Nuclear Shapes, Islands of Inversion, and all that . . .

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Outline

- The context; SM-CI
- The roots of quadrupole collectivity: Elliott's SU(3)
- The Islands of Inversion
- The meaning of nuclear shape

Basic elements of the SM-CI approach

- A valence space, computationally tractable, encompassing the targeted physics.
- An effective interaction for the valence space, that usually is expressed as a set of single particle energies and two body matrix elements (TBME)
- Shell Model codes to build and diagonalize the (huge dimensional) matrices involved, or other, approximated, mean field based methods (MCSM, DNO-SM, etc) to solve the secular problem.
- The present generation of SM codes includes, Antoine, Nushell, and K-shell, among others.

A game changer; the monopole multipole decomposition

The effective hamiltonian can be decomposed in two parts

- $H = \mathcal{H}_m + \mathcal{H}_M$. **Monopole and Multipole.**
- $\mathcal{H}_m$ **determines the spherical mean field and its evolution (aka shell evolution). It requires the explicit inclusion of 3B forces to comply with experiment.**
- $\mathcal{H}_M$ **contains the terms responsible for the correlations** *i.e.* **pairing, quadrupole, etc. This part is correctly given by the realistic two body effective interactions.**

Quadrupole Collectivity: Elliott's SU(3)

In 1958, using group theoretical methods, Elliott solved the problem of a quadrupole-quadrupole interaction in a full major Harmonic Oscillator (HO) shell.

$$H = H_0 + \chi(Q^{(2)} \cdot Q^{(2)})$$

He demonstrated that the most bound solution is the one with maximum deformation. On top of the ground state a perfect rotational band is built.

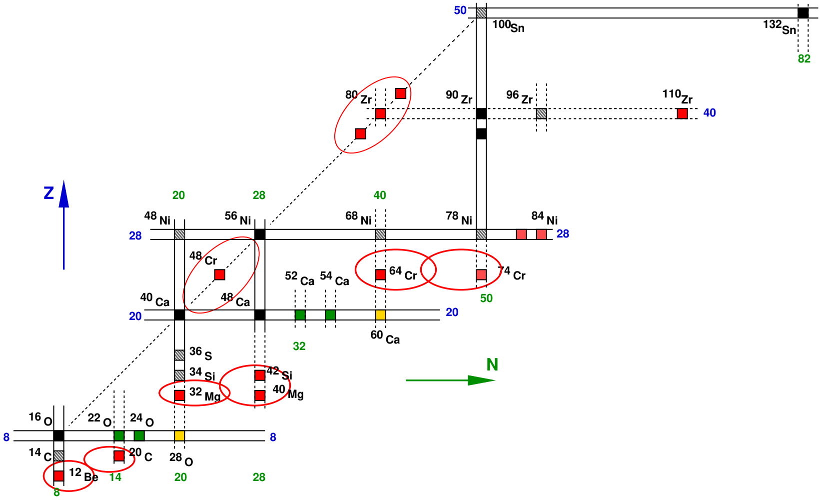
The problem of the description of the deformed nuclear rotors in the laboratory frame was formally solved.

Variants of Elliott's SU(3)

As it was realized later, there are variants of SU(3) which provide similar solutions. Pseudo-SU(3) applies when the relevant valence space contains the orbits of a major HO shell except the orbit with larger angular momentum (the so called intruder orbit) and they are quasi-degenerated. Quasi-SU(3), if the space comprises the intruder orbit and its quadrupole partners with $\Delta j = \Delta l = 2, 4, \ldots$.

Given that the quadrupole-quadrupole term is dominant in the effective NN interaction, Elliott's model and its variants, Pseudo-SU3 and Quasi-SU3, provide the heuristic toolkit to delineate the minimal valence spaces in which the quadrupole collectivity (deformation) can develop.

