Baryon-Rich State of QCD Matter

~Nuclear Matter and Quark Matter~

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A review is in preparation with C. Sasaki

Main Goals

- Intuitive understanding of the possible structure of the QCD phase diagram, extracting common features of the effective model approaches in the market.

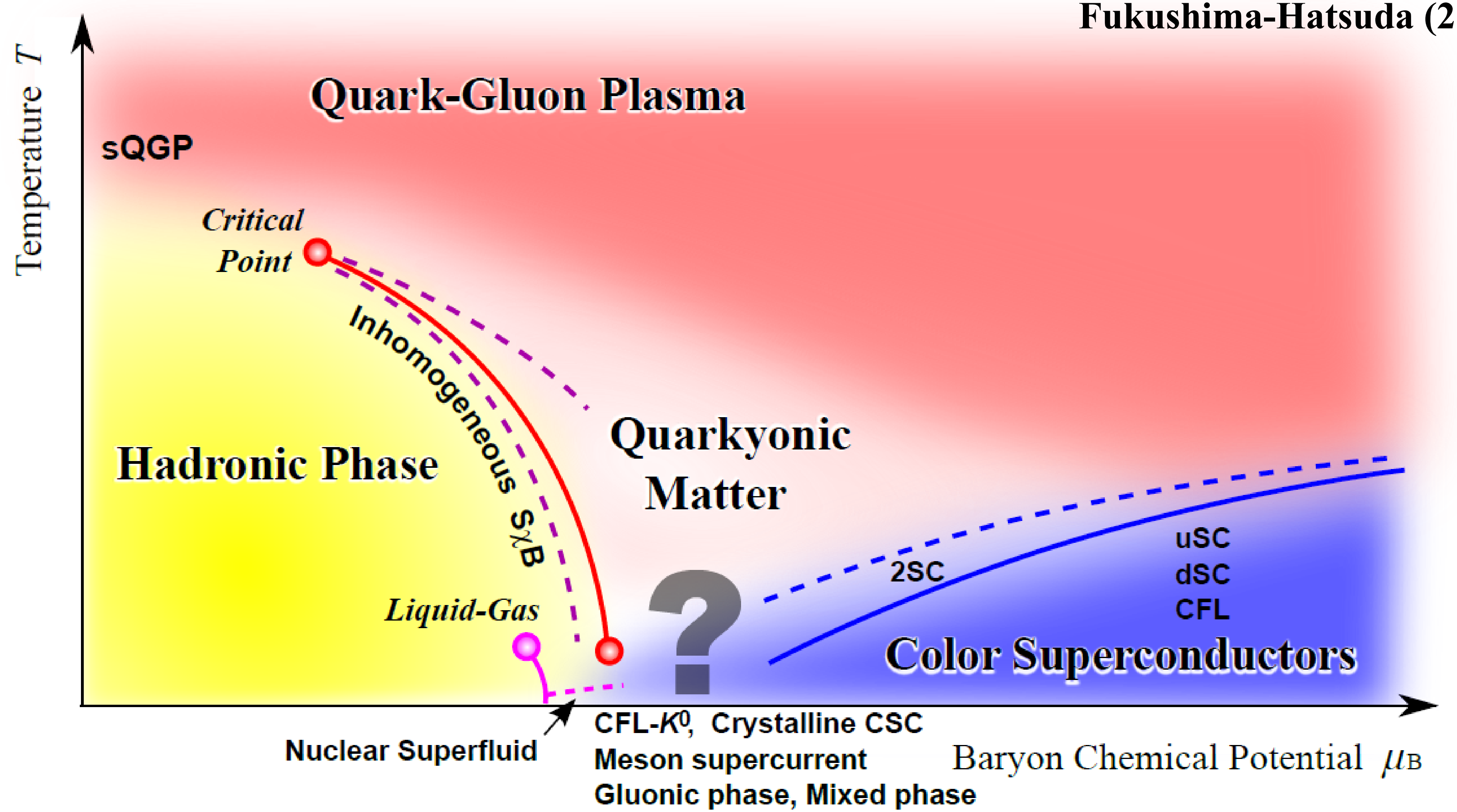
- General mechanisms that favor and hinder a 1st-order phase transition and cause inhomogeneous states.

- How to think about confinement / deconfinement ?
  First step: Construction of the Polyakov-loop potential in a first-principle-like calculation.

Motivation toward the Higher Density

1st-order? CP? Inhomogeneity? Quarkyonic?

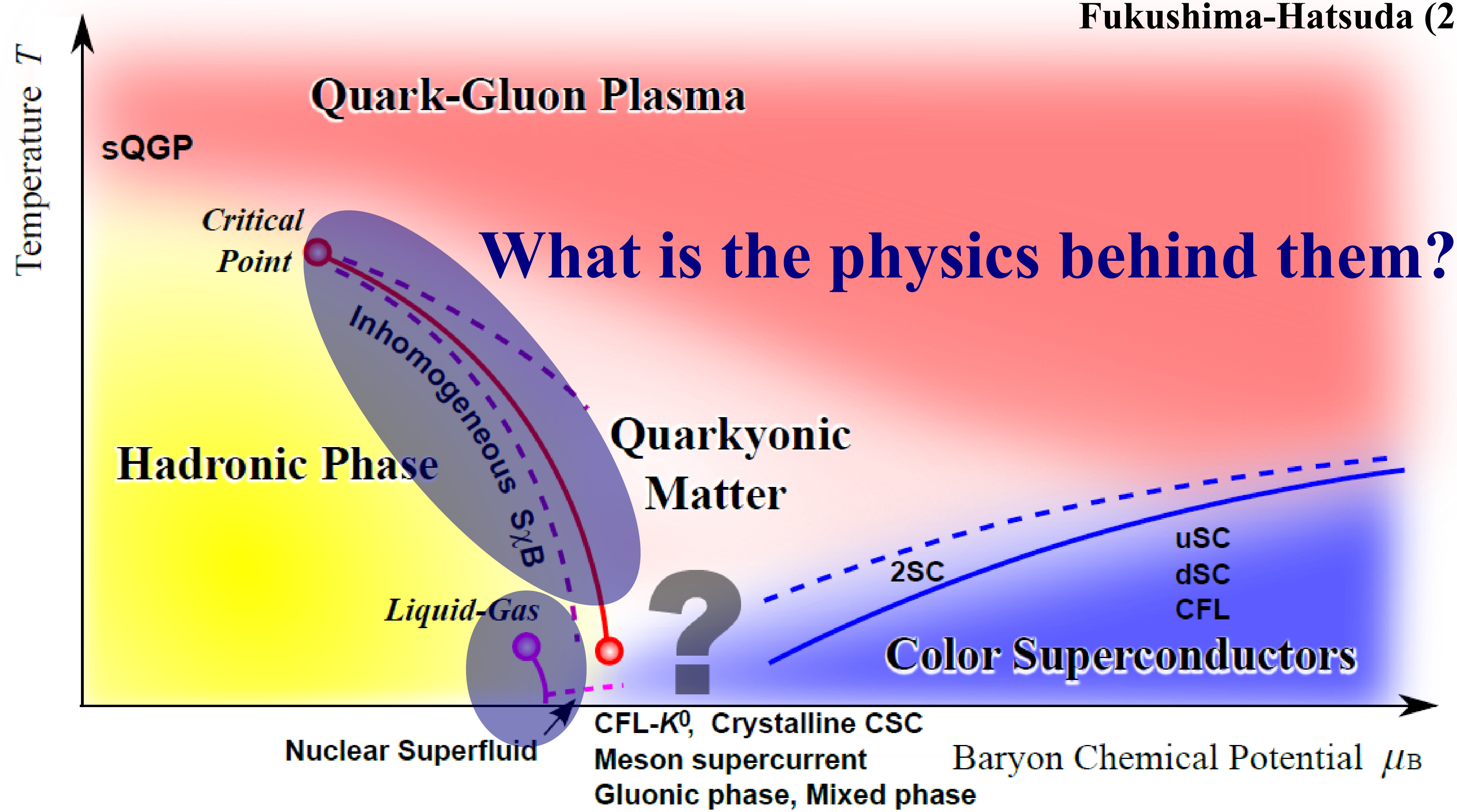
Fukushima-Hatsuda (2010)



1st-order? Inhomogeneity?

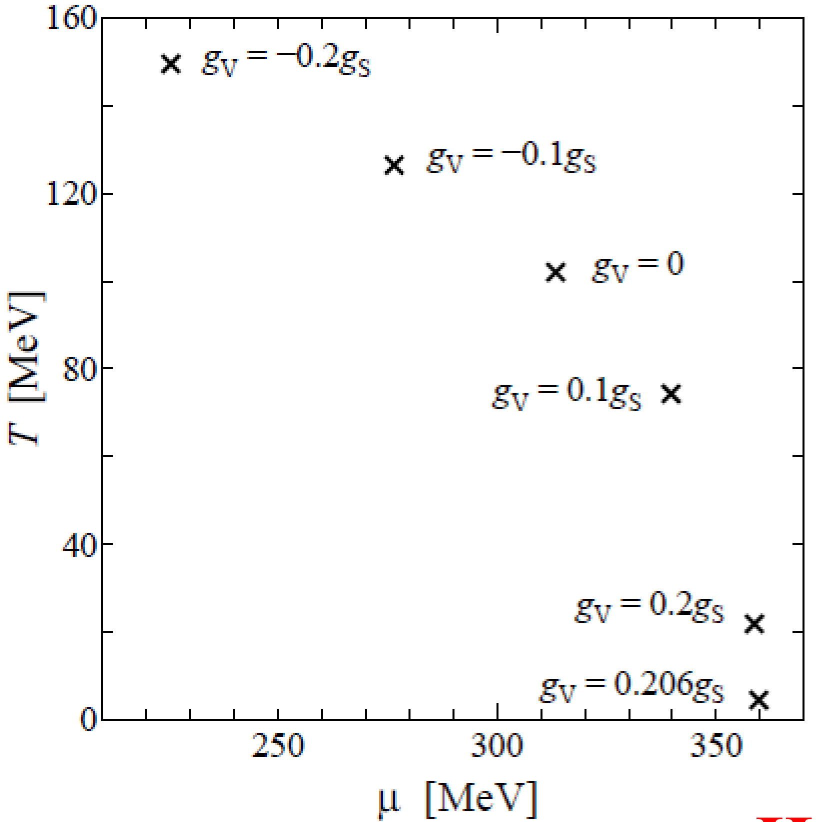
Intuitive explanation ?

Fukushima-Hatsuda (2010)



QCD Critical Point or Not ?

Weak vector interaction washes it out → Unlikely!



Fukushima (2008)

$$L_v = -g_v(\bar{\psi}\gamma_\mu\psi)(\bar{\psi}\gamma^\mu\psi)$$

Why such important?

