The thermal model: current status, limitations and future

Hadron yields in central nucleus-nucleus collisions and the QCD phase diagram

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- The statistical (thermal) model and the thermal fits
- Thermal fits and the QCD phase diagram
- The model ingredients (syst.uncert.) and predictions on (hyper)nuclei
- Conclusions

Largely based on: Andronic, Braun-Munzinger, Redlich, Stachel, Nature 561 (2018) 321 also known as: statistical / hadron resonance gas / statistical hadronization model

...is in a way the simplest model

the analysis of hadron yields within the thermal model provides a "snapshot" of a nucleus-nucleus collision at chemical freeze-out
(the earliest in the collision timeline we can look with hadronic observables)
test hypothesis of hadron abundancies in equilibrium

...but the devil is in the details ...one needs:

- a complete hadron spectrum (all species of hadrons, see PDG, extra states?)
- canonical approach at low energies (and smaller systems)
- to understand the data well (Ex.: control fractions from weak decays)

grand canonical partition function for specie (hadron) i :

$$\ln Z_i = \frac{V g_i}{2\pi^2} \int_0^\infty \pm p^2 \mathrm{d}p \ln[1 \pm \exp(-(E_i - \mu_i)/T)]$$

$g_i = (2J_i + 1)$ spin degeneracy factor; T temperature;
$E_i = \sqrt{p^2 + m_i^2}$ total energy; (+) for fermions (–) for bosons
$\mu_i = \mu_B B_i + \mu_{I_3} I_{3i} + \mu_S S_i + \mu_C C_i$ chemical potentials

μ ensure conservation (on average) of quantum numbers, fixed by "initial conditions"

i) isospin: $\sum_i n_i I_{3i} / \sum_i n_i B_i = I_3^{tot}/N_B^{tot}, \quad N_B^{tot} \sim \mu_B$
I_3^{tot}, N_B^{tot} isospin and baryon number of the system (=0 at high energies)

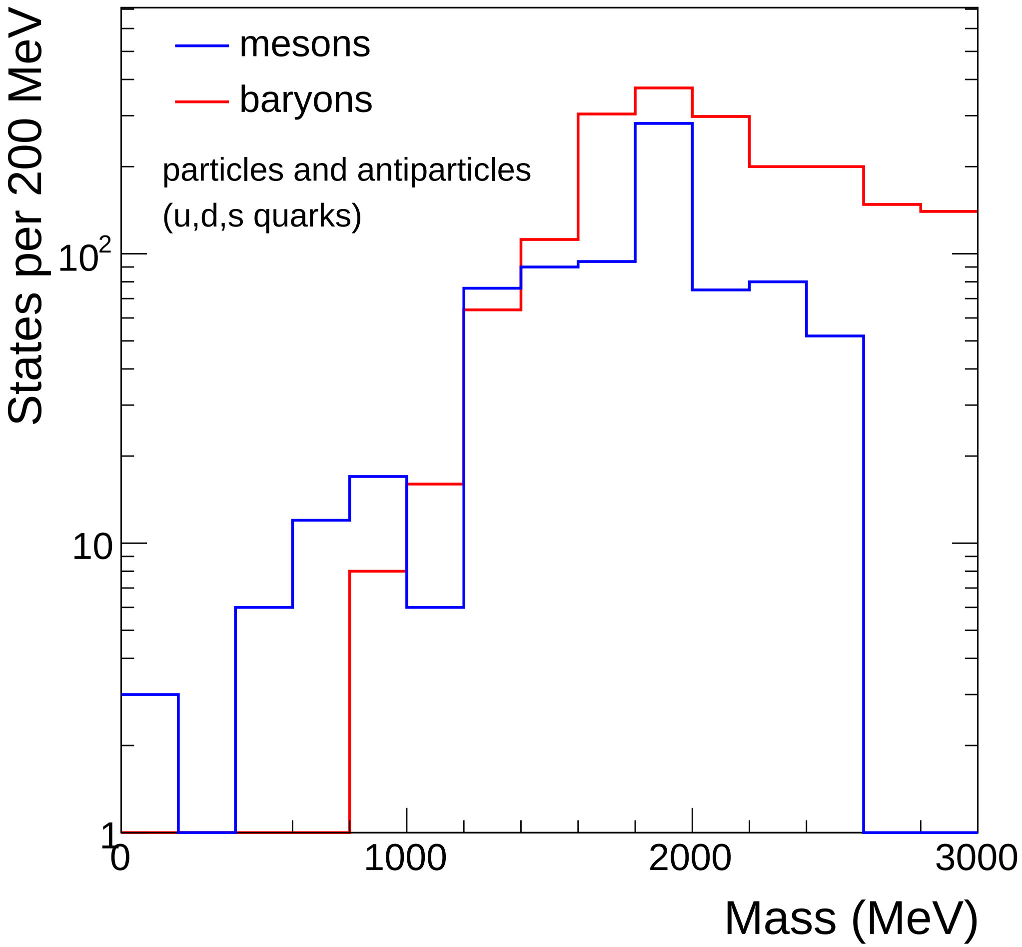
ii) strangeness: $\sum_i n_i S_i = 0$

iii) charm: $\sum_i n_i C_i = 0$.

Model input: hadron spectrum

...embodies low-energy QCD ...*vacuum masses*

well-known for $m < 2$ GeV; many confirmed states above 2 GeV, still incomplete



for high m, BR not well known, but can be reasonably guessed

T found to be robust in fits with spectrum truncated above 1.8 GeV

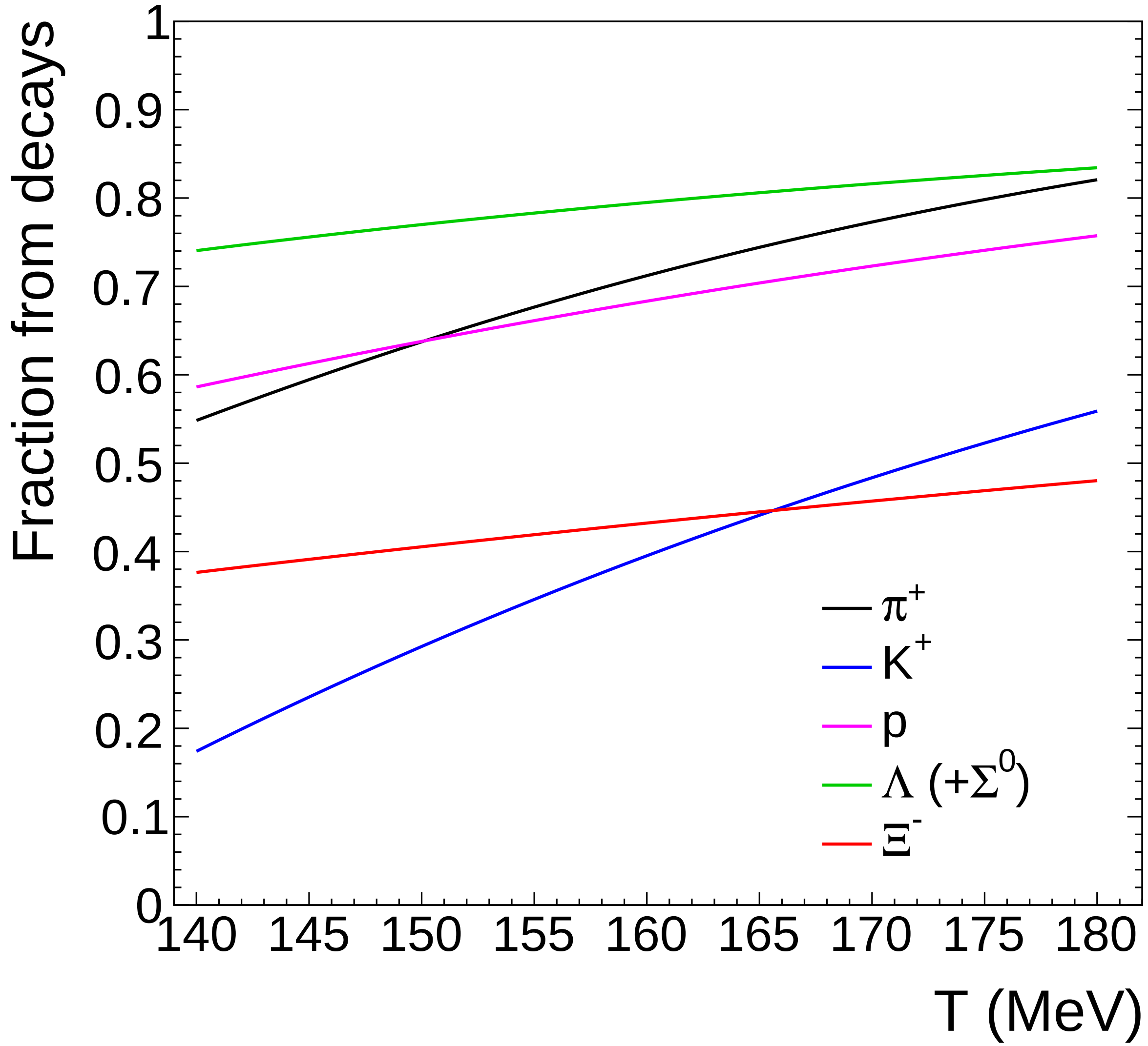
$$\rho(m) = c \cdot m^{-a} \exp\left(m/T_H\right)$$

$T_H \simeq 180$ MeV (max T for hadrons)

$(2J+1)$ counted in

Decays (feed-down)

(almost all) hadrons are subject to strong and electromagnetic decays



$\Delta \to p(n) + \pi$, $\rho \to \pi + \pi$

$\Sigma^0 \to \Lambda + \gamma$

weak decays can be treated as well ...to account for the exact experimental situation

contribution of resonances is significant (and particle-dependent)

(plot for μ_B=0)

Considering widths of resonances

$$n_i = N_i/V = -\frac{T}{V}\frac{\partial \ln Z_i}{\partial \mu} = \frac{g_i}{2\pi^2}\int_0^\infty \frac{p^2 \mathrm{d}p}{\exp[(E_i - \mu_i)/T] \pm 1}$$

$$n_i = \frac{g_i}{2\pi^2}\frac{1}{N_{BW}}\int_{M_{thr}}^\infty \mathrm{d}m \int_0^\infty \frac{\Gamma_i^2}{(m - m_i)^2 + \Gamma_i^2/4} \cdot \frac{p^2 \mathrm{d}p}{\exp[(E_i^m - \mu_i)/T] \pm 1}$$

M_{thr} threshold mass for the decay channel.

