

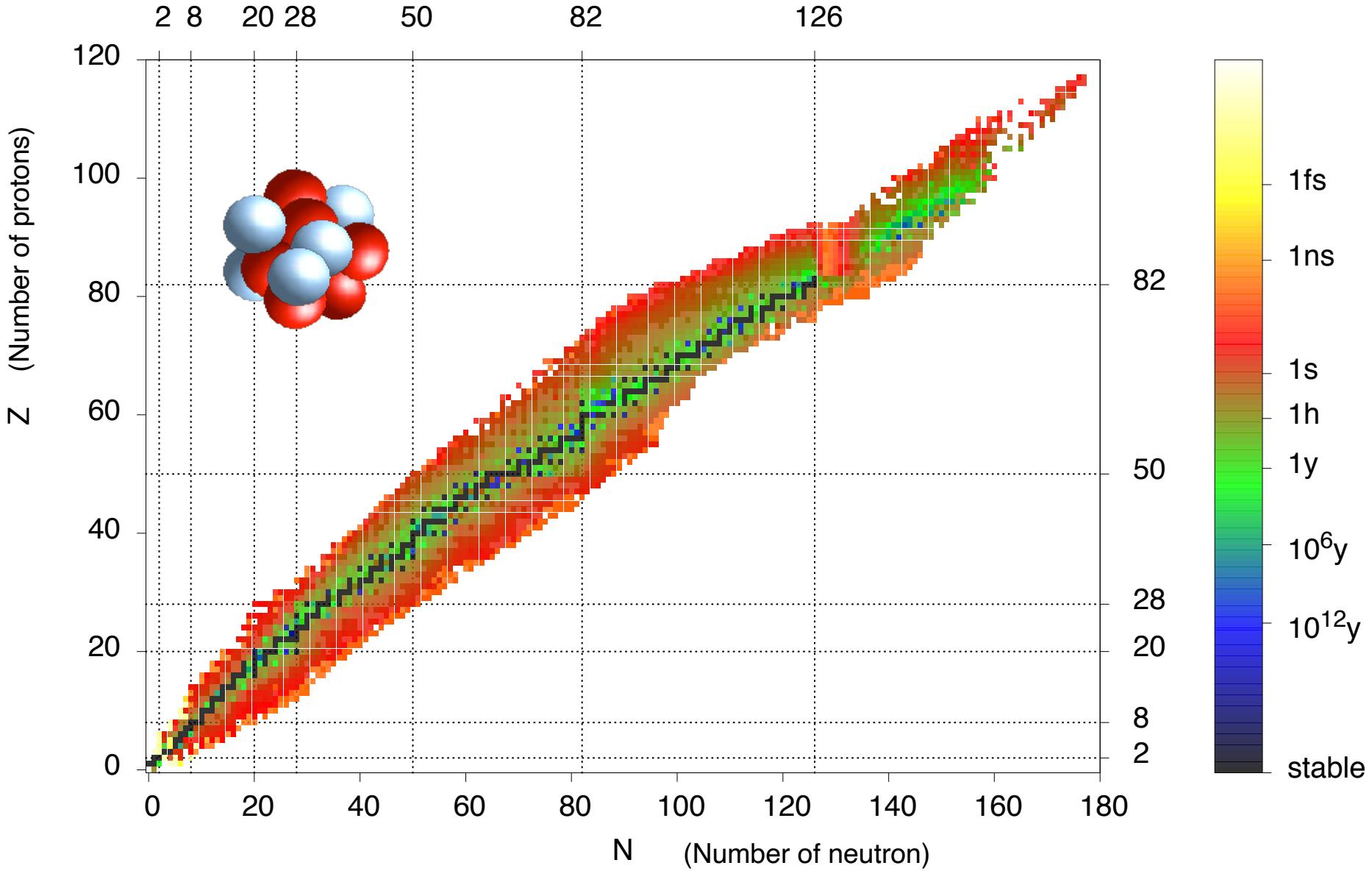
# *Time dependent dynamics of nuclear many-body states*

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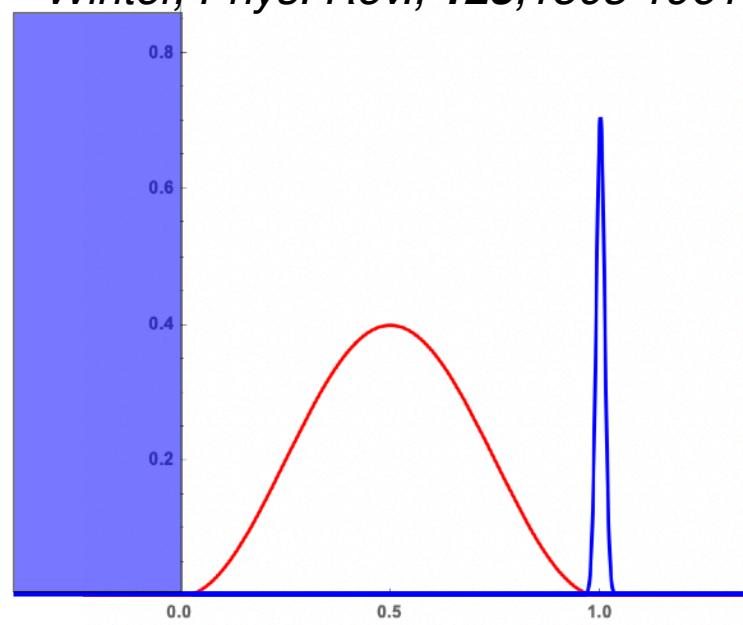
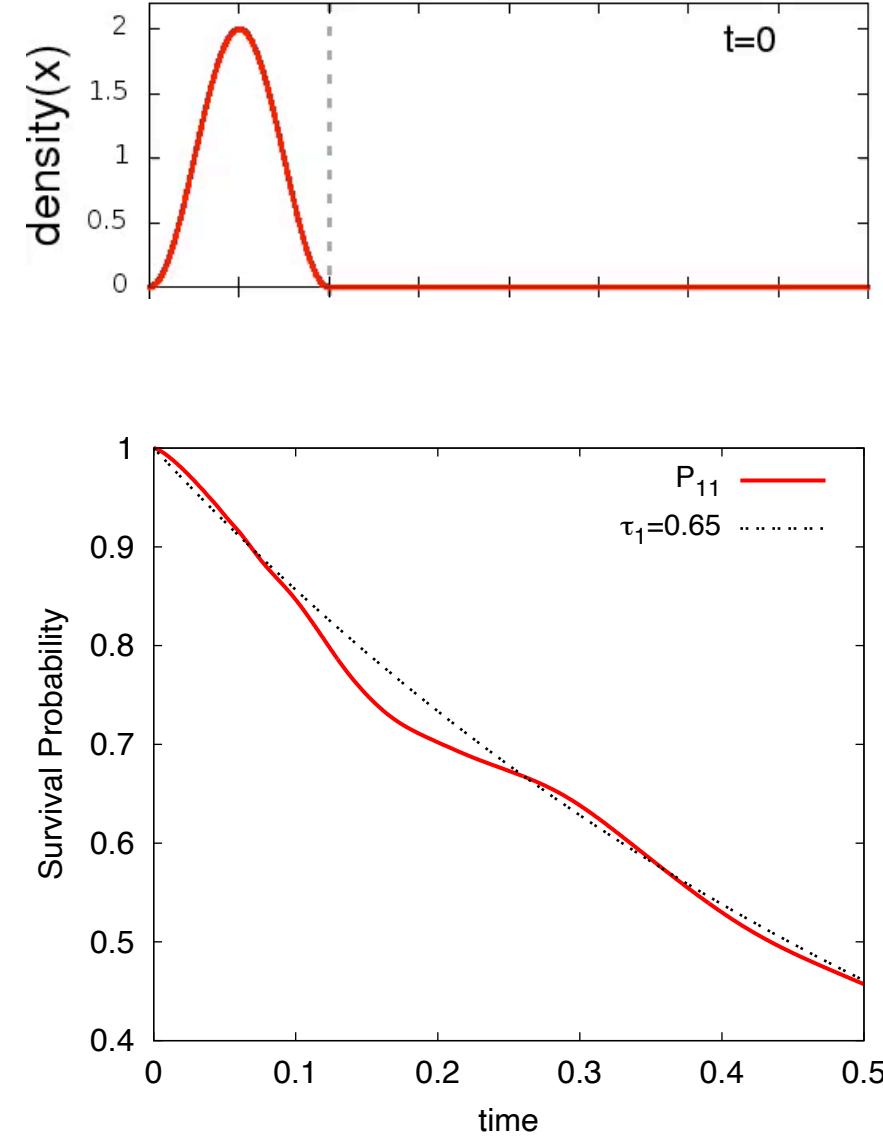


