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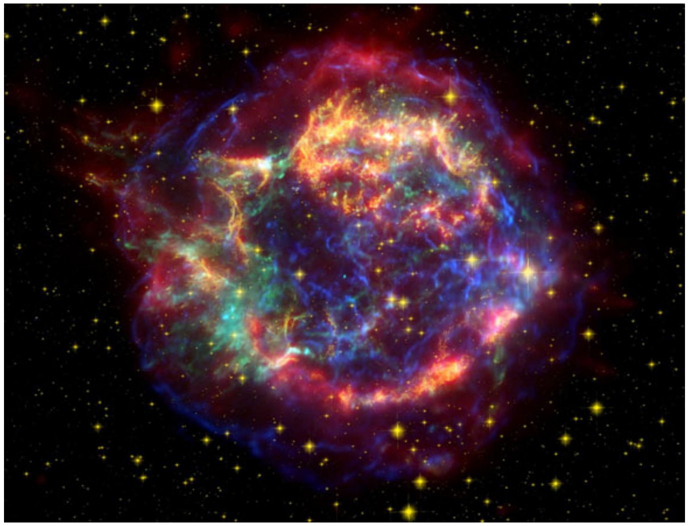
Quark superfluidity in the two-fluid formalism

M.G. Alford, S.K. Mallavarapu, A. Schmitt, S. Stetina, in preparation

- Motivation: hydrodynamics of CFL
- Superfluids as two-component fluids
- Link microscopic physics with hydro
  - $T = 0$: one fluid
  - $T > 0$: two fluids

• Motivation: hydrodynamics in compact stars

- What are compact stars made of?
  Are they ...
  ... neutron stars?
  ... hybrid stars?
  ... quark stars?



Cas A, Chandra X-Ray Observatory

- For various properties, need hydrodynamics:
  - r-mode instability e.g., N. Andersson, Astrophys. J. 502, 708 (1998)
  - pulsar glitches e.g., B. Link, MNRAS 422, 1640 (2012)
  - magnetohydrodynamics e.g., P. D. Lasky, B. Zink, K. D. Kokkotas, arXiv:1203.3590
  - asteroseismology e.g., L. Samuelsson, N. Andersson, MNRAS 374, 256 (2007)

• CFL quark matter in the core of a compact star?

Color-flavor locked (CFL) quark matter ...
M. Alford, K. Rajagopal, F. Wilczek, NPB 537, 443 (1999)

... is the ground state of asymptotically dense, 3-flavor quark matter

... breaks color, chiral, and baryon number symmetries spontaneously

$$SU(3)_c \times SU(3)_L \times SU(3)_R \times U(1)_B \rightarrow SU(3)_{c+L+R} \times \mathbb{Z}_2$$

→ octet of pseudo-Goldstones K^0, $K^{\pm}$, π^0, ...
→ (exactly massless) Goldstone ϕ ("phonon")

... has a kaon condensate for finite m_s ("CFL-K^0")

→ $U(1)_S$ broken spontaneously
→ pseudo-Goldstone ($U(1)_S$ expl. broken by weak interactions)

- **Towards the hydrodynamics of CFL ...**

Astrophysicist: How many fluid components does CFL have?

Particle physicist: ???

Astrophysicist: Is CFL a superfluid?

Particle physicist: Yes, CFL breaks $U(1)_B$.

Astrophysicist: ???

Particle physicist: CFL-K^0 also breaks $U(1)_S$, but that's only an approximate symmetry.

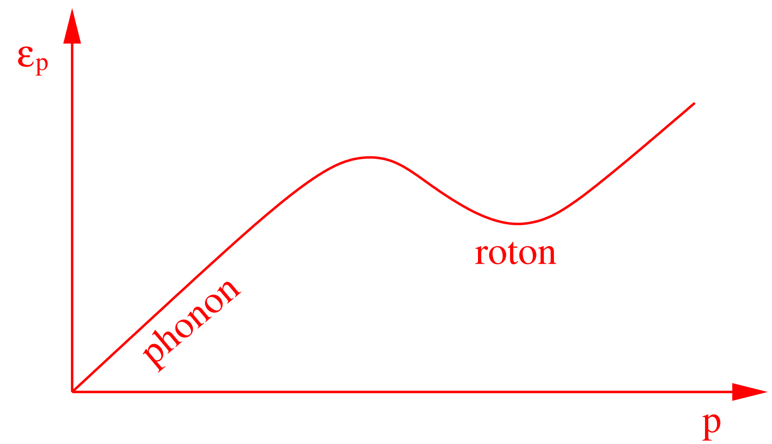
Astrophysicist: ???