Example: for $\Delta^{++} \rightarrow p + \pi^+$, M_{thr}=1.068 GeV ($m_{\Delta^{++}}$=1.232 GeV)

Important mainly at "low" temperatures ($T \lesssim$150 MeV)

Hadron densities

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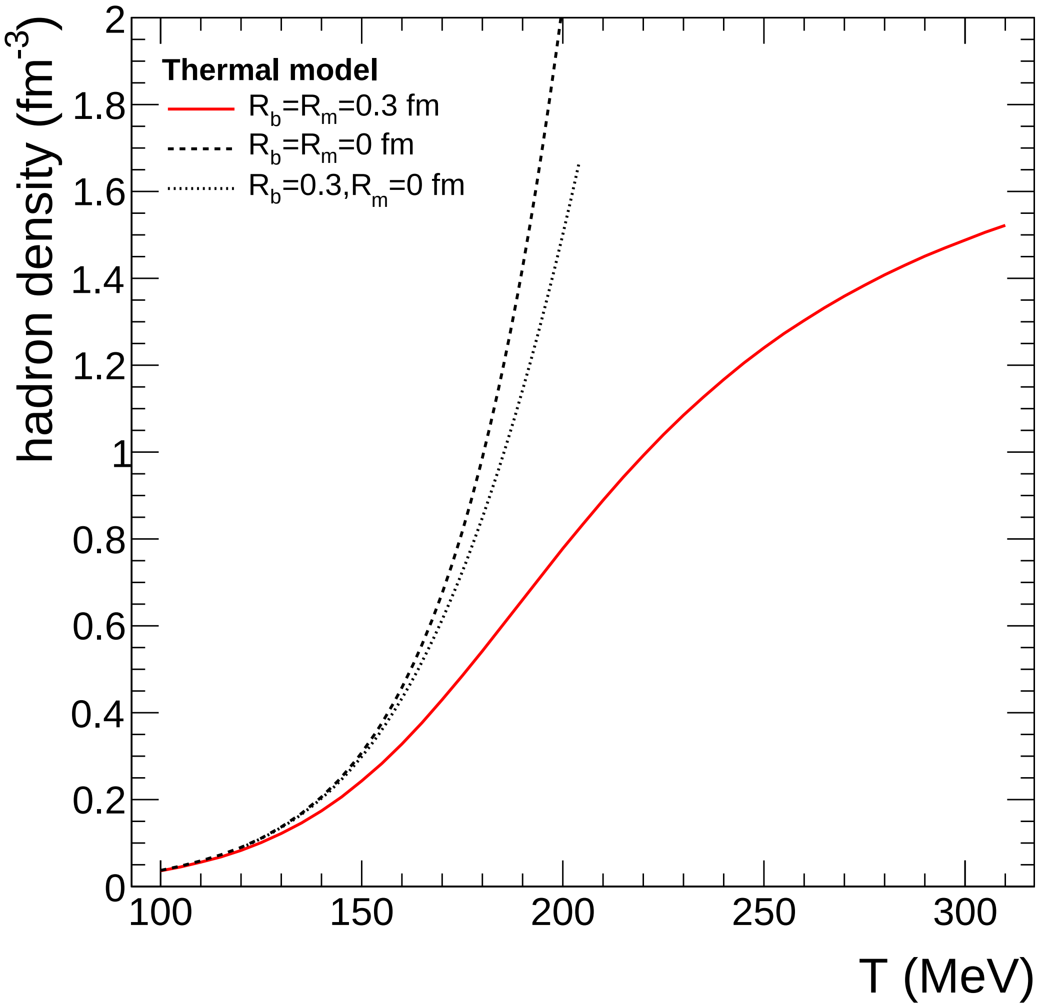
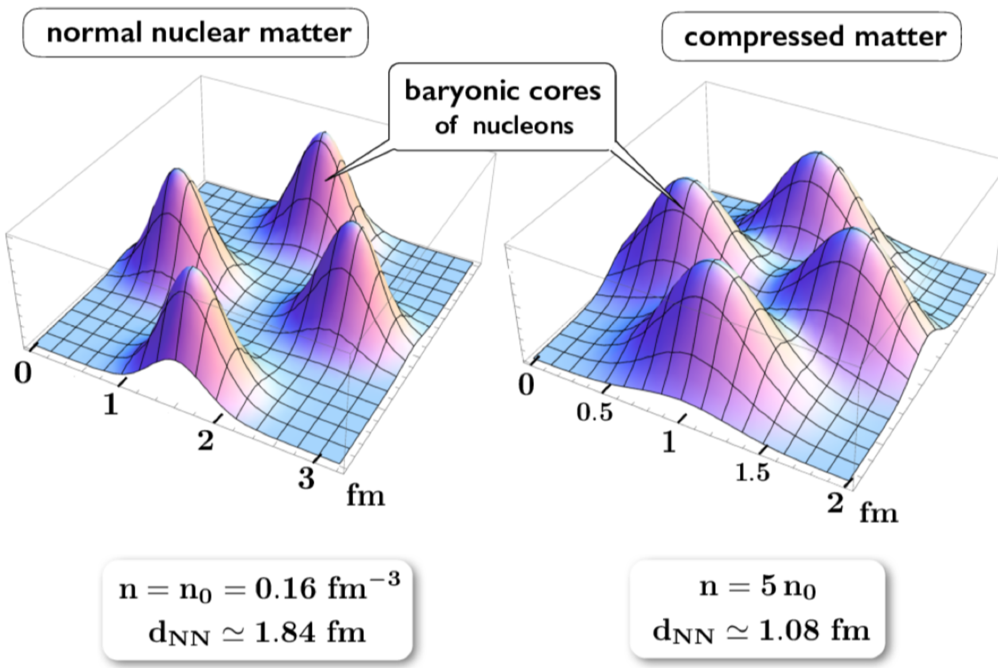




Weise, arXiv:1811.09682

(baryons: gaussians, r=0.5 fm)

"hadron gas": a dense system (also nuclear matter is rather a liquid than a gas)

(the usual case is $R_{baryon} = R_{meson} = 0.3$ fm ...hard-sphere repulsion)

Air at NTP: intermolecule distance $\simeq$50 $\times$ molecule size

Canonical correction ("canonical suppression")

needed whenever the abundance of hadrons with a given quantum number is very small ...so that one needs to enforce exact quantum-number conservation

in AA collisions:

strangeness at low energies

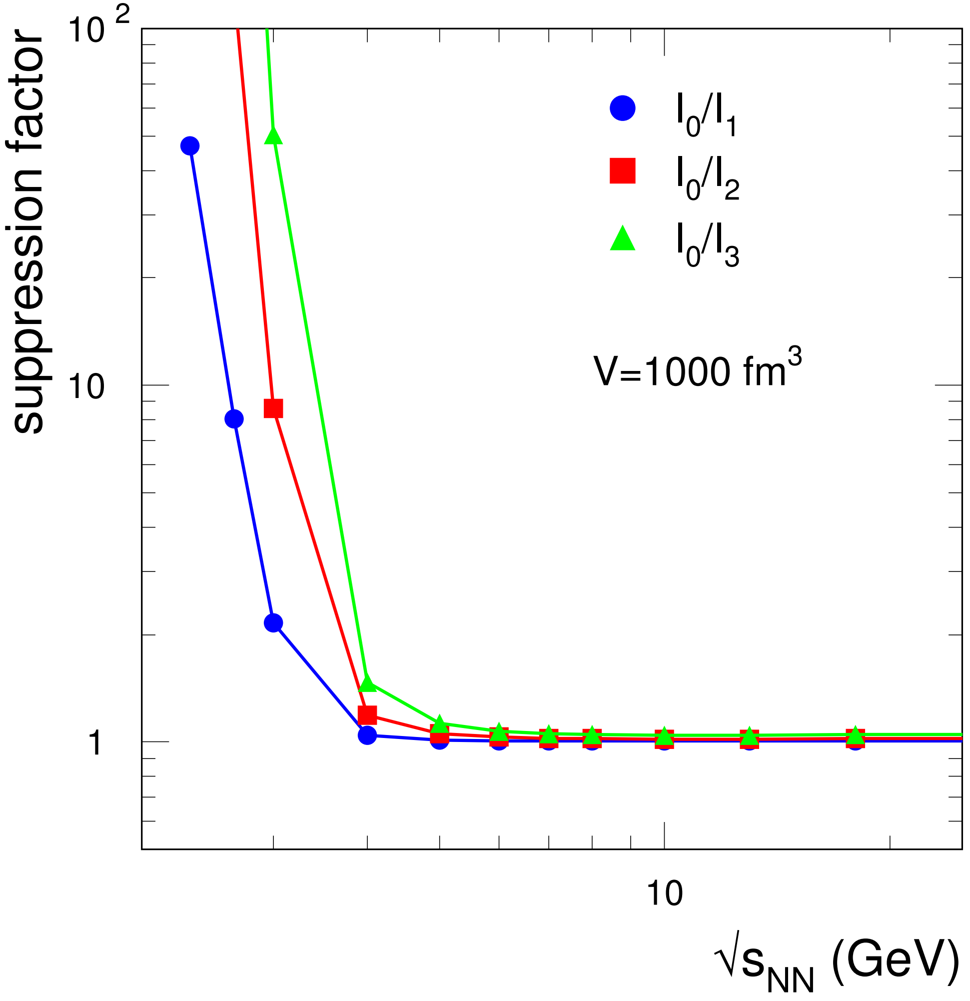
$$n^C_{i,S} = n^{GC}_{i,S} \cdot \frac{I_s(N_S)}{I_0(N_S)}$$

$N_S = V_c \cdot \sum S \cdot n_{i,S}$, total amount of strangeness-carrying hadrons (part.+antipart.)

$$n^C_{K,\Lambda} = n^{GC}_{K,\Lambda} \cdot \frac{I_1(N_S)}{I_0(N_S)},$$

$$n^C_{\phi} = n^{GC}_{\phi}$$

...negligible for $\sqrt{s_{NN}}$ >5 GeV



for V_C=1000 fm^3 (2005) ...largish

Strangeness suppression factor, γ_s

...a non-thermal fit parameter, to check possible non-thermal production of strangeness

for a hadron carrying "absolute" strangeness $s = |S - \bar{S}|$: $n_i \rightarrow n_i \gamma_s^s$

Examples: $K^{\pm}$ $(u\bar{s},\ \bar{u}s)$: $n_K \gamma_s$, Λ (uds): $n_\Lambda \gamma_s$,

$\Xi(dss)$: $n_\Xi \gamma_s^2$, $\Omega(sss)$: $n_\Omega \gamma_s^3$, $\phi(s\bar{s})$: $n_\phi \gamma_s^2$

in principle, usage of γ_s is to be avoided if one tests the basic thermal model

even as some models employ it ($\Rightarrow \gamma_s = 0.6 - 0.8$), all agree that it is not needed at RHIC, LHC energies (for central collisions)

here (central AA collisions) we fix γ_s=1

Thermal fits of hadron abundances

$$n_i = N_i/V = -\frac{T}{V}\frac{\partial \ln Z_i}{\partial \mu} = \frac{g_i}{2\pi^2}\int_0^\infty \frac{p^2 \mathrm{d}p}{\exp[(E_i-\mu_i)/T] \pm 1}$$

