BCS-BEC crossover in 2D spin-orbit coupled Fermi gases

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Quark Gluon Plasma meets Cold Atoms, Episode III
Hirschegg, Austria
August 25-31, 2012

Outline

- Introduction
- Spin-orbit coupling effect:
  BCS-BEC crossover, or actually
  BCS/BEC-RBEC crossover
- Zeeman field effect:
  Quantum phase transition
- Conclusion

Introduction

- Spin-orbit coupling in solid-state systems, e.g., Rashba SOC & Dresselhaus SOC

$$\mathcal{H}_{\mathrm{SO}}^{\mathrm{R}} = \lambda_{\mathrm{R}}(\hat{p}_x\sigma_y - \hat{p}_y\sigma_x)$$

$$\mathcal{H}_{\mathrm{SO}}^{\mathrm{D}} = \lambda_{\mathrm{D}}(\hat{p}_x\sigma_y + \hat{p}_y\sigma_x)$$

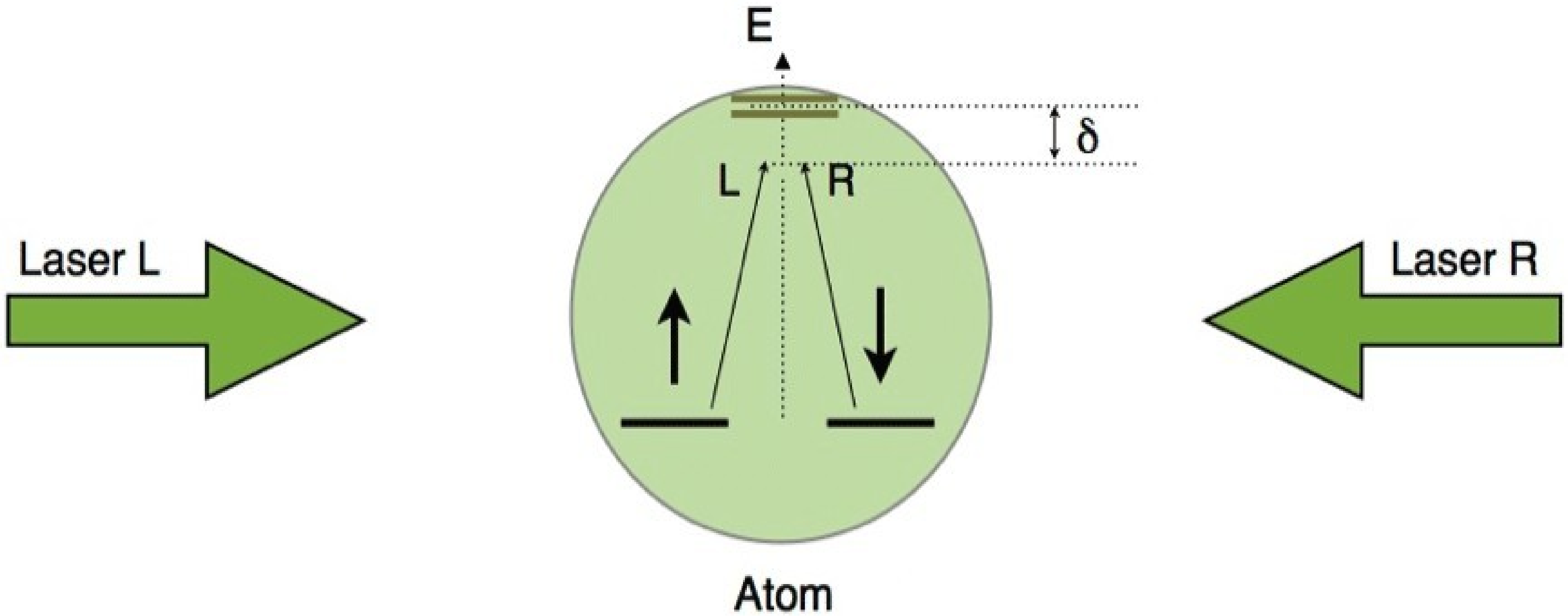
- Spin-orbit coupling for neutral atoms? Yes!
- First experimental realization of spin-orbit coupled Fermi gases

HIGHLIGHTED ARTICLES

Gathered here for your convenience are articles that have been highlighted in *Physics*, selected as an Editors' Suggestion

Spin-Orbit Coupled Degenerate Fermi Gases
Pengjun Wang, Zeng-Qiang Yu, Zhengkun Fu, Jiao Miao, Lianghui Huang, Shijie Chai, Hui Zhai, and Jing Zhang
Published 27 August 2012 *(5 pages)*
095301 [**View** PDF (1,472 kB)]
See accompanying *Physics* Viewpoint

Spin-Injection Spectroscopy of a Spin-Orbit Coupled Fermi Gas
Lawrence W. Cheuk, Ariel T. Sommer, Zoran Hadzibabic, Tarik Yefsah, Waseem S. Bakr, and Martin W. Zwierlein
Published 27 August 2012 *(5 pages)*
095302 [**View** PDF (2,037 kB)]
See accompanying *Physics* Viewpoint



APS/Erich J. Mueller

Figure 1: Scheme for generating spin-orbit coupling in a neutral, ultracold atomic gas. Two counterpropagating laser beams couple two spin states by a resonant stimulated two-photon Raman transition: an atom in a spin-up (↑) state is excited to a virtual level by absorbing a photon from the left beam, then flips to the spin-down (↓) state by emitting another photon into the right beam. The lasers are detuned by a frequency δ from an excited multiplet. This stimulated Raman process results in a momentum kick to the atom, leading to single-particle eigenstates where spin and momentum are entangled.

