Searching for $f_1(1285)$ in proton-proton collisions

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Physics opportunities with proton beams at SIS100
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Introduction

In this talk we will be concerned with central exclusive production (CEP) of axial-vector $f_1(1285)$ meson ($J^{PC} = 1^{++}$) in proton-(anti)proton collisions at c.m. energies:

- low: HADES (pp) and PANDA ($p\bar{p}$) at FAIR ← ***Lebiedowicz, Nachtmann, Salabura, Szczurek,* PRD 104 (2021) 034031**
- intermediate (WA102) and high (RHIC, LHC) ← *Lebiedowicz, Leutgeb, Nachtmann, Rebhan, Szczurek,* PRD 102 (2020) 114003

The $f_1(1285)$ meson was measured

- in two-photon interactions in e^+e^- reactions (MARKII, TPC/Two-Gamma, L3)
  [A. Szczurek, PRD 102 (2020) 113015 ← on production of f_1 mesons at e^+e^- collisions with double-tagging as a way to constrain the axial meson light-by-light contribution to the muon g-2 and hyperfine splitting of muonic hydrogen]
- in photoproduction process $\gamma p \to f_1 p$ (CLAS Collaboration)
- in CEP pp collisions for c.m. energies 12.7 and 29.1 GeV (WA102)
  and for 13 TeV at the LHC (ATLAS-ALFA) [R. Sikora, CERN-THESIS-2020-235]

Why study the $pp \to pp f_1(1285)$ process?

- What is underlying production mechanism of f_1 at near threshold and at LHC?
  - Poorly known V-V-f_1 (V = ρ^0, ω) and pomeron-pomeron-f_1 coupling strengths and vertex (form factors)
  - Can it be described in holographic QCD?
- What is underlying decay mechanism?
  - e.g., $f_1(1285) \to 4\pi$ decay via $\rho\rho$ or/and $\pi a_1(1260)$ with $a_1 \to \rho\pi \to 3\pi$
  - transition form factors e.g., $\gamma^*\gamma^* \to f_1$, $f_1 \to \gamma\gamma^* \to \gamma e^+e^-$ [see, e.g., Zanke, Hoferichter, Kubis, JHEP 07 (2021) 106]
  - What is the nature of the $f_1(1285)$? For instance, is it a normal $q\bar{q}$ state or $\bar{K}K^*$ molecule?
    see: *Aceti, Dias, Oset, EPJA 51 (2015) 48; Aceti, Xie, Oset, PLB 750 (2015) 609*
- What is optimal observation channel of $f_1(1285)$?
  - $f_1(1285)$ vs $\eta(1295)$

Particle	J^P	overall	$N\gamma$	$N\pi$	$N\sigma$	$N\eta$	$N\rho$	$N\omega$	$N\eta'$
N	$1/2^+$	****							
$N(1440)$	$1/2^+$	****	****	****	***				
$N(1520)$	$3/2^-$	****	****	****	**	****			
$N(1535)$	$1/2^-$	****	****	****	*	****	?	?	
$N(1650)$	$1/2^-$	****	****	****	*	****			
$N(1675)$	$5/2^-$	****	****	****	***	*			
$N(1680)$	$5/2^+$	****	****	****	***	*			
$N(1700)$	$3/2^-$	***	**	***	*	*	*		
$N(1710)$	$1/2^+$	****	****	****		***	*	*	
$N(1720)$	$3/2^+$	****	****	****	*	*	*	*	
$N(1860)$	$5/2^+$	**	*	**	*	*			
$N(1875)$	$3/2^-$	***	**	**	**	*	*	*	
$N(1880)$	$1/2^+$	***	**	*	*	*		**	
$N(1895)$	$1/2^-$	****	****	*	*	****	*	*	****
$N(1900)$	$3/2^+$	****	****	**	*	*		*	**
$N(1990)$	$7/2^+$	**	**	**		*			
$N(2000)$	$5/2^+$	**	**	*	*	*		*	
$N(2040)$	$3/2^+$	*		*					
$N(2060)$	$5/2^-$	***	***	**	*	*	*	*	
$N(2100)$	$1/2^+$	***	**	***	**	*	*	*	**
$N(2120)$	$3/2^-$	***	***	**	**			*	*
$N(2190)$	$7/2^-$	****	****	****	**	*	*	*	
$N(2220)$	$9/2^+$	****	**	****		*			
$N(2250)$	$9/2^-$	****	**	****		*			
$N(2300)$	$1/2^+$	**	?	**			?	?	$N f_1(1285)$
$N(2570)$	$5/2^-$	**		**					
$N(2600)$	$11/2^-$	***		***					
$N(2700)$	$13/2^+$	**		**					

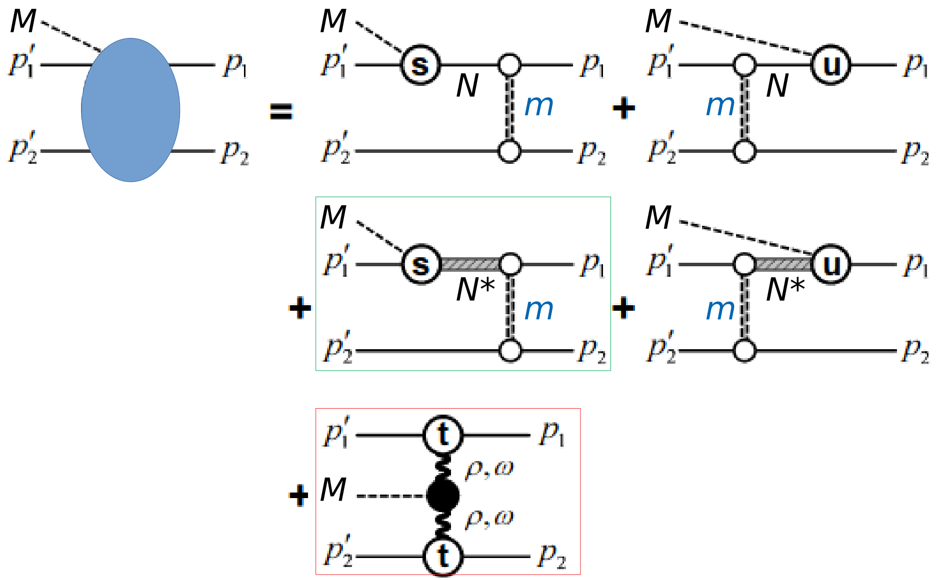
**** Existence is certain.
*** Existence is very likely.
** Evidence of existence is fair.
* Evidence of existence is poor.

← status of the N resonances (N^*) and their decays from PDG

Basic production mechanisms for $p_1 p_2 \to p_1' p_2' M$

$M = \eta, \eta'(958), f_1(1285)$

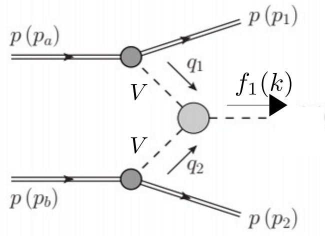
$m = \pi, \sigma, \eta, \rho^0, \omega$



VV-fusion mechanism

$p(p_a,\lambda_a) + p(p_b,\lambda_b) \to p(p_1,\lambda_1) + f_1(k,\lambda_{f_1}) + p(p_2,\lambda_2)$

$p_{a,b}$, $p_{1,2}$ and $\lambda_{a,b}$, $\lambda_{1,2} = \pm\frac{1}{2}$: the four-momenta and helicities of protons
k and $\lambda_{f_1} = 0, \pm 1$: the four-momentum and helicity of the f_1 meson



$q_1 = p_a - p_1, \quad q_2 = p_b - p_2, \quad k = q_1 + q_2$
$t_1 = q_1^2, \quad t_2 = q_2^2, \quad m_{f_1}^2 = k^2$
$s = (p_a + p_b)^2 = (p_1 + p_2 + k)^2$, c.m. energy squared
$s_1 = (p_1 + k)^2, \quad s_2 = (p_2 + k)^2$

VV-fusion amplitude: $\mathcal{M}^{(VV\text{ fusion})}_{pp\to ppf_1} = \mathcal{M}^{(\rho\rho\text{ fusion})}_{pp\to ppf_1} + \mathcal{M}^{(\omega\omega\text{ fusion})}_{pp\to ppf_1}$

$$\begin{aligned}\mathcal{M}^{(VV\text{ fusion})}_{\lambda_a\lambda_b\to\lambda_1\lambda_2\lambda_{f_1}} = \; & (-i)\,(\epsilon^{\alpha}(\lambda_{f_1}))^*\,\bar{u}(p_1,\lambda_1) i\Gamma^{(Vpp)}_{\mu_1}(p_1,p_a) u(p_a,\lambda_a) \\ & \times i\tilde{\Delta}^{(V)\,\mu_1\nu_1}(s_1,t_1)\, i\Gamma^{(VVf_1)}_{\nu_1\nu_2\alpha}(q_1,q_2)\, i\tilde{\Delta}^{(V)\,\nu_2\mu_2}(s_2,t_2) \\ & \times \bar{u}(p_2,\lambda_2) i\Gamma^{(Vpp)}_{\mu_2}(p_2,p_b) u(p_b,\lambda_b)\end{aligned}$$

$$i\Gamma^{(Vpp)}_{\mu}(p',p) = -i\Gamma^{(V\bar{p}\bar{p})}_{\mu}(p',p) = -i g_{Vpp}\, F_{VNN}(t)\left[\gamma_\mu - i\frac{\kappa_V}{2m_p}\sigma_{\mu\nu}(p-p')^{\nu}\right]$$

$g_{\rho pp} = 3.0\,, \quad \kappa_\rho = 6.1\,, \quad g_{\omega pp} = 9.0\,, \quad \kappa_\omega = 0$

κ_V: tensor-to-vector coupling ratio, $\kappa_V = f_{VNN}/g_{VNN}$

$$F_{VNN}(t) = \frac{\Lambda^2_{VNN} - m_V^2}{\Lambda^2_{VNN} - t}$$

For the proton-antiproton collisions we have

$$\begin{aligned}\bar{u}(p_2,\lambda_2) i\Gamma^{(Vpp)}_{\mu_2}(p_2,p_b) u(p_b,\lambda_b) \;\to\; & \bar{v}(p_b,\lambda_b) i\Gamma^{(V\bar{p}\bar{p})}_{\mu_2}(p_2,p_b) v(p_2,\lambda_2) \\ & = -\bar{u}(p_2,\lambda_2) i\Gamma^{(Vpp)}_{\mu_2}(p_2,p_b) u(p_b,\lambda_b)\end{aligned}$$

$$\mathcal{M}^{(VV\text{ fusion})}_{p\bar{p}\to p\bar{p}M} = -\mathcal{M}^{(VV\text{ fusion})}_{pp\to ppM}$$

The standard form of the vector-meson propagator:

$$i\Delta^{(V)}_{\mu\nu}(q) = i\left(-g_{\mu\nu} + \frac{q_\mu q_\nu}{q^2 + i\epsilon}\right)\Delta^{(V)}_T(q^2) - i\frac{q_\mu q_\nu}{q^2 + i\epsilon}\Delta^{(V)}_L(q^2)$$

$$\Delta^{(V)}_T(t) = (t - m_V^2)^{-1}$$

For higher values of s_1 and s_2 we must take into account reggeization:

$$\Delta^{(V)}_T(t_i) \to \tilde{\Delta}^{(V)}_T(s_i,t_i) = \Delta^{(V)}_T(t_i)\left(\exp(i\phi(s_i))\,\frac{s_i}{s_{\rm thr}}\right)^{\alpha_V(t_i)-1}$$

$$\phi(s_i) = \frac{\pi}{2}\exp\left(\frac{s_{\rm thr} - s_i}{s_{\rm thr}}\right) - \frac{\pi}{2}$$

where $s_{\rm thr}$ is the lowest value of s_i possible here: $s_{\rm thr} = (m_p + m_{f_1})^2$
We use the linear form for the vector meson Regge trajectories :
$\alpha_V(t) = \alpha_V(0) + \alpha'_V\,t\,, \quad \alpha_V(0) = 0.5\,, \quad \alpha'_V = 0.9\ \text{GeV}^{-2}$

