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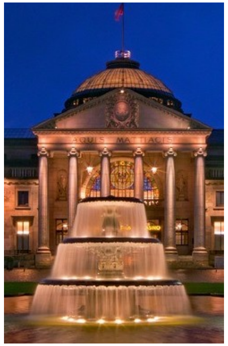

DREB24

Ab initio effective potentials for proton elastic scattering based on NCSM nonlocal densities

Charlotte Elster

Thanks to collaborators:

R.B. Baker, M. Burrows S.P. Weppner, K. Launey, P. Maris, G. Popa

Supported by

U.S. DEPARTMENT OF ENERGY
Office of Science

NERSC National Energy Research Scientific Computing Center

Department of physics + astronomy

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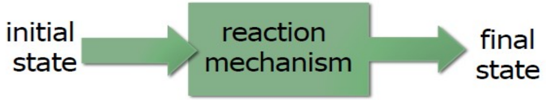
How do we learn about nuclei: Reactions

Exotic Nuclei are usually short lived:

Have to be studied with reactions in inverse kinematics

e.g. direct reaction:



Physics extracted from direct reactions -- Elastic Scattering:

Traditionally used to extract optical potentials, rms radii, density distributions

Eur. Phys. J. A **15**, 27–33 (2002)
DOI 10.1140/epja/i2001-10219-7

Nuclear-matter distributions of halo nuclei from elastic proton scattering in inverse kinematics

P. Egelhof[1,a], G.D. Alkhazov[2], M.N. Andronenko[2], A. Bauchet[1], A.V. Dobrovolsky[1,2], S. Fritz[1], G.E. Gavrilov[2], H. Geissel[1], C. Gross[1], A.V. Khanzadeev[2], G.A. Korolev[2], G. Kraus[1], A.A. Lobodenko[2], G. Münzenberg[1], M. Mutterer[3], S.R. Neumaier[1], T. Schäfer[1], C. Scheidenberger[1], D.M. Seliverstov[2], N.A. Timofeev[2], A.A. Vorobyov[2], and V.I. Yatsoura[2]

[1] Gesellschaft für Schwerionenforschung (GSI), D-64291 Darmstadt, Germany
[2] Petersburg Nuclear Physics Institute (PNPI), RU-188300 Gatchina, Russia
[3] Institut für Kernphysik (IKP), Technische Universität, D-64289 Darmstadt, Germany

Matter distributions for $^{6,8}He$ and $^{6,8,9,11}Li$ measured

Reactions with exotic nuclei

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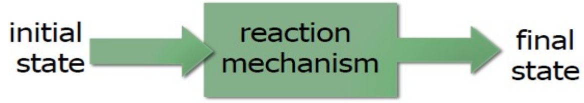
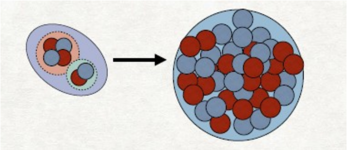
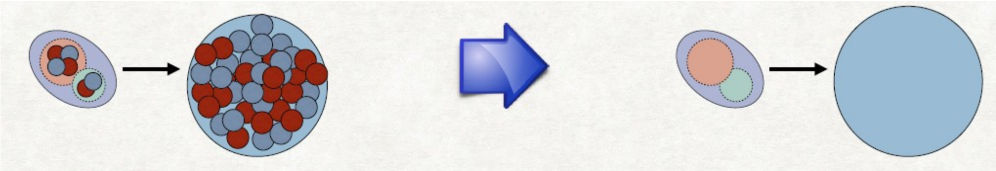


Many-body problem



Quantum mechanical scattering problem

In the continuum, theory can solve the few-body problem exactly.



few-body problem with effective interactions

Effective interactions from *ab initio* methods

Start from many-body Hamiltonian with 2 (and 3) body forces

Theoretical foundations laid by Feshbach and Watson in the 1950s

Feshbach:

effective nA interaction via Green's function from solution of many body problem using basis function expansion, e.g. SCGF, CCGF (current truncation to singles and doubles)

energy ~ 10 MeV

Watson:

► **Multiple scattering expansion, e.g. spectator expansion**

(current truncation to two active particles)

Spectator Expansion:

Siciliano, Thaler (1977)

Picklesimer, Thaler (1981)

Chinn, Elster, Thaler, Weppner (1995)

Expansion in:

- **particles active in the reaction**
- **antisymmetrized in active particles**

Intended for "fast reaction", i.e. ≳ 80 MeV

physics + astronomy (Department of) — OHIO UNIVERSITY 1804

Building up an *ab initio* framework

Ingredients:

1) **realistic nuclear interaction**

   must describe target and projectile

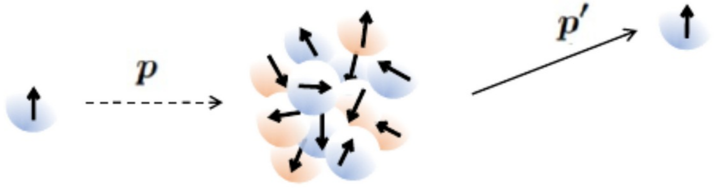
2) **controllable structure framework**

   here, no-core shell model (NCSM and SA-NCSM

3) **controllable reaction framework**

   here, spectator expansion

4) **a way to connect everything**



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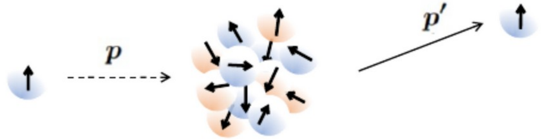

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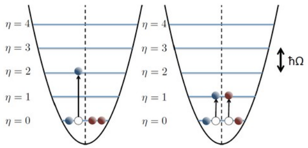

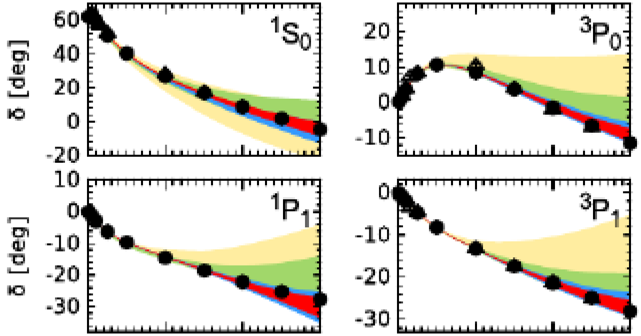

	$E(^3H)$	$E(^3He)$	$E(^4He)$	$r_p(^4He)$
NNLO	−8.249	−7.501	−27.759	1.43(8)
NNLO+NNN	−8.469	−7.722	−28.417	1.43(8)
Experiment	−8.482	−7.717	−28.296	1.467(13)

