Nonlinear Hydrodynamics in a Cold Fermi Gas with Tunable Interactions

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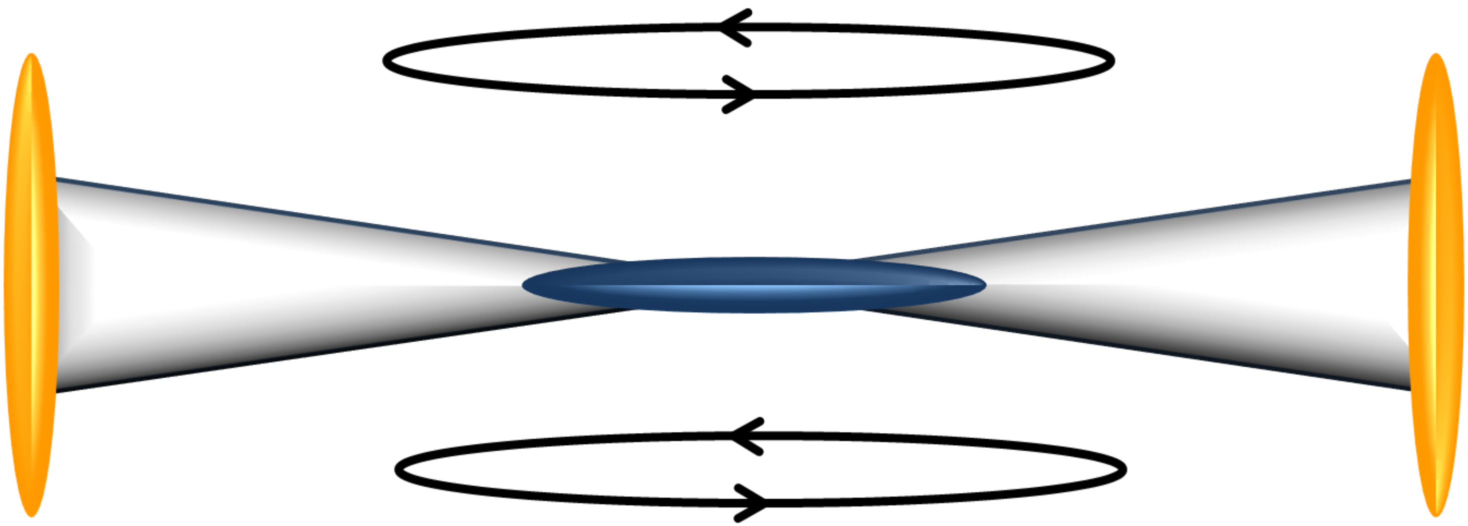
Organization

- Optically trapped Fermi gas with magnetically tunable interactions
- Nonlinear Hydrodynamics in trapped gasses
- Experiments
  - Sound velocity in the BEC-BSC Crossover
  - Shock waves in a unitary Fermi gas
- Future Outlook

Optically Trapped Fermi Gas

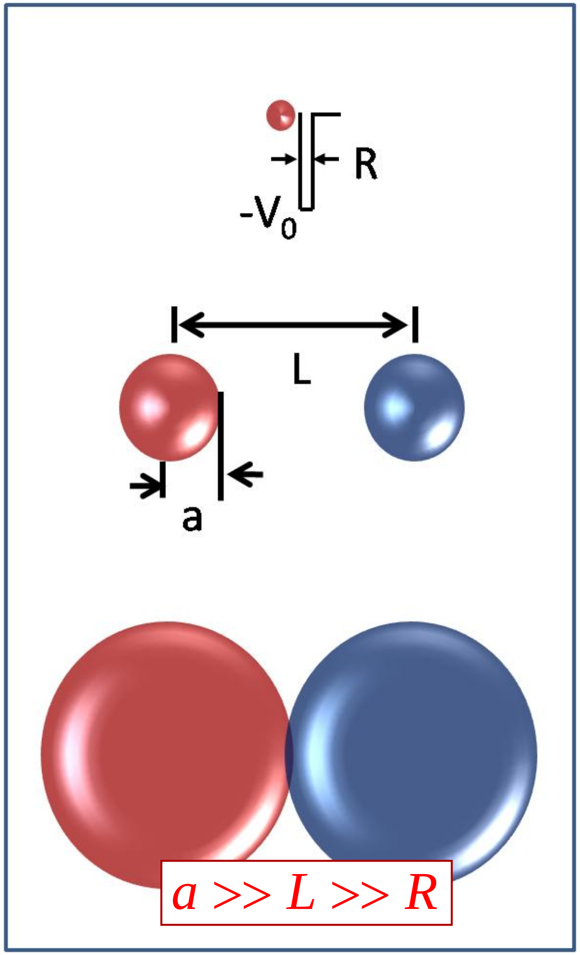
- The system is comprised of the two lowest spin states of ^6Li trapped at the focus of a CO_2 laser beam in an applied external magnetic field.