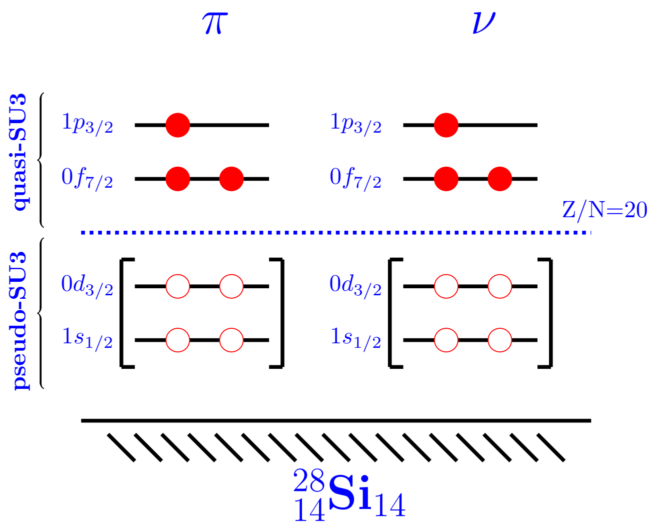
The nuclear landscape up to doubly magic ^{132}Sn



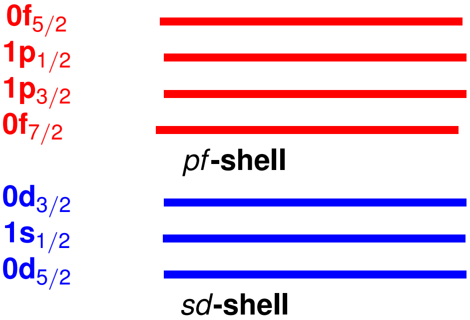
The Islands of Inversion (IoI) and SU(3)

- It is now common knowledge that the IoI's occur when the neutron magic gaps in the very neutron rich isotopes are quenched, provided the intruder configurations maximize their correlation energy, mostly of quadrupole type.
- Indeed, for that to happen, the orbits close to the Fermi level must pertain to some of the SU(3) variants mentioned above.

The IoI's at N/Z=20 and Nilsson-SU3

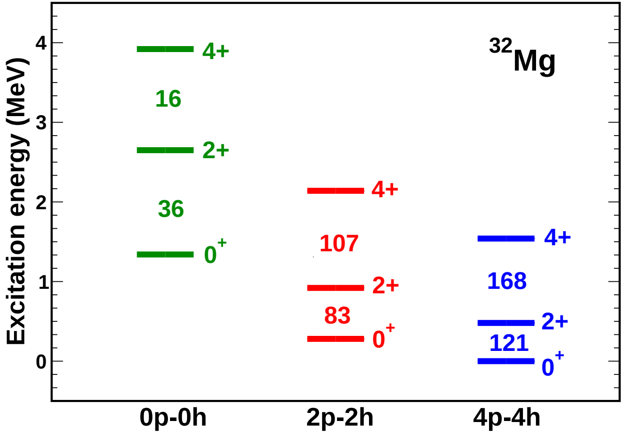


N=20 to N=28. The Valence Space; *sd-pf*



EFFECTIVE INTERACTION **SDPF-U-MIX**

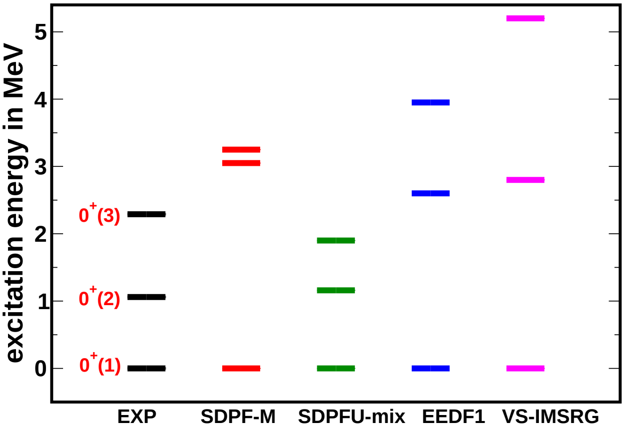
^{32}Mg; Semi-Magic, Deformed and Super-deformed configurations. B(E2)'s in e^2fm^4