# Atomic nucleus and open quantum system

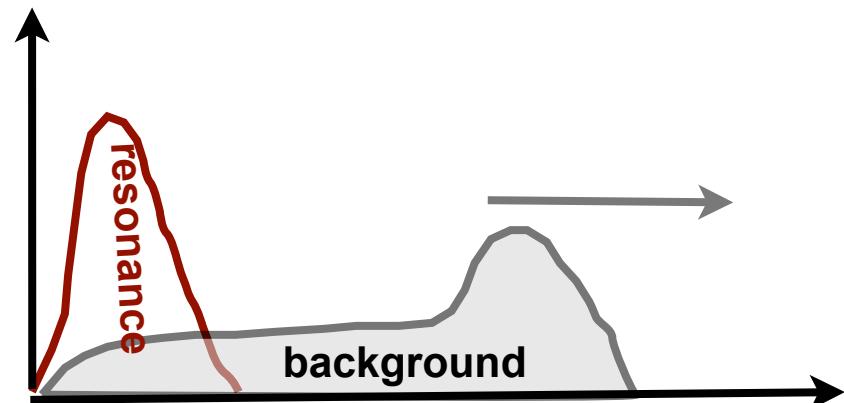
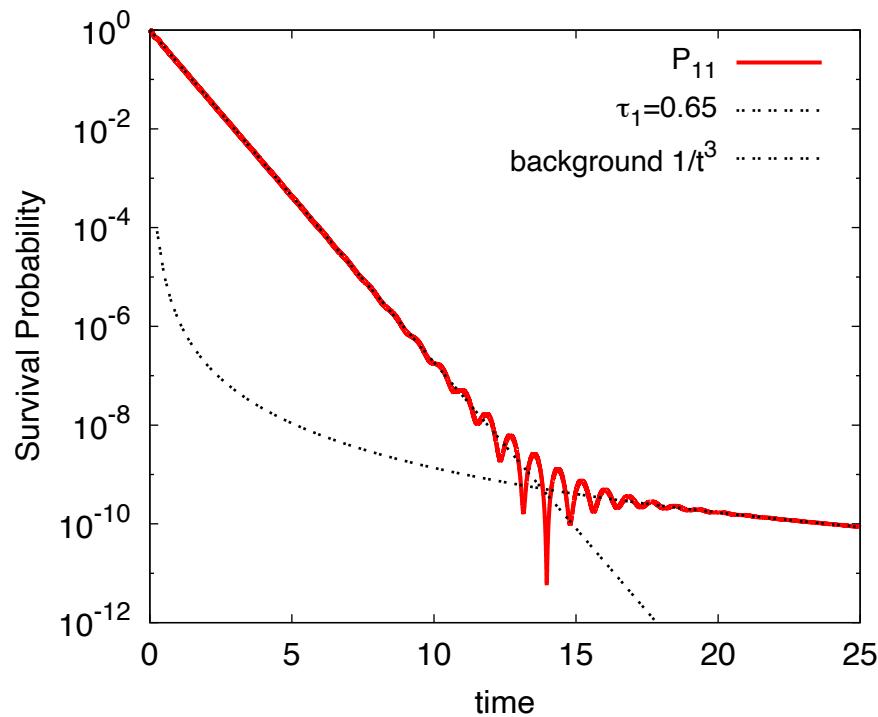
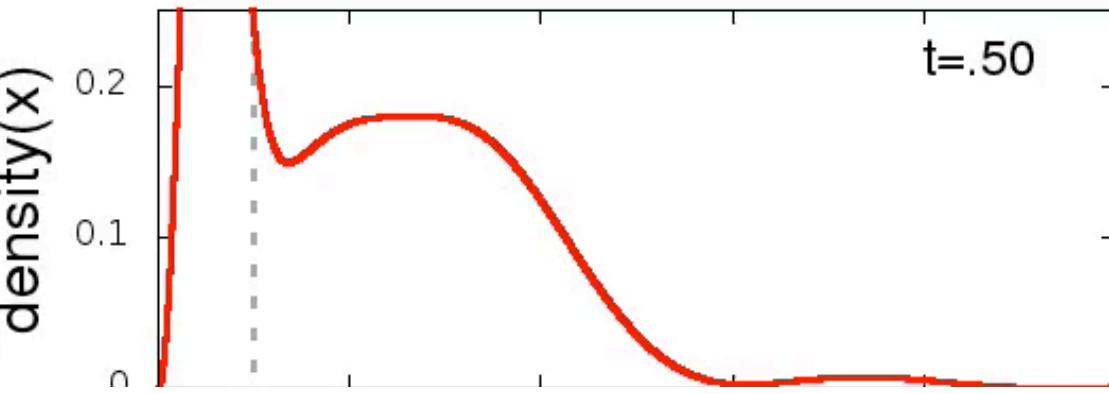


# Time dependence of decay

Winter, Phys. Rev., 123, 1503 1961.



## Winter's model: Dynamics at remote times



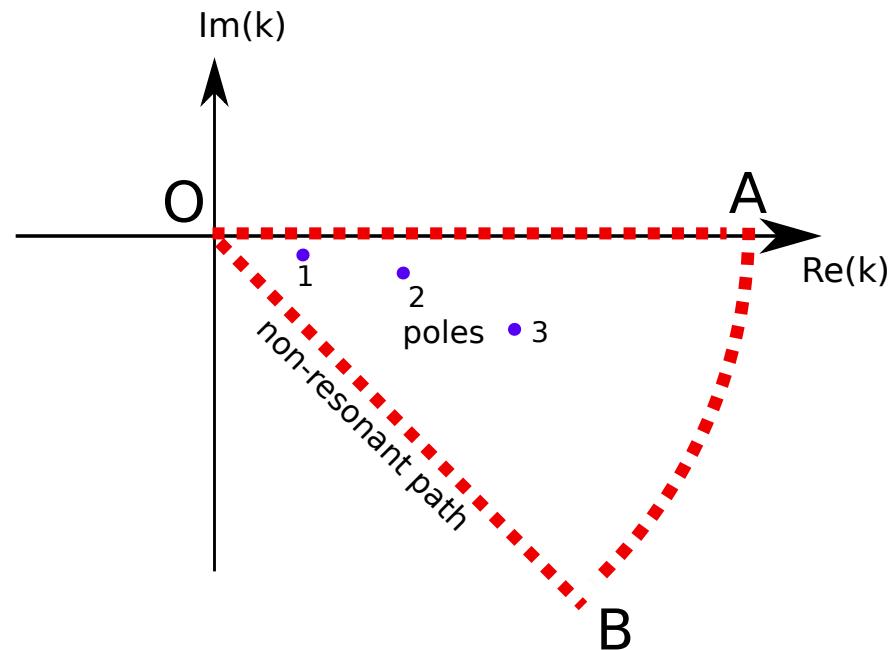
# Internal dynamics in decaying system

## Winter's model

$$M_{nn'}(t) = \langle n | e^{-iHt} | n' \rangle = \int_0^\infty e^{-ik^2 t} \langle n | k \rangle \langle k | n' \rangle dk.$$