E. J. Mueller, *Physics* 5, 96 (2012)

Model Hamiltonian

$$H_s = \int d\mathbf{r}\psi^\dagger(\mathbf{r})\left(\frac{\hat{\mathbf{p}}^2}{2M} - \mu + \mathcal{H}_{SO} + \mathcal{H}_Z\right)\psi(\mathbf{r}),$$

$$H_{int} = -U\int d\mathbf{r}\ \psi_\uparrow^\dagger(\mathbf{r})\psi_\downarrow^\dagger(\mathbf{r})\psi_\downarrow(\mathbf{r})\psi_\uparrow(\mathbf{r}).$$

Spin-orbit coupling

$$\mathcal{H}_{SO} = \lambda_R(\hat{p}_x\sigma_y - \hat{p}_y\sigma_x) + \lambda_D(\hat{p}_x\sigma_y + \hat{p}_y\sigma_x)$$

Zeeman fields

$$\mathcal{H}_Z = \frac{\Omega_R}{2}\sigma_z + \frac{\delta}{2}\sigma_y$$

Motivations

- Spin-orbit coupling effects on the BCS-BEC crossover
- Zeeman field effects in the presence of strong spin-orbit coupling
- We consider the case

$$\lambda_{\mathrm{D}} = 0, \quad \delta = 0,$$
$$\lambda_{\mathrm{R}} = \lambda, \quad \frac{\Omega_{\mathrm{R}}}{2} = h$$

Functional Path Integral

$$\mathcal{Z} = \int \mathcal{D}\psi \mathcal{D}\bar{\psi} \exp\{-\mathcal{S}[\psi, \bar{\psi}]\}$$

$$\mathcal{S}[\psi, \bar{\psi}] = \int_0^\beta d\tau \int d^3\mathbf{r} \bar{\psi}\partial_\tau \psi + \int_0^\beta d\tau H(\psi, \bar{\psi})$$

Hubbard-Stratonovich trans. & Integrating out fermions →

$$\mathcal{Z} = \int \mathcal{D}\Phi \mathcal{D}\Phi^\dagger \exp\{-\mathcal{S}_{\text{eff}}[\Phi, \Phi^\dagger]\}$$

$$\mathcal{S}_{\text{eff}}[\Phi, \Phi^\dagger] = \frac{1}{U}\int dx |\Phi(x)|^2 - \frac{1}{2}\mathrm{Tr}\ln[\mathbf{G}^{-1}(x, x')]$$

→ **Mean field + Fluctuations**

Spin-orbit coupling effect

- Two-body problem: binding energy & molecule effective mass $U^{-1} = \sum_{\mathbf{k}} (2\epsilon_{\mathbf{k}} + \epsilon_B)^{-1}$

$$\ln \frac{E_B}{\epsilon_B} = \frac{2\lambda}{\sqrt{E_B - \lambda^2}} \arctan \frac{\lambda}{\sqrt{E_B - \lambda^2}}$$

$$\frac{2m}{m_B} = 1 - \frac{1}{2\kappa} \frac{2\sqrt{\kappa - 1} - (\kappa - 2)\left(\frac{\pi}{2} - \arctan \frac{\kappa - 2}{2\sqrt{\kappa - 1}}\right)}{2\sqrt{\kappa - 1} + \left(\frac{\pi}{2} - \arctan \frac{\kappa - 2}{2\sqrt{\kappa - 1}}\right)}$$

$$\kappa = E_B / \lambda^2$$

He & Huang, PRL108, 145302(2012)