Ekström et al., PRL 110, 192502 (2013)

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Watson multiple scattering series

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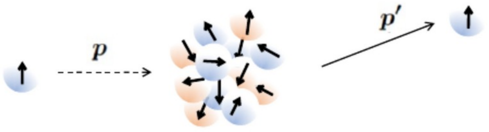

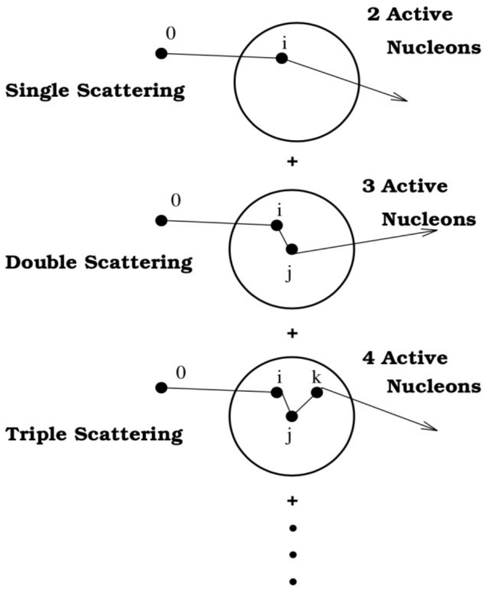

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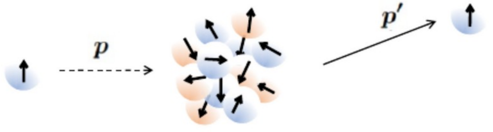

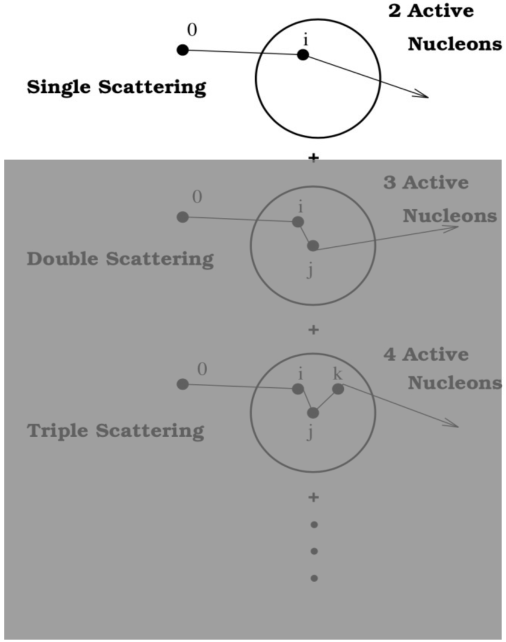

Leading order in spectator expansion can be computed *ab initio*

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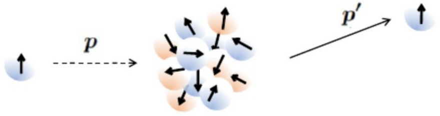
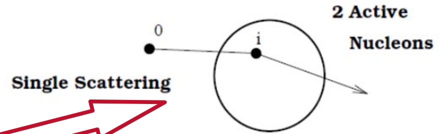
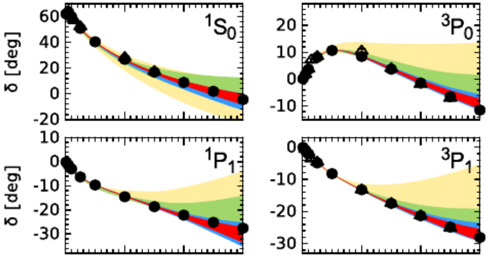
Framework for *ab initio* Elastic Scattering

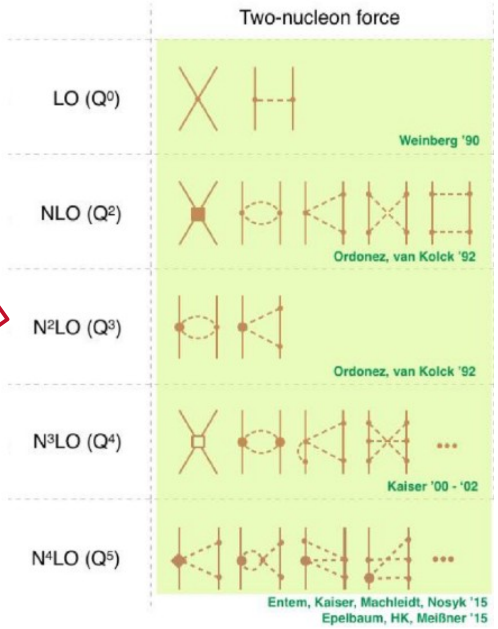


Same NN force in all parts

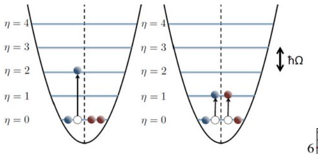
Reaction theory:
spectator expansion

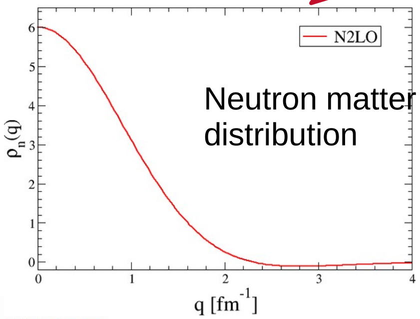






Structure theory:
No-Core Shell Model





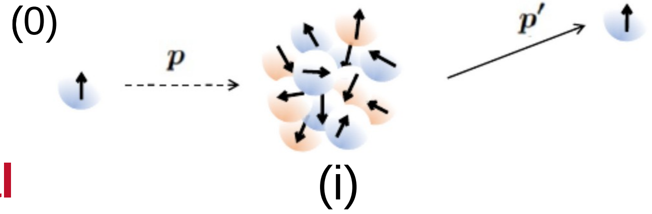
Neutron matter
distribution

Calculations shown with NNLO$_{opt}$ chiral interaction
A. Ekstrom et al. PRL 110, 192502 (2013)

Setting up the *ab initio* framework

(effective interaction) = (thing that puts them together) x (reaction information) x (structure information)

