Theory for Knockout Reactions

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Introduction

experimental study of nuclear structure

- knockout reactions at ’high’ energies
  - knockout of nucleons
    → single-particle structure, spectroscopic factors
  - knockout of pairs of nucleons
    → NN correlations, momentum distributions
  - knockout of clusters
    → many-body correlations, cluster degrees of freedom

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  - direct and indirect kinematics
  - quasi-free scattering conditions
→ ’simple’ theoretical description

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Is everything OK with the theory? Not really . . .

Knockout Reactions I

- different types of measurements
  - knockout of nucleon or cluster x from target $T = c + x$ with proton beam and detection of p and x in final state

$$\boxed{p + T \to p + x + c}$$

'direct' kinematics, exclusive cross section

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- theoretical description: theory of direct reactions

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  - inelastic breakup = stripping σ_{str}: excitation of target
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- high-energy reactions: eikonal description
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- main issues:
  - justification of cross section formulas, approximations
  - description of interactions (optical potentials, nucleon-nucleon collisions, ...)
  - higher-order processes (core destruction, ...)
  - nuclear structure input

Cross Sections I

- 'standard' expressions (see, e.g., J.A. Tostevin, NPA 682 (2001) 320c)
  - cross section for stripping of single nucleon *x*

$$\sigma_{\rm str} = \frac{1}{2j+1}\sum_{m}\int d^{2}b_{xT}\ \langle\phi^{*}_{jm}\left|[1-|S_{xT}(\vec{b}_{xT})|^{2}]|S_{cT}(\vec{b}_{cT})|^{2}\right|\phi_{jm}\rangle$$

  - cross section for diffractive dissociation

$$\sigma_{\rm dd} = \frac{1}{2j+1}\sum_{m}\int d^{2}b_{xT}\ \left(\langle\phi_{jm}\left||1-S_{xT}S_{cT}|^{2}\right|\phi_{jm}\rangle - \sum_{m'}\left|\langle\phi_{jm'}\,|1-S_{xT}S_{cT}|\,\phi_{jm}\rangle\right|^{2}\right)$$

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with

  - single-nucleon wave functions $\phi_{jm}(\vec{r})$
  - eikonal 'S-matrices' $S_{ij}(\vec{b}_{ij}) = \exp(i\chi_{ij}) = \exp\left(-\frac{i}{\hbar v_{ij}} \int_{-\infty}^{\infty} dz \, U_{ij}(\vec{b}_{ij}, z)\right)$
  - optical potentials U_{ij}
  - radius $\vec{r} = (\vec{\rho}, z)$, impact parameter $b_{cT} = |\vec{\rho} - \vec{b}_{xT}|$

Cross Sections II

- justification of expressions for cross section
  $\Rightarrow$ reconsider derivation in formal theory of direct reactions
  (see, e.g., M.S. Hussein & K.W. McVoy, NPA 445 (1985) 124, . . .)
  - use of completeness relations, spectator approximation, . . .
    $\rightarrow$ not discussed here

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- description of interaction in eikonal phase factor χ_{ij}
  - parametrized/systematic nucleon-nucleus optical potentials
    (e.g., global Dirac potentials, E.D. Cooper et al., PRC 80 (2009) 034605)
  - single/double-folding potentials with effective interactions (M3Y, ...)
  - 't$\rho_1\rho_2$' approximation → nucleon-nucleon interactions

$$U_{12}(E,\vec{r}) = \int d^3r' \; t_{NN} \; \rho_1(\vec{r}') \; \rho_2(\vec{r}-\vec{r}')$$

with NN scattering matrix element t_{NN} and densities ρ_1, ρ_2 (parameterization?),
Pauli correction for in-medium scattering?, isospin dependence?

  - differences to be explored

Cross Sections III

- parameterization of NN scattering amplitude
  ⇒ often used form using optical theorem (E. Kujawski et al., PRL 21 (1968) 583, ...)

$$\boxed{f_{NN} = \frac{k}{4\pi}\sigma_{NN}(i+\alpha)\exp(-\beta q^2) \quad \Rightarrow \quad t_{NN} = -\frac{2\pi\hbar^2}{\mu} f_{NN}}$$