* Consistent with chiral symmetry
* Non-zero mean-field ρ at finite μ (FRG? D-S?)
* ρ = correct order parameter than σ
* Making the EoS harder (demanded to sustain the two-$M_\odot$ neutron star)

How to understand this intuitively?
Two different explanations

Mechanism of the 1st-order Transition

Free energy vs the dynamical quark mass (T=0)

$$\Omega[M]/V = -\int_0^{\mu} d\mu' \rho(\mu')$$

Matter part favors M=0
(ρ is then the largest)

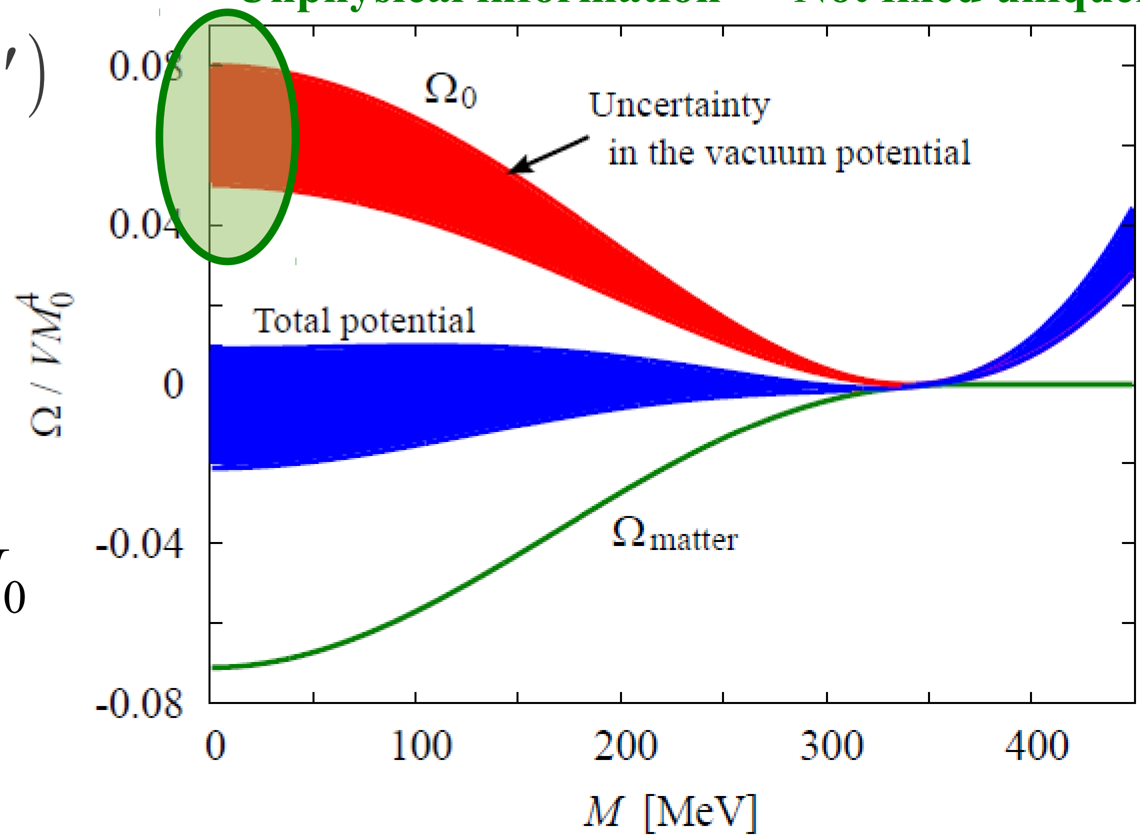
+

Vacuum part favors $M = M_0$

↓

Double-well shape
1st-order phase transition

Unphysical information ← Not fixed uniquely

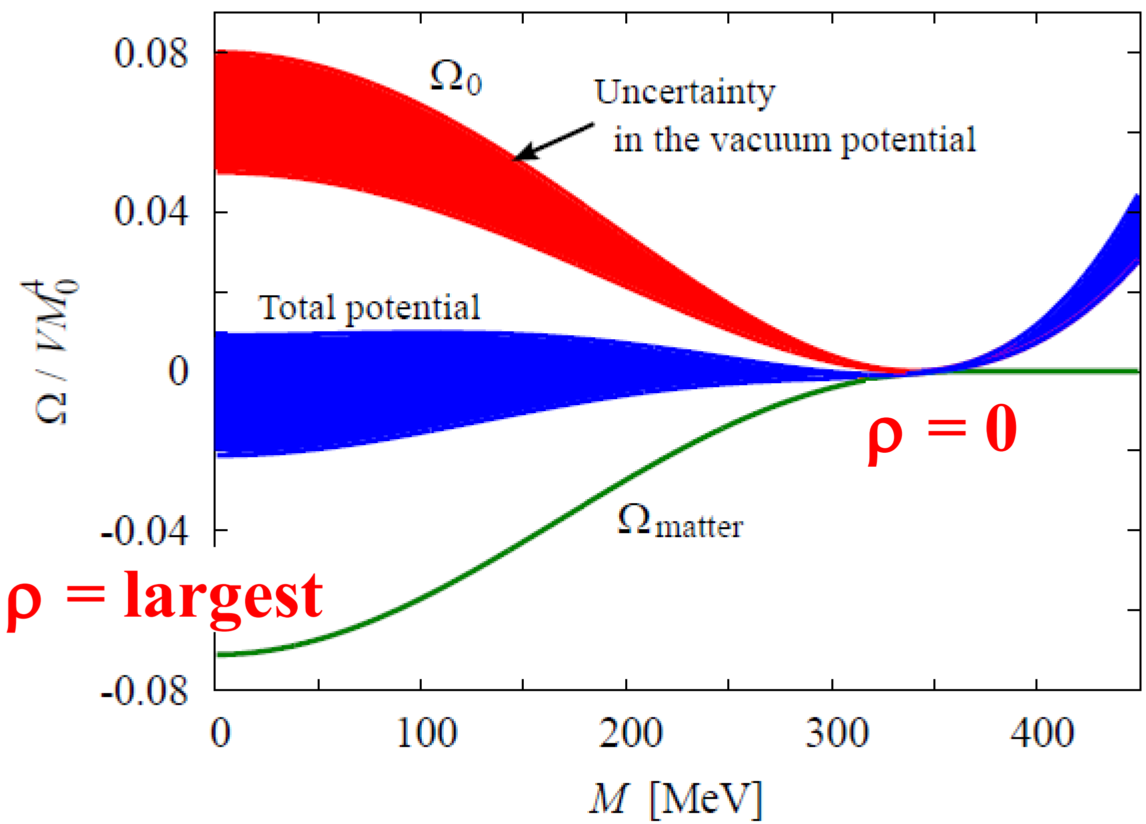


Simple and robust mechanism

Vector Interaction

Vector interaction in the mean-field approx.

$$L_v = -g_v(\bar{\psi}\gamma_\mu\psi)(\bar{\psi}\gamma^\mu\psi) \rightarrow \Delta\Omega = g_v\rho^2$$



Pushed up at $M = 0$

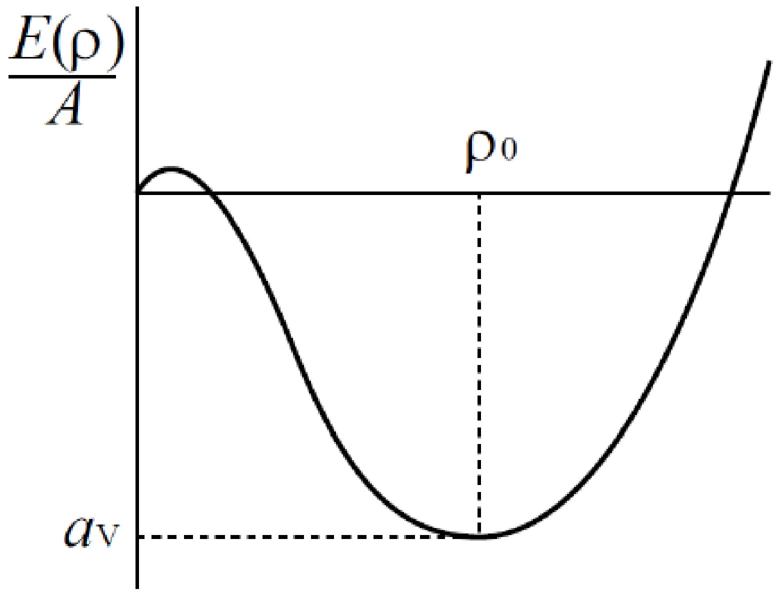
With some $g_v > 0$
the double-well shape gone

No 1st-order transition and
no critical point

Another Picture

Self-bound fermionic system → 1st-order

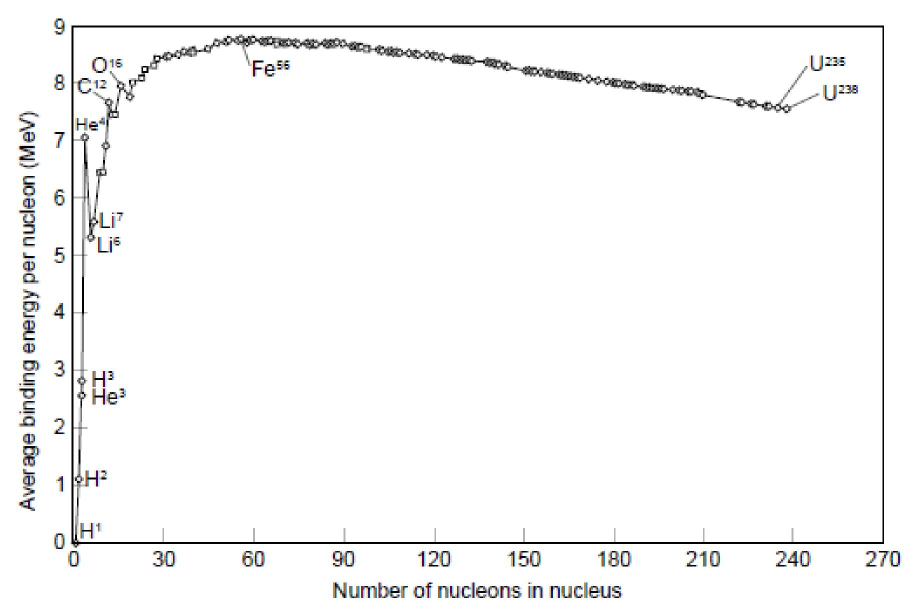
Schematic picture of the (symmetric) nuclear saturation curve



$$\rho_0 = 0.17\,\mathrm{fm}^{-3}$$
$$a_v = 16.3\,\mathrm{MeV}$$

Weizsäcker-Bethe mass formula

$$B = a_v A - a_s A^{2/3} - a_c \frac{Z^2}{A^{1/3}} - a_{\mathrm{sym}} \frac{(N-Z)^2}{A} + \delta_A$$



