

# Spectroscopy of light nuclei through $\chi$ EFT-based PGCM calculations

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  - ◇ Development of new methods but also **revisiting old ones**
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Egido, *Physica Scripta* 91, 073003 (2016)  
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→ technical know-how
- Many advantages
  - ◊ Efficient at capturing static correlations (e.g. deformation)
  - ◊ Respects the symmetries of  $H$
  - ◊ Access to excited states and various observables
  - ◊ Gentle scaling: mean field  $\times$  large prefactor

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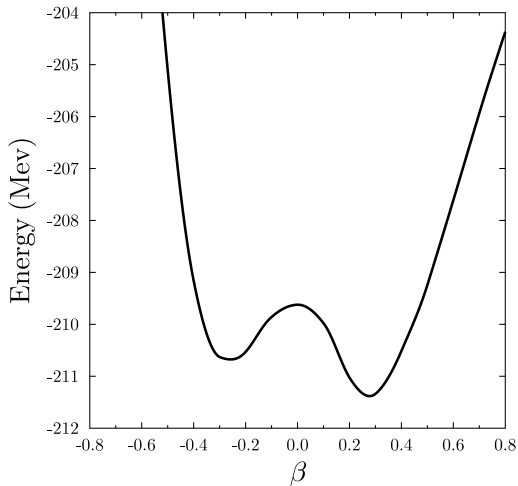
- Reference states  $\{|\Phi_i\rangle\}$   $\rightarrow$  capture collective correlations

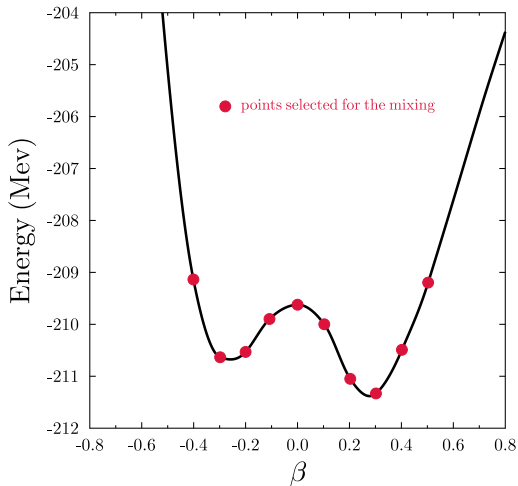
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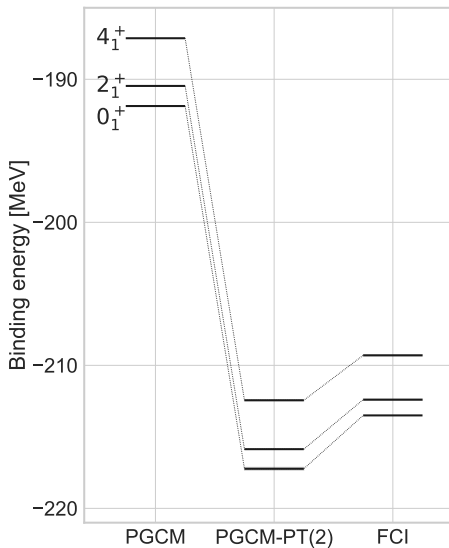
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- New formalism: **PGCM - Perturbation Theory (PGCM-PT)**

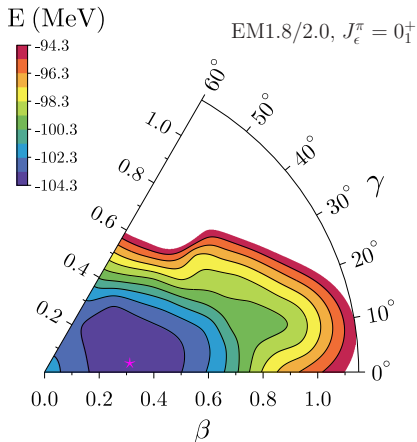
Frosini, EPJA 58, 62 (2022); Frosini, EPJA 58, 63 (2022); Frosini, EPJA 58, 64 (2022)

- ◇ Multi-reference perturbation theory on top of a PGCM reference state
- ◇ Includes missing dynamical correlations

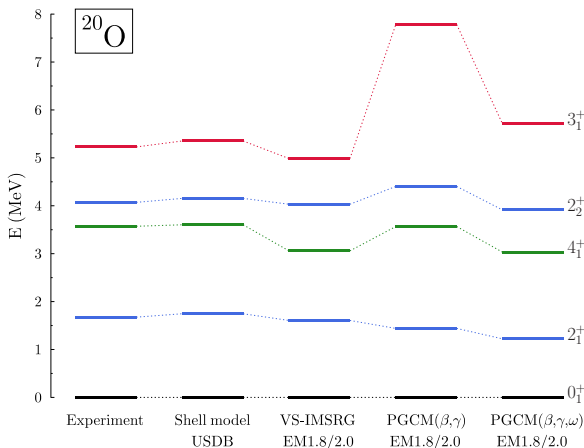


- $^{20}\text{O}$ : Recent experimental results compared with *ab initio* calculations  
Zanon, PRL 131, 262501 (2023)
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- Details of the calculation
  - ◇ Hamiltonians: EM1.8/2.0, Hüther N3LO  
Hebeler, PRC 83, 031301 (2011); Hüther, PLB 808, 135651 (2020)
  - ◇ Rank-reduction of 3N to an effective 2N  
Frosini, EPJA 57, 151 (2021)
  - ◇  $e_{\text{max}} = 6$  (7 HO shells),  $e_{3\text{max}} = 18$ ,  $\hbar\omega = 12$
  - ◇ Collective coordinates: triaxial deformations ( $\beta$ ,  $\gamma$ ), cranking ( $\omega$ )