Three Length Scales

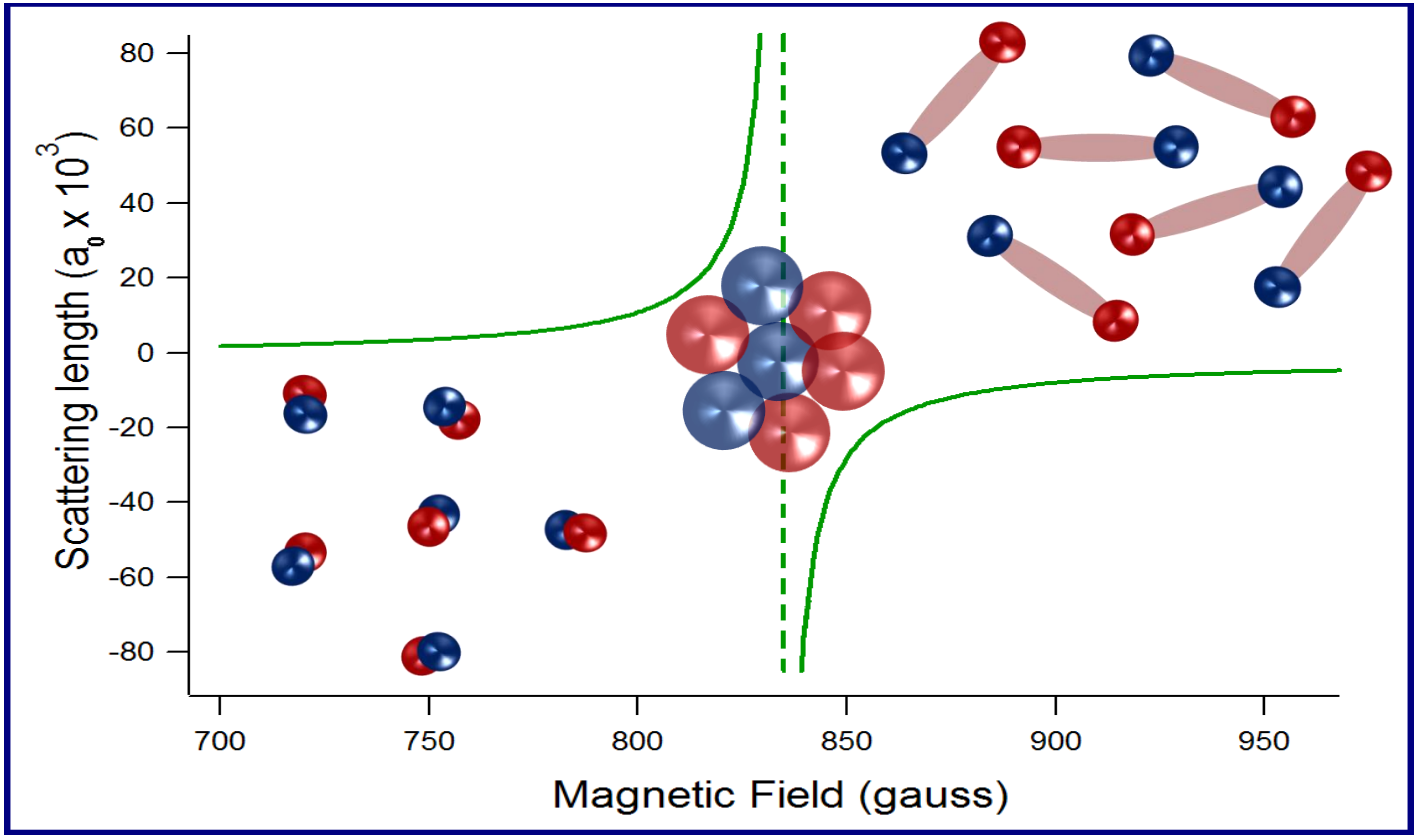
- Three length scales:
  - R is the range of the atomic potential
  - L is the interparticle spacing
  - a is the scattering length

- For our gas, L >> R always!
- a can be tuned by applying an external magnetic field.
- On resonance, all thermodynamic parameters are determined from density ($1/L^3$) and temperature.



Magnetically Tunable Interactions

- Fermi gases with magnetically tunable interactions provide a new universal medium for studies of hydrodynamics in quantum matter.



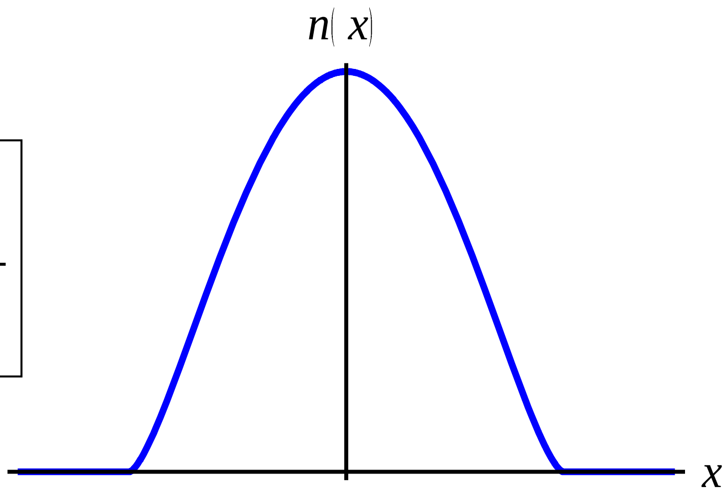
Zero Temperature Equilibrium Gas

Pressure — Gibbs-Duhem relation — Local Density Approximation

$$\nabla P(\mathbf{x}) = -n\nabla V(\mathbf{x}), \quad dP = n\,d\mu + s\,dT, \quad \mu(\mathbf{x}) = \mu_0 - V(\mathbf{x})$$

For a harmonic trapping potential: $V(\mathbf{x}) = \frac{m}{2}\sum \omega_i^2 x_i^2 \Rightarrow$

$$n(\mathbf{x}) = n_0\left(1 - \sum \frac{x_i^2}{R_i^2}\right)^{1/\gamma}, \quad R_i^2 = \frac{2\mu}{m\omega_i^2}$$



Density Distribution

Equation of State as a Function of Interaction Strength

The chemical as a function of atomic density is our equation of state.

$$\mu = C\, n^{\gamma}$$

The global chemical potential is the local chemical potential at the center of the trap.

$$\mu_0 = \mu(0)$$

The ground state global chemical potential and the density scaling depend on the interaction strength

$$\mu_0(k_F a), \quad \gamma(k_F a),$$

–Weakly Interacting Fermi gas: $k_F a \rightarrow 0^-$ $\quad \mu_0 = E_F \quad \gamma = 2/3$

–Strongly interacting Fermi gas: $k_F a \rightarrow \pm\infty$ $\quad \mu_0 = (1+\beta)E_F \quad \gamma = 2/3$

–Weakly Interacting Bose gas: $k_F a \rightarrow 0^+$

$$\mu_0 = \frac{1}{4}\left(\frac{5}{2} k_F a_{mol}(a)\right)^{2/5} E_F \qquad \gamma = 1$$

