Ultrasoft Fermionic Mode in Hot QCD

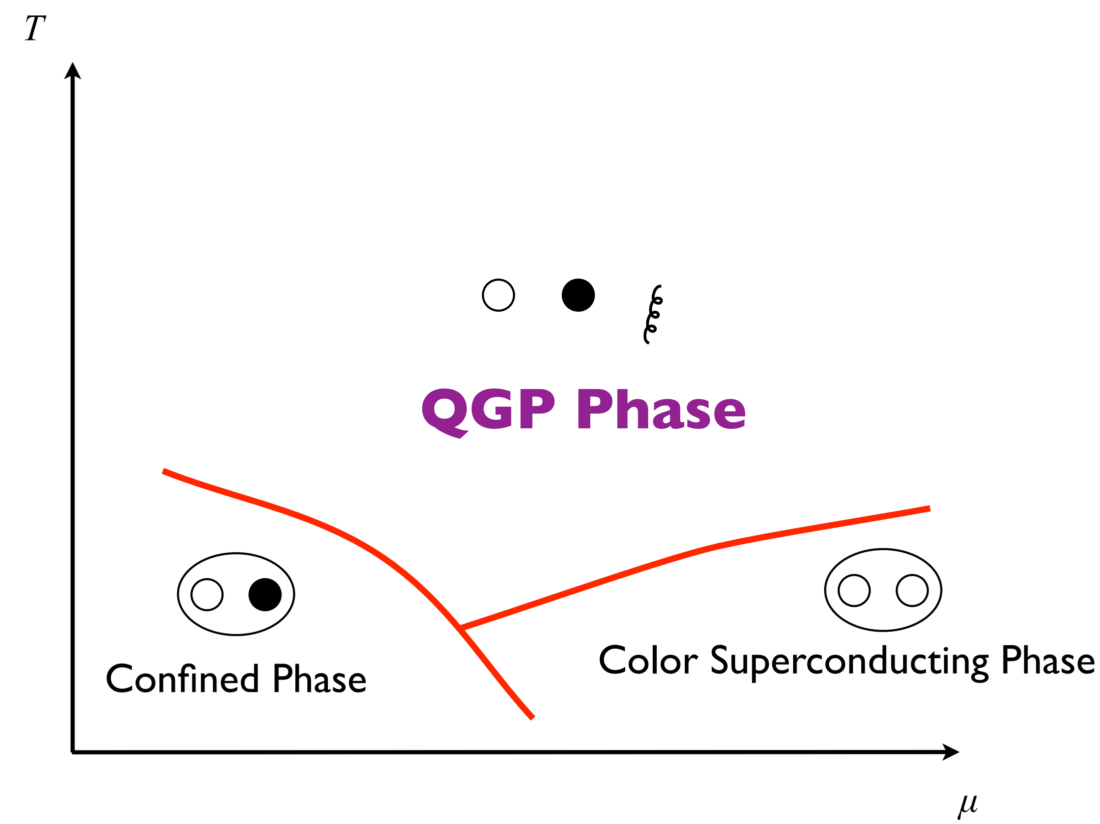
Daisuke Satow (Saclay CEA, France / Kyoto Univ. Japan)
Collaborators: Yoshimasa Hidaka (RIKEN, Japan), Teiji Kunihiro (Kyoto Univ. Japan)

Y. Hidaka, **D. S.**, and T. Kunihiro, NPA **876**, 93 (2012)
D. S., Y. Hidaka, PRD **85**, 116009 (2012).

Introduction

motivation: **quark collective excitation** in QGP phase

(cf. bosonic oscillation: plasmon)



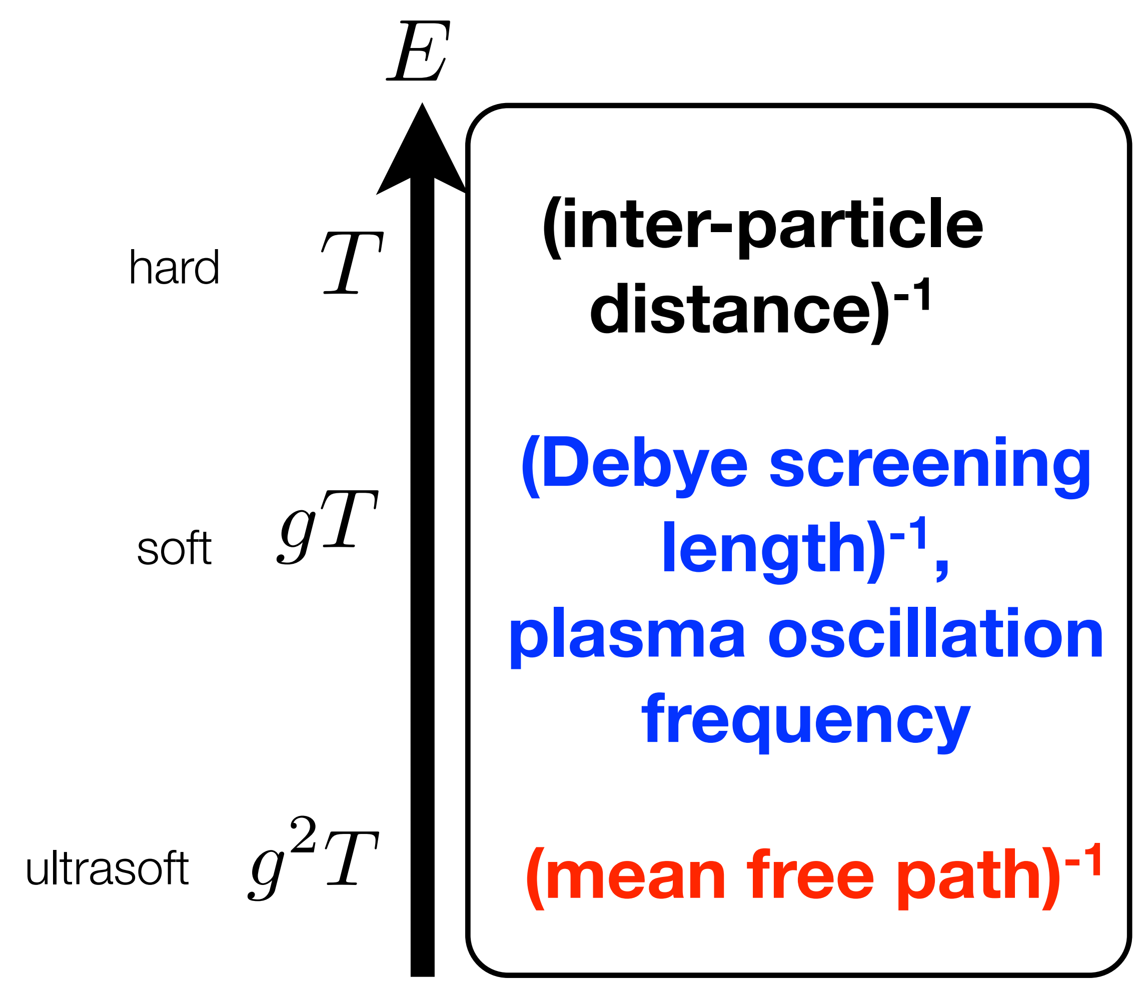
- How many excitation modes?

- How are the dispersion relation, damping rate, strength?

Introduction

This is nontrivial task;
Bare-particle picture is generally **invalid** even at weak coupling ($g << 1$).

energy hierarchy at high temperature ($T >> \Lambda_{QCD}$, m)