Simple Mean-field Model

σ−ω model (Walecka model)

This naturally incorporates the vector interaction

$$L=\bar{\psi}[i\gamma_\mu\partial^\mu+\underbrace{(\mu_B-g_\omega\omega^0)}_{\mu_B^*}\gamma_0-\underbrace{(M_N-g_\sigma\sigma)}_{M_N^*}]\psi-\frac{1}{2}m_\sigma^2\sigma^2+\frac{1}{2}m_\omega^2(\omega^0)^2$$

$$\Omega/V=-4\int\frac{d^3p}{(2\pi)^3}\left[T\ln[1+e^{-(\omega-\mu_B^*)/T}]+T\ln[1+e^{-(\omega+\mu_B^*)/T}]\right]$$

$$+\frac{m_\sigma^2(M_N^*-M_N)^2}{2g_\sigma^2}-\frac{m_\omega^2(\mu_B^*-\mu_B)^2}{2g_\omega^2}$$

$$\frac{\partial\Omega}{\partial M_N^*}=\frac{\partial\Omega}{\partial\mu_B^*}=0$$

$$\left.\frac{\varepsilon}{\rho}\right|_{\rho=\rho_0}-M_N=-16.3\,\text{MeV}\quad,\quad\left.\frac{d(\varepsilon/\rho)}{d\rho}\right|_{\rho=\rho_0}=0$$

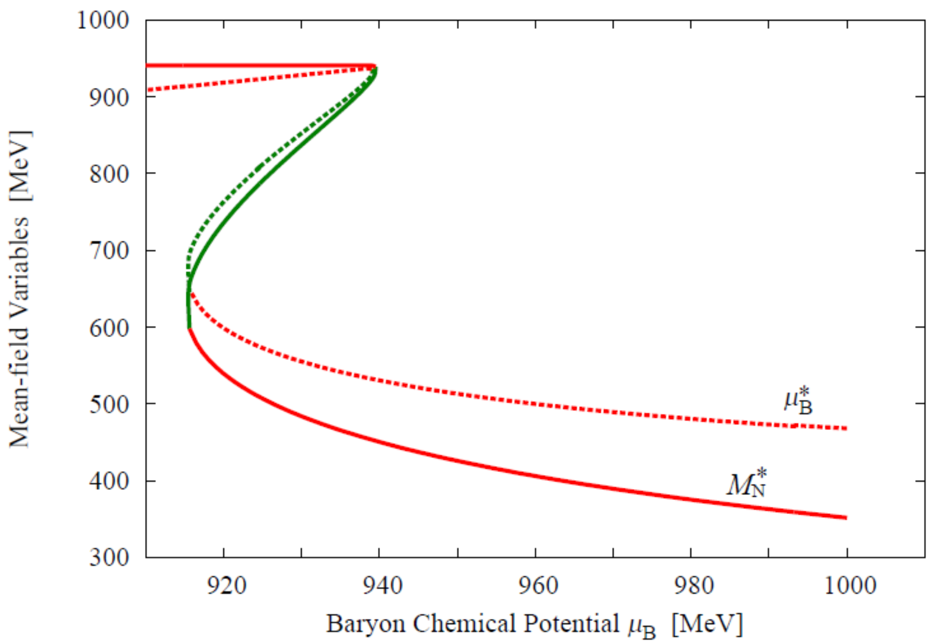
Compressibility needs the potential terms, etc...

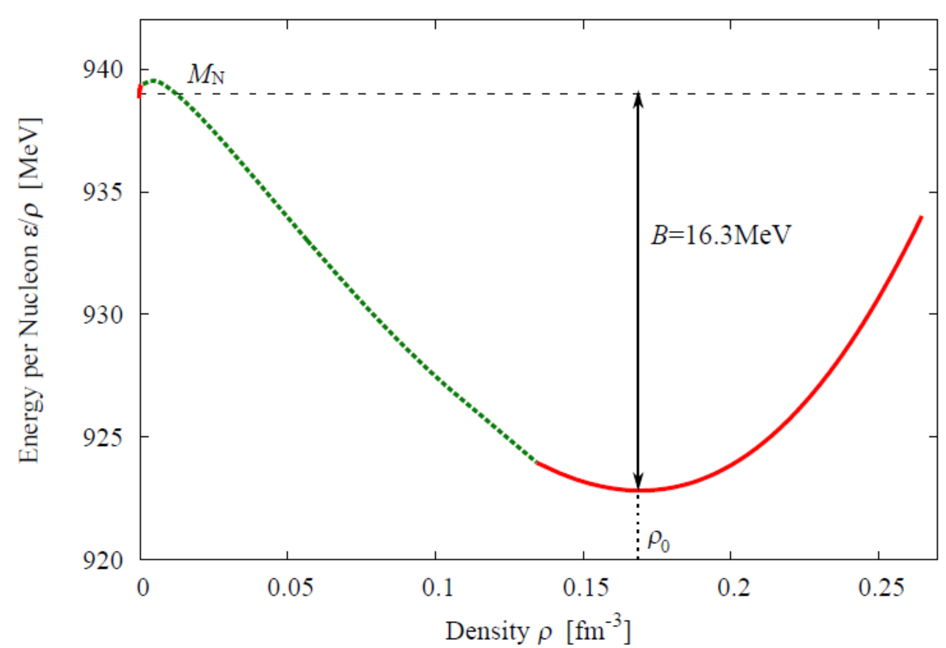
$$M_N=939\,\text{MeV},\quad m_\sigma=550\,\text{MeV},\quad m_\omega=783\,\text{MeV},\quad g_s=10.3,\quad g_\omega=12.7$$

Buballa (1996)

Mean-field Solution

Mean-field variables





At the saturation point

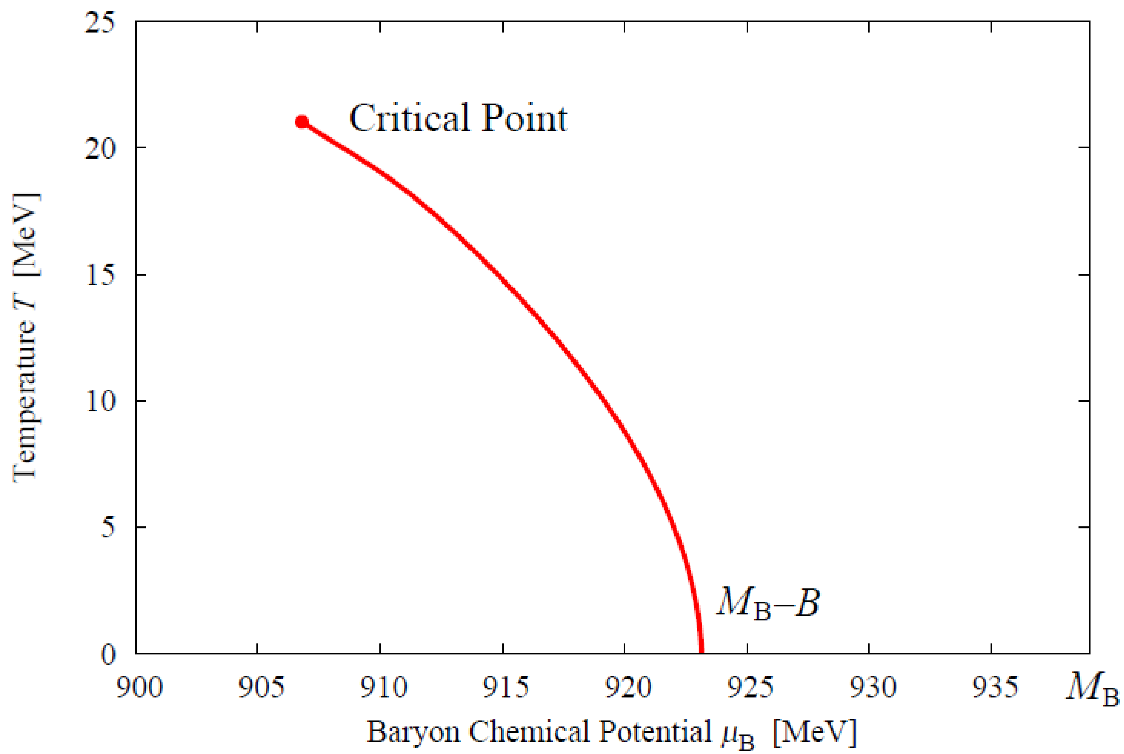
$$\frac{d(\varepsilon/\rho)}{d\rho} = \frac{\mu_B}{\rho} - \frac{\varepsilon}{\rho^2} = \frac{p}{\rho^2} = 0$$

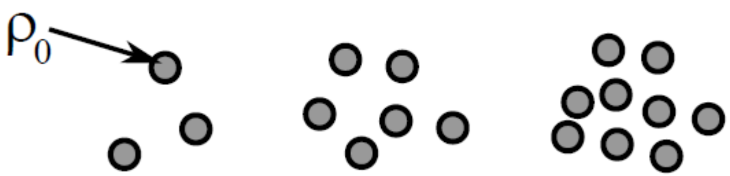
→

1st-order phase transition
Liquid-gas transition
Chiral phase transition

Critical Point of Nuclear Matter

Its existence is undoubtable





How to realize the density

$\rho < \rho_0$

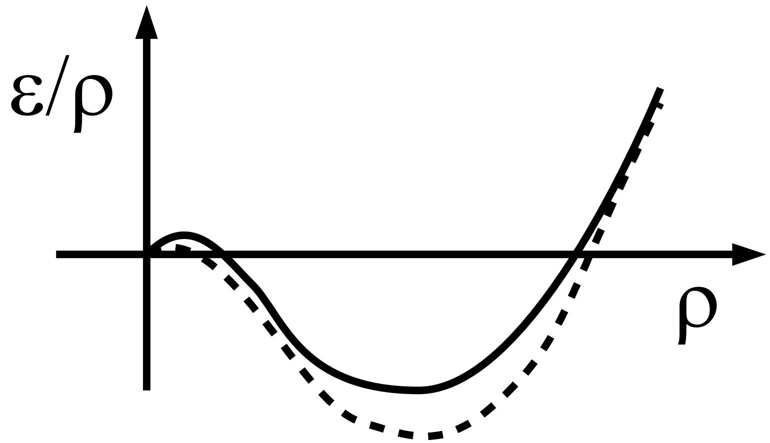
when $\rho = \rho_0$ is the most stable?

This should be realized as a mixed phase (or a non-trivial configuration depending on the surface energy).

Pion (thermal) loops are not important yet at such low temperature.