Nonlinear Hydrodynamics

Continuity: $\partial_t n = -\nabla \cdot (n\,\mathbf{v})$

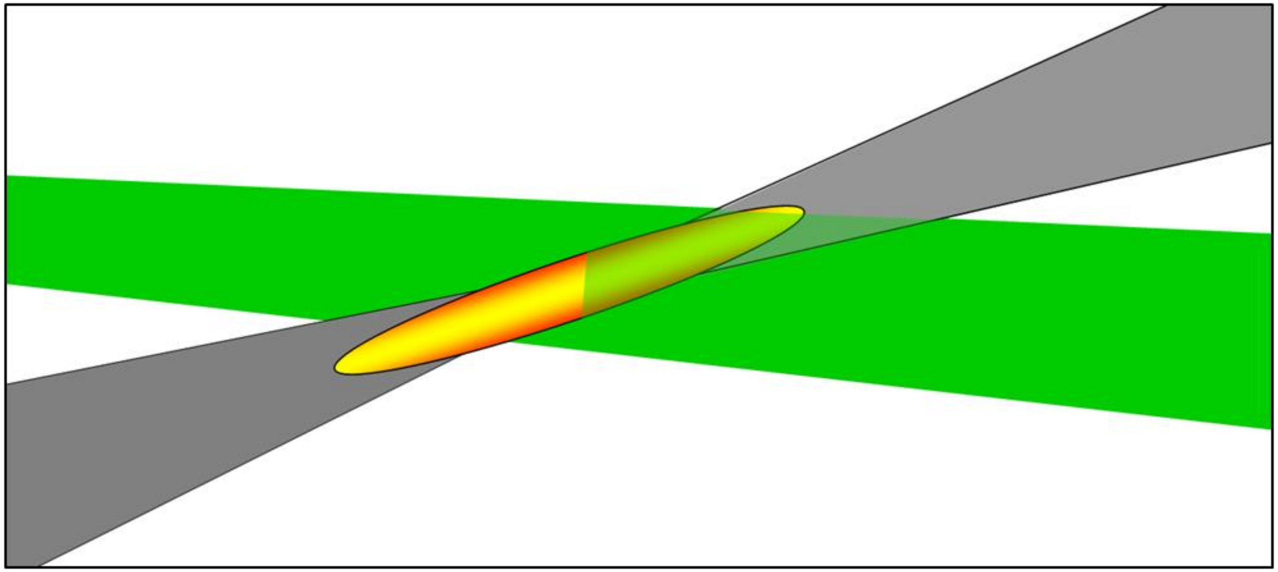
Euler: $m\partial_t \mathbf{v} = \nabla\left[C\,n^{\gamma} + V + \frac{1}{2}m\mathbf{v}^2\right]$

$\mathbf{v}(\partial_x \mathbf{v})$	$\partial_{xx}\mathbf{v}$	$\partial_{xxx}\mathbf{v}$
Nonlinear	Dissipation	Dispersion

- Continuity and Euler equations are hydrodynamic.
- Nonlinearity arises from the gradient of kinetic energy
- Higher order derivatives of velocity lead to dissipative and dispersive effects

Exciting a Sound Pulse

- A blue-detuned beam at 532nm is shaped to formed a sheet potential.
- Atoms at the center of the trap are repelled.



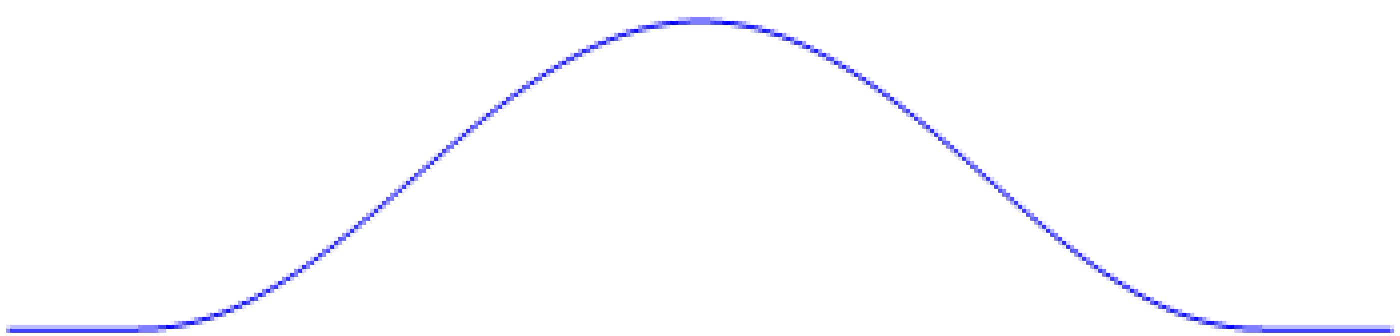
Sound Velocity

For small perturbations, $n = n_0 + \delta n$ we can solve for the sound velocity as a function of the density.