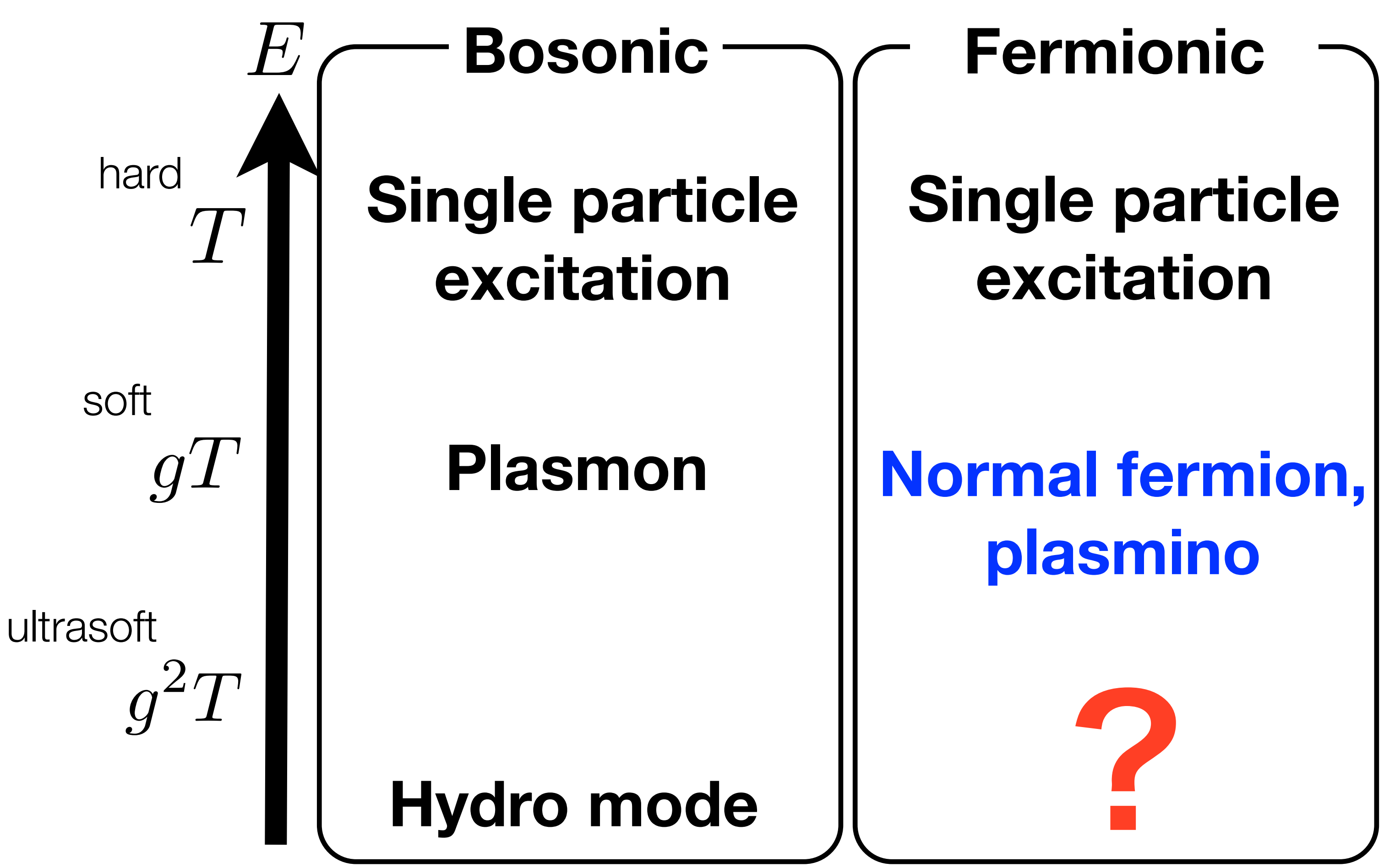
Because

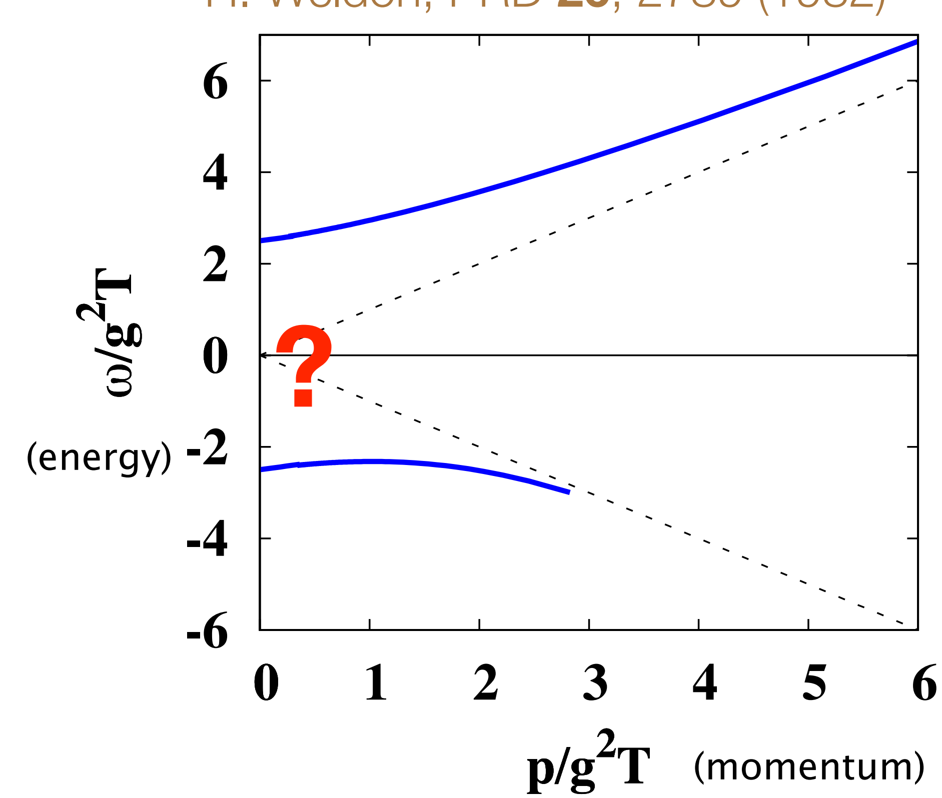
- many-body effect becomes non-negligible when (energy) ~ gT
- interaction effect such as collision becomes non-negligible when (energy) ~ g^2T **(<u>mesoscopic scale</u>)**

Ultrasoft scale is **not well investigated** even at weak coupling ($g << 1$).