Quark Matter

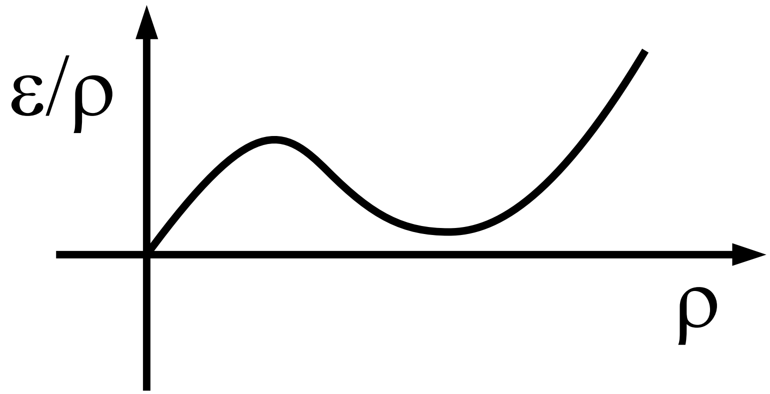
Is it a self-bound system? → Quark droplet?



1st-order phase transition
→ QCD critical point

Any more stable state would exhibit the 1st-order one too.

Even when the quark droplet is only meta-stable

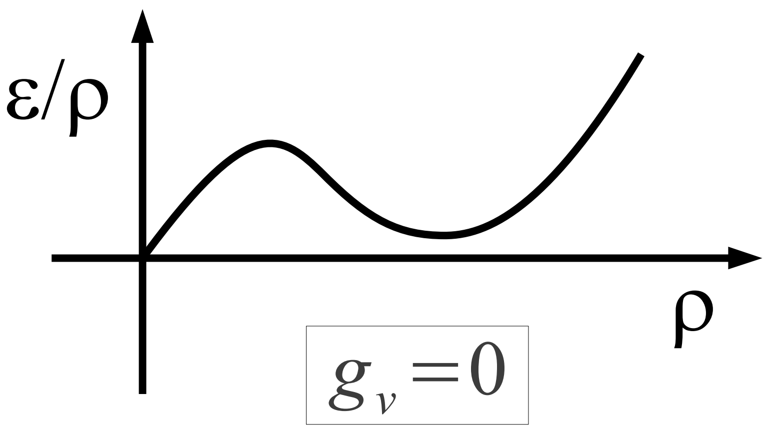


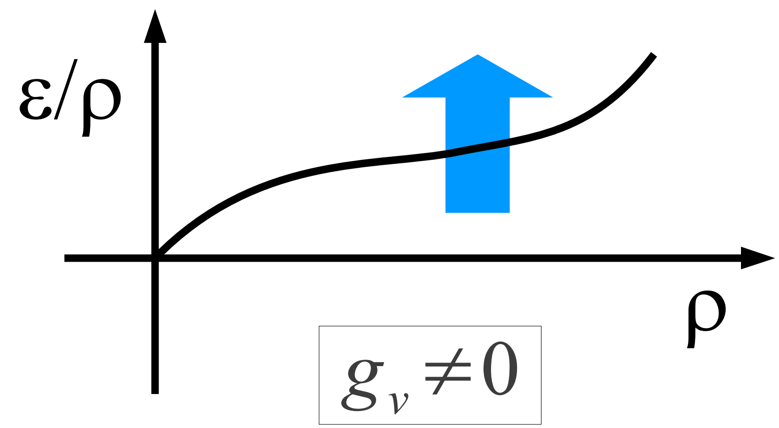
$$\frac{d\,(\varepsilon/\rho)}{d\,\rho} = \frac{\mu_B}{\rho} - \frac{\varepsilon}{\rho^2} = \frac{p}{\rho^2} = 0$$

1st-order phase transition
→ QCD critical point

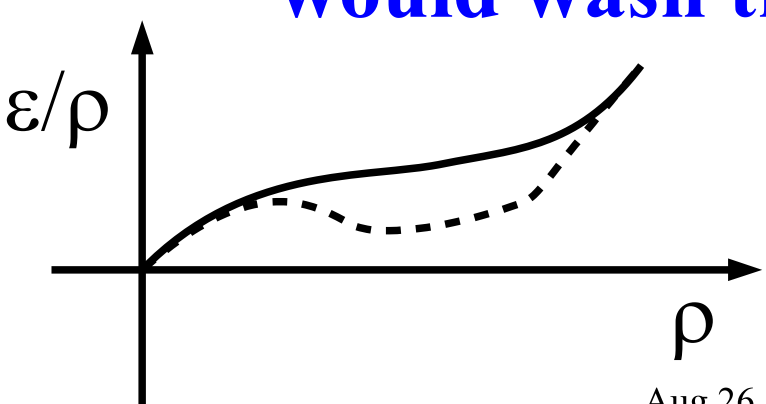
Vector Interaction Again

$$L_v = -g_v(\bar{\psi}\gamma_\mu\psi)(\bar{\psi}\gamma^\mu\psi) \;\rightarrow\; \Delta\Omega = g_v\rho^2$$





It is obvious at a glance that the vector interaction would wash the 1st-order transition out.



Is there any chance to find another branch of solution?

Inhomogeneity: Simplest Case

Chiral spiral in one direction

Deryagin-Grigoriev-Rubakov (1992)

$$\psi(x) = e^{i\gamma_5 \tau_3 q z} \psi'(x) \quad \text{with} \quad \chi = \langle \bar{\psi}' \psi' \rangle$$

$$\Rightarrow \quad \langle \bar{\psi} \psi \rangle = \chi \cos(2qz)$$
$$\langle \bar{\psi} \gamma_5 \tau_3 \psi \rangle = \chi \sin(2qz)$$

Buballa-Carignano
Soliton
2D

Quasi-particle dispersion relation

$$\omega = \sqrt{p_\perp^2 + \left(\sqrt{p_z^2 + M^2} \pm q\right)^2}$$

The system can develop a density
however large *M* is if *q* ~ *M* is chosen!

c.f. (1+1)-dimensional System

Dirac Lagrangian in (1+1) dimensions

$$L=\bar{\psi}\left[i(\partial_4-\mu)\gamma_4+i\partial_z\gamma_z\right]\psi$$
$$=\bar{\psi}'\left[i\partial_4\gamma_4+i\partial_z\gamma_z\right]\psi'$$

$$\psi=e^{-i\mu\gamma_5 z}\psi'$$

Thermodynamic potential

$$\Omega/V=-\int_{-\Lambda+\mu}^{\Lambda-\mu}\frac{dp}{2\pi}\frac{|\varepsilon(p)|}{2}-\int_{-\Lambda-\mu}^{\Lambda+\mu}\frac{dp}{2\pi}\frac{|\varepsilon(p)|}{2}+\cdots$$
$$=\Omega(\mu=0)/V-\frac{\mu^2}{2\pi}$$

Surface integral: Anomaly origin
No suppression by M

Push down the energy as compared to the homogeneous case:

$$\Omega(\mu=0)/V+\left[-\frac{p_F\mu}{2\pi}+\frac{M^2}{2\pi}\ln\left|\frac{p_F+\mu}{M}\right|\right]\underline{\theta(\mu-M)}\quad\Rightarrow\quad n=\frac{p_F}{\pi}\theta(\mu-M)$$

Competing Terms

- **From the matter (density effect)** $\omega=\sqrt{p_\perp^2+(\sqrt{p_z^2+M^2}\pm q)^2}$

$$-\int_0^\mu d\mu'\rho(\mu')-4N_cN_fT\int\frac{d^3p}{(2\pi)^3}\ln\left(1+e^{-\omega/T}\right) \Rightarrow \quad q\sim M\to\infty$$

- **From the vacuum (chiral symmetry breaking)**

$$a\left(M_0^2-M^2\right)-bM \qquad (b\sim\text{bare quark mass}) \Rightarrow \quad M\sim M_0$$

