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Probing the size of single-particle orbitals
in neutron-rich calcium isotopes
from quasi-free scattering missing momentum

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DREB – Direct Reactions with Exotic Beams

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The charge radii of neutron-rich Ca isotopes

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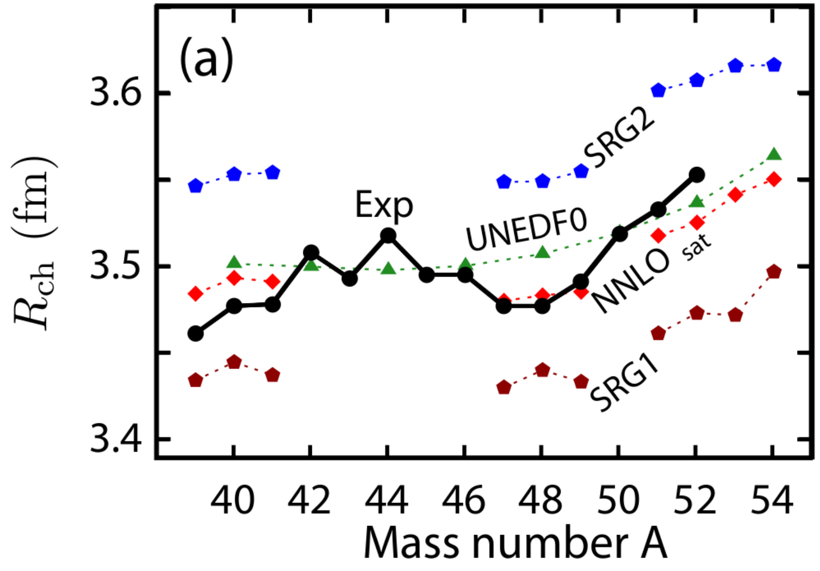
- Charge radii measurements via isotope shift method
- Steep linear increase from N = 28 to N=32

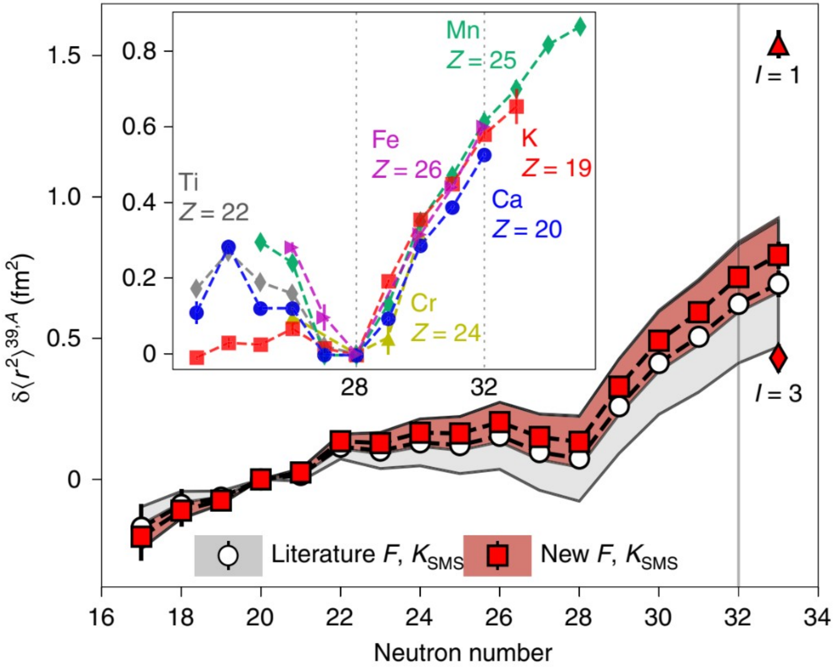
Bonnard et al. proposed a **0.7 fm larger $1p_{3/2}$** ($1p_{1/2}$) neutron orbital than $0f_{7/2}$ ($0f_{5/2}$)

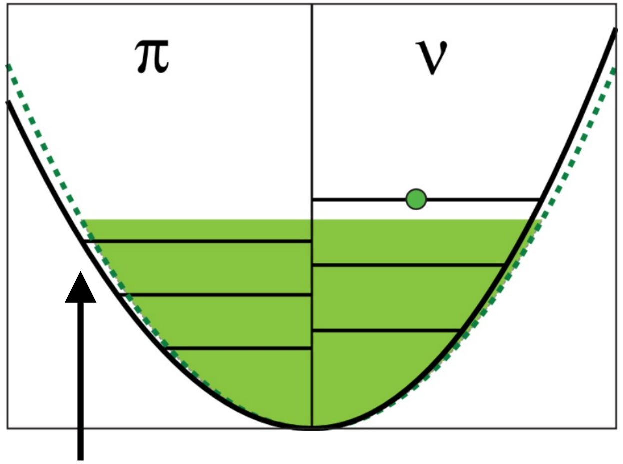
Charge radii Ca



Charge radii K

no "kink"





Charge radius increases as neutrons are added

References

R. F. Garcia Ruiz et al., Nature Physics 12 (2016)
Á. Koszorús et al., Nature Physics 17 (2021)
J. Bonnard et al., Phys. Rev. Lett. 116 (2016)

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The matter radii of neutron-rich Ca isotopes

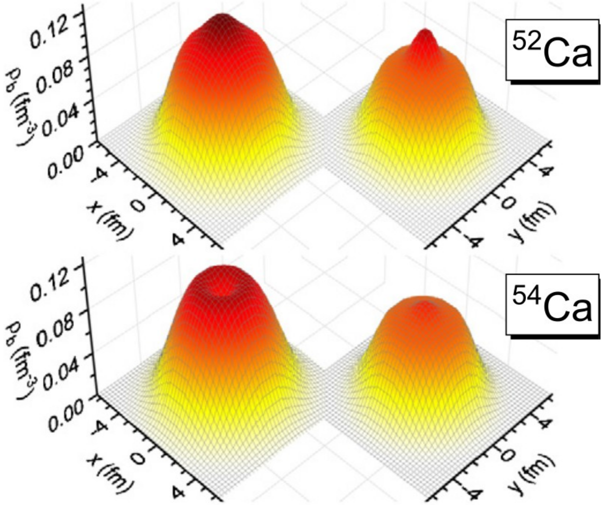
- Matter radii measurements via interaction cross section measurements
- Steep linear increase from N = 28 to N=32
- Neutron radii contribute the most to the increase of matter radii, but protons also present an increase

  → **Swelling core** as one fills a nodal or j-lower orbit such as 1p or $0f_{5/2}$

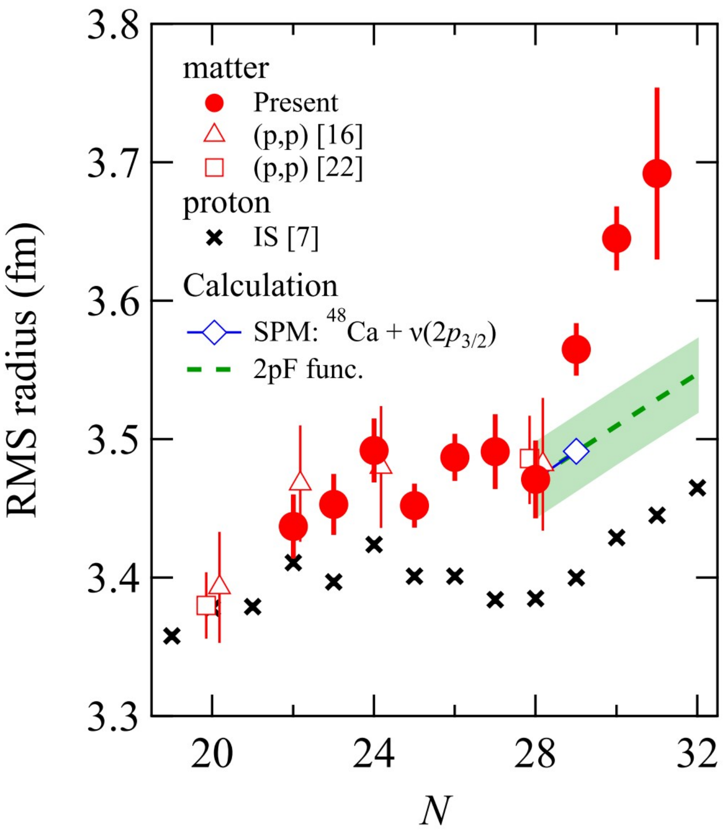
  W. Horiuchi et al. (2020)
- Strong interaction between $\nu p_{1/2}$ orbitals and neutron and proton $s_{1/2}$ orbitals

  → driving the $\mathbf{s_{1/2}}$ orbitals away from center

  J. Liu et al. (2020)



Matter radii Ca



References

M. Tanaka et al., Phys. Rev. Lett. 124 (2020)

W. Horiuchi et al., Phys. Rev. C 101 (2020)

J. Liu et al., Phys. Lett. B 806 (2020)

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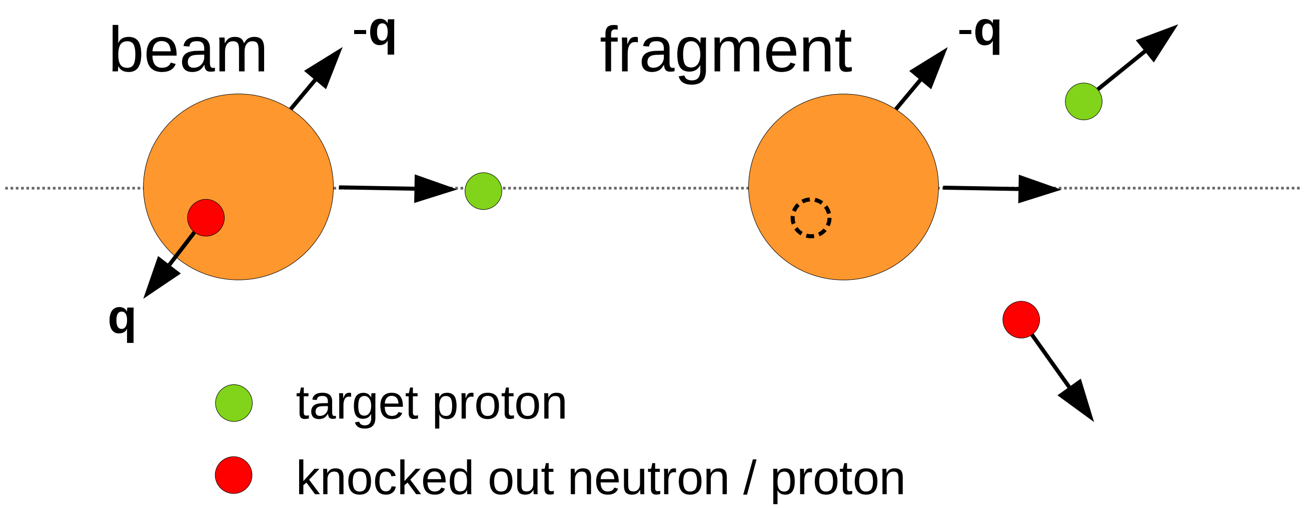
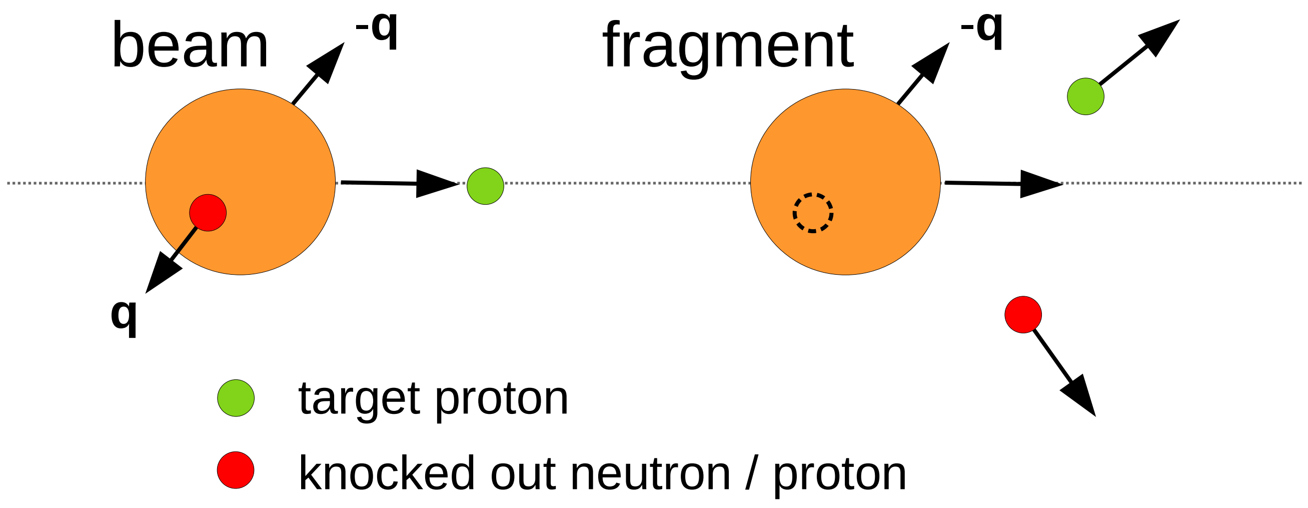