Introduction

Present status of excitations

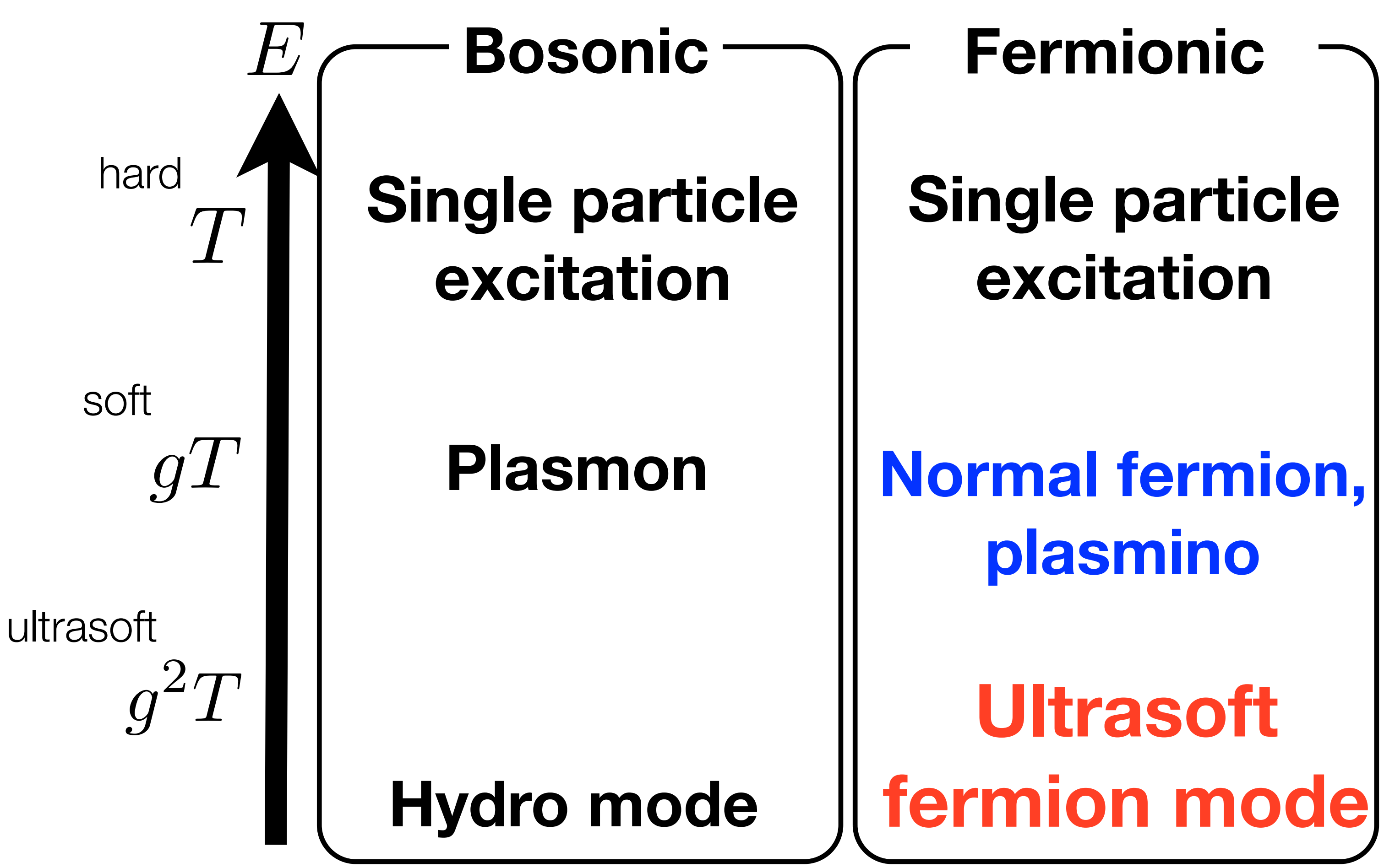




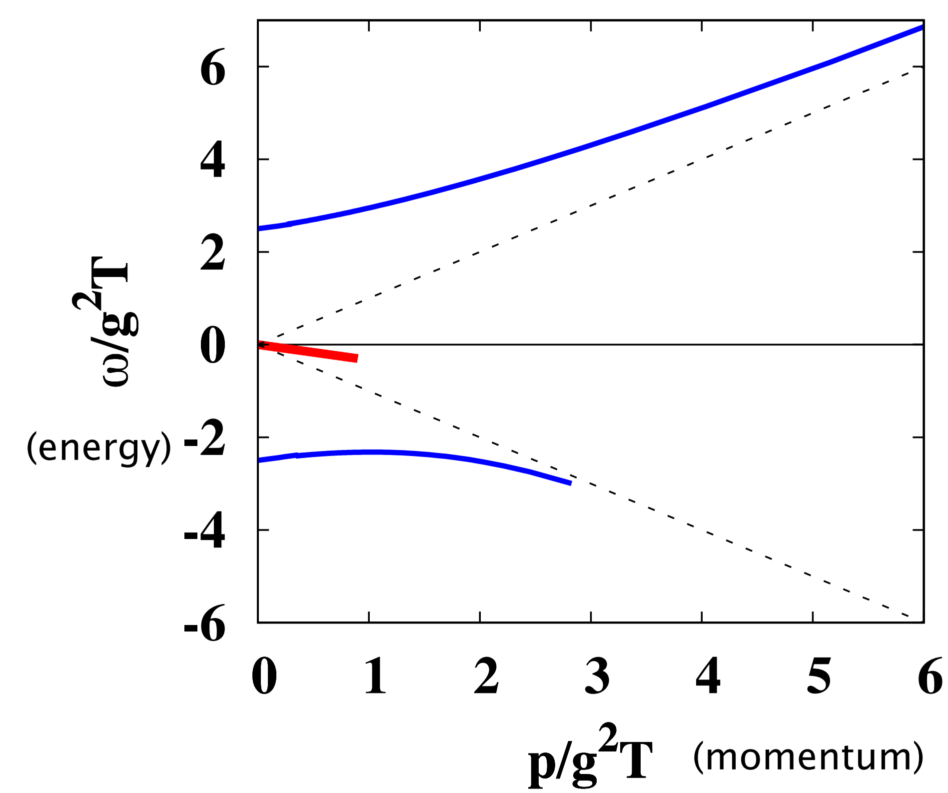
Dispersion relation
in fermionic sector

Introduction

Present status of excitations



H. Weldon, PRD **26**, 2789 (1982)



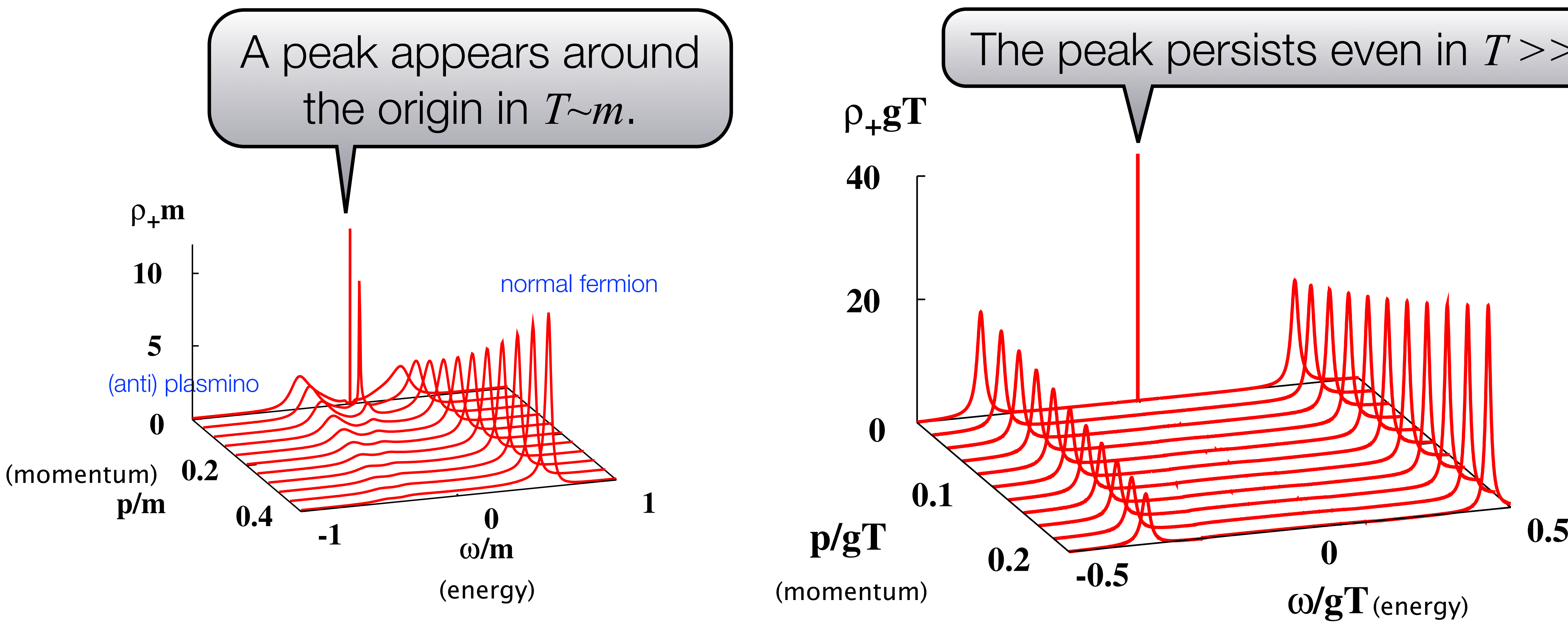
Dispersion relation in fermionic sector

Ultrasoft fermionic mode

Resummed perturbation: V. V. Lebedev and A. V. Smilga, Annals Phys. **202,** 229 (1990).
Schwinger-Dyson eq.: M. Harada and Y. Nemoto, PRD **78**, 014004 (2008), S. X. Qin, L. Chang, Y. X. Liu, and C. D. Roberts, PRD **84**, 014017 (2011).
NJL model: M. Kitazawa, T. Kunihiro and Y. Nemoto, PLB **633**, 269 (2006).

one-loop analysis (m: boson mass)

M. Kitazawa, T. Kunihiro and Y. Nemoto, PTP **117**, 103 (2007).
D. S., Y. Hidaka and T. Kunihiro, PRD **83** 045017 (2011).



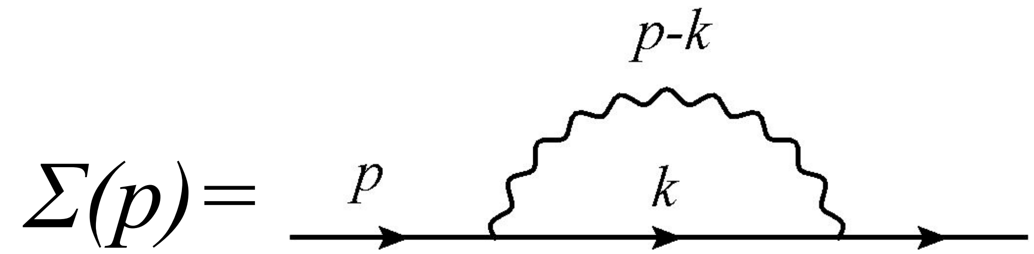
spectral function in fermionic sector

E. Braaten and R. D. Pisarski, Nucl. Phys. B **337**, 569 (1990)

Hard Thermal Loop (HTL) approximation

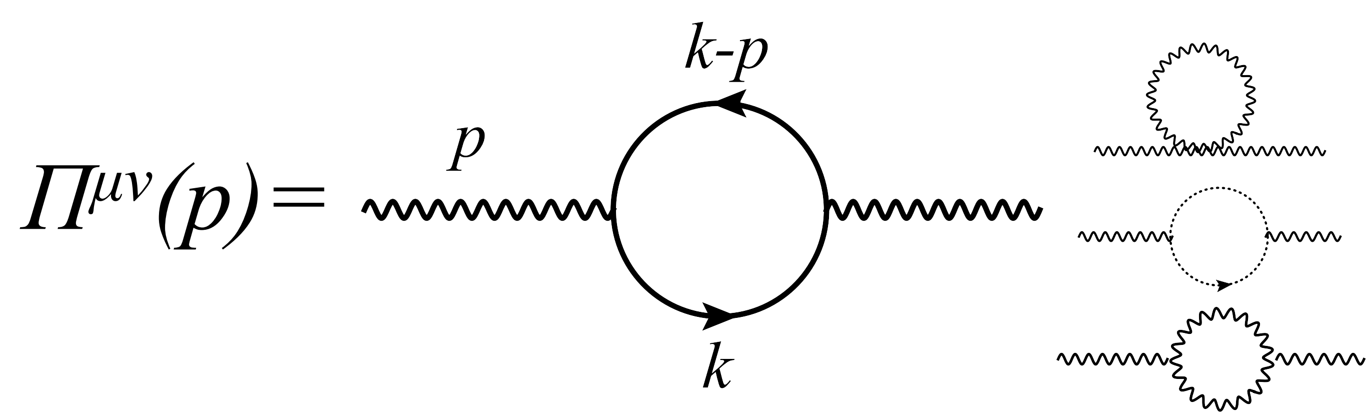
- valid when $p \sim gT$.