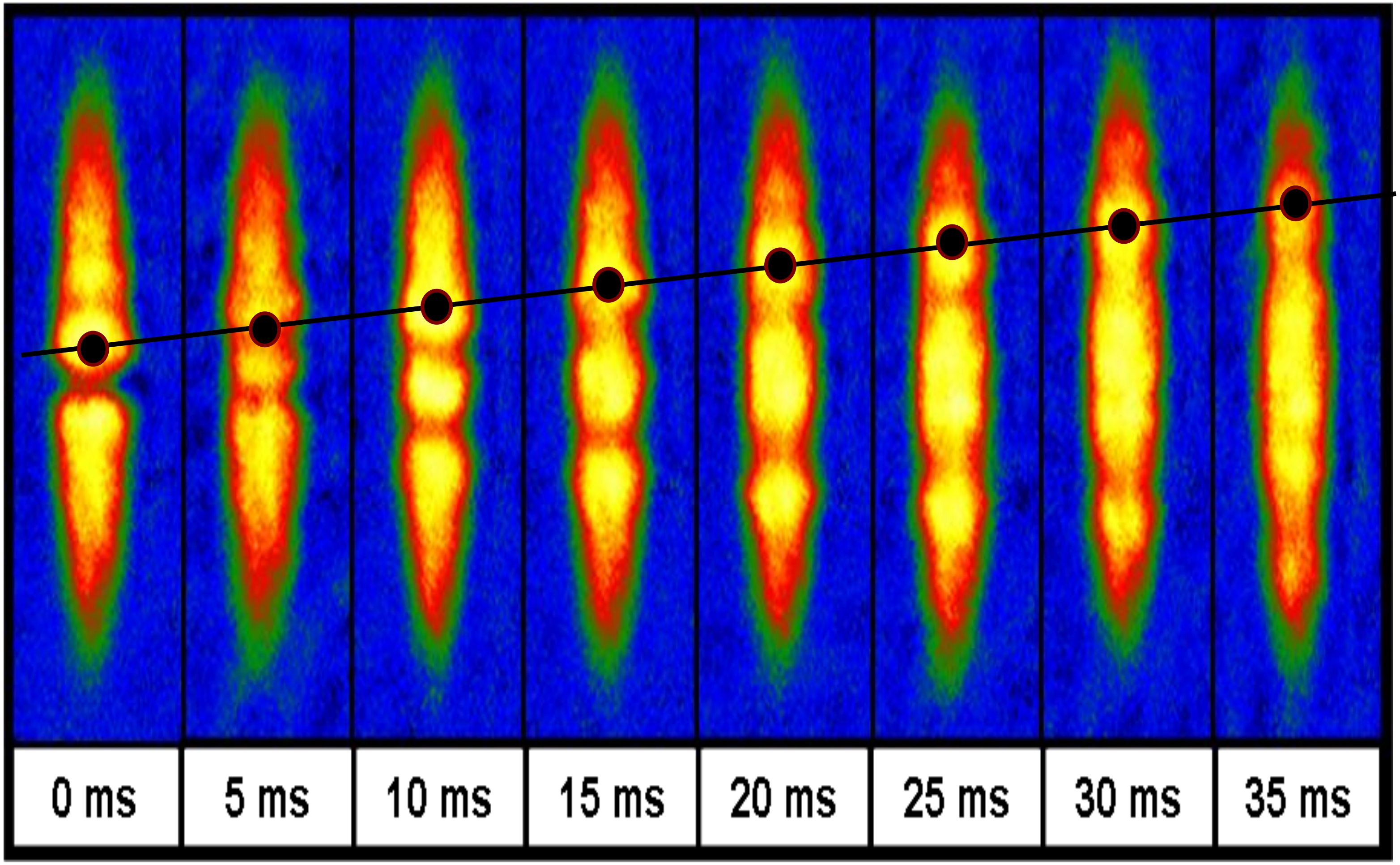
$$c(n) = \sqrt{\frac{1}{m}\frac{\partial P(n)}{\partial n}} = \sqrt{\frac{n}{m}\frac{\partial \mu(n)}{\partial n}}$$

For excitation of a plane wave:

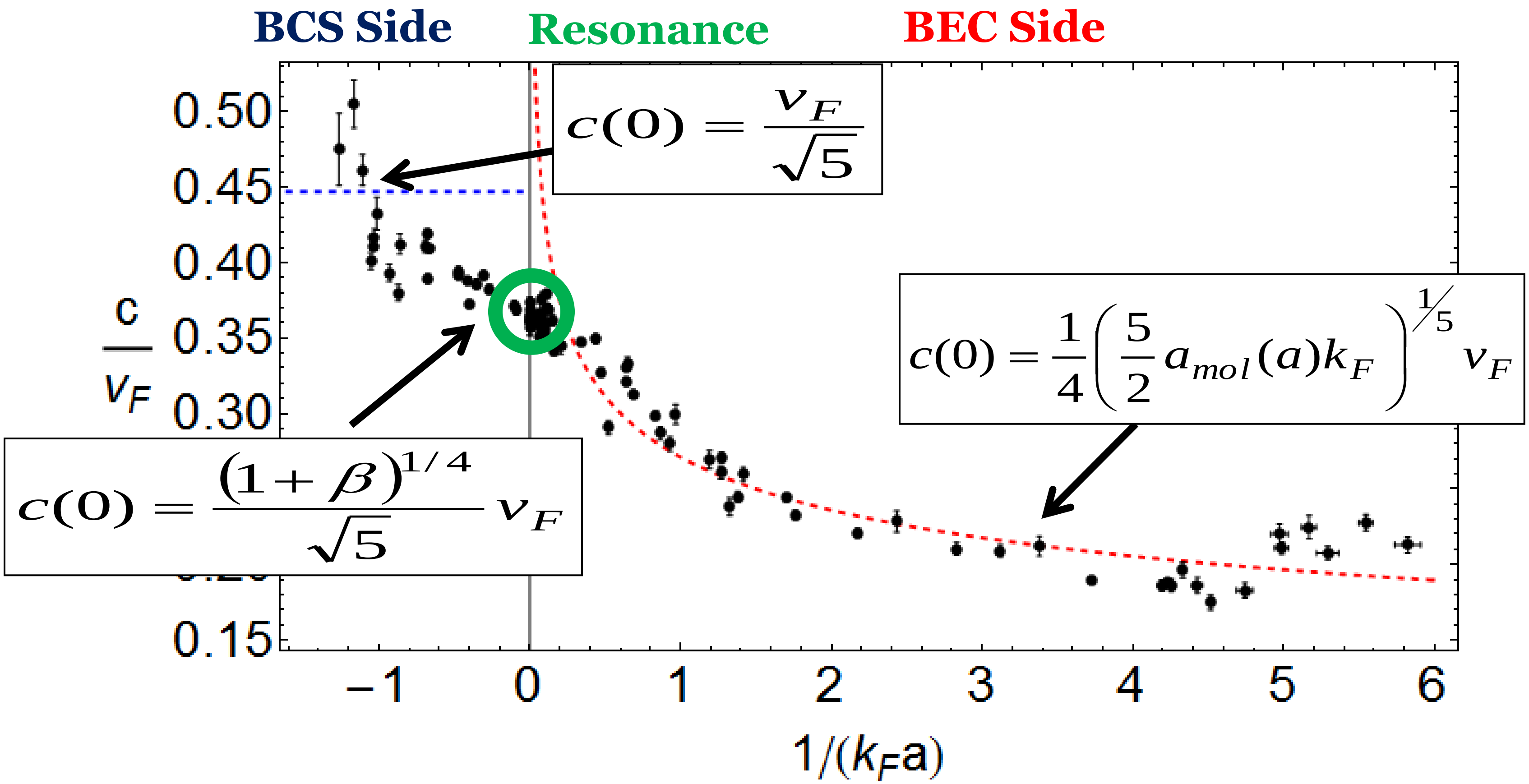
$$c(z) = \left(\frac{1}{m} \frac{\int n \, dxdy}{\int (\partial\mu/\partial n)^{-1} \, dxdy} \right) \Rightarrow c(0) = \sqrt{\frac{\gamma\mu_0}{m(1+\gamma)}}$$



