Coupled-channel systems for light baryons from and with QCD

Matthias F.M. Lutz

GSI Helmholtzzentrum für Schwerionenforschung GmbH

- **Chiral SU(3) expansions in QCD**
- **Chiral extrapolation for baryons masses**
- **Coupled-channel dynamics for charmed mesons (GPA)**
- **Summary and outlook**

arXiv:2301.06837, arXiv:2211.03508, arXiv:2209.10601, arXiv:1808.08695, arXiv:1801.10122

The chiral Lagrangian with baryon fields

$$\Phi = \sqrt{2}\begin{pmatrix} \frac{\pi^0}{\sqrt{2}} + \frac{\eta}{\sqrt{6}} & \pi^+ & K^+ \\ \pi^- & -\frac{\pi^0}{\sqrt{2}} + \frac{\eta}{\sqrt{6}} & K^0 \\ K^- & \overline{K^0} & -\frac{2}{\sqrt{6}}\,\eta \end{pmatrix}$$

Goldstone boson octet $(J^P = 0^-)$

baryon octet $(J^P = \frac{1}{2}^+)$

$$B = \begin{pmatrix} \frac{\Sigma^0}{\sqrt{2}} + \frac{\Lambda}{\sqrt{6}} & \Sigma^+ & p \\ \Sigma^- & -\frac{\Sigma^0}{\sqrt{2}} + \frac{\Lambda}{\sqrt{6}} & n \\ \Xi^- & \Xi^0 & -\frac{2}{\sqrt{6}}\,\Lambda \end{pmatrix}$$

✔ Leading order terms

covariant derivative $\partial_\mu = \partial_\mu + \cdots$

$$\begin{aligned} \mathcal{L} \;=\;& \mathrm{tr}\left\{\bar{B}(i\,\partial\cdot\gamma - M_{[8]})\,B\right\} + F\,\mathrm{tr}\left\{\bar{B}\,\gamma^\mu\gamma_5\,[i\,U_\mu, B]\right\} + D\,\mathrm{tr}\left\{\bar{B}\,\gamma^\mu\gamma_5\,\{i\,U_\mu,\, B\}\right\} \\ -\;& \mathrm{tr}\left\{\bar{B}_\mu\cdot\left((i\,\partial\cdot\gamma - M_{[10]})\,g^{\mu\nu} - i\,(\gamma^\mu\partial^\nu + \gamma^\nu\partial^\mu) + \gamma^\mu(i\,\partial\cdot\gamma + M_{[10]})\gamma^\nu\right)B_\nu\right\} \\ +\;& C\left(\mathrm{tr}\left\{(\bar{B}_\mu\cdot i\,U^\mu)\,B\right\} + \mathrm{h.c.}\right) + H\,\mathrm{tr}\left\{(\bar{B}^\mu\cdot\gamma_\nu\gamma_5\,B_\mu)\,i\,U^\nu\right\} \end{aligned}$$

- $U_\mu = \frac{1}{2}\,u^\dagger\left(\partial_\mu\,e^{i\,\frac{\Phi}{f}}\right)u^\dagger - \frac{i}{2}\,u^\dagger\,(v_\mu + a_\mu)\,u + \frac{i}{2}\,u\,(v_\mu - a_\mu)\,u^\dagger$ with $u = e^{i\,\frac{\Phi}{2\,f}}$
- from $B \to B' + e + \bar{\nu}_e$: $F \simeq 0.45$ and $D \simeq 0.80$
- from large-N_c: $H = 9\,F - 3\,D$ and $C = 2\,D$

Leading-order chiral interaction

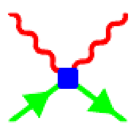
✔ **Leading-order interaction**

covariant derivative D_μ (local chiral SU(3) rotations) in kinetic term:

e.g. $\mathrm{tr}(\bar{B}_{[8]}\, i\, \gamma_\mu\, D^\mu B_{[8]})$ for baryon octet

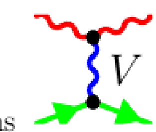
✔ **Weinberg–Tomazawa term for meson-baryon interaction**

$$\mathcal{L}_{\mathrm{WT}} = \frac{i}{8\,f^2}\,\mathrm{tr}\;\bar{B}_{[8]}\,\gamma_\mu \left[\left[\Phi_{[8]}, (\partial^\mu \Phi_{[8]})\right]_-, B_{[8]}\right]_- + \frac{3\,i}{8\,f^2}\,\mathrm{tr}\; g_{\alpha\beta}\left(\bar{B}^\alpha_{[10]}\,\gamma^\mu\; B^\beta_{[10]}\right)\cdot\left[\Phi_{[8]}\,, (\partial_\mu \Phi_{[8]})\right]_-$$



$$= \frac{C_{\alpha,\beta}}{4\,f^2}\left(\not{k}_\alpha + \not{k}_\beta\right)$$ $\longleftarrow$ linear in meson 4-momentum

- vector meson t-channel exchange $\sum_{\text{vector mesons}}$

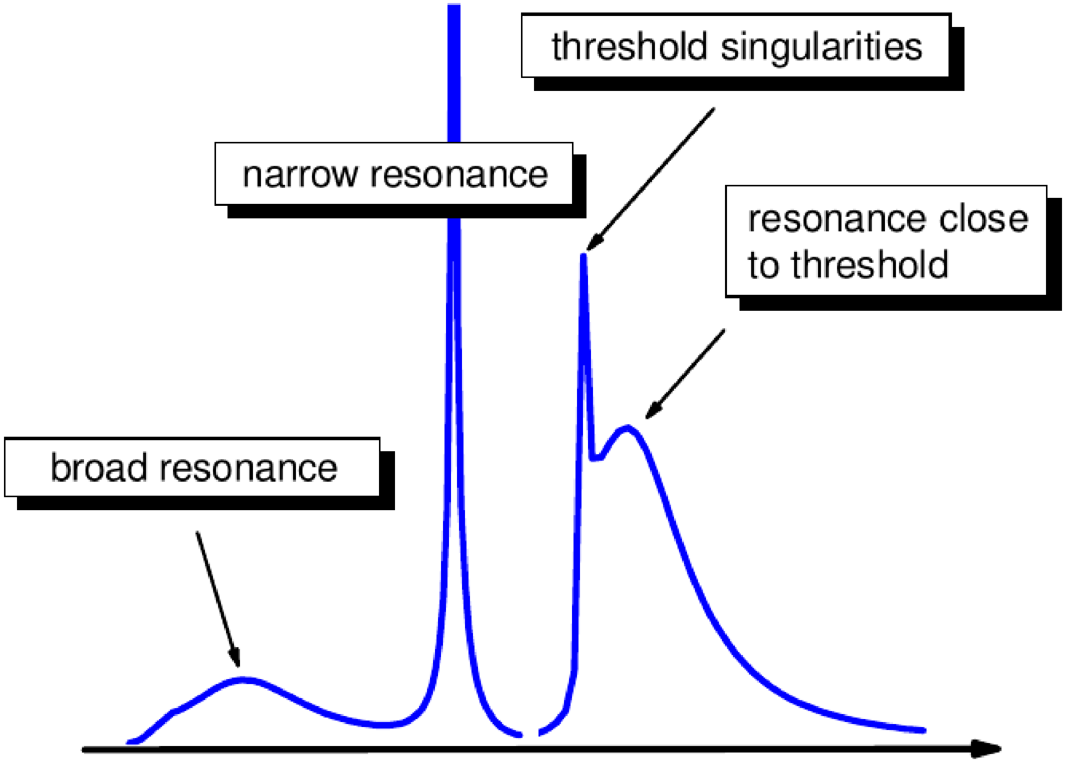


$$\Longrightarrow \quad \mathcal{K}\, C_{\alpha,\beta}\left(\not{k}_\alpha + \not{k}_\beta\right)$$