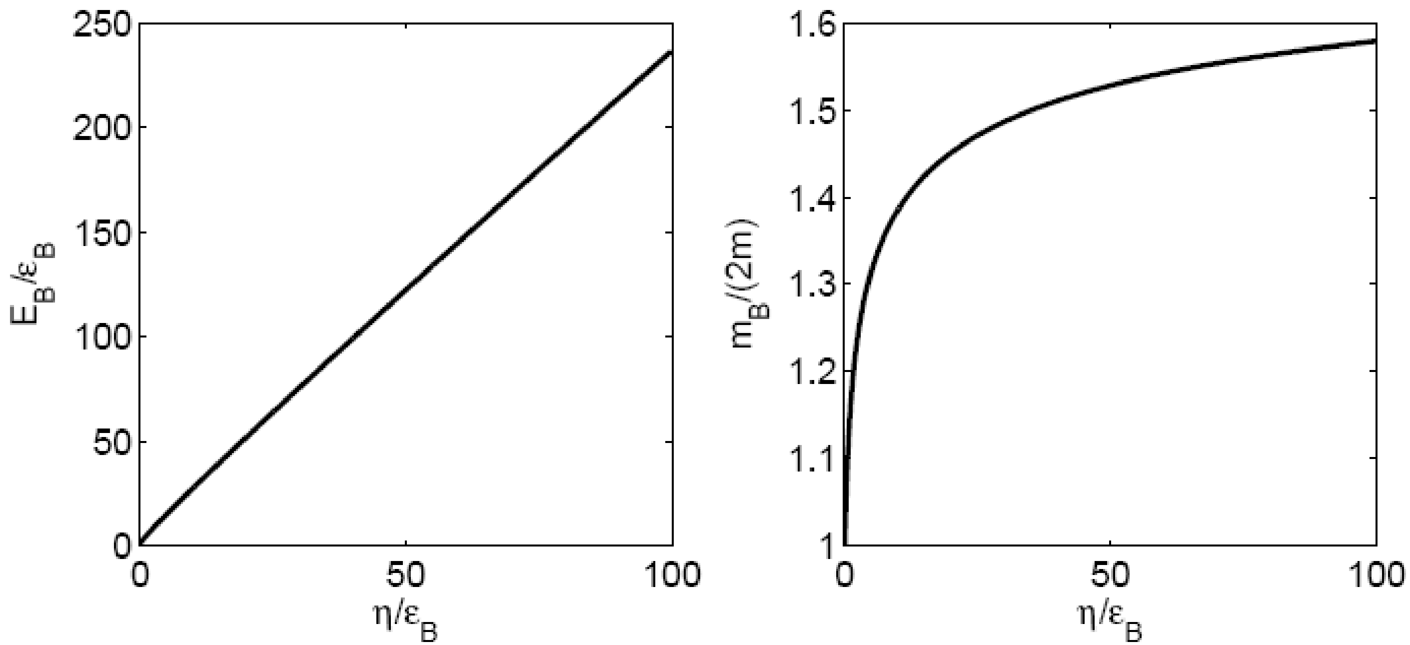


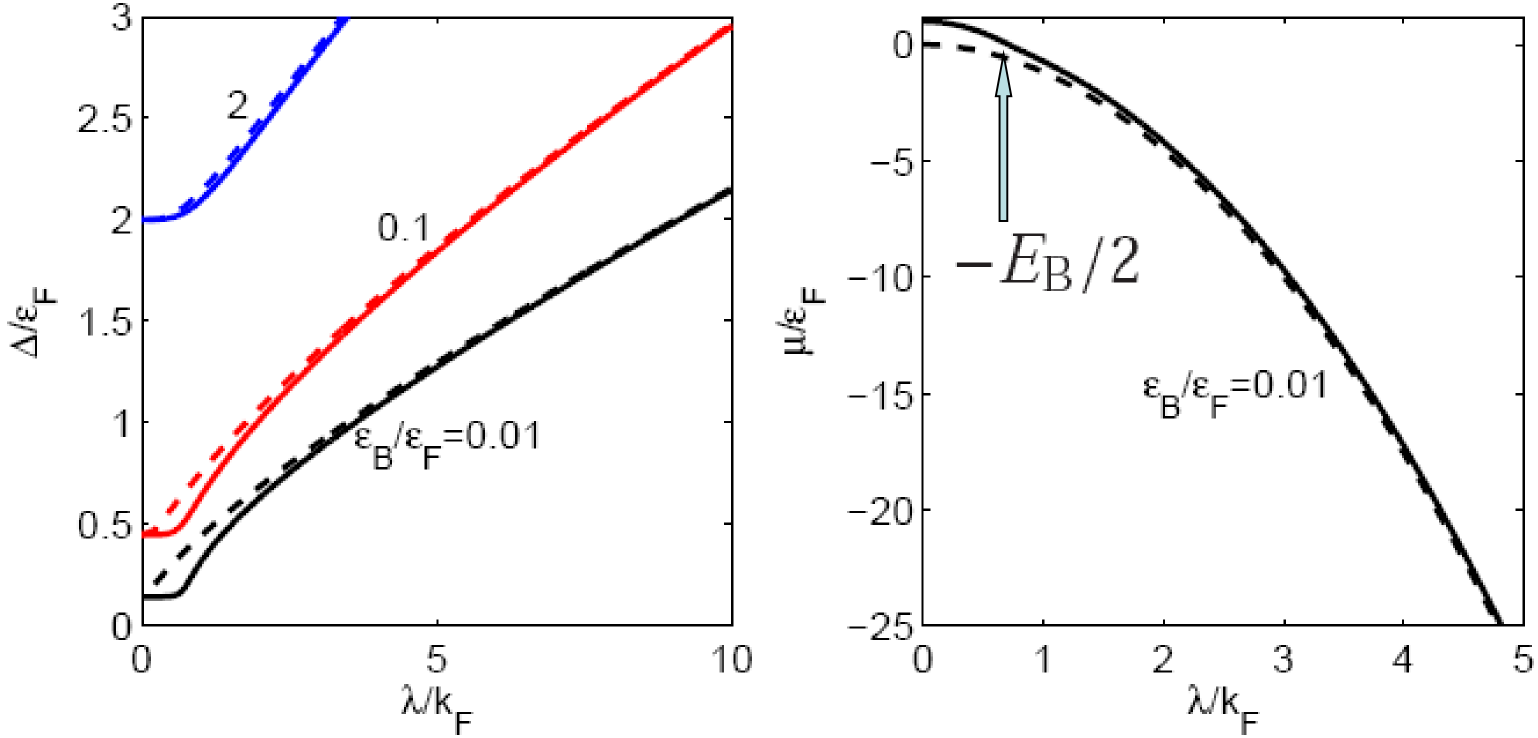
FIG. 1: The binding energy $E_{\rm B}$ (left, divided by $\epsilon_{\rm B}$) and the effective mass $m_{\rm B}$ (right, divided by $2m$) as functions of $\eta/\epsilon_{\rm B}$.