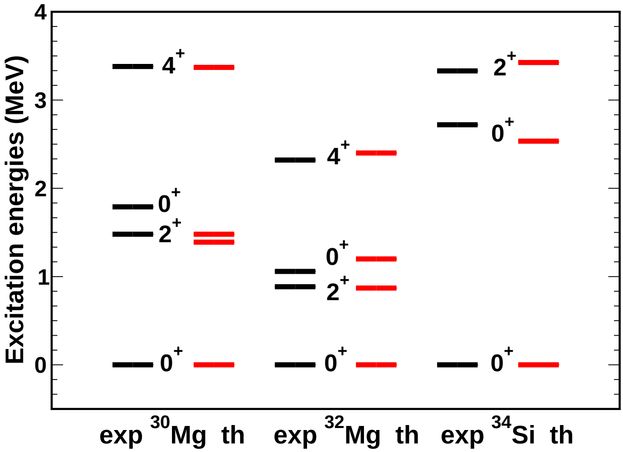
The remarkable structure of the three 0^+'s of ^{32}Mg

- They are rather weird; the ground state is 9% 0p-0h, 54% 2p-2h, and 35% 4p-4h, thus, it is a mixture of deformed and superdeformed prolate shapes and it makes sense to speak of shape mixing.
- However, the first excited 0^+ (K. Wimmer's state) has 33% 0p-0h, 12% 2p-2h, and 54% 4p-4h, a surprising hybrid of semi-magic and superdeformed, whose direct mixing matrix element is strictly zero. Clearly, it is not a case of shape mixing, could it be an example of *shape entanglement*?
- Finally the second excited 0^+ turns out to be an even mixture of semi-magic, deformed and super-deformed. Quite exotic as well ! More about ^{32}Mg later on.

To reproduce their excitation energies is a real challenge to theory



Shape Coexistence in ^{30}Mg and ^{34}Si
The Portal to the N=20 IoI



Nuclear shape: Quadrupole Invariants

- The only rigorous method to relate the intrinsic parameters to laboratory-frame observables is provided by the so-called quadrupole invariants Q^n of the second-rank quadrupole operator Q_2 introduced by Kumar.
- The calculation of β and γ requires the knowledge of the expectation values of the second- and third-order invariants defined, respectively, by $\hat{Q}^2 = \hat{Q} \cdot \hat{Q}$ and $\hat{Q}^3 = (\hat{Q} \times \hat{Q}) \cdot \hat{Q}$ (where $\hat{Q} \times \hat{Q}$ is the coupling of $\hat{Q}$ with itself to a second-rank operator).

Fluctuations in β and γ

Indeed, it is not very meaningful to assign effective values to β and γ without also studying their fluctuations. Our aim is to go beyond the extraction of effective values of these intrinsic parameters and obtain their variances.

With this goal, we calculate:

$$\sigma(\hat{Q}^2) = (\langle \hat{Q}^4 \rangle - \langle \hat{Q}^2 \rangle^2)^{1/2} \tag{1}$$

and

$$\sigma(\hat{Q}^3) = (\langle \hat{Q}^6 \rangle - \langle \hat{Q}^3 \rangle^2)^{1/2} \, . \tag{2}$$

See A. Poves, F. Nowacki and Y. Alhassid, PRC 101, 054307 (2020), for the details on how to compute them exactly in a shell model context.

Fluctuations in β and γ

The intrinsic quadrupole moment Q_0 and the effective (average) values of the Bohr-Mottelson shape parameters β and γ can be calculated from the expectation values of the second- and third-order invariants using

$$Q_0 = \sqrt{\frac{16\pi}{5}} \langle \hat{Q}^2 \rangle^{1/2} , \tag{3}$$

$$\beta = \frac{4\pi}{3 r_0^2} \frac{\langle \hat{Q}^2 \rangle^{1/2}}{A^{5/3}} , \tag{4}$$

with r_0=1.2 fm, and

$$\cos 3\gamma = -\sqrt{\frac{7}{2}} \frac{\langle \hat{Q}^3 \rangle}{\langle \hat{Q}^2 \rangle^{3/2}} \tag{5}$$

Fluctuations in β and γ

$$\frac{\Delta\beta}{\beta} = \frac{1}{2}\frac{\sigma\langle\hat{Q}^2\rangle}{\langle\hat{Q}^2\rangle} \; . \tag{6}$$

$$\frac{\sigma^2(\cos 3\gamma)}{(\overline{\cos 3\gamma})^2} = \frac{\sigma^2\langle\hat{Q}^3\rangle}{\langle\hat{Q}^3\rangle^2} + \frac{9}{4}\frac{\sigma^2\langle\hat{Q}^2\rangle}{\langle\hat{Q}^2\rangle^2} - 3\frac{\langle\hat{Q}^5\rangle - \langle\hat{Q}^3\rangle\langle\hat{Q}^2\rangle}{\langle\hat{Q}^3\rangle\langle\hat{Q}^2\rangle} \; . \tag{7}$$