✔ **Pion-decay constant** $f \Longleftrightarrow f_\pi/f \simeq 1.07 \pm 0.12$ $\qquad f_\pi \simeq 92.42 \pm 0.33$ MeV

Phys. Lett. B585 (2004) 243

Speed plots for multichannel scattering

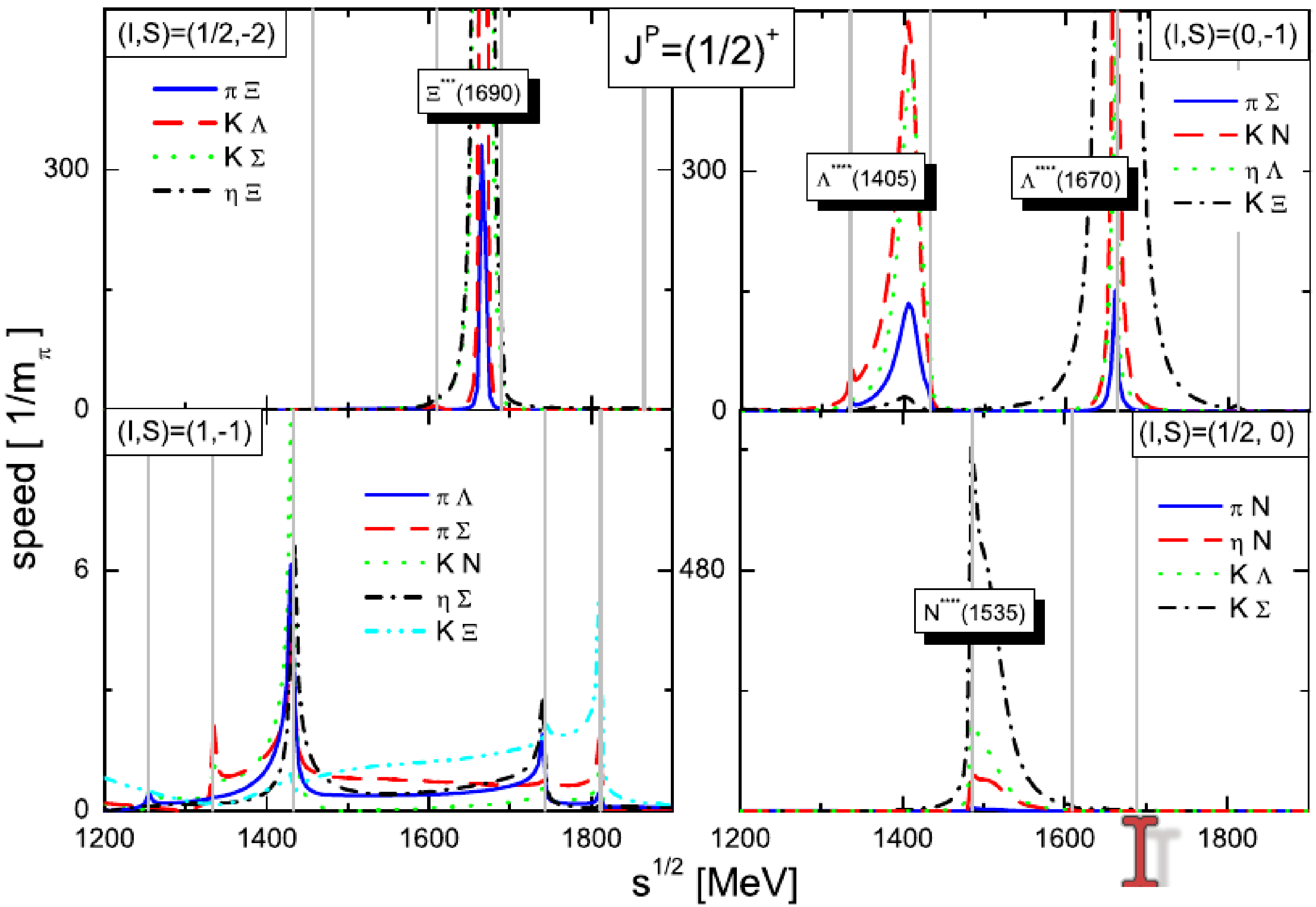


- generalization of time-delay for closed/open channels $\dfrac{d\delta(E)}{dE}$
- resonance position $\longleftarrow$ local maximum of the Speed
- for s-wave resonances - cusp effects at thresholds

✔ **analytic continuation of the S-matrix:** $S_{ab} = \delta_{ab} + 2\,i\,T_{ab}$

$$\mathrm{Speed}_{ab}^{(I,S)}(\sqrt{s}) = \left| \sum_c \left[\frac{d}{d\sqrt{s}} S_{ac}^{(I,S)}(\sqrt{s}) \right] \left(S_{cb}^{(I,S)}(\sqrt{s}) \right)^* \right|$$

$J^P = \frac{1}{2}^-$ baryon resonances: $8 \otimes 8 = 27 \oplus 10 \oplus \overline{10} \oplus 8 \oplus 8 \oplus 1$



Chiral forces

Attraction in both 8 and 1

Repulsion in 27plet

Repulsion in 10 and $\overline{10}$

✔ **Why does this work?**

- how to make it more quantitative?
- how to connect it to QCD?