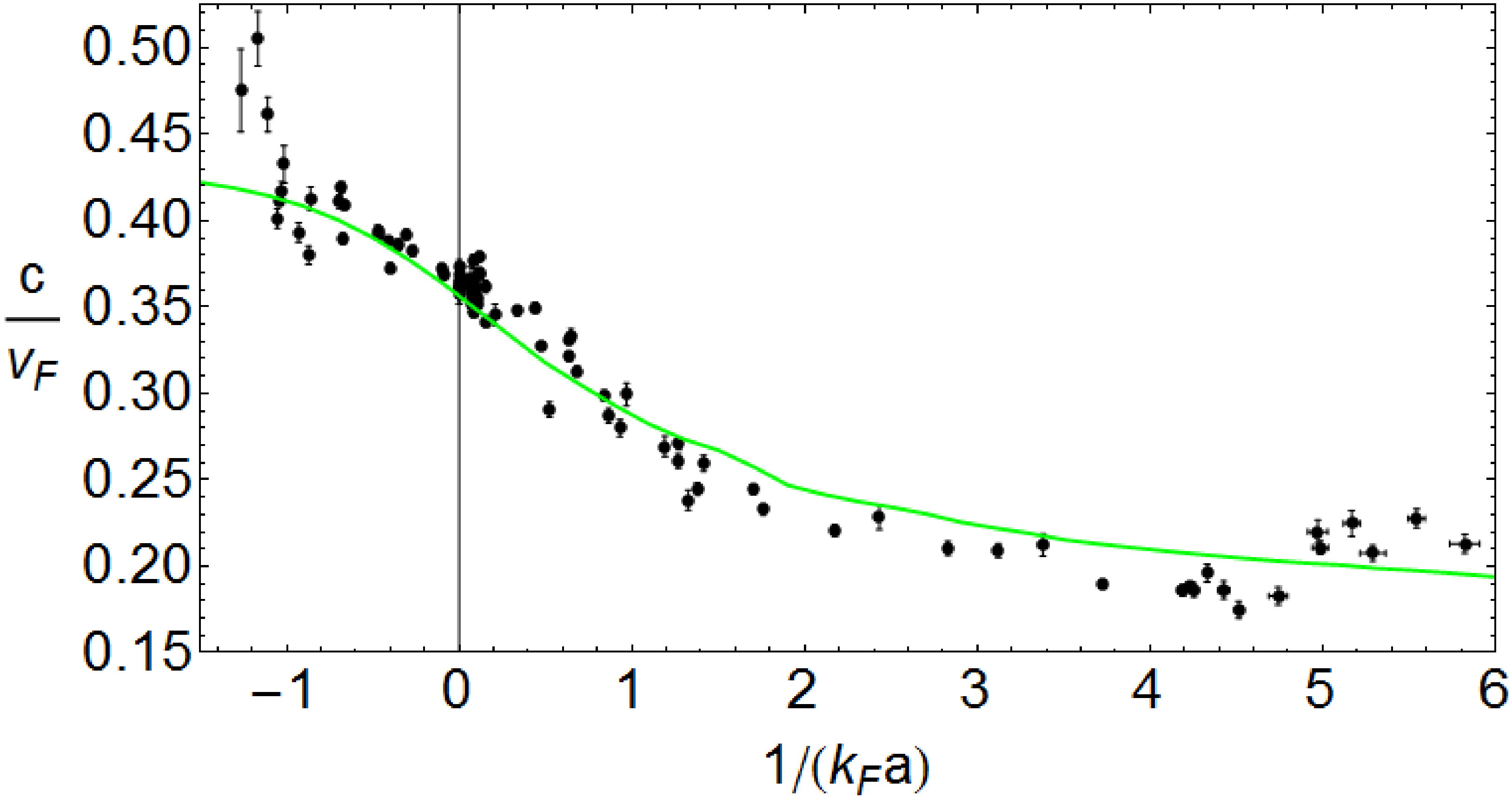
Measuring Speed of Sound



Sound Velocity Data versus Zero Temp theory in the limits a → ±0 ,∞



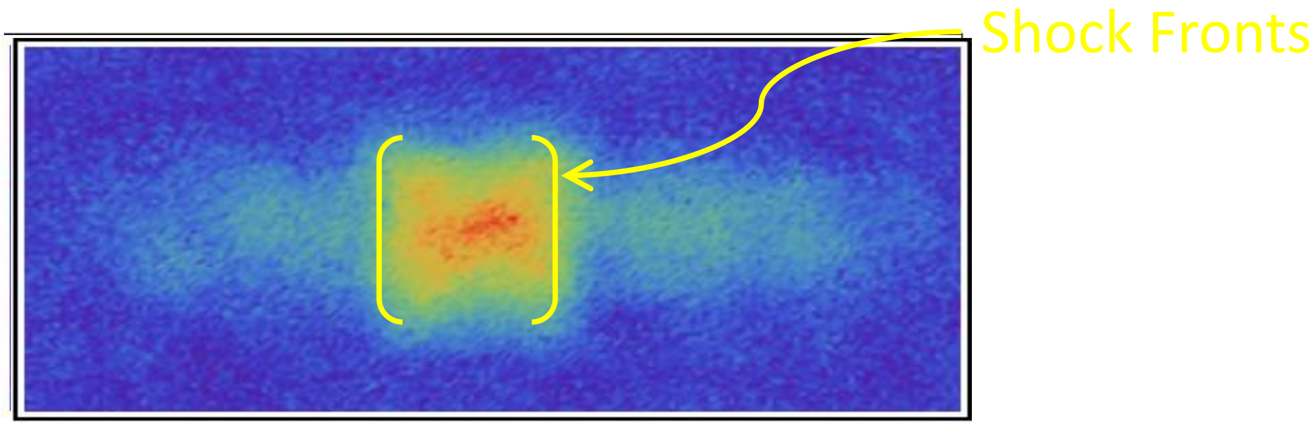
Non-Perturbative Quantum Monte Carlo Calculation



*Quantum Monte Carlo curve supplied by G. Astrakharchik (private communication)

Colliding Clouds Experiment

- Trapped gas is divided into two clouds with a repulsive optical potential.