The **momentum distribution of the fragment** particles are kinematically linked to the **momentum distribution (and wave-function) of the knocked out nucleon**

Probing the size of single-particle orbitals
in neutron-rich calcium isotopes
from quasi-free scattering missing momentum



The **momentum distribution of the fragment** particles are kinematically linked to the **momentum distribution (and wave-function) of the knocked out nucleon**

- Interpretation within DWIA framework
- Single-particle wave-function as bound state of Woods-Saxon potential

→ **RMS radii of single-particle orbitals**

Acknowledgments:

All DWIA calculations shown in this presentation are performed by **K. Yoshida** and **K. Ogata**

Shell model calculations performed by **F. Nowacki** and **A. Poves**

Ab initio input for comparison by **T. Miyagi**

SEASTAR collaboration for the experimental part

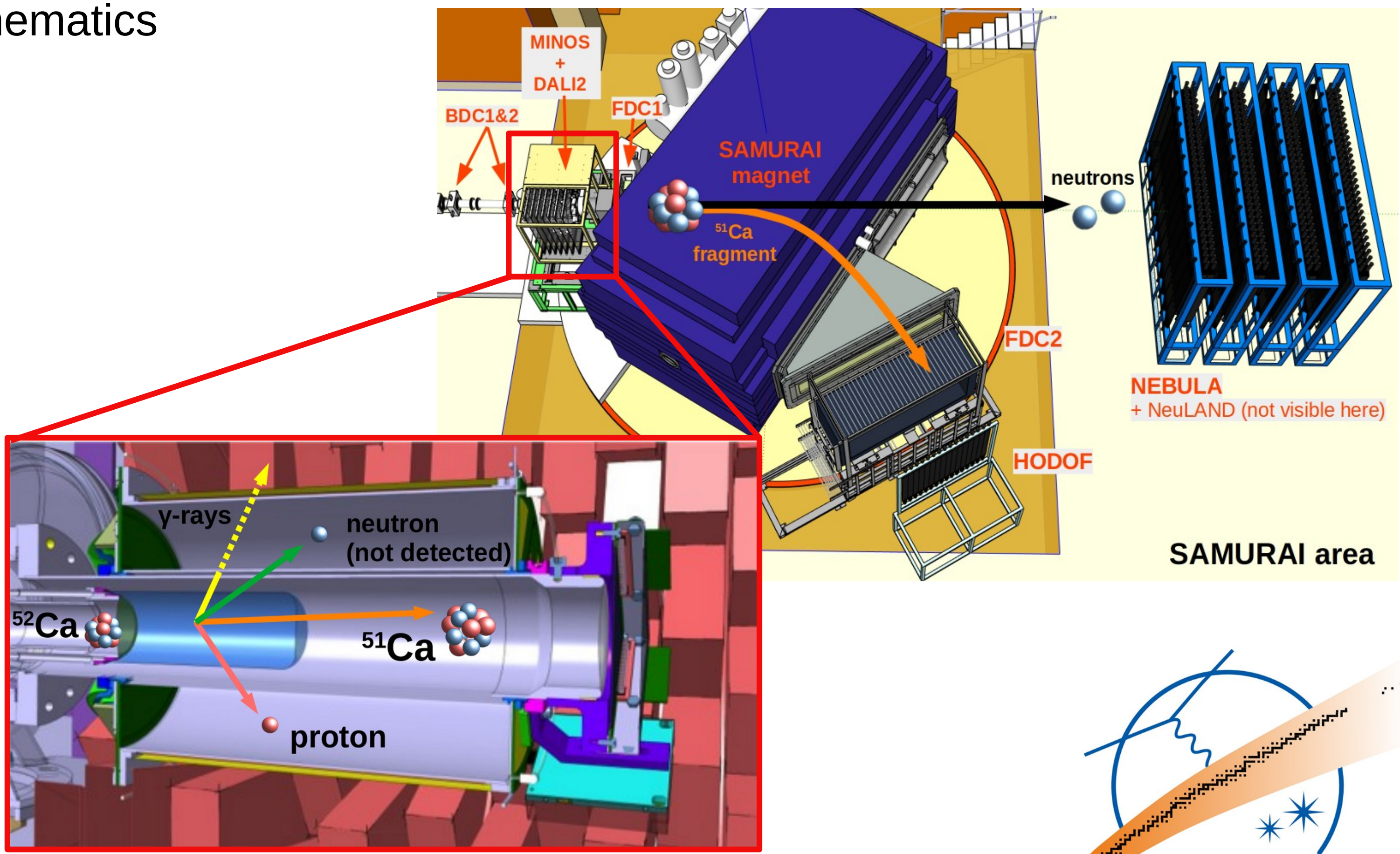
Experimental setup

- experiment at **RIBF, RIKEN**, SEASTAR3 campaign (2017)
- ^{70}**Zn primary beam** at 345 MeV/u
- In-flight **γ-ray spectroscopy** in inverse kinematics
- 15-cm **liquid H_2 target** (MINOS)
- 170 - 270 MeV/u beam energy at vertex
- MINOS TPC for vertex reconstruction
- **DALI2+** γ-ray detector array
- NeuLAND and NEBULA detectors

  used for subtracting the (p,p’) + n-evap.

Studied reactions:

^{52}Ca (p,pn)	^{52}Ca (p,2p)
^{53}Ca (p,pn)	^{53}Ca (p,2p)
^{54}Ca (p,pn)	^{54}Ca (p,2p)



References

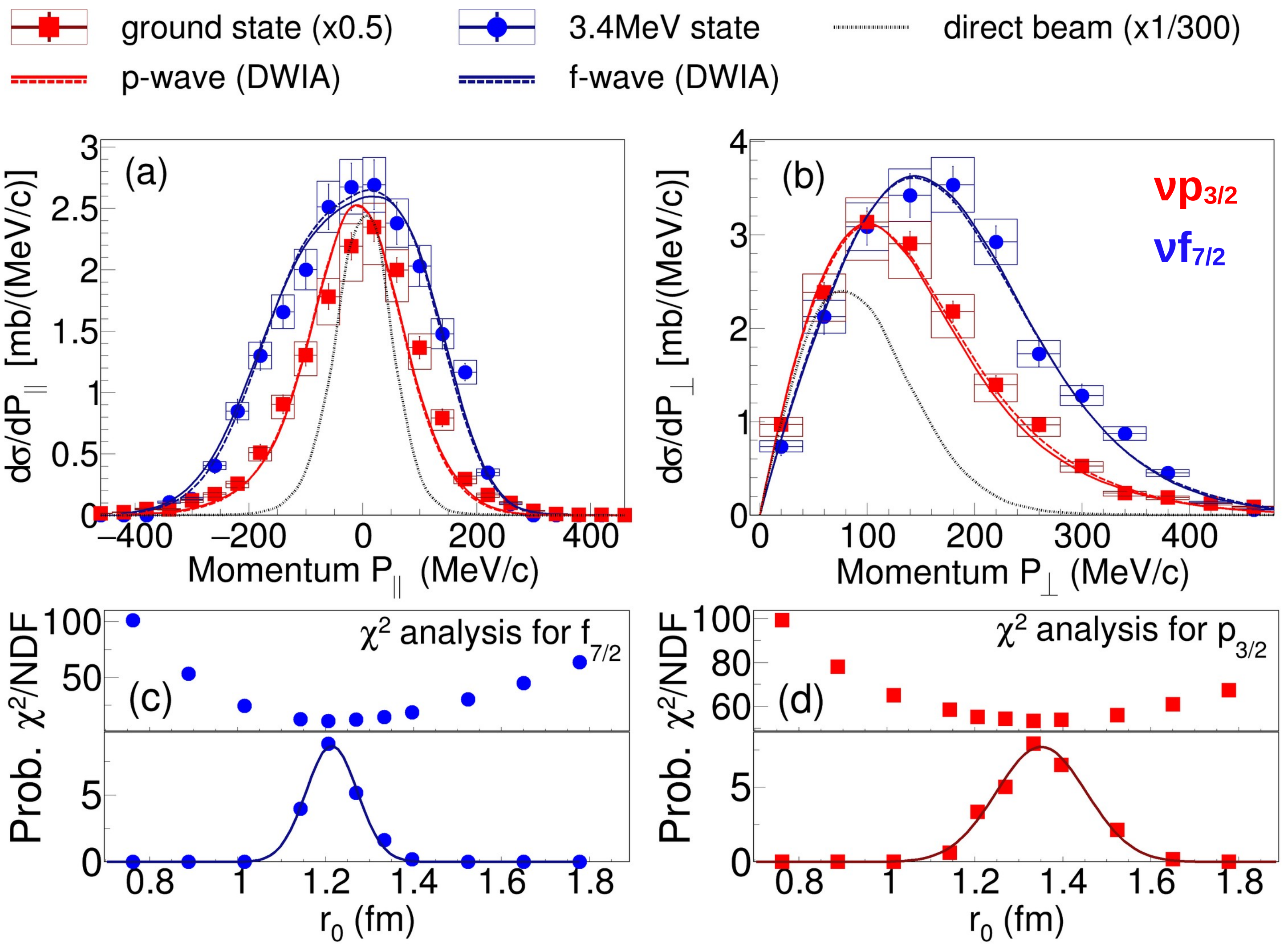
NeuLAND: K. Boretzky, et al., Nucl. Instr. Meth. Phys. Res. A 1014 (2021)

DALI2+: S. Takeuchi, et al., Nucl. Instr. Meth. Phys. Res. A 763 (2014)

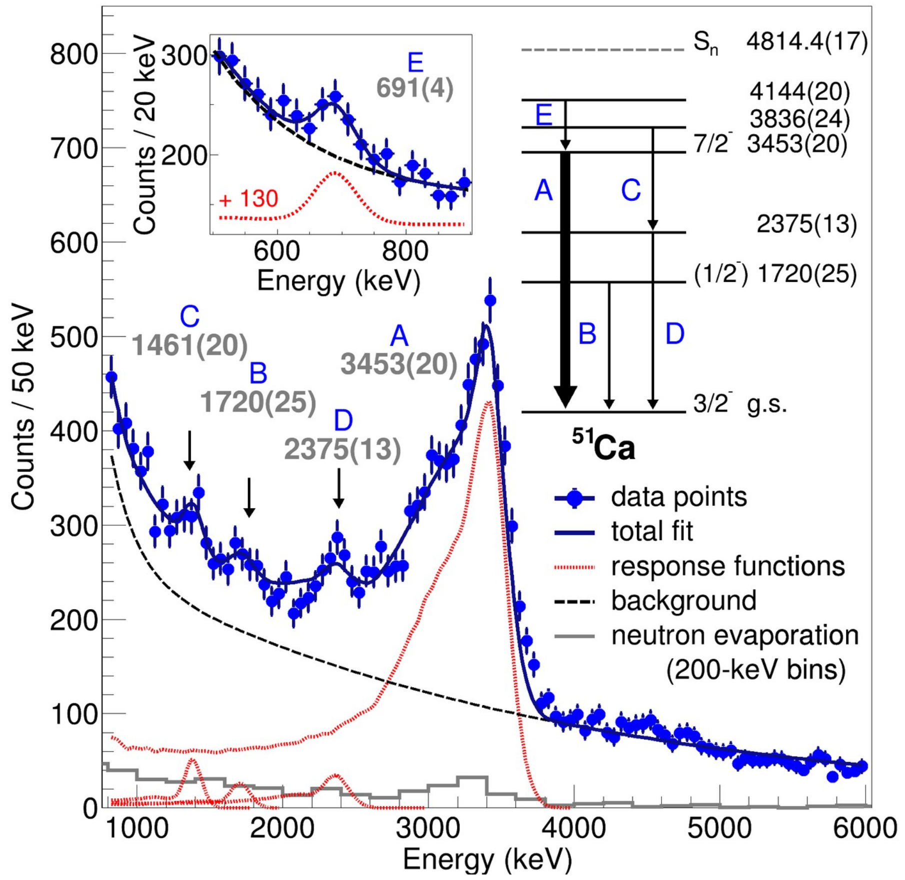
MINOS: A. Obertelli et al., Eur. Phys. J. A 50 (2014)