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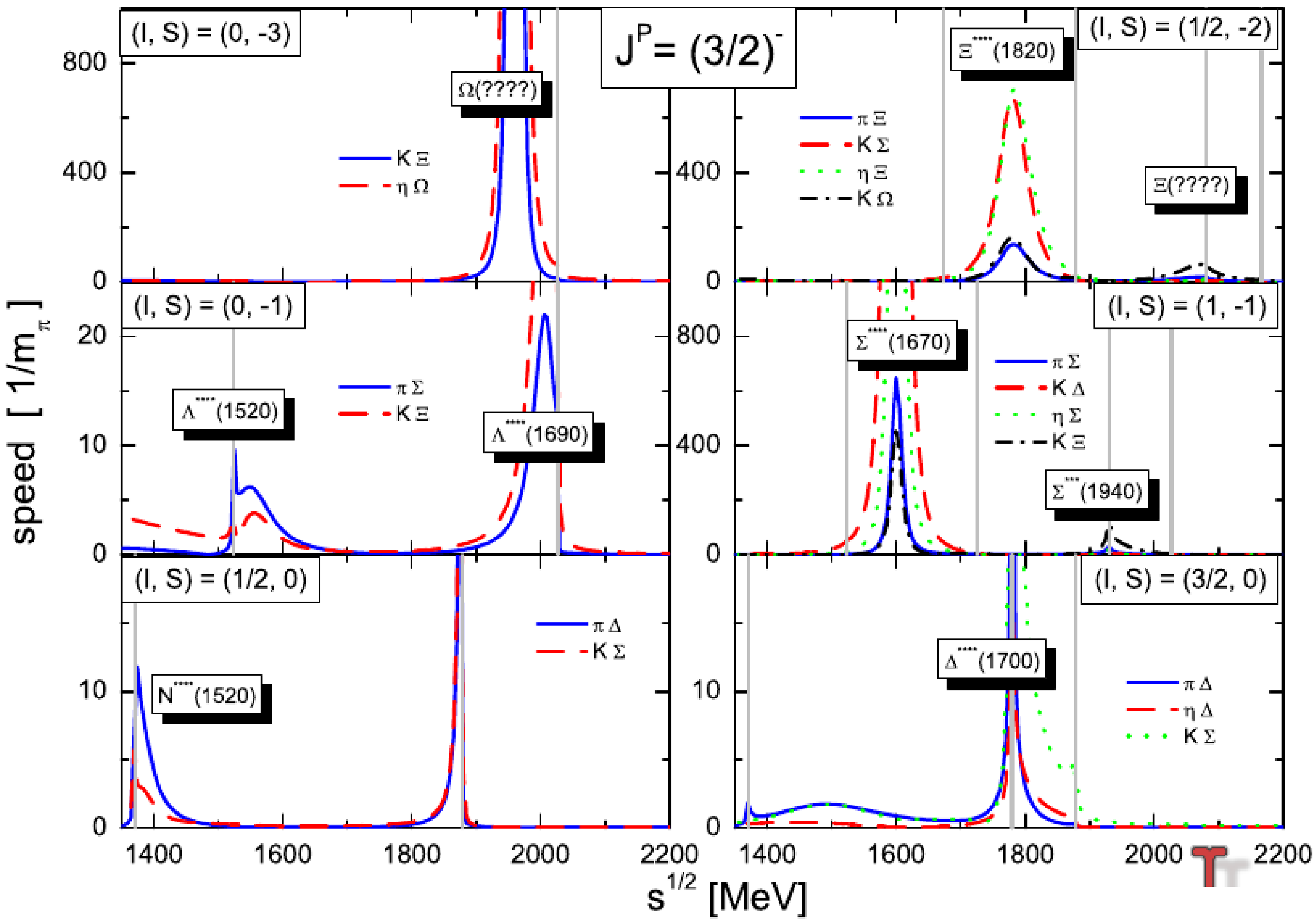
$J^P = \frac{3}{2}^-$ baryon resonances: $8 \otimes 10 = 35 \oplus 27 \oplus 10 \oplus 8$



Chiral forces

- Strong attraction 10 and 8
- Weak attraction in 27plet
- Repulsion in 35plet

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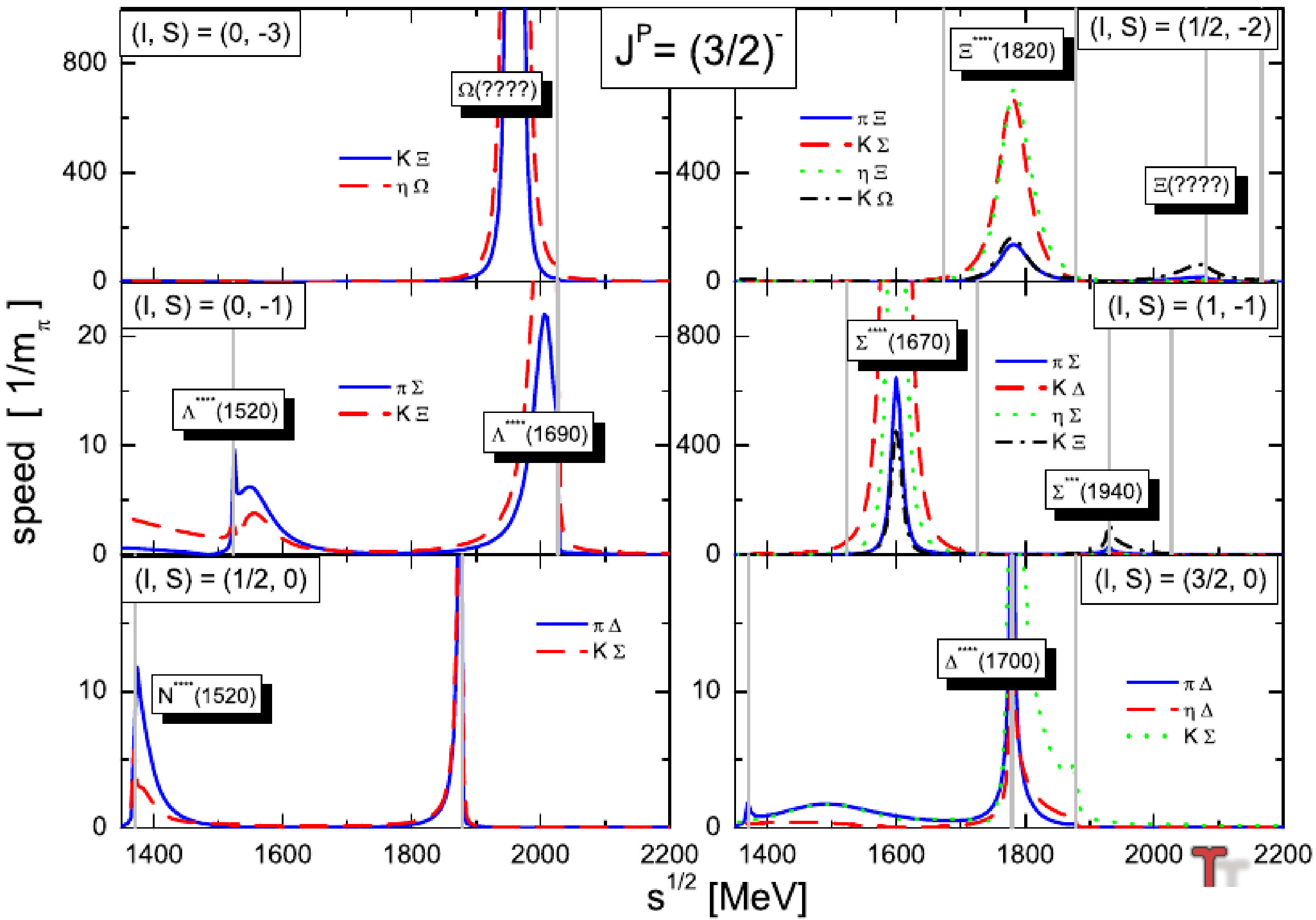