$$\eta = \lambda^2/2$$

➢ Ground state (T=0)

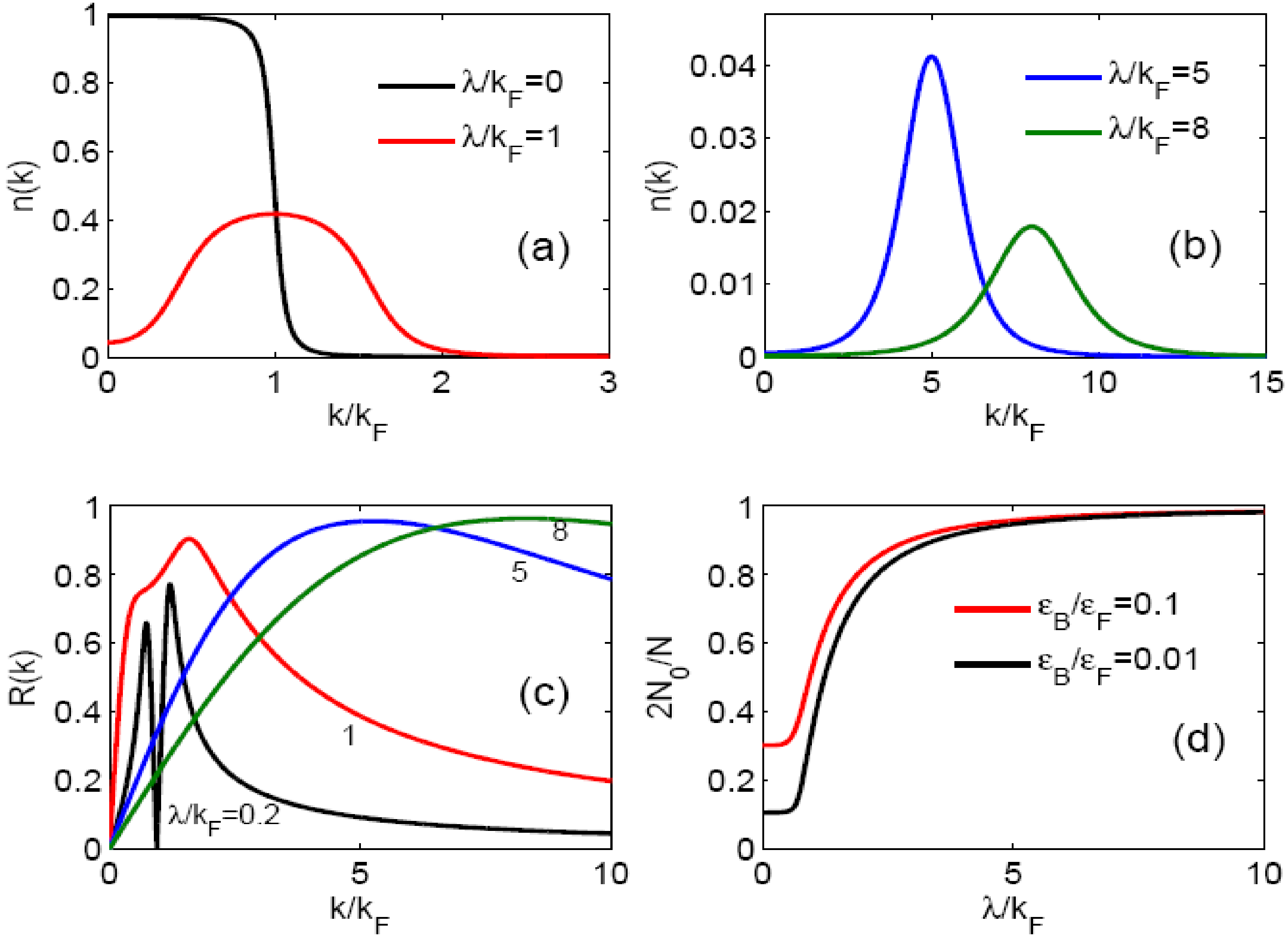
$$[\mu^2+\Delta^2]^{1/2}-\mu=\epsilon_{\rm B}\exp\left[2I_1\left(\mu/\eta,\Delta/\eta\right)\right],$$

$$[\mu^2+\Delta^2]^{1/2}+\mu=2\epsilon_{\rm F}-2\eta\left[1-I_2\left(\mu/\eta,\Delta/\eta\right)\right]$$



BCS-BEC crossover by tuning the SOC strength!

