Coalescence and Correlations

Maximilian Mahlein
TU Munich

Based on: arXiv:2404.03352 & EPJC. C 83, 804 (2023)

Wigner function formalism

Theory

- The coalescence probability can be expressed in terms of the distance and relative momentum of the two nucleons

$$p(\sigma, q) = \int d^3r_p d^3r_n h(r_n) h(r_p) W(q, r)$$

- With W(q,r) the Wigner function of the deuteron

$$W(\vec{q}, \vec{r}) = \int d^3\zeta \; \Psi(\vec{r} + \vec{\zeta}/2)\Psi^*(\vec{r} - \vec{\zeta}/2)e^{i\vec{q}\vec{\zeta}}$$

- And $h(r_i)$ the single particle spatial distribution

$$h(\vec{r}_n)h(\vec{r}_p) = \frac{1}{(2\pi\sigma^2)^3} \exp\left(- \frac{\vec{r}_n^{\,2} + \vec{r}_p^{\,2}}{2\sigma^2} \right)$$

📚MM et al .Eur.Phys.J.C 83 (2023) 9, 804

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$$W(\vec{q}, \vec{r}) = \int d^3\zeta \, \underbrace{\Psi(\vec{r} + \vec{\zeta}/2)}_{\text{Deuteron Wave function}} \Psi^*(\vec{r} - \vec{\zeta}/2) e^{i\vec{q}\vec{\zeta}}$$

- And $h(r_i)$ the single particle spatial distribution

$$h(\vec{r}_n) h(\vec{r}_p) = \underbrace{\frac{1}{(2\pi\sigma^2)^3}}_{\text{Gaussian Source}} \exp\left(-\frac{\vec{r}_n^{\,2} + \vec{r}_p^{\,2}}{2\sigma^2}\right)$$

📚MM et al .Eur.Phys.J.C 83 (2023) 9, 804

Wigner function formalism
Theory

- The coalescenc   elative
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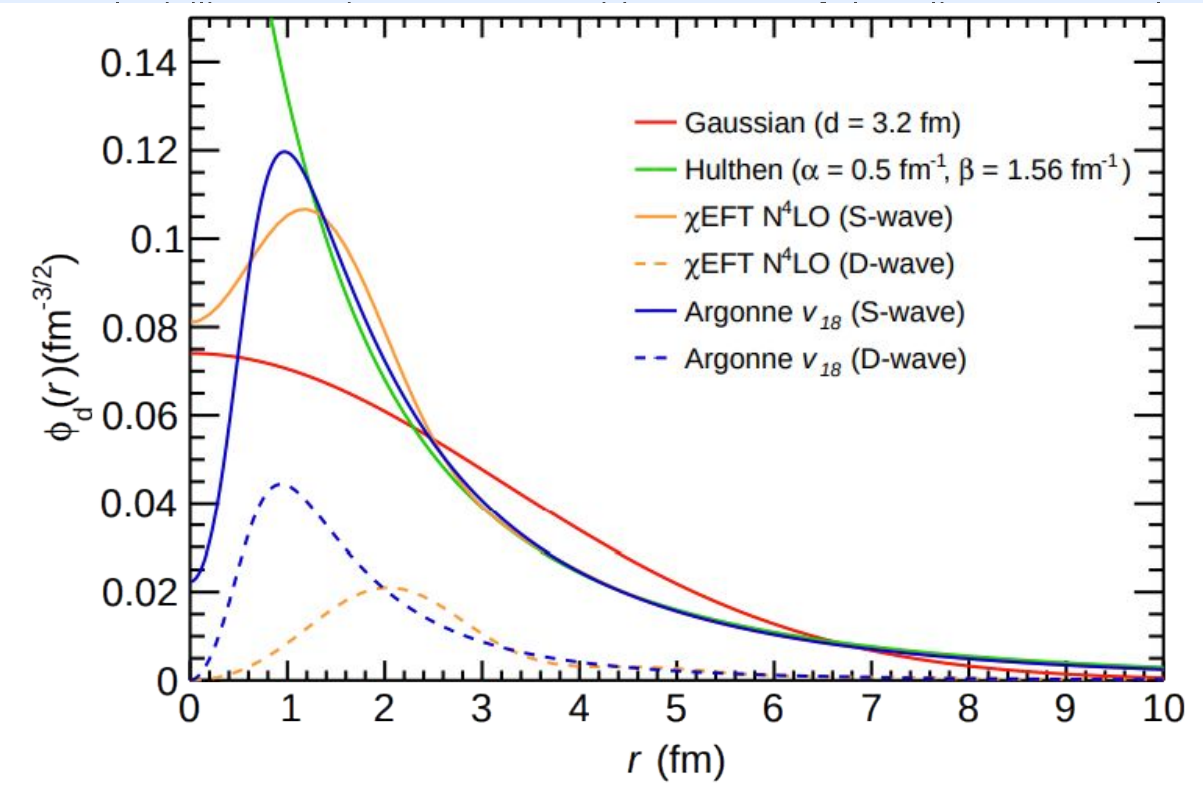

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Coalescence Results EPOS

Deuteron spectra

- Corrections to Protons, Source, Multiplicity
- Wavefunctions: Gaussian,Hulthén and Argonne v_{18}
- v_{18} reproduces data to ~10%

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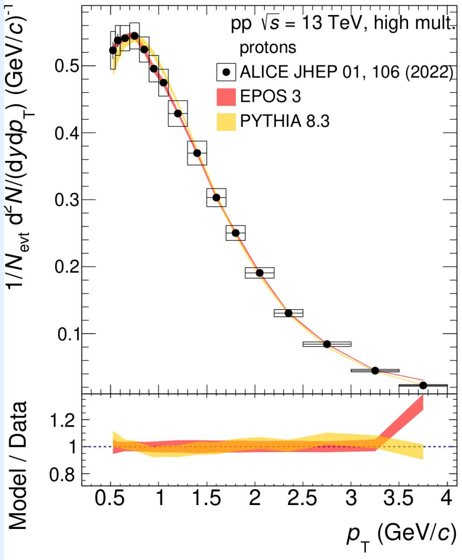