Fermion self-energy



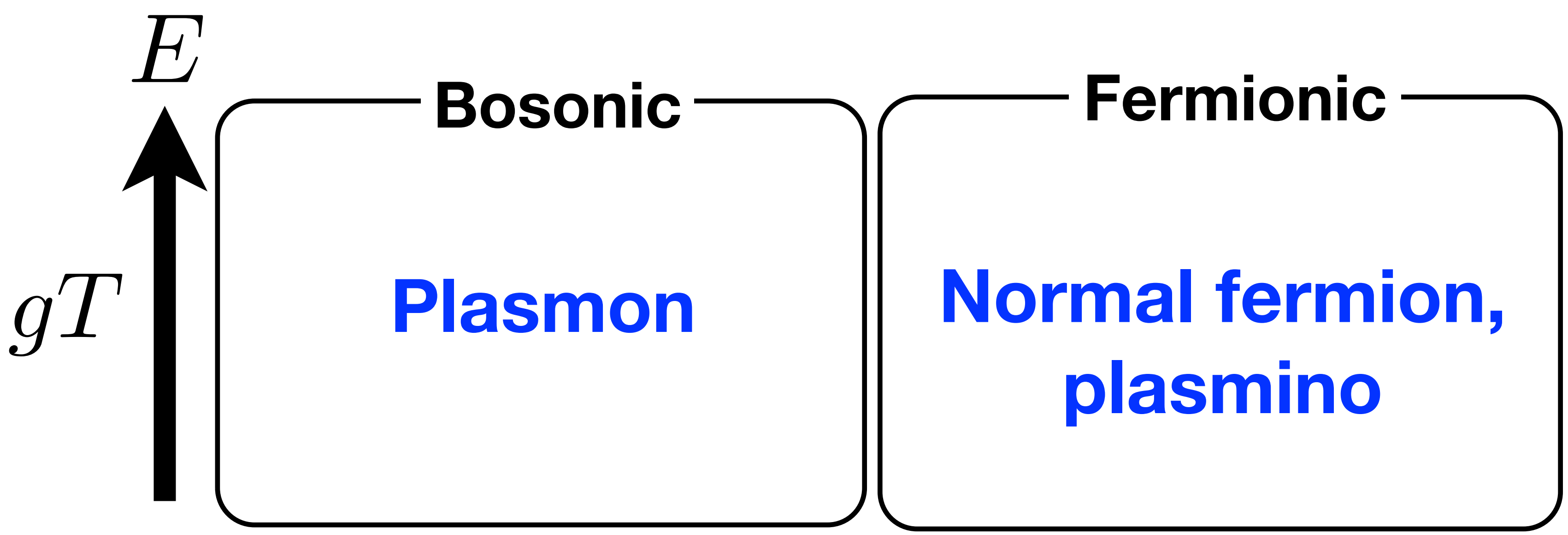
- one-loop diagram

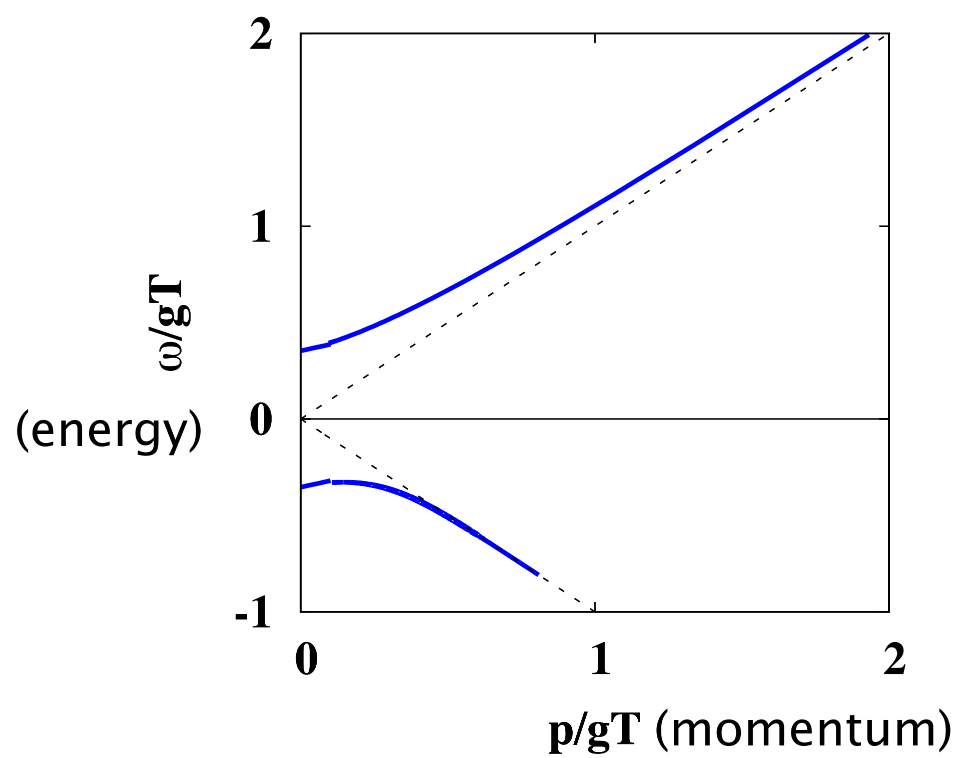
Boson self-energy



Result

H. A. Weldon, PRD **26**, 1394 (1982), 2789 (1982)

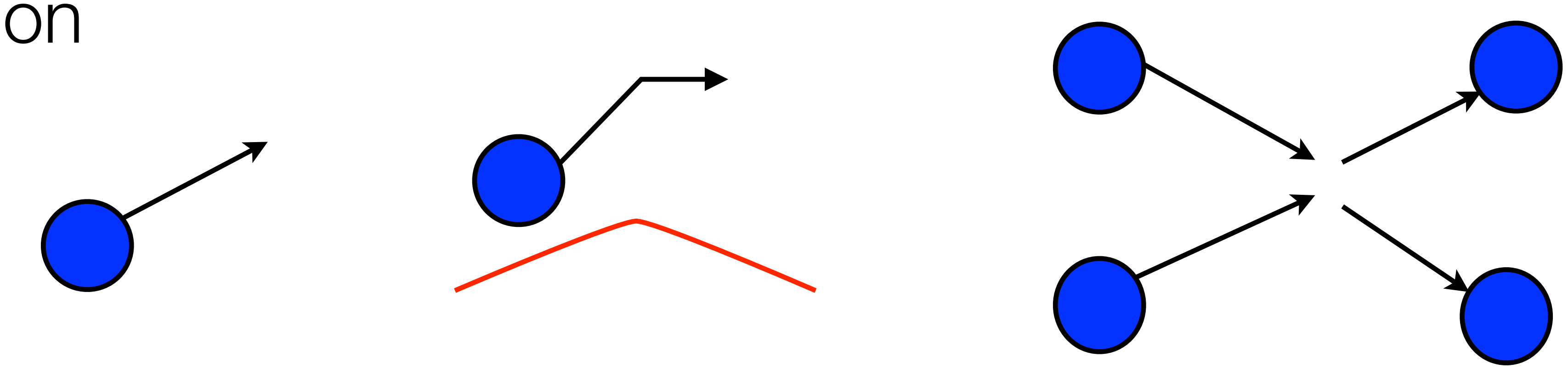




dispersion relation in fermionic sector

Vlasov equation

e.g. : fermion distribution



drift term

force term

collision term

Boltzmann eq.: $(v\cdot\partial_X + g(\boldsymbol{E}(X)+\boldsymbol{v}\times\boldsymbol{B}(X))\cdot\partial_{\boldsymbol{k}})n(X,\boldsymbol{k}) = C[n]$

$n(X,\boldsymbol{k})$: distribution function of quark
$v = (1, \boldsymbol{k}/|\boldsymbol{k}|)$: 4-velocity, $\boldsymbol{E}(X)$, $\boldsymbol{B}(X)$: field strength

Vlasov equation

The collision term in Boltzmann eq. is neglected.

Vlasov Eq.: $$(v \cdot \partial_X + g(\boldsymbol{E}(X) + \boldsymbol{v} \times \boldsymbol{B}(X)) \cdot \partial_{\boldsymbol{k}}) n(X, \boldsymbol{k}) = \cancel{C[n]}$$

$$\partial_X = O(gT) >> C[n] = O(g^2 T)$$

Self-energies calculated from Vlasov eq. coincides with that from HTL approximation.

J. P. Blaizot and E. Iancu, PRL **70**, 3376 (1993)

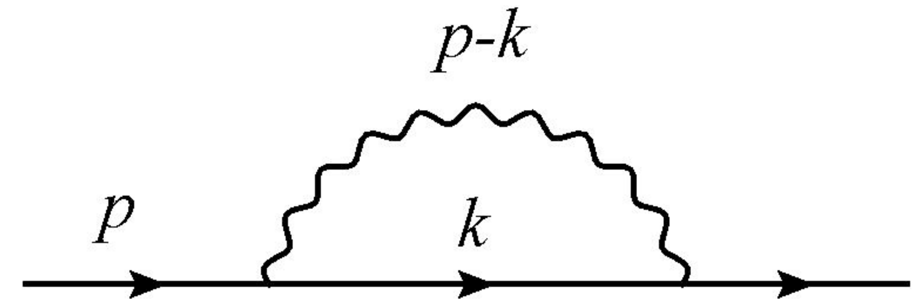
By contrast, the collision term is non-negligible at ultrasoft region($\partial_X = O(g^2 T)$)!

Pinch singularity

Reflecting this fact, the HTL approximation is not applicable in ultrasoft region($p \lesssim g^2T$).

1. $p \to 0$ limit can not be taken (pinch singularity)

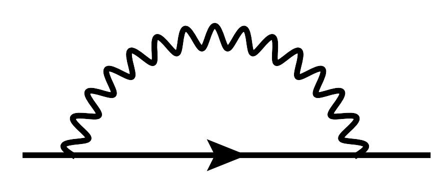
$$g^2\int\frac{d^4k}{(2\pi)^4}2\pi\theta(k^0)\delta(k^2)(N(k^0)+n(k^0))\frac{\not{k}}{2p\cdot k-p^2}\xrightarrow{p\to 0}\infty$$



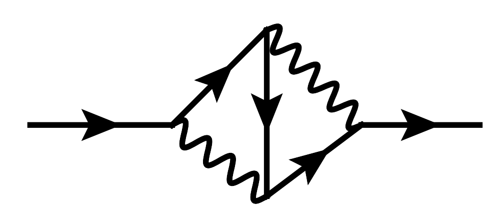
Pinch singularity in the computation of the transport coefficient:
S. Jeon, PRD **52**, 3591 (1995)

2. (one-loop) << (higher loop)

$$\frac{g^2}{p\cdot k} \qquad\qquad \left(\frac{g^2}{p\cdot k}\right)^2$$



$<<$



reorganizing perturbative expansion is necessary.

What we do

Analyze the quark spectrum in the ultrasoft region ($p \ll g^2 T$) with the resummed perturbation theory.