• Two-fluid picture of a superfluid (Helium-4) (page 1/2)

L. Tisza, Nature 141, 913 (1938); L. Landau, Phys. Rev. 60, 356 (1941)

relativistic: I.M. Khalatnikov, V.V. Lebedev, Phys. Lett. A 91, 70 (1982)

- "superfluid component":
  condensate, carries no entropy
- "normal component": excitations (Goldstone mode), carries entropy



- Hydrodynamic eqs.

  ⇒ two wave eqs.

$$\frac{\partial^2 \rho}{\partial t^2} = \Delta P$$

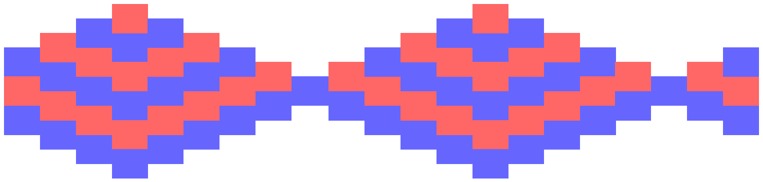
$$\frac{\partial^2 S}{\partial t^2} = \frac{S^2 \rho_s}{\rho_n} \Delta T$$

⇒ two sound velocities:

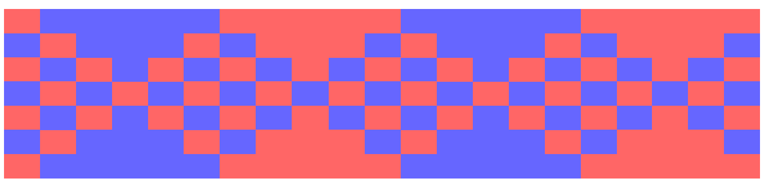
$$u_1 = \sqrt{\frac{\partial P}{\partial \rho}}\,, \quad u_2 = \sqrt{\frac{s^2 T \rho_s}{\rho c_V \rho_n}}$$

• Two-fluid picture of a superfluid (Helium-4) (page 2/2)

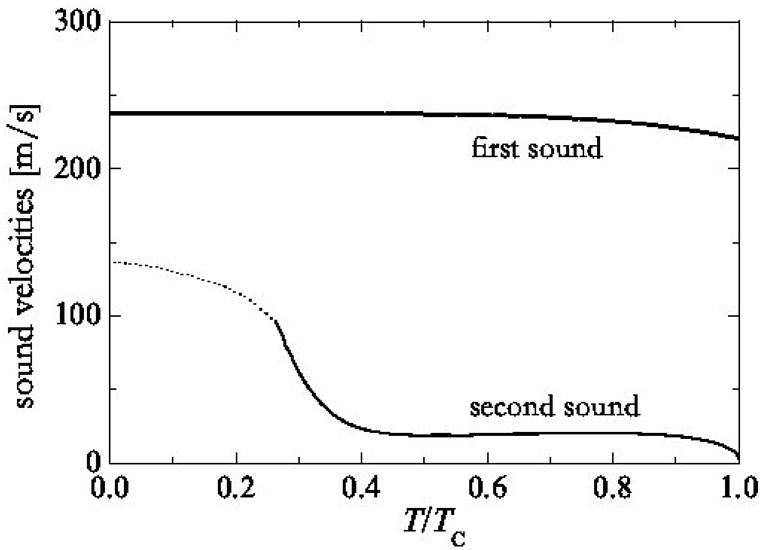
- 1st sound: total density oscillates



- 2nd sound: relative densities of **superfluid** and **normal** components oscillate



- exp. sound velocities of ^{4}He



E. Taylor *et al.*, PRA 80, 053601 (2009) according to K.R. Atkins *et al.* (1953); V.P. Peshkov (1960)

→ How does the two-fluid picture arise from a microscopic theory?