The pf-shell neutron orbitals of ^{52}Ca

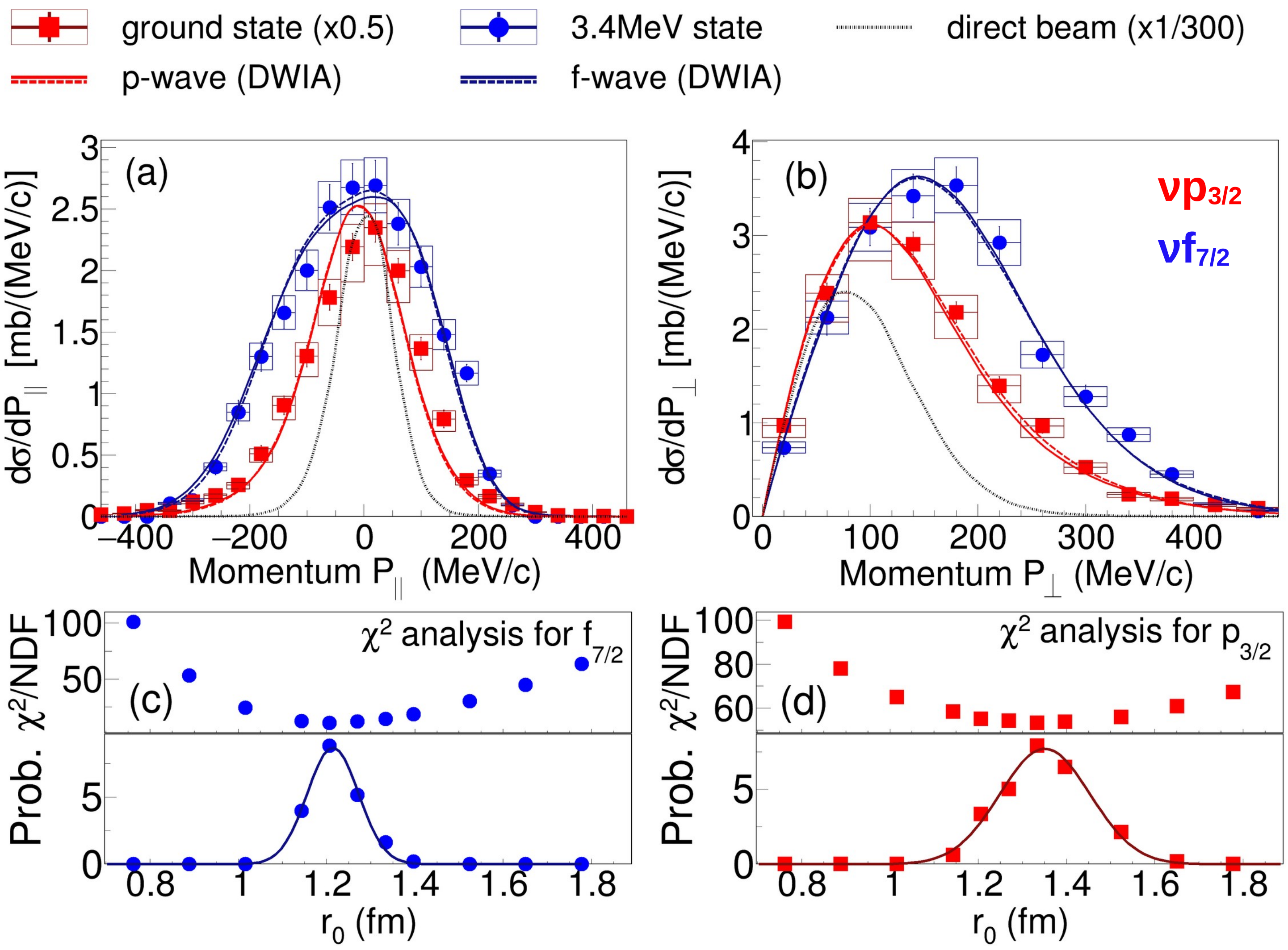
References:
M. Enciu et al., PRL 129 (2022)



Doppler-corrected γ-ray spectrum for ^{52}Ca(p,pn)



The pf-shell neutron orbitals of ^{52}Ca



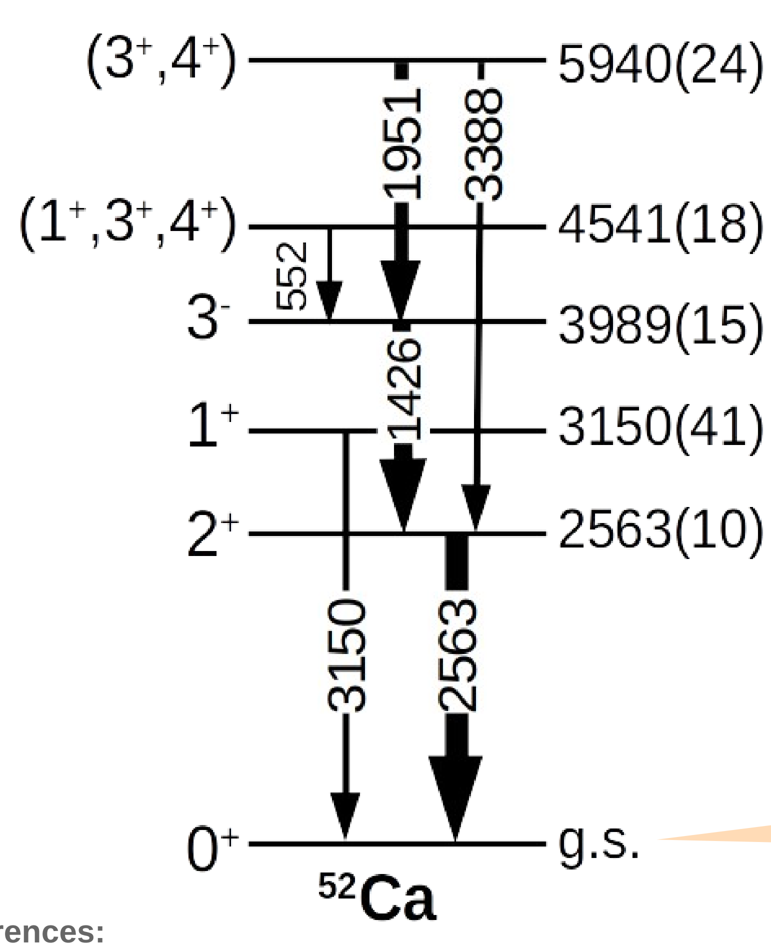
References:
M. Enciu et al., PRL 129 (2022)

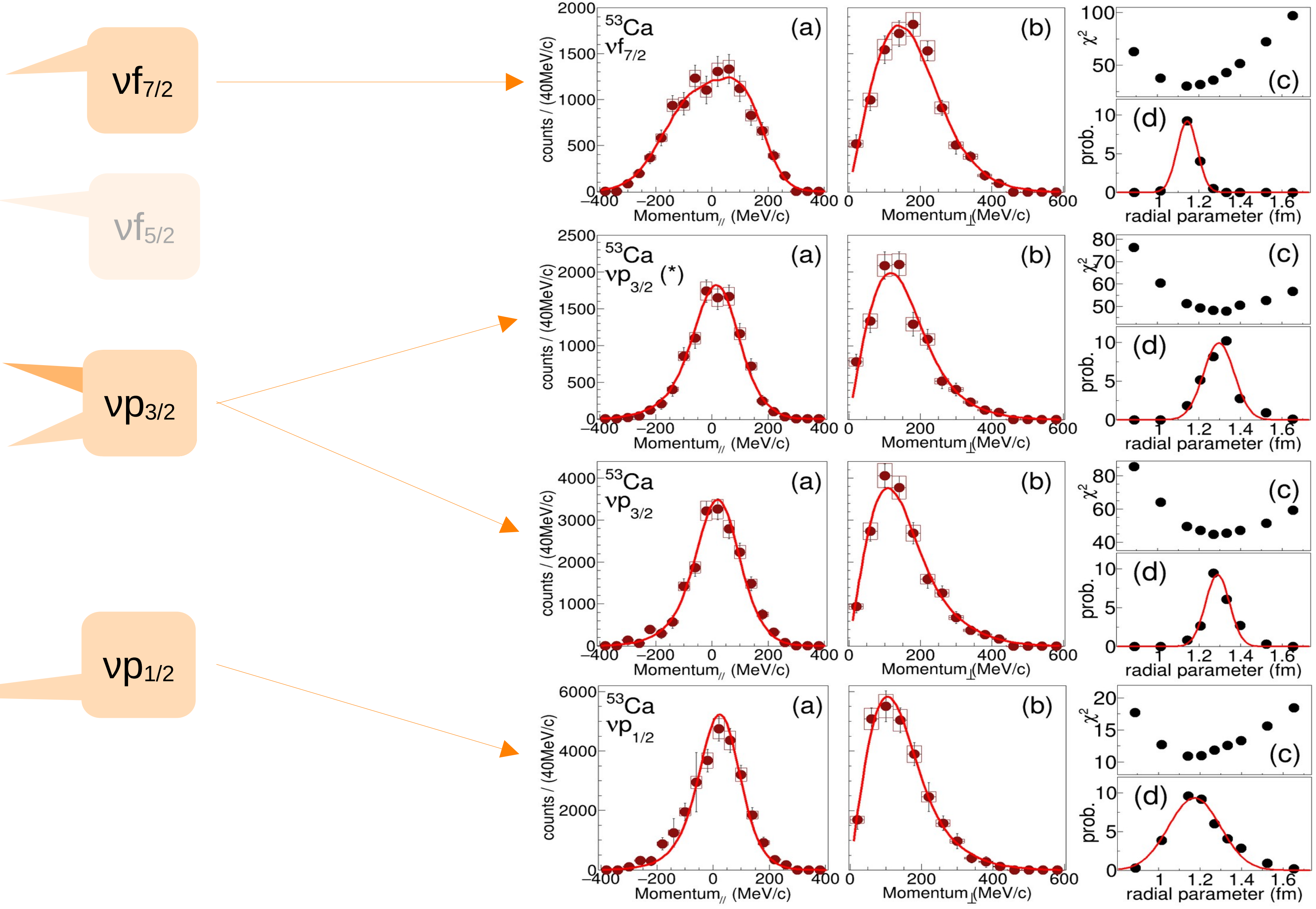
rms radii of single-particle (sp) neutron orbitals obtained by **variation of the radial parameter r_0 and χ^2 minimization**