➢Ground state (T=0): more



$R(k) = |\phi_{\uparrow\uparrow}(k)|/|\phi_{\uparrow\downarrow}(k)|$ $\quad$ $n_0 = \sum_{\mathbf{k}}[|\phi_{\uparrow\downarrow}(\mathbf{k})|^2 + |\phi_{\uparrow\uparrow}(\mathbf{k})|^2]$

BKT transition temperature

effective action for phase fluctuation

$$\frac{1}{2}\mathcal{J}\int d^2\mathbf{r}\,[\nabla\theta(\mathbf{r})]^2 \quad \text{phase stiffness } \mathcal{J} = \frac{\rho_s}{4m}$$

$$\Rightarrow \quad T_{\rm BKT} = \frac{\pi}{2}\mathcal{J}$$

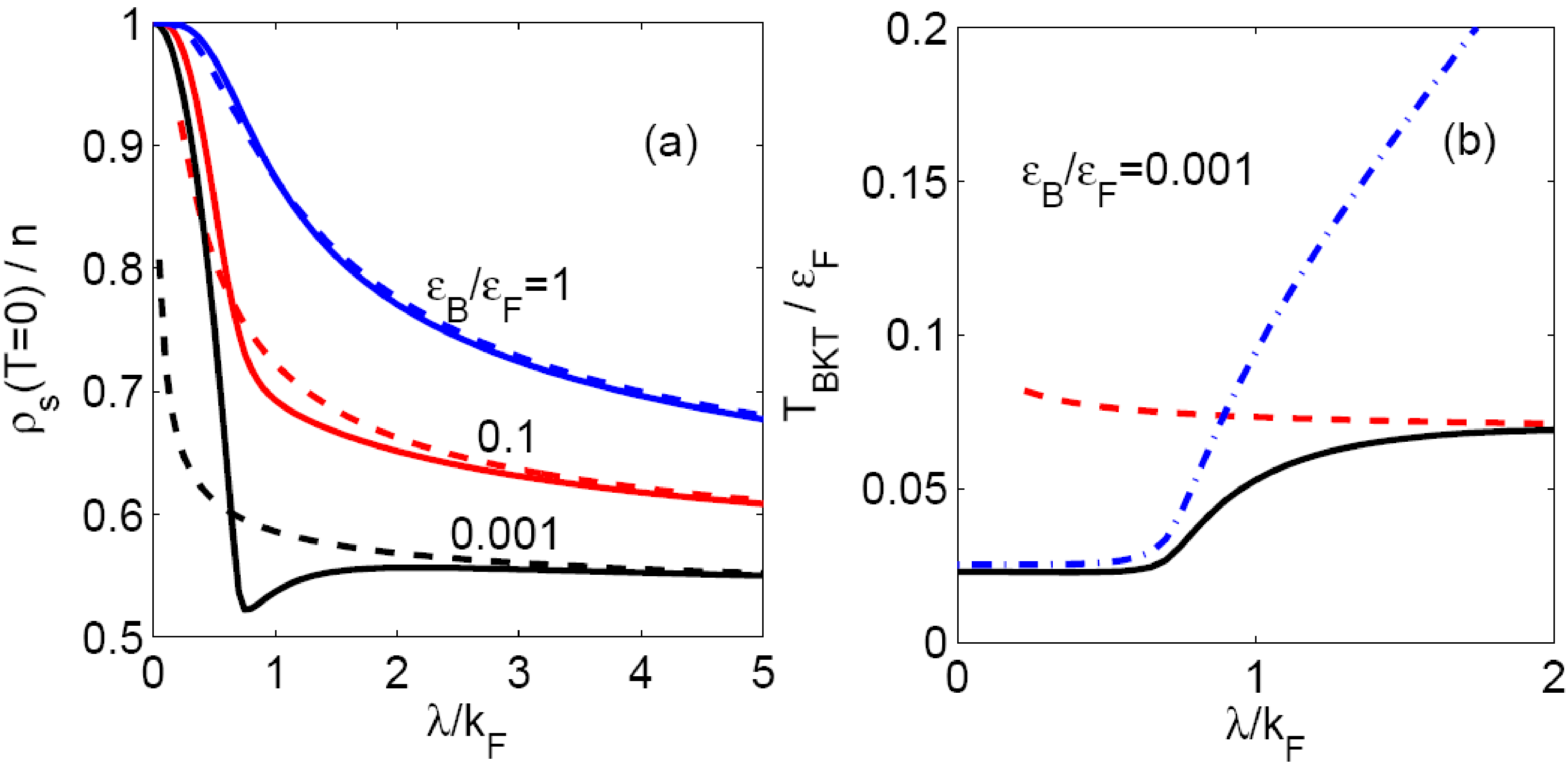
Superfluid density $\rho_s = n - \rho_1 - \rho_2$