Resumed Perturbation

V. V. Lebedev and A. V. Smilga, Annals Phys. **202,** 229 (1990)

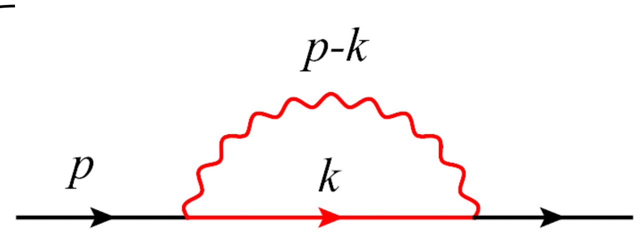
(1) thermal mass (m_f, m_b=$O(gT)$) and

decay width (ζ_f, ζ_b=$O(g^2T)$) are resummed.

m_f, ζ_f

m_b, ζ_b

→Pinch singularity is regularized.



$$g^2 \frac{\not{k}}{2p\cdot k + \delta m^2 + 2i\zeta k^0} \xrightarrow{p\to 0} O(g^0)$$

$\delta m^2 = m^2{}_b - m^2{}_f$, $\zeta = \zeta_f + \zeta_b$

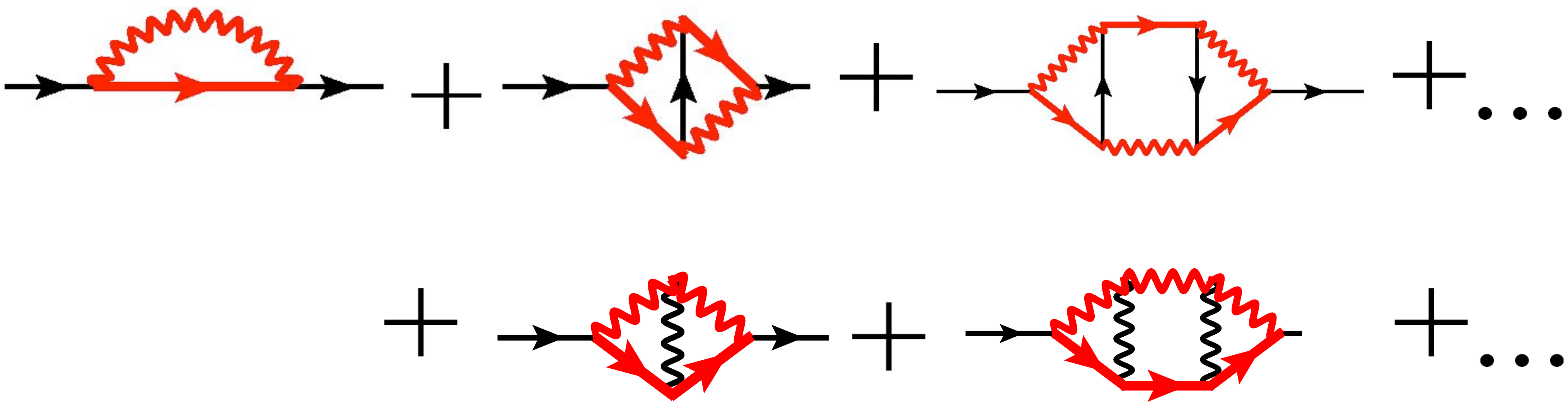
Resummed Perturbation

V. V. Lebedev and A. V. Smilga, Annals Phys. **202,** 229 (1990)

The singularity is regularized, but **all the ladder diagrams contribute at the same order**.

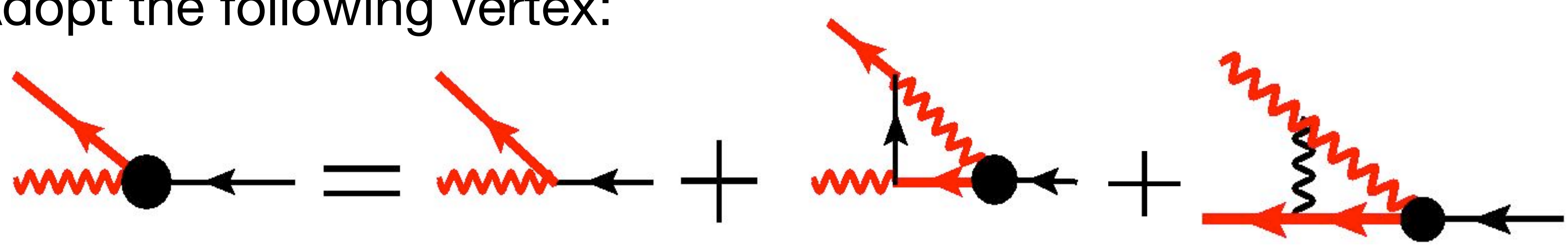
$$\frac{g^2}{2p\cdot k+\delta m^2+2i\zeta k^0}=O(g^0) \qquad \left(\frac{g^2}{2p\cdot k+\delta m^2+2i\zeta k^0}\right)^2=O(g^0)$$

(#vertex)/(#red line)=1



(2) Sum up all the ladder diagrams.

Adopt the following vertex:

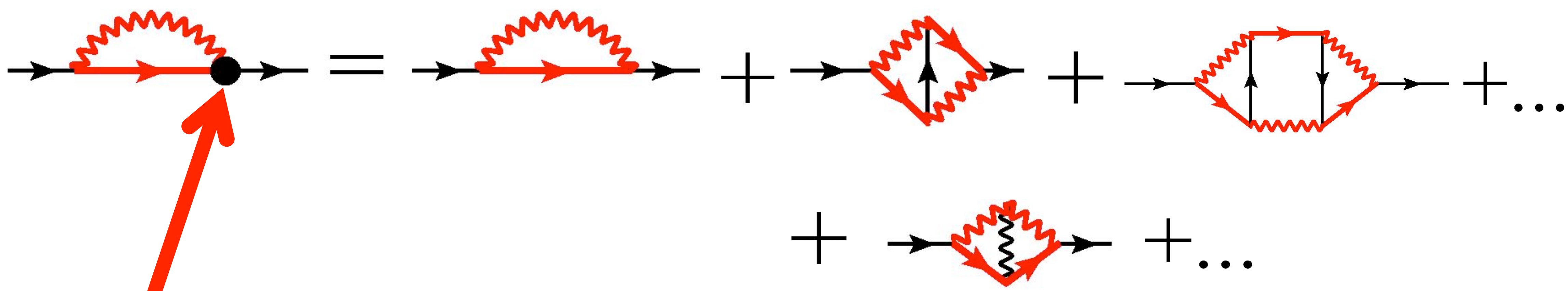


Resummed Perturbation

V. V. Lebedev and A. V. Smilga, Annals Phys. **202,** 229 (1990)

(1), (2)

$\Sigma(p) =$

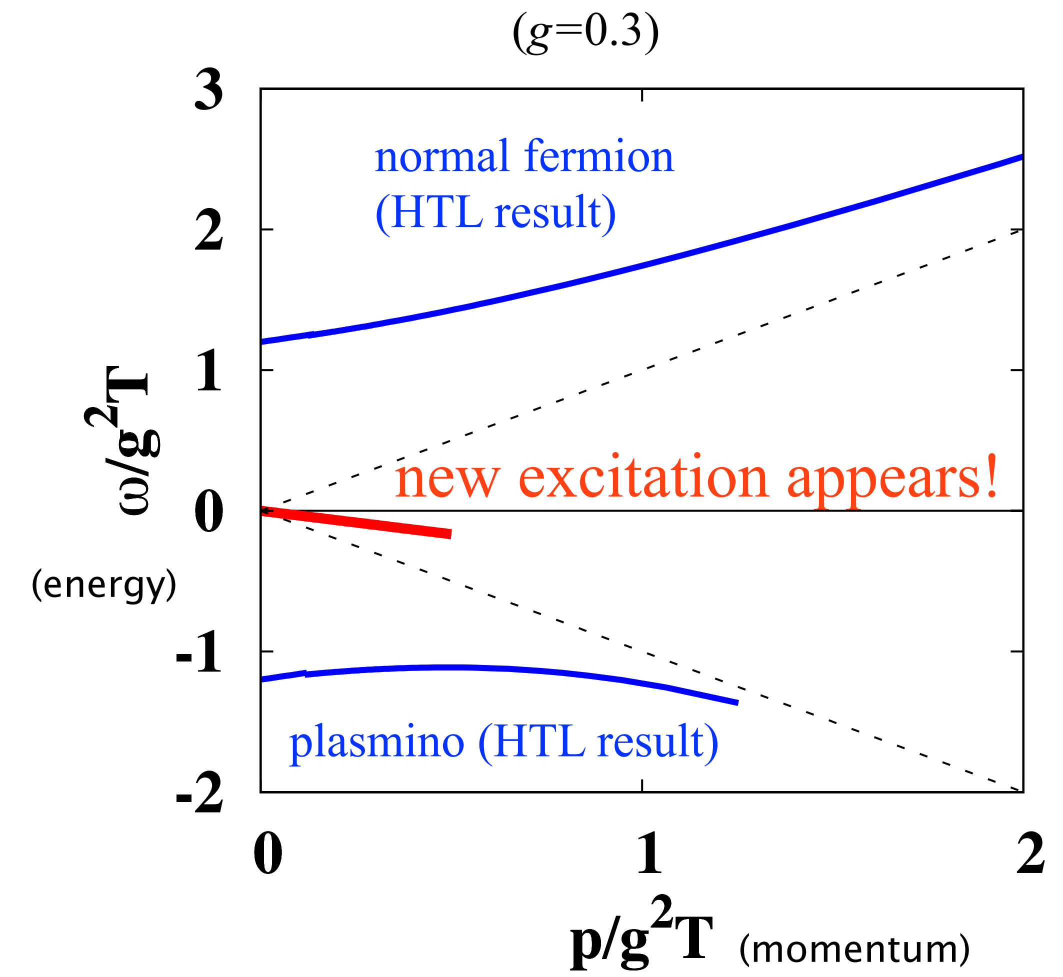


This diagram contains all the contribution at leading order. $(O(g^0))$

Results

We establish a novel excitation in ultrasoft ($p,\omega \lesssim g^2T$) region.