Chiral forces

- Attraction in 10 and 8
- Weak attraction in 27plet
- Repulsion in 35plet

✔ **Possibility of an exotic 27plet**

$$(\mathrm{I,S})_{[27]} = (1,1), (\tfrac{3}{2},0), (\tfrac{1}{2},0), (2,-1), (1,-1), (0,-1), (\tfrac{3}{2},-2), (\tfrac{1}{2},-2), (1,-3)$$

- depends on chiral correction terms

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Chiral symmetry breaking terms

$$
\begin{aligned}
\mathcal{L}_\chi^{(2)} &= 2\,b_0\,\mathrm{tr}\,\left(\bar{B}\,B\right)\mathrm{tr}\,(\chi_+) + 2\,b_D\,\mathrm{tr}\,\left(\bar{B}\,\{\chi_+,\,B\}\right) + 2\,b_F\,\mathrm{tr}\,\left(\bar{B}\,[\chi_+,\,B]\right) \\
&- 2\,d_0\,\mathrm{tr}\,\left(\bar{B}_\mu\cdot B^\mu\right)\mathrm{tr}\,(\chi_+) - 2\,d_D\,\mathrm{tr}\,\left(\left(\bar{B}_\mu\cdot B^\mu\right)\chi_+\right)
\end{aligned}
$$

$$
\chi_+ = \chi_0 - \frac{1}{8\,f^2}\left\{\Phi,\left\{\Phi,\,\chi_0\right\}\right\} + \mathcal{O}\left(\Phi^4\right)
$$

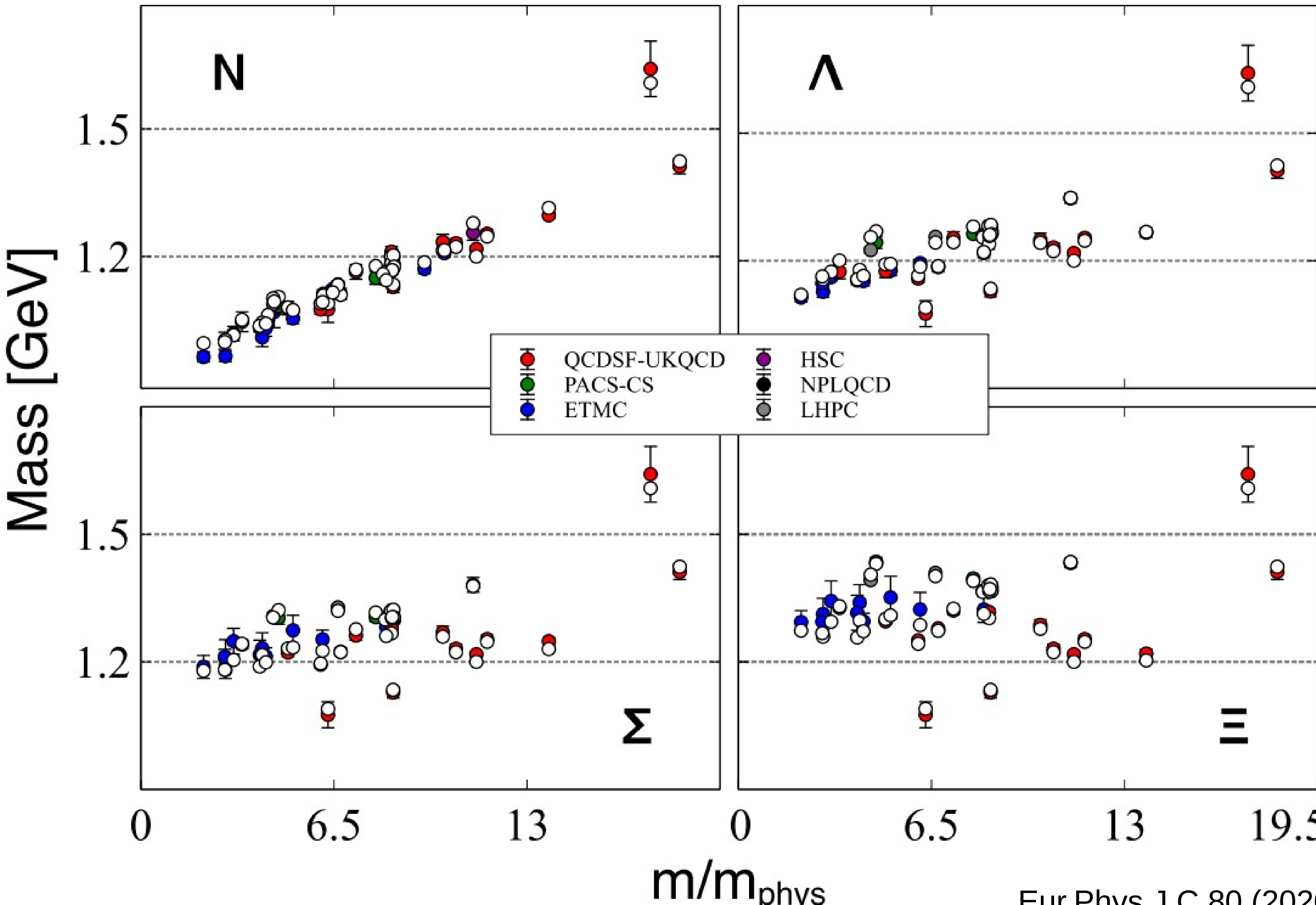
quark mass matrix $\qquad \chi_0 \sim \begin{pmatrix} m_u & 0 & 0 \\ 0 & m_d & 0 \\ 0 & 0 & m_s \end{pmatrix}$

✔ Relevance of low-energy parameters

- quark-mass dependence of the baryon masses $\leftrightarrow$ lattice QCD
- meson-baryon scattering $\leftrightarrow$ resonances in QCD
- nucleon sigma terms, $\langle N|\bar{u}\,u|N\rangle$, $\langle N|\bar{d}\,d|N\rangle$ and $\langle N|\bar{s}\,s|N\rangle$ relevant in WIMP scenarios – ATLAS