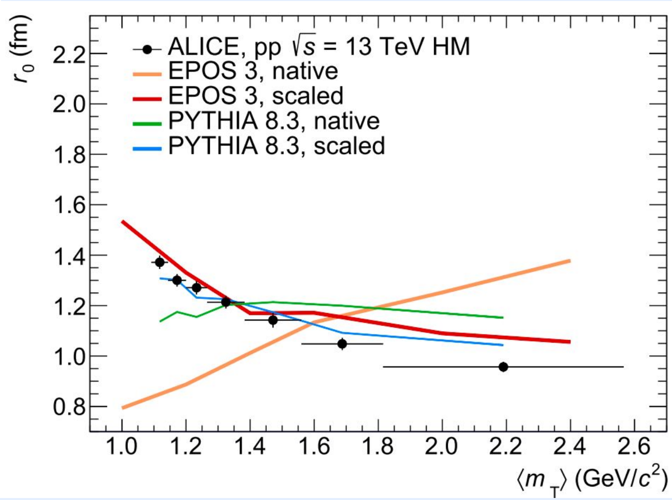

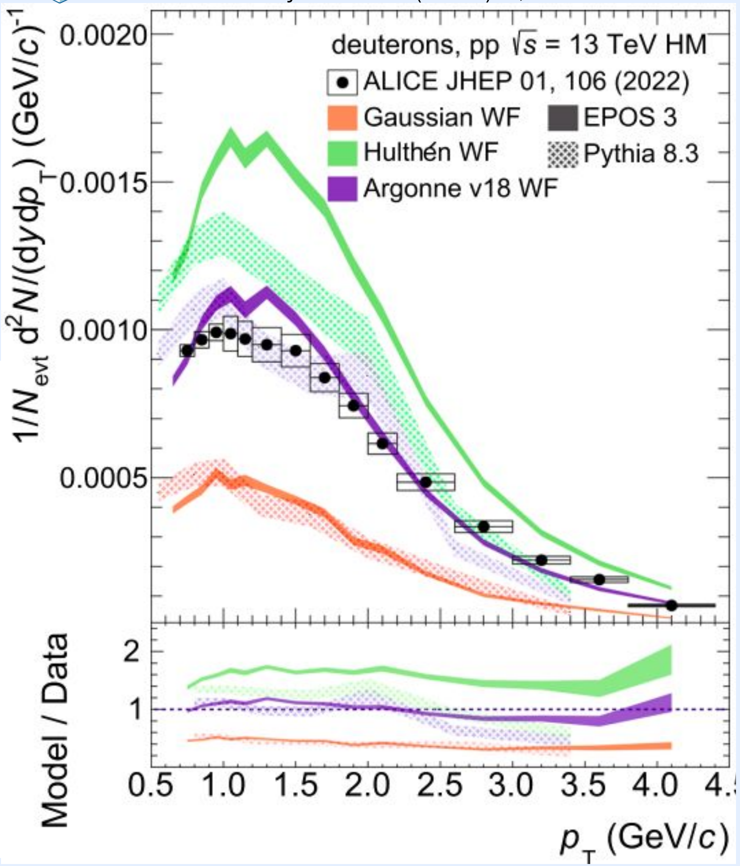

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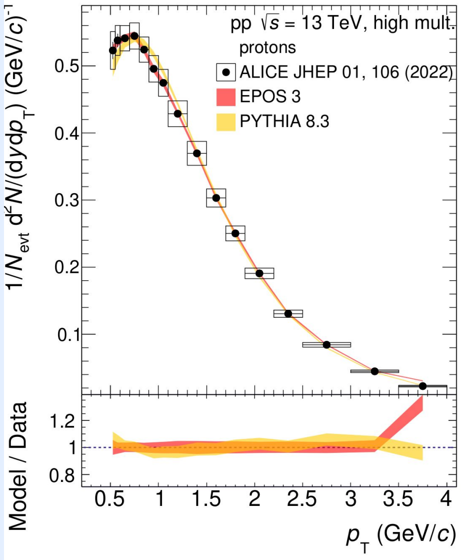
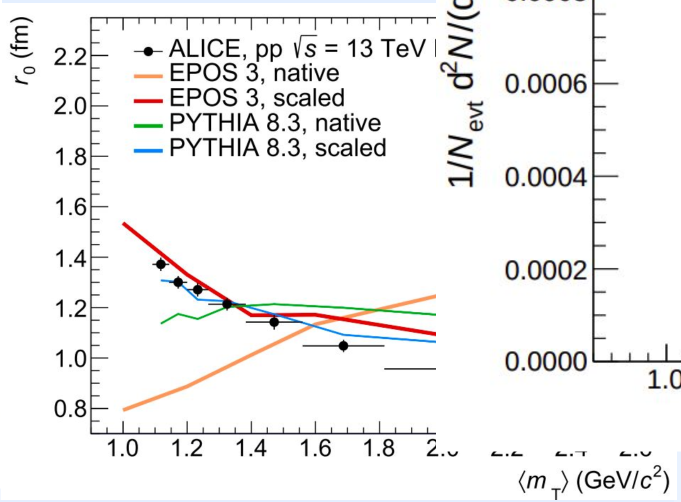
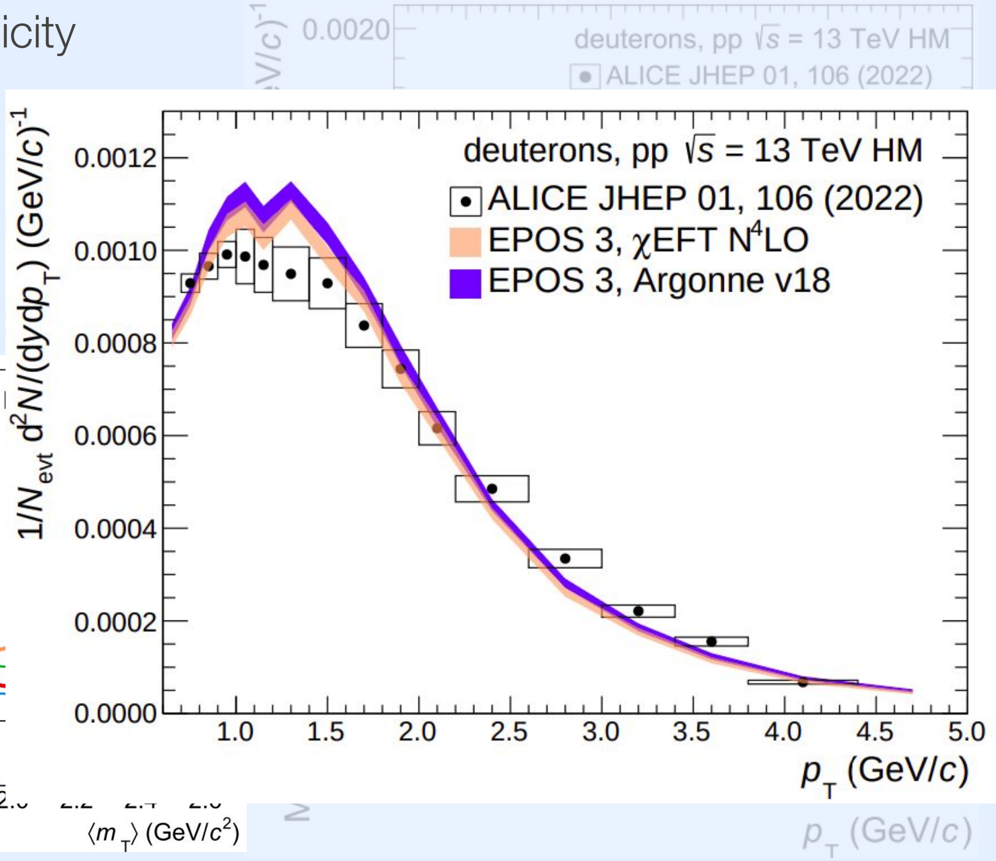
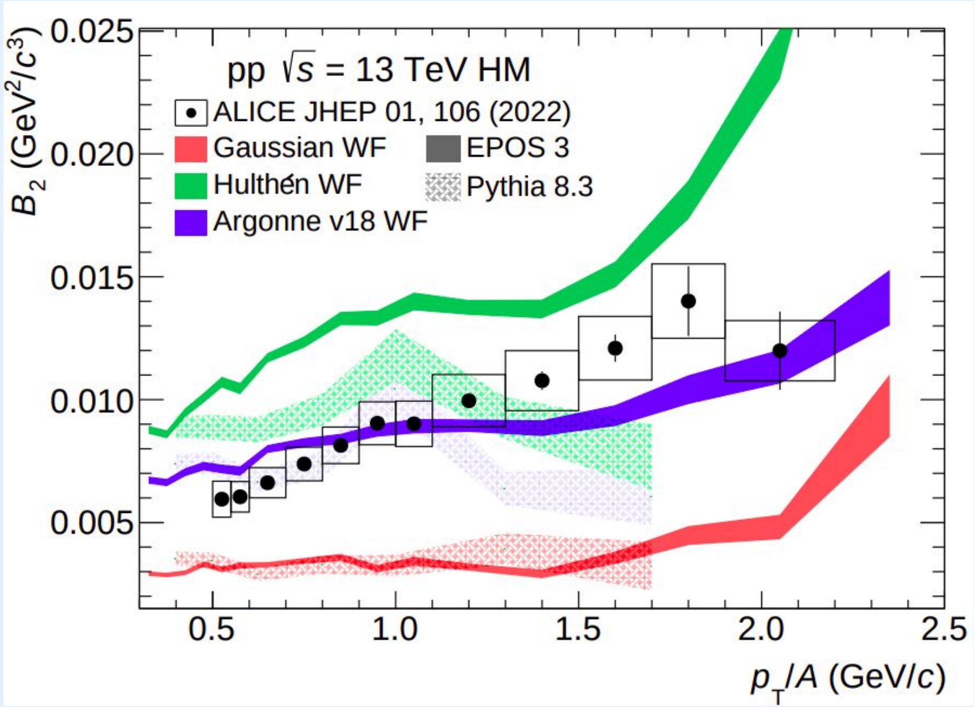