Notice that the covariance term in (7) requires the knowledge of $\langle\hat{Q}^5\rangle$. The range of γ values at 1σ is given by

$$\cos^{-1}(\cos 3\gamma \pm \sigma(\cos 3\gamma)) \tag{8}$$

The meaning and limits of the nuclear shape

- **Do the intrinsic shape parameters β and γ survive in the laboratory frame?**
- **β: yes, although nuclei are most often β-soft**
- **γ: rather not. The fluctuations in γ amount to 20°- 30°. In some cases the oblate or prolate character survives. In others, both sectors of the β-γ sextant are equally probable**
- **β and γ only have small fluctuations when the nucleus approaches the SU3 limit. In heavy, well deformed, nuclei this is surely the case.**

Spherical nuclei in the laboratory frame?

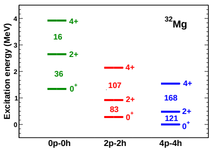
- It is a matter of how to define *spherical*, because, being strict, one should demand β=0, ergo $\langle \hat{Q}^2 \rangle$=0
- And this only happens for HO closed shells,
- Classical doubly magic nuclei do not comply with this condition
- ^{56}Ni β = 0.21$\pm$ 0.07 γ= 40.5° span 13° – 60°
- ^{48}Ca β = 0.15$\pm$ 0.05 γ= 33° span 0° – 60°
- With these huge fluctuations in β and γ, the concept of shape is meaningless
- Therefore, we are bound either to abandon or to redefine the label ”spherical nuclei”

The K-plots are a representation in the (β, γ) sextant of their average values and of the locus of their variances at 1σ.

^{32}Mg under the Kumar lens; the np-nh configurations

- **The 0p-0h (semi-magic) has $\frac{\sigma\langle\hat{Q}^2\rangle}{\langle\hat{Q}^2\rangle}$=0.48 and β=0.20$\pm$0.05, $\frac{\sigma\langle\hat{Q}^3\rangle}{\langle\hat{Q}^3\rangle}$=0.71, γ=16° with a spread 0° – 24° at 1σ**
- **The 2p-2h (normal deformed) has $\frac{\sigma\langle\hat{Q}^2\rangle}{\langle\hat{Q}^2\rangle}$=0.26 and β=0.48$\pm$0.06, $\frac{\sigma\langle\hat{Q}^3\rangle}{\langle\hat{Q}^3\rangle}$=0.86, γ=20° with a spread 9° – 27° at 1σ**
- **The 4p-4h (superdeformed) has $\frac{\sigma\langle\hat{Q}^2\rangle}{\langle\hat{Q}^2\rangle}$=0.21 and β=0.66$\pm$0.07, $\frac{\sigma\langle\hat{Q}^3\rangle}{\langle\hat{Q}^3\rangle}$=0.40 , γ=15° with a spread 7° – 21° at 1σ**

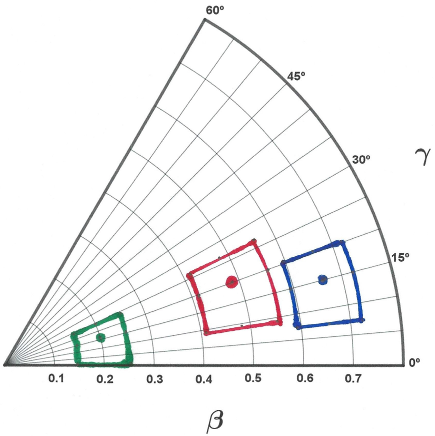
K-plots for ^{32}Mg; the np-nh configulations



$\mathbf{0}^+_{0p-0h}$

$\mathbf{0}^+_{2p-2h}$

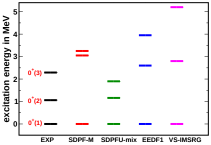
$\mathbf{0}^+_{4p-4h}$



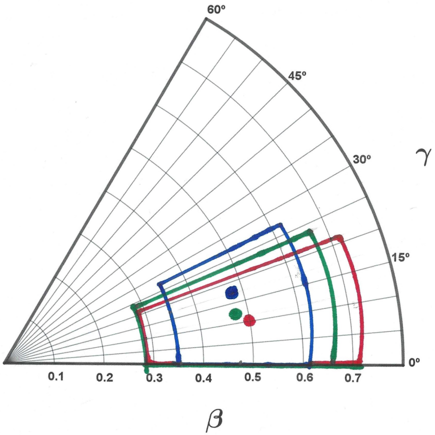
^{32}Mg under the Kumar lens; the physical states