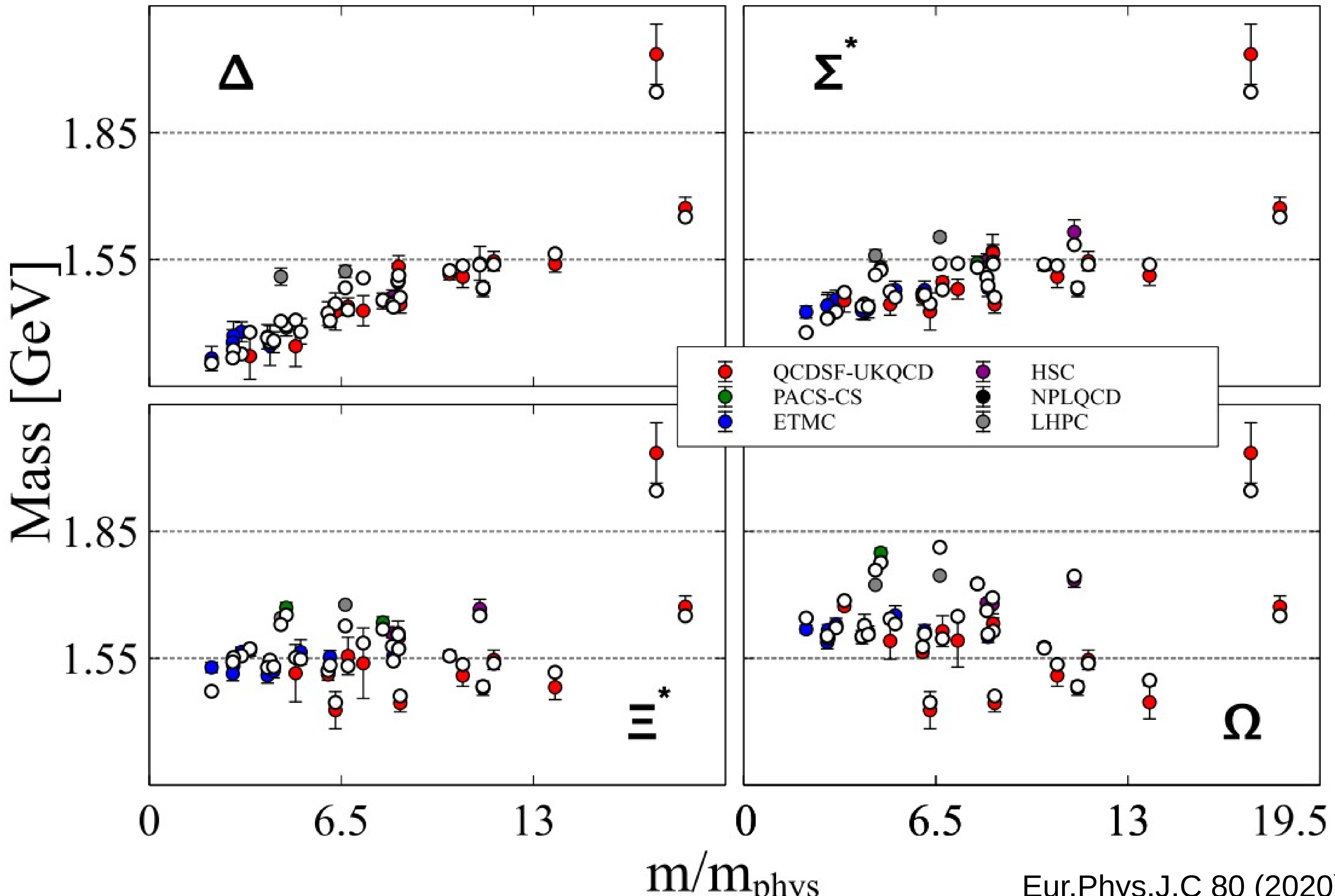
see e.g. Eur.Phys.J.C 78 (2018) 7

Quark-mass dependence of the baryon octet masses



Eur.Phys.J.C 80 (2020) 3

Quark-mass dependence of the baryon decuplet masses



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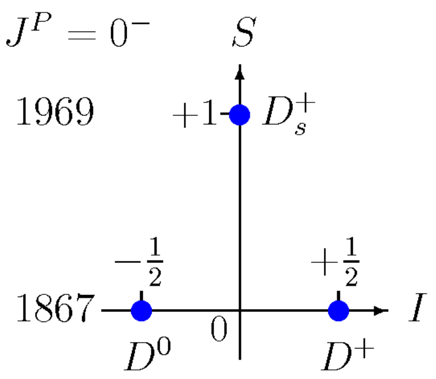
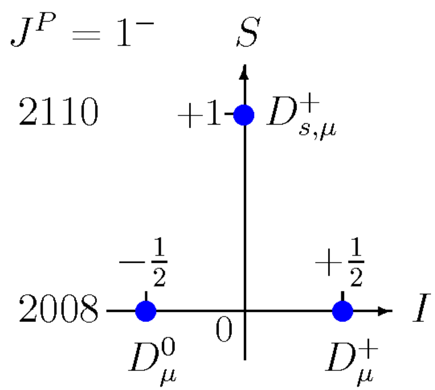
Chiral Lagrangian for charmed mesons

✔ **Heavy-light mesons** $(c\,\bar q)$ SU(3) anti-triplet $[\bar 3]$





$$\mathcal{L} = (\partial_\mu D)(\partial^\mu \bar D) - M^2\, D\, \bar D - (\partial_\mu D^{\mu\alpha})(\partial^\nu \bar D_{\nu\alpha}) + \frac{1}{2}\,\tilde M^2\, D^{\mu\alpha}\,\bar D_{\mu\alpha}$$
$$+\,2\,g_P\left\{D_{\mu\nu}\,U^\mu\,(\partial^\nu \bar D) - (\partial^\nu D)\,U^\mu\,\bar D_{\mu\nu}\right\}$$
$$-\,\frac{i}{2}\,\tilde g_P\,\epsilon^{\mu\nu\alpha\beta}\left\{D_{\mu\nu}\,U_\alpha\,(\partial^\tau \bar D_{\tau\beta}) + (\partial^\tau D_{\tau\beta})\,U_\alpha\,\bar D_{\mu\nu})\right\}$$

covariant derivative $\quad \partial_\mu \to \partial_\mu + \frac{1}{2}\,e^{-i\,\frac{\Phi}{2\,f}}\,\partial_\mu\,e^{+i\,\frac{\Phi}{2\,f}} + \frac{1}{2}\,e^{+i\,\frac{\Phi}{2\,f}}\,\partial_\mu\,e^{-i\,\frac{\Phi}{2\,f}}$

- chiral symmetry : $f \sim 90$ MeV chiral SU(3) limit value of f_π
- hadronic decay of $D^* \to D\pi$ implies $|g_P| = 0.57 \pm 0.07$
- heavy-quark spin symmetry : $\tilde g_P = g_P$ and $M = \tilde M$ as $m_c \to \infty$

Coupled-channel scattering with long range forces

$$T^J_{ab}(s) = U^J_{ab}(s) + \sum_{c,d} \int_{\mu^2_{thr}}^{\infty} \frac{d\bar{s}}{\pi} \frac{s - \mu^2_M}{\bar{s} - \mu^2_M} \frac{T^J_{ac}(\bar{s})\, \rho^J_{cd}(\bar{s})\, T^{J*}_{db}(\bar{s})}{\bar{s} - s - i\epsilon}$$