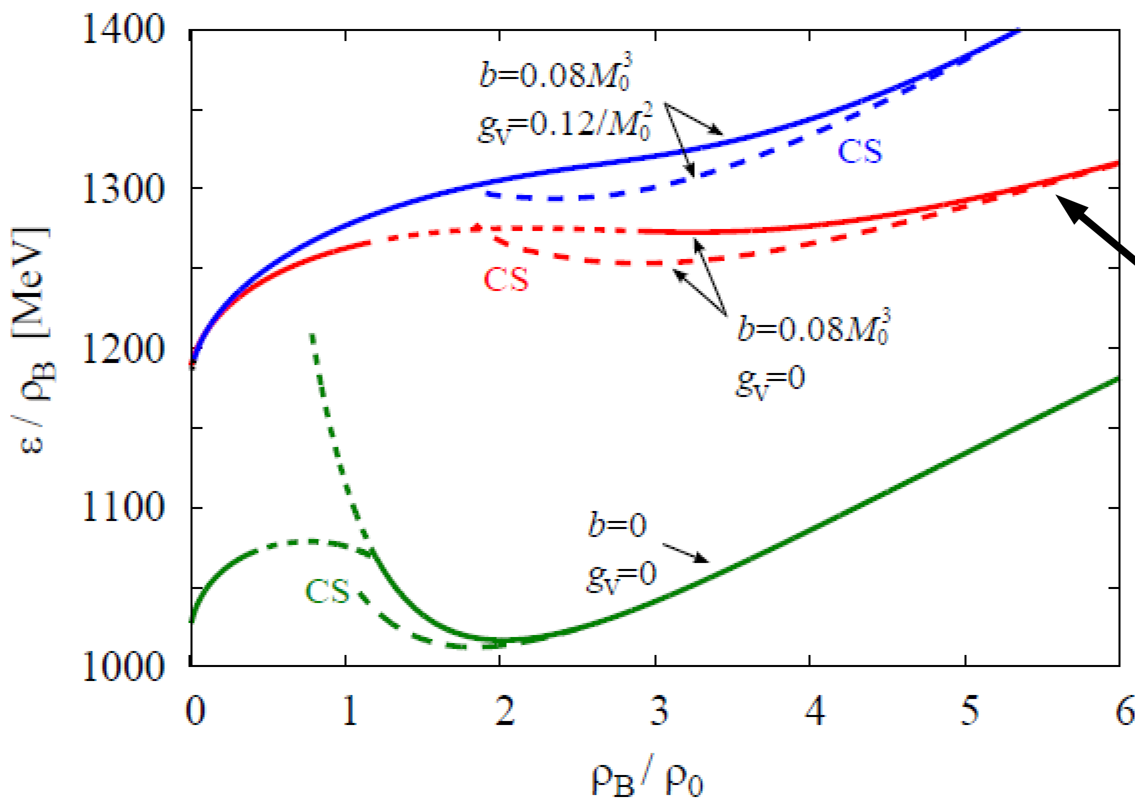
- **From the vacuum (kinetic term)**

$$(\alpha M^2+\beta b)q^2 \Rightarrow \quad q\sim 0$$

- **From the interaction (vector-type)**

$$g_v\rho^2 \Rightarrow \quad M\neq 0$$

Saturation Curves



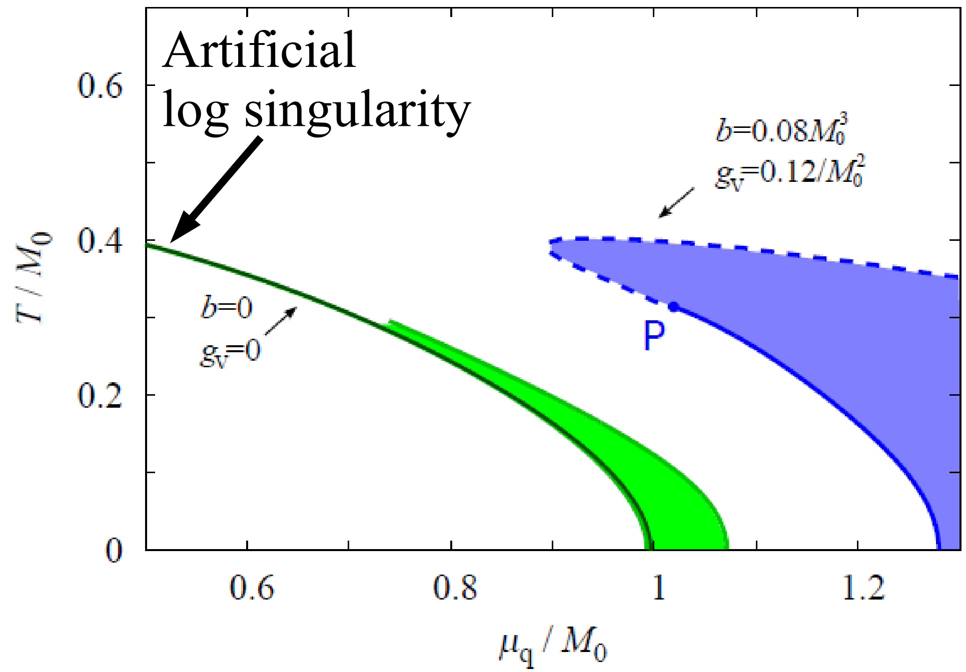
$a = 0.05$ (~LSM)
$M_0 = 340$ MeV
$\alpha = 0.25$
$\beta = 0.25 / M_0$

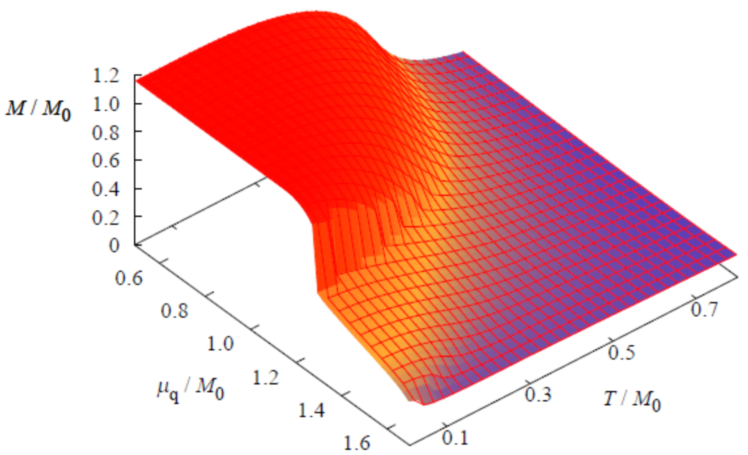
~ Conventional QCD phase diagram

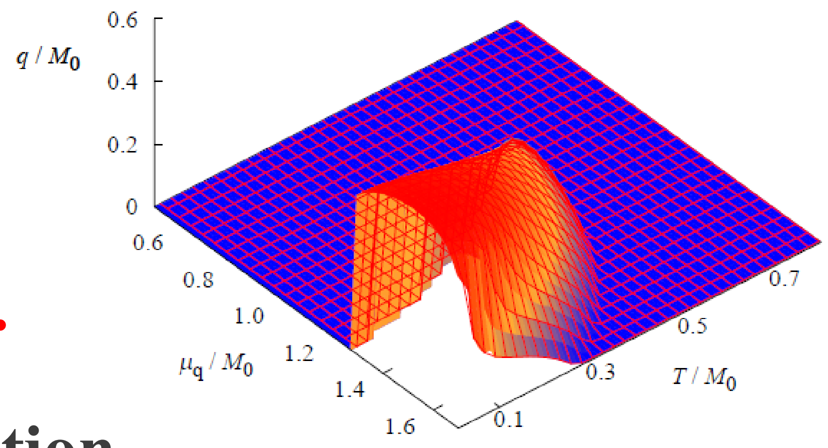
Fukushima (2012)

It is natural (but not necessary) that the 1st-order transition with a smaller energy occurs at smaller density.
Less affected by the vector interaction then.

Phase Diagram







Inhomogeneity survives even with g_v that washes the CP out.

Note that the order of the transition and P are not a robust conclusion...

$q \sim M$

Artificial Log Singularity

Fermionic fluctuations at finite *T*

$$\int^{\Lambda} \frac{d^3 p}{(2\pi)^3}\,\omega = \frac{\Lambda^4}{16\pi^2}\left[\sqrt{1+\xi^2}\,(2+\xi^2) - \frac{\xi^4}{2}\ln\left|\frac{\sqrt{1+\xi^2}-1}{\sqrt{1+\xi^2}+1}\right|\right] \qquad \xi = \frac{M}{\Lambda}$$

This log singularity is exactly canceled by finite-*T* contribution

Skokov-Friman-Nakano-Redlich-Schaefer (2010)

Magnetic Catalysis at finite *T*

$$\int^{\Lambda} \frac{d^3 p}{(2\pi)^3}\,\omega = \frac{|eB|}{2\pi}\cdot\frac{\Lambda^2}{2}\left[1 + \left(\ln\frac{2}{\xi} + \frac{1}{2}\right)\xi^2\right]$$

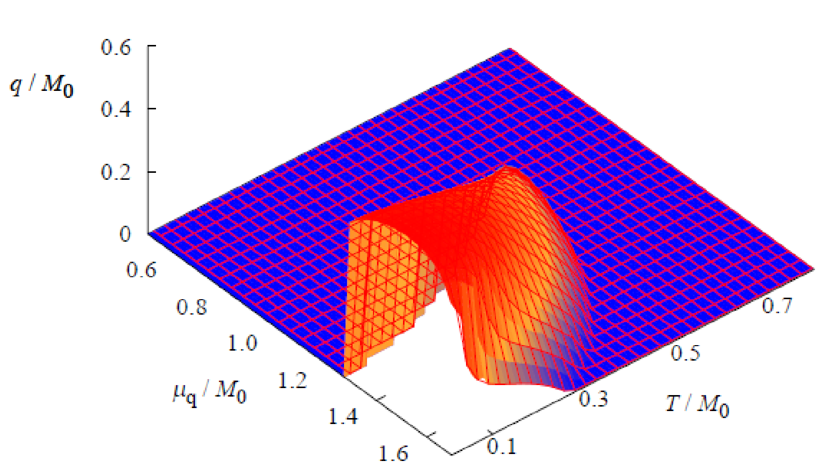
Klimenko
Gusynin-Miransky-Shovkovy

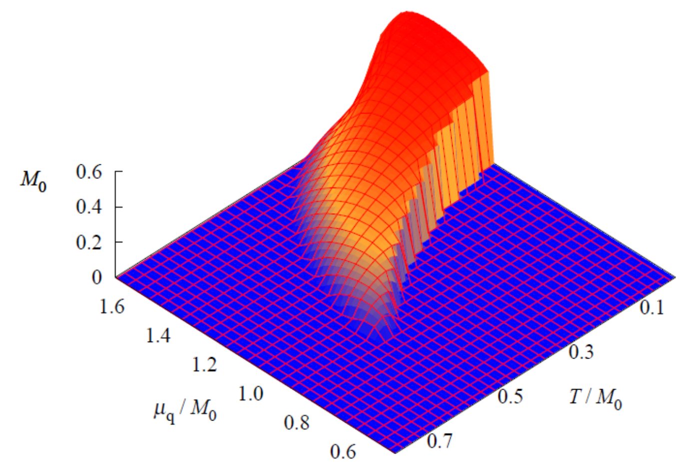
Chiral symmetry is always spontaneously broken in *B*

This log singularity is exactly canceled by finite-*T* contribution because its origin is the IR singularity

Fukushima-Pawlowski (2012)

Structure of the Island





In the small density side (cliff):

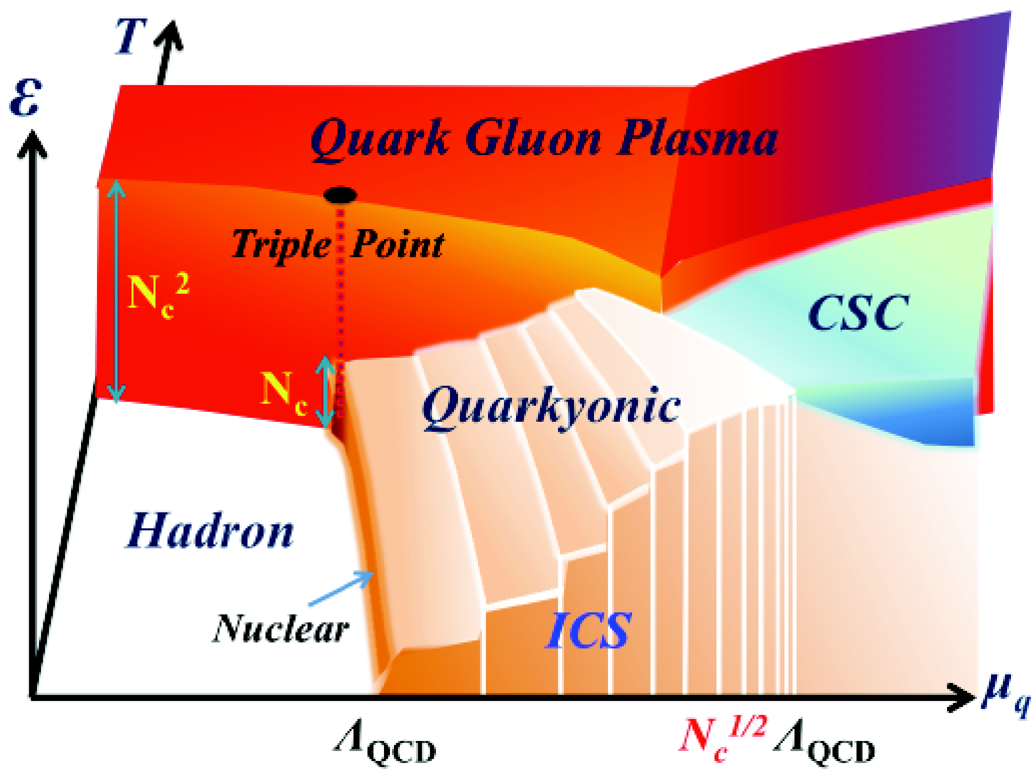
Big energy gain (or jump in the wave-number) because M is still large.