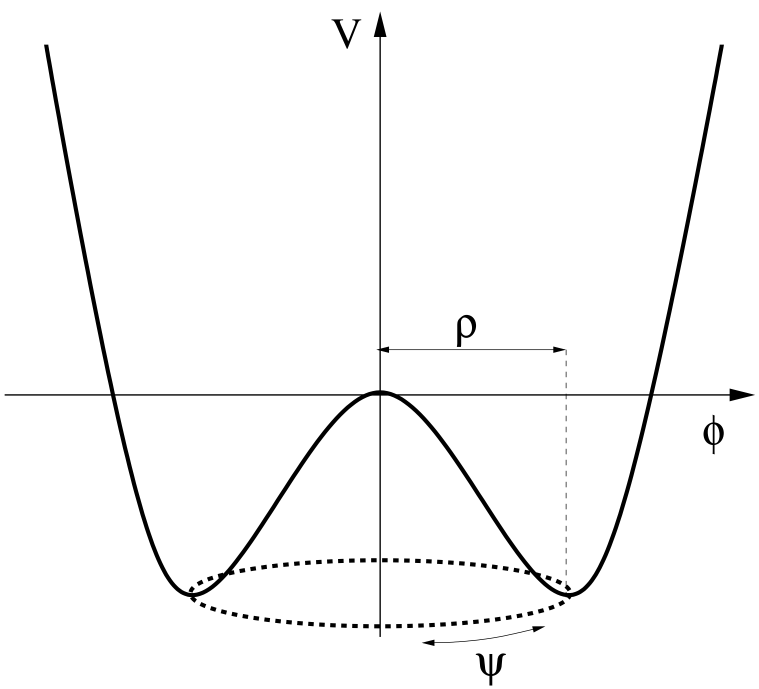
• Bose condensation and superfluid velocity (page 1/2)

- start with simplest case: φ^4 model
  $\rightarrow$ from chiral Lagrangian for CFL mesons
  Bedaque, Schäfer, NPA 697, 802 (2002);
  Alford, Braby, Schmitt, JPG 35, 025002 (2008)

$$\mathcal{L} = (\partial\varphi)^2 - m^2|\varphi|^2 - \lambda|\varphi|^4$$

$$m^2 = m_{K^0}^2 = \frac{m_s^2 - m_d^2}{2\mu}$$

$$\lambda = \frac{4\mu_{K^0}^2 - m_{K^0}^2}{6f_\pi^2}$$



- $\varphi \rightarrow \phi + \varphi$, condensate $\phi = \frac{\rho}{\sqrt{2}} e^{i\psi}$
- first step: no fluctuations ($T = 0$)
- minimize $V(\rho) = -\mathcal{L}$

$$\rho^2 = \frac{(\partial\psi)^2 - m^2}{\lambda}$$

(assumption: ρ, $\partial\psi$ const.)

• Bose condensation and superfluid velocity (page 2/2)

- "translation" at zero temperature (single fluid!) $(m=0)$

	Field-theoretically	**Hydrodynamically**
j^μ	$\frac{(\partial\psi)^2}{\lambda}\partial^\mu\psi$	nv^μ
$T^{\mu\nu}$	$\frac{(\partial\psi)^2}{\lambda}\partial^\mu\psi\partial^\nu\psi - g^{\mu\nu}\mathcal{L}$	$(\epsilon+P)v^\mu v^\nu - g^{\mu\nu}P$

- With $\epsilon + P = \mu n$:

$$P = \frac{(\partial\psi)^4}{4\lambda}\,, \qquad \epsilon = \frac{3(\partial\psi)^4}{4\lambda}$$

$$\mu = |\partial\psi|\,, \qquad n = \frac{|\partial\psi|^3}{\lambda}$$

- **superfluid velocity**

$$\boxed{v^\mu = \frac{\partial^\mu\psi}{\mu}}$$

$\Rightarrow$ superfluid is curl-free,

$$\nabla\times\mathbf{v}_s = 0$$

- **From one fluid ($T = 0$) to two fluids ($T > 0$)**

- qualitative change:
  - one fluid: $\exists$ frame in which pressure is isotropic
  - two fluids: pressure anisotropic $\forall$ frames

- formulation in terms of superfluid and normal fluid:

$$j^\mu = n_s v^\mu + n_n u^\mu$$

$$T^{\mu\nu} = (\epsilon_s + P_s) v^\mu v^\nu - g^{\mu\nu} P_s + (\epsilon_n + P_n) u^\mu u^\nu - g^{\mu\nu} P_n$$

D. T. Son, Int. J. Mod. Phys. A 16S1C, 1284 (2001)

- formulation in terms of entropy current and conserved current:

I.M. Khalatnikov and V.V. Lebedev, Phys. Lett. 91A, 70 (1982)

B. Carter and I. M. Khalatnikov, PRD 45, 4536 (1992)

• Microscopic calculation at nonzero T (page 1/2)