✔ **Derive $T^J_{ab}(s)$ from the Chiral Lagrangian (GPA)**

✔ **$T^J_{ab}(s)$ is computed in terms of non-linear integral equations**

- use perturbation theory for $U^J_{ab}(s)$ followed by a conformal expansion

$$U(s) = U_{\rm close-by}(s) + U_{\rm far-distant}(s)$$

$$\text{with} \qquad U_{\rm far-distant}(s) = \sum_k c_k\, \xi^k(s)$$

arXiv:2209.10601, arXiv:1212.3057, arXiv:1106.2230v2, arXiv:1003.3426v2

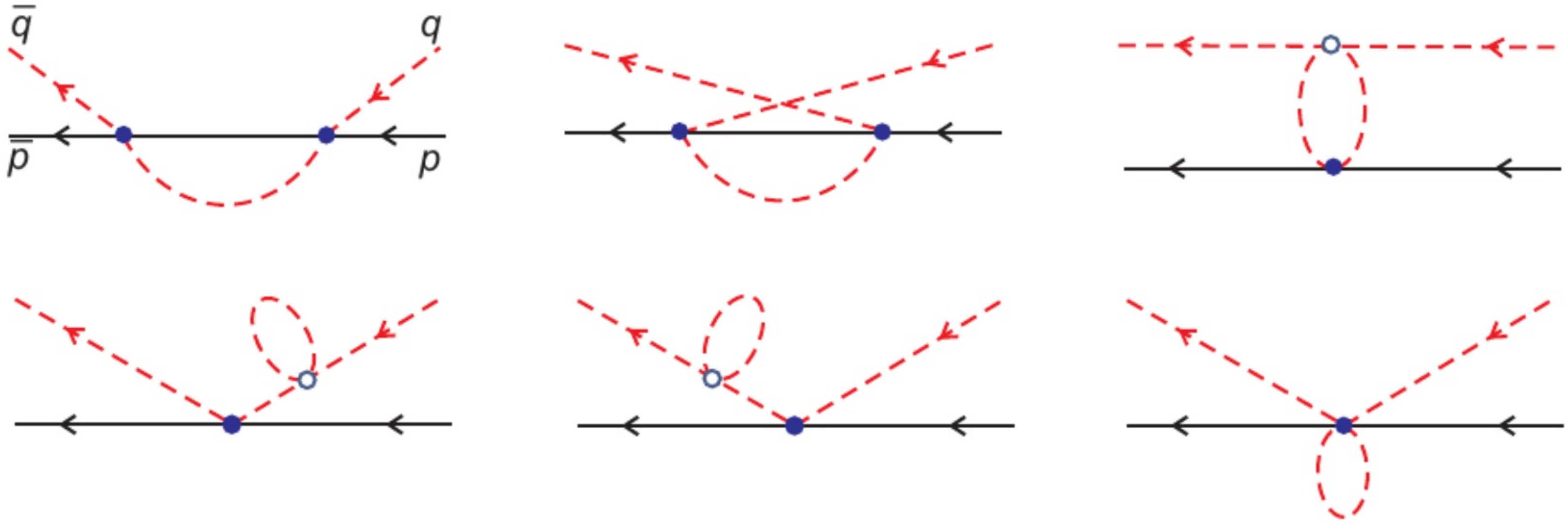
Coupled-channel scattering with long range forces

$$T^J_{ab}(s) = U^J_{ab}(s) + \sum_{c,d} \int_{\mu^2_{thr}}^{\infty} \frac{d\bar{s}}{\pi} \frac{s-\mu_M^2}{\bar{s}-\mu_M^2} \frac{T^J_{ac}(\bar{s})\rho^J_{cd}(\bar{s})T^{J*}_{db}(\bar{s})}{\bar{s}-s-i\epsilon}$$

✔ **Derive $T^J_{ab}(s)$ from the Chiral Lagrangian (GPA)**

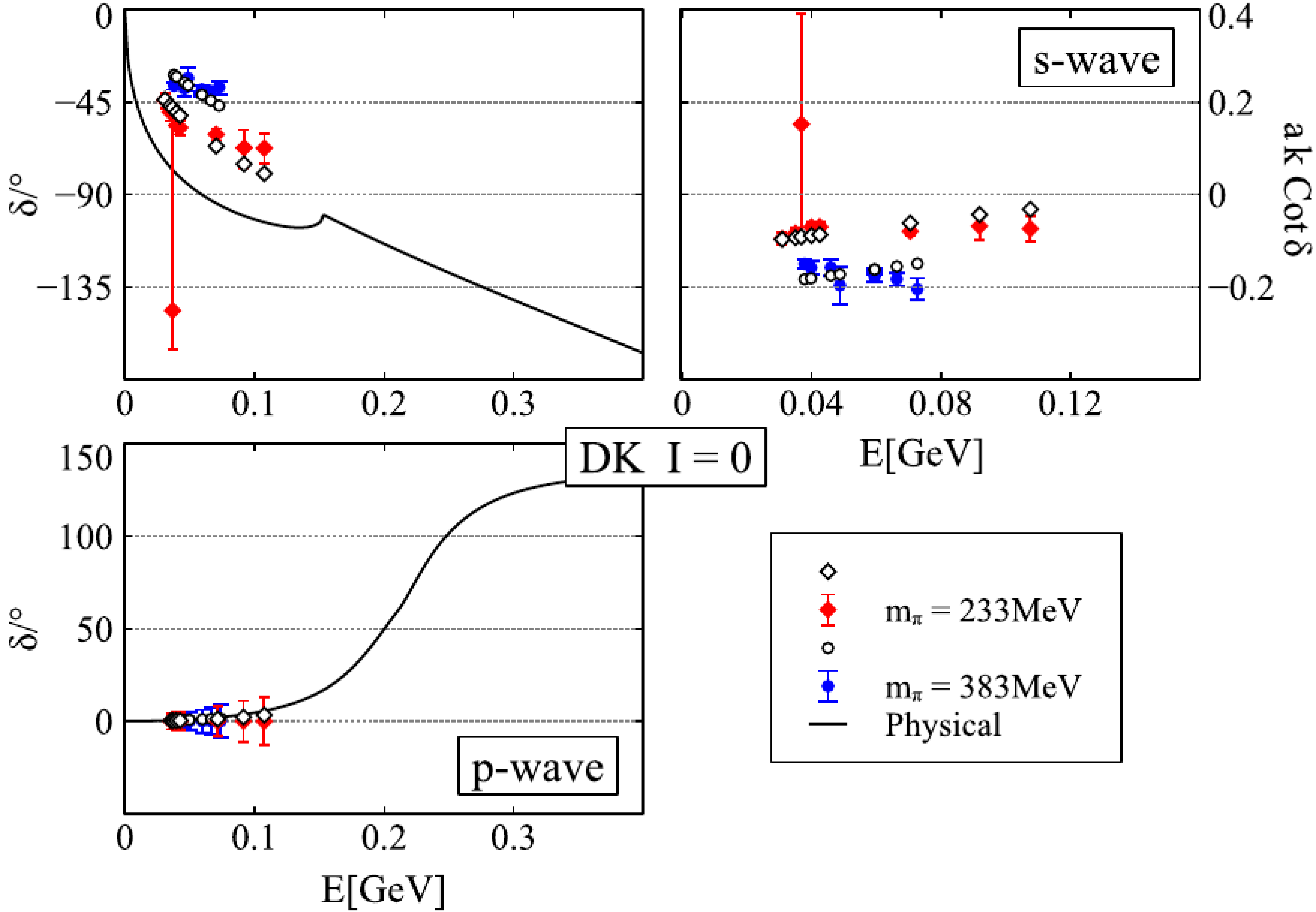
✔ **$T^J_{ab}(s)$ is computed in terms of non-linear integral equations**