Latest PDG hadron mass spectrum ...quasi-complete up to m=2 GeV;
our code: 555 species (including fragments, charm and bottom hadrons)

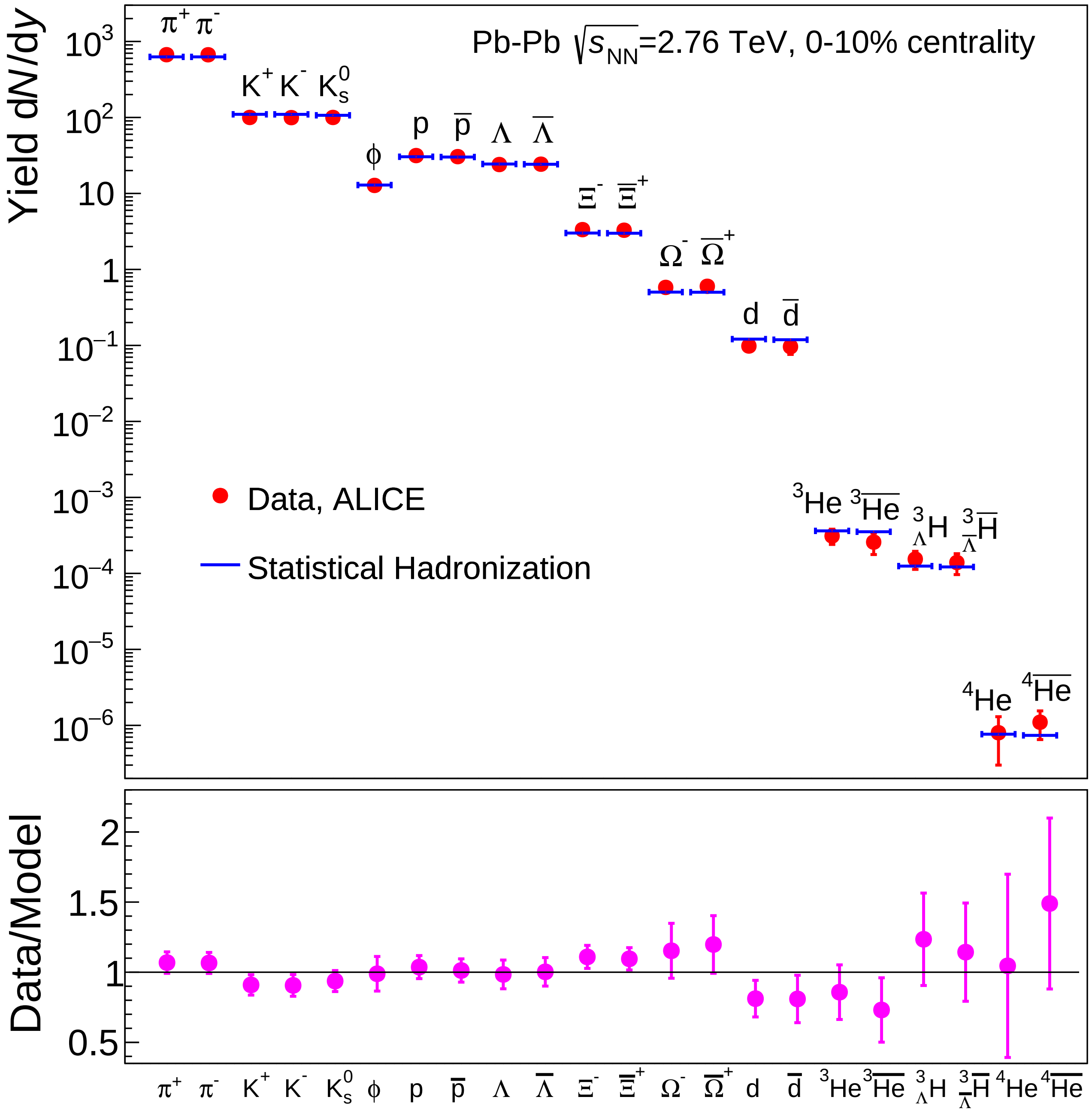
for resonances, the width is considered in calculations

canonical treatment whenever needed (small abundances)

Minimize: $\chi^2 = \sum_i \frac{(N_i^{exp} - N_i^{therm})^2}{\sigma_i^2}$

N_i hadron yield, σ_i experimental uncertainty (stat.+syst.)

$\Rightarrow$ (T, μ_B, V) *...tests chemical freeze-out* (chemical equilibrium)



matter and antimatter produced in equal amounts

$T_{CF} = 156.6 \pm 1.7$ MeV
$\mu_B = 0.7 \pm 3.8$ MeV
$V_{\Delta y=1} = 4175 \pm 380$ fm^3

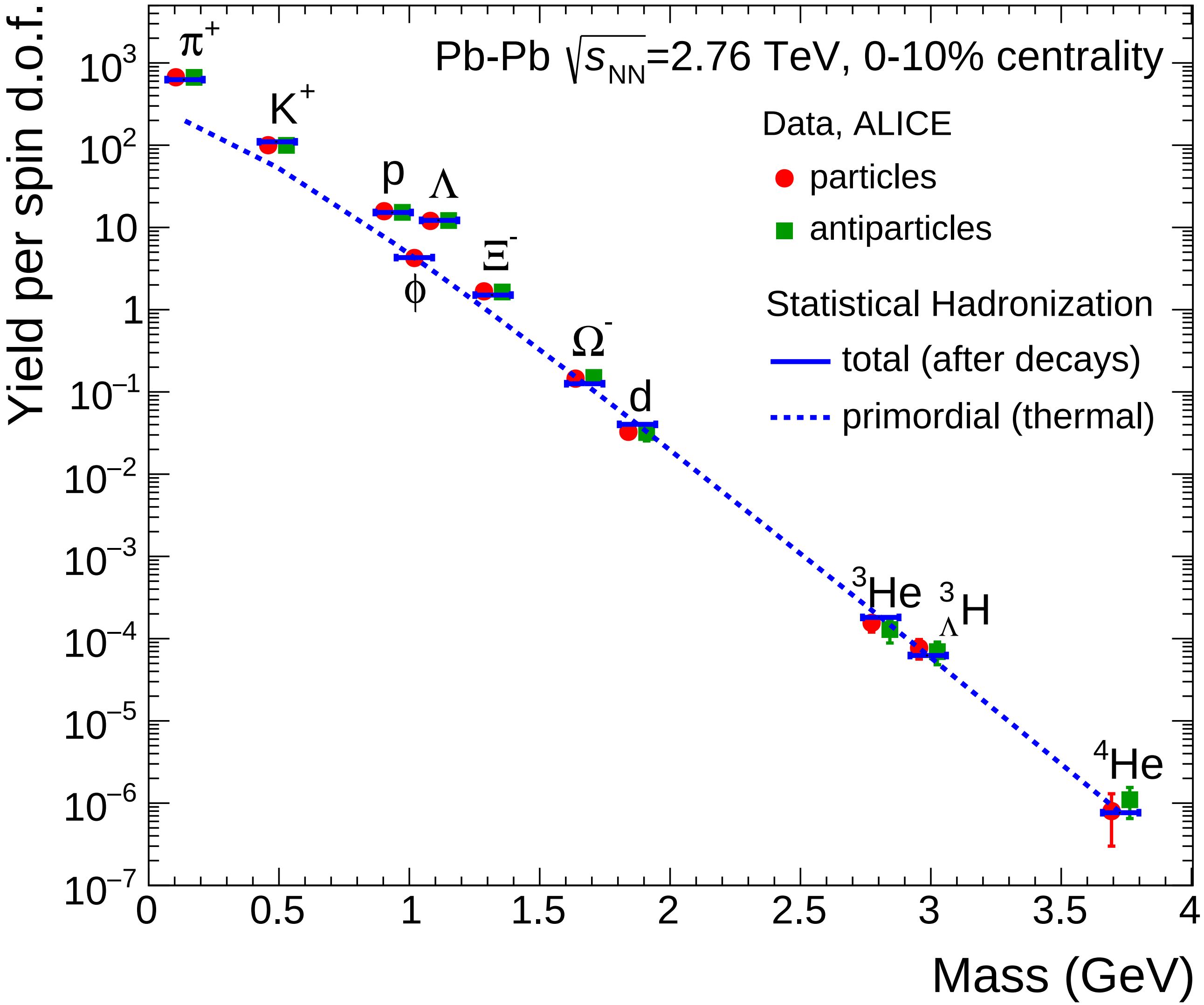
$\chi^2/N_{df} = 16.7/19$

S-matrix treatment

remarkably, loosely-bound objects are also well described
($^3_\Lambda$H with 25% B.R.)

hadronization as bags of quarks and gluons?

Model uncertainties 1. Hadron spectrum



contribution of resonances is significant (and particle-dependent)

Fit of ϕ, Ω, d, ^{3}He, ${}^3_\Lambda$H, ^{4}He:

$T_{CF} = 156 \pm 2.5$ MeV

$(\chi^2/N_{df} = 7.4/8)$

Fit of nuclei (d, ^{3}He, ^{4}He):

$T_{CF} = 159 \pm 5$ MeV

3-4 MeV upper bound of systematic uncertainty due to hadron spectrum

Model uncertainties 2. Interactions

hadron eigenvolumes ...to mimick interactions (beyond low-density, Dashen-Ma)

we consider that $R_{meson} = 0.3, R_{baryon} = 0.3$ *fm is a reasonable case*

point-like hadrons lead to same T, but volume larger by 20-25%

an extreme case, $R_{meson} = 0, R_{baryon} = 0.3$ fm leads to
$T = 161.0 \pm 2.0$ MeV, $\mu_B = 0$ *fixed*, $V = 3470 \pm 280$ fm^3

NB: in this case, the result is rather sensitive on the set of hadrons in the fit
for instance, using hadrons up to Ω, cannot constrain T (unphysically large)
Vovchenko, Stöcker (et al.), JPG 44 (2017) 055103, arXiv:1606.06350

...and anything else can be imagined, see (R dependent on mass & strangeness)
Alba, Vovchenko, Gorenstein, Stöcker, NPA 974 (2018) 22, etc.