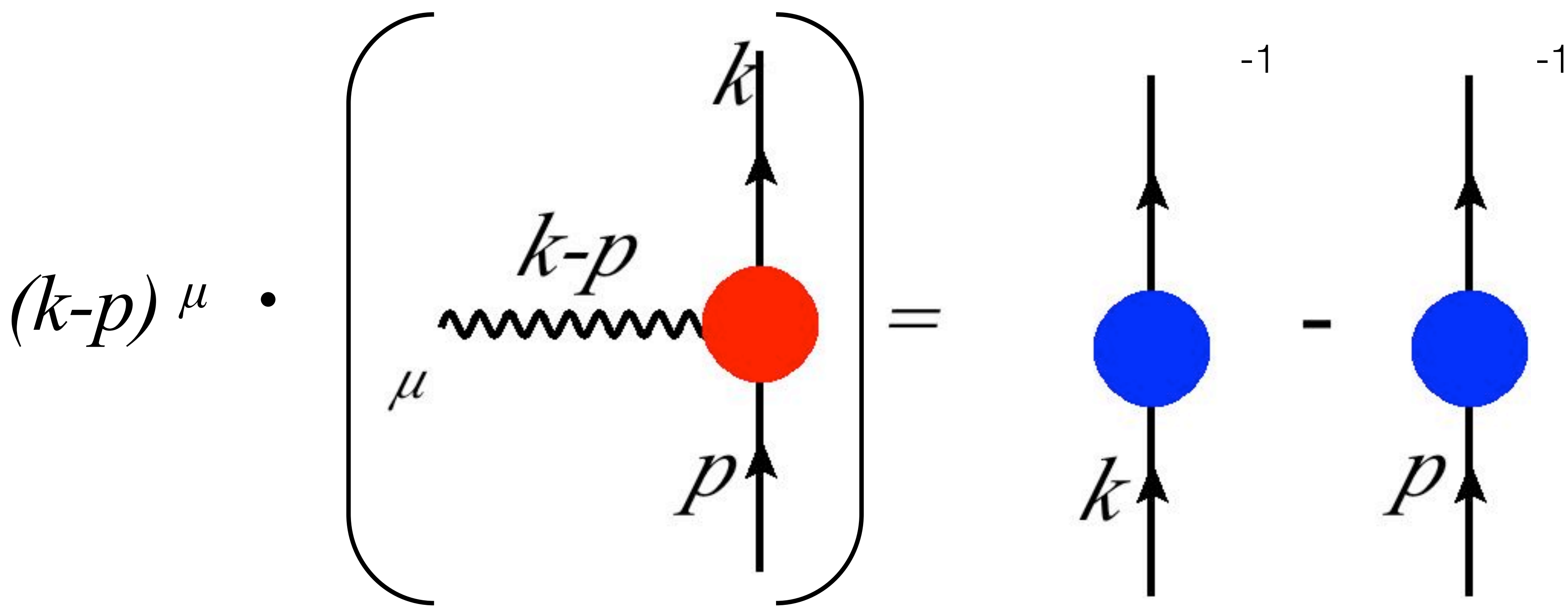
dispersion relation	$\mathrm{Re}\,\omega = -p/3$
Decay width	$\mathrm{Im}\,\omega = \zeta = O(g^2T)$
Residue	$\begin{cases} \dfrac{g^2}{144\pi^2} & \text{QED} \\ g^2\dfrac{(4+N_f)^2}{48\pi^2} & \text{QCD} \end{cases}$



dispersion relation in fermionic sector

Consistency check using gauge symmetry

Ward-Takahashi identity: derived from the gauge symmetry

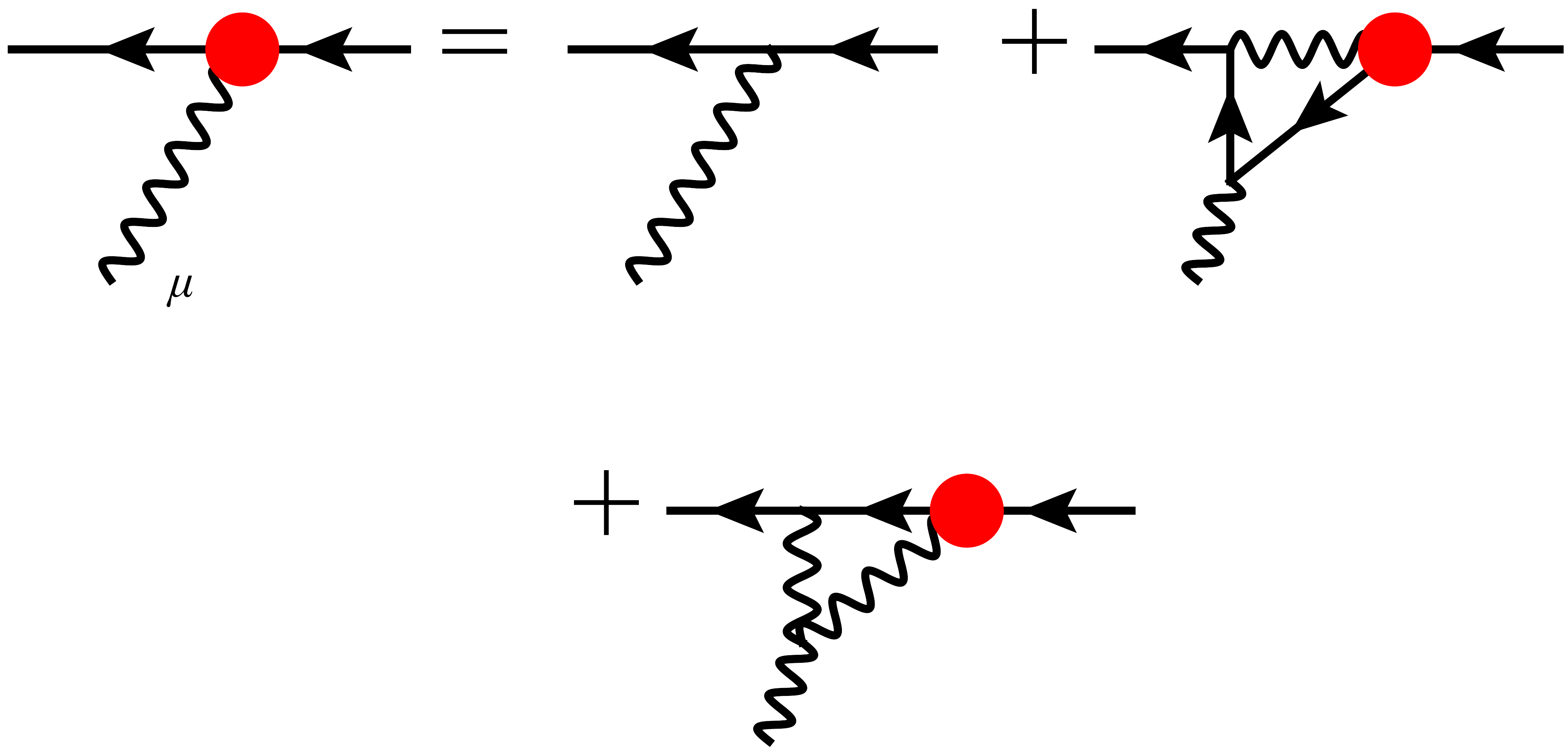


$$S^{-1}(k) \simeq \not{k} \qquad -S^{-1}(p) \simeq \Sigma(p)$$

vertex and self-energy are related.

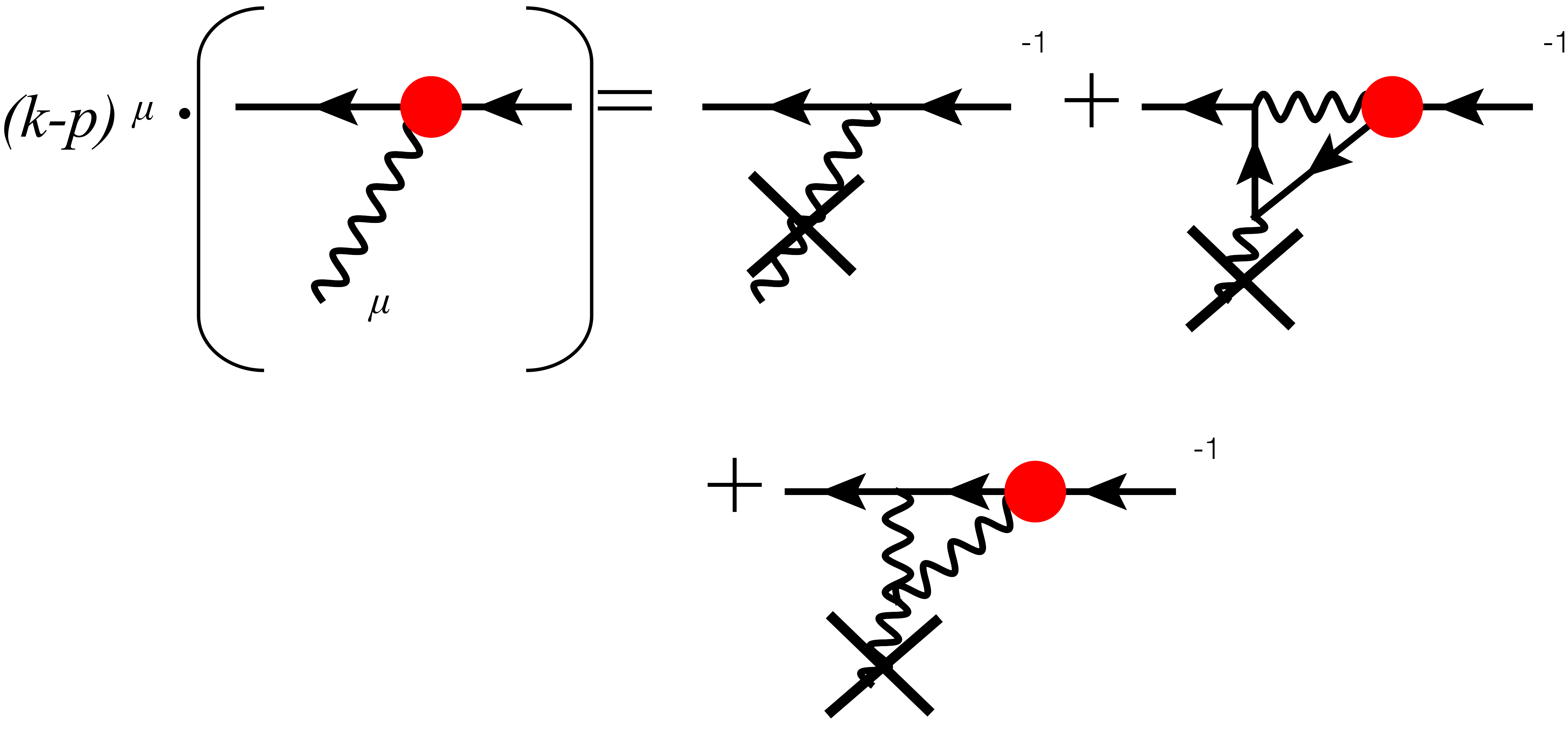
Consistency check using gauge symmetry

self-consistent eq. :