Resulting rms radii:
$\nu f_{7/2}$: 4.13(14) fm
$\nu p_{3/2}$: 4.74(18) fm
Difference: **0.61(23) fm**

from HFB, SKM:
$\nu f_{7/2}$: 4.12 fm
$\nu p_{3/2}$: 4.49 fm
Difference: 0.37 fm

The pf-shell neutron orbitals of ^{53}Ca

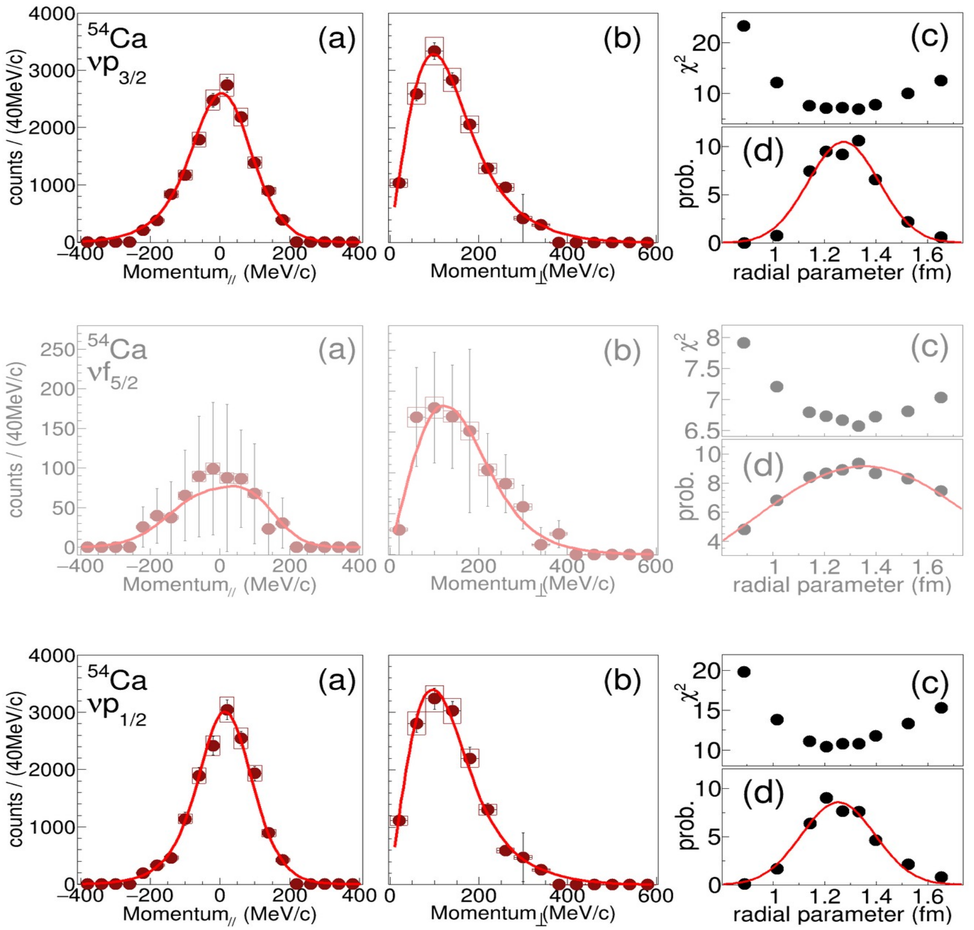




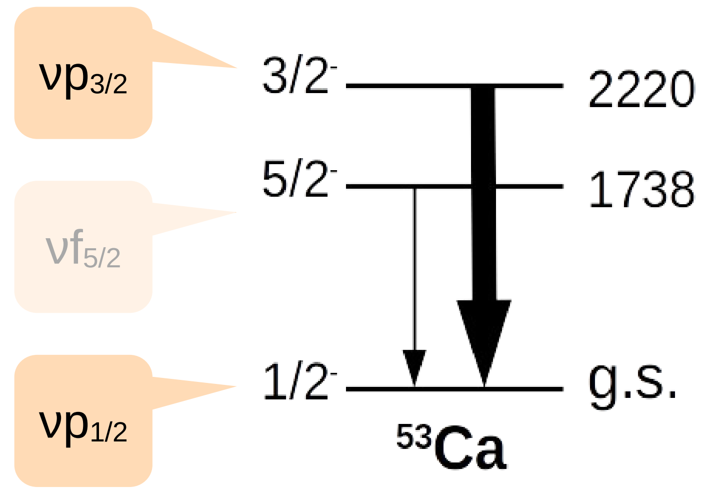
References:

A. Huck et al., Phys. Rev. C 31, 2226–2237 (1985).
A. Gade et al., Phys. Rev. C 74, 021302 (2006).
M. Rejmund et al., Phys. Rev. C 76, 021304 (2007).
F. Perrot et al., Phys. Rev. C 74, 014313 (2006).

The pf-shell neutron orbitals of ^{54}Ca



References:
S. Chen et al., Phys. Rev. Lett, 123, 142501 (2019).



^{54}Ca(p,pn) already studied by Chen et al.

The sd-shell proton orbitals of ^{52}Ca and ^{54}Ca

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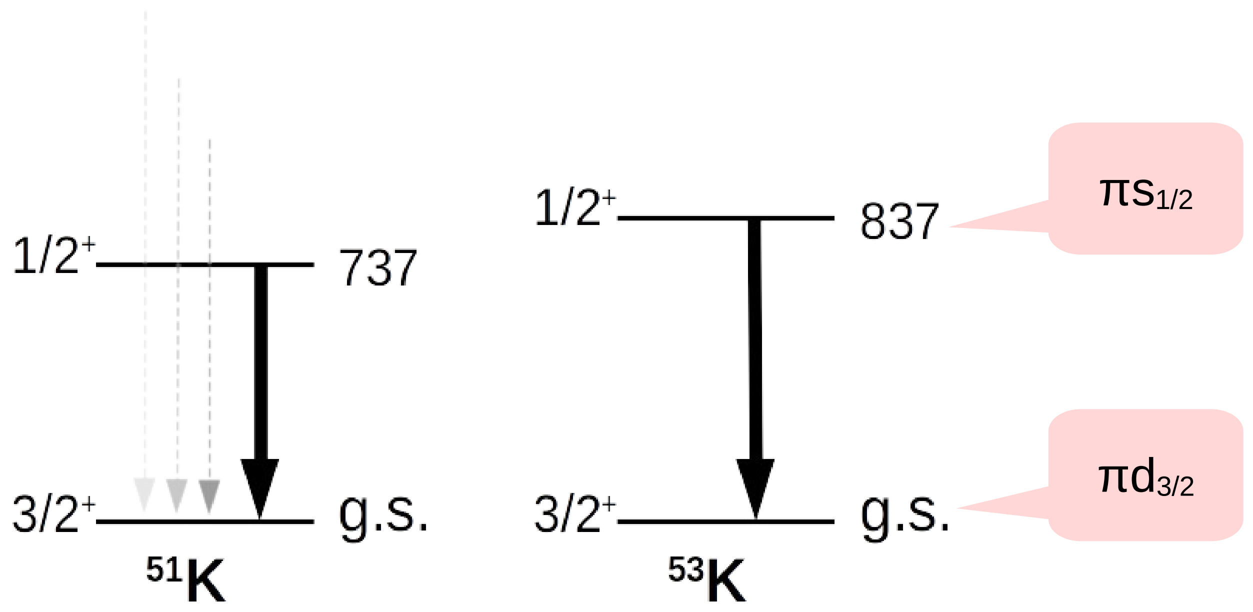
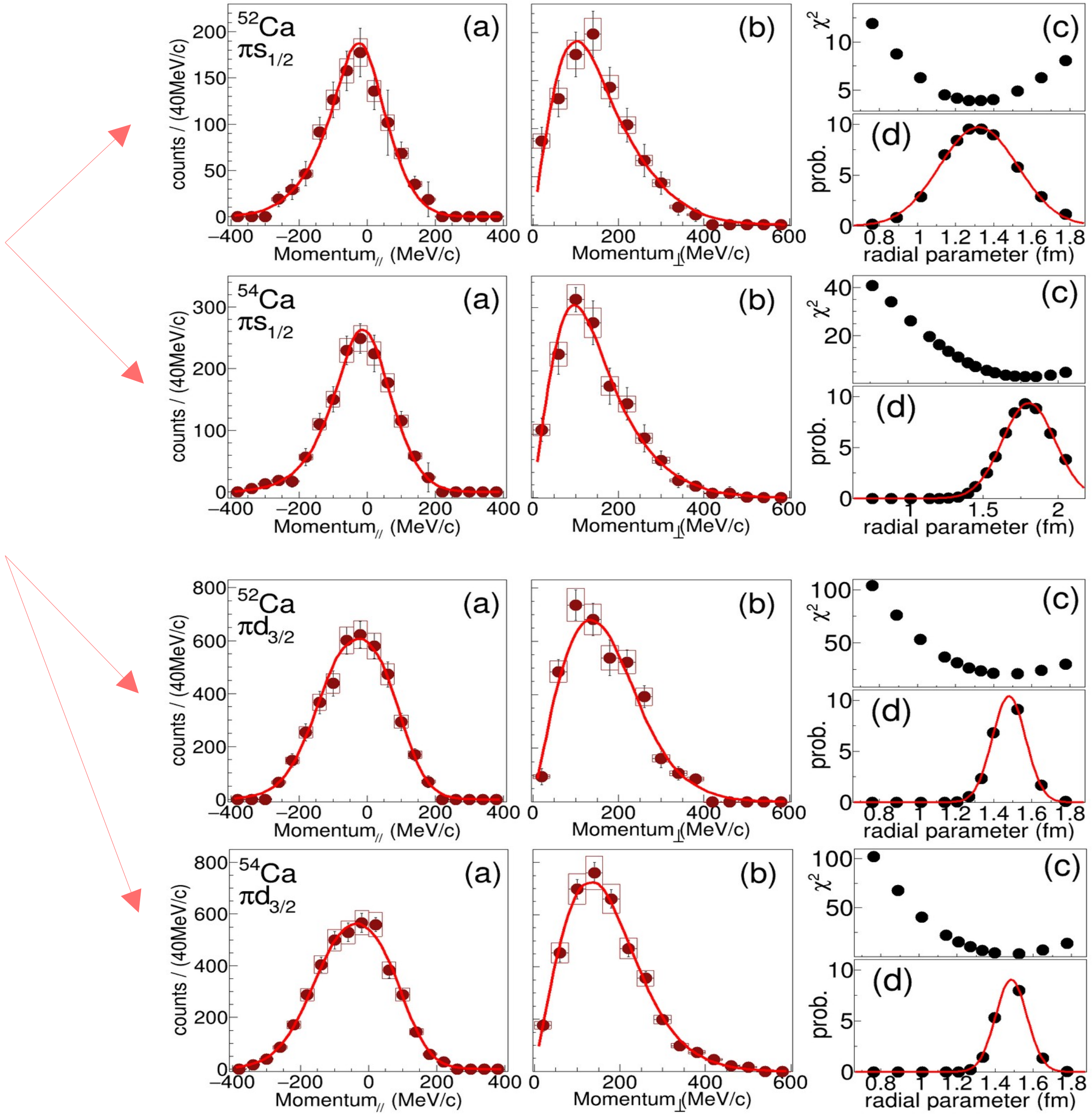


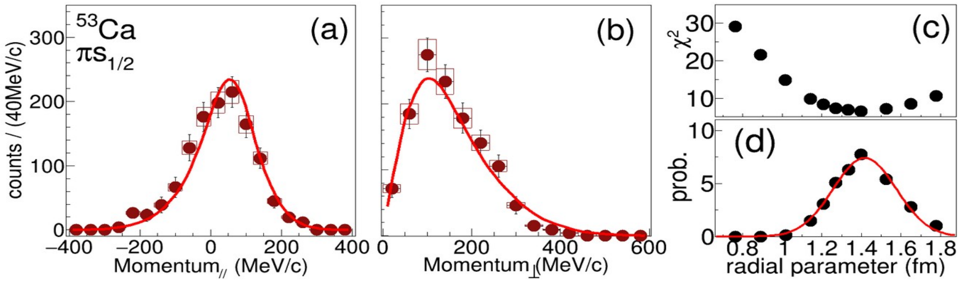


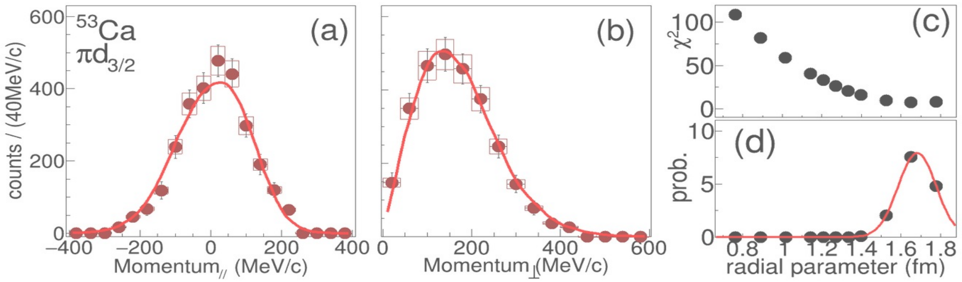
^{52}Ca(p,2p) and ^{54}Ca(p,2p) already studied by Sun et al.