NN interaction represented by **Wolfenstein amplitudes**



$$
\begin{aligned}
\overline{M}(\boldsymbol{q},\boldsymbol{\mathcal{K}}_{NN},\epsilon) = \; & A(\boldsymbol{q},\boldsymbol{\mathcal{K}}_{NN},\epsilon)\mathbf{1}\otimes\mathbf{1} \\
& + iC(\boldsymbol{q},\boldsymbol{\mathcal{K}}_{NN},\epsilon)\left(\boldsymbol{\sigma}^{(0)}\cdot\hat{\boldsymbol{n}}\right)\otimes\mathbf{1} \\
& + iC(\boldsymbol{q},\boldsymbol{\mathcal{K}}_{NN},\epsilon)\,\mathbf{1}\otimes\left(\boldsymbol{\sigma}^{(i)}\cdot\hat{\boldsymbol{n}}\right) \\
& + M(\boldsymbol{q},\boldsymbol{\mathcal{K}}_{NN},\epsilon)(\boldsymbol{\sigma}^{(0)}\cdot\hat{\boldsymbol{n}})\otimes(\boldsymbol{\sigma}^{(i)}\cdot\hat{\boldsymbol{n}}) \\
& + [G(\boldsymbol{q},\boldsymbol{\mathcal{K}}_{NN},\epsilon) - H(\boldsymbol{q},\boldsymbol{\mathcal{K}}_{NN},\epsilon)]\,(\boldsymbol{\sigma}^{(0)}\cdot\hat{\boldsymbol{q}})\otimes(\boldsymbol{\sigma}^{(i)}\cdot\hat{\boldsymbol{q}}) \\
& + [G(\boldsymbol{q},\boldsymbol{\mathcal{K}}_{NN},\epsilon) + H(\boldsymbol{q},\boldsymbol{\mathcal{K}}_{NN},\epsilon)]\,(\boldsymbol{\sigma}^{(0)}\cdot\hat{\boldsymbol{\mathcal{K}}})\otimes(\boldsymbol{\sigma}^{(i)}\cdot\hat{\boldsymbol{\mathcal{K}}}) \\
& + D(\boldsymbol{q},\boldsymbol{\mathcal{K}}_{NN},\epsilon)\left[(\boldsymbol{\sigma}^{(0)}\cdot\hat{\boldsymbol{q}})\otimes(\boldsymbol{\sigma}^{(i)}\cdot\hat{\boldsymbol{\mathcal{K}}}) + (\boldsymbol{\sigma}^{(0)}\cdot\hat{\boldsymbol{\mathcal{K}}})\otimes(\boldsymbol{\sigma}^{(i)}\cdot\hat{\boldsymbol{q}})\right]
\end{aligned}
$$

$$
\boldsymbol{q} = \boldsymbol{p}' - \boldsymbol{p}, \quad \boldsymbol{\mathcal{K}} = \frac{1}{2}(\boldsymbol{p}' + \boldsymbol{p}), \quad \hat{\boldsymbol{n}} = \frac{\boldsymbol{\mathcal{K}}\times\boldsymbol{q}}{|\boldsymbol{\mathcal{K}}\times\boldsymbol{q}|}
$$

A: central
C: spin-orbit

M, G, H: tensor

Most general structure of NN amplitudes consistent with invariance principles

Setting up the *ab initio* framework

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NN interaction represented by **Wolfenstein amplitudes**

A: central
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& + [G(\boldsymbol{q},\boldsymbol{\mathcal{K}}_{NN},\epsilon)-H(\boldsymbol{q},\boldsymbol{\mathcal{K}}_{NN},\epsilon)]\,(\boldsymbol{\sigma}^{(0)}\cdot\hat{\boldsymbol{q}})\otimes \\
& + [G(\boldsymbol{q},\boldsymbol{\mathcal{K}}_{NN},\epsilon)+H(\boldsymbol{q},\boldsymbol{\mathcal{K}}_{NN},\epsilon)]\,(\boldsymbol{\sigma}^{(0)}\cdot\hat{\boldsymbol{\mathcal{K}}})\otimes \\
& + D(\boldsymbol{q},\boldsymbol{\mathcal{K}}_{NN},\epsilon)\left[(\boldsymbol{\sigma}^{(0)}\cdot\hat{\boldsymbol{q}})\otimes(\boldsymbol{\sigma}^{(i)}\cdot\hat{\boldsymbol{\mathcal{K}}})+(\boldsymbol{\sigma}^{(0)}\right.
\end{aligned}
$$

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\boldsymbol{q} = \boldsymbol{p}'-\boldsymbol{p}
\qquad
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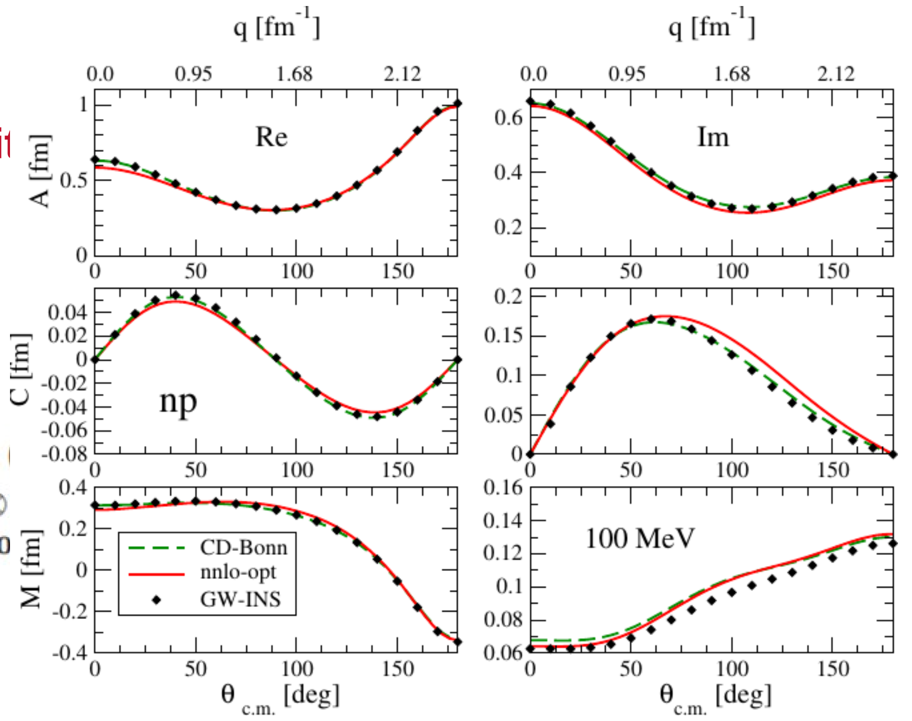