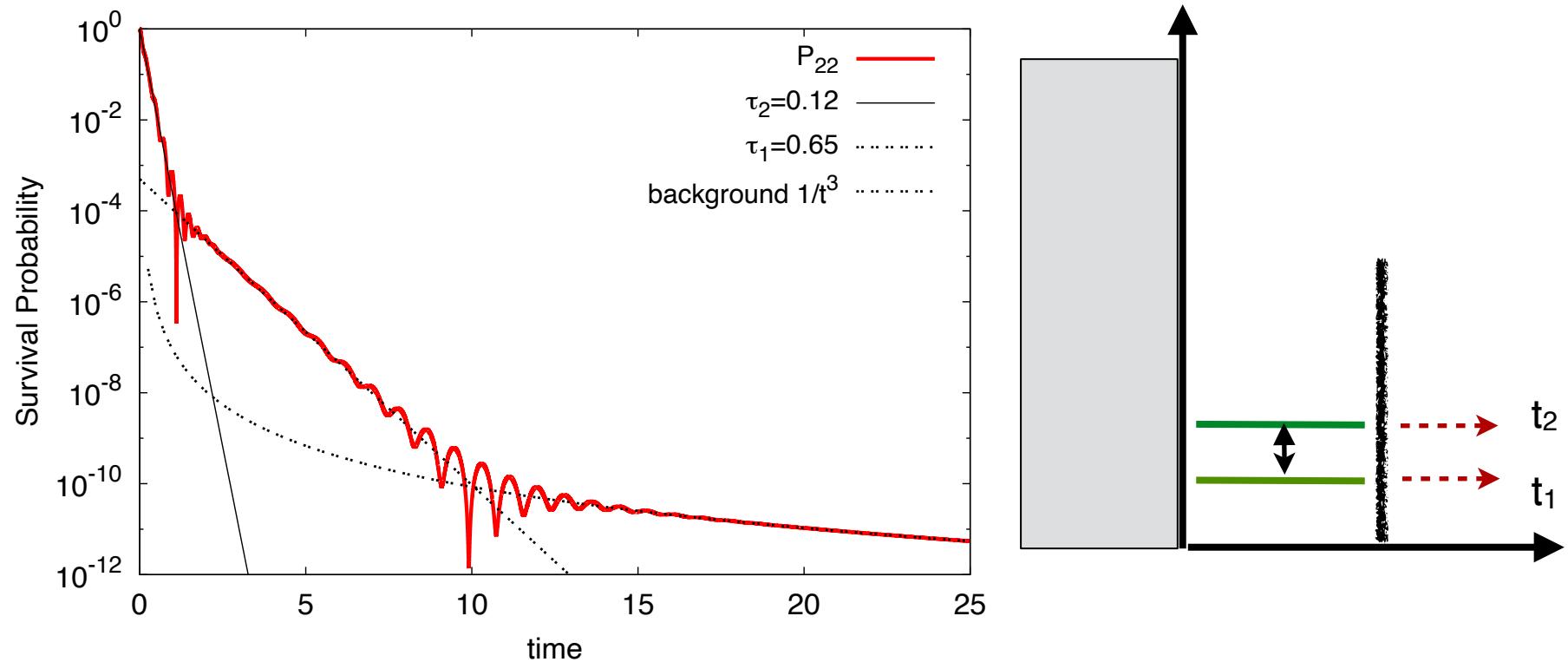
$$M_{nn'}(t) = \sum_{k_r} M_{nn'}^{(r)} e^{-ik_r^2 t} + M_{nn'}^{(NR)}(t) \quad M_{nn'}^{(r)}(t) = -2\pi i \operatorname{Res}(k_r)$$

$$M_{nn'}^{(NR)}(t) = \frac{1+i}{\pi^{5/2}\sqrt{2}} (1+G)^2 \frac{1}{nn'} \frac{1}{t^{3/2}}.$$



# Internal dynamics in decaying system

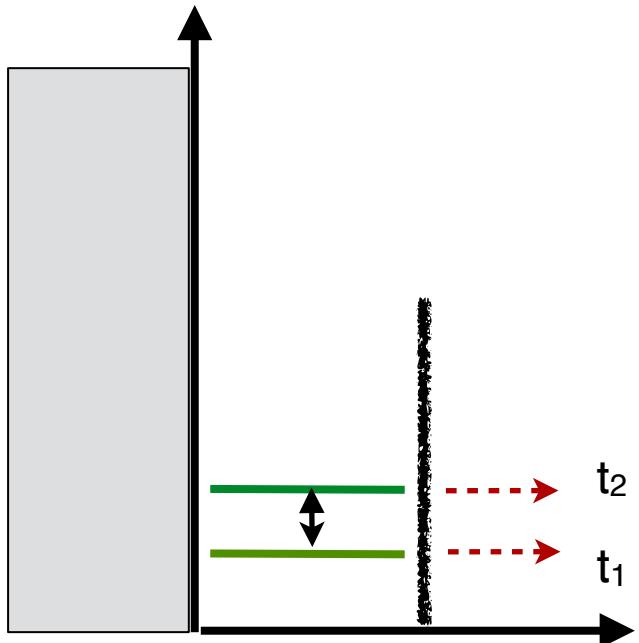
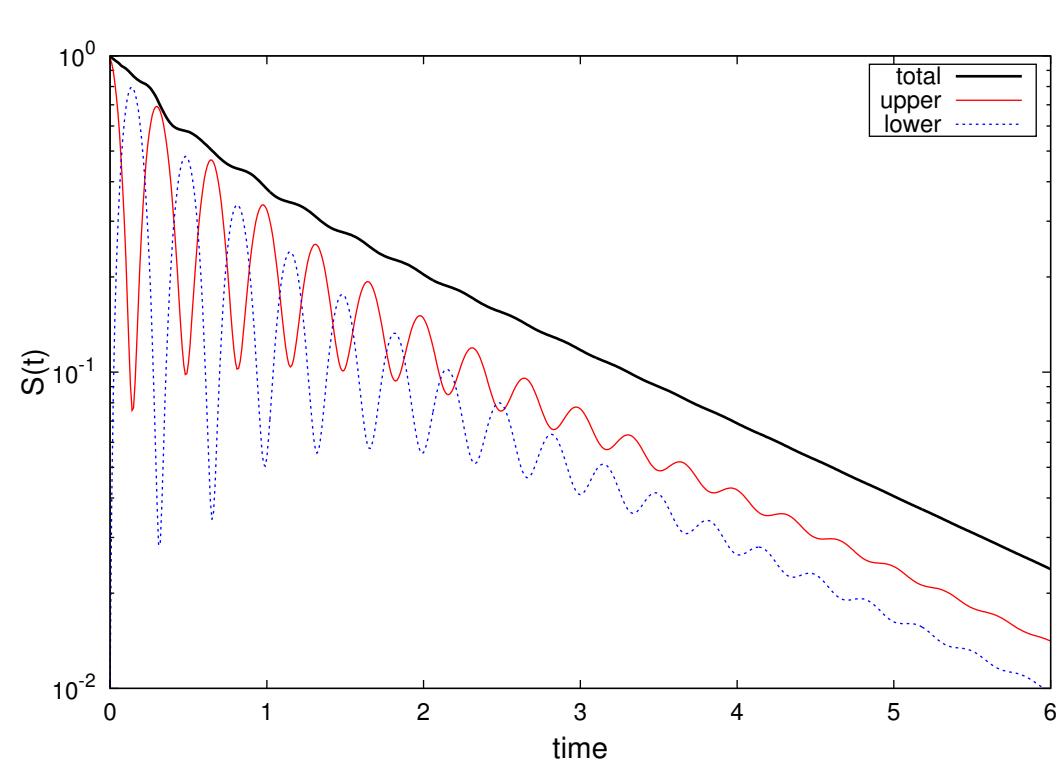
## Winter's model



M. Peshkin, A. Volya, and V. Zelevinsky, “Non-exponential and oscillatory decays in quantum mechanics,” *EPL*, vol. 107, no. 4, p. 40001, 2014.