- calculation for all $T \leq T_c$ needs self-consistent formalism;
  2PI (no superflow): M. G. Alford, M. Braby, A. Schmitt, JPG 35, 025002 (2008)

- here: one-loop (small T) effective action

$$\frac{T}{V}\Gamma_{\rm eff} = \frac{(\partial\psi)^4}{4\lambda} - \frac{1}{2}\frac{T}{V}\sum_k \mathrm{Tr}\ln\frac{S^{-1}(k)}{T^2}$$

- inverse tree-level propagator (at the $T = 0$ stationary point)

$$S^{-1}(k) = \begin{pmatrix} -k^2 + 2(\partial\psi)^2 & 2ik\cdot\partial\psi \\ -2ik\cdot\partial\psi & -k^2 \end{pmatrix}$$

- anisotropic phonon dispersion (→ first sound)

$$\epsilon(\theta, k) = \frac{f(\theta)}{\sqrt{3}}\,k + \ldots\,, \qquad f(\theta) = \frac{\sqrt{1-\mathbf{v}_s^2}\sqrt{1-\frac{\mathbf{v}_s^2}{3}(1+2\cos^2\theta)} + \frac{2|\mathbf{v}_s|}{\sqrt{3}}\cos\theta}{1-\frac{\mathbf{v}_s^2}{3}}$$

• Microscopic calculation at nonzero T (page 2/2)

- compute current and stress-energy tensor

$$j^\mu = \frac{\sigma^2}{\lambda}\partial^\mu\psi - \frac{1}{2}\frac{T}{V}\sum_k \mathrm{Tr}\left[S\frac{\partial S^{-1}}{\partial\,\partial_\mu\psi}\right]$$

$$T^{\mu\nu} = \frac{(\partial\psi)^2}{\lambda}\partial^\mu\psi\partial^\nu\psi - g^{\mu\nu}\frac{(\partial\psi)^4}{4\lambda} - \frac{T}{V}\sum_k \mathrm{Tr}\left[S\,\frac{\partial S^{-1}}{\partial g^{\mu\nu}} - u^\mu u^\nu\right]$$

[where $u^\mu = (1,0,0,0)$]

- can be evaluated analytically for small T, $|\mathbf{v}_s|$, e.g.,

$$T^{00} = \frac{\mu^4}{4\lambda}(3-2\mathbf{v}_s^2) + \frac{\pi^2}{10\sqrt{3}}(3+4\mathbf{v}_s^2)\,T^4 - \frac{4\pi^2}{21\sqrt{3}}(3+19\mathbf{v}_s^2)\,\frac{T^6}{\mu^2} + \ldots$$

• Relativistic two-fluid formalism (page 1/2)

- write stress-energy tensor as

$$T^{\mu\nu} = -g^{\mu\nu}\Psi + j^{\mu}\partial^{\nu}\psi + s^{\mu}\Theta^{\nu}$$

- **"generalized pressure"** Ψ:
  - Ψ is transverse pressure in superfluid and normal rest frames
  - Ψ depends on momenta $\partial^{\mu}\psi$, Θ^{μ}

$$\Psi = \Psi[(\partial\psi)^2, \Theta^2, \partial\psi \cdot \Theta]$$

- **"generalized energy density"** $\Lambda \equiv -\Psi + j \cdot \partial\psi + s \cdot \Theta$
  - Λ is Legendre transform of Ψ
  - Λ depends on currents j^{μ}, s^{μ}

$$\Lambda = \Lambda[j^2, s^2, j \cdot s]$$

• Relativistic two-fluid formalism (page 2/2)

$$j^\mu = \frac{\partial \Psi}{\partial(\partial_\mu \psi)} = \mathcal{B}\,\partial^\mu\psi + \mathcal{A}\,\Theta^\mu$$

$$s^\mu = \frac{\partial \Psi}{\partial \Theta_\mu} = \mathcal{A}\,\partial^\mu\psi + \mathcal{C}\,\Theta^\mu$$

$$\mathcal{B} = 2\frac{\partial\Psi}{\partial(\partial\psi)^2}\,, \quad \mathcal{C} = 2\frac{\partial\Psi}{\partial\Theta^2}$$

$$\mathcal{A} = \frac{\partial\Psi}{\partial(\partial\psi\cdot\Theta)}$$

"entrainment coefficient"