M, G, H: tensor

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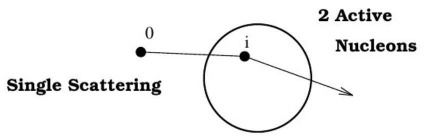
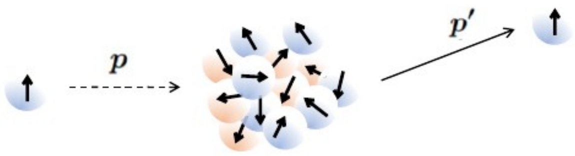
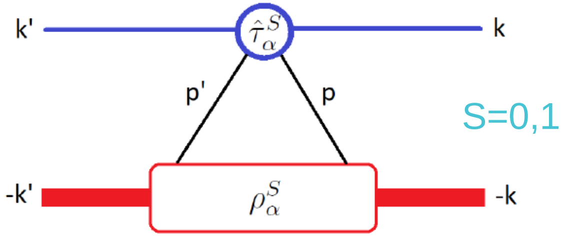
Computing the leading order effective potential



(effective interaction) = (thing that puts them together) x (reaction information) x (structure information)



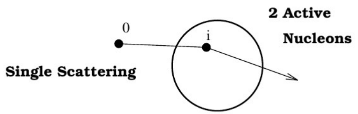


scatter off nucleus with 0^+ ground state

$$\widehat{U}_p(\boldsymbol{q},\boldsymbol{\mathcal{K}}_{NA},\epsilon) = \sum_{\alpha=n,p}\sum_{S=0}^{1}\int d^3\mathcal{K}\ \ \eta(\boldsymbol{q},\boldsymbol{\mathcal{K}},\boldsymbol{\mathcal{K}}_{NA})\ \hat{\tau}^S_{p,\alpha}\left(\boldsymbol{q},\tfrac{1}{2}\left(\tfrac{A+1}{A}\boldsymbol{\mathcal{K}}_{NA}-\boldsymbol{\mathcal{K}}\right);\varepsilon\right)$$

$$\rho^S_\alpha\left(\boldsymbol{\mathcal{K}}-\tfrac{A-1}{A}\tfrac{\boldsymbol{q}}{2},\boldsymbol{\mathcal{K}}+\tfrac{A-1}{A}\tfrac{\boldsymbol{q}}{2}\right),$$

Computing the leading order effective potential



(effective interaction) = (thing that puts them together) x (reaction information) x (structure information)

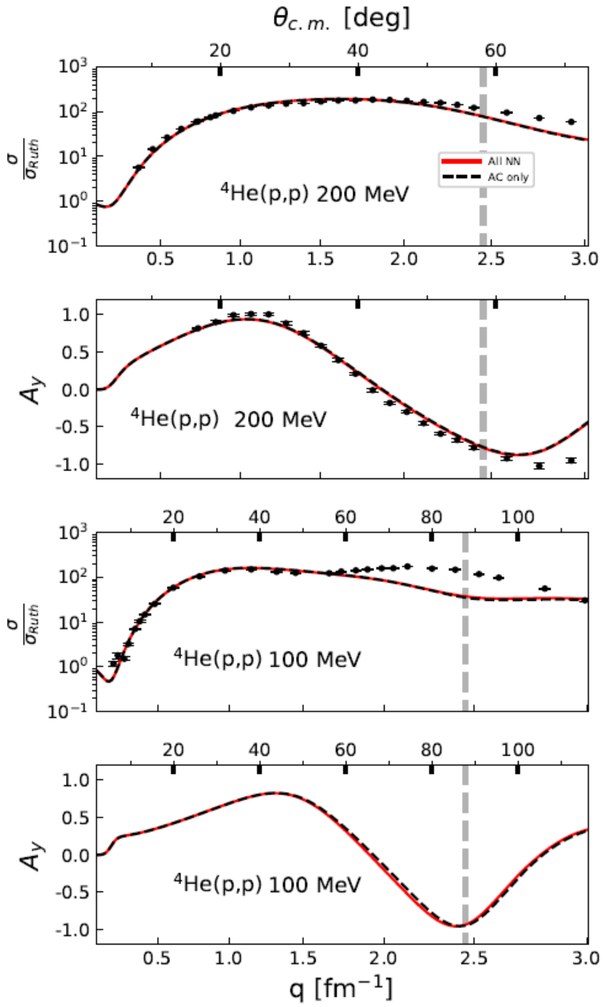
matter distribution = density

$$
\begin{aligned}
&\widehat{U}_p(\boldsymbol{q}, \boldsymbol{\mathcal{K}}_{NA}, \epsilon) \\
&= \sum_{\alpha=p,n} \int d^3\mathcal{K} \eta(\boldsymbol{q}, \boldsymbol{\mathcal{K}}_{NA}, \epsilon)\, A_{p,\alpha}\left(\boldsymbol{q}, \frac{1}{2}\left(\frac{A+1}{A}\boldsymbol{\mathcal{K}}_{NA} + \boldsymbol{\mathcal{K}}\right), \epsilon\right) \rho_\alpha^{S=0}(\boldsymbol{P}', \boldsymbol{P}) \\
&+ i(\boldsymbol{\sigma}^{(0)} \cdot \hat{\boldsymbol{n}}) \sum_{\alpha=N,Z} \int d^3\mathcal{K} \eta(\boldsymbol{q}, \boldsymbol{\mathcal{K}}_{NA}, \epsilon)\, C_{p,\alpha}\left(\boldsymbol{q}, \frac{1}{2}\left(\frac{A+1}{A}\boldsymbol{\mathcal{K}}_{NA} + \boldsymbol{\mathcal{K}}\right), \epsilon\right) \rho_\alpha^{S=0}(\boldsymbol{P}', \boldsymbol{P}) \\
&+ i \sum_{\alpha=N,Z} \int d^3\mathcal{K} \eta(\boldsymbol{q}, \boldsymbol{\mathcal{K}}_{NA}, \epsilon)\, C_{p,\alpha}\left(\boldsymbol{q}, \frac{1}{2}\left(\frac{A+1}{A}\boldsymbol{\mathcal{K}}_{NA} + \boldsymbol{\mathcal{K}}\right), \epsilon\right) S_{n,\alpha}(\boldsymbol{P}', \boldsymbol{P}) \cos\beta \\
&+ i(\boldsymbol{\sigma}^{(0)} \cdot \hat{\boldsymbol{n}}) \sum_{\alpha=N,Z} \int d^3\mathcal{K} \eta(\boldsymbol{q}, \boldsymbol{\mathcal{K}}_{NA}, \epsilon)\, M_{p,\alpha}\left(\boldsymbol{q}, \frac{1}{2}\left(\frac{A+1}{A}\boldsymbol{\mathcal{K}}_{NA} + \boldsymbol{\mathcal{K}}\right), \epsilon\right) S_{n,\alpha}(\boldsymbol{P}', \boldsymbol{P}) \cos\beta
\end{aligned}
$$

spin-projected momentum distribution

with $\mathcal{P}' = \left(\mathcal{K} - \frac{A-1}{A}\frac{q}{2}\right)$ and $\mathcal{P} = \left(\mathcal{K} + \frac{A-1}{A}\frac{q}{2}\right)$