# Internal dynamics in decaying system

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# Time-dependent Continuum Shell Model Approach

- Expand Using evolution operator in Chebyshev polynomials

$$\exp(-iHt) = \sum_{n=0}^{\infty} (-i)^n (2 - \delta_{n0}) J_n(t) T_n(H)$$

- Chebyshev polynomial  $T_n[\cos(\theta)] = \cos(n\theta)$

- Use iterative relation and matrix-vector multiplication to generate

$$|\lambda_n\rangle = T_n(H)|\lambda\rangle$$

$$|\lambda_0\rangle = |\lambda\rangle, \quad |\lambda_1\rangle = H|\lambda\rangle \quad |\lambda_{n+1}\rangle = 2H|\lambda_n\rangle - |\lambda_{n-1}\rangle$$

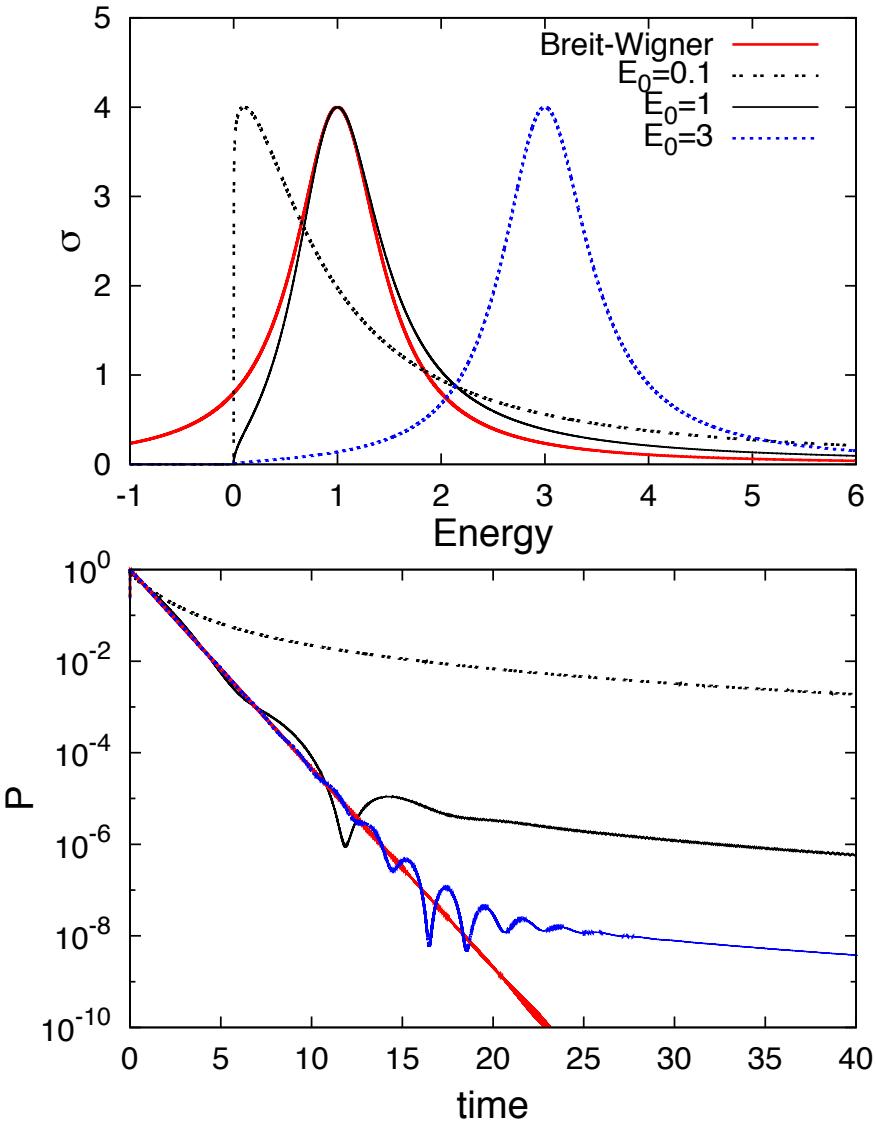
$$\langle \lambda' | T_{n+m}(H) | \lambda \rangle = 2\langle \lambda'_m | \lambda_n \rangle - \langle \lambda' | \lambda_{n-m} \rangle, \quad n \geq m$$

- Use FFT to find return to energy representation

# Probing the Non-exponential Decay Regime in Open Quantum Systems

- Broad threshold resonance ( ${}^9\text{N}, {}^9\text{He}$ )
  - Pronounced non-exponentiality
  - Very short half-life
- Three-body decay ( ${}^6\text{Be}, {}^{13}\text{Li}, {}^{16}\text{Be}$ )
  - Nucleon-nucleon correlations
  - Energy dependence
- Overlapping resonances ( ${}^{13}\text{C}, {}^{13}\text{N}$ )
  - Interference, pronounced non-exponentiality
  - Superradiance