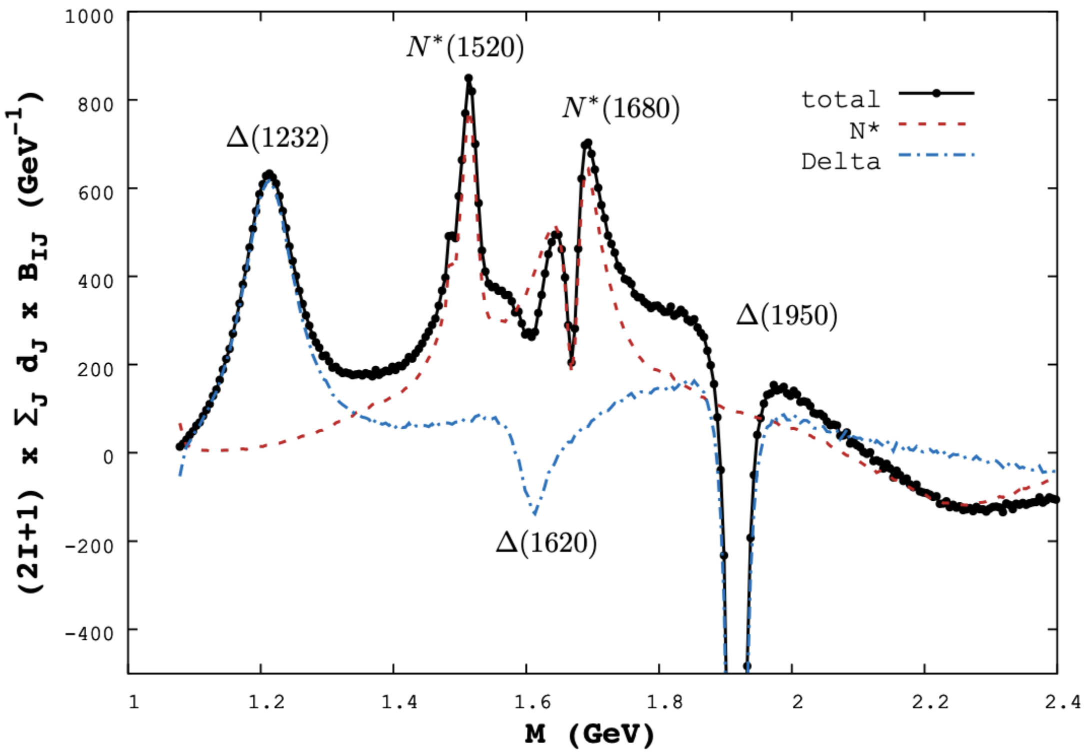
Energy-dependent Breit-Wigner resonance widths:
Vovchenko, Gorenstein, Stöcker, PRC 98 (2018) 034906

non-strange baryon sector treated in S-matrix formalism (P.M.Lo):
πN scattering phase shifts, *including non-resonant contributions*

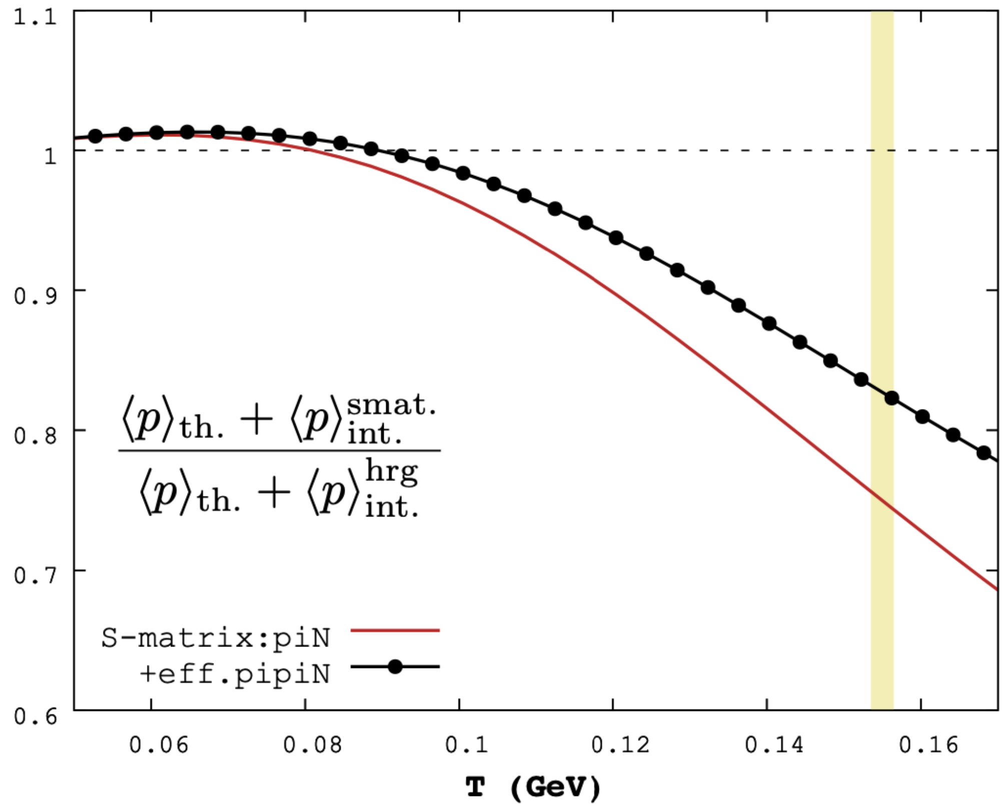
PLB 792 (2019) 304

...for now only at the LHC ($\mu_B \simeq$0)

GWU/SAID PWA $\rightarrow$ effective spectral f.



correction of proton dens.



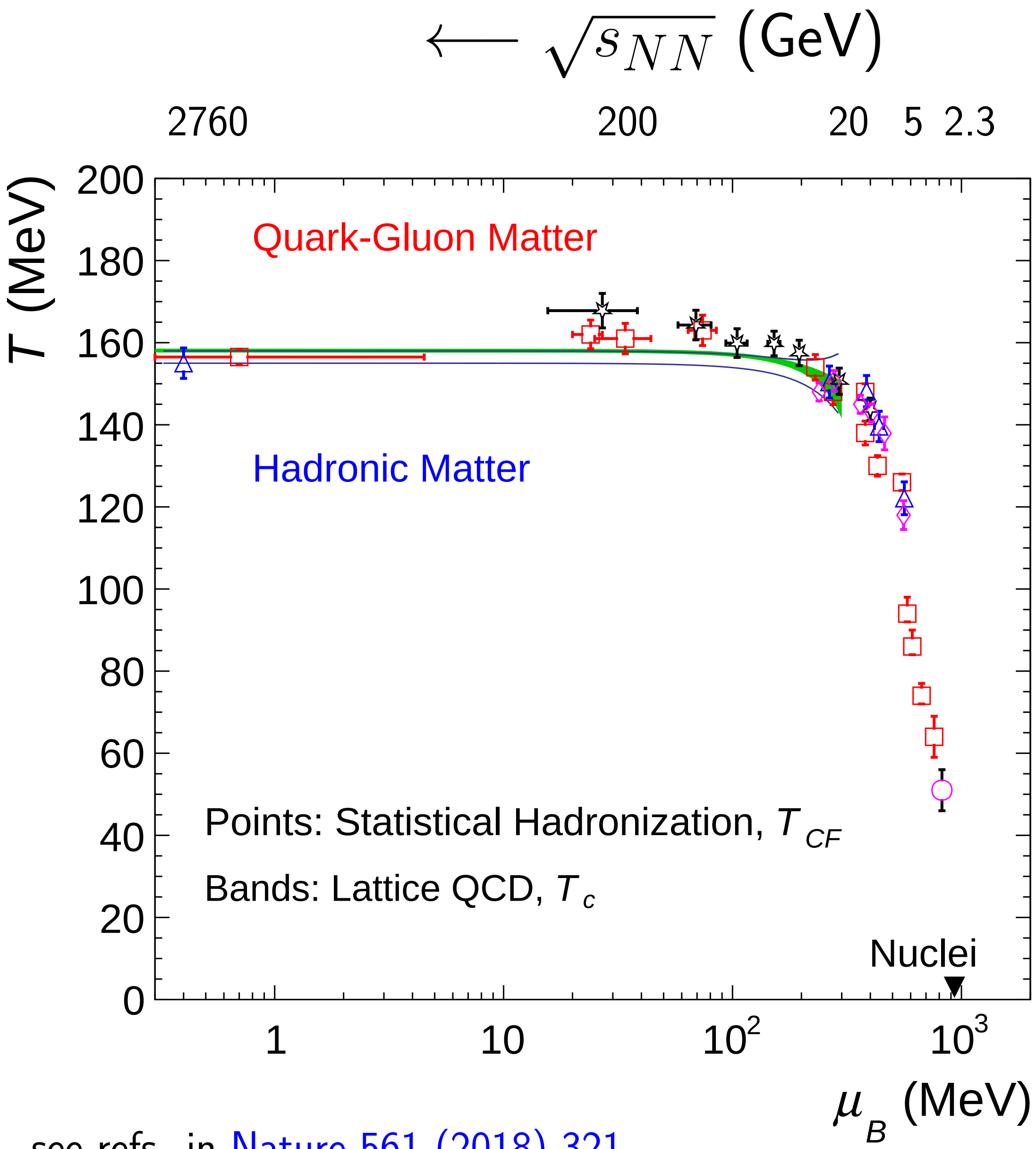
$\pi\pi$N: effective, from matching to χ_{BQ} in LQCD solved the so-called "proton puzzle" (too many protons in the statistical model)

for T=156.5 MeV, proton yield decreased by 17% compared to point-like

still missing: strange baryon sector (∼7% of protons from Λ^*, Σ^*, T-dep.)
PRC 98 (2018) 044910

NB: presence of resonances implies interaction
(this is why moderate $R = 0.3$ fm is a reasonable choice)

The phase diagram of QCD



at LHC, remarkable "coincidence" with Lattice QCD results

at LHC ($\mu_B \simeq 0$): purely-produced (anti)matter ($m = E/c^2$), as in the Early Universe