- The repulsive potential is extinguished, the two clouds accelerate towards each other and collide

Dimensionally Reduced Hydrodynamics

1D density and chemical potential:

$$\mu_{3D} = \mu_0 - V(\mathbf{x}) \propto n_{3D}^{2/3} \quad \Rightarrow \quad n_{3D} \propto [\mu_0 - \tfrac{1}{2} m\overline{\omega}^2 r^2]^{3/2}$$

$$n_{1D}(z) = \iint dxdy\, n_{3D}(x,y,z) \propto [\mu_0 - \frac{1}{2} m\omega_z^2 z^2]^{5/2} \Rightarrow \mu_{1D} \propto n_{1D}{}^{2/5}$$

1D hydrodynamic equations:

$$\partial_t n = -\partial_z (nv)$$

$$\partial_t v = -\partial_z \left(\frac{v^2}{2} + C_{1D} n^{2/5} + \frac{1}{2} \omega_z^2 z^2 \right)$$

$$\mu_{1D} = C_{1D} n_{1D}^{2/5}$$

$$C_{1D} \propto \hbar\omega_\perp l_\perp{}^{2/5}$$

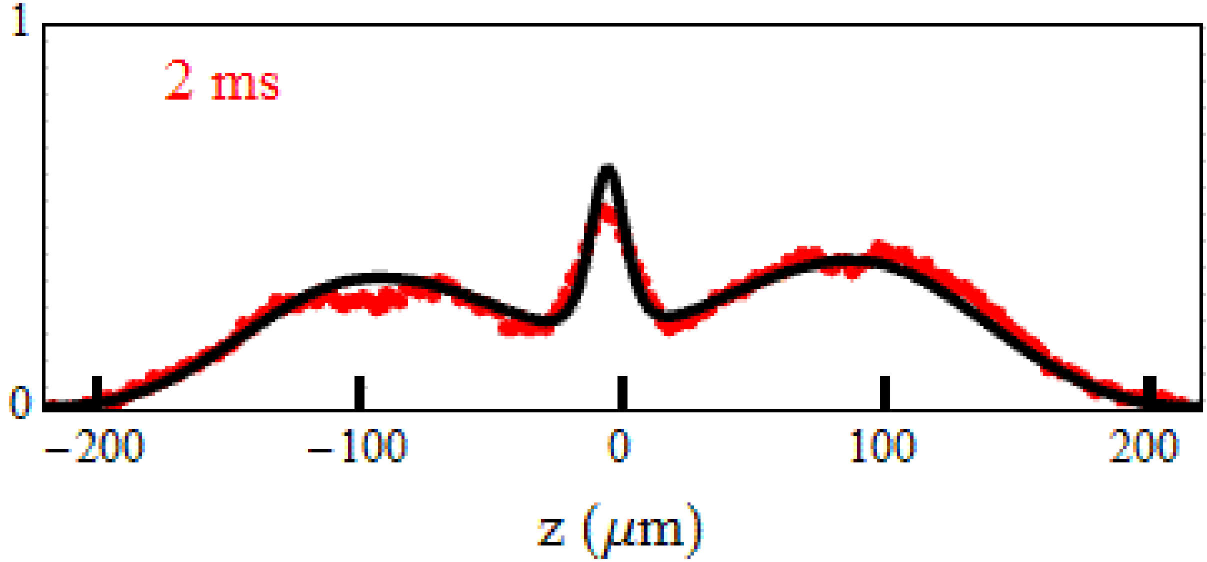
$$l_\perp = \sqrt{\frac{\hbar}{m\omega_\perp}}$$

We numerically simulate the shock wave experiment in 1D with a phenomenological dissipative term, the kinematic viscocity

$$\upsilon \frac{\partial_z (n \partial_z v_z)}{n}$$

Numerical Simulation

- We perform a 1D numerical simulation and compare to the integrated density profiles.

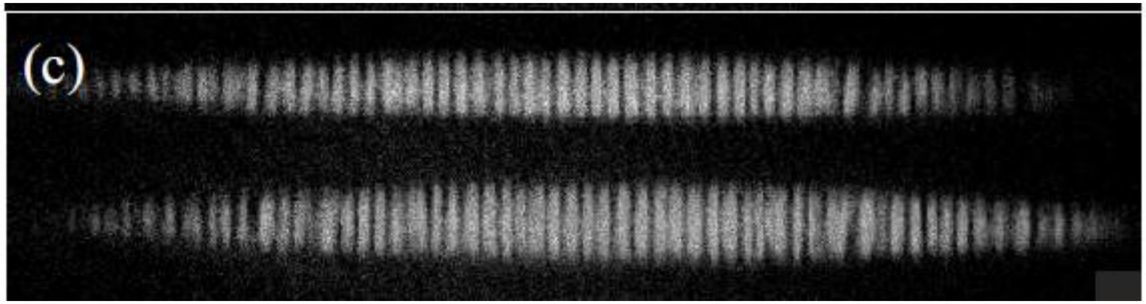


- Data is well fit with a kinematic viscosity as the only fitting parameter: $\upsilon = 10\hbar / m$

Dispersion in a Bose-Einstein condensate

-One order higher dispersive terms such as the quantum pressures appear leading to fringe type patterns in BEC's

$$\nabla\left(\frac{\hbar^2}{2m}\frac{\nabla^2\sqrt{n}}{\sqrt{n}}\right)$$



*S. Middelkamp, J.J. Chang, C. Hamner, R. Carretero-Gonzalez, P.G. Kevrekidis, V. Achilleos, D.J. Frantzeskakis, P. Schmelcher, P. Engels, Dynamics of dark-bright solitons in cigar-shaped Bose-Einstein condensates

