EMMI workshop

Quark Gluon Plasma meets Cold Atoms – Episode III

Hirschegg, Austria, August 25–31, 2012

Quantum magnetism of mass-imbalanced fermionic mixtures

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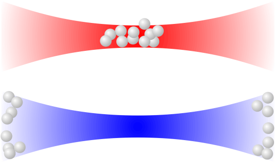
FOR 801

GOETHE UNIVERSITÄT FRANKFURT AM MAIN

Introduction: a) ultracold atoms in optical lattices

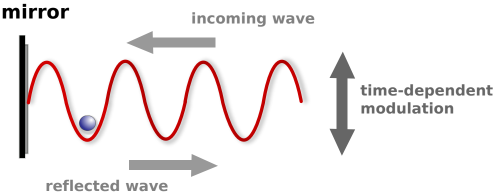
- dipole potential: $V_{dip} = \dfrac{\hbar\Omega_R^2\delta}{\delta^2 + \Gamma_e^2/4}$



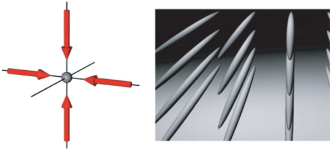
red detuning
$\delta < 0$

blue detuning
$\delta > 0$

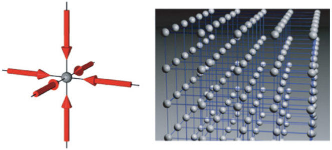
- standing wave (1d lattice):



- 2d lattices:



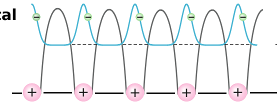
- 3d lattices:



I. Bloch *et al.*, Rev. Mod. Phys. **80**, 885 (2008)

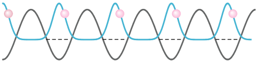
b) ultracold atoms in optical lattices: advantages

Real crystal



vs.

Optical lattice



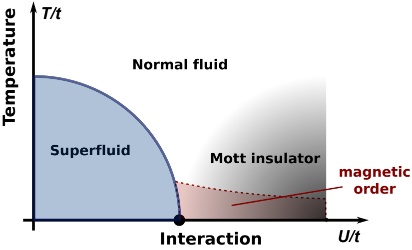
	electrons in crystals	atoms in optical lattices	
	tunability	tunability	source
statistics	fermions	bosons/fermions	isotope choice
hopping ampl.	tunable	easily tunable	laser intensity
interaction	repulsive or weak attractive, short/long-range	attractive/repulsive weak/strong short/long-range	Feshbach resonances Feshbach resonances atomic dipole moment
disorder	always present	absent/present (speckle/box)	additional lasers
mixtures	two-component [spin-$\uparrow,\downarrow$: SU(2)]	multi-comp., bose-fermi [SU(2)..SU(2)$\times$SU(6)..]	different species, hyperfine states
imbalances	only population (very hard)	population, mass	different at. densities diff. species, laser int.
geometries	cubic, triangular, hexagonal, . . .	cubic, triangular, hexagonal; any (2d)	setup of lasers; holographic projections

c) magnetic ordering of ultracold atomic mixtures

At low temperatures (i.e. at low entropies) it is often energetically favorable for a many-body system to have a ground state with the broken symmetry. This is the case for mixtures of 2 and more atomic species (both bosons and fermions) in simple lattice geometries (cubic, square, ...) with integer N_{tot} per site.

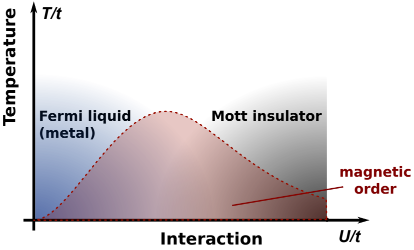
- two-component repulsively-interacting mixtures in a cubic lattice:

– bosons:



[Y. Li *et al.*, PRA **85**, 023624 '12]