- conservation equations $\partial_\mu T^{\mu\nu} = 0, \quad \partial_\mu j^\mu = 0$ become

$$\partial_\mu j^\mu = 0\,, \qquad \partial_\mu s^\mu = 0\,, \qquad s_\mu \underbrace{(\partial^\mu\Theta^\nu - \partial^\nu\Theta^\mu)}_{\text{"vorticity"}} = 0$$

- in "mixed" form, we recover superfluid/normal formulation

$$T^{\mu\nu} = -g^{\mu\nu}\Psi + \frac{\mathcal{B}\mathcal{C} - \mathcal{A}^2}{\mathcal{C}}\partial^\mu\psi\partial^\nu\psi + \frac{1}{\mathcal{C}}s^\mu s^\nu$$

- **Connect microscopic calculation with hydro**
  - microscopic calculation done in "normal rest frame" $s^\mu = (s^0, 0, 0, 0)$
  - one can then show that

$$\boxed{\frac{T}{V}\Gamma_{\rm eff} = \Psi}$$

  - 8 independent degrees of freedom from 16 $(\partial^\mu\psi,\ \Theta^\mu,\ j^\mu,\ s^\mu)$

$$(\mu, \mu v_s^i, T) = (\partial^0\psi, \partial^i\psi, \Theta^0) \quad + \quad \text{constraint } s^i = 0$$

  - obtain current j^μ and entropy s^0 microscopically
  - determine $\mathcal{A}$, $\mathcal{B}$, $\mathcal{C}$, (and Θ^i), for instance

$$\mathcal{A} = \frac{s^0}{\partial^0\psi}\left[j^0 + \frac{\vec{j}\cdot\nabla\psi}{(\nabla\psi)^2}\partial^0\psi\right]\left[j^0 + \frac{\vec{j}\cdot\nabla\psi}{(\nabla\psi)^2}\partial^0\psi + s^0\Theta^0\right]^{-1}$$

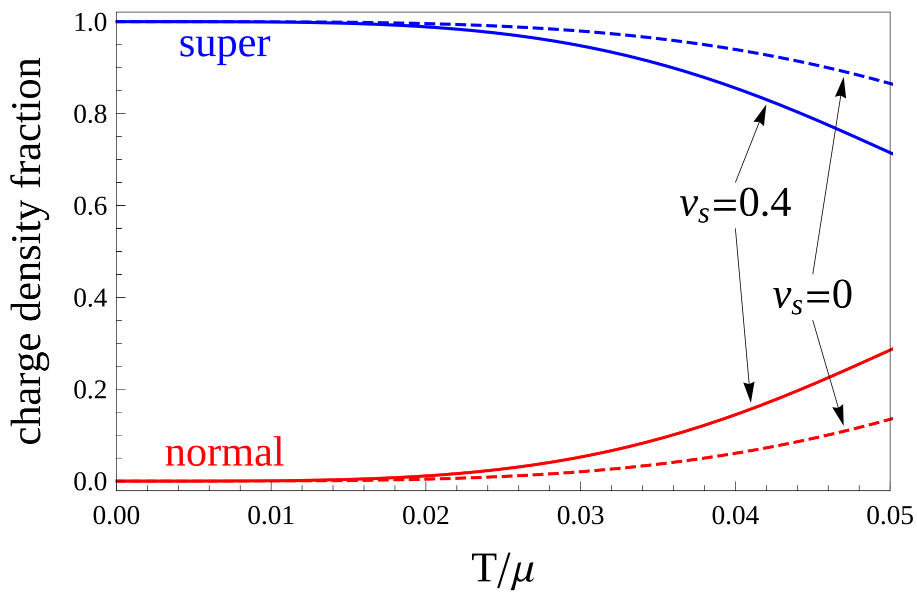
etc.

• Compute properties of the superfluid (page 1/2)

- superfluid and normal charge densities (measured in normal rest frame)

$$n_s = \mu \frac{\mathcal{BC} - \mathcal{A}^2}{\mathcal{C}} = \frac{\mu^3}{\lambda}(1 - \mathbf{v}_s^2) - \frac{4\pi^2}{5\sqrt{3}}\left(1 + \frac{47}{6}\mathbf{v}_s^2\right)\frac{T^4}{\mu} + \ldots$$

$$n_n = s\frac{\mathcal{A}}{\mathcal{C}} = \frac{4\pi^2}{5\sqrt{3}}\left(1 + \frac{41}{6}\mathbf{v}_s^2\right)\frac{T^4}{\mu} + \ldots$$