- The ground state 0_1^+ has $\frac{\sigma\langle\hat{Q}^2\rangle}{\langle\hat{Q}^2\rangle}$=0.62 and β=0.48$\pm$0.13, $\frac{\sigma\langle\hat{Q}^3\rangle}{\langle\hat{Q}^3\rangle}$=1.16, γ=17° with a spread 0° – 27° at 1σ
- The excited 0_2^+ has $\frac{\sigma\langle\hat{Q}^2\rangle}{\langle\hat{Q}^2\rangle}$=0.88 and β=0.50$\pm$0.21, $\frac{\sigma\langle\hat{Q}^3\rangle}{\langle\hat{Q}^3\rangle}$=1.03, γ=10° with a spread 0° – 21° at 1σ
- The excited 0_3^+ has $\frac{\sigma\langle\hat{Q}^2\rangle}{\langle\hat{Q}^2\rangle}$=0.77 and β=0.47$\pm$0.18, $\frac{\sigma\langle\hat{Q}^3\rangle}{\langle\hat{Q}^3\rangle}$=1.15, γ=12° with a spread 0° – 23° at 1σ

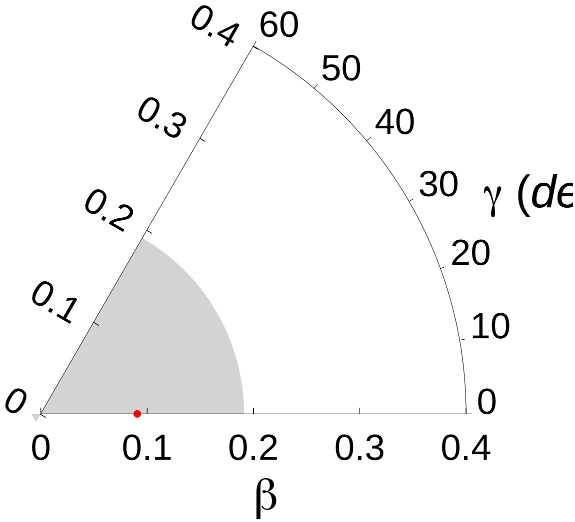
K-plots for ^{32}Mg; the three lowest 0^+ states



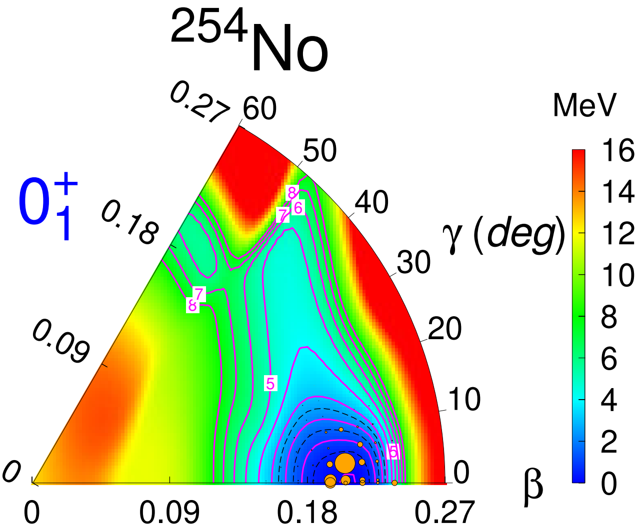
0_1^+
0_2^+
0_3^+



Doubly magic ^{40}Ca



Superheavy ^{254}No; D. Dao and F. Nowacki, DNO-SM



Finale

Thanks for your attention !

Work done in collaboration with Y. Alhassid, E. Caurier, S. M. Lenzi, F. Nowacki, K. Sieja and A. P. Zuker

More about these and other related topics in:
The neutron rich edge of the nuclear landscape,
F. Nowacki, A. Obertelli, and A. Poves
PPNP 120 (2021) 103866