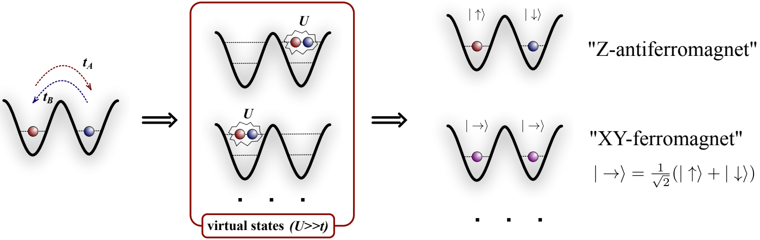
– fermions:



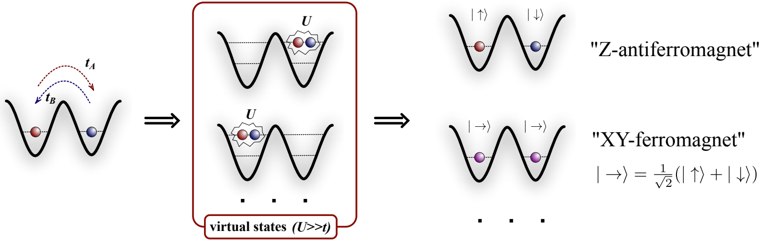
[P.R. Kent *et al.*, PRB **72**, 060411 '05]

In the Mott-insulator (MI) region, a long-range magnetic order is governed by the second order tunneling processes, $J_{\rm magn} \propto t^2/U$

d) magnetic couplings and types of order at T=0



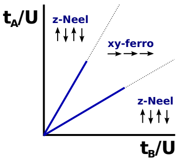
d) magnetic couplings and types of order at T=0



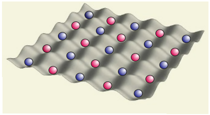
- bosons:

eff. Hamiltonian: $\hat{\mathcal{H}}_{\text{eff}} = J_z \sum_{\langle ij \rangle} \hat{S}_i^Z \hat{S}_j^Z + J_\perp \sum_{\langle ij \rangle} (\hat{S}_i^X \hat{S}_j^X + \hat{S}_i^Y \hat{S}_j^Y) - h \sum_i S_i^Z$

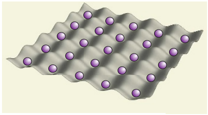
magnetic couplings: $J_z = 2\frac{t_A^2 + t_B^2}{U_{AB}} - \frac{4t_A^2}{U_{AA}} - \frac{4t_B^2}{U_{BB}}$, $J_\perp = -\frac{4t_A t_B}{U_{AB}}$



Z-antiferromagnet



XY-ferromagnet



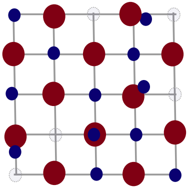
E. Altman, W. Hofstetter, E. Demler, M.D. Lukin, *New J. Phys.* **5**, 2003

A. Kuklov and B. Svistunov *Phys. Rev. Lett.* **90**, 2003

- fermions: will be discussed below in detail

Ultracold fermionic mixtures with mass imbalance in optical lattices

System under study



□ mixtures of two types of fermions,
possible experimental realizations in optical lattices:

- ^{6}Li–^{40}K mixture; [Taglieber *et al.*, PRL **100**, 010401 '08]
- mixtures of alkaline-earth atoms (^{171}Yb, ^{173}Yb, ^{87}Sr, ...); [Taie et al, PRL **105**, 190401 10; ...]
- state-sensitive optical lattices. [Mandel, *et al.*, PRL **91**, '03]

□ Fermi-Hubbard Hamiltonian (repulsive interactions, $U > 0$):

$$\hat{\mathcal{H}} = -t_A \sum_{\langle i,j \rangle} (\hat{a}_i^\dagger \hat{a}_j + \mathrm{H.c.}) - t_B \sum_{\langle i,j \rangle} (\hat{b}_i^\dagger \hat{b}_j + \mathrm{H.c.}) + U \sum_i \hat{n}_{iA} \hat{n}_{iB} + \sum_i \sum_{\alpha = A,B} (V_i - \mu_\alpha) \hat{n}_{i\alpha},$$

- $t_A \neq t_B$: hopping (="mass") imbalance;
- $N_A \neq N_B$: population imbalance (depends on μ_A, μ_B, and V_i).