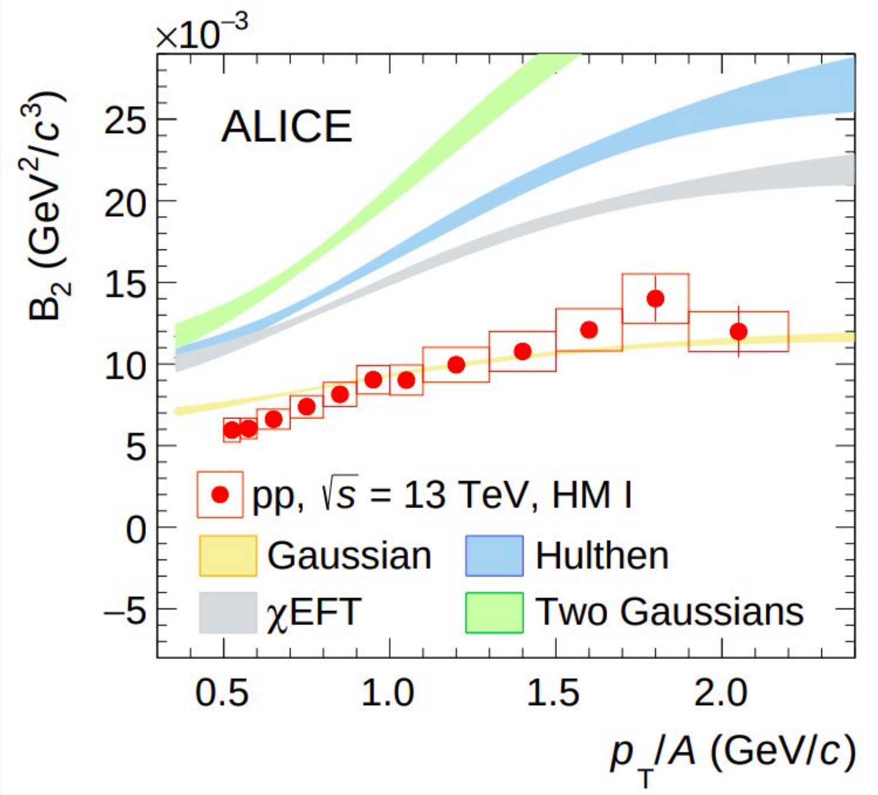






Comparison of B_2 with previous works





Coalescence Results EPOS

Angular correlations

- Delta Phi of pp (pn) pairs
- Not reproduced by EPOS or Pythia
- No real control over these behaviours in general purpose event generators