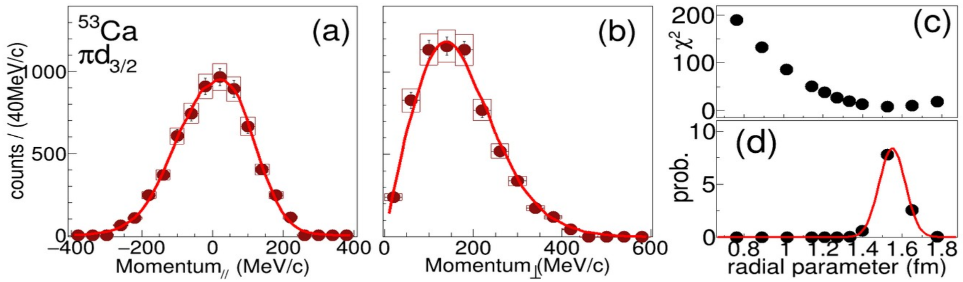
References:

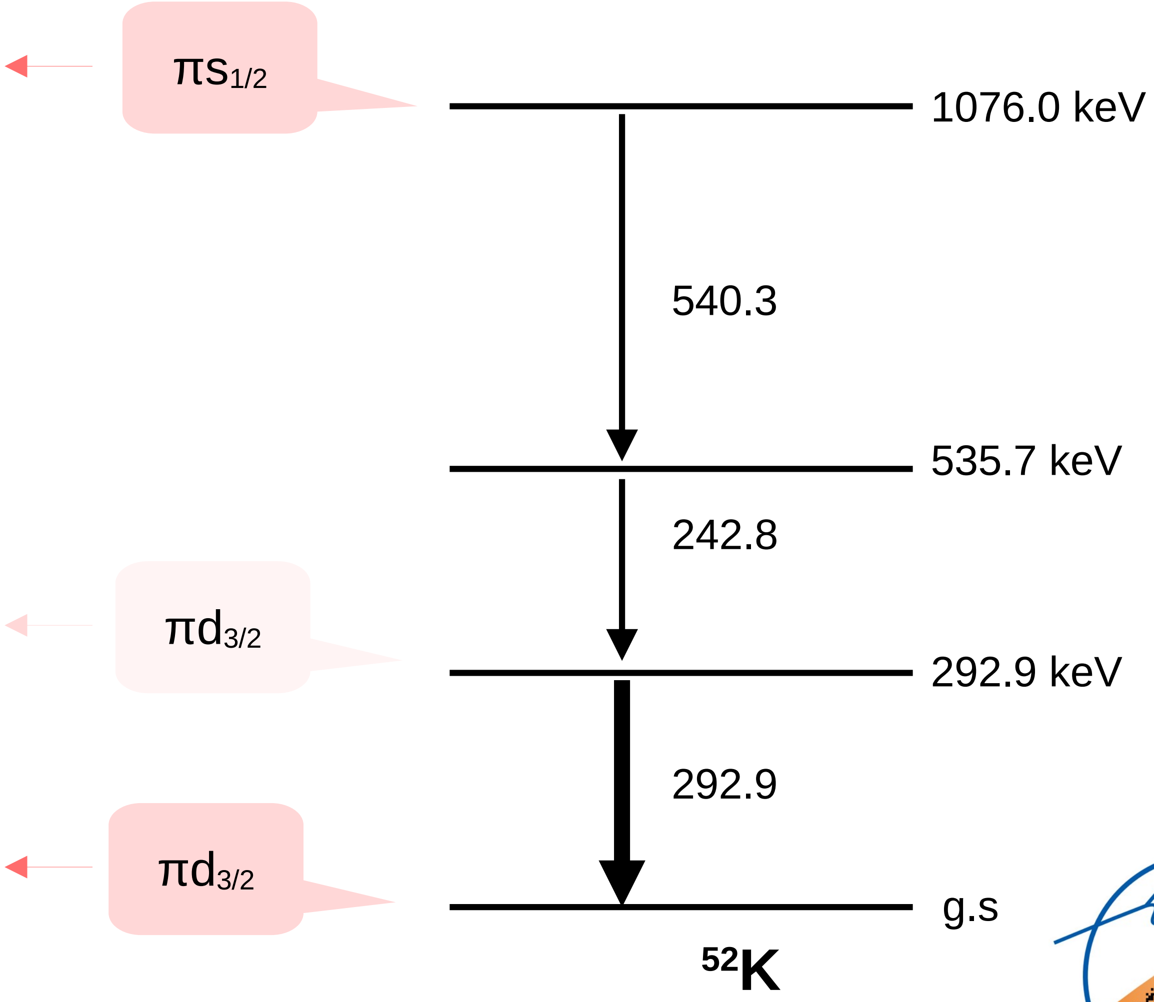
Y.L. Sun et al., Phys. Lett. B 802 (2020).

The sd-shell proton orbitals of ^{53}Ca

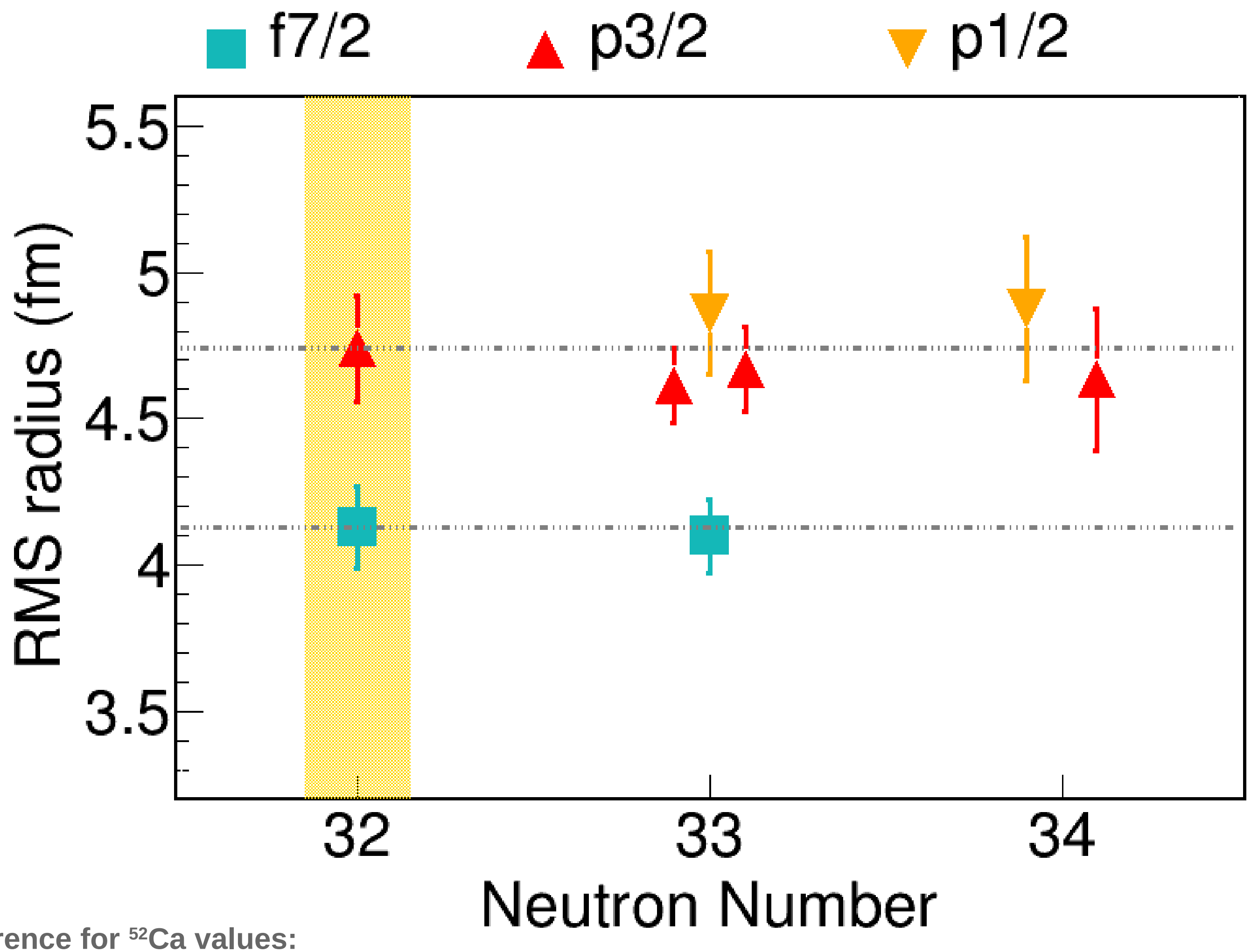








Rms radii of single-particle neutron orbitals



- **systematic difference of 0.5 up to 0.75 fm between the $f_{7/2}$ and p orbitals** for ^{53}Ca and ^{54}Ca as found for ^{52}Ca

^{52}Ca	$\nu f_{7/2}$:	4.13(14) fm
	$\nu p_{3/2}$:	4.74(18) fm
^{53}Ca	$\nu f_{7/2}$:	4.10(13) fm
	$\nu p_{3/2}$:	4.61(13) fm
	$\nu p_{1/2}$:	4.86(21) fm
	$\nu p_{1/2}$:	4.67(15) fm
^{54}Ca	$\nu p_{3/2}$:	4.64(24) fm
	$\nu p_{1/2}$:	4.88(24) fm

Reference for ^{52}Ca values:
M. Enciu et al., PRL 129 (2022)

Rms radii of single-particle proton orbitals

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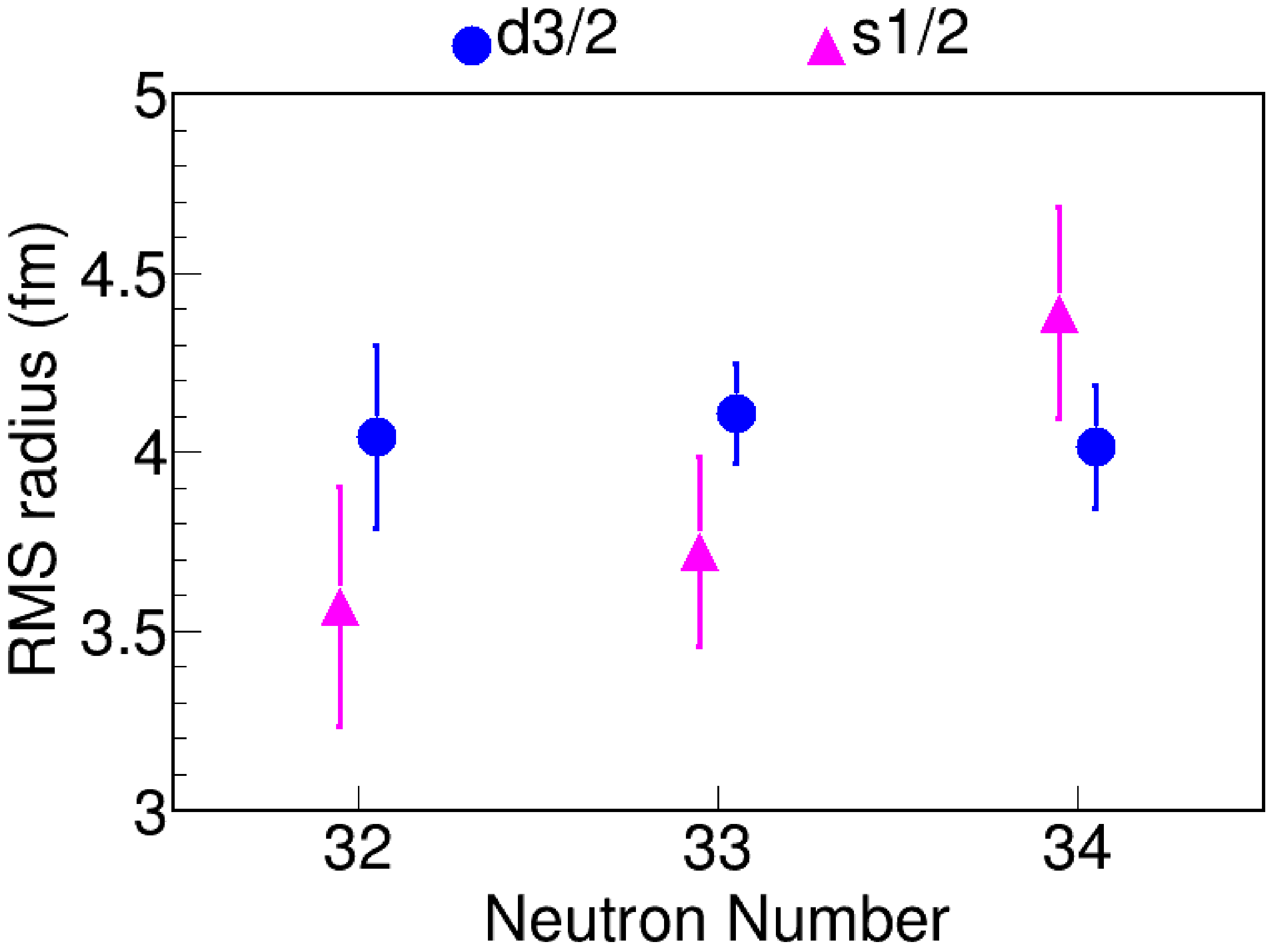