^{4}He N_{max}=18



$$\vec{q}_{nn} = \vec{q}_{nA} = \vec{q}$$

$$q \approx 480 \text{ MeV} = 2.45 \text{ fm}^{-1}$$

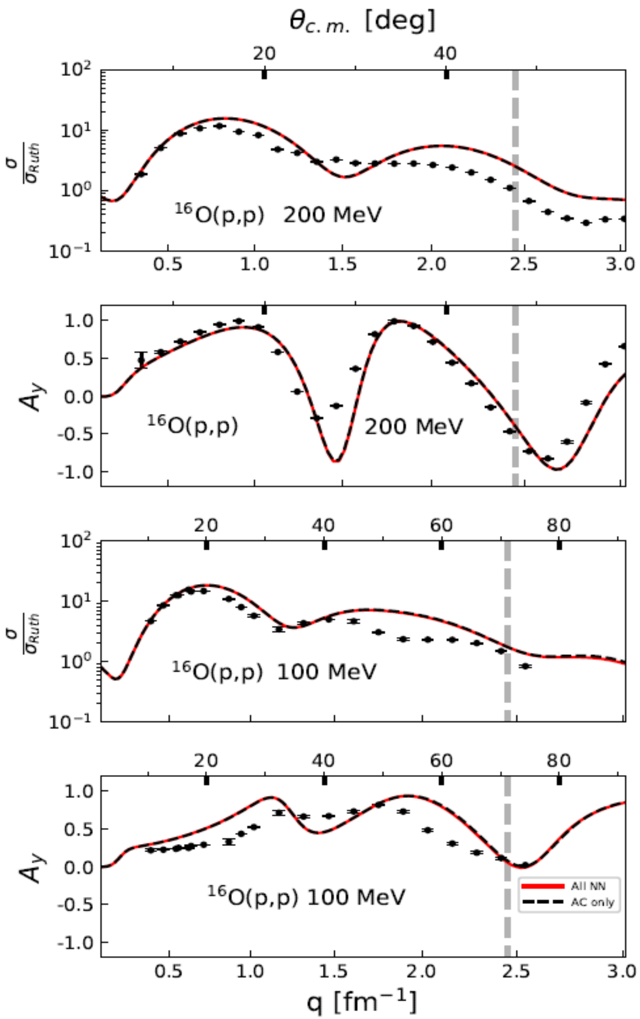
ħω=20 MeV

closed-shell nuclei

NNLO$_{opt}$
Chiral interaction

A. Ekstrom et al.
PRL 110, 192502 (2013)

N_{max}=10 ^{16}O



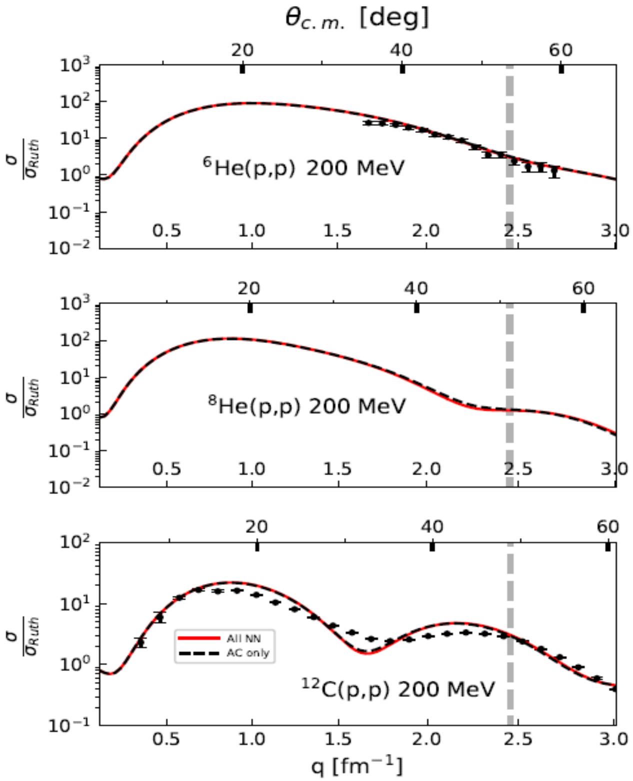
Department of
physic

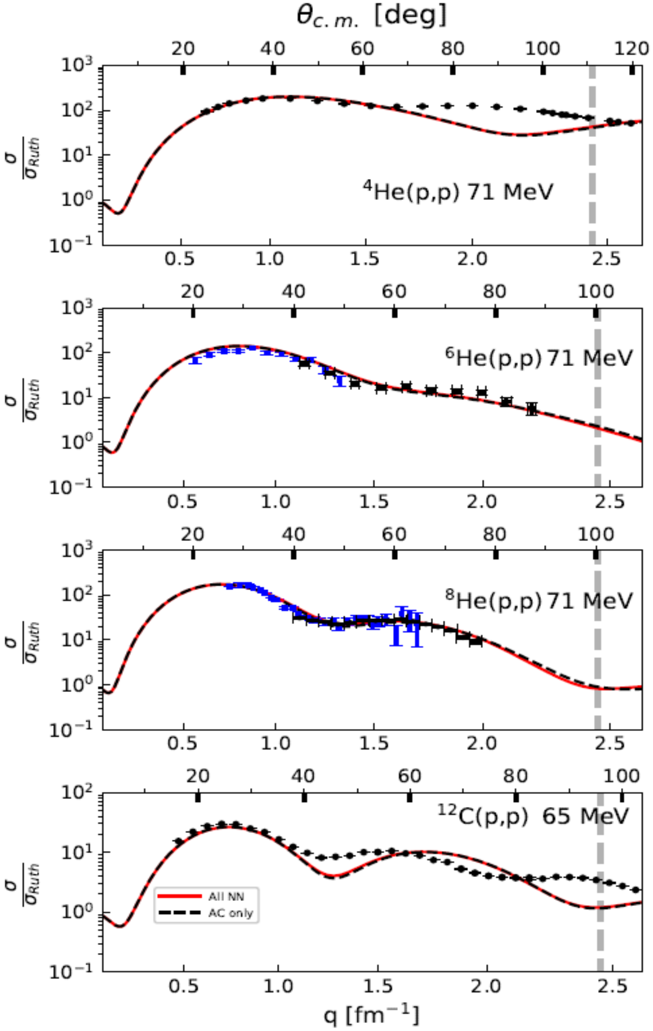
Open-shell nuclei

NNLO$_{opt}$
Chiral
interaction

N_{max}=18

N_{max}=12





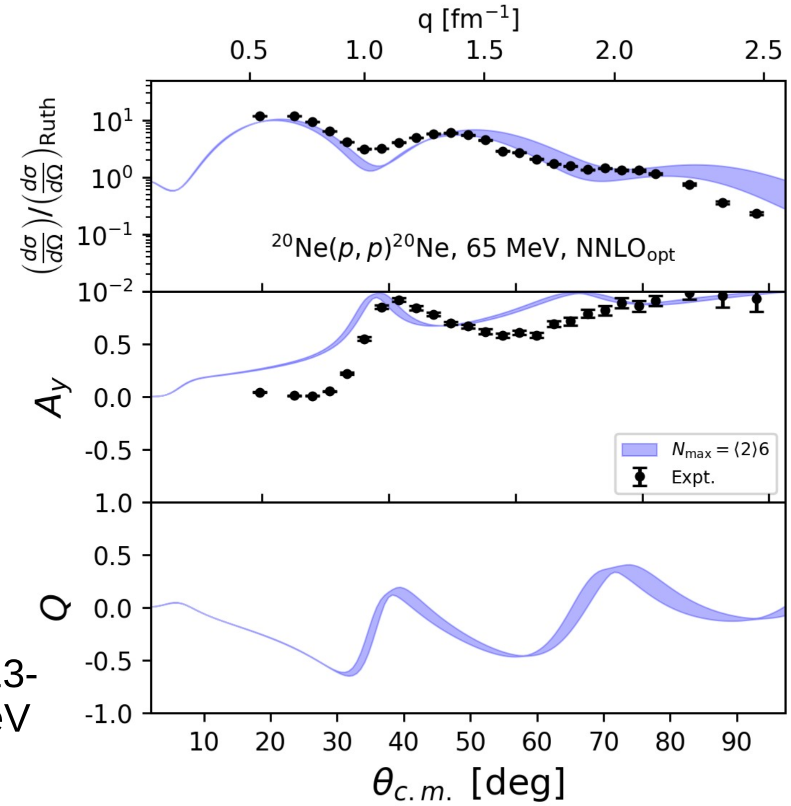
N_{max}=18

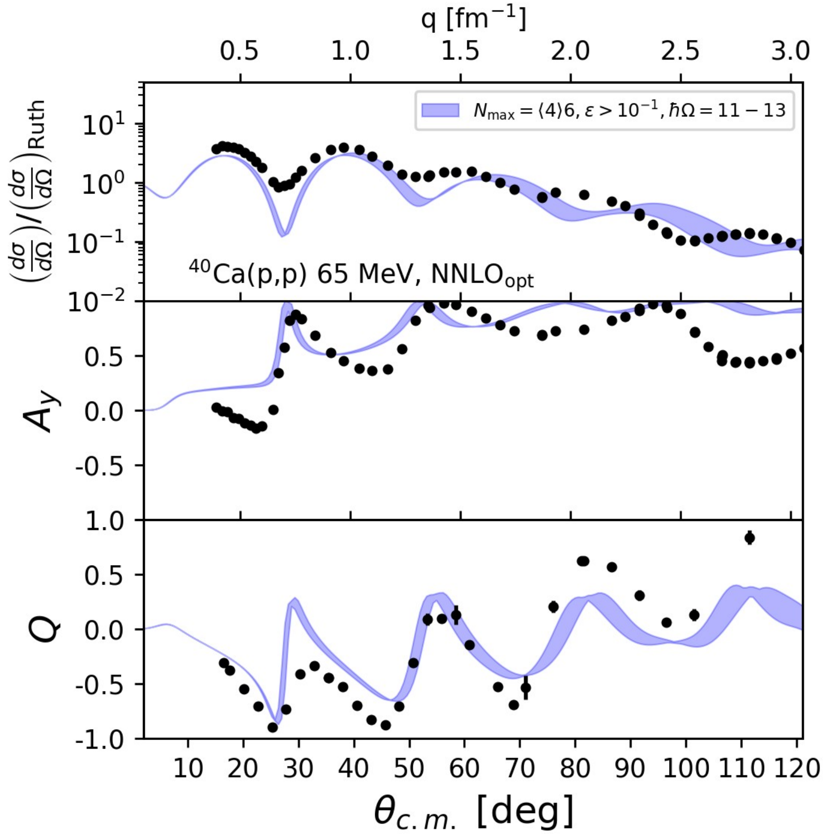
N_{max}=12

ħω=20 MeV

Beyond NCSM: SA-NCSM One-Body Densities