VVf_1 coupling , corresponds to (l, S) = (2,2)

$$\mathcal{L}'_{VVf_1}(x) = \frac{1}{M_0^4} g_{VVf_1}\left(V_{\kappa\lambda}(x)\overleftrightarrow{\partial}_\mu\overleftrightarrow{\partial}_\nu V_{\rho\sigma}(x)\right)\left(\partial_\alpha U_\beta(x) - \partial_\beta U_\alpha(x)\right) g^{\kappa\rho} g^{\mu\sigma}\varepsilon^{\lambda\nu\alpha\beta}$$

$V_{\kappa\lambda}(x) = \partial_\kappa V_\lambda(x) - \partial_\lambda V_\kappa(x)$, $U_\alpha(x)$ and $V_\kappa(x)$ are the fields of the f_1 and the vector meson V, $M_0 \equiv 1$ GeV and g_{VVf_1} is a dimensionless coupling constant

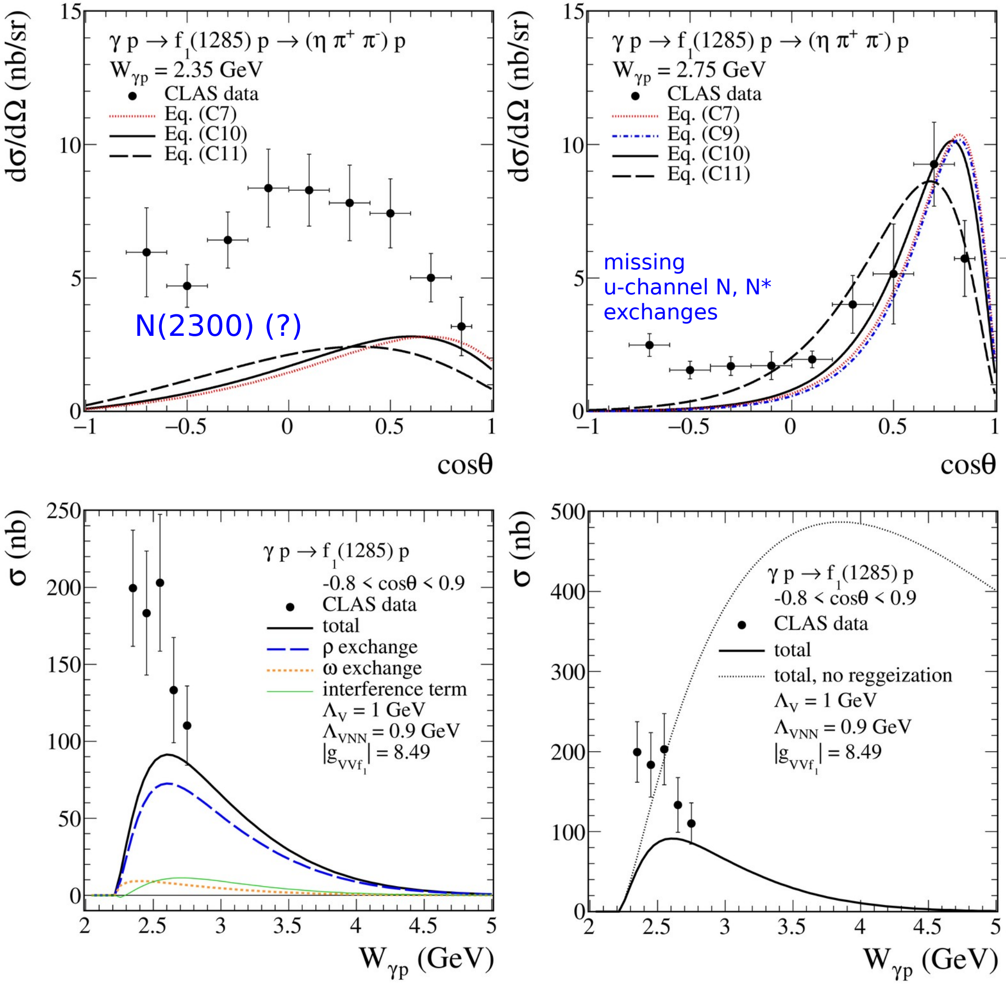
$$\begin{aligned}i\Gamma^{(VVf_1)}_{\mu\nu\alpha}(q_1,q_2) = \; & \frac{2g_{VVf_1}}{M_0^4}\big[(q_1-q_2)^{\rho}(q_1-q_2)^{\sigma}\varepsilon_{\lambda\sigma\alpha\beta}\,k^{\beta} \\ & \times(q_{1\kappa}\,\delta^{\lambda}{}_{\mu} - q_1^{\lambda}\,g_{\kappa\mu})(q_2^{\kappa}\,g_{\rho\nu} - q_{2\rho}\,\delta^{\kappa}{}_{\nu}) + (q_1\leftrightarrow q_2, \mu\leftrightarrow\nu)\big] \\ & \times F^{(VVf_1)}(q_1^2,q_2^2,k^2)\end{aligned}$$

satisfies gauge invariance relations: $\Gamma^{(VVf_1)}_{\mu\nu\alpha}(q_1,q_2)\,q_1^{\mu} = 0\,, \Gamma^{(VVf_1)}_{\mu\nu\alpha}(q_1,q_2)\,q_2^{\nu} = 0$
and $\Gamma^{(VVf_1)}_{\mu\nu\alpha}(q_1,q_2)k^{\alpha} = 0$

$$F^{(VVf_1)}(q_1^2,q_2^2,m_{f_1}^2) = \tilde{F}_V(q_1^2)\tilde{F}_V(q_2^2)F(m_{f_1}^2) = \frac{\Lambda_V^4}{\Lambda_V^4 + (t_1 - m_V^2)^2}\,\frac{\Lambda_V^4}{\Lambda_V^4 + (t_2 - m_V^2)^2}$$

with $F(m_{f_1}^2) = 1$

Results



CLAS data:
R. Dickson et al. (CLAS Collaboration), PRC 93 (2016) 065202

- The $\rho\rho f_1$ coupling constant is extracted from the radiative decay rate $f_1 \to \rho^0\gamma$ using the VMD approach.

$$\text{from PDG}: \quad \Gamma(f_1(1285) \to \gamma\rho^0) = 1384.7^{+305.1}_{-283.1}\ \text{keV}$$

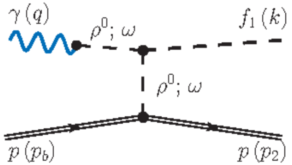
$$\text{from CLAS}: \quad \Gamma(f_1(1285) \to \gamma\rho^0) = (453 \pm 177)\ \text{keV}$$

we use

We consider decay $f_1 \to \rho^0\gamma \to \pi^+\pi^-\gamma$ taking ρ^0 mass distribution.
We estimate the cotoff parameter Λ_ρ in the $f_1\rho\rho$ form factor:

$$F_{\rho\rho f_1}(k_\rho^2, k_\gamma^2, k^2) = F_{\rho\rho f_1}(k_\rho^2, 0, m_{f_1}^2) = \tilde{F}_\rho(k_\rho^2)\tilde{F}_\rho(0)F(m_{f_1}^2) = \tilde{F}_\rho(k_\rho^2)\tilde{F}_\rho(0)$$

- Photoproduction process:



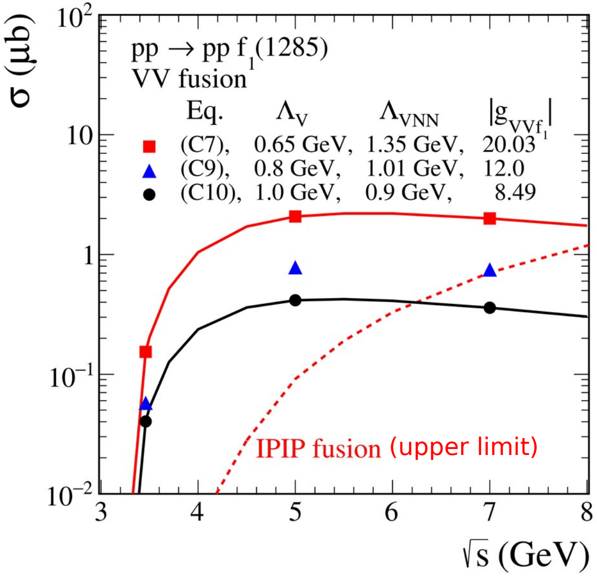
- We assume $g_{\omega\omega f_1} = g_{\rho\rho f_1}$ based on arguments from the quark model and VMD. We assume $\Lambda_\rho = \Lambda_\omega = \Lambda_V$ and $\Lambda_{\rho NN} = \Lambda_{\omega NN} = \Lambda_{VNN}$.
- Reggeization effect included
- The t-channel V-exchange mechanism play a crucial role in reproducing the forward-peaked angular distributions, especially at higher energies. From the comparison of differential cross sections to the CLAS data we estimate:

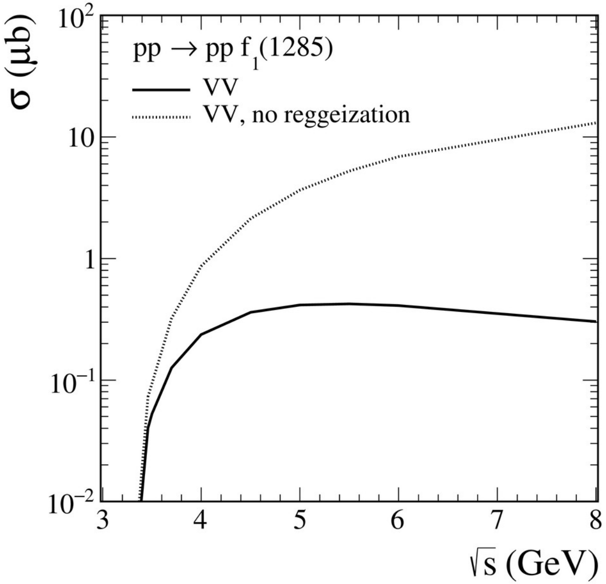
$$\text{(C7)}: \Lambda_{VNN} = 1.35\ \text{GeV for}\ \Lambda_V = 0.65\ \text{GeV}, |g_{VVf_1}| = 20.03$$
$$\text{(C9)}: \Lambda_{VNN} = 1.01\ \text{GeV for}\ \Lambda_V = 0.8\ \text{GeV}, |g_{VVf_1}| = 12.0$$
$$\text{(C10)}: \Lambda_{VNN} = 0.9\ \text{GeV for}\ \Lambda_V = 1.0\ \text{GeV}, |g_{VVf_1}| = 8.49$$
$$\text{(C11)}: \Lambda_{VNN} = 0.834\ \text{GeV for}\ \Lambda_V = 1.5\ \text{GeV}, |g_{VVf_1}| = 6.59$$

(C11) is excluded due to small Λ_{VNN}, we stay with (C7) – (C10)

- Missing N* resonances and s/u-channel proton exchange
  Possible N(2300) contribution
  → postulated in *Y.-Y. Wang et al., PRD 95 (2017) 096015*

Results





No data for the $pp \to pp\, f_1$ and $p\bar{p} \to p\bar{p}\, f_1$ reactions at low energies

← Integrated cross section for VV→ f1 fusion with different parameters.