$\mu_B > 0$: more matter, from "remnants" of the colliding nuclei

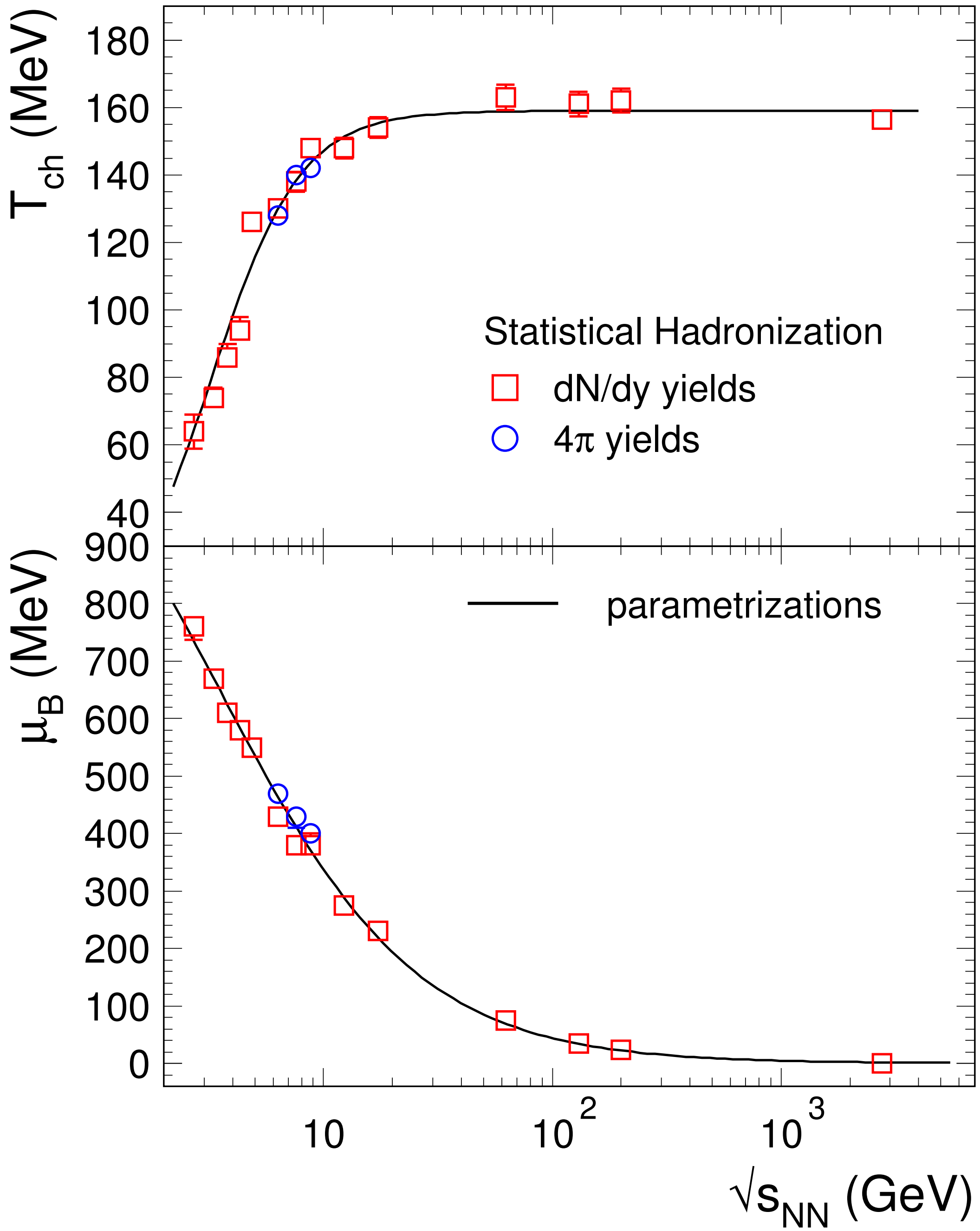
$\mu_B \gtrsim 400$ MeV: *the critical point awaiting discovery*

(RHIC BES / FAIR)

see refs. in Nature 561 (2018) 321

points: independent analyses of same data → "model/code uncert." are small

Energy dependence of T, μ_B (central collisions)



thermal fits exhibit a limiting temperature:

$$T_{lim} = 158.4 \pm 1.4 \text{ MeV}$$

$$T_{CF} = T_{lim} \frac{1}{1+\exp(2.60-\ln(\sqrt{s_{NN}}(\mathrm{GeV}))/0.45)}$$

$$\mu_B[\mathrm{MeV}] = \frac{1307.5}{1+0.288\sqrt{s_{NN}}(\mathrm{GeV})}$$

NPA 772 (2006) 167, PLB 673 (2009) 142

μ_B is a measure of the net-baryon density, or matter-antimatter asymmetry

determined by the "stopping" of the colliding nuclei

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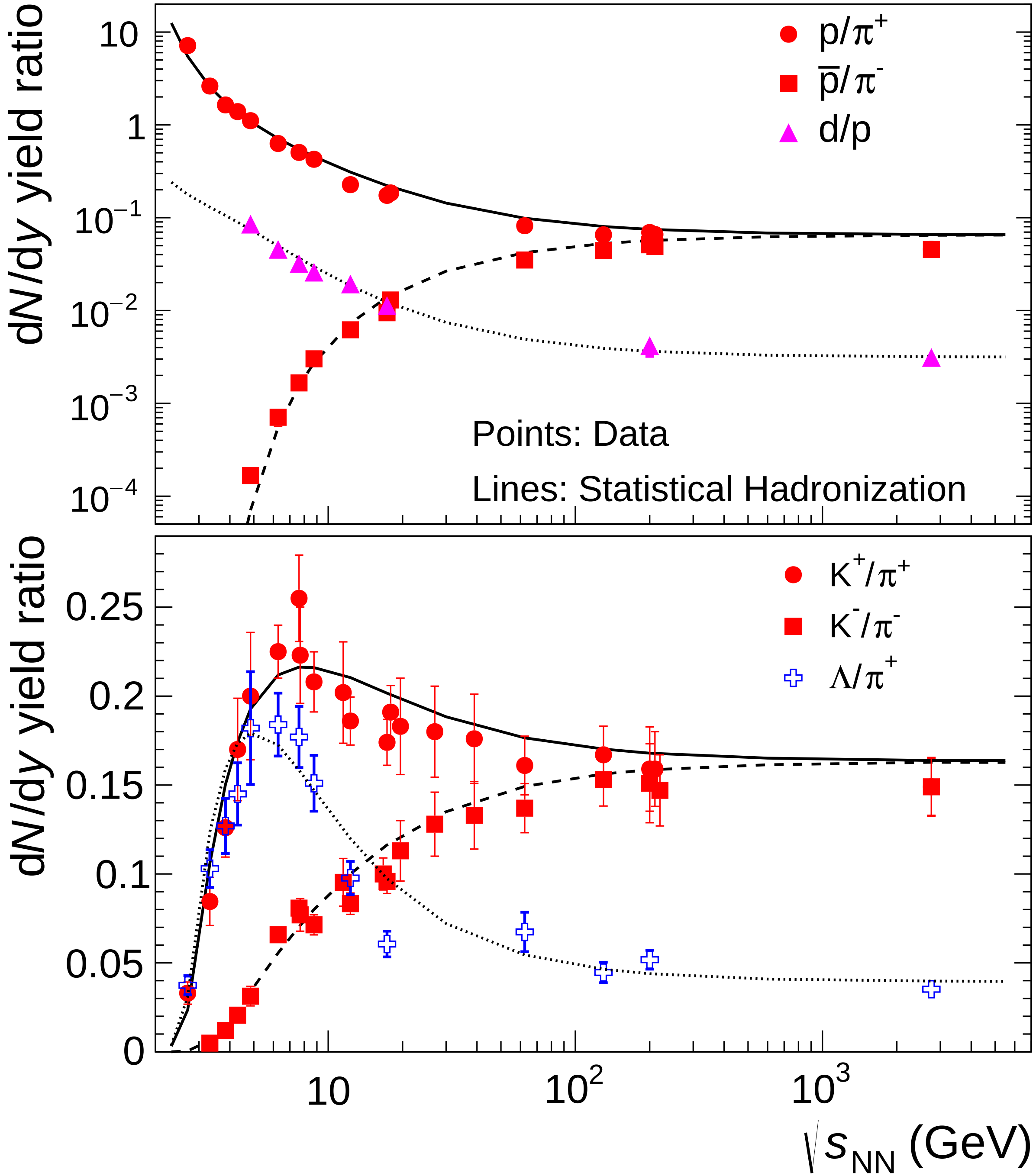


Data:

AGS: E895, E864, E866, E917, E877

SPS: NA49, NA44

RHIC: STAR, BRAHMS

LHC: ALICE

NB: no contribution from weak decays

no S-matrix correction $(\mathrm{p}, \overline{\mathrm{p}})$

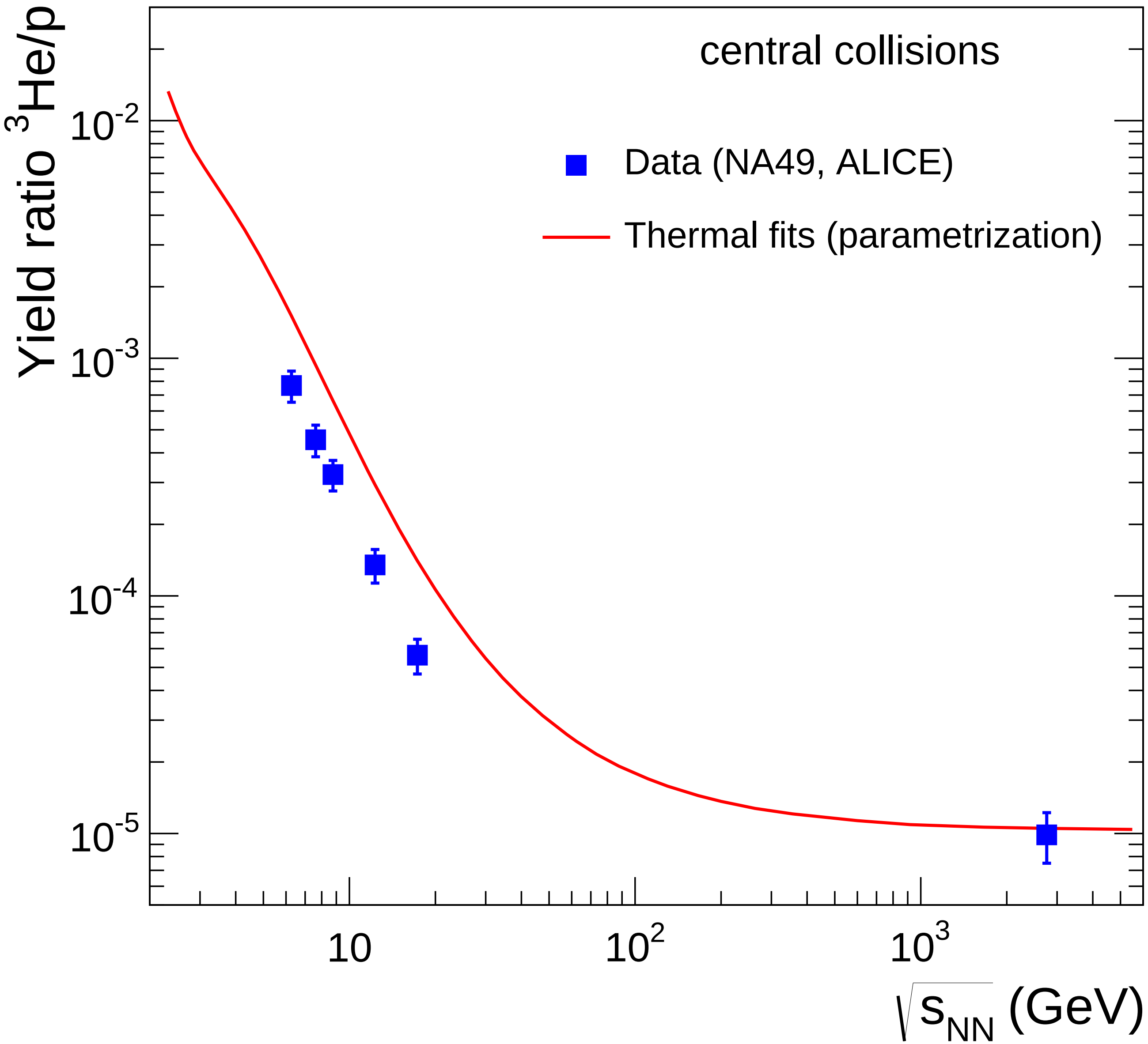
d/p ratio is well described for all energies