(effect exaggerated by choosing λ very large)

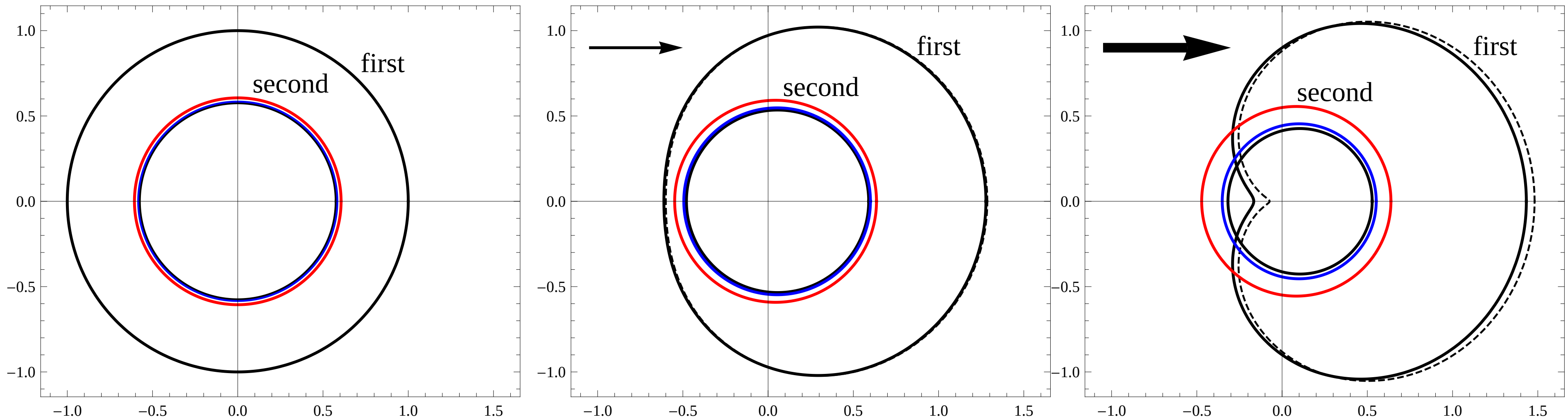
- one fluid gets converted into the other by heating

• Compute properties of the superfluid (page 2/2)

- sound velocities (measured in normal rest frame)

$$u_1 = \frac{1}{\sqrt{3}}\left(1+\frac{2\cos\theta}{\sqrt{3}}|\mathbf{v}_s| - \frac{1+\cos^2\theta}{3}\mathbf{v}_s^2\right)+\ldots$$

$$u_2 = \frac{1}{3}\left(1+\frac{\cos\theta}{3}|\mathbf{v}_s| - \frac{17-\cos^2\theta}{18}\mathbf{v}_s^2\right)+\frac{20}{21}\left[1-\frac{4\cos\theta}{3}|\mathbf{v}_s| + \left(\frac{2702}{243}-\frac{\cos^2\theta}{2}\right)\mathbf{v}_s^2\right]\left(\frac{\pi T}{\mu}\right)^2+\ldots$$



$|\mathbf{v}_s| = 0$ $\quad$ $|\mathbf{v}_s| = \frac{0.5}{\sqrt{3}}$ $\quad$ $|\mathbf{v}_s| = \frac{0.95}{\sqrt{3}}$

• Summary

- The hydrodynamics of CFL is nontrivial ...
  ... and poses fundamental questions regarding
  relativistic superfluid hydrodynamics and its
  microscopic, field-theoretical description.

- For the case of a φ^4 model we have connected the
  microscopic theory (at finite T) with the
  two-fluid formalisms of Son and Khalatnikov/Lebedev

• Outlook

- go beyond small-T expansion
  - solve stationarity eqs with superflow numerically
  - compute superfluid density etc for all $T < T_c$
- how does the picture change with approximate (not exact) $U(1)_S$ symmetry? is superfluidity lost completely?
- start from fermionic microscopic theory to account for $U(1)_B$
- put all this together for hydrodynamics of CFL-K^0
- include dissipation
  - various viscosity coefficients already computed

    C. Manuel, A. Dobado and F. J. Llanes-Estrada, JHEP 0509, 076 (2005)

    M. G. Alford, M. Braby and A. Schmitt, JPG 35, 115007 (2008)
  - three bulk viscosity coefficients due to two fluids

    M. Mannarelli and C. Manuel, PRD 81, 043002 (2010)