In our procedure of extracting the model parameters from the CLAS data the dominant sensitivity of cross section is on coupling constants not on the cut-off parameters in form factors.

Reggeization effect must be included, it reduces cross section by a factor of 1.8 already for HADES

HADES: $\sqrt{s} = 3.46$ GeV	$\sigma(pp \to pp f_1) = 0.04 - 0.15\,\mu$b
PANDA: $\sqrt{s} = 5.0$ GeV	$\sigma(p\bar{p} \to p\bar{p} f_1) = 0.41 - 2.07\,\mu$b
SIS100: $\sqrt{s} = 7.61$ GeV	$\sigma(pp \to pp f_1) = 0.33 - 1.84\,\mu$b

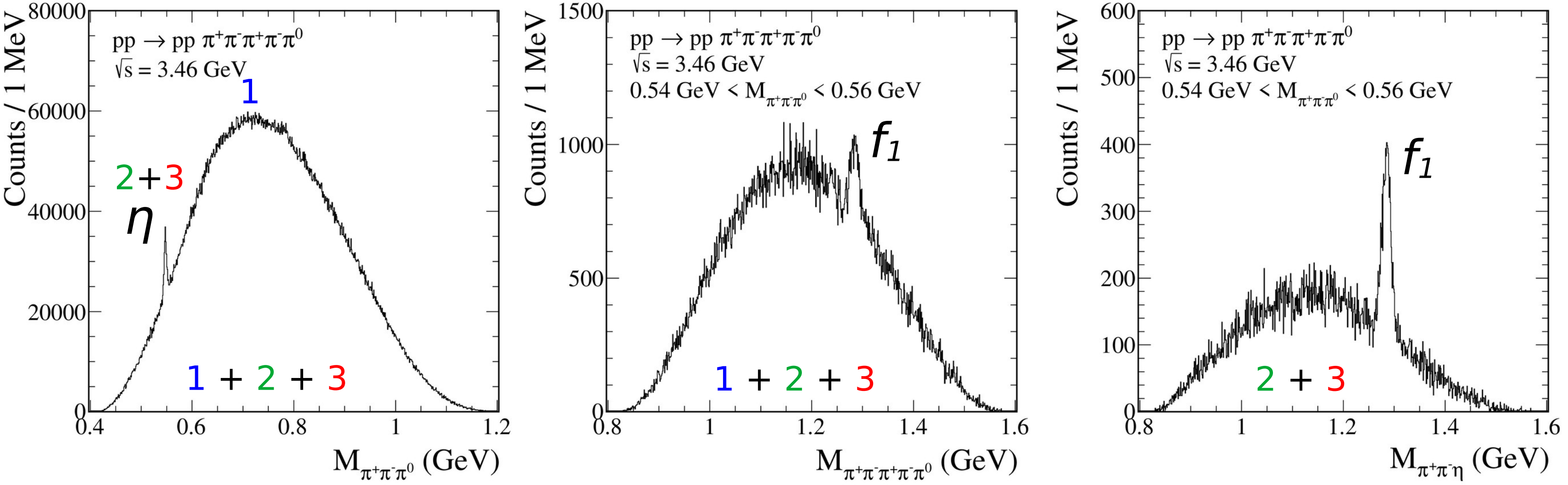
Diffractive contribution (IPIP fusion) is very small for the HADES and PANDA energy range → IPIP-fusion contribution should be regarded as an upper limit *[PL, Leutgeb, Nachtmann, Rebhan, Szczurek, PRD 102 (2020) 114003]*.
If at the WA102 c.m. energy (29.1 GeV) there are important contributions from subleading Reggeon exchanges (IP f_{2IR}, f_{2IR} IP, $f_{2IR}\, f_{2IR}$, $a_{2IR}\, a_{2IR}$, $\omega_{IR}\omega_{IR}$, $\rho_{IR}\, \rho_{IR}$, etc.) the IPIP contribution could be smaller (by a factor of up to 4).

Barberis et al. (WA102 Collaboration), PLB 440 (1998) 225:

$pp \to pp f_1(1285)$	$|x_{F,M}| \leqslant 0.2$
$\sqrt{s} = 12.7$ GeV	$\sigma_{\rm exp} = (6.86 \pm 1.31)\ \mu$b
$\sqrt{s} = 29.1$ GeV	$\sigma_{\rm exp} = (6.92 \pm 0.89)\ \mu$b

Results Optimal observation channel of $f_1(1285)$

Simulations for HADES experiment for $\sqrt{s} = 3.46$ GeV
using PLUTO MC generator. $P = 5.36$ GeV



	Contribution	Cross section (μb)	
1	$pp \to pp\pi^+\pi^-\pi^+\pi^-\pi^0$	88	$\sigma = (88 \pm 14)$ μb [1], $P = 5.5$ GeV
			$\sigma = (90 \pm 30)$ μb [2] for $pp \to pp\pi^+\pi^-\omega$ at $P = 6.92$ GeV
2	$pp \to pp\pi^+\pi^-\eta(\to \pi^+\pi^-\pi^0)$	0.18	estimates via double N^* production (via π^0 exchange) $pp \to N(1440)N(1535)$ and $pp \to N(1535)N(1535)$
3	$pp \to ppf_1[\to \pi^+\pi^-\eta(\to \pi^+\pi^-\pi^0)]$	0.012	$\sigma = 3.2 - 12.4$ nb, see (C7)–(C10), from $VV \to f_1$ fusion mechanism

The narrow width of the η meson allows to set a mass cut on the $\pi^+\pi^-\pi^0$ invariant mass and suppresses the multi-pion background efficiently.

[1] G. Alexander et al., Phys. Rev. 154 (1967) 1284
[2] S. Danieli et al., Nucl. Phys. B27 (1971) 157

Experimental data for P = 24 GeV ($\sqrt{s} = 6.84$ GeV)

$\sigma(pp \to pp\pi^+\pi^-\pi^+\pi^-\pi^0) = (660 \pm 130)\ \mu b$ [3]; $\sigma(pp \to pp\pi^+\pi^-\omega) = (200 \pm 40)\ \mu b$ [4]

and our predictions for $f_1(1285)$ signal at SIS100 ($\sqrt{s} = 7.61$ GeV)

$$\sigma(pp \to ppf_1[\to \pi^+\pi^-\eta(\to \pi^+\pi^-\pi^0)]) = 0.03 - 0.15\ \mu b$$

[3] Blobel et al., NPB 135 (1978) 379
[4] Blobel et al., NPB 111 (1976) 397

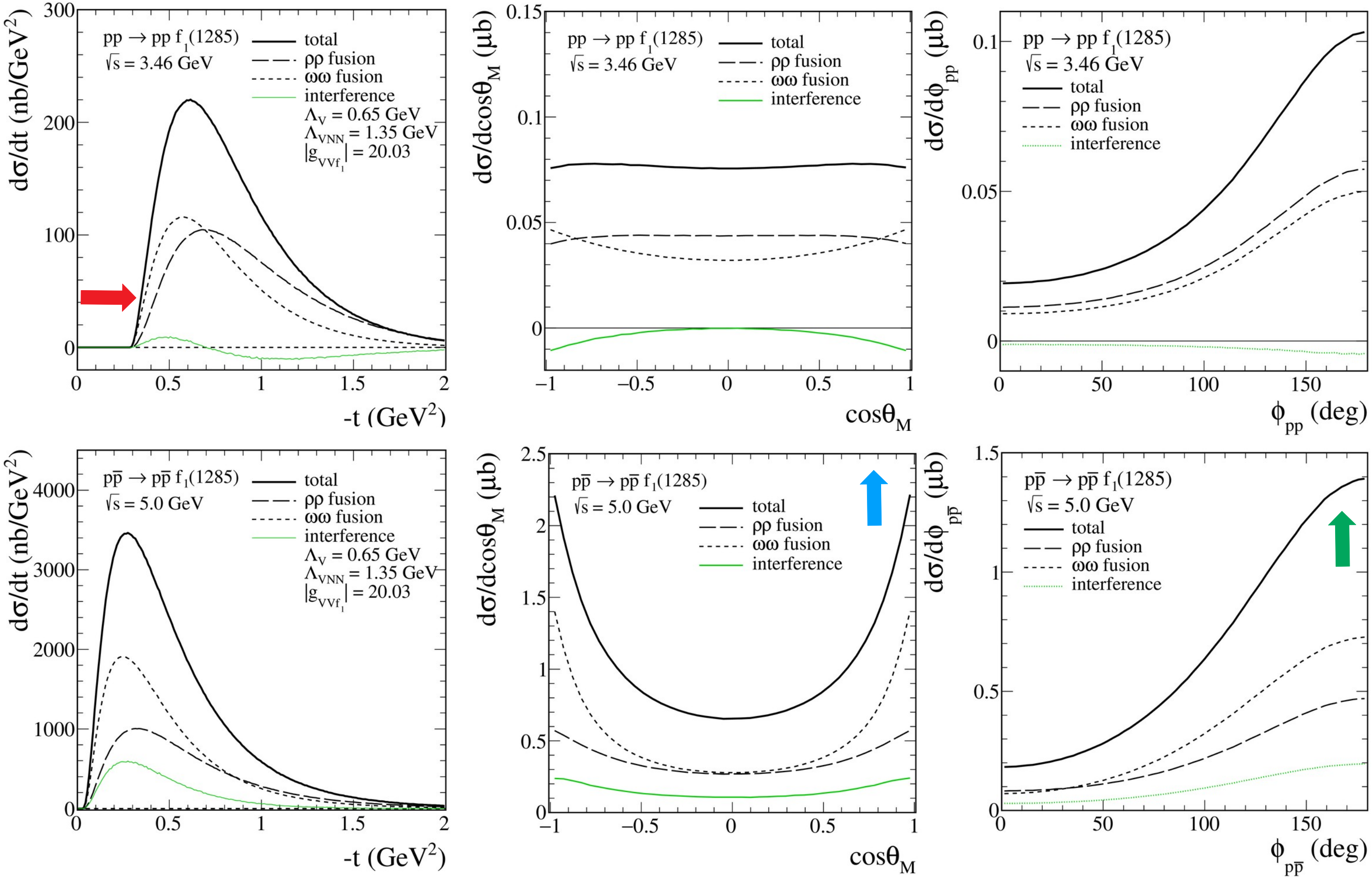
$\mathcal{BR}(\omega(782) \to \pi^+\pi^-\pi^0) = (89.3 \pm 0.6)$ %

$\mathcal{BR}(\eta \to \pi^+\pi^-\pi^0) = (22.92 \pm 0.28)$ %

$\mathcal{BR}(f_1(1285) \to \pi^+\pi^-\eta) = (35 \pm 15)$ %

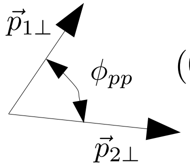
Results

$\sqrt{s}$ = 3.46 GeV (top) and 5.0 GeV (bottom)