- Minimum only slightly triaxial:  $\beta \approx 0.31$ ,  $\gamma \approx 6^\circ$



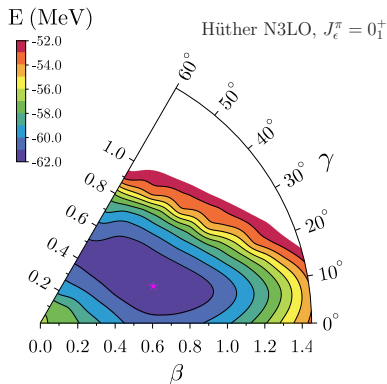
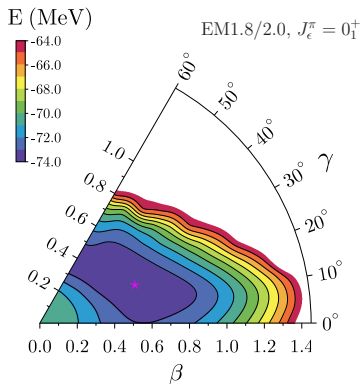
[Data partly taken from Zanon, PRL 131, 262501 (2023)]

- PGCM( $\beta, \gamma, \omega$ ) consistent with VS-IMSRG calculations

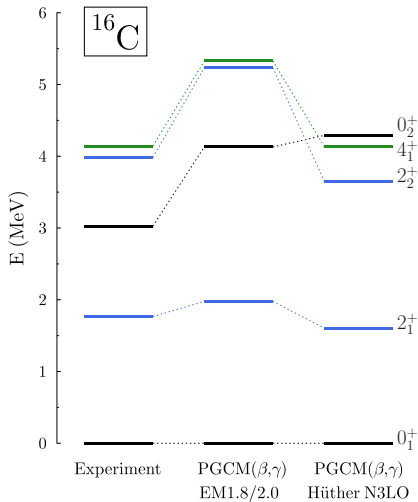
$B(E2)$ ( $e^2\text{fm}^4$ )	Experiment	Shell model USDB	VS-IMSRG EM1.8/2.0	PGCM( $\beta, \gamma$ ) EM1.8/2.0	PGCM( $\beta, \gamma, \omega$ ) EM1.8/2.0
$2_1^+ \rightarrow 0_1^+$	5.9(2)	3.25	0.89	1.59	1.33
$2_2^+ \rightarrow 0_1^+$	1.3(2)	0.77	0.20	0.45	0.45
$2_2^+ \rightarrow 2_1^+$	4(2)	0.0005	0.07	0.01	0.006
$3_1^+ \rightarrow 2_1^+$	0.32(7)	0.57	0.17	1.13	0.34

- Theory does not reproduce experimental data  
(not shown here but works slightly better for  $B(M1)$ )
- Cranking does not change much the transition probabilities  
→ but better for  $3_1^+$

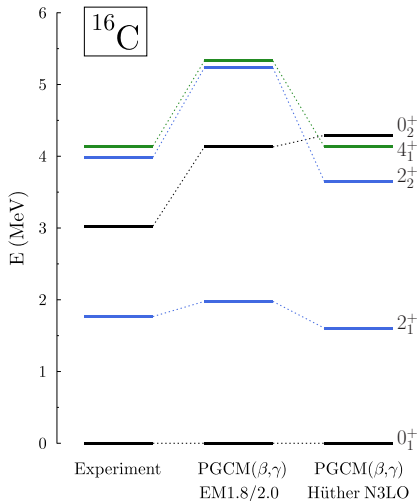




- Similar energy surfaces but EM1.8/2.0 more rigid
- Triaxial minima: 0.55, 22° (EM1.8/2.0) and 0.63, 18° (Hütter N3LO)



- Preliminary predictions, e.g. need to add cranking



Quantity	EM1.8/2.0	Hüther N3LO
$\mu(2_1^+)$	+0.11	+0.19
$\mu(2_2^+)$	+0.54	+0.94
$\mu(4_1^+)$	-0.30	-0.03
$Q_s(2_1^+)$	-3.4	-4.0
$Q_s(2_2^+)$	+4.0	+4.4
$Q_s(4_1^+)$	-5.5	-8.1
$B(E2: 2_1^+ \rightarrow 0_1^+)$	3.2	5.3
$B(E2: 4_1^+ \rightarrow 2_1^+)$	2.2	3.0
$B(E2: 2_2^+ \rightarrow 2_1^+)$	3.6	7.3
$B(E2: 0_2^+ \rightarrow 2_1^+)$	2.3	4.1

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- Developments of PGCM in the *ab initio* context
  - ◇ Use of  $\chi$ EFT-based Hamiltonians
  - ◇ Formulation of PGCM-PT
  - ◇ Work remains to make the whole scheme more systematic/controllable

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- Useful tool to describe low-energy spectroscopy
  - ◊ Captures important collective correlations
  - ◊ Conserves symmetries and associated selection rules for transitions
  - ◊ Access to various observables of interest ( $J^\pi$ ,  $E_{\text{exc}}$ ,  $Q_s$ ,  $\mu$ ,  $B(T\lambda)$ , ...)
  
- Good description of  $^{20}\text{O}$  and  $^{16}\text{C}$  spectra



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