"structures" described by SHM

...determined by strangeness conservation

Λ/π peak reflects increasing T and decreasing μ_B

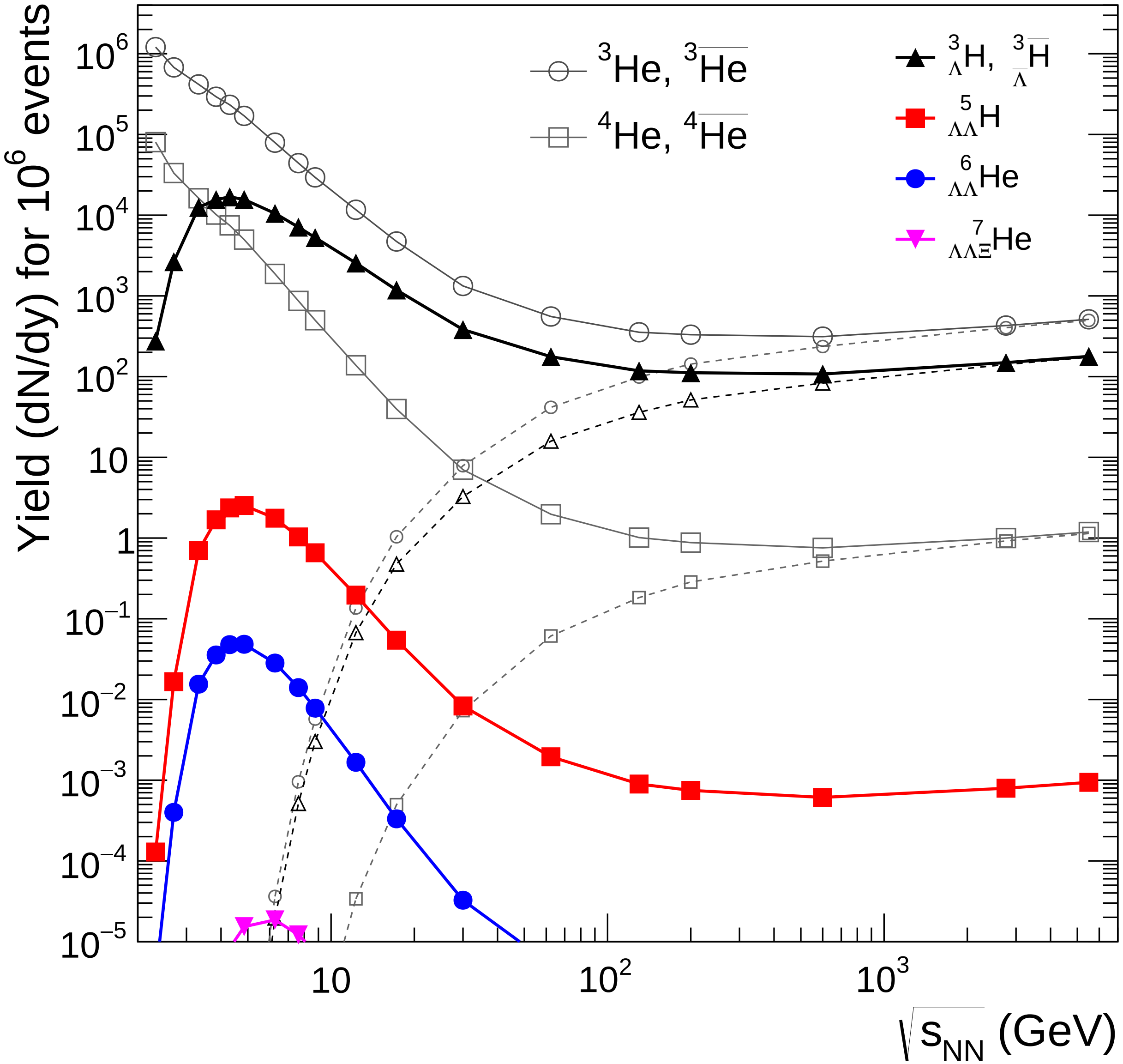
^{3}He/p ratio well described at the LHC, but not at lower energies



ALICE 5.02 TeV:
$(0.7 \pm 0.1) \cdot 10^{-5}$

Complex objects

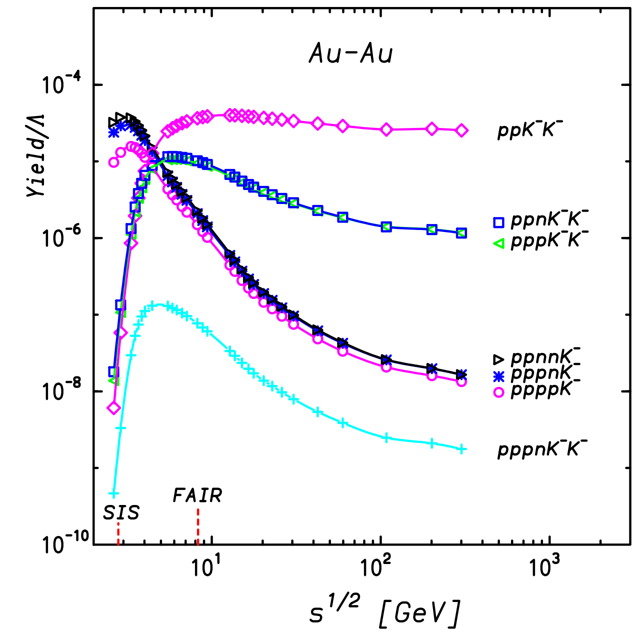
...are copiously produced at low (RHIC-BES/FAIR) energies



...some to be discovered

maybe also nucleon-K^- clusters?

AA, PBM, K.Redlich, NPA 765 (2006) 211



AA, PBM, JS, HS, PLB 697 (2011) 203

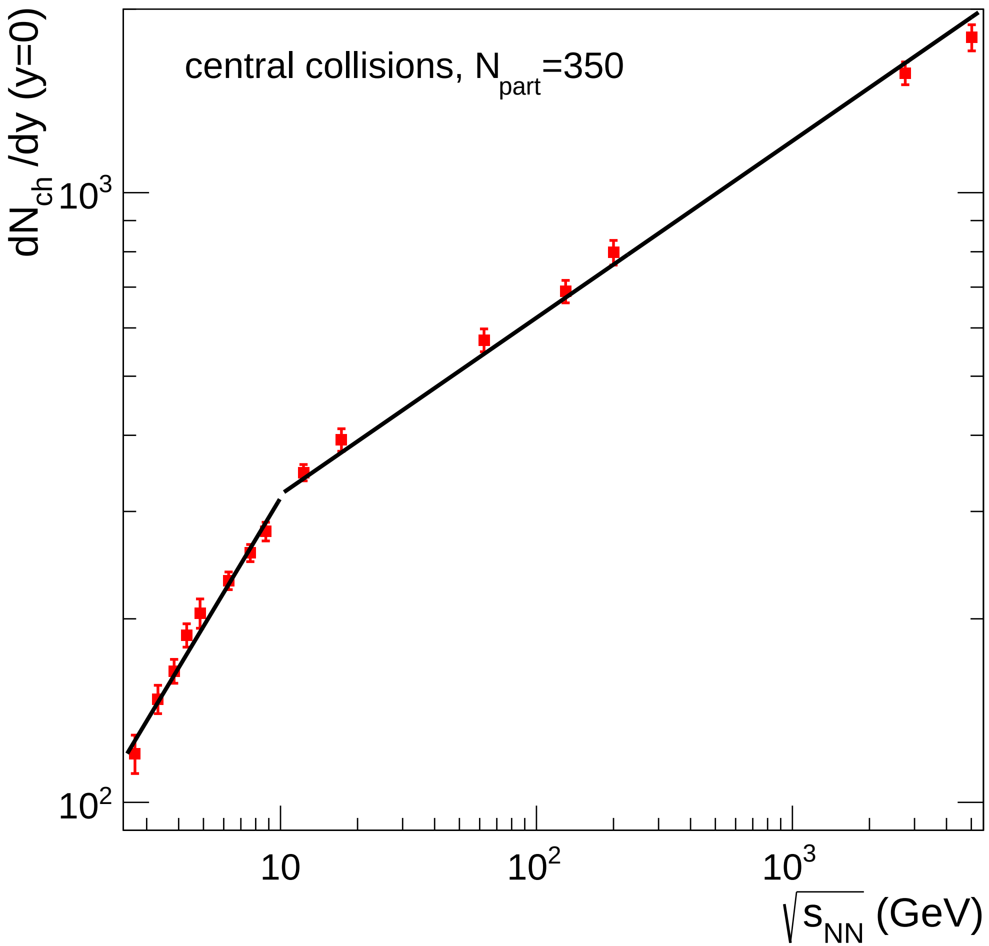
Updating the strangeness canonical suppression

Up until now: the (strangeness) canonical volume V_C=1000 fm^3

We currently have solid indications that V_{can} is multiplicity dependent.