θ_M is the angle between $\vec{k}$ and $\vec{p}_a$

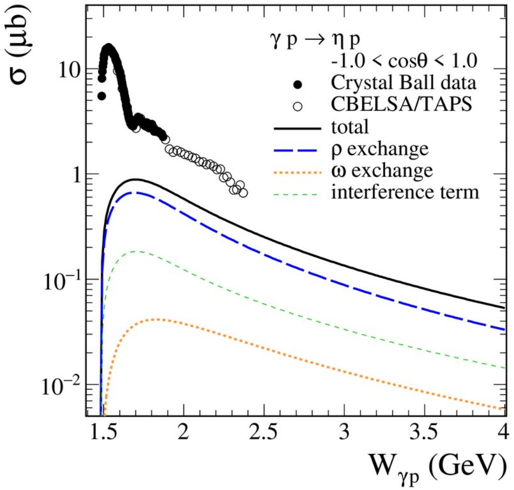
- At near threshold energy (HADES) the values of small $|t_1|$ and $|t_2|$ are not accessible kinematically
- HADES and PANDA experiments have a good opportunity to study physics of large four-momentum transfer squared $|t_{1,2}|$ → probes corresponding form factors at relatively large values of $|t_{1,2}|$ and far from their on mass-shell values $t_{1,2} = m_V^2$ at where they were normalised
- $\rho^0\rho^0$- and $\omega\omega$-fusion processes have different kinematic dependences. Both terms play similar role. With increasing c.m. energy the averages of $|t_{1,2}|$ decrease (damping by form factors), hence the $\omega\omega$ term becomes more important
- We predict a strong preference for the outgoing nucleons to be produced with their transverse momenta being back-to-back, $d\sigma/d\phi_{pp}$ at $\phi_{pp} = \pi$

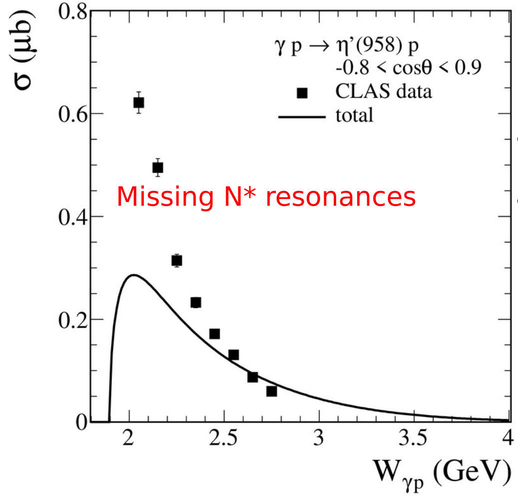


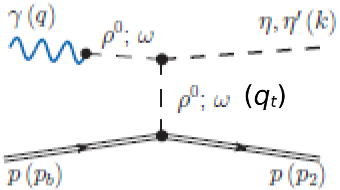
$(0 \leqslant \phi_{pp} \leqslant \pi)$

Results

$\gamma p \to \eta p$ and $\gamma p \to \eta'(958) p$



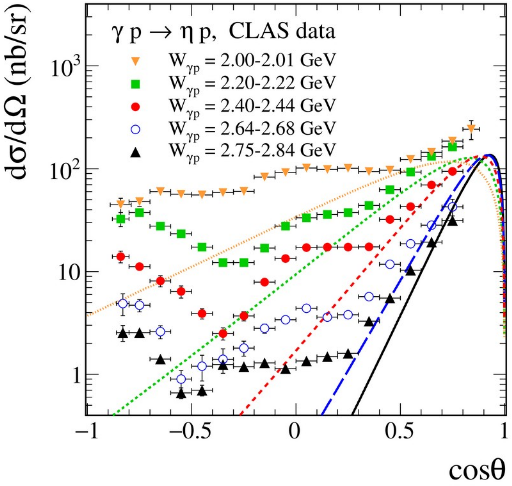


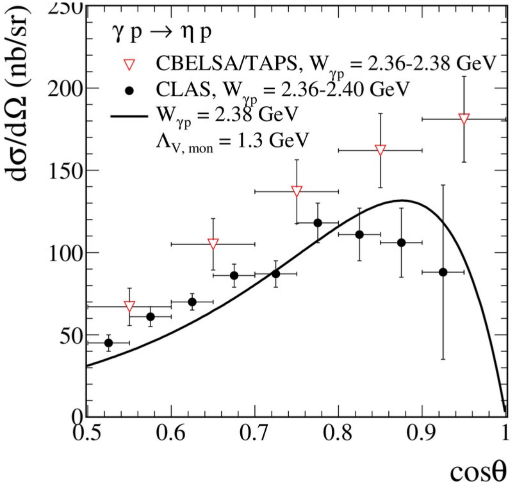


- Coupling constants obtained from radiative decay rates η' → Vγ and V → ηγ
- $$i\Gamma^{(\gamma V \widetilde{M})}_{\mu\nu}(q, q_t) = -ie\frac{g_{\gamma V \widetilde{M}}}{m_V}\, \varepsilon_{\mu\nu\alpha\beta}\, q^{\alpha} q_t^{\beta}\, F^{(\gamma V \widetilde{M})}(q^2, q_t^2, k^2)$$

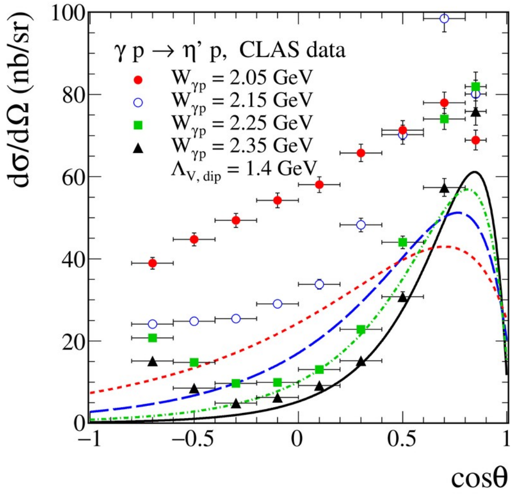
$$F^{(\gamma V \widetilde{M})}(0, q_t^2, m_{\widetilde{M}}^2) = \left(\frac{\Lambda_V^2 - m_V^2}{\Lambda_V^2 - q_t^2}\right)^n$$

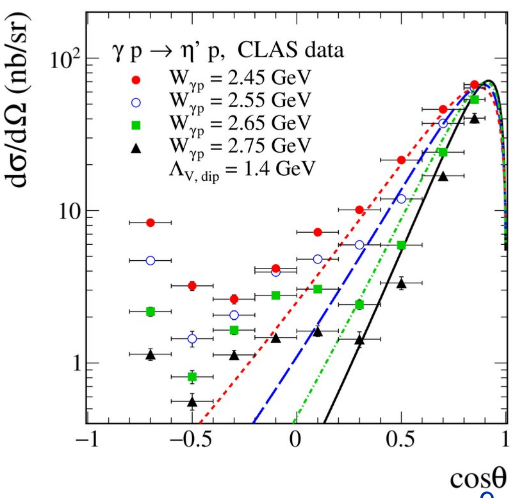
where $n = 1$ for $\widetilde{M} = \eta$ and $n = 2$ for $\widetilde{M} = \eta'(958)$.





Reggeized V-meson exchange mechanism is dominant at higher energies and forward angles.

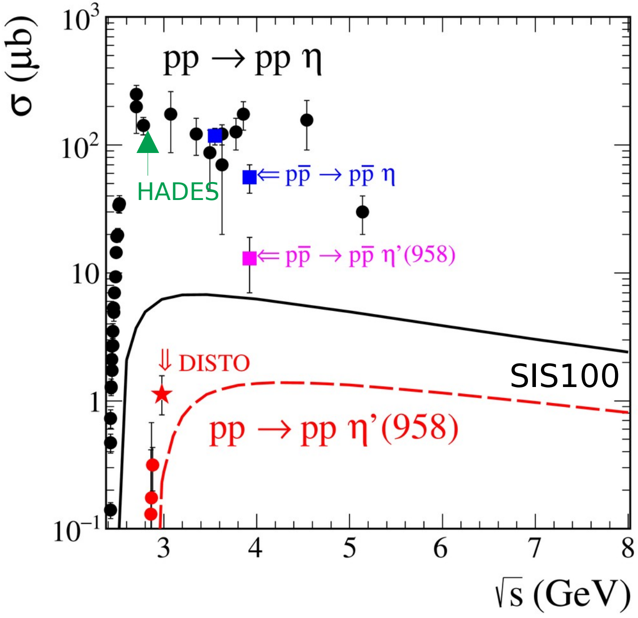




Crystal Ball Collaboration, E.F. McNicoll et al., PRC 82 (2010) 035208
CBELSA/TAPS Collaboration, V. crede et al., PRC 80 (2009) 055202
CLAS Collaboration, M. Williams et al., PRC 80 (2009) 045213, T. Hu et al., PRC 102 (2020)065203; ***for η': R. Dickson et al., PRC 93 (2016) 065202***

Results

Exclusive production of pseudoscalar mesons



← Total cross-sections for *pp → pp η* (black points) and *pp → pp η'* (red points). There are data also from pp̄ interactions.

Results for the VV-fusion mechanism fast rises from threshold, reaches a maximum at $\sqrt{s}$ = 3.5 (4.2) GeV for *η* (*η'*), and then decreases (reggeization). Our predictions are at $\sqrt{s}$ = 3.46 (7.61) GeV :

$$\sigma(pp \to pp\eta) = 6.8\,(2.6)\,\mu\text{b}$$
$$\sigma(pp \to pp\eta'(958)) = 1.1\,(0.9)\,\mu\text{b}$$
$$\sigma(pp \to pp[\eta'(958) \to \pi^+\pi^-\eta(\to \pi^+\pi^-\pi^0)]) = 0.11\,(0.09)\,\mu\text{b}$$

Basic production mechanism at low energies → single excitation of N* resonances, at WA76, WA102 energies → Reggeon/Pomeron-fusion mechanism

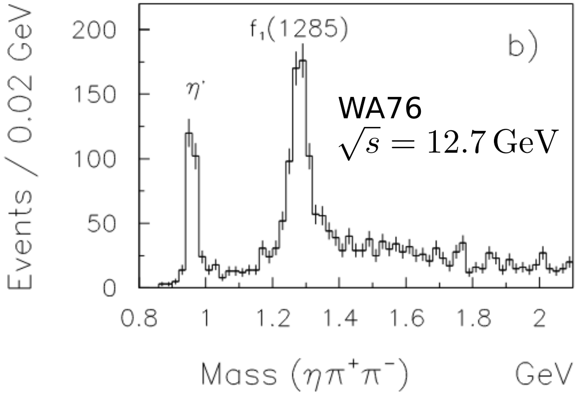
	$pp \to pp\eta$	$pp \to pp\eta'$
From WA102 at $\sqrt{s} = 29.1$ GeV	$\sigma_{\rm exp} = (3.86 \pm 0.37)\ \mu$b	$\sigma_{\rm exp} = (1.72 \pm 0.18)\ \mu$b

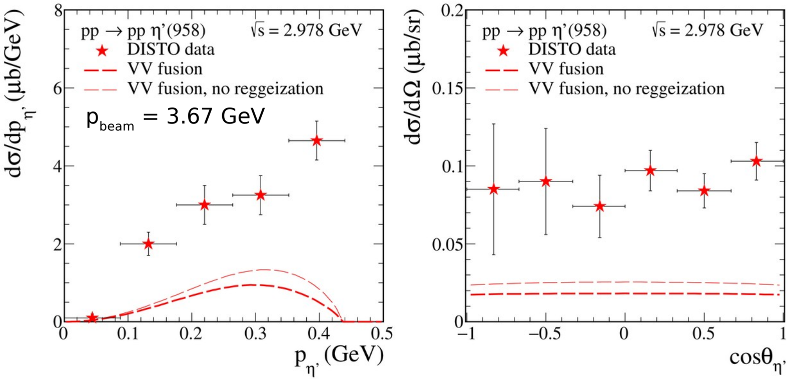
and for *pp → ppη'*:

$$\frac{\sigma(\sqrt{s}=12.7\,\text{GeV})}{\sigma(\sqrt{s}=23.8\,\text{GeV})} = 0.2 \pm 0.05$$

$$\frac{\sigma(\sqrt{s}=29.1\,\text{GeV})}{\sigma(\sqrt{s}=12.7\,\text{GeV})} = 0.72 \pm 0.16$$