pseudospin Hamiltonian

- Fermi-Hubbard Hamiltonian:

$$\hat{\mathcal{H}} = -t_A \sum_{\langle i,j \rangle} (\hat{a}_i^\dagger \hat{a}_j + h.c.) - t_B \sum_{\langle i,j \rangle} (\hat{b}_i^\dagger \hat{b}_j + h.c.)$$
$$+U \sum_i \hat{n}_{iA} \hat{n}_{iB} + \sum_i \sum_{\alpha = A,B} (V_i - \mu_\alpha) \hat{n}_{i\alpha},$$

Schrieffer–Wolff transformations

$t_{A,B} \ll U$

$n_{Ai} + n_{Bi} \approx 1$

- Effective Hamiltonian:

$$\hat{\mathcal{H}}_{\text{eff}} = J_{\parallel} \sum_{\langle ij \rangle} \hat{S}_i^Z \hat{S}_j^Z + J_{\perp} \sum_{\langle ij \rangle} (\hat{S}_i^X \hat{S}_j^X + \hat{S}_i^Y \hat{S}_j^Y) - \Delta\mu \sum_i S_i^z,$$

[anisotropic Heisenberg (XXZ) model]

- for $t_A \neq t_B$: $J_{\parallel} = 2(t_A^2 + t_B^2)/U$ is always larger than $J_{\perp} = 4 t_A t_B / U$

due to the imbalance in hopping amplitudes, $SU(2)$ spin symmetry is reduced to a lower $Z_2 \times U(1)$ symmetry [Cazalilla *et al.*, PRL **95**, 226402 '05]

consequences: ground states, LRO

anisotropic Heisenberg (XXZ) model: **interplay between different orderings**

$$\hat{\mathcal{H}}_{\text{eff}} = J_{\parallel} \sum_{\langle ij \rangle} \hat{S}_i^Z \hat{S}_j^Z + J_{\perp} \sum_{\langle ij \rangle} (\hat{S}_i^X \hat{S}_j^X + \hat{S}_i^Y \hat{S}_j^Y) - \Delta\mu \sum_i S_i^z, \quad (J_{\parallel} > J_{\perp}).$$

□ small pop.imb. & large mas.imb., $\Delta\mu \ll (J_{\parallel} - J_{\perp})$:

• the ground state at $T = 0$ is Z-antiferromagnet (ferrimagnet);
• at large mass imbalance, $t_A \gg t_B \Rightarrow J_{\parallel} \gg J_{\perp}$, one arrives at the Ising model;
• the excitation spectrum is gapped $\Rightarrow$ possibility for a true long-range Z-AF order at $T > 0$ in low dimensions ($d < 3$).

□ large pop.imb. & small mas.imb., $\Delta\mu \gg (J_{\parallel} - J_{\perp})$:

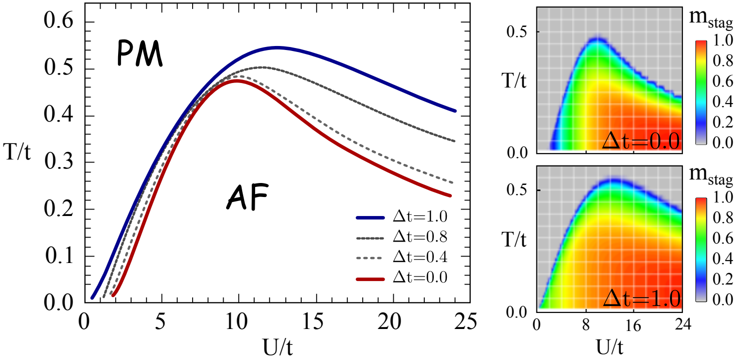
• the ground state at $T = 0$ is XY-antiferromagnet (canted AF);
• Mermin-Wagner theorem forbids a true long-range order in XY-plane at $T > 0$ in low dimensions ($d < 3$).