In the large density side (beach):

Small energy gain and smoothly approaches an inhomogeneous chiral-symmetric state.

Patch Problem and Successive Phase Transitions

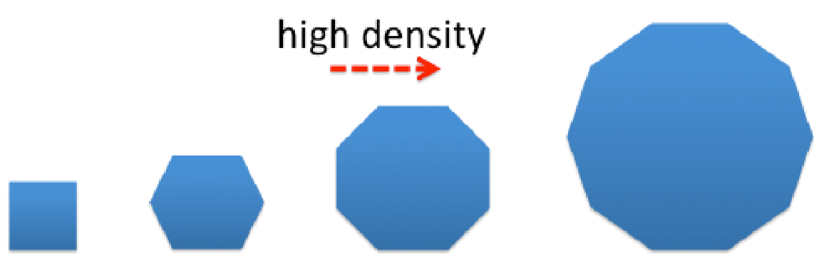
Kojo-Hidaka-Fukushima-McLerran-Pisarski (2011)

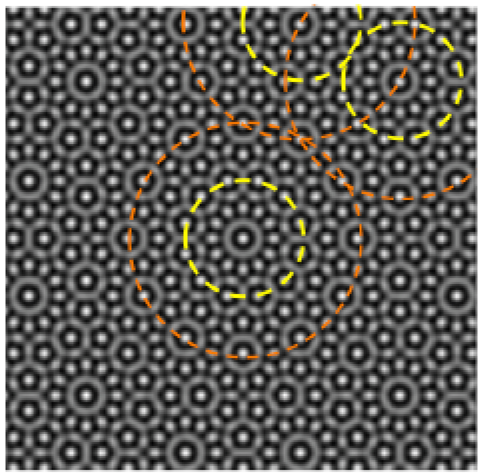


1-D modulation = 1 patch

How to cover the Fermi surface by patches ?

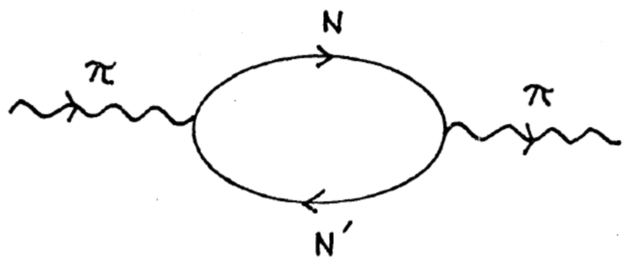
Quasi-crystal ?





cf. p-wave Pion Condensation

$$\Pi(\omega, k) \rightarrow D^{-1}(\omega=0, k=k_c)=0 \text{ at } \rho=\rho_c$$



Landau-Migdal (short-range) interaction

$$f + g\,\boldsymbol{\sigma}_1\cdot\boldsymbol{\sigma}_2 + f'\,\boldsymbol{\tau}_1\cdot\boldsymbol{\tau}_2 + g'(\boldsymbol{\sigma}_1\cdot\boldsymbol{\sigma}_2)(\boldsymbol{\tau}_1\cdot\boldsymbol{\tau}_2)$$

Should include Δ for quantitative calculations

OPEP

$$V = \frac{m_\pi^2}{3}\frac{g^2}{4\pi}\boldsymbol{\tau}_1\cdot\boldsymbol{\tau}_2\left[\boldsymbol{\sigma}_1\cdot\boldsymbol{\sigma}_2\frac{e^{-m_\pi r}}{r} + S_{12}\left(1+\frac{3}{m_\pi r}+\frac{3}{(m_\pi r)^2}\right)\frac{e^{-m_\pi r}}{r}\right]$$

$$-\frac{g^2}{3}(\boldsymbol{\sigma}_1\cdot\boldsymbol{\sigma}_2)(\boldsymbol{\tau}_1\cdot\boldsymbol{\tau}_2)$$

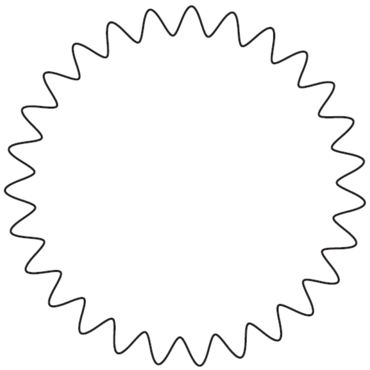
Large g' kills the pion cond.
→ Gamow-Teller resonance
Majority thinks negative,
but some people still believe.

Similar analysis desirable
in the RPA level in quark matter

Confinement / Deconfinement

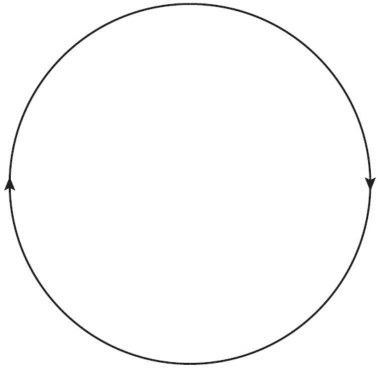
Polyakov loop potential

$\Omega[\Phi]$: pure YM thermodynamics



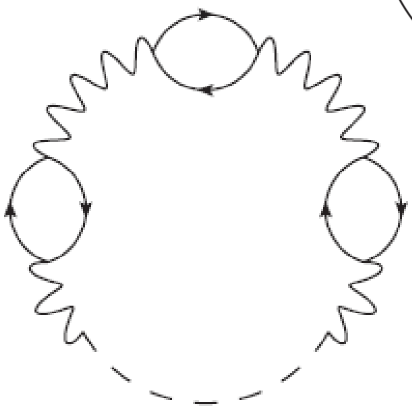
Coupling to the Polyakov loop

$$\mathrm{tr}\ln\left(1+L\,e^{-(E-\mu)/T}\right)+\mathrm{tr}\ln\left(1+L^{\dagger}e^{-(E+\mu)/T}\right)$$



Back-reaction

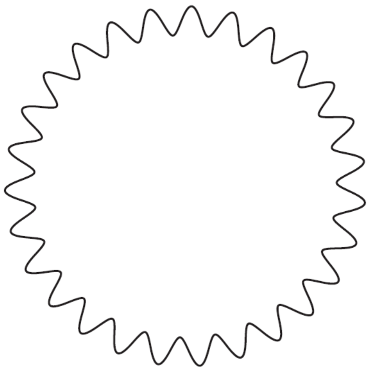
Important at finite μ or *B*
No systematic approach yet



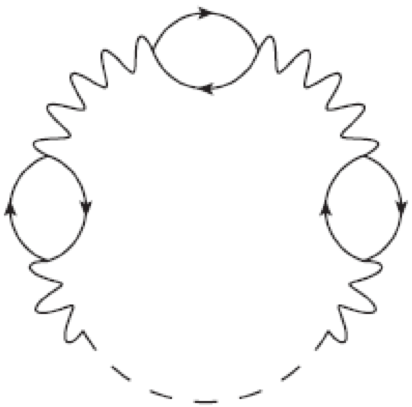
Confinement / Deconfinement

Polyakov loop potential

$\Omega[\Phi]$: pure YM thermodynamics



Back-reaction



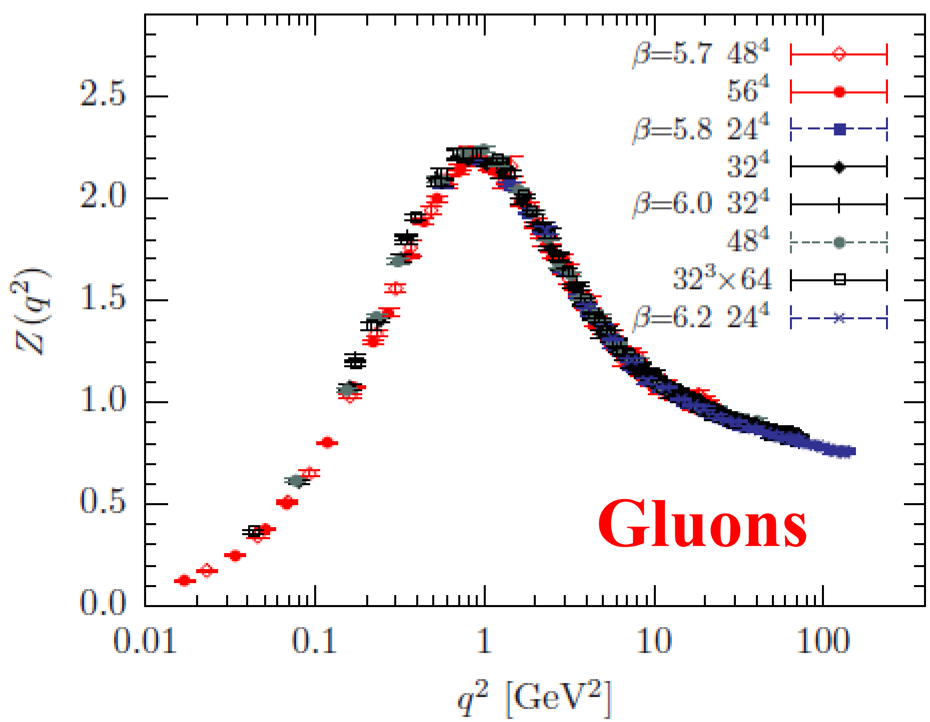
One way to deal with this is to take the large-*Nc* limit where the reaction drops out $\longrightarrow$ Quarkyonic Matter

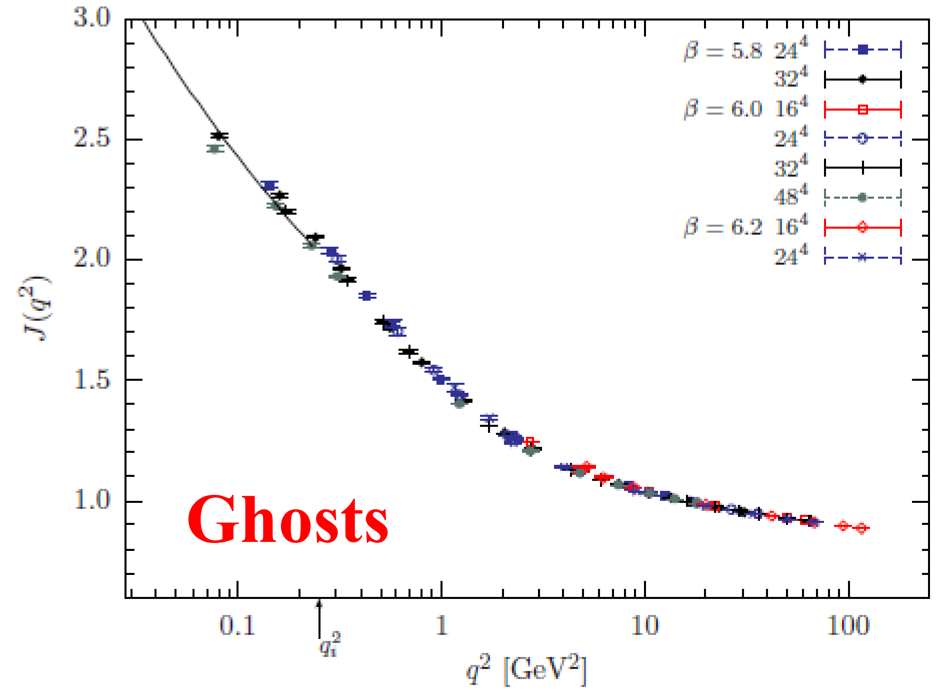
Need to establish how to get the Polyakov loop potential based on a *first-principle-like* calculation and, at the same time, in a *reasonably tractable* way.