The WA76 experiment was able to reconstruct the $\eta\pi^+\pi^-$ mass spectrum using the decay $\eta \to \pi^+\pi^-(\pi^0)_{\rm missing}$. There the *η'* and $f_1(1285)$ are seen →





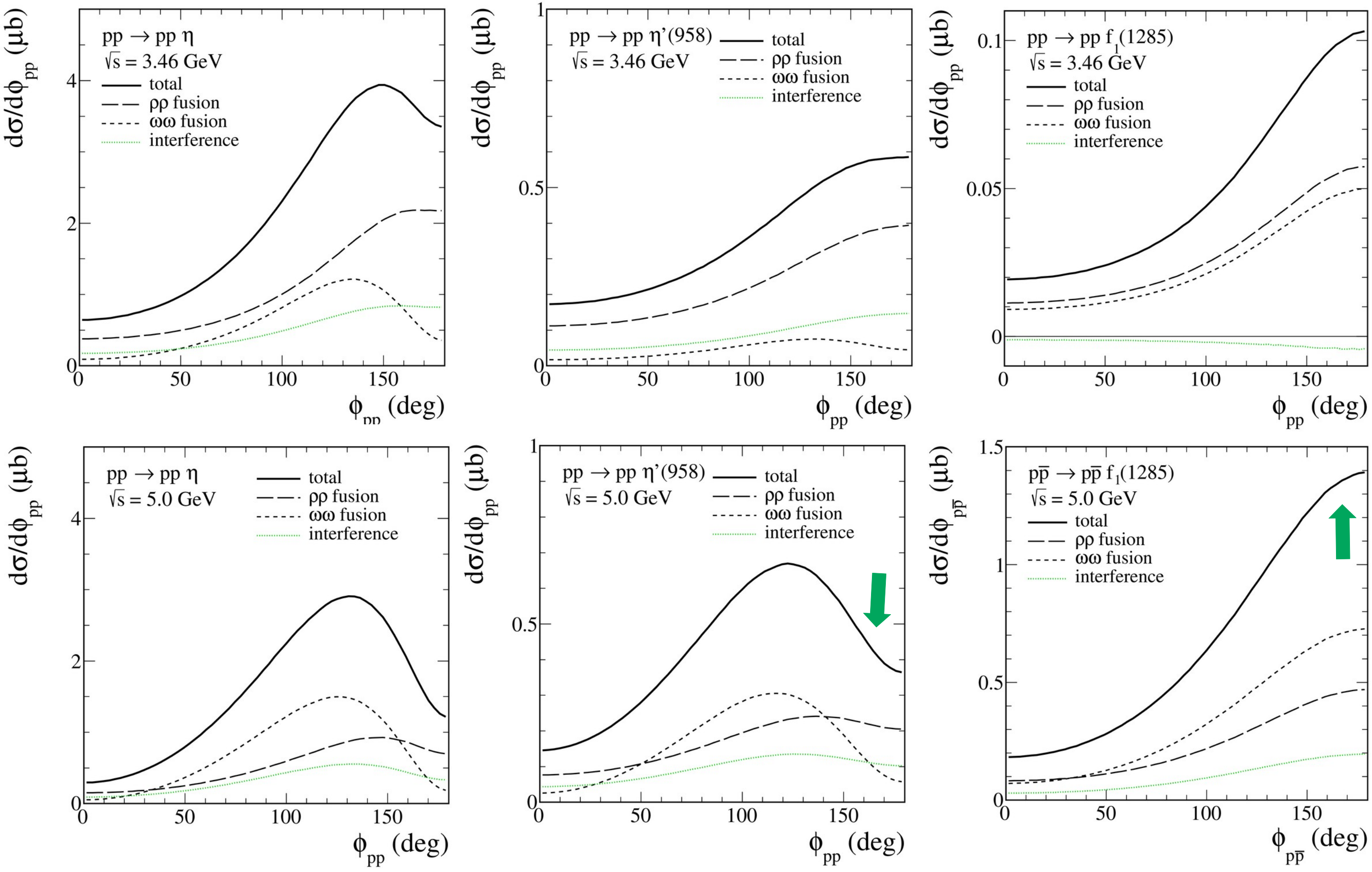
Experiment SATURNE-213 *F. Balestra et al. (DISTO Collaboration) PLB 491 (2000) 29*

The ratio of total cross section for *η'* and *η* is $R = \sigma_{pp\to pp\eta'}/\sigma_{pp\to pp\eta} = (0.83 \pm 0.11^{+0.23}_{-0.18}) \times 10^{-2}$

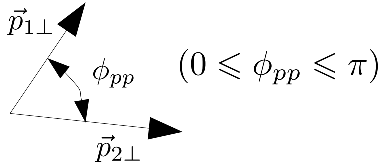
F. Close, A. Kirk, Z.Phys.C 76 (1997) 469
WA102, Phys.Lett. B 467 (1999) 165
A. Kirk, Phys. Lett. B 489 (2000) 29

Results

$\sqrt{s}$ = 3.46 GeV (top) and 5.0 GeV (bottom)



- Since $f_1(1285)$ and $\eta(1295)$ are close in mass and both decaying to $\pi^+\pi^-\eta$ channel, care must be taken for potential overlap of these resonances with each other in the measurement
- $\eta(1295)$ has about 2 times larger total width than $f_1(1285)$.
- In order to distinguish both resonances the distribution in azimuthal angle may be used



- With the couplings of V to protons we see that the helicity flipping tensor coupling of the ρ to the protons is large whereas the tensor coupling of the ω is small (taken to be zero)
- At higher energies, available in the future at PANDA and SIS100, ωω fusion giving $\eta(1295)$ should dominate over ρρ fusion
- The distribution for $\eta(1295)$ should (nearly) vanish for $\phi_{pp} = 0$ and π

Results Other decay channels?

$$\mathcal{BR}(f_1(1285) \to \pi^+\pi^-\pi^+\pi^-) = (10.9 \pm 0.6)\ \%$$

PDG: $\mathcal{BR}(f_1(1285) \to \rho^0\gamma) = (6.1 \pm 1.0)\ \%$

CLAS: $\mathcal{BR}(f_1(1285) \to \rho^0\gamma) = (2.5^{+0.7}_{-0.8})\ \%$

$$\mathcal{BR}(f_1(1285) \to K\bar{K}\pi) = (9.0 \pm 0.4)\ \%$$

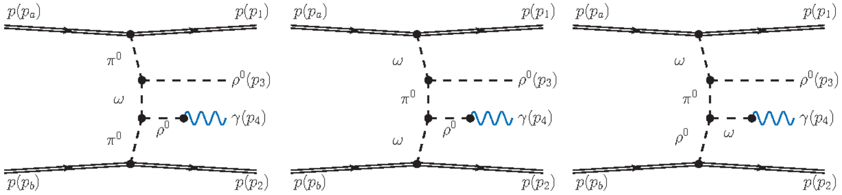
$pp \to pp\pi^+\pi^+\pi^-\pi^-$

- For the **4π channel** it may be difficult to identify the $f_1(1285)$ due to large continuum background e.g. pp → N(1440)N(1440) → Nππ Nππ
  → we have found that fusion mechanisms for the $\rho^0\rho^0$ production: π^0-ω-π^0 and ω-π^0-ω exchanges (treated with exact 2 → 4 kinematics) give much smaller background cross sections

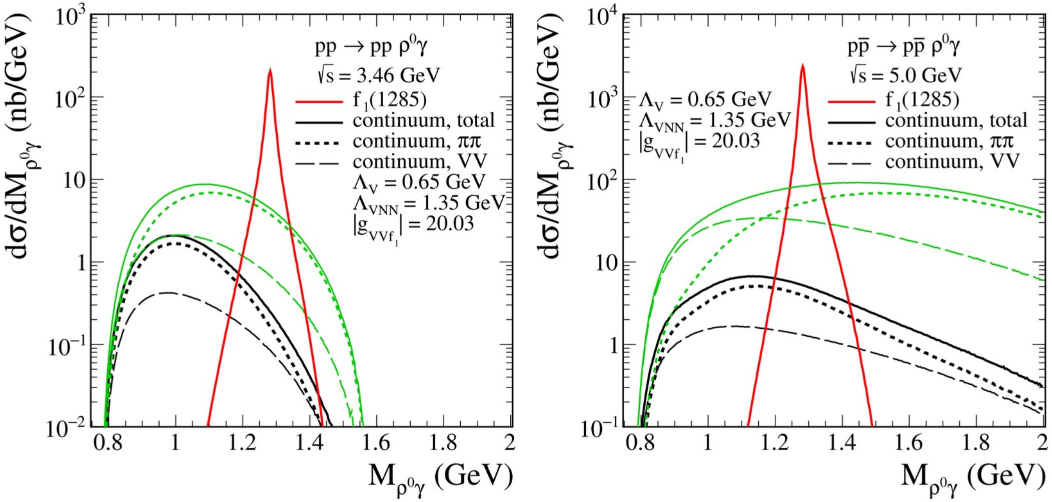
  $\sigma^{4\pi}_{back} \sim 227\,\mu\text{b}$ [1], $\sigma^{4\pi}_{f_1} = 16$ nb
- The **ργ channel** should be much better suited

  The $\rho^0\gamma$ channel should be much better suited. There, however, dominant background channel $pp\pi^+\pi^-\pi^0$ is of the order of 2 mb [1] and ρ^0 is so broad that it will not provide sufficient reductions (as it is the case in η decay channel)

[1] G. Alexander et al., Phys. Rev. 154 (1967) 1284



$+ (p_a \leftrightarrow p_b, p_1 \leftrightarrow p_2)$



The ππ-continuum contribution is larger than the VV-continuum term.
In both cases the f_1 resonance is clearly visible, even without the reggeization (green lines) in the continuum processes.

We get: for $\sqrt{s} = 3.46$ GeV : $\sigma_{pp\to pp(f_1\to\rho^0\gamma)} = 5.38$ nb

for $\sqrt{s} = 5.0$ GeV : $\sigma_{p\bar{p}\to p\bar{p}(f_1\to\rho^0\gamma)} = 62.86$ nb ← 10 x larger

For our exploratory study we have neglected interference effects between the background ργ and the signal f_1 → ργ processes.
We have also neglected the background processes due to bremsstrahlung of γ and ρ^0 from the nucleon lines.

Conclusions

***(I) Lebiedowicz, Nachtmann, Salabura, Szczurek,* PRD 104 (2021) 034031**

- We have given predictions for experiments with HADES and PANDA at FAIR.

  We have estimated that HADES should allow the identification of $f_1(1285)$ in the $\pi^+\pi^- \eta$ channel.
  We are looking forward to first experimental results on production of $f_1(1285)$ in *pp* collisions at HADES.

  We shall learn from f_1 CEP at low energies about the $\rho\rho f_1$ and $\omega\omega f_1$ coupling strengths.

  Comparison of the full model (including nucleonic contributions, *N* resonances, *VV* fusion, *IR IR* fusion etc.) with experimental results from SIS100 (total and differential cross sections) should help to learn more about the production mechanism of η, η', $f_1(1285)$.