NNLO$_{opt}$ chiral potential





ħω=13-17 MeV

ħω=11-13 MeV

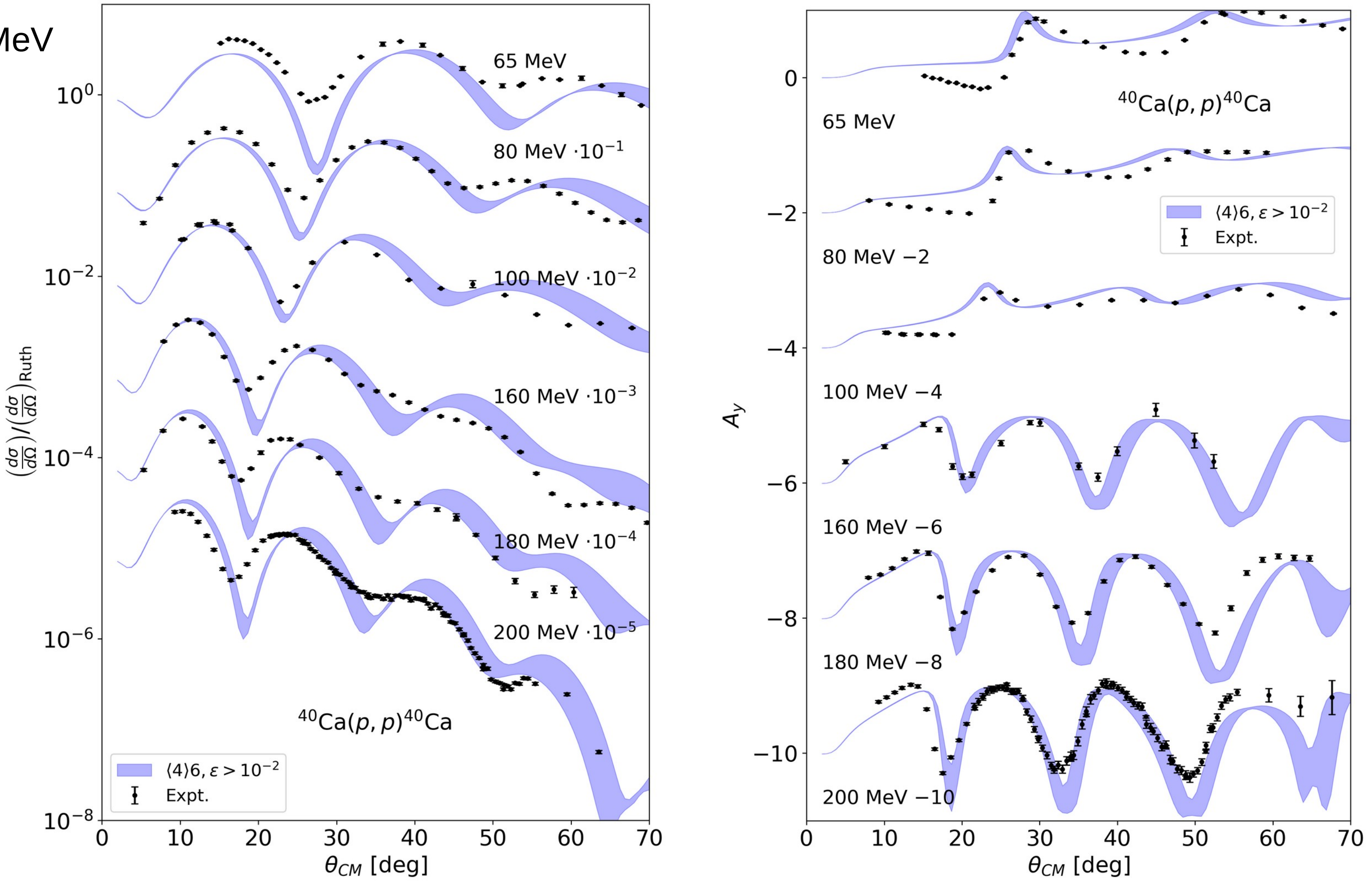
Baker, Elster, Dytrych, Launey arXiv 2404.03106 [nucl-th]

rms calc. 3.04 – 3.25 fm
rms exp. 3.48 fm

NNLO$_{opt}$ chiral potential

Beyond NCSM: SA-NCSM One-Body Densities

ħω=11- 13 MeV



What we learned so far:

- **Consistent approach to p+A effective interaction in leading order multiple scattering expansion is possible.**

  (spin of projectile and struck target nucleon treated consistently)

- **In the multiple scattering approach the leading order term can be calculated consistently *ab initio* for light nuclei based on NCSM**