$$\rho_1 = (\lambda/8\pi)\sum_{\alpha=\pm}\int_0^\infty dk\alpha(\xi_k^\alpha + \Delta^2/\xi_k)[1 - 2f(E_k^\alpha)]/E_k^\alpha$$

$$\rho_2 = -(1/4\pi)\sum_{\alpha=\pm}\int_0^\infty kdk(k+\alpha\lambda)^2 f'(E_k^\alpha)$$

$$E_{\mathbf{k}}^{\pm} = [(\xi_{\mathbf{k}}^{\pm})^2 + \Delta^2]^{1/2}$$

$$\xi_{\mathbf{k}}^{\pm} = \xi_{\mathbf{k}} \pm \lambda|\mathbf{k}|$$

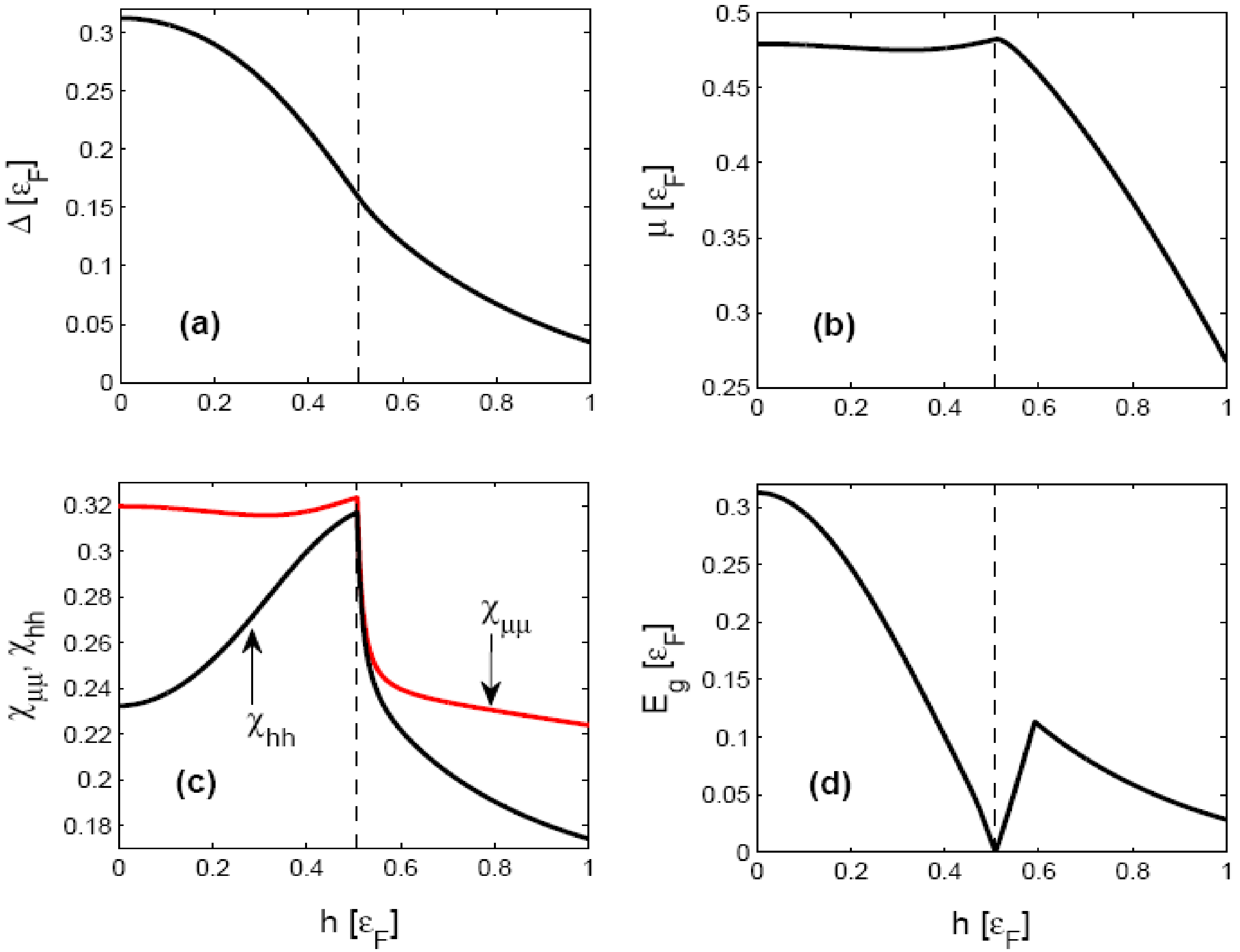


For large SOC, we have analytically

$$\rho_s(T \ll T_\Delta) \simeq \frac{2m}{m_{\rm B}} n, \quad \mathcal{J}(T \ll T_\Delta) \simeq \frac{n_{\rm B}}{m_{\rm B}}$$

Also true for 3D: He & Huang, PRB86, 014511(2012)

Zeeman field effects



He & Huang, arXiv: 1207. 5685

➢ Quantum phase transition? Yes!