# Time-dependent picture

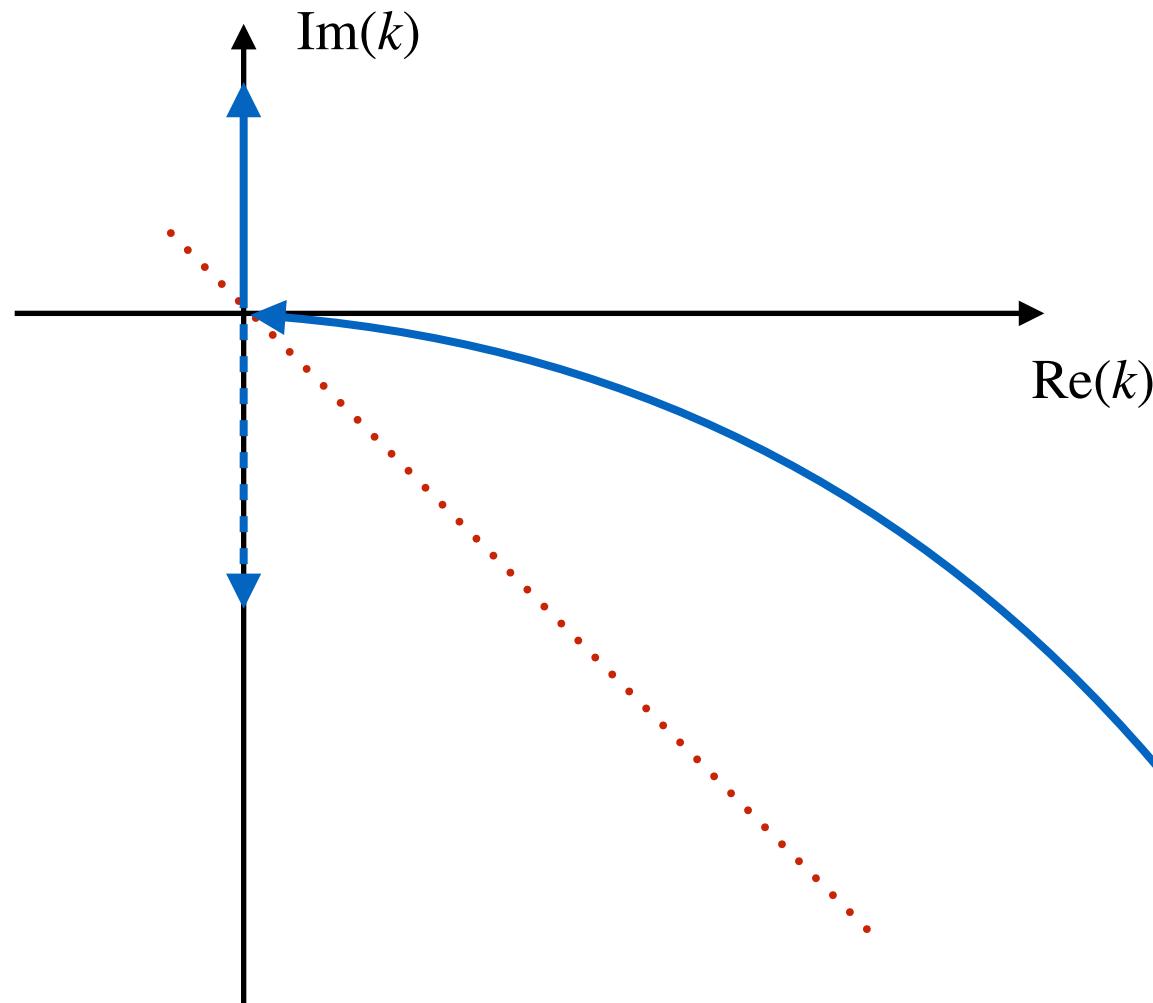


$$\mathcal{G} = \frac{1}{E - E_o + i/2 \Gamma(E)}$$

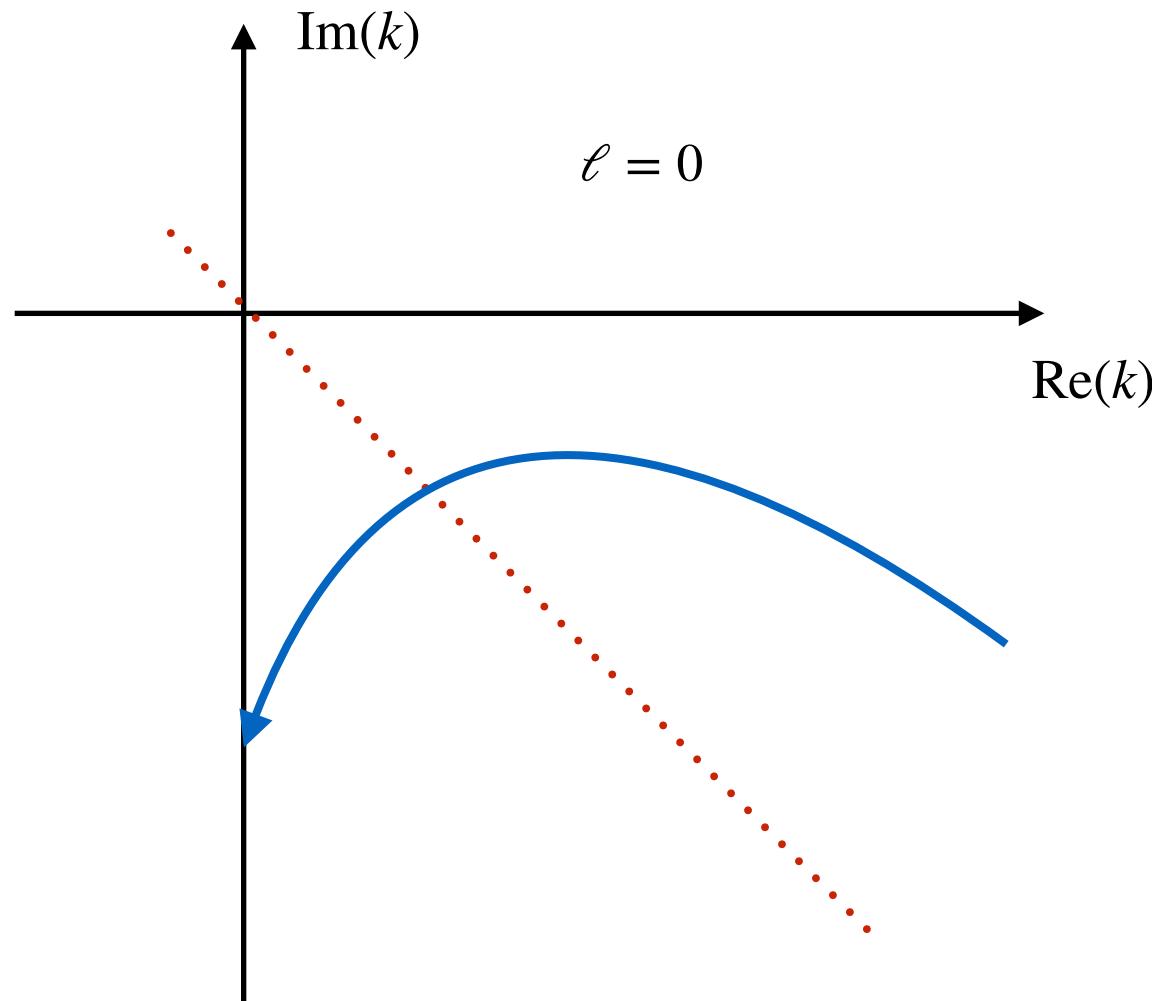
$$\Gamma(E) \propto \sqrt{E}$$

Power-law remote decay rate!

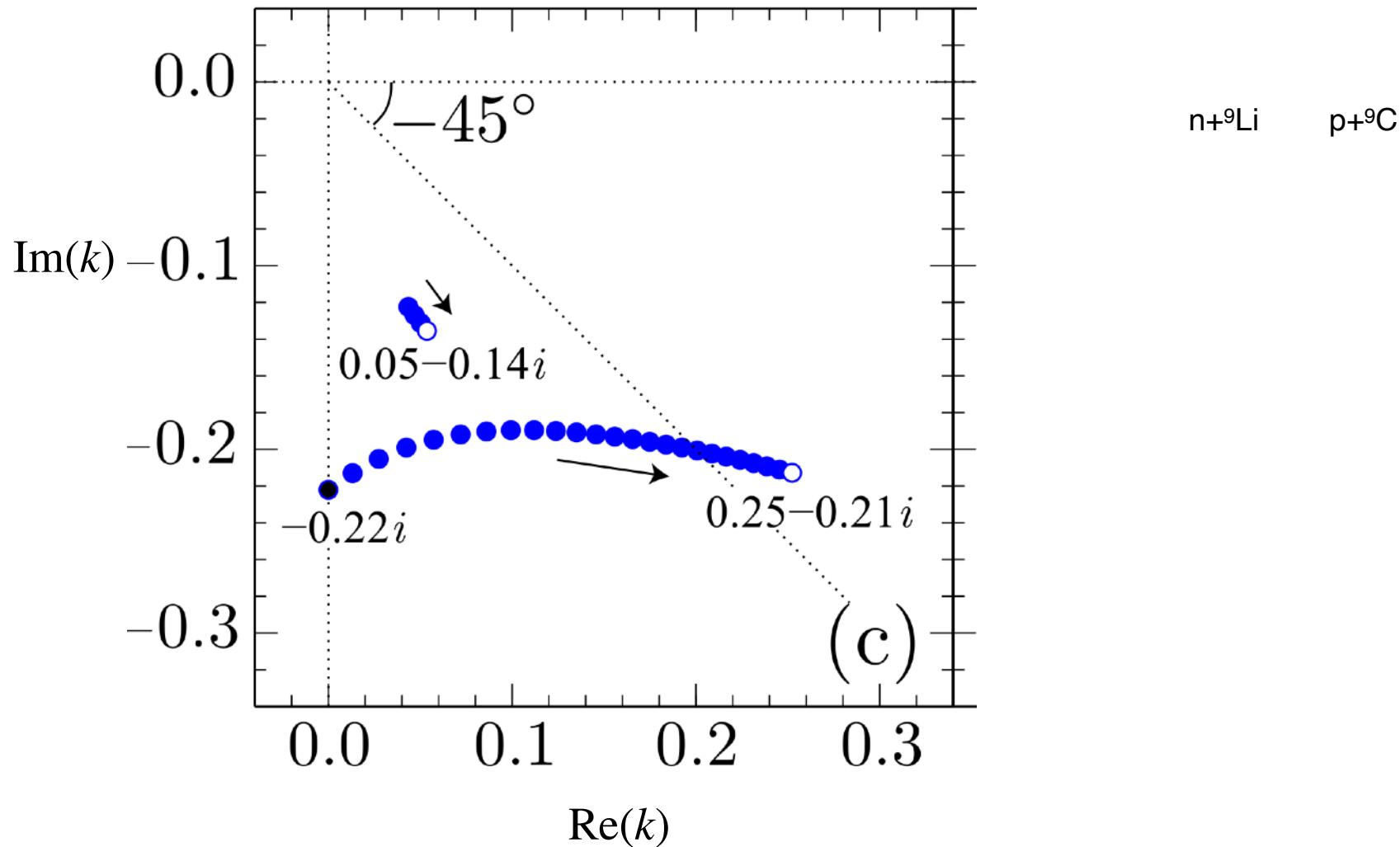
# Resonance to bound state pole dynamics