***(II) Lebiedowicz, Leutgeb, Nachtmann, Rebhan, Szczurek,* PRD 102 (2020) 114003**

- We have discussed CEP of $f_1(1285)$ in *pp* collisions at high energies in the tensor-pomeron approach.
  Different forms of the *IP-IP-*f_1 coupling are possible. We obtain a good description of the WA102 data for the $pp \to pp\, f_1(1285)$ reaction assuming that the reaction (at c.m. energy 29.1 GeV) is dominated by *IP* exchange. We have included - very important - absorptive corrections.

  We have given predictions for experiments at the LHC.

  Experimental studies of single meson CEP reactions will give many *IP-IP-M* coupling parameters.
  Their theoretical calculation is a challenging problem of nonperturbative QCD.

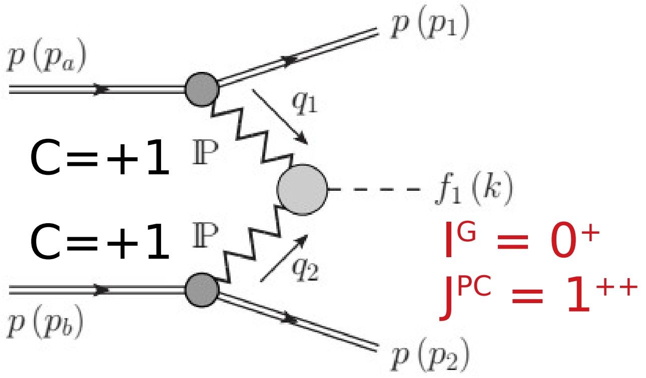
Thank you for your attention

Pomeron-Pomeron fusion mechanism

$p\,(p_a) + p\,(p_b) \to p\,(p_1) + f_1\,(k) + p\,(p_2)$

At high energies double pomeron (IP) exchange is dominant production mechanism of the $f_1(1285)$

see: Lebiedowicz, Leutgeb, Nachtmann, Rebhan, Szczurek, PRD 102 (2020) 114003



We treat our reaction in the tensor-pomeron approach
[Ewerz, Maniatis, Nachtmann, Ann. Phys. 342 (2014) 31]
The pomeron and the charge conjugation C=+1 reggeons are described as effective rank 2 symmetric tensor exchanges. The odderon and the C= -1 reggeons are described as effective vector exchanges.

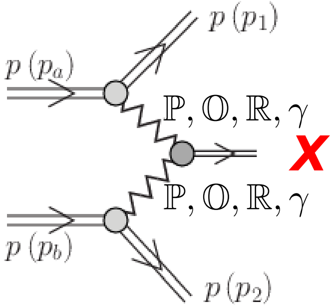
This approach has a good basis from nonperturbative QCD considerations.
The IP exchange can be understood as a coherent sum of exchanges of spin 2+4+6+ ...
[Nachtmann, Ann. Phys. 209 (1991) 436]

A tensor character of the pomeron is also preferred in holographic QCD, see e.g.,
Brower, Polchinski, Strassler, Tan, JHEP 12 (2007) 005
Domokos, Harvey, Mann, PRD 80 (2009) 126015
Iatrakis, Ramamurti, Shuryak, PRD 94 (2016) 045005

Applications of the tensor-pomeron and vector-odderon model

- ***$\gamma\, p \to \pi^+ \pi^- p$*** *Bolz, Ewerz, Maniatis, Nachtmann, Sauter, Schöning, JHEP 01 (2015) 151*
  ← interference between $\gamma p \to (\rho^0 \to \pi^+ \pi^-)p$ (IP exchange) and $\gamma p \to (f_2(1270) \to \pi^+ \pi^-)p$ (O exchange) processes and as a consequence $\pi^+\pi^-$ charge asymmetries
- **Photoproduction and low *x* DIS** *Britzger, Ewerz, Glazov, Nachtmann, Schmitt, PRD100 (2019) 114007*
  ← a "vector pomeron" decouples completely in the total photoabsorption cross section and in the structure functions of DIS
- **Helicity in proton-proton elastic scattering and the spin structure of the pomeron**
  Ewerz, P.L., Nachtmann, Szczurek, PLB 763 (2016) 382 ← studying the ratio r_5 of single-helicity-flip to non-flip amplitudes we found that the STAR data [L. Adamczyk *et al.*, PLB 719 (2013) 62] are compatible with the tensor pomeron ansatz while they clearly exclude a scalar character of the pomeron

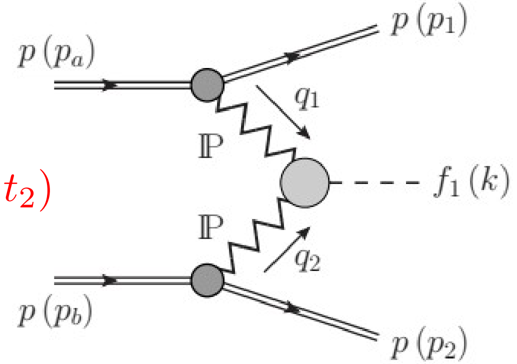
- **Central Exclusive Production (CEP),** $p\,p \to p\,p\,X$, *P.L., Nachtmann, Szczurek:*



X: η, η', f_0 — *Ann. Phys. 344 (2014) 301*
ρ^0 — *PRD91 (2015) 074023*
$\pi^+\pi^-$, f_0, f_2 ($\to \pi^+\pi^-$) — *PRD93 (2016) 054015, PRD101 (2020) 034008*
$\pi^+\pi^-\pi^+\pi^-$, $\rho^0\rho^0$ — *PRD94 (2016) 034017*
ρ^0 with proton diss. — *PRD95 (2017) 034036*
$p\bar{p}$ — *PRD97 (2018) 094027*
K^+K^- — *PRD98 (2018) 014001*
$\phi \to K^+K^-$, $\mu^+\mu^-$ — *PRD101 (2020) 094012* } odderon exchange
$\phi\phi \to K^+K^-K^+K^-$ — *PRD99 (2019) 094034* } odderon exchange

$f_1(1285)$ *P.L., Leutgeb, Nachtmann, Rebhan, Szczurek, PRD102 (2020) 114003*
$K^{*0}\bar{K}^{*0}$, continuum vs $f_2(1950)$ *P.L., PRD103 (2021) 054039*

The Born-level amplitude within the tensor-pomeron approach:



$$\mathcal{M}^{\rm Born}_{\lambda_a\lambda_b\to\lambda_1\lambda_2\lambda_{f_1}} = (-i)\,(\epsilon^{\mu}(\lambda_{f_1}))^*\,\bar{u}(p_1,\lambda_1)i\Gamma^{(I\!\!Ppp)}_{\mu_1\nu_1}(p_1,p_a)u(p_a,\lambda_a)$$
$$\times i\Delta^{(I\!\!P)\,\mu_1\nu_1,\alpha_1\beta_1}(s_1,t_1)\,i\Gamma^{(I\!\!PI\!\!Pf_1)}_{\alpha_1\beta_1,\alpha_2\beta_2,\mu}(q_1,q_2)\,i\Delta^{(I\!\!P)\,\alpha_2\beta_2,\mu_2\nu_2}(s_2,t_2)$$
$$\times\bar{u}(p_2,\lambda_2)i\Gamma^{(I\!\!Ppp)}_{\mu_2\nu_2}(p_2,p_b)u(p_b,\lambda_b)$$

with terms of the effective pomeron propagator and the pomeron-proton vertex

$$i\Delta^{(I\!\!P)}_{\mu\nu,\kappa\lambda}(s,t) = \frac{1}{4s}\left(g_{\mu\kappa}g_{\nu\lambda}+g_{\mu\lambda}g_{\nu\kappa}-\frac{1}{2}g_{\mu\nu}g_{\kappa\lambda}\right)(-is\alpha'_{I\!\!P})^{\alpha_{I\!\!P}(t)-1}$$

$$i\Gamma^{(I\!\!Ppp)}_{\mu\nu}(p',p) = -i3\beta_{I\!\!PNN}F_1\big((p'-p)^2\big)\left\{\frac{1}{2}[\gamma_\mu(p'+p)_\nu+\gamma_\nu(p'+p)_\mu]-\frac{1}{4}g_{\mu\nu}(\not{p}'+\not{p})\right\}$$

$$\alpha_{I\!\!P}(t)=\alpha_{I\!\!P}(0)+\alpha'_{I\!\!P}t\,,\quad \alpha_{I\!\!P}(0)=1.0808,\quad \alpha'_{I\!\!P}=0.25\,\mathrm{GeV}^{-2}$$
$$\beta_{I\!\!PNN}=1.87\ \mathrm{GeV}^{-1},\qquad F_1(t):\ \text{Dirac form factor of the proton}$$

Ewerz, Maniatis, Nachtmann, Ann. Phys. 342 (2014) 31

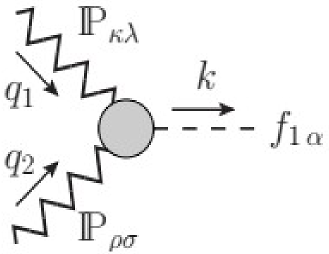
Absorption effects:

$$\mathcal{M}_{pp\to ppf_1} = \mathcal{M}^{\rm Born}_{pp\to ppf_1}+\mathcal{M}^{pp-\rm rescattering}_{pp\to ppf_1}$$

$$\mathcal{M}^{pp-\rm rescattering}_{pp\to ppf_1}(s,\vec{p}_{1\perp},\vec{p}_{2\perp}) = \frac{i}{8\pi^2 s}\int d^2\vec{k}_\perp\mathcal{M}^{\rm Born}_{pp\to ppf_1}(s,\vec{p}_{1\perp}-\vec{k}_\perp,\vec{p}_{2\perp}+\vec{k}_\perp)\,\mathcal{M}^{I\!\!P-\rm exchange}_{pp\to pp}(s,-\vec{k}^2_\perp)$$

where $\vec{k}_\perp$ is the transverse momentum carried around the loop

IP IP f1 coupling



coupling Lagrangian $\mathcal{L}^{(I\!\!P I\!\!P f_1)}$

↓

"bare" vertex function $\mathrm{i}\Gamma^{(I\!\!P I\!\!P f_1)}_{\kappa\lambda,\rho\sigma,\alpha}(q_1, q_2)\,|_{\mathrm{bare}}$

↓ CEP reaction

vertex function supplemented by suitable form factor

$$i\Gamma^{(I\!\!P I\!\!P f_1)}_{\kappa\lambda,\rho\sigma,\alpha}(q_1, q_2) = i\Gamma^{(I\!\!P I\!\!P f_1)}_{\kappa\lambda,\rho\sigma,\alpha}(q_1, q_2)\,|_{\mathrm{bare}}\, \tilde{F}_{I\!\!P I\!\!P f_1}(q_1^2, q_2^2, k^2)$$

For the on-shell meson we have set $k^2 = m_{f_1}^2$.

$$\tilde{F}^{(I\!\!P I\!\!P f_1)}(t_1, t_2, m_{f_1}^2) = F_M(t_1)F_M(t_2), \quad F_M(t) = \frac{1}{1 - t/\Lambda_0^2}, \quad \Lambda_0^2 = 0.5\ \mathrm{GeV}^2$$

or

$$\tilde{F}^{(I\!\!P I\!\!P f_1)}(t_1, t_2, m_{f_1}^2) = \exp\left(\frac{t_1 + t_2}{\Lambda_E^2}\right)$$

where the cutoff constant Λ_E should be adjusted to experimental data

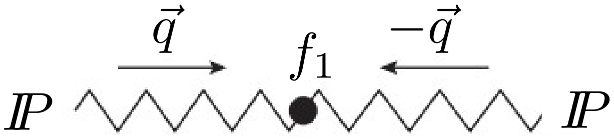
We follow two strategies for constructing coupling Lagrangian:

(1) Phenomenological approach. First we consider a fictitious process:
the fusion of two "real spin 2 pomerons" (or tensor glueballs) of mass m giving an f_1 meson of $J^{PC} = 1^{++}$

$$I\!\!P(m, \epsilon_1) + I\!\!P(m, \epsilon_2) \to f_1(m_{f_1}, \epsilon)$$

$\epsilon_{1,2}$: polarisation tensors, ϵ : polarisation vector

We work in the rest system of the f_1 meson:



The spin 2 of these "real pomerons" can be combined to a total spin S ($0 \leq S \leq 4$) and this must be combined with the orbital angular momentum ℓ to give $J^{PC} = 1^{++}$ of the f_1 state.
There are exactly two possibilities: (ℓ,S) = (2,2) and (4,4).
Corresponding couplings are:

$$\mathcal{L}^{(2,2)}_{I\!\!P I\!\!P f_1} = \frac{g'_{I\!\!P I\!\!P f_1}}{32\, M_0^2} \left(I\!\!P_{\kappa\lambda} \overleftrightarrow{\partial}_\mu \overleftrightarrow{\partial}_\nu\, I\!\!P_{\rho\sigma} \right) \left(\partial_\alpha U_\beta - \partial_\beta U_\alpha \right) \Gamma^{(8)\, \kappa\lambda,\rho\sigma,\mu\nu,\alpha\beta}$$

$$\mathcal{L}^{(4,4)}_{I\!\!P I\!\!P f_1} = \frac{g''_{I\!\!P I\!\!P f_1}}{24 \times 32\, M_0^4} \left(I\!\!P_{\kappa\lambda} \overleftrightarrow{\partial}_{\mu_1} \overleftrightarrow{\partial}_{\mu_2} \overleftrightarrow{\partial}_{\mu_3} \overleftrightarrow{\partial}_{\mu_4}\, I\!\!P_{\rho\sigma} \right) \left(\partial_\alpha U_\beta - \partial_\beta U_\alpha \right) \Gamma^{(10)\, \kappa\lambda,\rho\sigma,\mu_1\mu_2\mu_3\mu_4,\alpha\beta}$$

Here $M_0 \equiv 1$ GeV, $g'_{I\!\!P I\!\!P f_1}, g''_{I\!\!P I\!\!P f_1}$: dimensionless coupling parameters,
$I\!\!P_{\kappa\lambda}$ effective pomeron field, U_α f_1 field, $\overleftrightarrow{\partial}_\mu = \overrightarrow{\partial}_\mu - \overleftarrow{\partial}_\mu$ asymmetric derivative,
and $\Gamma^{(8)}, \Gamma^{(10)}$ are known tensor functions.

(2) Holographic QCD approach using the Sakai-Sugimoto model.
There, the *IP IP* f_1 coupling can be derived from the bulk Chern-Simons (CS) term requiring consistency of supergravity and the gravitational anomaly.

$$\mathcal{L}^{\rm CS} = \varkappa'\, U_\alpha\, \varepsilon^{\alpha\beta\gamma\delta}\, I\!P^{\mu}{}_{\beta}\, \partial_\delta I\!P_{\gamma\mu} + \varkappa''\, U_\alpha \varepsilon^{\alpha\beta\gamma\delta} \left(\partial_\nu P^{\mu}{}_{\beta}\right)\left(\partial_\delta\partial_\mu I\!P^{\nu}{}_{\gamma} - \partial_\delta\partial^\nu I\!P_{\gamma\mu}\right)$$

$\varkappa'$: dimensionless, $\varkappa''$: dimension GeV^{-2}

Sakai, Sugimoto, Prog. Theor. Phys. 113 (2005) 843; 114 (2005) 1083,
Leutgeb, Rebhan, PRD 101 (2020) 114015

For our fictitious reaction with real pomerons there is strict equivalence $\mathcal{L}^{\rm CS} \widehat{=} \mathcal{L}^{(2,2)} + \mathcal{L}^{(4,4)}$
if the couplings satisfy:

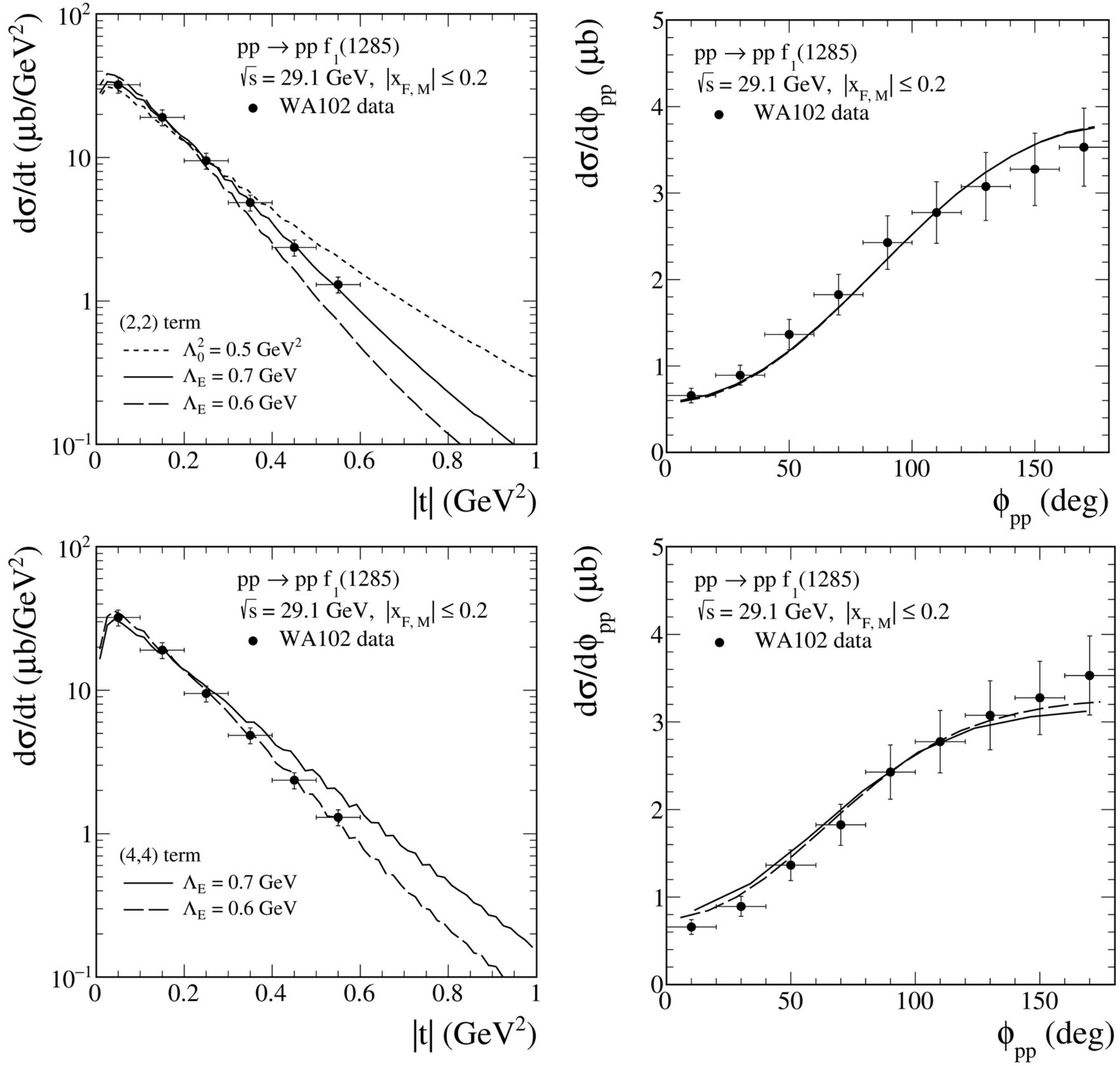
$$g'_{I\!P I\!P f_1} = -\varkappa' \frac{M_0^2}{k^2} - \varkappa'' \frac{M_0^2(k^2-2m^2)}{2k^2}$$

$$g''_{I\!P I\!P f_1} = \varkappa'' \frac{2M_0^4}{k^2}$$

where k^2 is invariant mass squared of the resonance f_1.

For the CEP reaction the pomerons have invariant mass squared t_1, $t_2 < 0$ instead of m^2 and, in general, $t_1 \neq t_2$.
Replacing above $2m^2 \to t_1 + t_2$ we expect for small $|t_1|$ and $|t_2|$ still approximate equivalence to hold.
This is confirmed by explicit numerical studies.

Comparison with experimental results from WA102@CERN



Data: D. Barberis et al. (WA102 Collaboration), PLB 440 (1998) 225

	$\sqrt{s} = 29.1$ GeV, $\lvert x_{F,M} \rvert \leqslant 0.2$
$f_1(1285)$	$\sigma_{\rm exp} = (6919 \pm 886)$ nb

Phenomenological approach

← $(\ell,S) = (2,2)$ term only
$|g'_{I\!P I\!P f_1}| = 4.89$

← $(\ell,S) = (4,4)$ term only
$|g''_{I\!P I\!P f_1}| = 10.31$

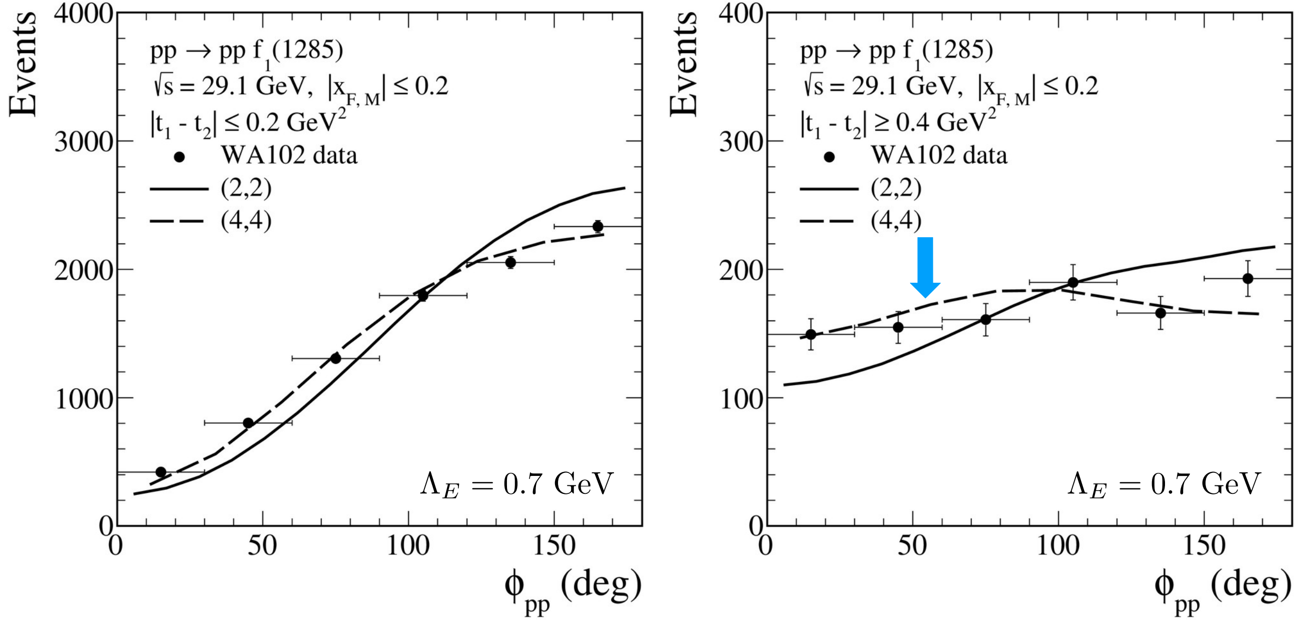
- We get a reasonable description of WA102 data with $\Lambda_E = 0.7$ GeV
- Absorption effects included $\langle S^2 \rangle = \sigma_{abs}/\sigma_{Born} \approx 0.5$-$0.7$ depending on the kinematics