see study for pp, p-Pb at the LHC Cleymans, Lo, Redlich, Sharma, PRC 103 (2021) 014904

$$V_C[\mathrm{fm}^3] = 12.3 + 3 \times \frac{\mathrm{d}N_{ch}}{\mathrm{d}\eta}$$



...derive this vs. energy

Updating the canonical suppression

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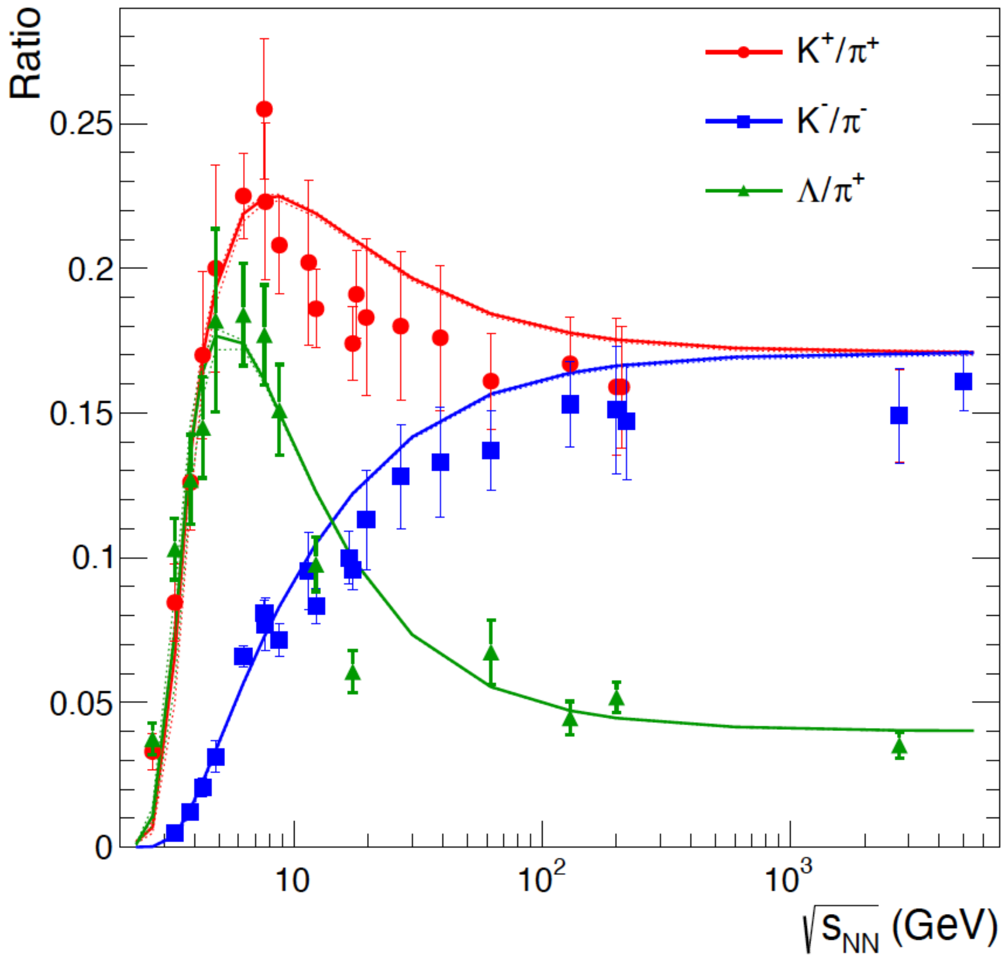
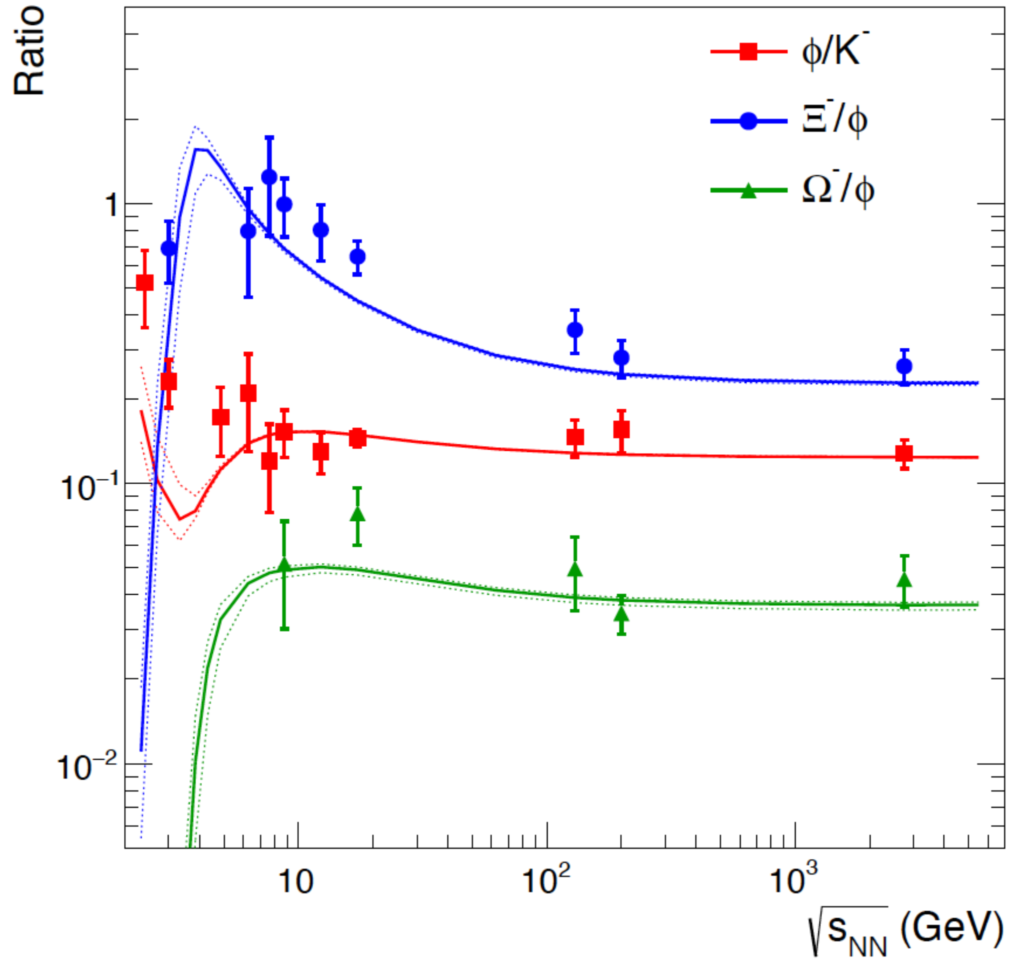
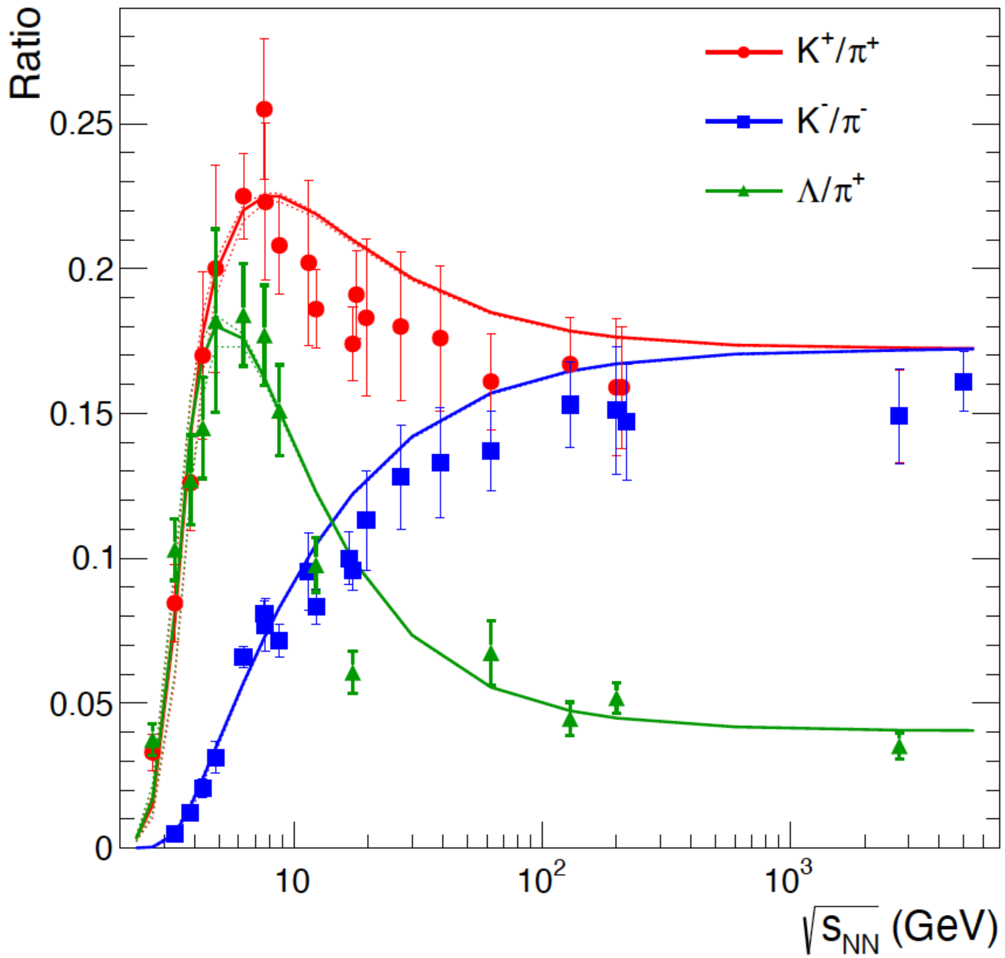
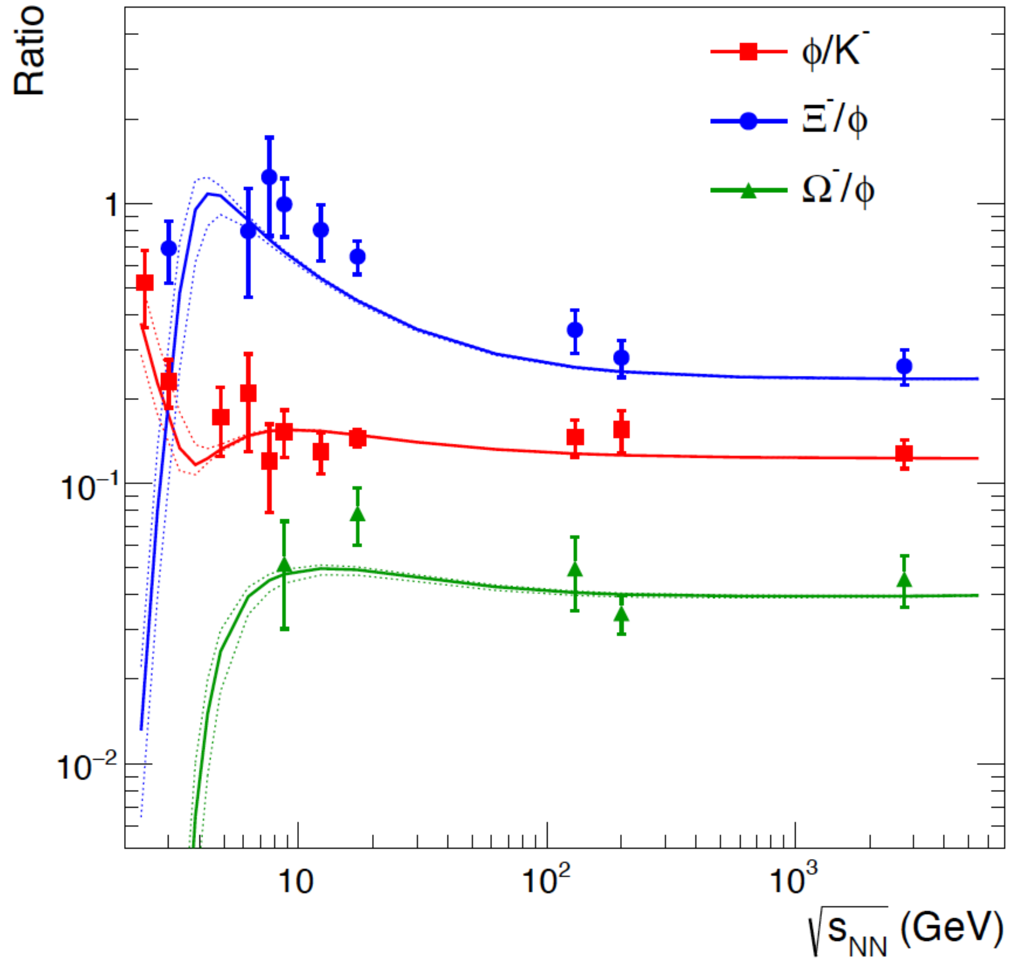