# Resonance to bound state pole dynamics



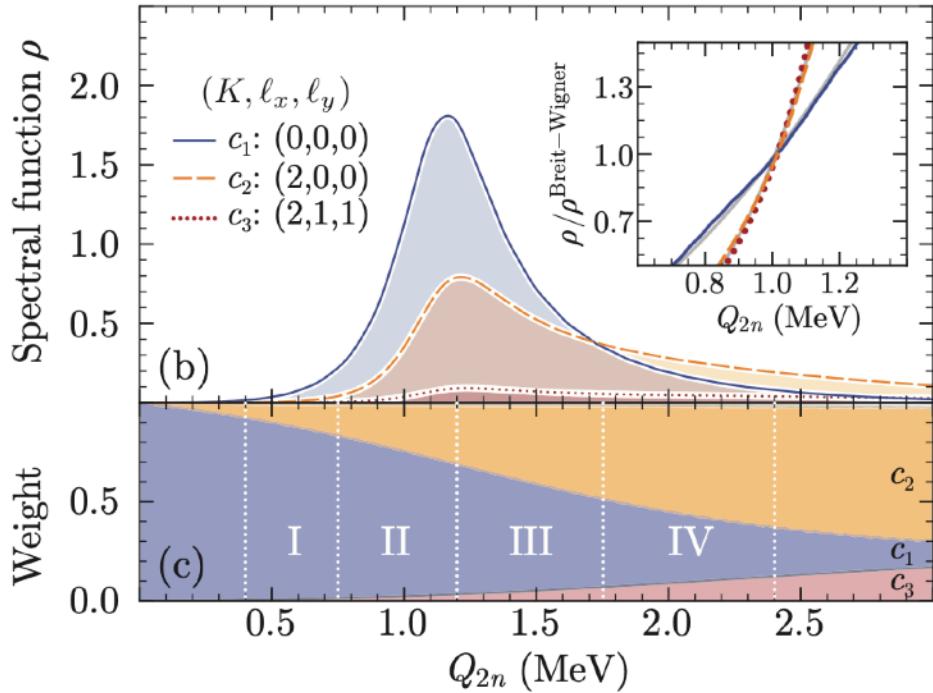
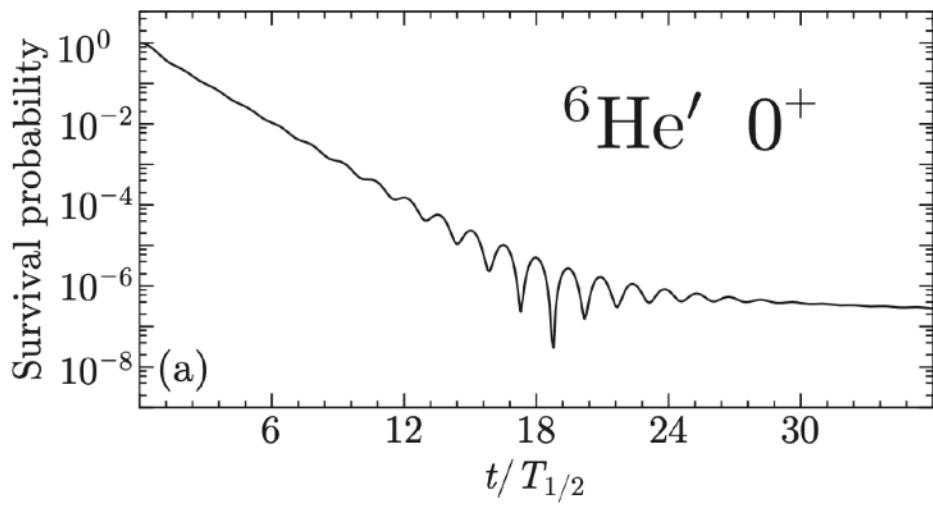
# Resonance to bound state pole dynamics



# Probing the Non-exponential Decay Regime in Open Quantum Systems

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# Time-dependent picture Two-neutron decay

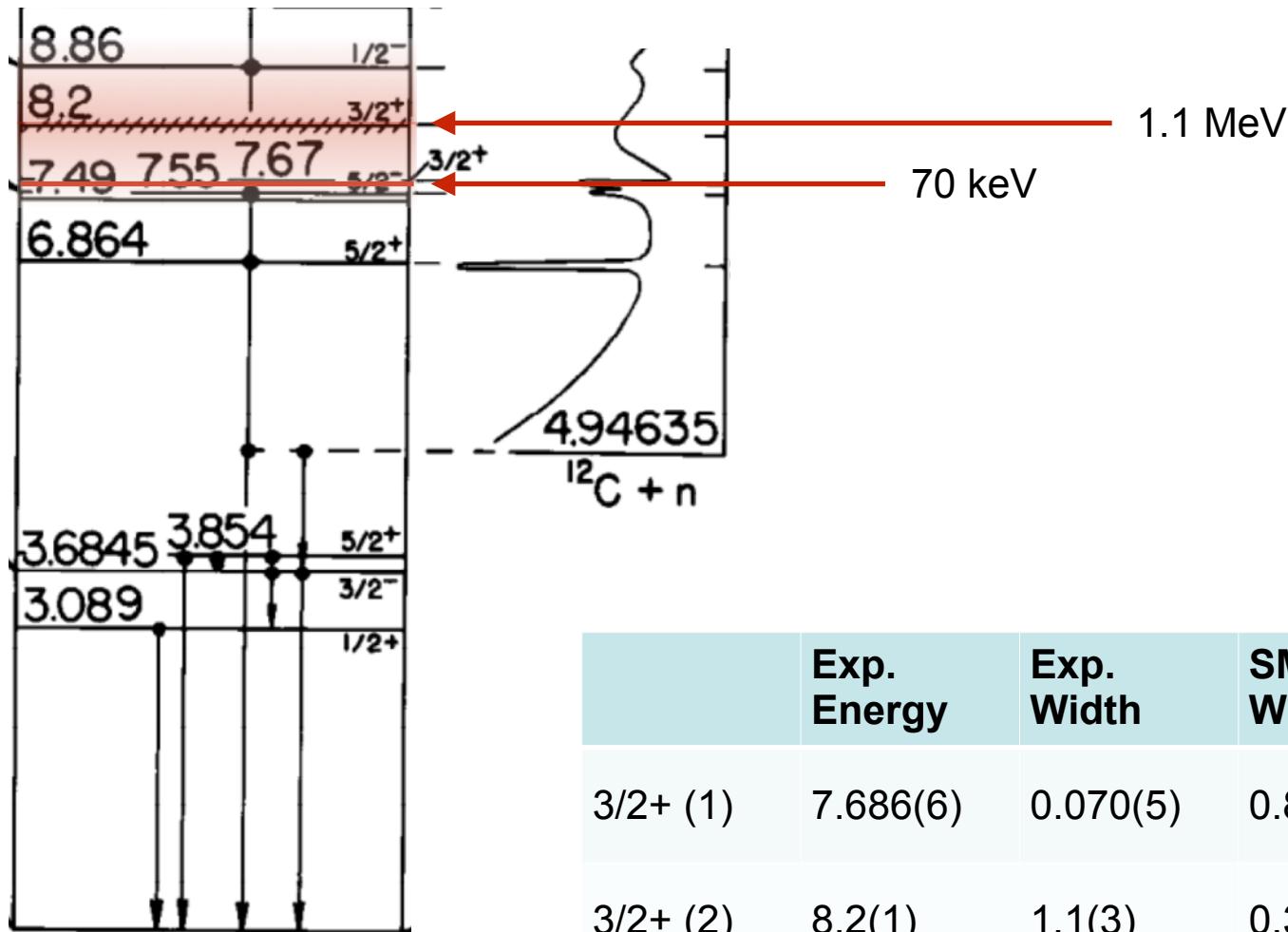


S. M. Wang, W. Nazarewicz, A. Volya, and Y. G. Ma,  
“Probing the Non-exponential Decay Regime in Open  
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# Probing the Non-exponential Decay Regime in Open Quantum Systems

