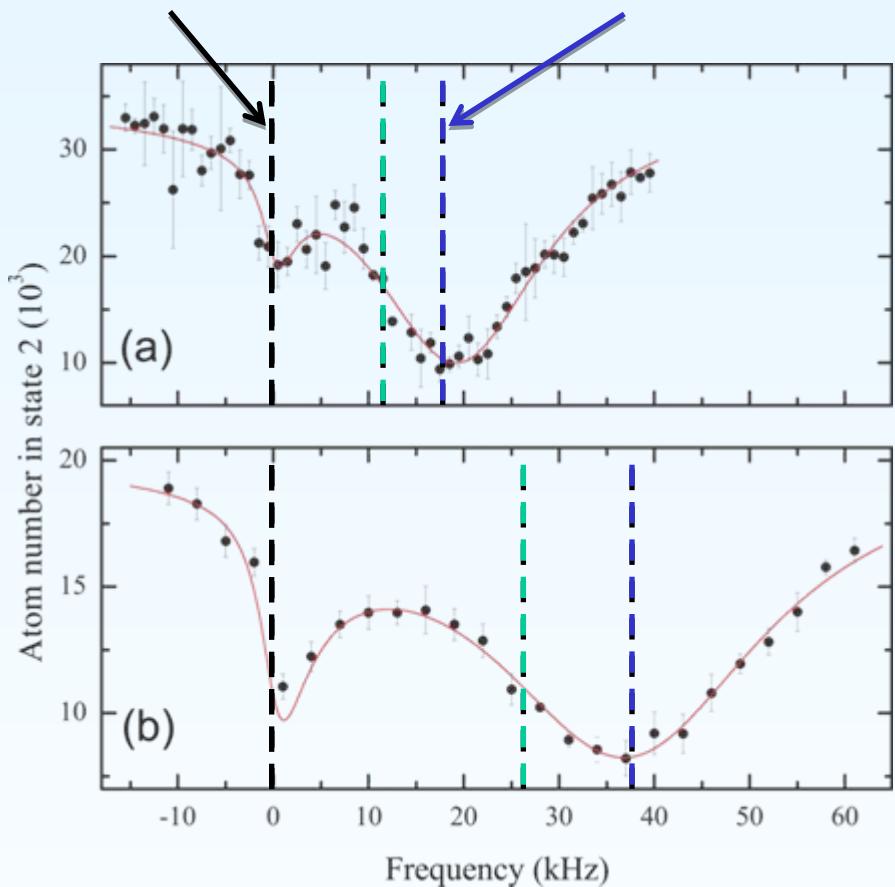
B (G)	v_z (kHz)	$\Delta\nu_{\text{measure}}$ (kHz)	$\Delta\nu_{\text{polaron}}$ (kHz)
809	24	18.7	18.3
810	85	37.1	37.0
842	24.5	10.1	9.7
842	82.5	27.2	26.7
832	24.5	12.3	11.6
832	82	28.3	29.1

RF 12-to-13 spectrum at 832G

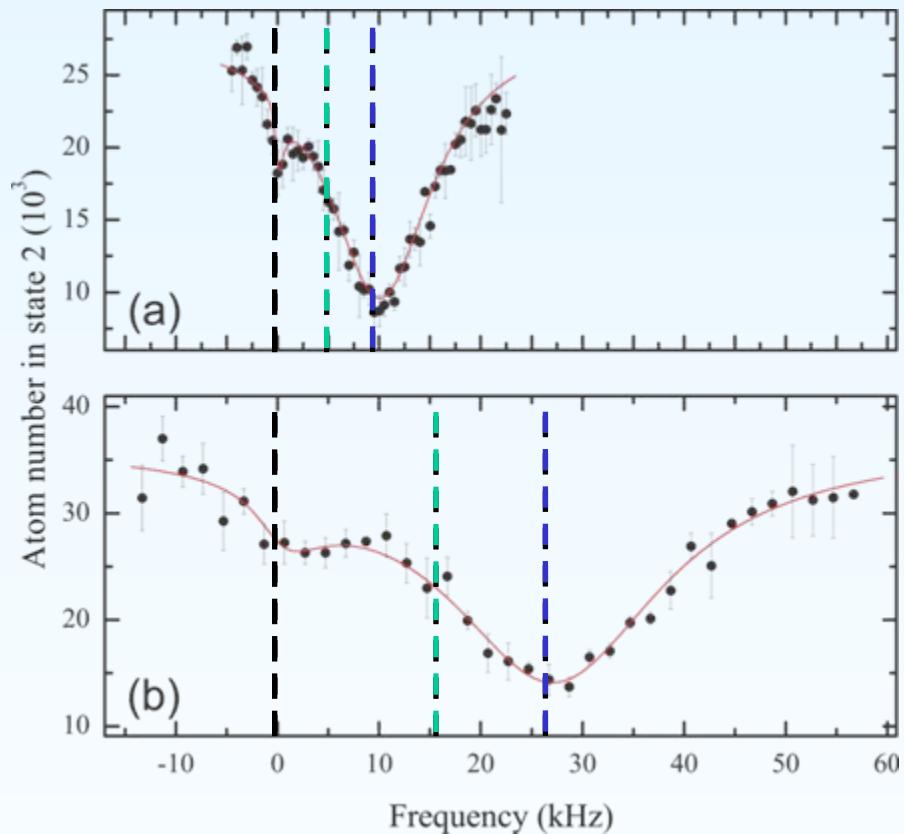


RF 12-to-13 spectra at 811G & 842G

Bare atomic transition



Polaron to polaron transition



Summary

- Thermodynamics of strongly-interacting Fermi gases:
 - Tests of non-perturbative many-body theory/Temperature
- Transport: Minimum viscosity hydrodynamics:
 - Shear viscosity versus reduced temperature
 - Bulk viscosity vanishes for high temperature expansion
 - Minimum η/s 5 times the minimum viscosity conjecture
- Quasi-2D Fermi gas
 - Many-body physics — dimers versus polarons
- Future
 - Dependence of shear and bulk viscosity on interaction strength
 - Nonlinear hydrodynamics and shock waves
 - Optical control of interactions and dispersion

JETLAB Team 2012

PHYSICS