□ $\Delta\mu \sim (J_{\parallel} - J_{\perp})$: **phase transition (1st order)**

critical temperature enhancement ($\Delta\mu = 0$)

□ **dynamical mean-field theory (DMFT) analysis:**

– *TU* diagram at half-filling, $\mu = U/2$

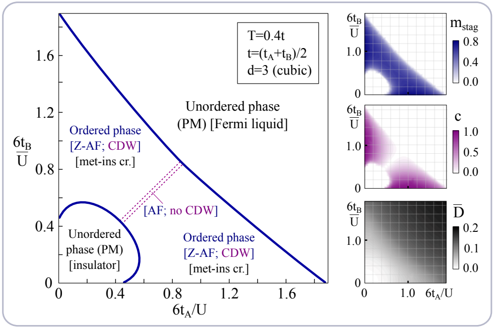


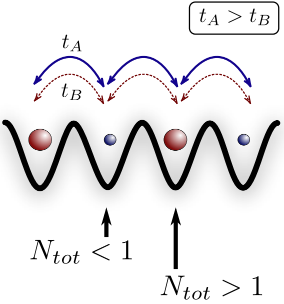
$$t = (t_A + t_B)/2, \qquad \Delta t = (t_A - t_B)/(t_A + t_B).$$

hopping (mass) imbalance results in growth of the ordered phase

additional type of order: CDW ($\Delta\mu = 0$)

– finite-temperature t_A/t_B phase diagram at half-filling:



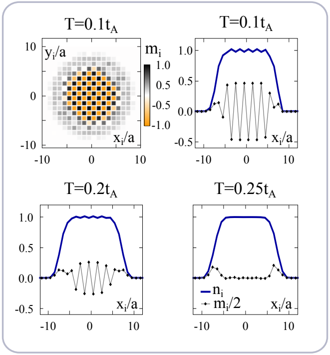


charge-density wave (CDW): two adjacent sites have different values of double- and zero-occupancy. $D_i = \langle \hat{n}_{Ai}\hat{n}_{Bi} \rangle$, $K_i = \langle (1 - \hat{n}_{Ai})(1 - \hat{n}_{Bi}) \rangle$, $c = |D - K|/(D + K)$, $\overline{D} = (D + K)/2$.

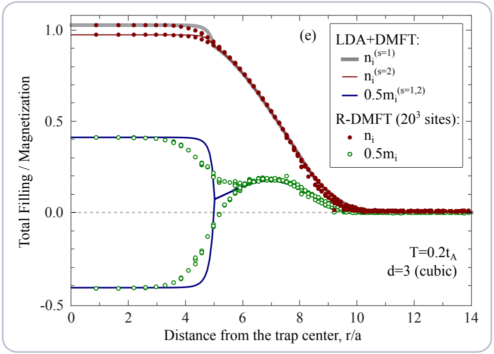
CDW is present only in the ordered region with a finite imbalance

real-space analysis for a trap ($\Delta\mu = 0$)

- 2d optical lattice (R-DMFT):



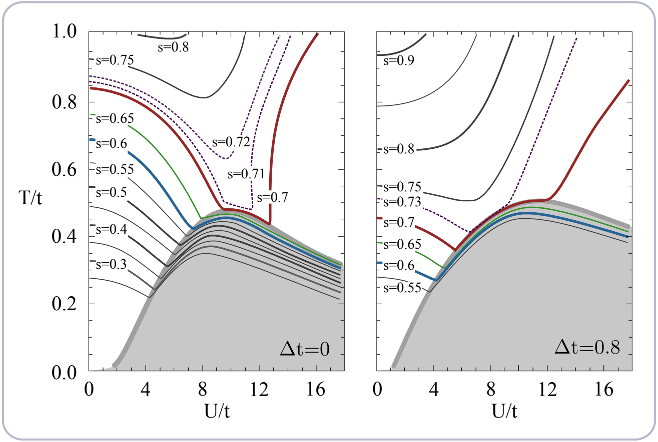
- 3d optical lattice [(R- *vs.* LDA+) DMFT]:



- CDW is clearly seen in the bulk region with AF ordering;
- "ferromagnetic" ring emerges from a wider distribution of a lighter component;
- R-DMFT better describes the detailed structure in the intermediate region than LDA+DMFT (interplay between AFM-bulk and FM-shell).

entropy analysis for a homogeneous system ($\Delta\mu = 0$)

– entropy calculations are based on the Maxwell relation in the integral form, $s(\mu_0, T) = \int_{-\infty}^{\mu_0} (\partial n/\partial T) d\mu$.