Comparison with experimental results from WA102@CERN

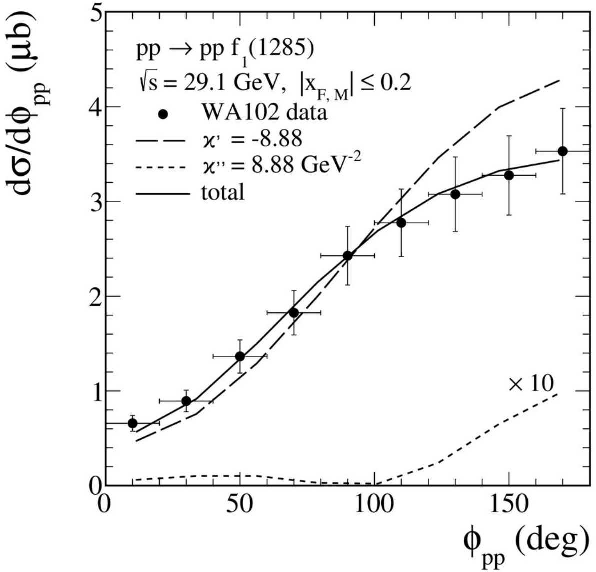
Comparison with data from: A. Kirk (WA102 Collaboration), Nucl. Phys. A 663 (2000) 608

The theoretical results have been normalized to the mean value of the number of events

Phenomenological approach



- An almost 'flat' distribution at large values of $|t_1 - t_2|$ can be observed
  → absorption effects play a significant role there,
  large damping of cross section at higher values of ϕ_{pp}
- It seems that the $(\ell,S) = (4,4)$ term best reproduces the shape of the WA102 data



Holographic QCD approach

← Fit to WA102 data using the Chern-Simons (CS) coupling.

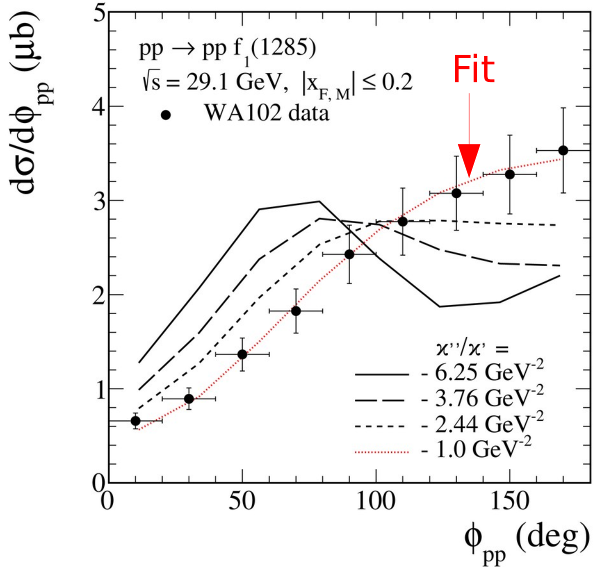
The relation between the (ℓ,S) and CS forms of the couplings:

With $\varkappa' = -8.88, \quad \varkappa''/\varkappa' = -1.0\ \text{GeV}^{-2}$

and setting $t_1 = t_2 = -0.1\ \text{GeV}^2$

we get: $g'_{I\!\!PI\!\!Pf_1} = 0.42, \quad g''_{I\!\!PI\!\!Pf_1} = 10.81$

This CS coupling corresponds practically to a pure (ℓ,S) = (4,4) coupling!



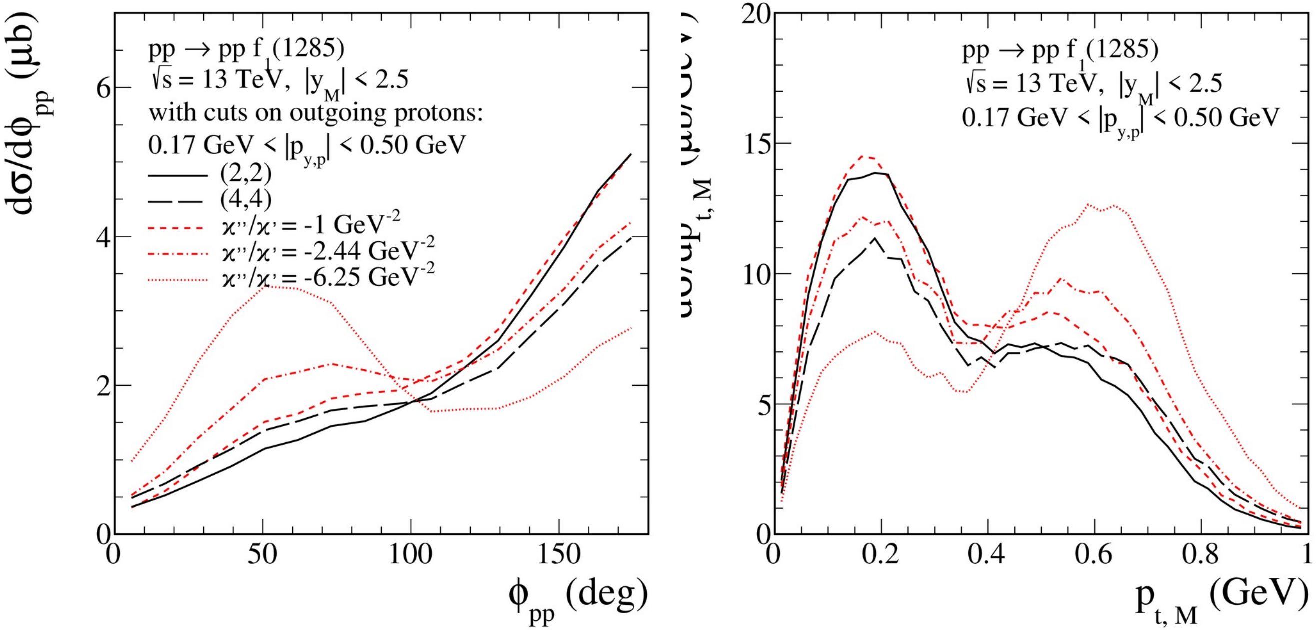
The prediction for $\varkappa''/\varkappa'$ obtained in the Sakai-Sugimoto model:

$$\varkappa''/\varkappa' = -5.631/M_{KK}^2 = -(6.25, 3.76, 2.44)\ \text{GeV}^{-2}$$

$$\text{for} \quad M_{KK} = (949, 1224, 1519)\ \text{MeV}$$

Usually M_{KK} (Kaluza-Klein mass scale) is fixed by matching the mass of the lowest vector meson to that of the physical ρ meson, leading to $M_{KK} = 949$ MeV.
However, this choice leads to tensor glueball mass which is too low, $M_T \approx 1.5$ GeV.
The standard pomeron trajectory corresponds to $M_T \approx 1.9$ GeV,
whereas lattice gauge theory indicates $M_T \approx 2.4$ GeV.

Predictions for the LHC experiments



- The contribution with $\varkappa''/\varkappa' = -6.25\ \mathrm{GeV}^{-2}$ gives a significantly different shape
- The absorption effects are included, $\langle S^2 \rangle \approx 0.35$. They decrease the distribut. mostly at higher values of ϕ_{pp} and at smaller values of $p_{t,M}$ (and also $|t|$). This could be tested in ATLAS-ALFA experiment when both protons are measured

Cross sections in μb for $pp \to pp f_1(1285)$ for $\sqrt{s} = 13$ TeV:

Contribution	Parameters $\Lambda_E = 0.7$ GeV,	$\lvert y_{f_1} \rvert < 1.0$	$\lvert y_{f_1} \rvert < 2.5$	$\lvert y_{f_1} \rvert < 2.5$, $0.17 < \lvert p_{y,p} \rvert < 0.50$ GeV	$2.0 < y_{f_1} < 4.5$
$(l,S) = (2,2)$	$g'_{I\!PI\!Pf_1} = 4.89$	14.8	37.5	6.46	18.9
$(l,S) = (4,4)$	$g''_{I\!PI\!Pf_1} = 10.31$	13.8	34.0	6.06	18.1
$(\varkappa', \varkappa'')$	$\varkappa''/\varkappa' = -6.25\ \mathrm{GeV}^{-2}$	18.6	45.8	7.14	23.1
$(\varkappa', \varkappa'')$	$\varkappa''/\varkappa' = -2.44\ \mathrm{GeV}^{-2}$	17.5	43.4	7.10	22.1
$(\varkappa', \varkappa'')$	$\varkappa''/\varkappa' = -1.0\ \mathrm{GeV}^{-2}$	16.6	41.0	7.09	20.5

Predictions for the LHC experiments

- One of the most prominent decay modes of the $f_1(1285)$ is $f_1(1285) \to \pi^+\pi^-\pi^+\pi^-$
- There $f_1(1285)$ and $f_2(1270)$ are close in mass.
  We obtain for $\sqrt{s} = 13$ TeV and $|\mathrm{y}_M| < 2.5$:

$$\sigma_{pp\to pp f_1(1285)} \times \mathcal{BR}(f_1(1285) \to 2\pi^+2\pi^-) = 34.0\ \mu\mathrm{b} \times 0.109 = 3.7\ \mu\mathrm{b}$$

$$\sigma_{pp\to pp f_2(1270)} \times \mathcal{BR}(f_2(1270) \to 2\pi^+2\pi^-) = 11.3\ \mu\mathrm{b} \times 0.028 = 0.3\ \mu\mathrm{b}$$

← CEP of $f_2(1270)$: Lebiedowicz et al., PRD 93 (2016) 054015, PRD 101 (2020) 034008

- As the $f_1(1285)$ has a much narrower width than the $f_2(1270)$ it would be seen in the M(4π) distribution as a peak on top of broader $f_2(1270)$ and of the 4π continuum background

- $f_1(1285)$ is seen in the preliminary ATLAS-ALFA results for $pp \to pp\pi^+\pi^-\pi^+\pi^-$ at $\sqrt{s} = 13$ TeV and for $|\eta_\pi| < 2.5, p_{t,\pi} > 0.1\,\mathrm{GeV}, \max(p_{t,\pi}) > 0.2\,\mathrm{GeV}, 0.17\,\mathrm{GeV} < |p_{y,p}| < 0.5\,\mathrm{GeV}$

[R. Sikora, CERN-THESIS-2020-235]

- Theoretical studies of the reaction *pp* → *pp 4π* including both the resonances and continuum contributions within the tensor-pomeron approach → in progress:

4π production via the intermediate σσ and *ρρ states: Lebiedowicz, Nachtmann, Szczurek, PRD 94 (2016) 034017*
4π continuum: *Kycia, Lebiedowicz, Szczurek, Turnau, PRD 95 (2017) 094020*
$f_1(1285)$ production: *Lebiedowicz, Leutgeb, Nachtmann, Rebhan, Szczurek, PRD 102 (2020) 114003*

using GenEx MC generator for exclusive reactions and DECAY MC library for the decay of a particle with ROOT compatibility:
GenEx MC, *Kycia, Chwastowski, Staszewski, Turnau, Commun. Comput. Phys. 24 (2018) 860*
DECAY MC, *Kycia, Lebiedowicz, Szczurek, Commun. Comput. Phys. 30 (2021) 942*