^{52}Ca $\pi d_{3/2}$: 4.04(26) fm
$\pi s_{1/2}$: 3.57(33) fm
^{53}Ca $\pi d_{3/2}$: 4.10(13) fm
$\pi s_{1/2}$: 3.72(26) fm
^{54}Ca $\pi d_{3/2}$: 4.02(17) fm
$\pi s_{1/2}$: 4.39(29) fm

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Rms radii of single-particle proton orbitals

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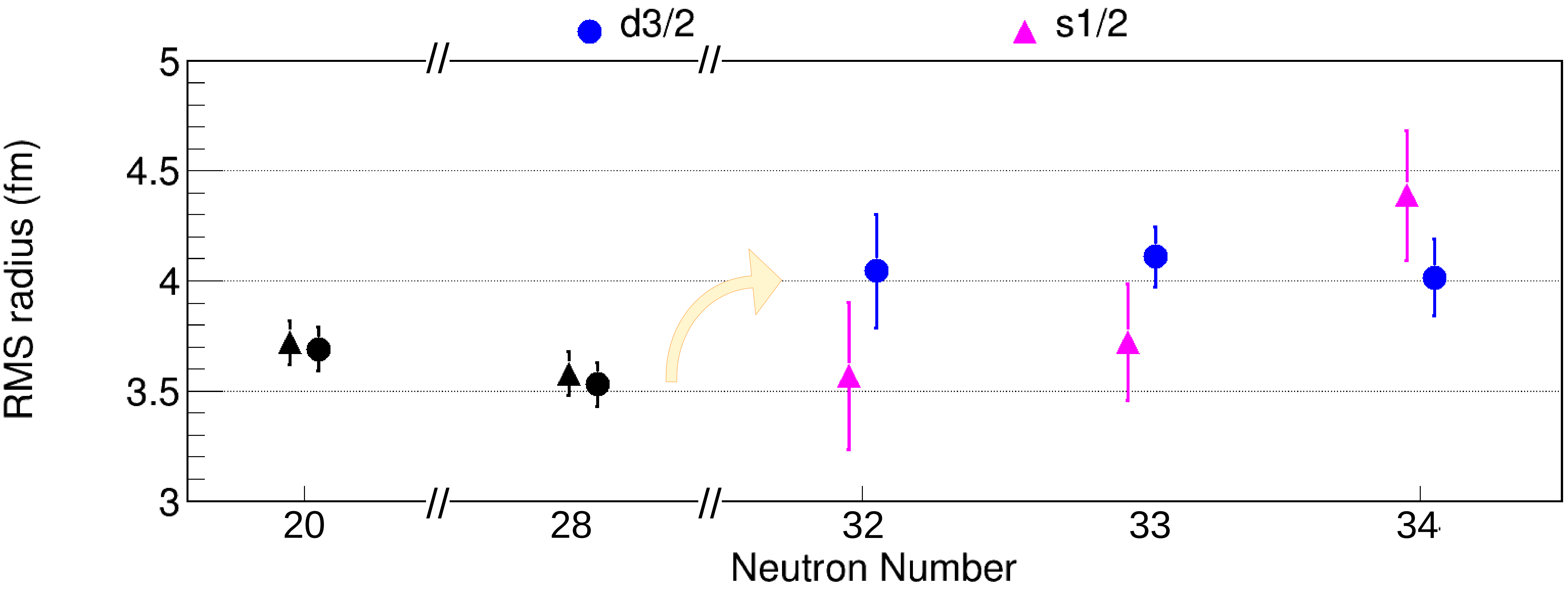

^{40}Ca and ^{48}Ca data from:
G. J. Kramer, "The proton spectral function of ^{40}Ca and ^{48}Ca studied with the (e,e'p) reaction", PhD Thesis, Amsterdam Univ. (Netherlands), 1990

Uncertainty estimations and checks

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- **uncertainties of rms radii dominated by experimental statistical uncertainties**
  - larger uncertainties for (p,2p) with lower statistics
- **choice of potential:**
  - how the shape of momentum distribution changes
  - and what is the impact on the determined rms radii?
- why do we only perform an **1-dimensional r_0 variation**?
  - comparison 1D r_0 variation vs 2D r_0,a_0 variation of the Woods-Saxon potential parameters
- how do momentum distributions change when using **HFB wavefunction**
  - or **transition amplitudes from state-of-the-art ab initio calculations**?

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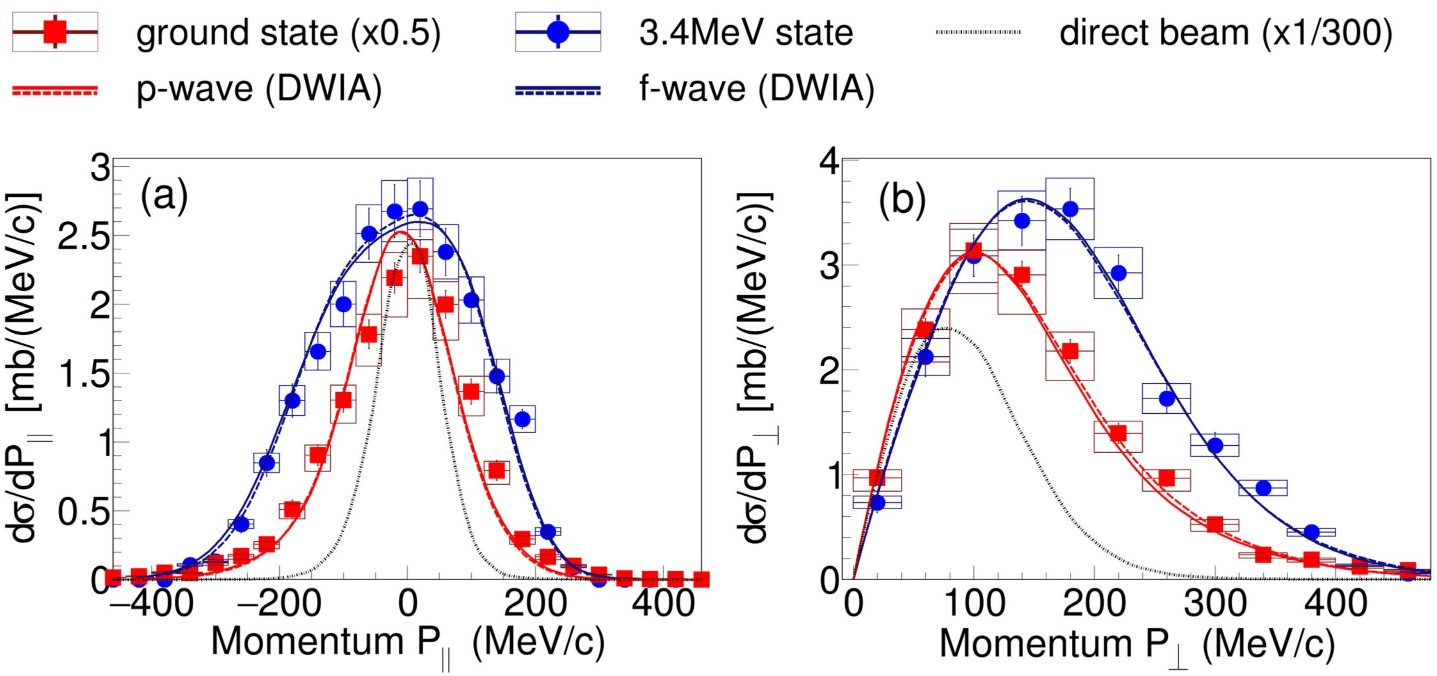
Choice of optical potential

used as input for the DWIA calculations



From ^{52}Ca $\nu p_{3/2}$ and $\nu f_{7/2}$:

- **4.5% relative difference** for the momentum distributions for the considered energy range and (r_0, a_0) combinations

From ^{53}Ca $\nu p_{3/2}$:

impact on r_0 and rms radii evaluated

- **0.10 fm for r_0 (8.1%)**
- **0.16 fm for rms radius (3.5%)**

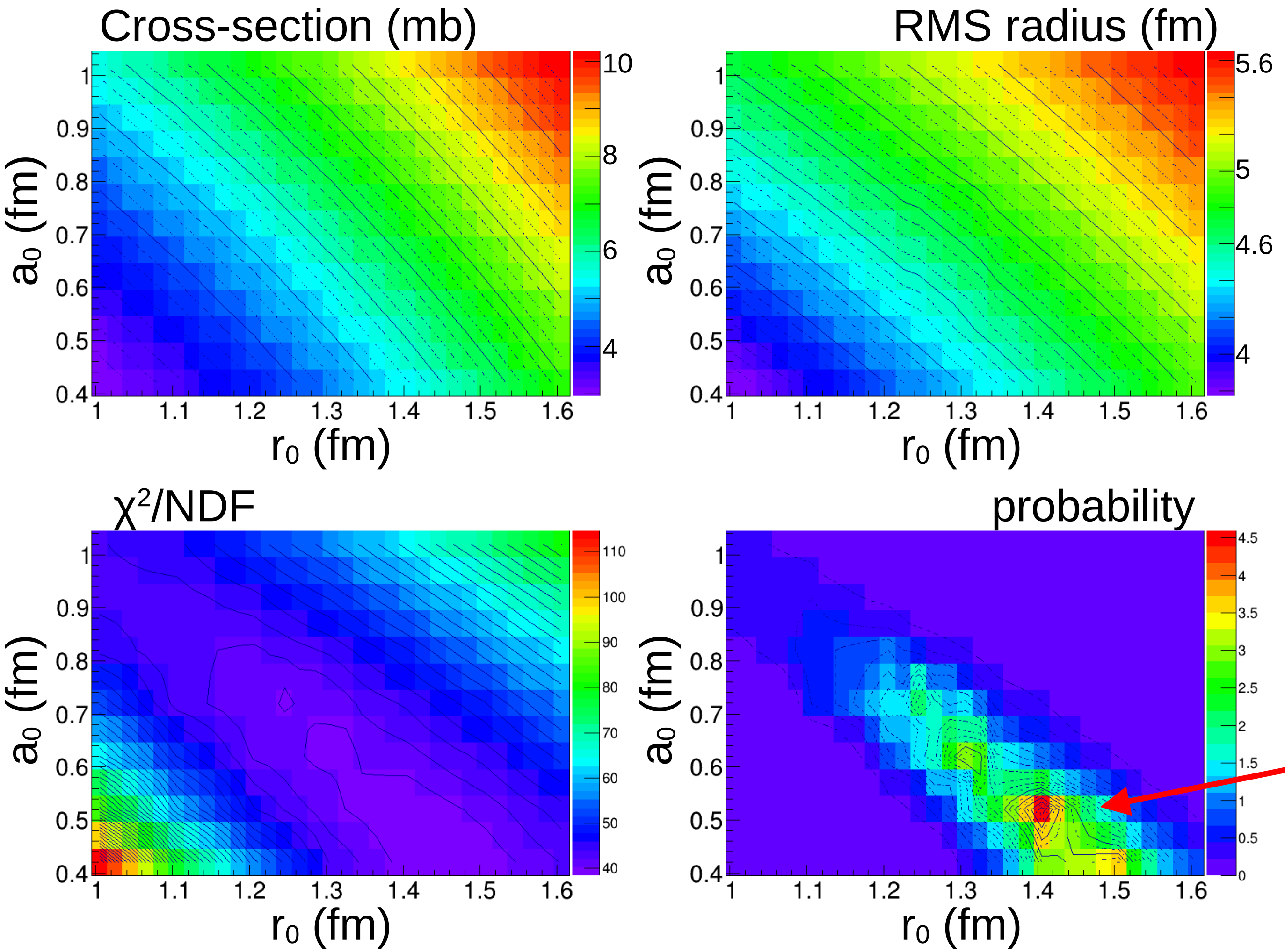
Plots: ^{52}Ca $\nu p_{3/2}$ and $\nu f_{7/2}$

Folding (solid line) and Dirac (EDAD1, dashed line)