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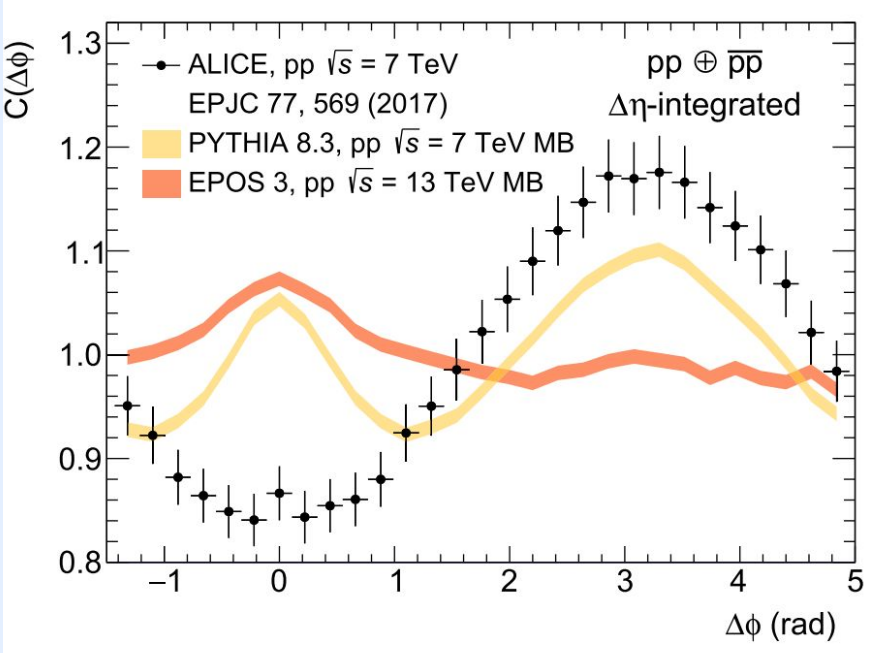
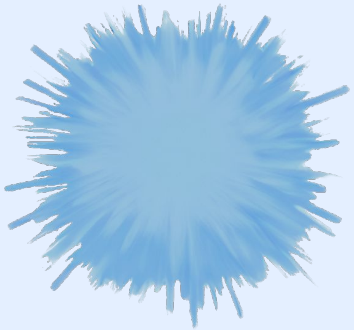


Careful: rather technical!

The ToMCCA Model

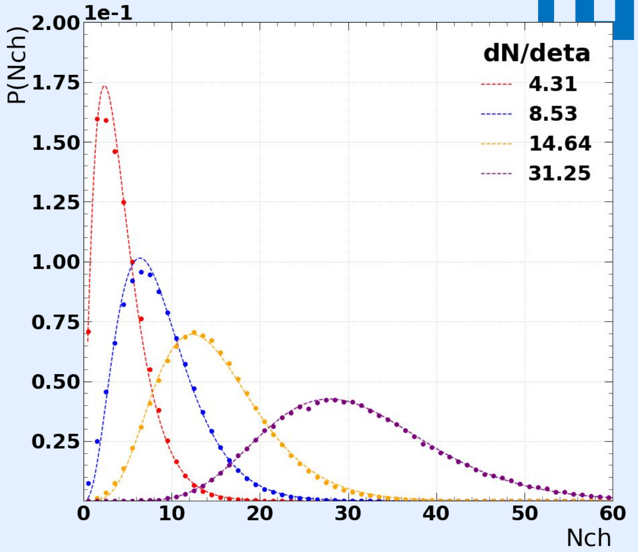
Event Loop

Event Loop:



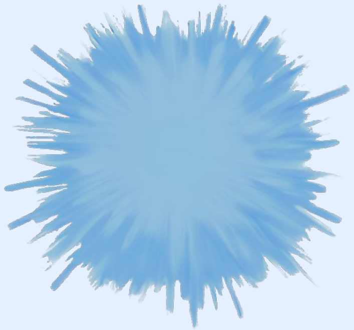
The ToMCCA Model

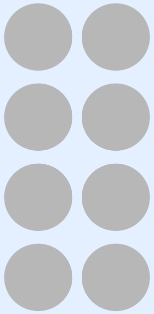
Event Loop



<u>Event Loop:</u>

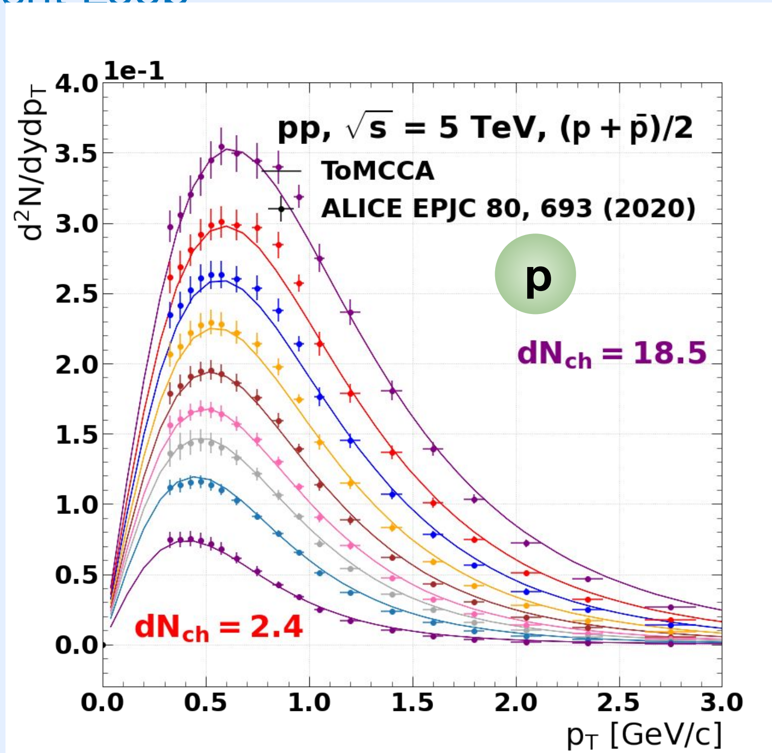
- ➢ Get number of charged particles
  1. Poissonian distribution with given mean
  2. dN/deta measurements by ALICE
  3. *Event generator output (Fitted using Erlang dist)*





The ToMCCA Model

Event Loop



Event Loop:

- Get number of charged particles
- Get proton yield
- Get neutron yield

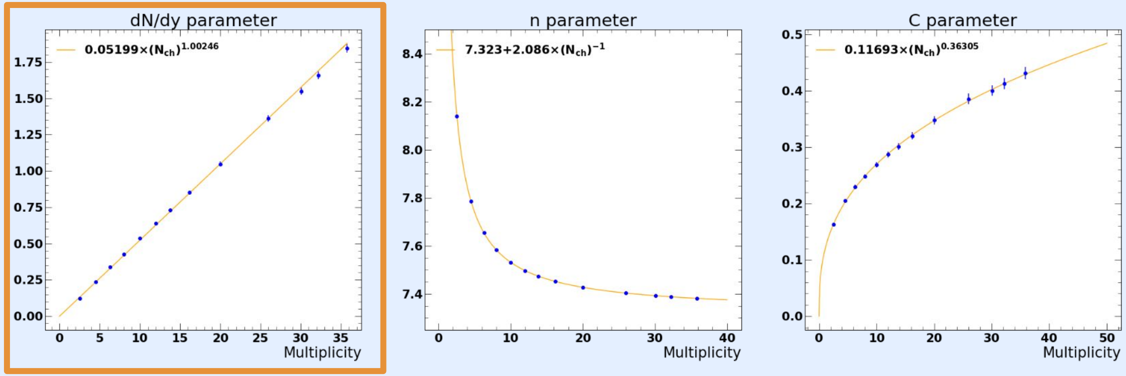
Fit all proton spectra for 5 TeV and HM using a Lévy Tsallis:

$$\frac{d^2N}{dydp_T} = \frac{dN}{dy}\,\frac{p_T(n-1)(n-2)}{nC[nC+m_p(n-2)]}\left(1+\frac{m_T-m_p}{nC}\right)^{-n}$$

Yield parameter!

The ToMCCA Model

Event Loop



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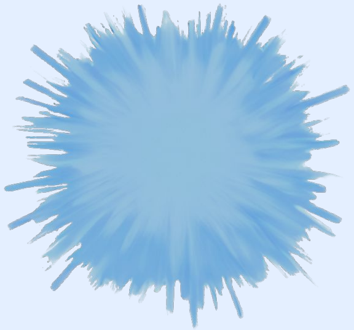
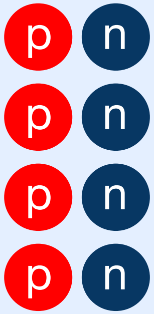
Yield parameter!

Full parameterization as a function of multiplicity





The ToMCCA Model

Event Loop

Event Loop:

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- ➢ Get neutron yield

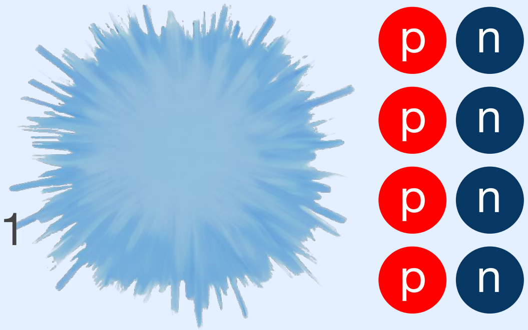
Pearson correlation coefficient

$\rho_{pn} = \langle (n_p - \langle n_p \rangle)(n_n - \langle n_n \rangle) \rangle / \sqrt{\kappa_{2p} \kappa_{2n}}$

	Uncorrelated Emission	String Fragmentation	Quark recombination	Tuned Emission
ρ_{pn}	0	-0.052	-0.058	-0.024

Neutron suppression: $\alpha(\langle n_p \rangle - n_p) + 1$

$\rightarrow \alpha(N_{ch}) = 1.508\ (N_{ch})^{-1}$

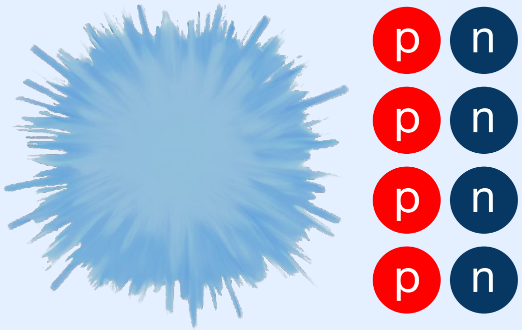


The ToMCCA Model

Event Loop

Event Loop:

- ➢ Get number of charged particles
- ➢ Get proton yield
- ➢ Get neutron yield
- ➢ ⟳ Loop over all protons



The ToMCCA Model

Event Loop

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- ➢ Get number of charged particles
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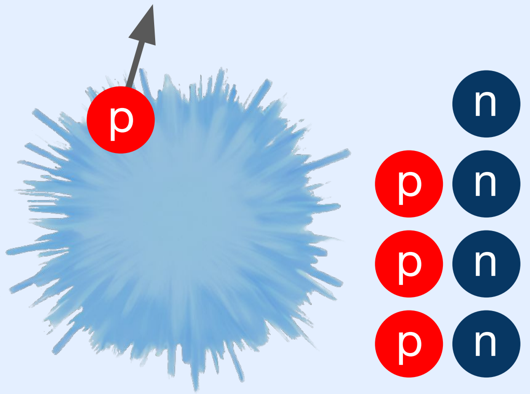
  Get 3D momentum of proton

  - Draw p_T from parameterization
  - Draw flat rapidity y=[-0.5,0.5]
  - Draw random ϕ=[0,2π)



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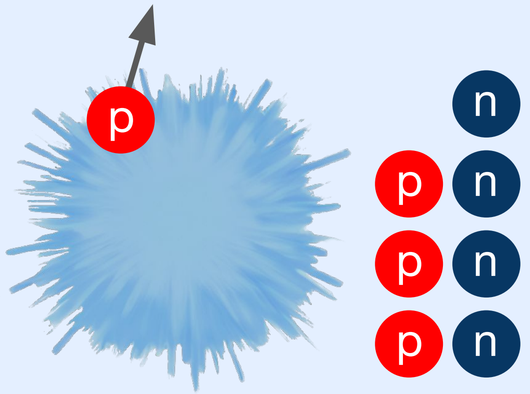


The ToMCCA Model

Event Loop

Event Loop:

Get number of charged particles

Get proton yield

Get neutron yield

⟳ Loop over all protons

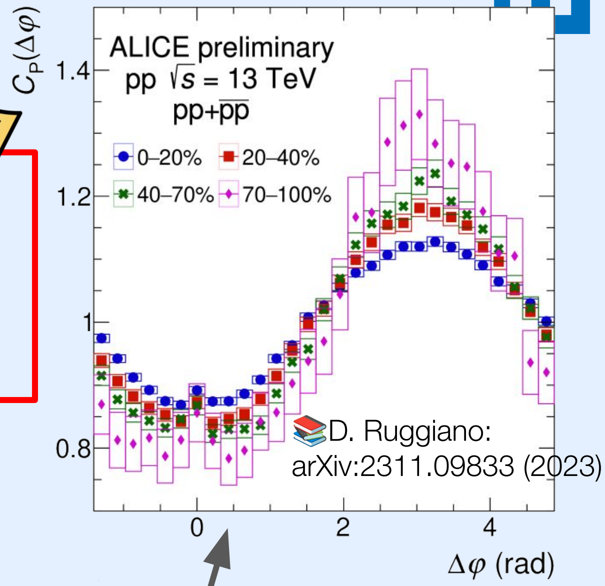
Get 3D momentum of proton

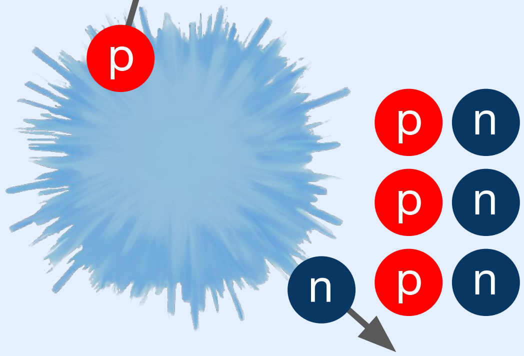
⟳ Loop over all neutrons

Get 3D momentum of neutron

- Draw p_T from parameterization
- Draw flat rapidity y=[-0.5,0.5]
- Draw random Δϕ from ALICE measurement (*20-40%*)

Not included so far:
$\Delta\phi(N_{ch})$ and $\Delta\phi(p_T)$
Included:
Scaling from
pp→pΛ





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Event Loop

Event Loop:

- Get number of charged particles
- Get proton yield
- Get neutron yield
- ⟳ Loop over all protons
  - Get 3D momentum of proton
  - ⟳ Loop over all neutrons
    - Get 3D momentum of neutron
      - Draw p_T from parameterization
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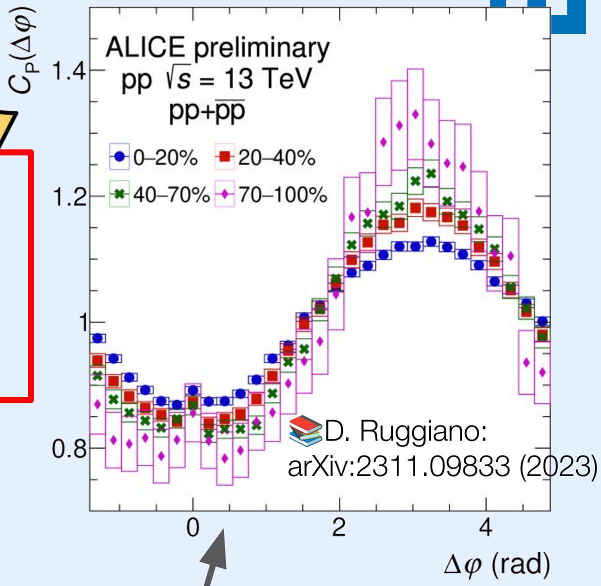

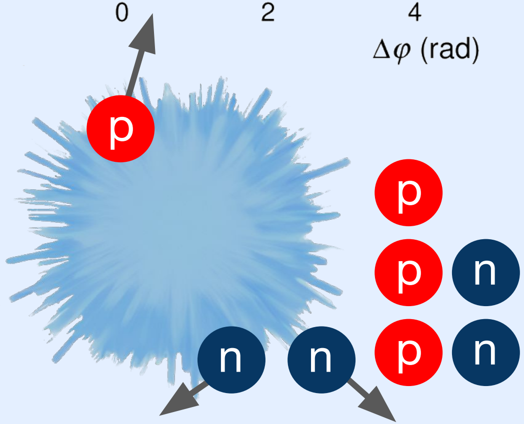

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⟳ Loop over all neutrons
Get 3D momentum of neutron
- Draw p_T from parameterization
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Discuss!

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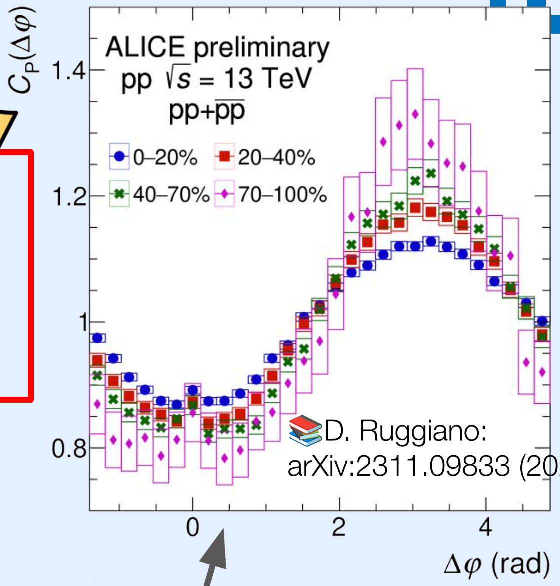
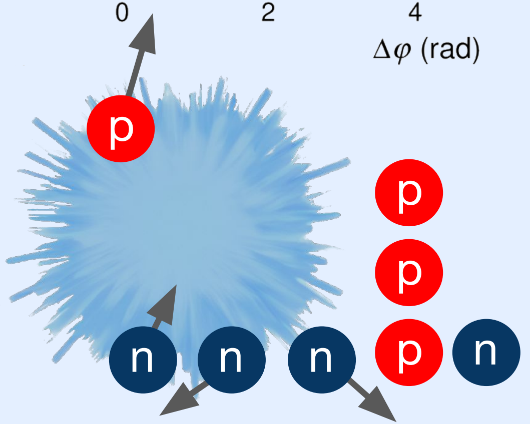


📚D. Ruggiano: arXiv:2311.09833 (2023)