(1) Phase transition higher than second order
(2) Non-analyticities from gapless quasiparticles

Quasiparticle dispersions

$$E_{\mathbf{k}}^{\alpha} = [E_{\mathbf{k}}^2 + \eta_{\mathbf{k}}^2 + 2\alpha\zeta_{\mathbf{k}}]^{1/2} \qquad \alpha = \pm$$

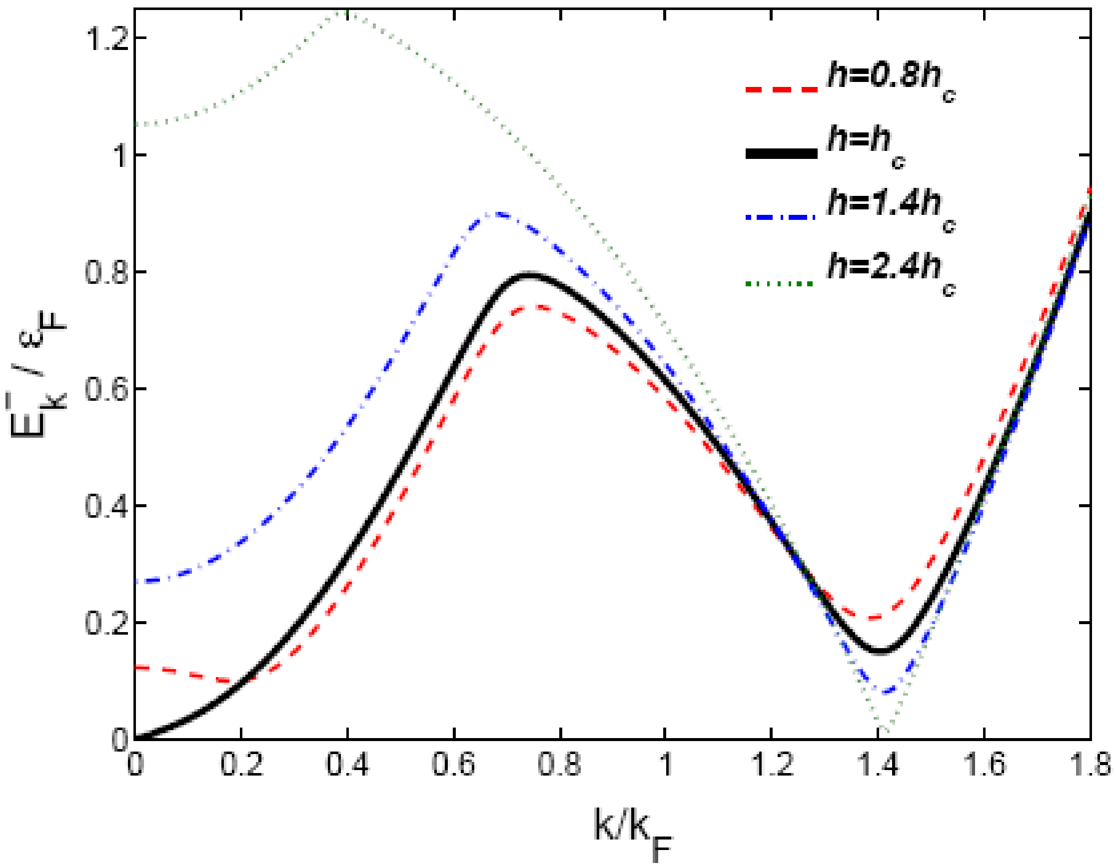
where $E_{\mathbf{k}} = [\xi_{\mathbf{k}}^2 + \Delta^2]^{1/2} \qquad \eta_{\mathbf{k}} = [\lambda^2\mathbf{k}^2 + h^2]^{1/2}$

$$\zeta_{\mathbf{k}} = [\xi_{\mathbf{k}}^2\eta_{\mathbf{k}}^2 + h^2\Delta^2]^{1/2} \qquad \xi_{\mathbf{k}} = \mathbf{k}^2/2 - \mu$$

$$\Rightarrow \quad (E_{\mathbf{k}}^{+})^2(E_{\mathbf{k}}^{-})^2 = (E_{\mathbf{k}}^2 - \eta_{\mathbf{k}}^2)^2 + 4\lambda^2\mathbf{k}^2\Delta^2$$

Gapless only at $C_0 = \mu^2 + \Delta^2 - h^2 = 0$

where $E_{\mathbf{k}}^{-} = v_c|\mathbf{k}| + O(|\mathbf{k}|^2)$



$$h_c = \sqrt{\mu^2 + \Delta^2}$$

➢Non-analyticities

$$\chi_{\mu\mu} = -\frac{\partial^2 \Omega(\mu, h)}{\partial \mu^2}, \quad \chi_{hh} = -\frac{\partial^2 \Omega(\mu, h)}{\partial h^2}$$