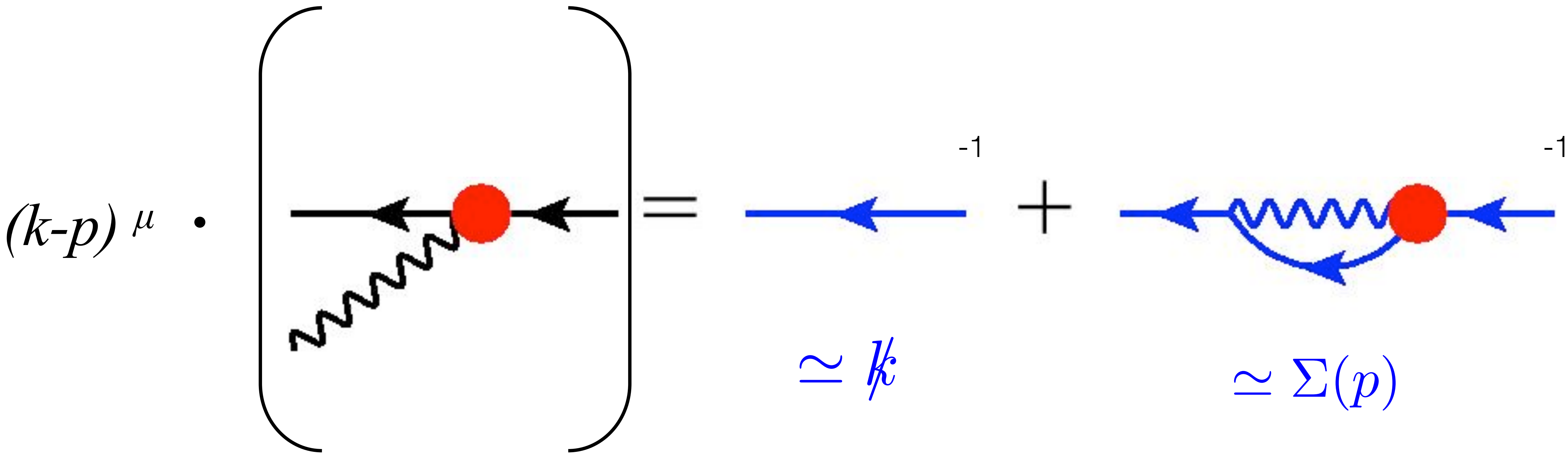
Consistency check using gauge symmetry

self-consistent eq. :



Consistency check using gauge symmetry

self-consistent eq. ⇨ W-T identity



Our vertex and self-energy satisfy the identity; we passed the consistency-check!

Robustness of ultrasoft fermion mode

How robust?

(model: Yukawa model, QED/QCD, NJL
approach in QED/QCD: resummed perturbation, S-D eq.)

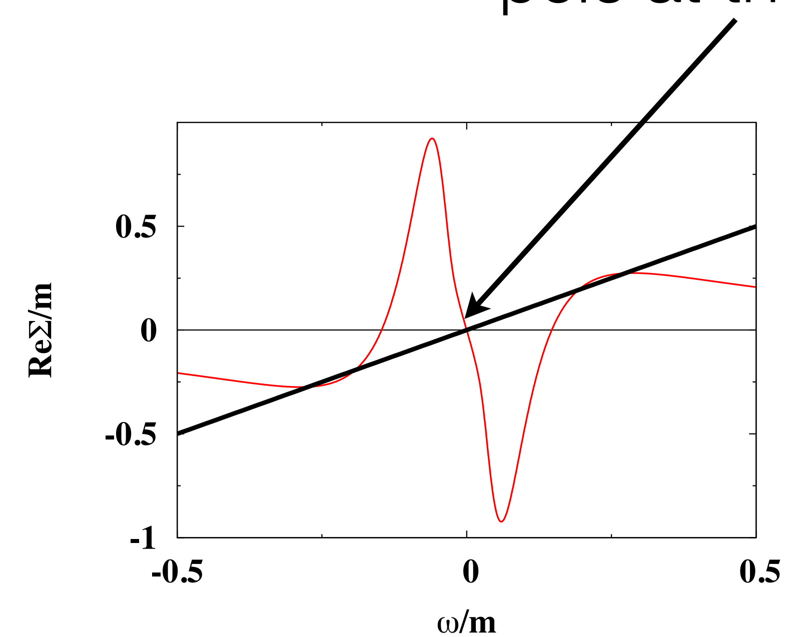
Schwinger-Dyson eq.: M. Harada and Y. Nemoto, PRD **78**, 014004 (2008),
S. X. Qin, L. Chang, Y. X. Liu, and C. D. Roberts, PRD **84**, 014017 (2011).
NJL model: M. Kitazawa, T. Kunihiro and Y. Nemoto, PLB **633**, 269 (2006).

Chiral, Charge symmetry

H. A. Weldon, PRD **61**, 036003 (2000)

$$S^R(p^0, \mathbf{0}) = -\frac{\gamma^0}{p^0 - \Sigma(p^0, \mathbf{0})}$$

Since $\mathrm{Re}\Sigma$ is odd, p^0-$\mathrm{Re}\Sigma$ is zero at p^0=0



Robustness of ultrasoft fermion mode

finite bare fermion mass ⇒ ~~chiral symmetry~~

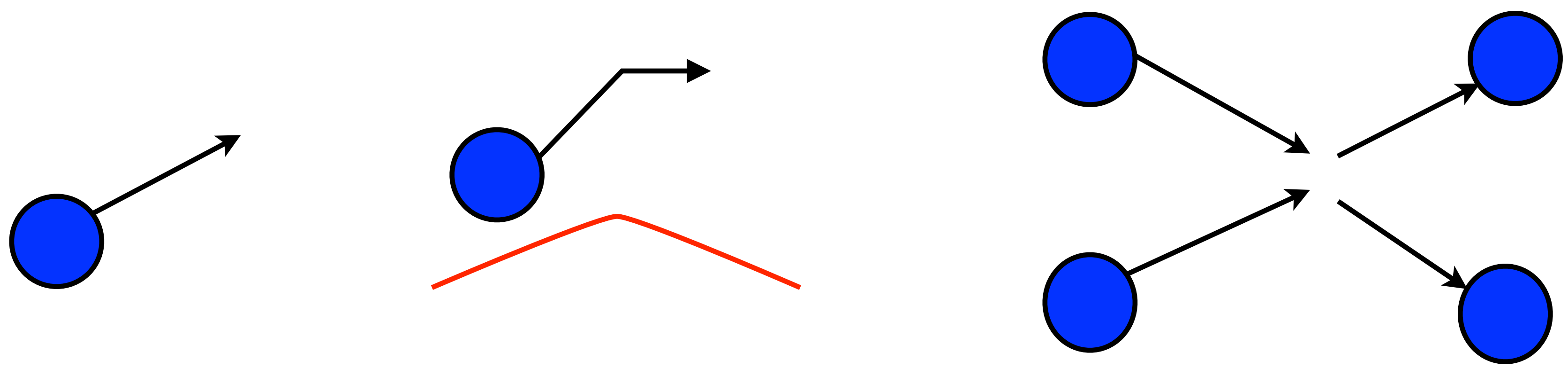
M. Kitazawa, T. Kunihiro, K. Mitsutani and Y. Nemoto, PRD **77**, 045034 (2008).