Deconfinement from Confinement

Confinement understood from the non-perturbative propagators of gluons and ghosts in the Landau gauge

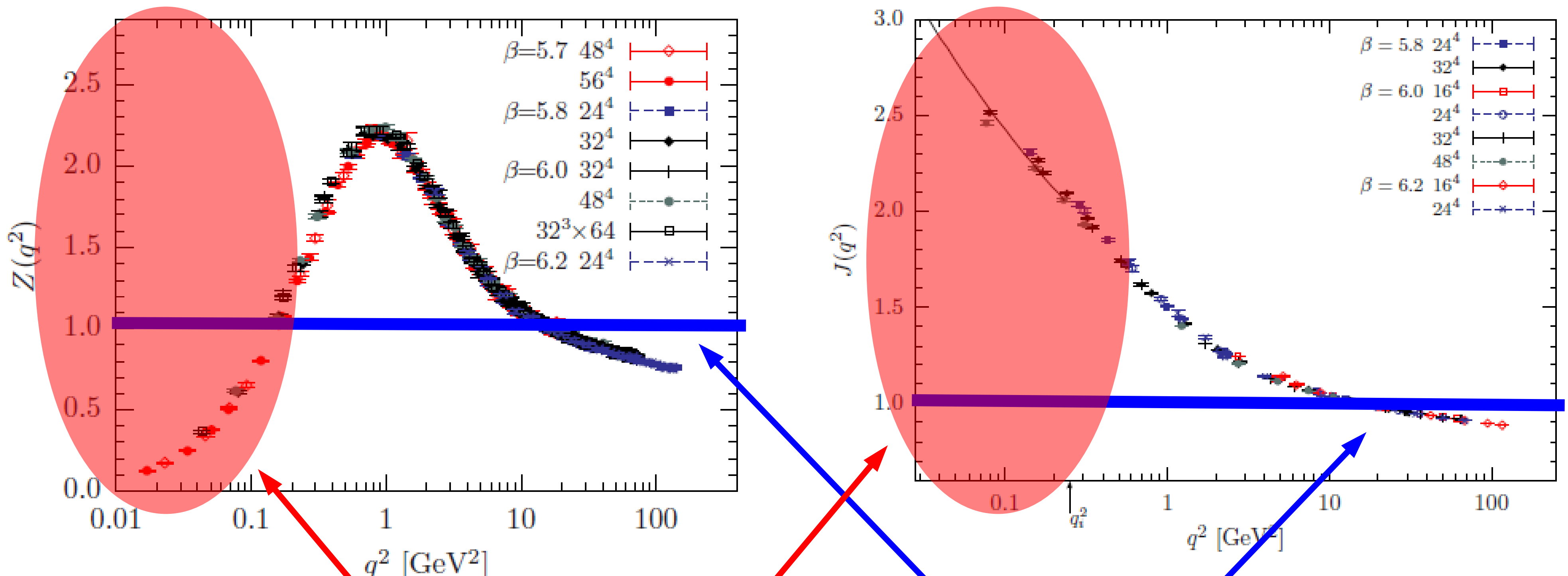
Ilgenfritz-Muller-Preussker-Sternbeck-Schiller-Bogolubsky (2007)





Behavior of the "dressing functions" (propagator residue)

Confinement at Low T



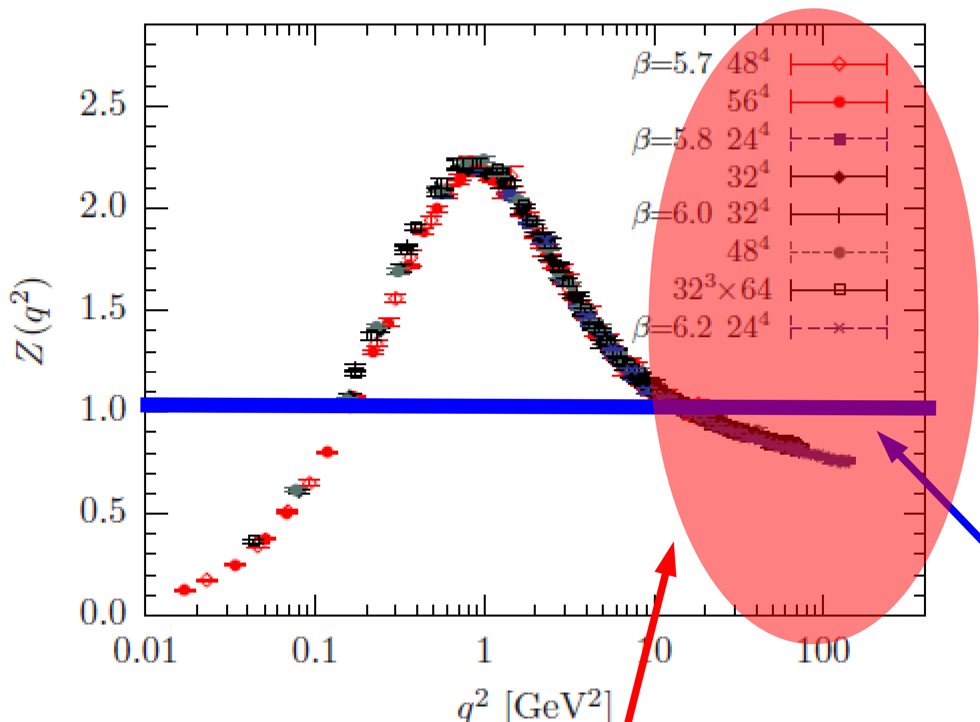
Tree-level Limit

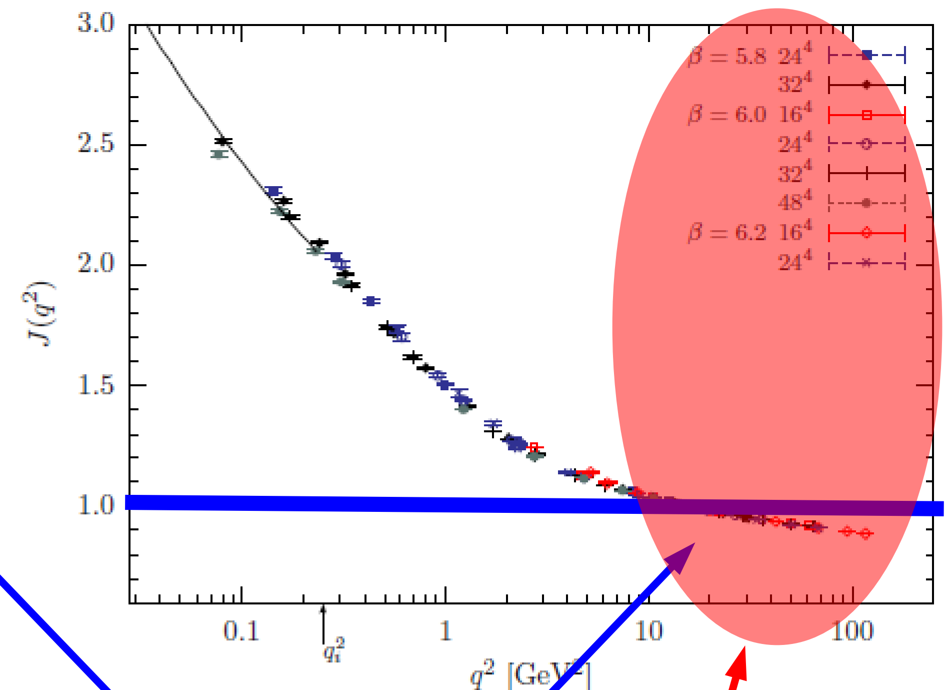
Ghost Dominance → Color Confinement

(c.f. Kugo-Ojima / Gribov-Zwanziger)

Braun-Gies-Pawlowski (2007)

Deconfinement at High T





Tree-level Limit

All Excitations with $p \sim 2\pi T$ → Perturbative Limit

Two Transverse Gluons (unphysical ones canceled)

Braun-Gies-Pawlowski (2007)

2PI (CJT) Formalism

Effective Action

$$\Gamma = \frac{1}{2}\,\mathrm{tr}\ln G^{-1} - \frac{1}{2}\,\mathrm{tr}\ln\left(G^{-1} - G_0^{-1}\right)G + \Gamma_2[G]$$

Once the full propagator is know, the effective action (or the pressure) is calculable from the above.

$$\Gamma \simeq \frac{1}{2}\,\mathrm{tr}\ln G^{-1}$$

Reasonable approximation
if the quasi-particle picture makes sense.
(c.f. Hartree approximation)

In principle, improvable by evaluating the 2PI diagrams
(Controllable approximation)