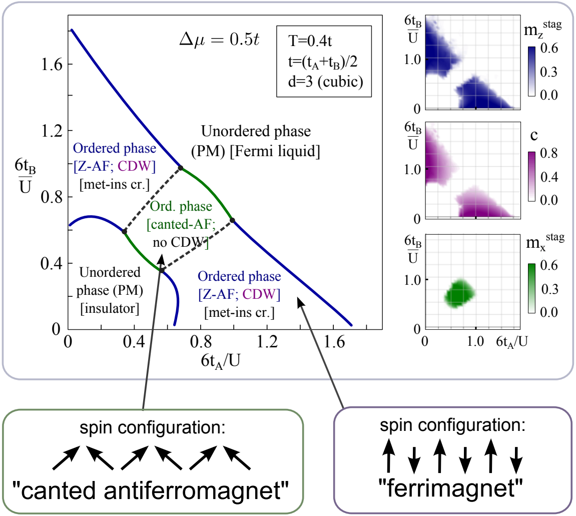


• mass-imbalanced mixtures allow a much closer approach (at equal given entropy) to the critical region;

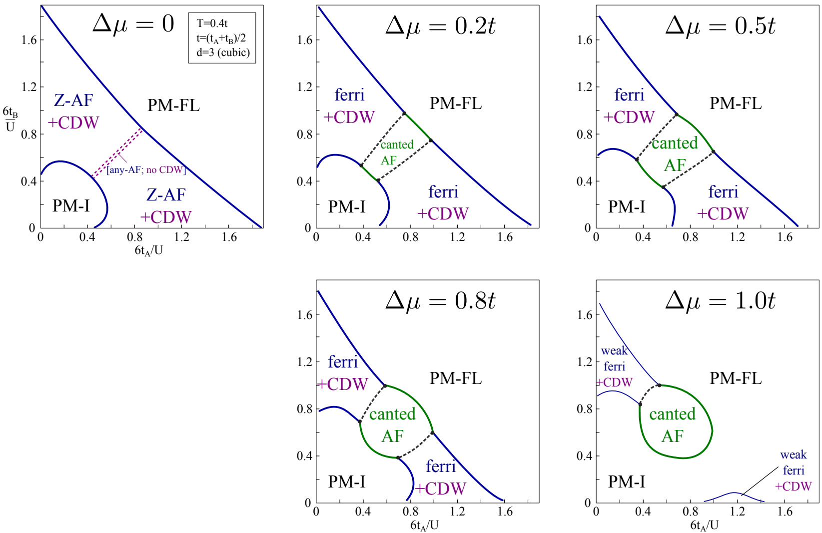
• in the Fermi-liquid region the entropy increases with hopping imbalance.

including population imbalance: $\Delta\mu \neq 0$

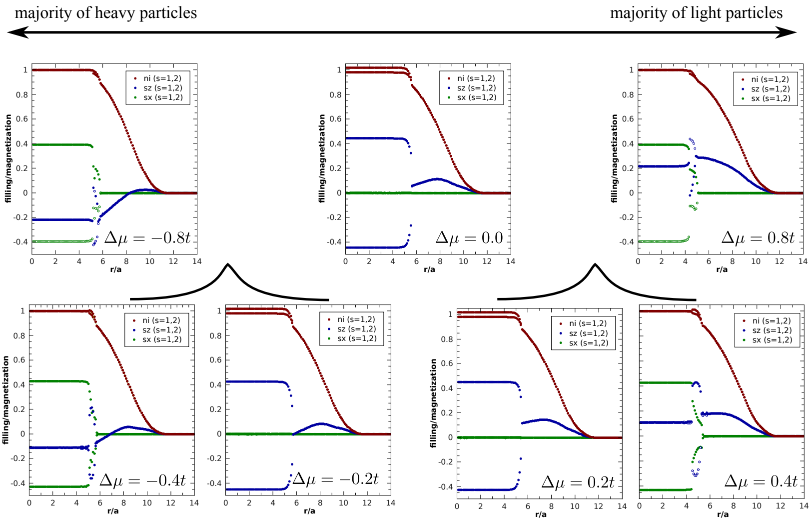
– eff. Hamiltonian: $\mathcal{H}_{\text{eff}} = J_{\|} \sum_{\langle ij \rangle} S_i^z S_j^z + J_{\perp} \sum_{\langle ij \rangle} (S_i^x S_j^x + S_i^y S_j^y) - \Delta\mu \sum_i S_i^z$