3D Model With Dispersive Terms

Extended Thomas-Fermi (ETF) density functional approach:

$$E[n] = \int d^3\mathrm{r}\ \mathcal{E}(n, \nabla n)$$

$$\mathcal{E}(n, \nabla n) = \lambda \frac{\hbar^2}{8m} \frac{(\nabla n)^2}{n} + \xi \frac{3}{5} \frac{\hbar^2}{2m} (3\pi^2)^{5/3} n^{5/3} + U(\mathbf{r})\, n$$

Where λ is a phenomenological parameter scalling a gradient term first introduced by von Weizsäcker (1939) to treat surface effects in nuclei. The hydrodynamic equations under this approach are:

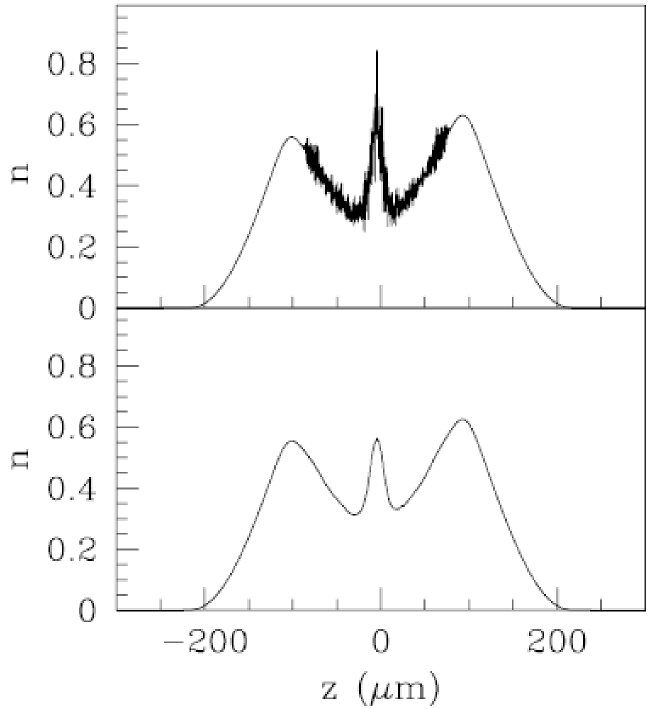
$$\frac{\partial n}{\partial t} + \nabla \cdot (n\mathbf{v}) = 0\ ;$$

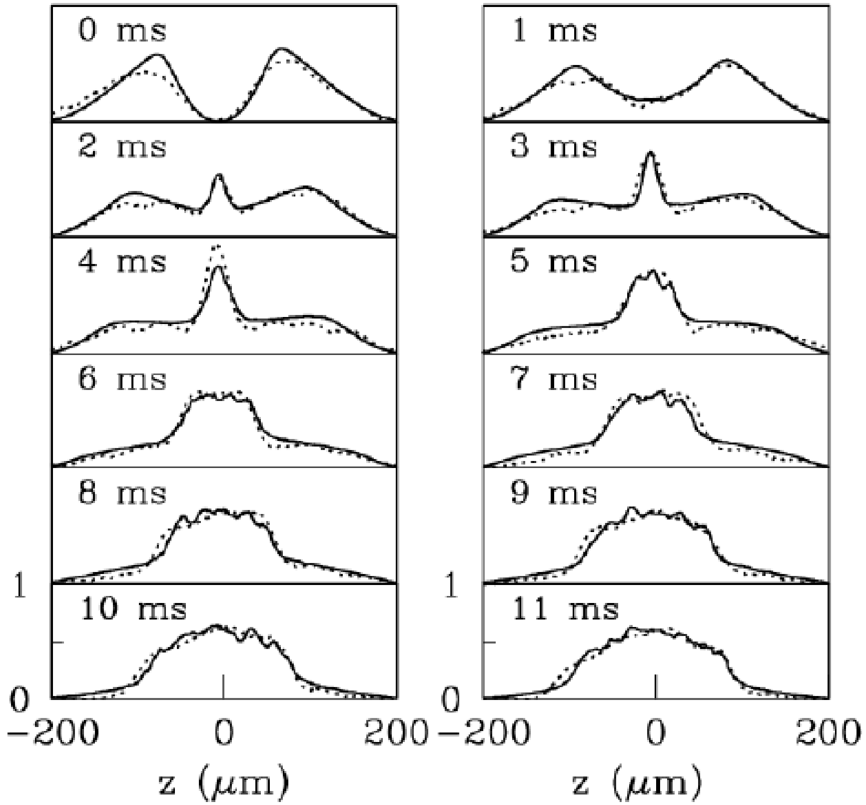
$$m \frac{\partial \mathbf{v}}{\partial t} + \nabla \left[\frac{m}{2} v^2 + \frac{\partial \mathcal{E}}{\partial n} - \nabla \cdot \frac{\partial \mathcal{E}}{\partial (\nabla n)} \right] = 0$$

*L. Salasnich, "Supersonic and subsonic shock waves in the unitary Fermi gas" EPL 96, 40007 (2011).

Simulation Results

Simulation results report fast oscillations with wavelengths smaller than our experimental resolutions as well as good agreement with data for λ=1/4.