Practical Prescription

Gribov-Stingle form

$$D_L = \frac{1}{p^2} \qquad D_T^{(T)} = \frac{c_t}{p^2} \cdot \frac{d_t p^2 + 1}{p^2 + r_t^2} \qquad D_T^{(L)} = \frac{c_l}{p^2} \cdot \frac{d_l p^2 + 1}{p^2 + r_l^2}$$

$$D_C = \frac{d_g^{-1}}{p^2} \cdot \frac{d_g p^2 + 1}{p^2}$$

$$c_t = 5.5 \text{ GeV}^2, \ d_t = 0.152 \text{ GeV}^{-2}, \ r_t^2 = 0.847 \text{ GeV}^2,$$
$$c_l = 3.7 \text{ GeV}^2, \ d_l = 0.221 \text{ GeV}^{-2}, \ r_l^2 = 0.257 \text{ GeV}^2.$$

at $T = 0.86 T_c$

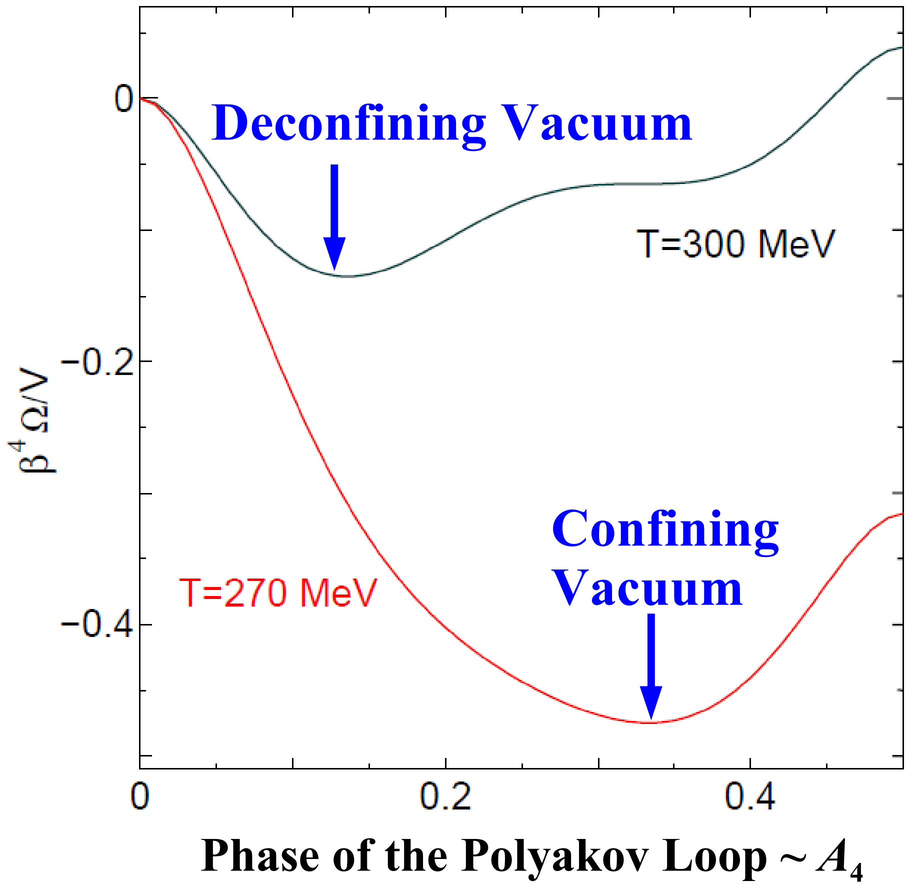
$$\mathrm{tr} \ln D_T^{(T)-1} = \mathrm{tr} \ln p^2 + \mathrm{tr} \ln (p^2 + r_t^2) - \mathrm{tr} \ln (p^2 + d_t^{-1})$$
$$= W_B(0, L_8) + W_B(r_t^2, L_8) - W_B(d_t^{-1}, L_8)$$

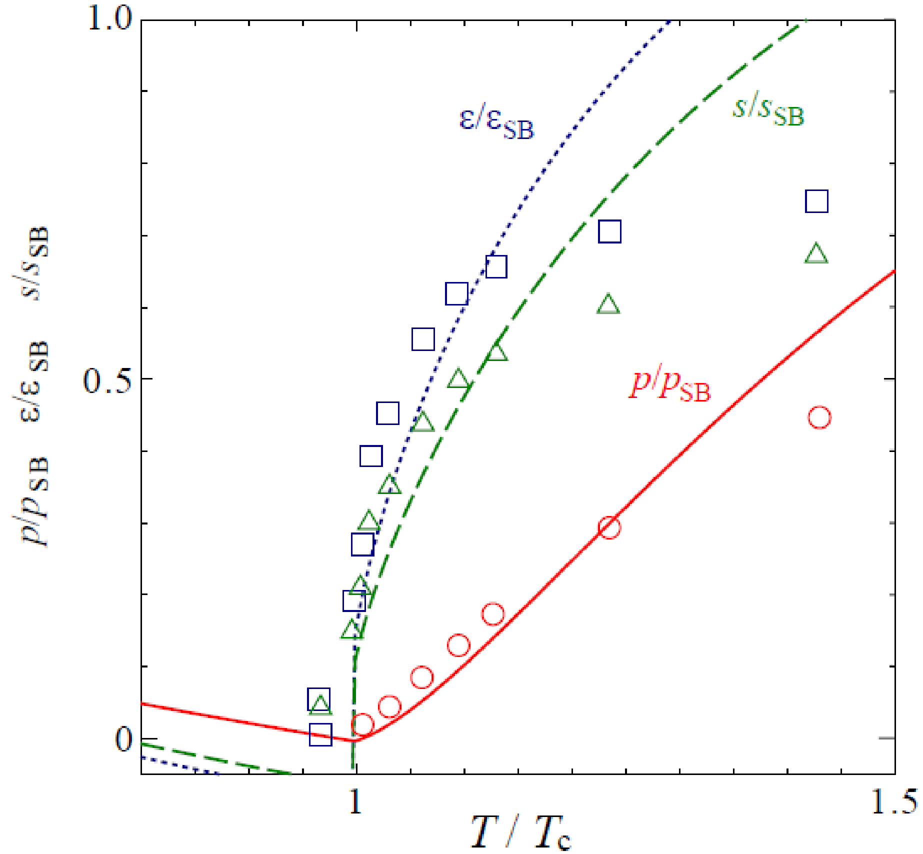
$$W_B(m^2, L_8) = -2V \int \frac{d^3 p}{(2\pi)^3} \, \mathrm{tr} \ln \left(1 - L_8 e^{-\sqrt{p^2 + m^2}/T}\right)$$

Finite!

Phase Transition from Propagators

$$\ln Z = -\frac{1}{2}\,\mathrm{tr}\ln D^{-1}_{\rm gluon} + \mathrm{tr}\ln D^{-1}_{\rm ghost} + \cdots$$

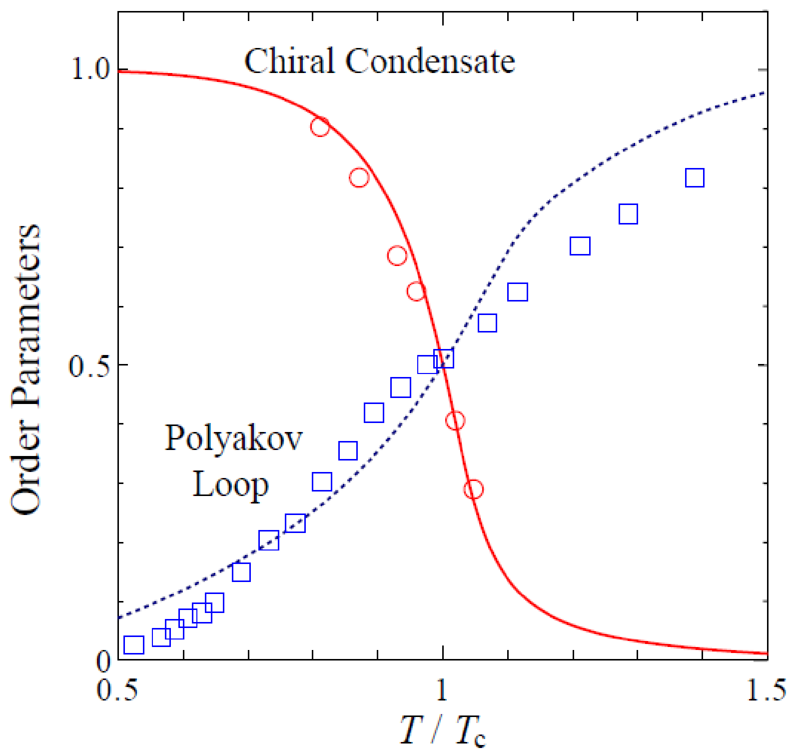




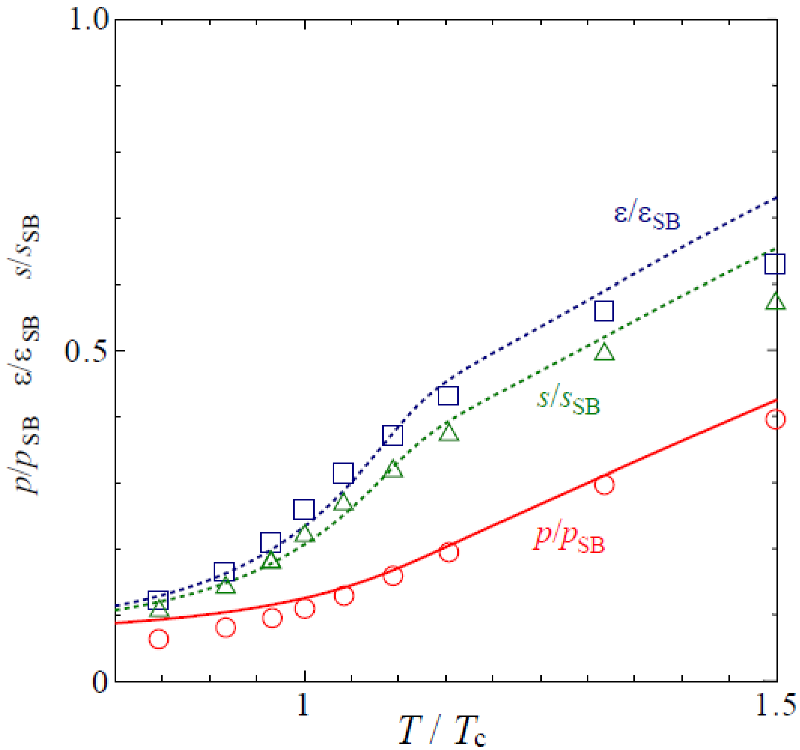
Confinement → Deconfinement

Fukushima-Kashiwa (2012)

Inclusion of Dynamical Quarks



Simultaneous Crossovers



Thermodynamics near T_c

First model computation with the first-principle-like Polyakov loop potential implemented → Promising!

KF-Kashiwa (2012)

Outlooks

How to introduce the back-reaction at finite μ or *B* ?

How to understand deconfinement at high μ ?

Indispensable to clarify the properties of Quarkyonic...

Non-trivial question; nobody knows the answer...?

Whether deconfinement is induced by B ?

Naïve insertion of the polarization leads to artifacts...

So far, deconfinement is not seen on the lattice...?

Interplay between μ and B ?

Sign problem is (partially) evaded! **(Fukushima-Hayata-Hidaka)**

Inhomogeneous ground states even more favored...?