- use perturbation theory for $U^J_{ab}(s)$ followed by a conformal expansion
- truncate $U^J_{ab}(s)$ at the one-loop level and fit LEC to Lattice QCD data



arXiv:2209.10601

Lattice ensembles from HSC



arXiv:2209.10601
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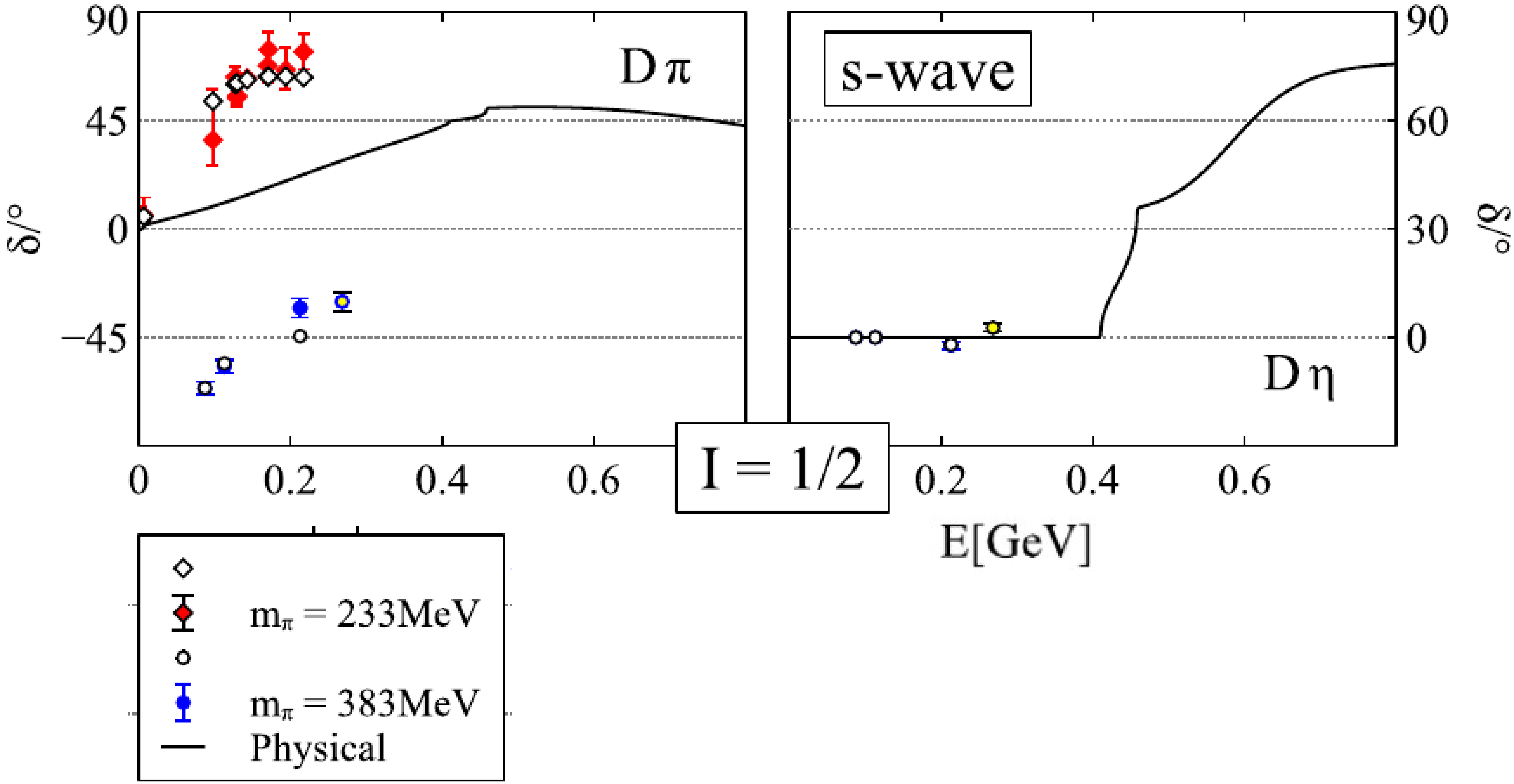


- at unphysical quark masses (published HSC ensemble)
- black line: prediction of phase shifts at physical quark masses
- dashed red points were predicted (by now results from HSC)

arXiv:2209.10601
arXiv:1801.10122

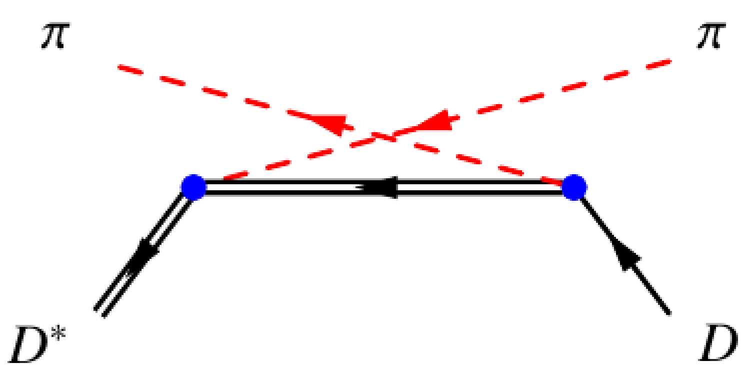
Anomalous thresholds and coupled-channel unitarity

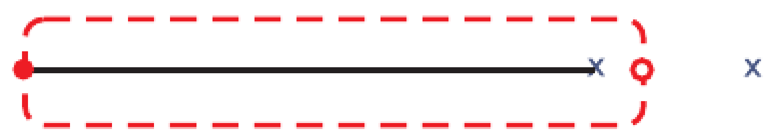
✔ Consider p-wave scattering of πD with $I = 1/2$

- couples to a p-wave πD^* channel

✔ Anomalous threshold occurs at physical masses

- assume first $M_{D^*} < M_D + m_\pi$ (can be tuned on Lattice QCD ensembles)
- for $m_\pi = 150$ MeV we find a normal system
- for $m_\pi = 145$ MeV we find an anomalous reaction $\pi D \to \pi D^*$