$$V_C[\text{fm}^3] = (2.0 \pm 0.6) \times \frac{\mathrm{d}N_{ch}}{\mathrm{d}y}$$

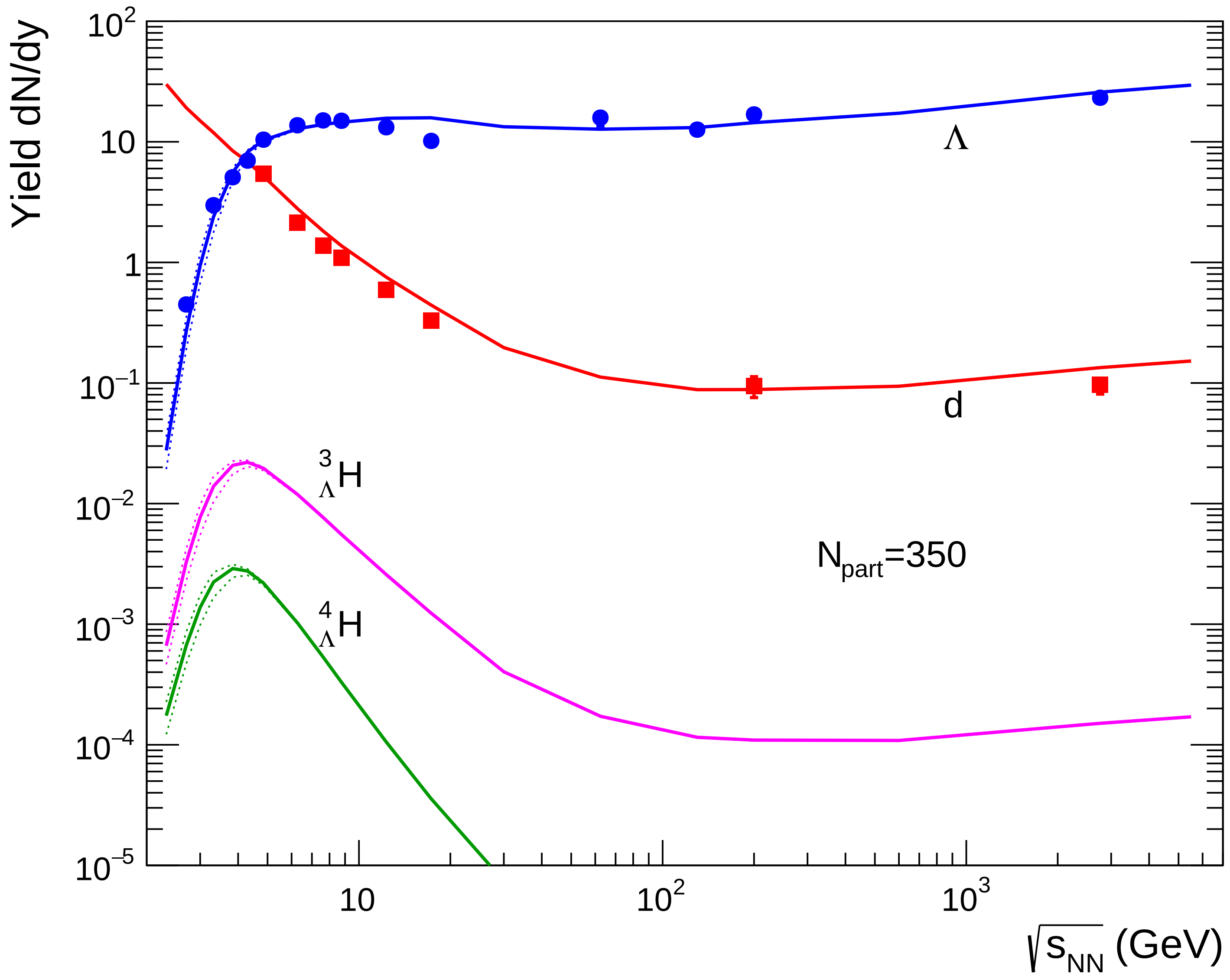




$$V_C[\mathrm{fm}^3] = (1.1 \pm 0.3) \times \frac{\mathrm{d}N_{ch}}{\mathrm{d}y}$$

...with a tick (∼10%) less steep decrease of T towards lower energies

Hypernuclei (2022, for STAR)



$^{4}_{\Lambda}$H with excited state (→4x larger yield) ...(thanks, Benjamin:)

Summary (on the light-quark sector)

- abundance of hadrons with light quarks consistent with chemical equilibration
- there is a variety of approaches ... *a personal bias: the "minimal model"*

  a minimal set of parameters, means a well-constrained model
- the thermal model provides a simple way to access the QCD phase boundary

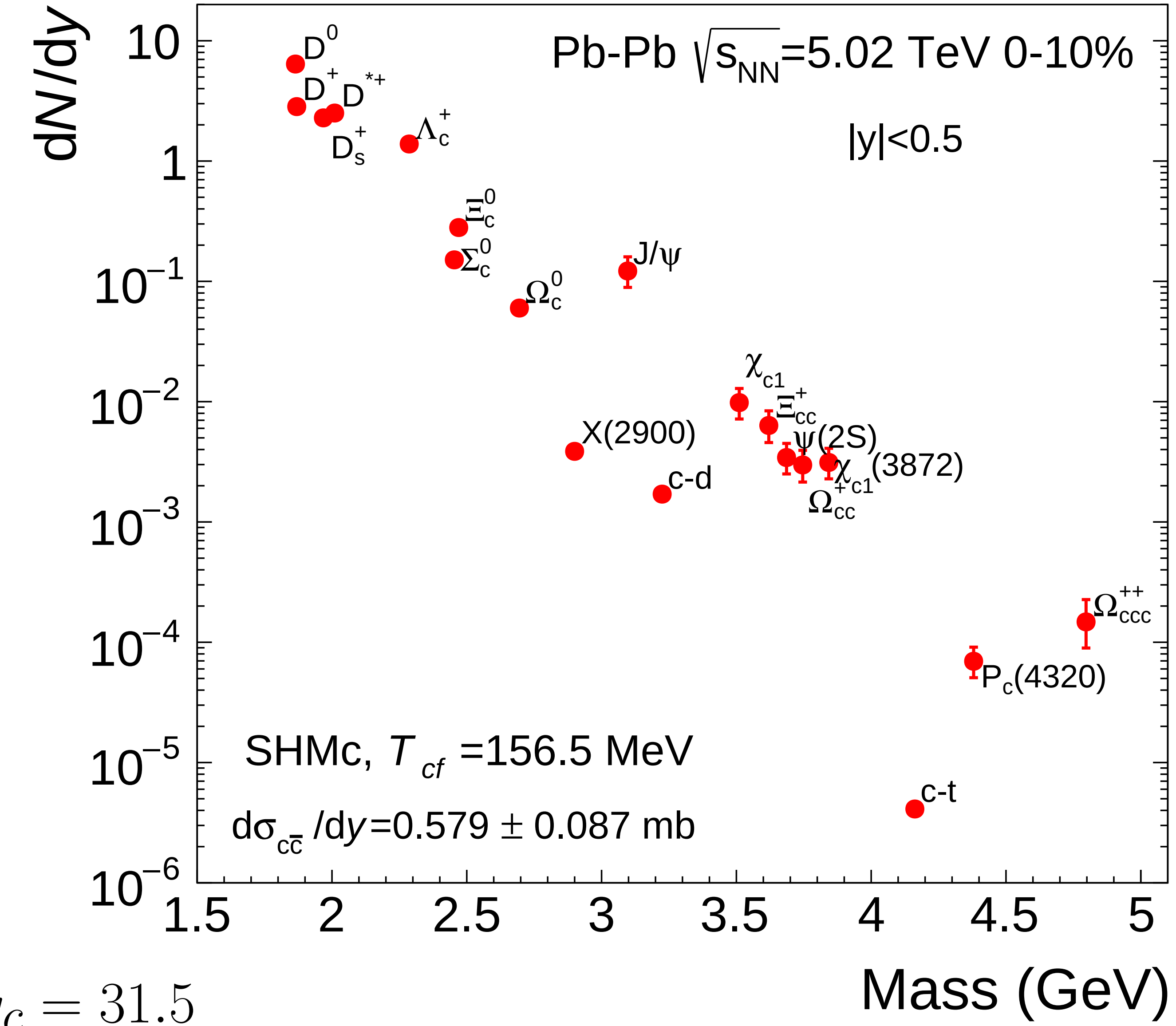
  ...at high energies (at low energies canonical suppression needs more care)

  ...but is it more than a 1st order description (of loosely-bound objects)?

  ...and what fundamental point does it make about hadronization?

  (statistical features dominate, but understanding still missing as a dynamical process)
- more insights from higher moments and from heavier (charm) quarks

SHMc: the full charm zoo



$T_{cc}^{+} \simeq$
$0.9 \cdot \chi_{c1}(3872)$

$X(6900) \sim 10^{-8}$

$\frac{\mathrm{d}N_{c\bar{c}}}{\mathrm{d}y} = 13.8$
$\rightarrow g_c = 31.5$

The power of the model: predicting the full suite of charmed hadrons