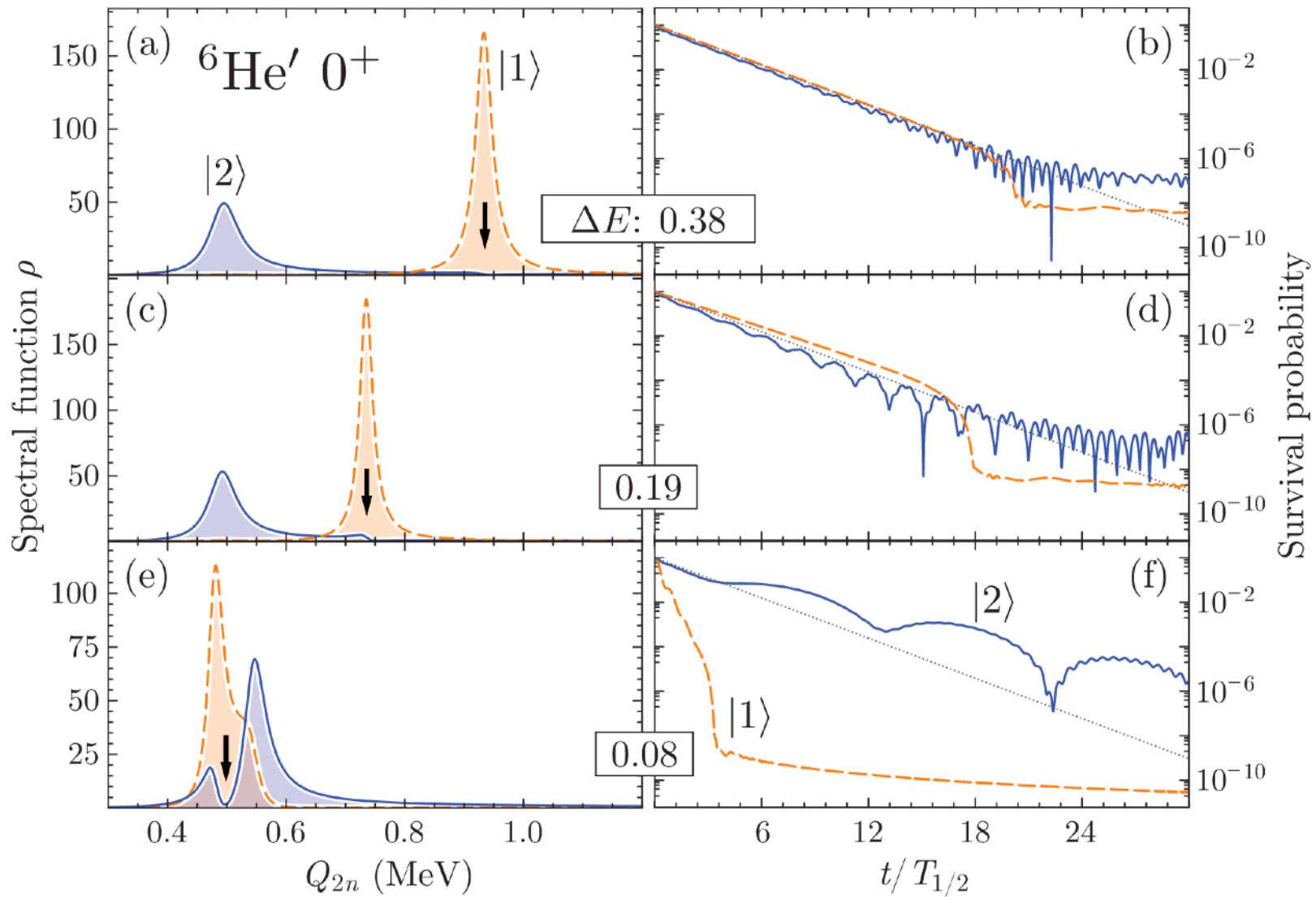
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# Superradiance in $^{13}\text{C}$

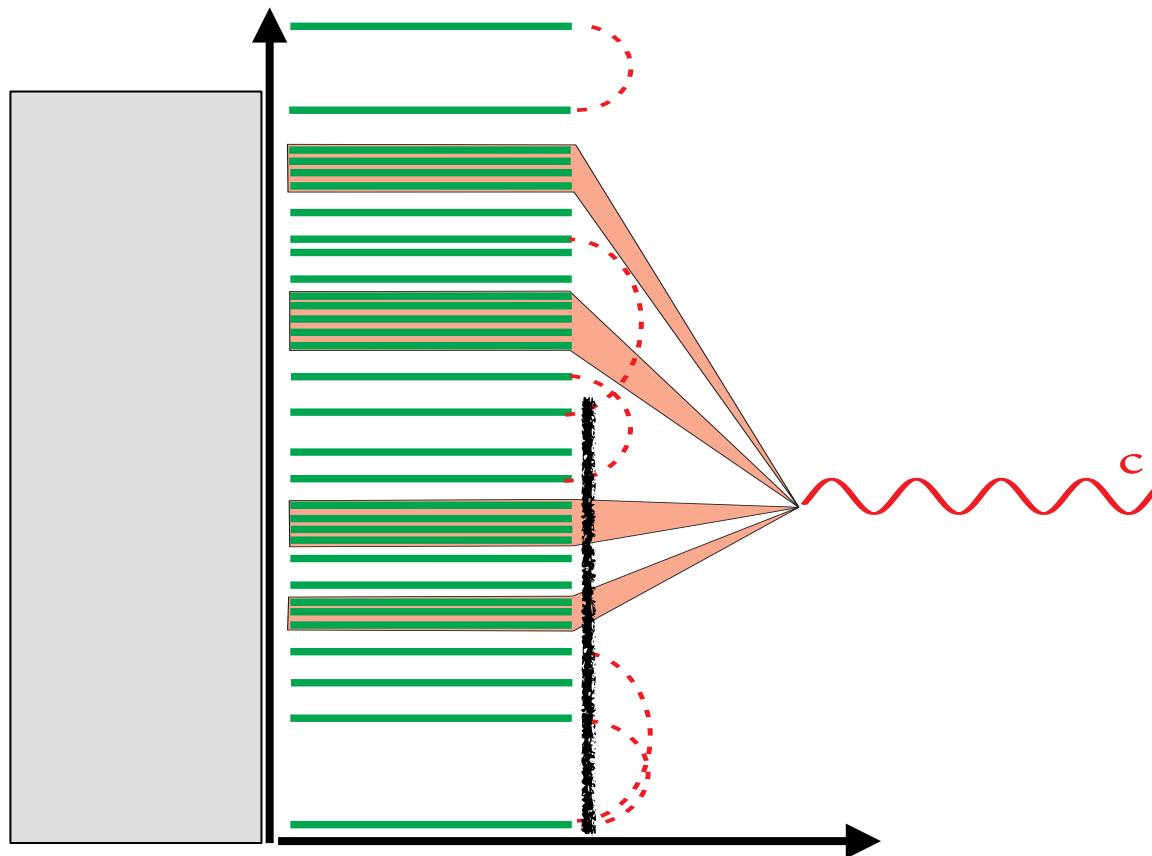


$^{13}\text{C}$

# Example of interference between resonances



# Coupling via continuum



# Universal law of relaxation

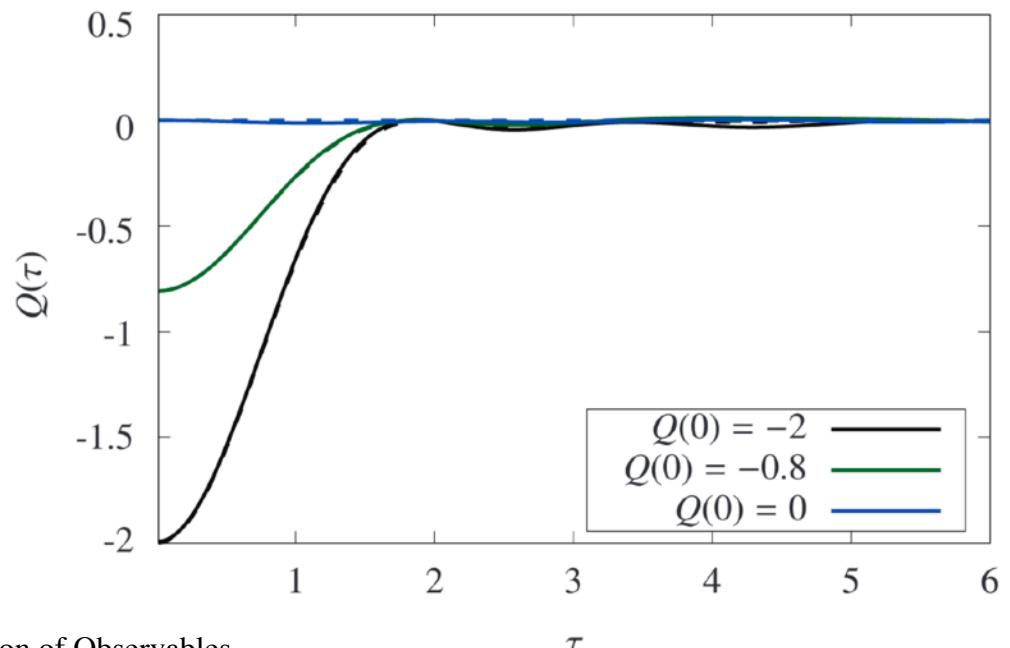
$$Q_a(t) = \langle \Psi_a(t) | \hat{Q} | \Psi_a(t) \rangle. \quad f_{a'a}(t) = \langle a' | e^{-i\hat{H}t} | a \rangle,$$

$$f_{aa'}(0) = \delta_{aa'}, \quad f_{aa'}(t) = f_{a'a}(-t),$$

$$\overline{f_{aa'}(t)} = \sum_n e^{-iE_n t} \overline{\langle a | n \rangle \langle n | a' \rangle} = \delta_{aa'} f(t), \quad Q(t) = Q(0) |f(t)|^2$$