arXiv:1808.08695
arXiv:1506.02375

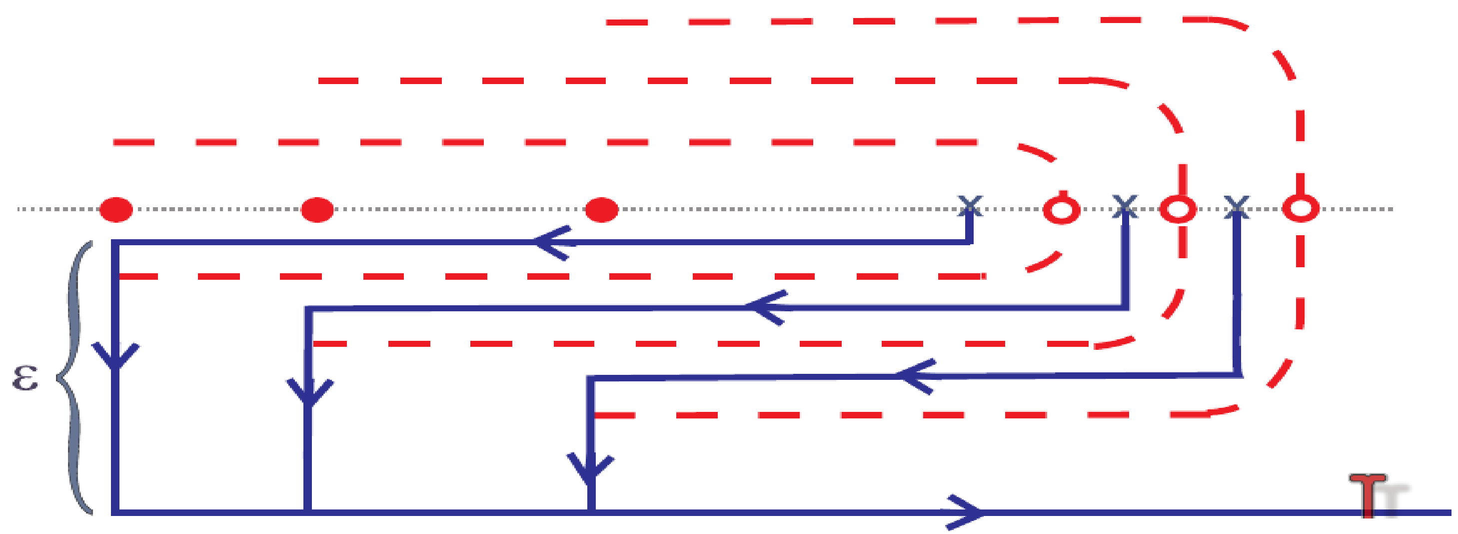
Anomalous thresholds and coupled-channel unitarity

$$T^J_{ab}(s) = U^J_{ab}(s) + \sum_{c,d} \int_{\mu^2_{thr}}^{\infty} \frac{d\bar{s}}{\pi} \frac{s-\mu_M^2}{\bar{s}-\mu_M^2} \frac{T^J_{ac}(\bar{s})\rho^J_{cd}(\bar{s})T^{J*}_{db}(\bar{s})}{\bar{s}-s-i\epsilon}$$

✔ **How to solve this equation with anomalous left-hand cut lines?**

✔ **Analytic continuation as implied by deformed s-channel cut lines**

- suggested already by Mandestam et al in the 1960s
- a first implementation in hadron physics only 2018



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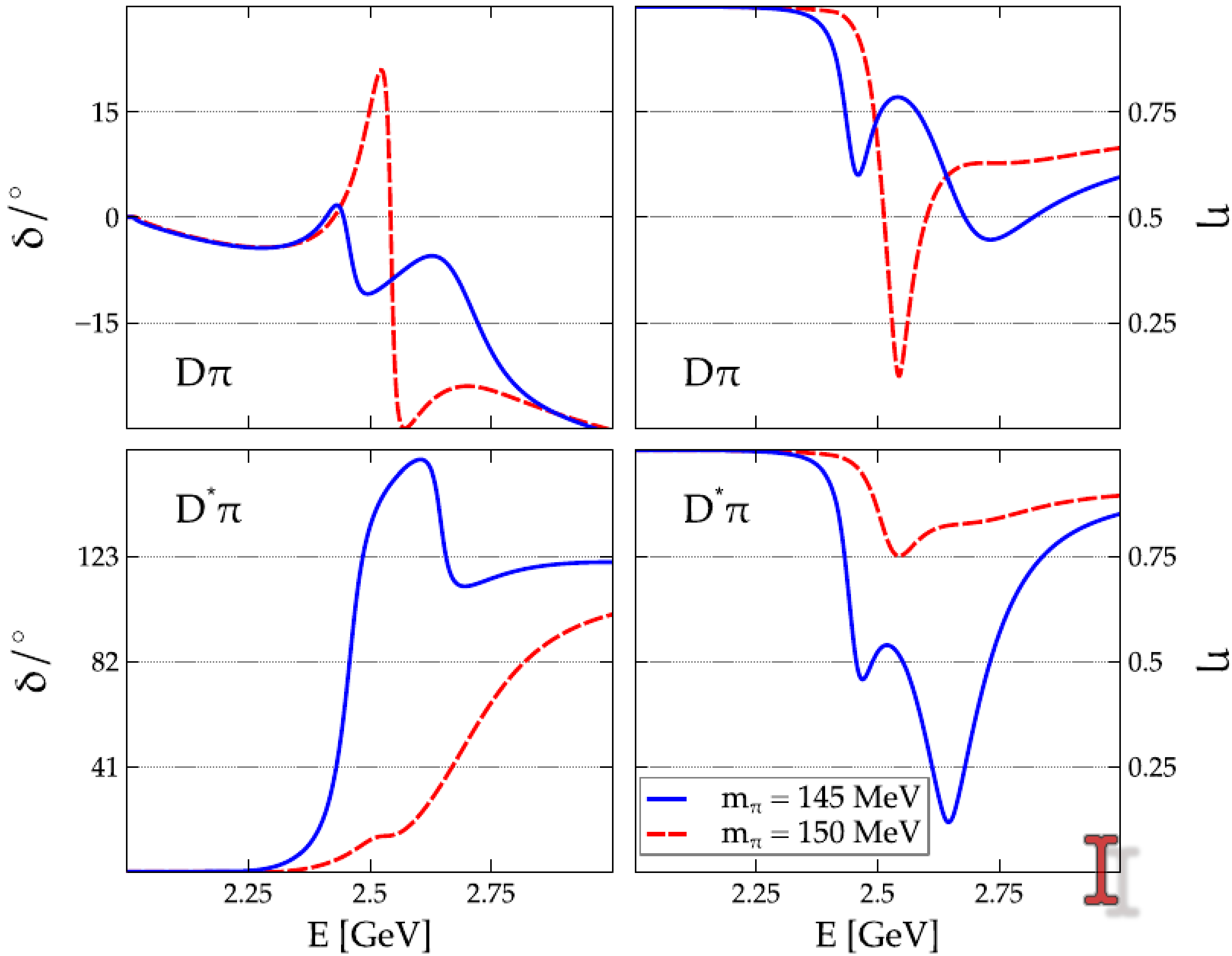
P-wave state from anomalous threshold effects



arXiv:2209.10601

Summary & Outlook

✔ Chiral extrapolation with up, down and strange quarks

- resummed χPT : use on-shell masses in the loops
  - chiral expansion is working
- coupled-channel computations based on the chiral Lagrangian
  - controlled access to the spectrum via conformal expansion
  - left-hand cut contributions are important
- predict a large number of low-energy constants from Lattice QCD simulations
  - the quark-mass dependence of phase shifts is not always trivial
  - anomalous threshold effects may cause the dynamic generation of p-wave states

✔ QCD spectroscopy with coupled-channel dynamcis

- curent QCD lattice data provide the counter terms for hadron-hadron scattering
- use as input in systematic coupled-channel computations
- analyze and predict the quark-mass dependence of hadron resonances in QCD
- use as input in amplitude analysis of experimental data sets

thanks to: Yonggoo Heo, Xiao-Yu Guo and Csaba Korpa