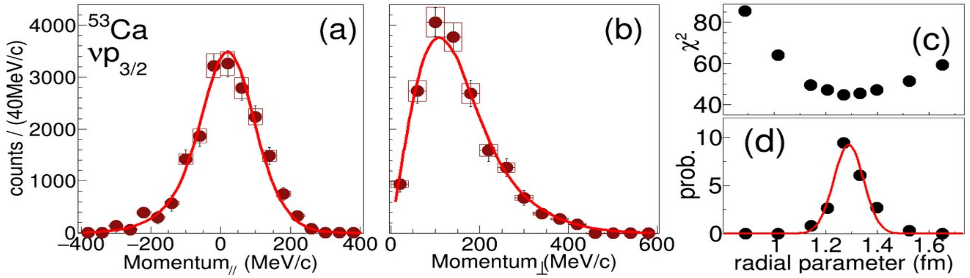
M. Enciu et al., PRL 129 (2022)

1D vs 2D variation: r_0 and a_0 for the WS potential

2D variation



1D variation ^{53}Ca (p,pn) [-ν$p_{3/2}$, 2.5MeV]



(r0,a0) pairs not unique

RMS radius results:

1D variation **4.67 fm**

for r_0 = 1.29 fm

and a_0 = 0.67 fm (fixed)

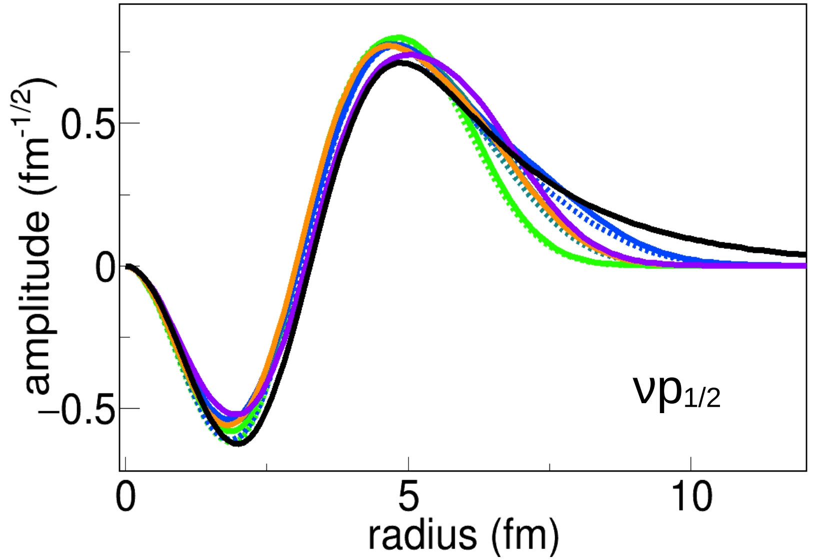
2D variation **4.663 fm**

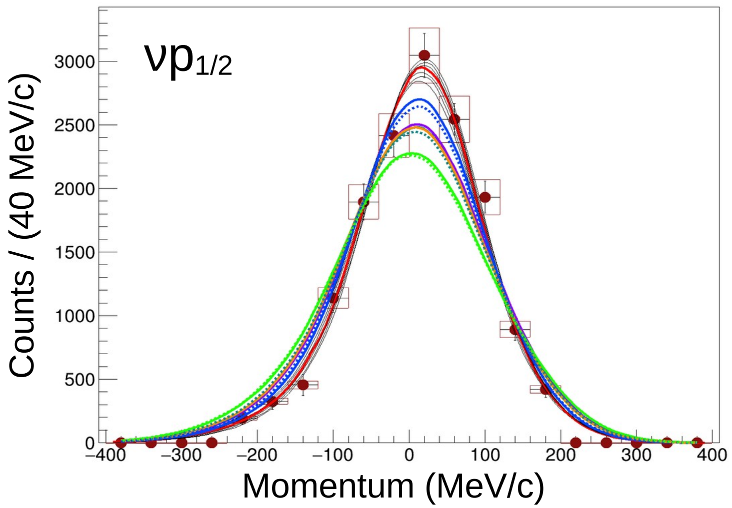
for r_0 = 1.406 fm

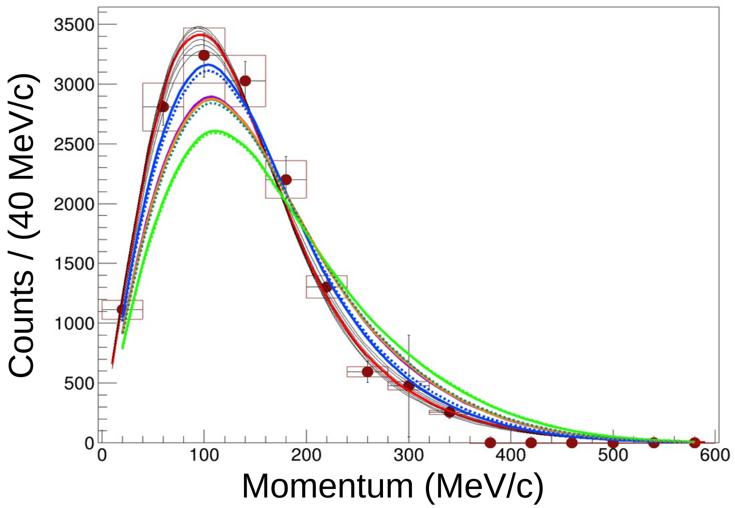
and a_0 = 0.52 fm

Woods-Saxon wavefunction vs ab initio input

ab initio amplitudes: <^{54}Ca(0$^+$)| a^+_{nlj} |^{53}Ca(J$^\pi$)>   used for the DWIA calculations for ^{54}Ca(p,pn)







Legend:

Woods-Saxon

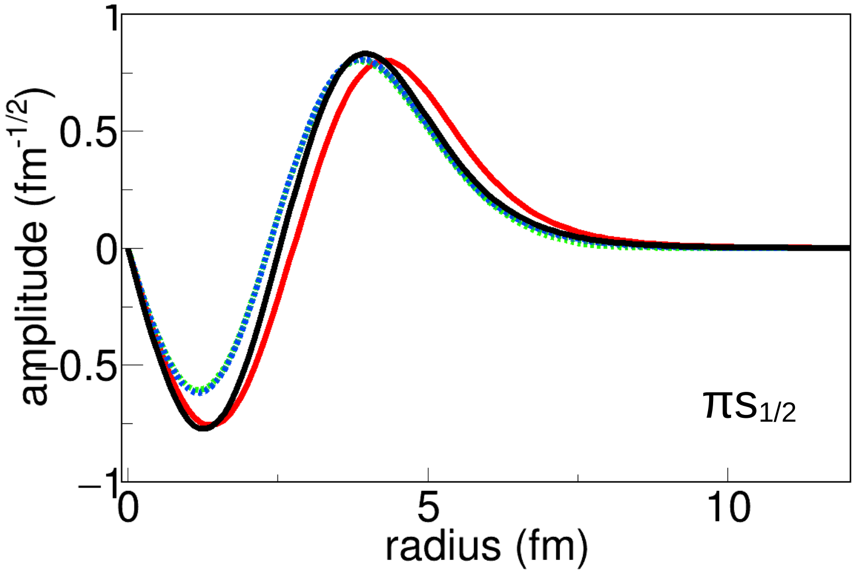
DN2LOGO394hw16	**N3LOEM500lnlhw16**
EM1.8-2.0hw12 (HF)	***-- EM1.8-2.0hw12 (NAT)***
EM1.8-2.0hw16 (HF)	***-- EM1.8-2.0hw16 (NAT)***
EM1.8-2.0hw20 (HF)	***-- EM1.8-2.0hw20 (NAT)***

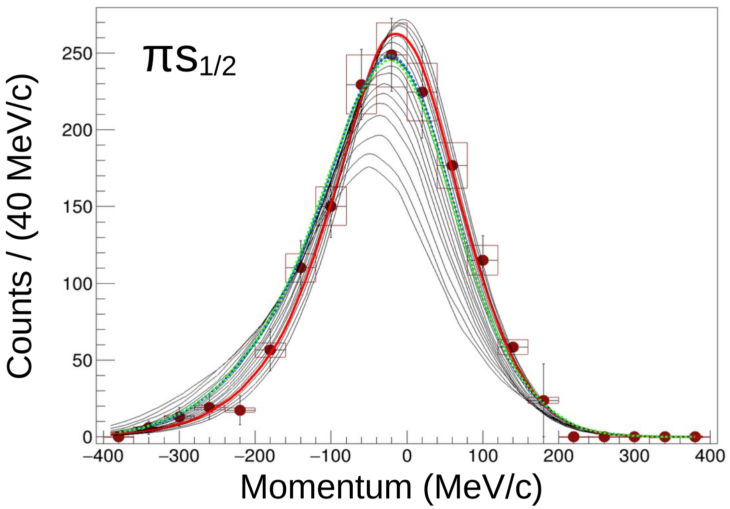
Similar results for ^{54}Ca $\nu p_{3/2}$ and $\nu p_{1/2}$ orbitals

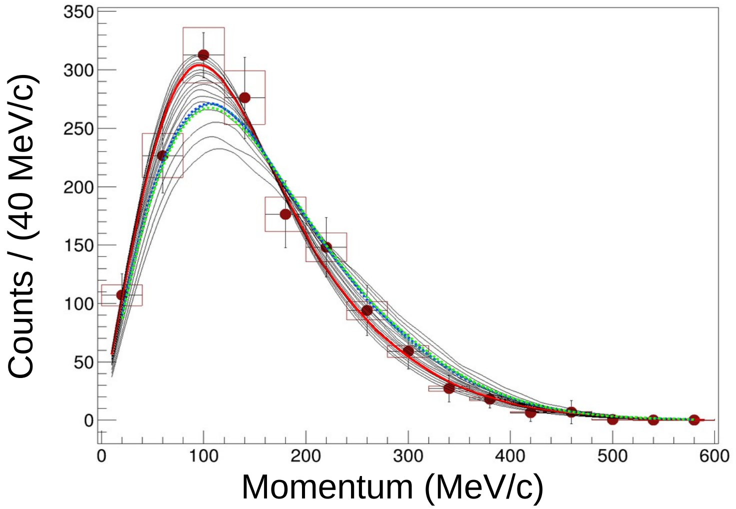
- frequency dependence
- wider momentum distributions
- lower rms radii
- poor fit to experimental data

Woods-Saxon wavefunction vs ab initio input

ab initio amplitudes: $\langle ^{54}Ca(0^+)| a^+_{nlj} |^{53}K(J^\pi) \rangle$ used for the DWIA calculations for $^{54}Ca(p,2p)$







Legend:

Woods-Saxon

-- EM1.8-2.0hw12 (NAT)