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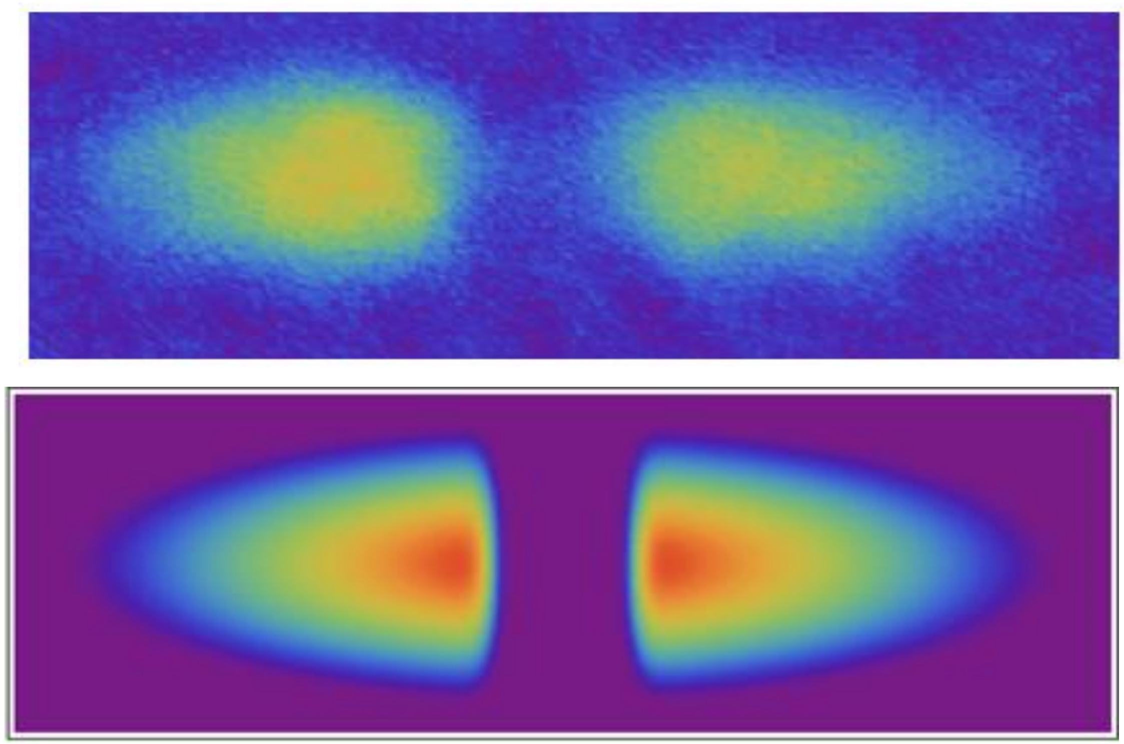
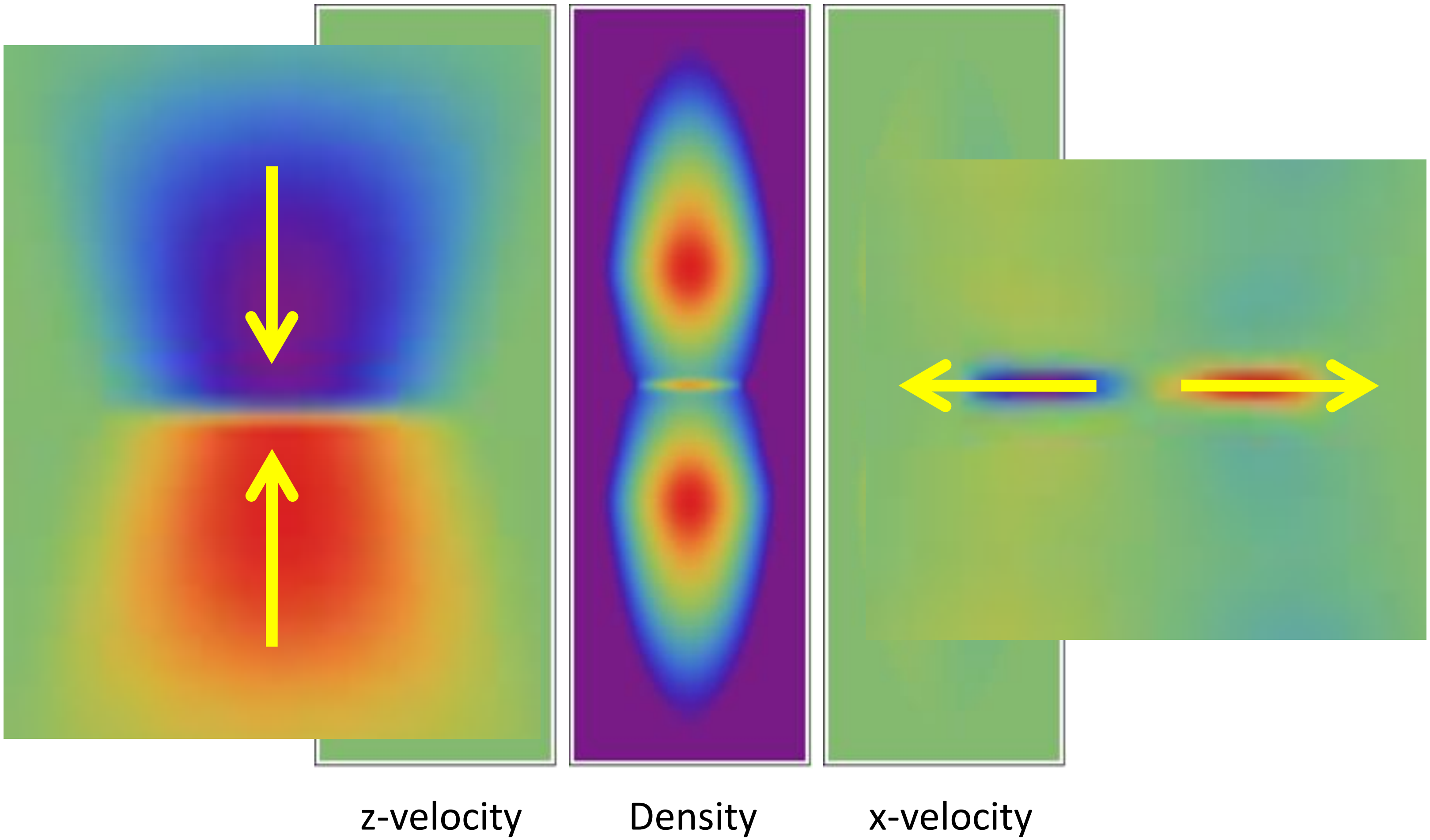
3D Simulation

- We can also use the hydrodynamic equations to numerically simulate the collision in 3-Dimensions



Velocity Fields



Exciting System to Explore Nonlinear Hydrodynamics

- As a function of interaction strength
  - Weakly Interacting BEC's are clearly dispersive
  - Strongly Interacting Fermi gas can be modeled with both dissipative and dispersive contributions.
- Shear and Bulk Viscosity
  - Classically shock waves dynamics are depend on bulk viscosity, but bulk viscosity vanishes for a Fermi gas on resonance.
- As a function of temperature
  - On resonance, our gas stays hydrodynamic over a broad temperature range.
  - What are the effects of local heating?

Thank You