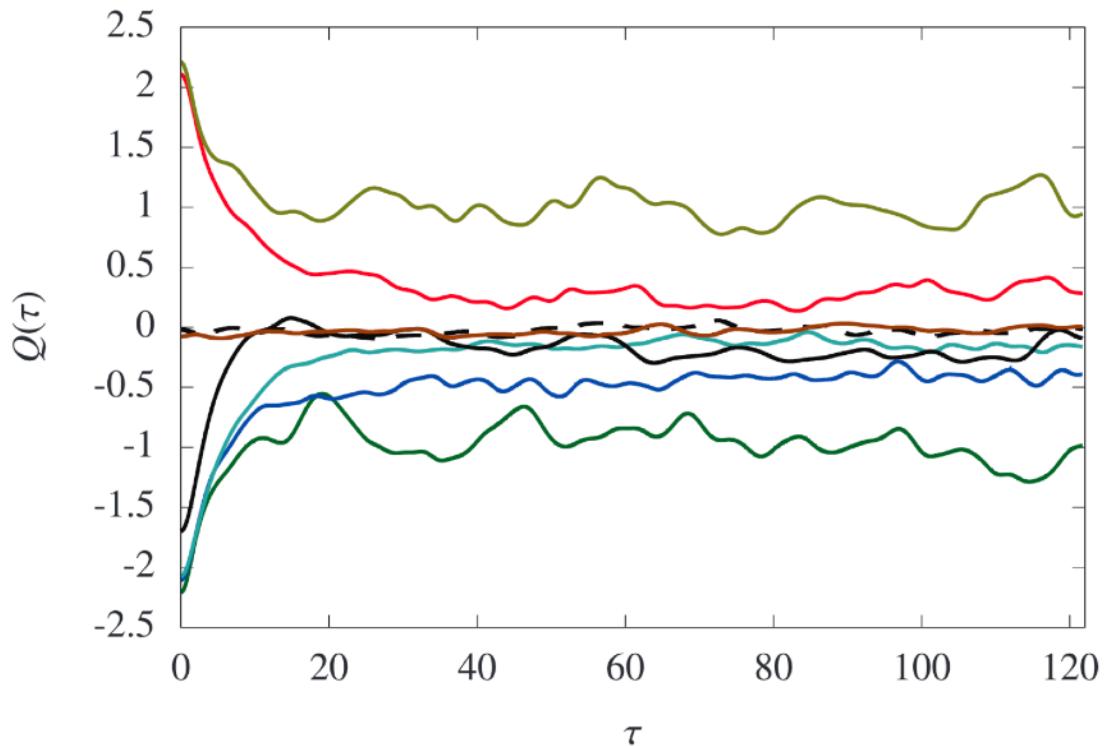
Under the above assumptions of full randomness, the function  $f(t)$  is universal

$$f(\tau) = \frac{1}{\tau} J_1(2\tau),$$



# Realistic shell model

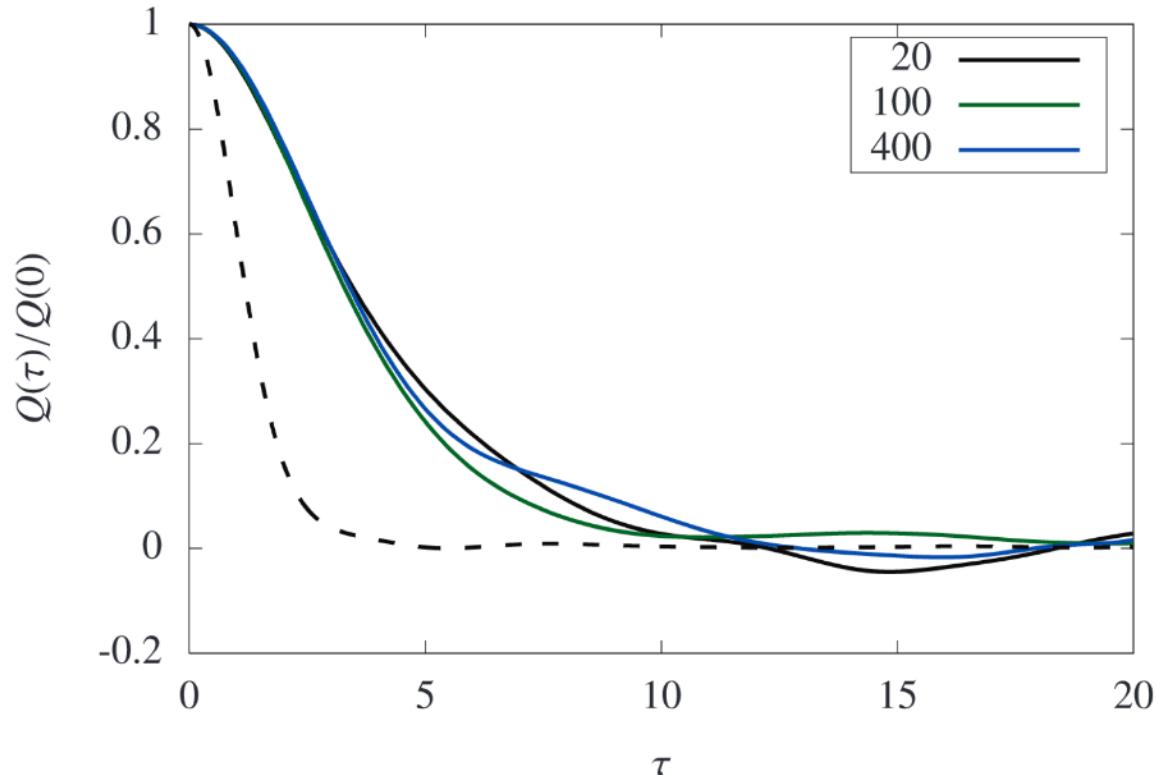
Realistic sd shell model  
USDB Hamiltonian  
Realistic quadrupole moment  
Spreading width  $\sim 10$  MeV



- Most states thermalize at zero
- Extreme states retain their average quadrupole moment
- Giant resonances
- Scars in chaotic dynamics

# Relaxation in realistic systems

Realistic sd shell model  
USDB Hamiltonian  
Realistic quadrupole moment  
Spreading width  $\sim 10$  MeV



Relaxation times in realistic systems  
is significantly longer.  
Extended lifetime of perturbation

A. Volya and V. Zelevinsky, Time-Dependent Relaxation of Observables  
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K. Ma, A. Volya, and K. Yang, *Eigenstate Thermalization and  
Disappearance of Quantum Many-Body Scar States in Weakly Interacting  
Fermion Systems*, Phys. Rev. B **106**, 214313 (2022).

# Conclusions

- Discussion of dynamics becomes central to exotic nuclear systems on the verge of stability.
- Exponential decay is not exact and is a result of delicate approximations.
- Poles of the scattering matrix differentiate the types of dynamics.
- The initial state (of real energy) matters, as do the reaction mechanism and memory effect.
- Experimental studies of correlations can be used to probe the state and its time evolution.
- There is strong competition between internal relaxation and decay.

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## Funding Acknowledgements:

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