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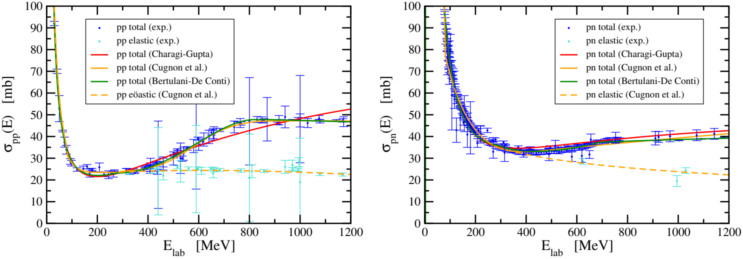
  - total NN cross section $\sigma_{NN} \rightarrow$ different parameterizations
    (exp. data: R. L. Workman et al. (Particle Data Group), Prog. Theo. Exp. Phys. 2022, 083C01)



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  - total NN cross section σ_{NN} → different parameterizations
  - parameters α, β and momentum transfer $\vec{q}$ for angular dependence
    → sometimes inconsistent usage
    → identical angular dependence of real and imaginary parts
    → α, β not independent,
    relation of elastic and total cross sections
    (see, e.g., W. Horiuchi et al., PRC 75 (2007) 044607)

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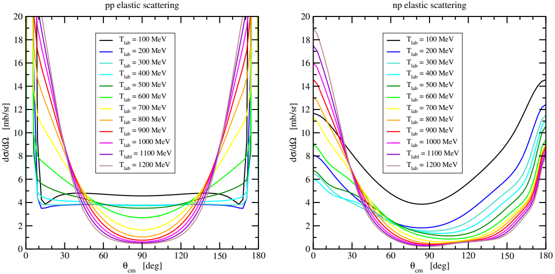

from SAID partial-wave analysis
(gwdac.phys.gwu.edu)

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  - modification of stripping cross section with additional factor
    (C.A. Bertulani, PLB 842 (2023) 138250)

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$$\text{with} \qquad \langle |S_{xc}|^2 \rangle = \frac{1}{\sigma_{NN}^{el}} \int d\Omega \, \frac{d\sigma_{NN}^{el}(\theta)}{d\Omega} |S_{xc}(\vec{b}_{xc}(\theta,\phi))|^2$$

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reaction	E_{beam} [MeV/nucl.]	$S_p(S_n)$ [MeV]	σ_{str} [mb]	$\sigma_{\mathrm{str}}^{\mathrm{mod}}$ [mb]	% change
^{9}B(^{7}Li,^{6}He)	80	9.98	20.49	19.01	-7.22
^{9}B(^{7}Li,^{6}Li)	120	7.25	24.23	22.16	-8.54
^{12}C(^{8}B,^{7}Be)	285	0.137	42.49	39.02	-8.17
^{12}C(^{9}C,^{8}B)	78	1.3	40.14	36.86	-8.17
^{12}C(^{9}Li,^{8}Li)	100	4.06	40.32	37.00	-8.23
^{9}Be(^{10}Be,^{9}Li)	80	19.64	35.33	32.47	-8.09
^{9}Be(^{10}Be,^{9}Be)	120	6.812	77.62	70.68	-8.94
^{9}Be(^{10}C,^{9}C)	120	21.28	44.57	40.50	-9.31
^{12}C(^{12}C,^{11}B)	250	15.95	64.68	58.70	-9.55
^{12}C(^{12}C,^{11}C)	250	18.72	74.16	67.10	-9.52
^{12}C(^{14}O,^{13}N)	305	1.531	37.45	33.99	-9.22
^{9}Be(^{14}O,^{13}O)	53	3.234	25.57	23.32	-8.80
^{12}C(^{16}O,^{15}N)	2100	22.04	46.90	42.48	-9.42
^{12}C(^{18}O,^{15}O)	2100	22.04	44.46	40.26	-9.45

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  - modified derivation of cross section with non-local effective densities

    (M. Gómez-Ramos, J. Gómez-Camacho, A.M. Moro, PLB 847 (2023) 138284)

    - reduction of cross section by 10 - 50%
    - strong dependence on binding energy/isospin

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  - density distributions
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    $\Rightarrow$ only selected nucleon states

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  - input from ab-initio calculations ?
  - wave functions, density distributions from energy density functionals (EDF)
    - development of new relativistic EDF
      (in collaboration with S. Shlomo, Texas A&M University, paper in preparation)
      $\Rightarrow$ improved description of nuclei

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  - similar to CDXS+ code for Coulomb dissociation
  - work in progress

Thank You for Your Attention!

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