  SA-NCSM is being explored for medium-mass nuclei

- **Some indication that the leading order the spectator expansion describes elastic scattering data better for open-shell (deformed) and exotic nuclei than densely packed closed shell nuclei**
  (good for providing predictions for optical potential fits in exotic regime)

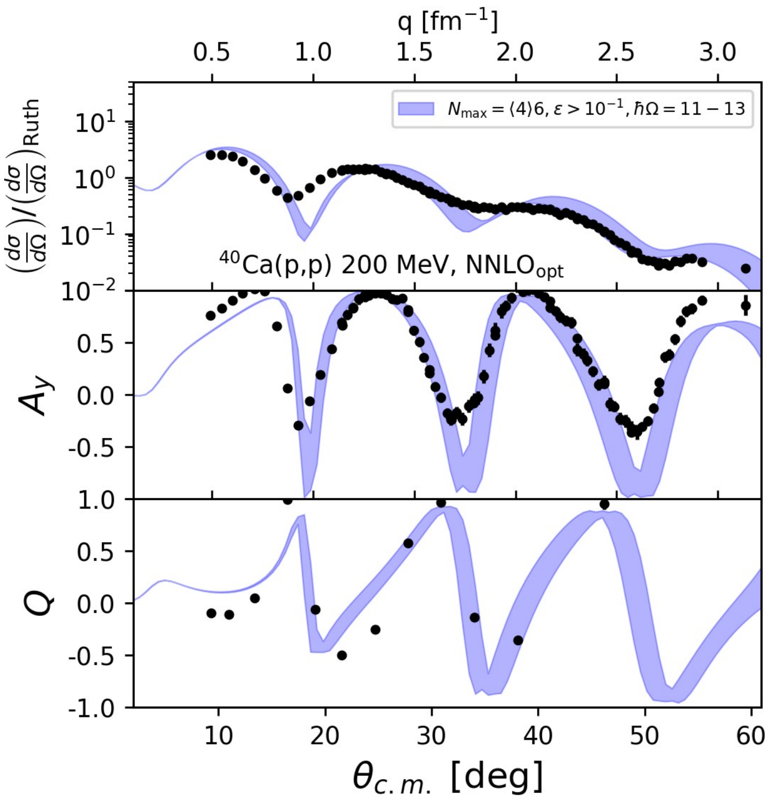
  We plan a study of Mg isotopes with the SA-NCSM – data at 65 MeV available

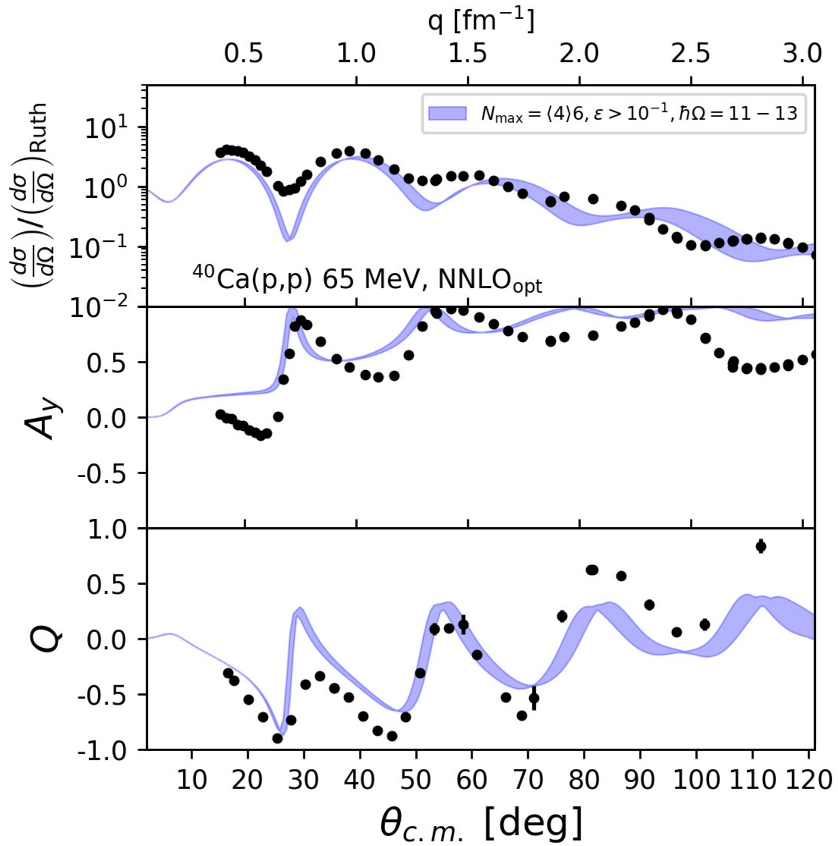
- **Ongoing work: move beyond nuclei with 0^+ ground states**
  Transition potentials for inelastic scattering