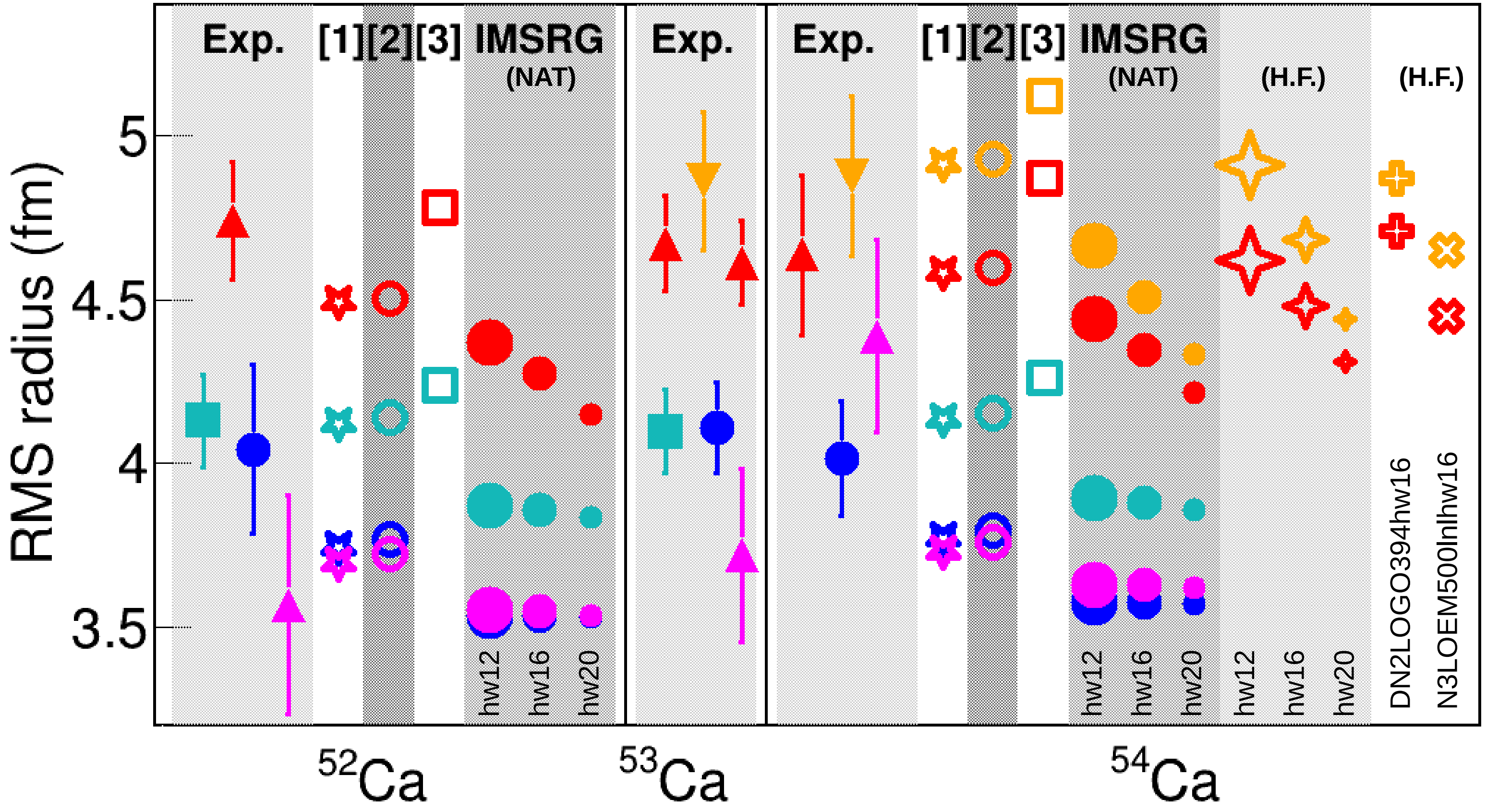
-- EM1.8-2.0hw16 (NAT)

-- EM1.8-2.0hw20 (NAT)

Similar results for $^{52,54}Ca$ $\pi d_{3/2}$ and $\pi s_{1/2}$ orbitals

- no frequency dependence
- wider momentum distributions
- lower rms radii
- poor fit to experimental data

Rms radii of sp proton and neutron orbitals



[1] HFB calculations (HFBRAD code) with the SKM interaction

[2] Results from private communication with W. Horiuchi based on published work: W. Horiuchi et al., Phys. Rev. C 101 (2020) [Mean Field calculations]

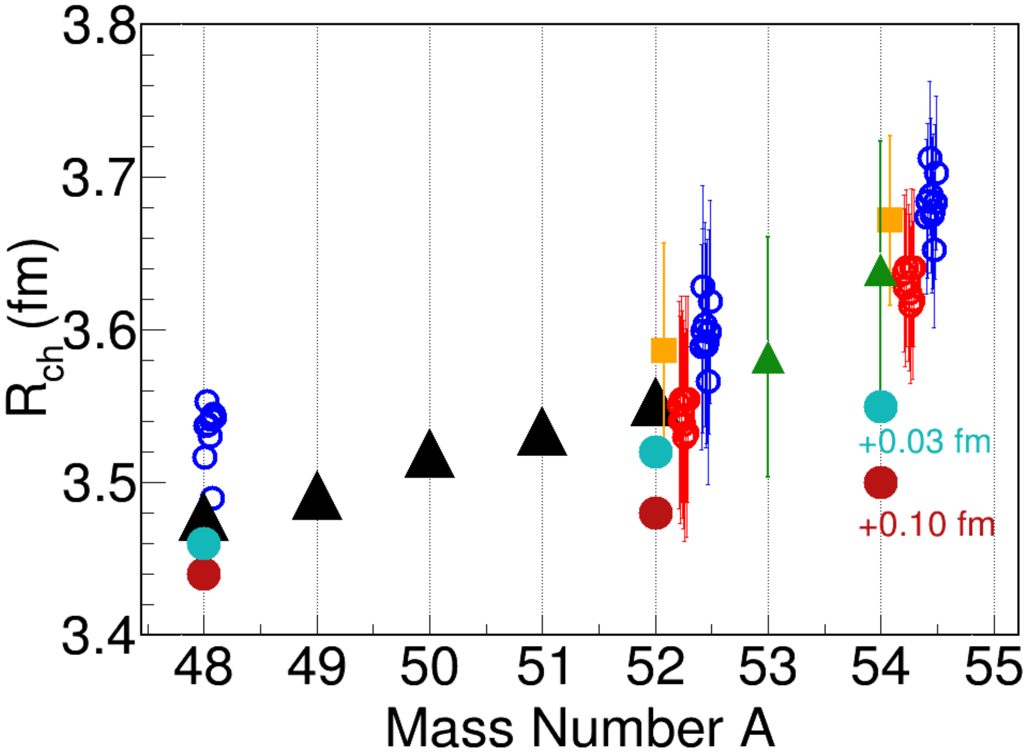
[3] Results from private communication with J. Liu (via H. Liu) based on published work: J. Liu et al., Phys. Lett. B 806 (2020) [Relativistic Hartree-Fock calculations]

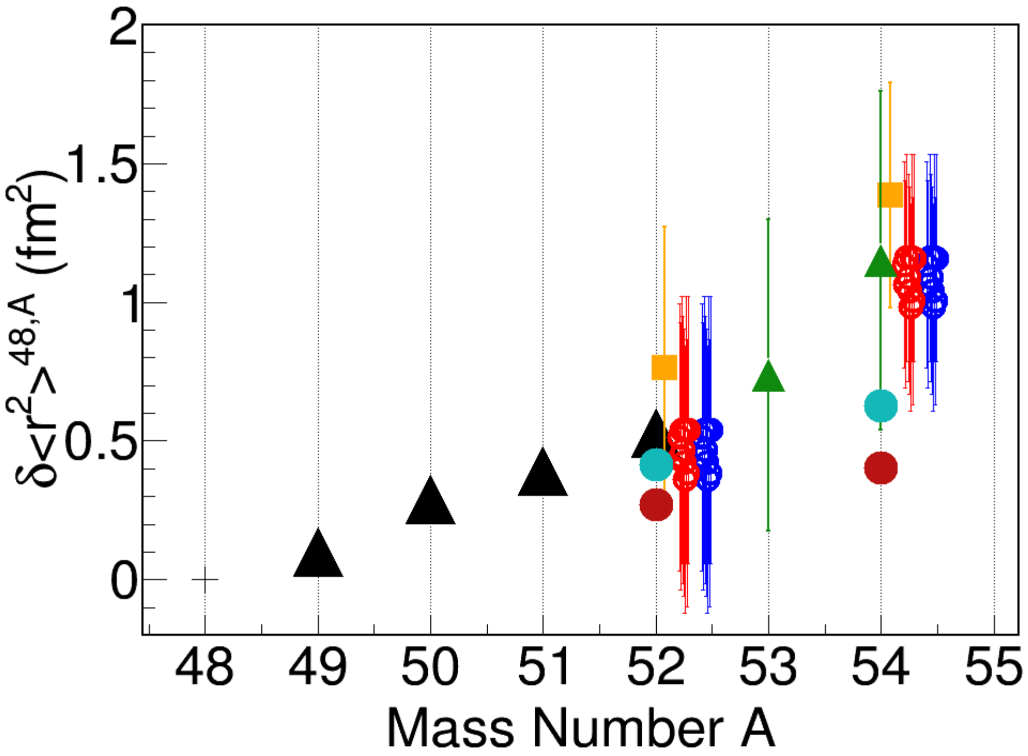
Neutrons:
- f7/2
- p3/2
- p1/2

Protons:
- d3/2
- s1/2

IMSRG (full) one-body level: preliminary, ongoing work M. Heinz, T. Miyagi, A. Schwenk, A. Tichai

Charge radii of Calcium isotopes





$R(^{48}Ca)_{HFB}$ – $(\pi sd)^{48Ca}_{HFB}$ + $(\pi sd)^{ACa}_{\text{present work}}$

$R(^{48}Ca)_{IS}$ – $(\pi sd)^{48Ca}_{HFB}$ + $(\pi sd)^{ACa}_{\text{present work}}$

$R(^{48}Ca)_{IS}$ – $(\pi sd)^{48Ca}_{(e,e'p)}$ + $(\pi sd)^{ACa}_{\text{present work}}$

$R(^{52}Ca)_{IS}$ – $(\pi sd)^{52Ca}_{\text{present work}}$ + $(\pi sd)^{ACa}_{\text{present work}}$

- ▲ Isotope Shift Measurements
- ○ ○ ■ ▲ Evaluations w/ sp rms radii
- ● DN2LOGO hw16
- ● EM1.8-2.0 hw16

HFB Interactions considered: SKM, SKM*, SIII, SII, Ska, Skb, SKI5, SLY4, SLY5; IMSRG calculations by M. Heinz

Summary and Conclusions

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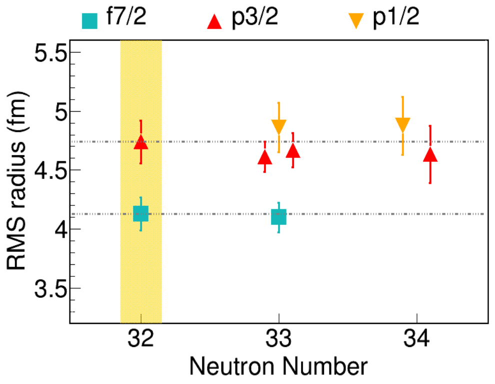
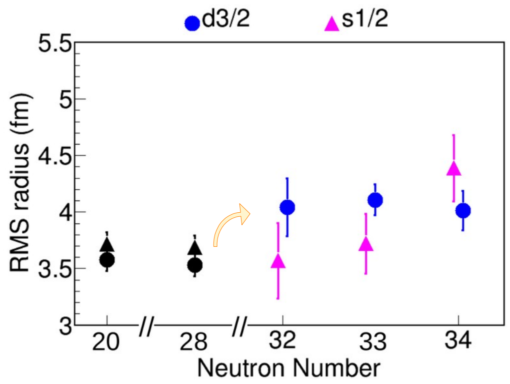
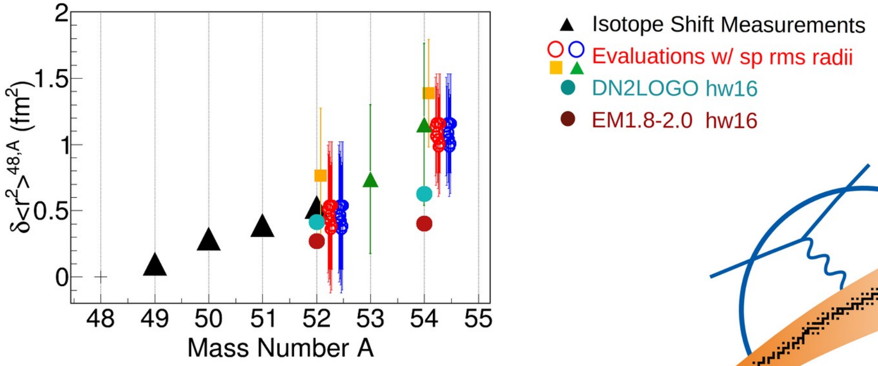
- **Calcium charge radii puzzle: steep increase from N=28 to N=32** (R. F. Garcia-Ruiz et al. 2016)
  - **matter radii** present the same behavior (M. Tanaka et al. 2020)
  - J. Bonnard et al. 2016: **νp orbitals larger than νf orbitals by 0.7fm** for explaining the charge radii
  - W. Horiuchi et al. 2020: **core swelling** as one fills the $\nu p_{1/2}$, $\nu p_{3/2}$ and $\nu f_{5/2}$ orbitals
  - J. Liu et al. 2023: density evolution from **^{52}Ca to ^{54}Ca**

- **Rms radii of proton and neutron orbitals in ^{52,53,54}Ca:**
  - Neutrons: large difference between **νp and νf orbitals**
  - Protons: an **increase in πsd** orbitals compared to ^{40,48}Ca

- From single-particle orbital **rms radii** to **charge radii:**
  - agreement with the prediction of Bonnard for νp-νf
  - the $\delta\langle r^2\rangle^{48,52}$ as well as R_{ch}^{52Ca} was reproduced
  - evaluations for ^{53}Ca and ^{54}Ca charge radii







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