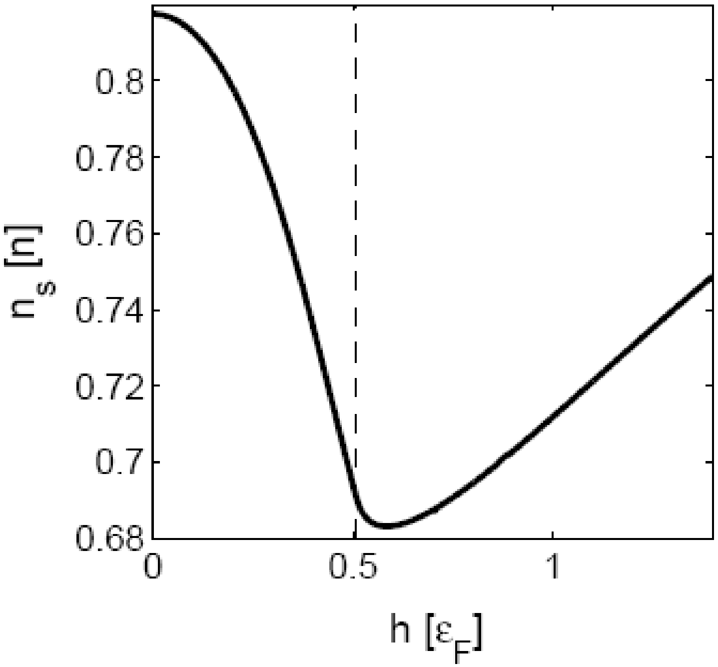
contain integrals like $\mathcal{I}_{ij} \sim \int_0^\infty k dk \frac{Q_i Q_j}{(E_{\mathbf{k}}^-)^3} g(k)$

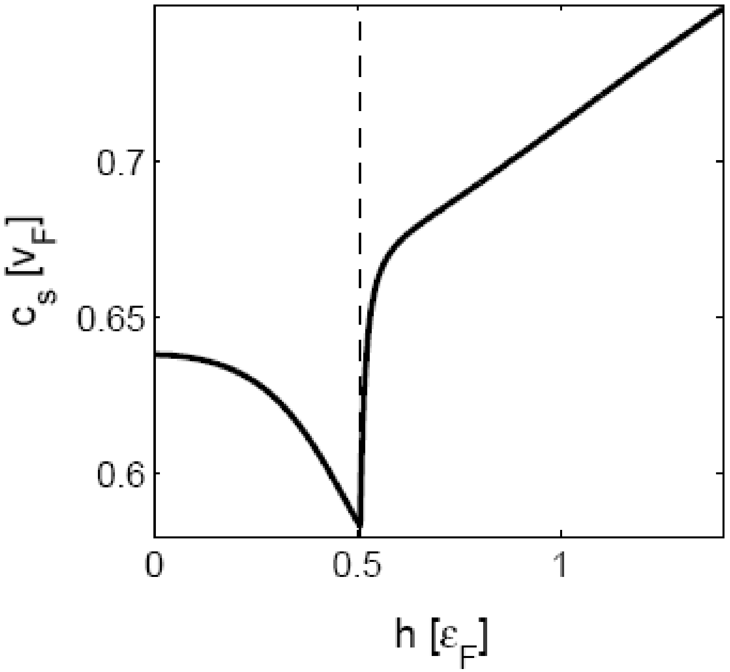
$$Q_i \text{ go as } k^2 \text{ for } k \to 0$$

l-th derivatives with respect to \mu and h contain integrals that go as

$$\int_0^\epsilon k dk \frac{k^{4-2l}}{k^3} = \int_0^\epsilon dk k^{2-2l}$$

➢Collective mode property also shows non-analyticity





$$h \rightarrow \infty, \quad n_s \rightarrow n \qquad\qquad c_s \rightarrow v_F \simeq \tilde{v}_F/\sqrt{2}$$

The system can be mapped to a weakly coupled p_x+ip_y superfluid at large h

Conclusions

- Unusual two-body bound states in the presence of SOC (rashbon)
- BCS-BEC crossover induced by tuning the strength of SOC
- Quantum phase transition at nonzero Zeeman field

Thanks for your attention!

Spin-orbit coupling

- Spin-orbit coupling from Dirac Equation

$$H = \frac{\mathbf{p}^2}{2m} + V(\mathbf{r}) + \frac{e\hbar}{mc}\boldsymbol{\sigma}\cdot\mathbf{B} + \boxed{\frac{\hbar}{4m^2c^2}\boldsymbol{\sigma}\cdot[\mathbf{p}\times\nabla V(\mathbf{r})]}$$

- Hydrogen atom

$$H_{\mathrm{SO}} = \frac{e^2}{4\pi\epsilon_0}\frac{\hbar}{4m^2c^2}\frac{\boldsymbol{\sigma}\cdot\mathbf{L}}{r^3}$$

- Solid state system: Rashba effect

$$\nabla V(\mathbf{r}) \rightarrow \alpha \mathbf{e}_z$$

$$H_{SO} = \lambda(\boldsymbol{\sigma} \times \mathbf{p}) \cdot \mathbf{e}_z = \lambda(\sigma_x p_y - \sigma_y p_x)$$

Rashba spin-orbit coupling

→ Breaks inversion symmetry
& lifts spin degeneracy

→ Plays important role in realizing topological insulators & topological superconductors/superfluids