evolution of t_A-t_B diagrams with $\Delta\mu$

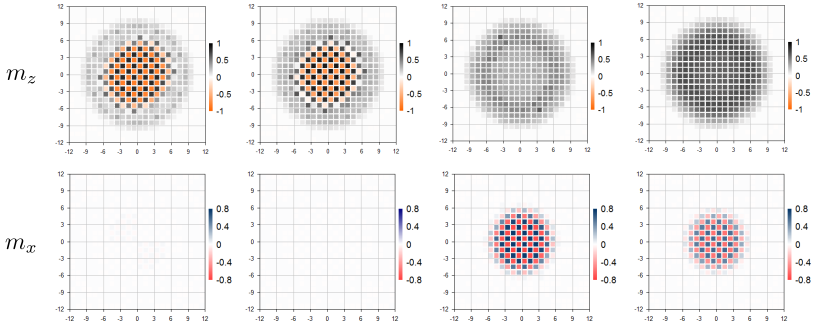


real-space analysis: LDA+DMFT (3d)



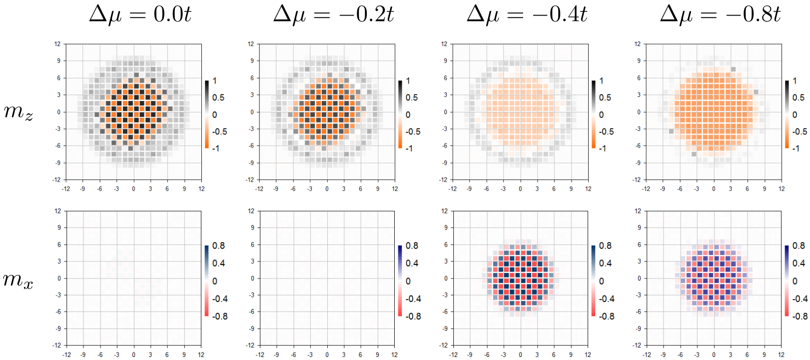
real-space analysis: R-DMFT (2d) [positive $\Delta\mu$]

$\Delta\mu = 0.0t$ | $\Delta\mu = 0.2t$ | $\Delta\mu = 0.4t$ | $\Delta\mu = 0.8t$



$\Delta\mu = 0.0t$	$\Delta\mu = 0.2t$	$\Delta\mu = 0.4t$	$\Delta\mu = 0.8t$
$N_A = 121$	$N_A = 135$	$N_A = 152$	$N_A = 181$
$N_B = 82$	$N_B = 69$	$N_B = 52$	$N_B = 27$

real-space analysis: R-DMFT (2d) [negative $\Delta\mu$]



$N_A = 121$
$N_B = 82$

$N_A = 108$
$N_B = 96$

$N_A = 87$
$N_B = 116$

$N_A = 57$
$N_B = 148$

Conclusions

- mass imbalance enhances the critical temperature;
- ordered phase can be approached with the higher entropy values;
- possibility for a long-range order in low dimensions;
- additional type of order: *charge-density wave* in the AF phase;
- rich phase diagram in the presence of both population and mass imbalance (canted-AF and ferrimagnetic ordering);
- real-space distributions for a trap: different orderings in the bulk, multiple-shell structures.

Special thanks to:

- Walter Hofstetter (Frankfurt);
- Daniel Cocks (Frankfurt);
- Michiel Snoek (Amsterdam).

More details:

A. Sotnikov *et al.*,
PRL **109**, 065301 (2012).
[arXiv:1203.4658]

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Thank you for your attention!