finite chemical potential ⇒ ~~charge symmetry~~

J. P. Blaizot and <u>**D. S.**</u>, to be submitted.

~~Ultrasoft fermion mode~~

Resummed perturbation as kinetic equation



drift term　　force term　　**collision term**

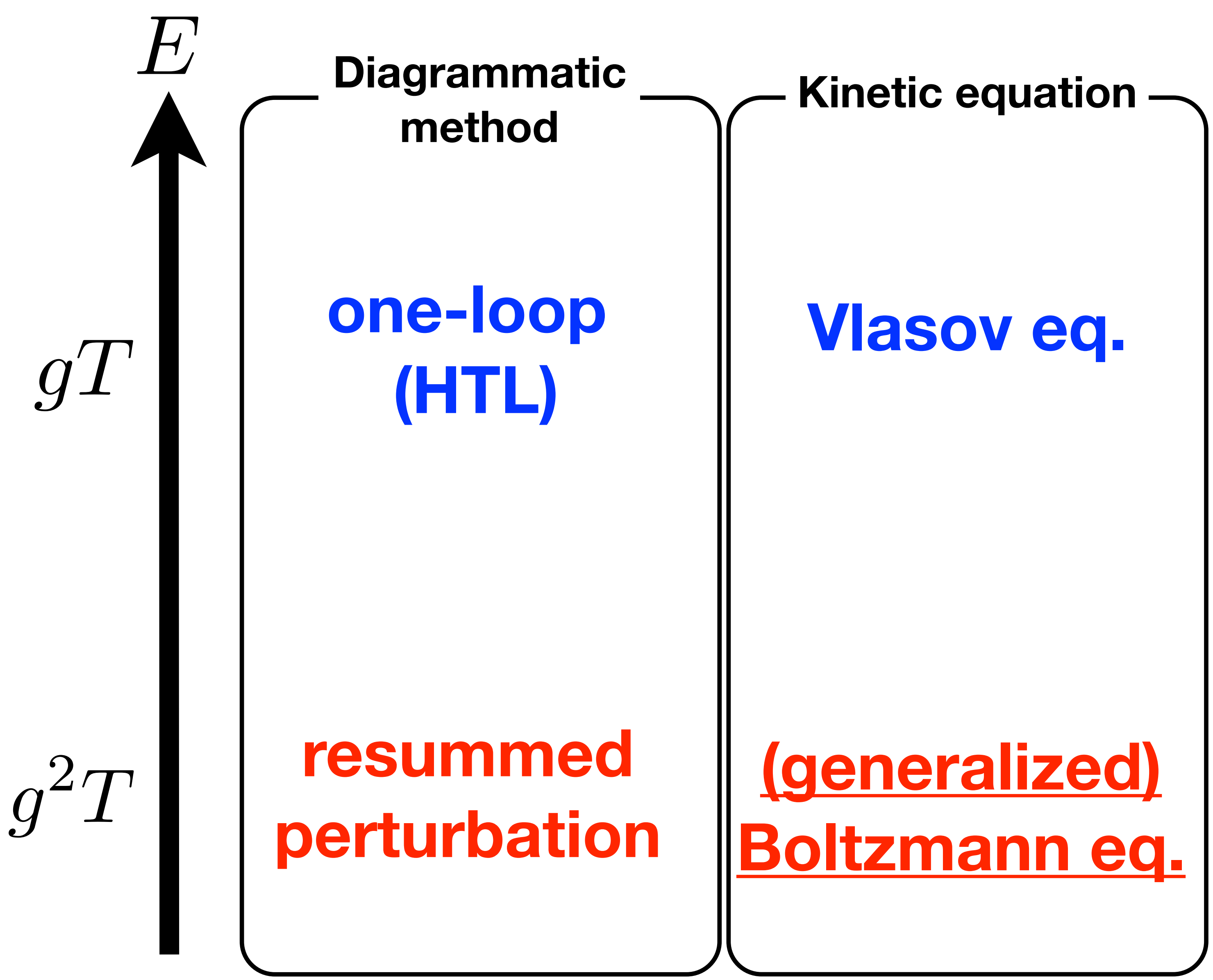
Boltzmann eq.: $\underbrace{(v\cdot\partial_X+g(\boldsymbol{E}(X)+\boldsymbol{v}\times\boldsymbol{B}(X))\cdot\partial_{\boldsymbol{k}})}_{\text{HTL terms}}n(X,\boldsymbol{k})=C[n]$

interaction term

$n(X,\boldsymbol{k})$: distribution function of quark
$v=(1,\boldsymbol{k}/|\boldsymbol{k}|)$: 4-velocity, $\boldsymbol{E}(X)$, $\boldsymbol{B}(X)$: field strength

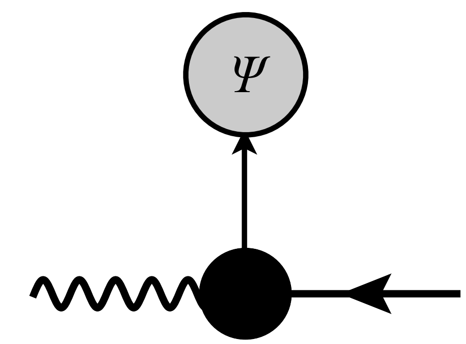
Resummed perturbation as kinetic equation

cf: bosonic sector
J. P. Blaizot and E. Iancu, Nucl. Phys. B **570** 326 (2000).



“Generalized” Boltzmann equation

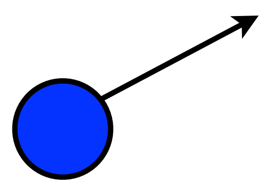
The equation which determines
the strength of quark→gluon process
(unlike the Boltzmann equation, which determines
the distribution function of particles)

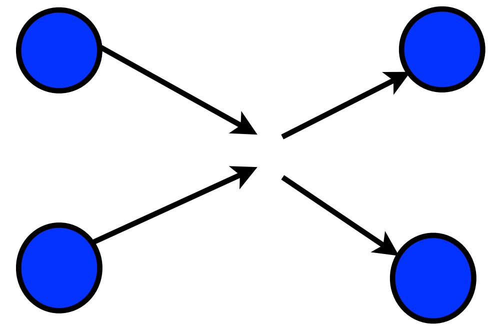


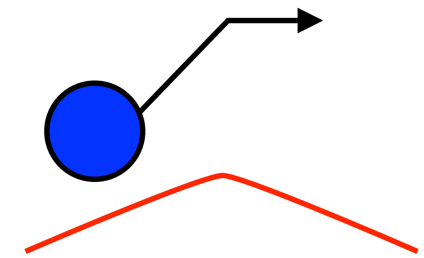
However, we call the resultant equation
(generalized) “**kinetic equation**”.

∵ The equation has the similar structure/derivation
to the Boltzmann equation.

Result







drift term

mass difference term

collision term

force term

$$(2iv\cdot\partial_X \pm \delta m^2/|\mathbf{k}| + 2i\zeta)\not{\Lambda}_\pm(k,X) = 2g\not{\psi}(N(|\mathbf{k}|)+n(|\mathbf{k}|))\Psi(X) + \dots$$

(same order: ∂_X, $\delta m^2/|\boldsymbol{k}|$, $\zeta = O(g^2T)$)

$\Lambda(\boldsymbol{k}, X)$: $K^\mu(k,X) \equiv 2\pi\delta(k^2)(\theta(k^0)\Lambda_+^\mu(\mathbf{k},X)+\theta(-k^0)\Lambda_-^\mu(-\mathbf{k},X))$

It corresponds to the deviation of the distribution function, $\delta n(\boldsymbol{k},X)$

N (n): Bose (Fermi) distribution function

mass difference term and **collision term** appear in addition to Vlasov eq. (HTL approximation) terms.

(resummation of thermal mass and decay width)

Result

Furthermore, the force term is corrected.

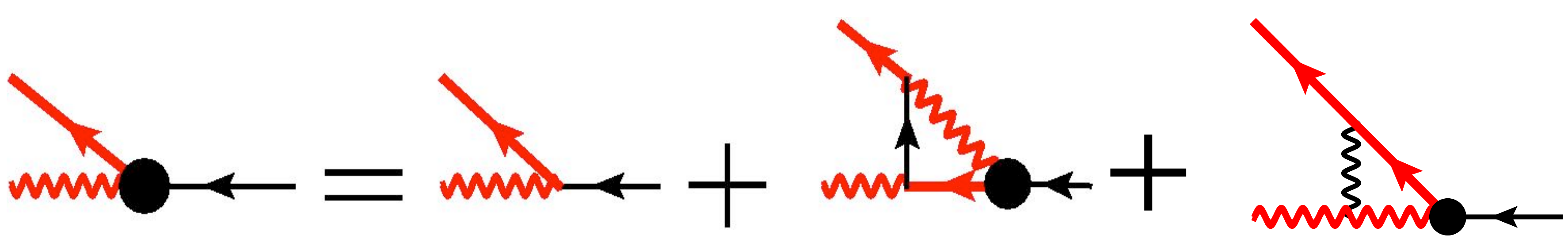
$$(2iv\cdot\partial_X + \delta m^2/|\mathbf{k}| + 2i\zeta)\not\Lambda_\pm(k,X) = 2g\not v(N(|\mathbf{k}|)+n(|\mathbf{k}|))\Big(\Psi(X) - \frac{g}{2}C_f\gamma_i P_T^{ij}(v)\sum_{s=\pm}\int\frac{d^3\mathbf{k}'}{(2\pi)^3}\frac{1}{2|\mathbf{k}'|}\frac{sv^\alpha\gamma_j \pm v'_j\gamma^\alpha}{|\mathbf{k}||\mathbf{k}'|v\cdot v'}\Lambda_{s\alpha}(\mathbf{k}',X)\Big)$$

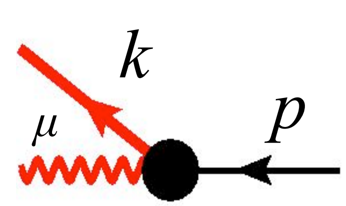
force correction

$C_f = N^2\text{-}1/(2N)$

$P_T^{ij}(k) = \delta_{ij} - \hat{k}_i\hat{k}_j$

summing up ladder diagram:



$(K^\mu(k,p)$ ⇔ 

)

This equation reproduce the resummed perturbation.

Summary

- We established the **novel fermionic mode** in ultrasoft ($<<g^2T$) region with a resummed perturbation.
- We obtained the expression of the **dispersion relation, decay width** and **residue**.
- The obtained vertex and the fermion self-energy satisfy the **Ward-Takahashi identity**.
- We discussed the relation between the **chiral** and **charge symmetry** and the ultrasoft fermion mode.