Backup slides

Beyond NCSM: SA-NCSM One-Body Densities

NNLO$_{opt}$ chiral potential





What about different chiral NN interactions ?

Description of NN data below ~ 130 MeV almost identical

NNLO$_{\text{opt}}$

Fitted to about 125 MeV NN E_{lab}

PRL **110**, 192502 (2013) PHYSICAL REVIEW LETTERS week ending 10 MAY 2013

Optimized Chiral Nucleon-Nucleon Interaction at Next-to-Next-to-Leading Order

A. Ekström,[1,2] G. Baardsen,[1] C. Forssén,[3] G. Hagen,[4,5] M. Hjorth-Jensen,[1,2,6] G. R. Jansen,[4,5] R. Machleidt,[7] W. Nazarewicz,[5,4,8] T. Papenbrock,[5,4] J. Sarich,[9] and S. M. Wild[9]

LENPIC – SCS

Semi-local coordinate space regulator R=1 fm
Sometimes referred to as EKM
(fitted up to about 300 MeV NN E_{lab})

PRL **115**, 122301 (2015) PHYSICAL REVIEW LETTERS week ending 18 SEPTEMBER 2015

Precision Nucleon-Nucleon Potential at Fifth Order in the Chiral Expansion

E. Epelbaum,[1] H. Krebs,[1] and U.-G. Meißner[2,3,4]

Physics Letters B 761 (2016) 87–91

N3LO *NN* interaction adjusted to light nuclei in *ab exitu* approach

A.M. Shirokov[a,b,c,*], I.J. Shin[d], Y. Kim[d], M. Sosonkina[e], P. Maris[b], J.P. Vary[b]

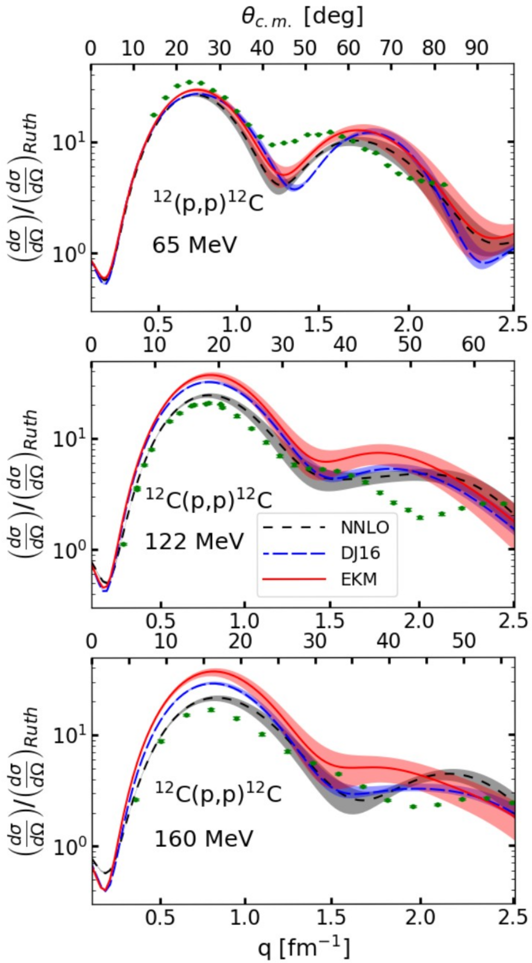
Daejeon 16

Starts from Idaho N3LO, applies SRG transformation
And represents in HO basis
On-shell equivalent to Idaho N3LO (fitted to about 300 MeV NN E_{lab})

Baker, Burrows, Elster, Launey, Maris, Popa, Weppner, PRC 108, 044617 (2023)

Differential Cross Sections

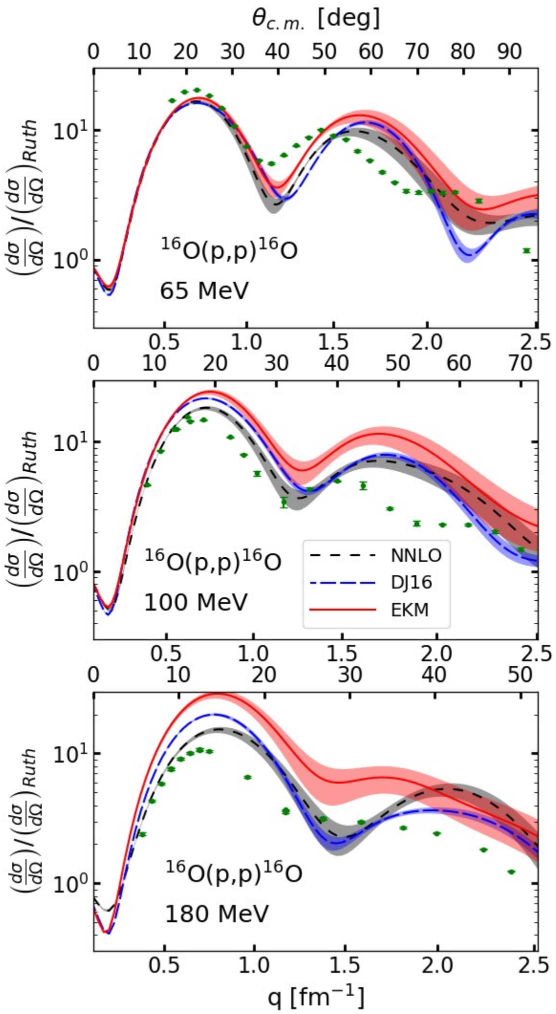
^{12}C



Bands indicate Dependence on Oscillator parameter

Energy dependence for small momentum transfer is different

^{16}O



Baker, Burrows, Elster, Maris, Popa, Weppner, PRC 108, 044617 (2023)

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