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Get number of charged particles
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Get neutron yield
⟳ Loop over all protons
  Get 3D momentum of proton
  ⟳ Loop over all neutrons
    Get 3D momentum of neutron
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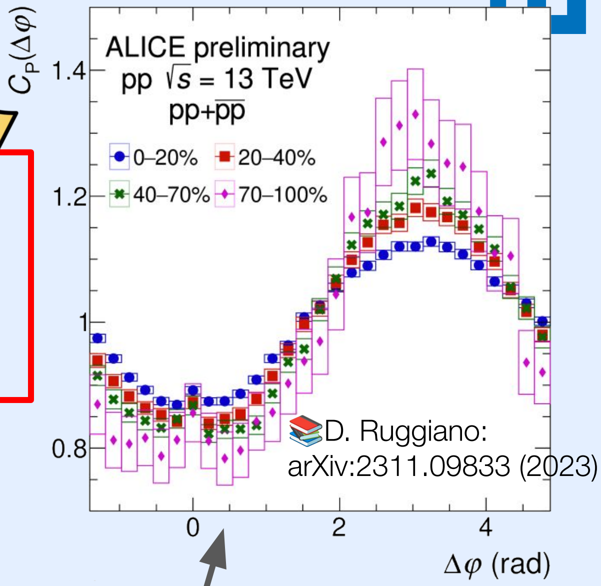
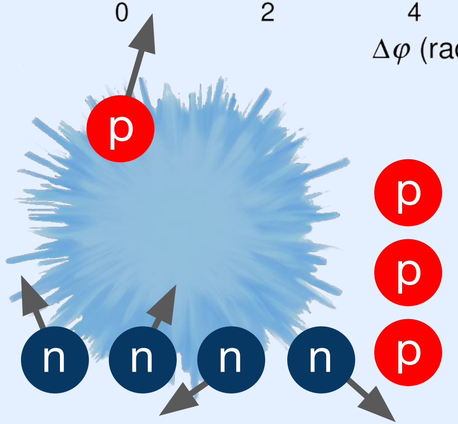


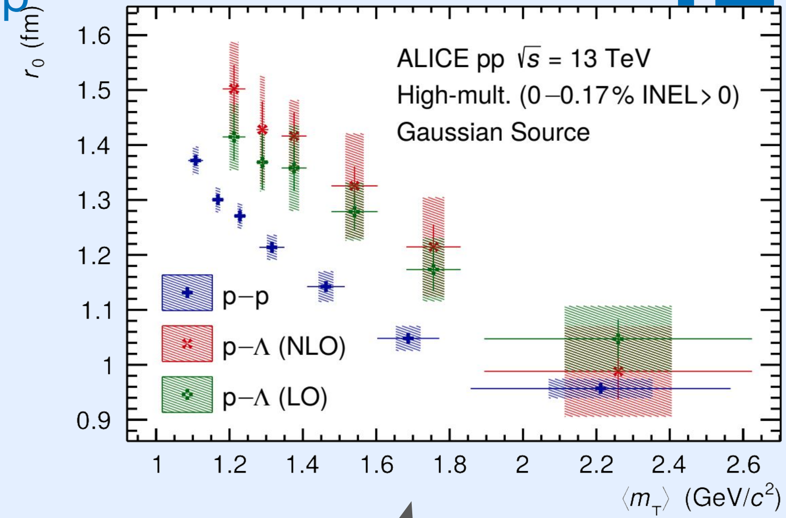


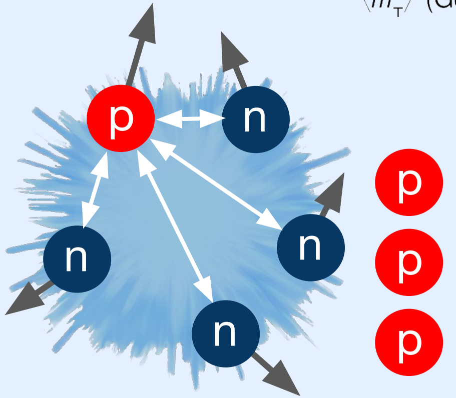
The ToMCCA Model

Event Loop

Event Loop:

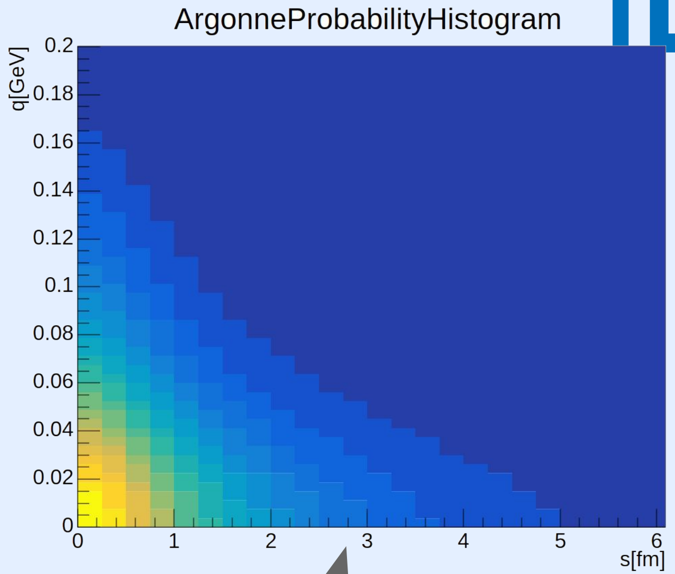
Get number of charged particles
Get proton yield
Get neutron yield
⟳ Loop over all protons
Get 3D momentum of proton
⟳ Loop over all neutrons
Get 3D momentum of neutron
Get source size





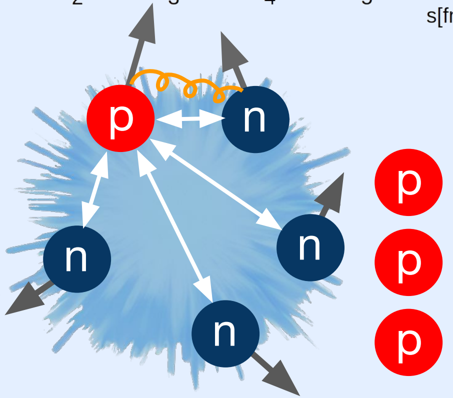
The ToMCCA Model

Event Loop



Event Loop:

- Get number of charged particles
- Get proton yield
- Get neutron yield
- ⟳ Loop over all protons
    - Get 3D momentum of proton
    - ⟳ Loop over all neutrons
        - Get 3D momentum of neutron
        - Get source size
        - Apply coalescence condition



The ToMCCA Model

Event Loop

Event Loop:

Get number of charged particles
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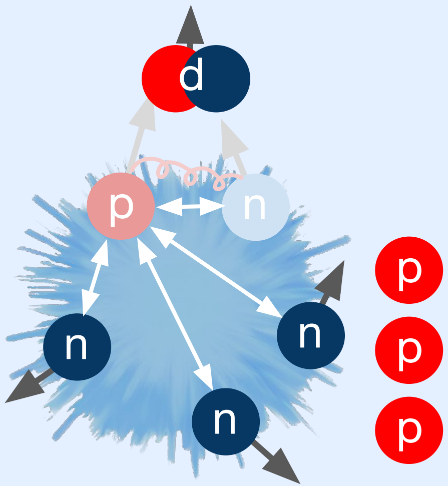
Get neutron yield
⟳ Loop over all protons
  Get 3D momentum of proton
  ⟳ Loop over all neutrons
    Get 3D momentum of neutron
    Get source size
    Apply coalescence condition



The ToMCCA Model

Event Loop

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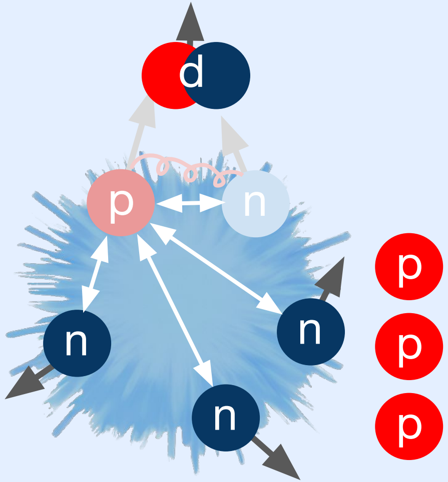
Get number of charged particles
Get proton yield
Get neutron yield
⟳ Loop over all protons
  Get 3D momentum of proton
  ⟳ Loop over all neutrons
    Get 3D momentum of neutron
    Get source size
    Apply coalescence condition
      ✅ make deuteron, number of neutrons -1
      ❎ try next neutron



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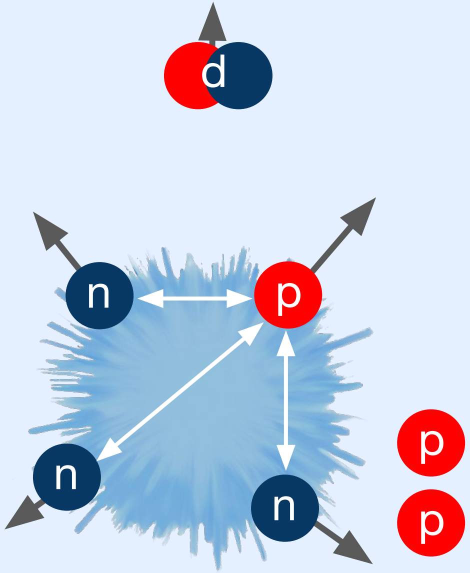

The ToMCCA Model

Event Loop

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⟳ Loop over all protons
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  ⟳ Loop over all neutrons
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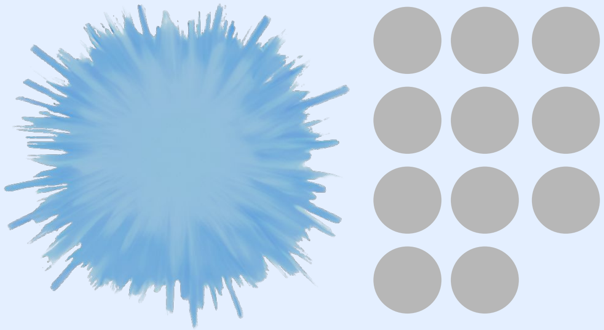
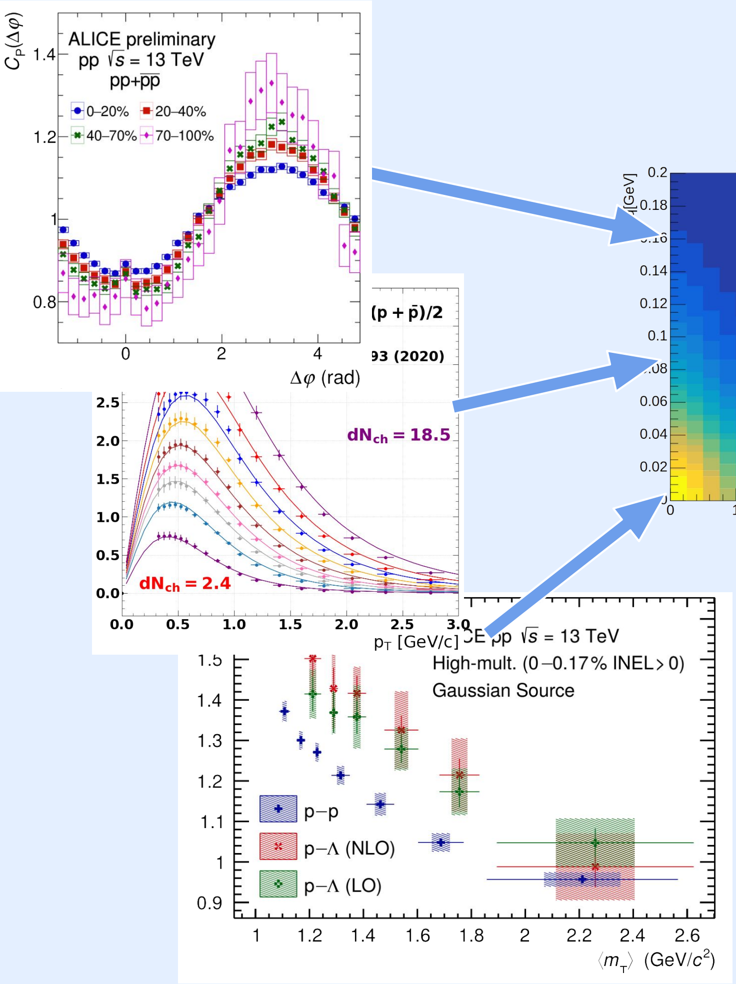
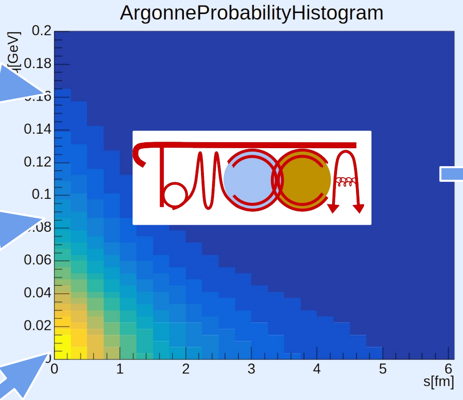
Next Event..



ToMCCA
HM Results





ToMCCA

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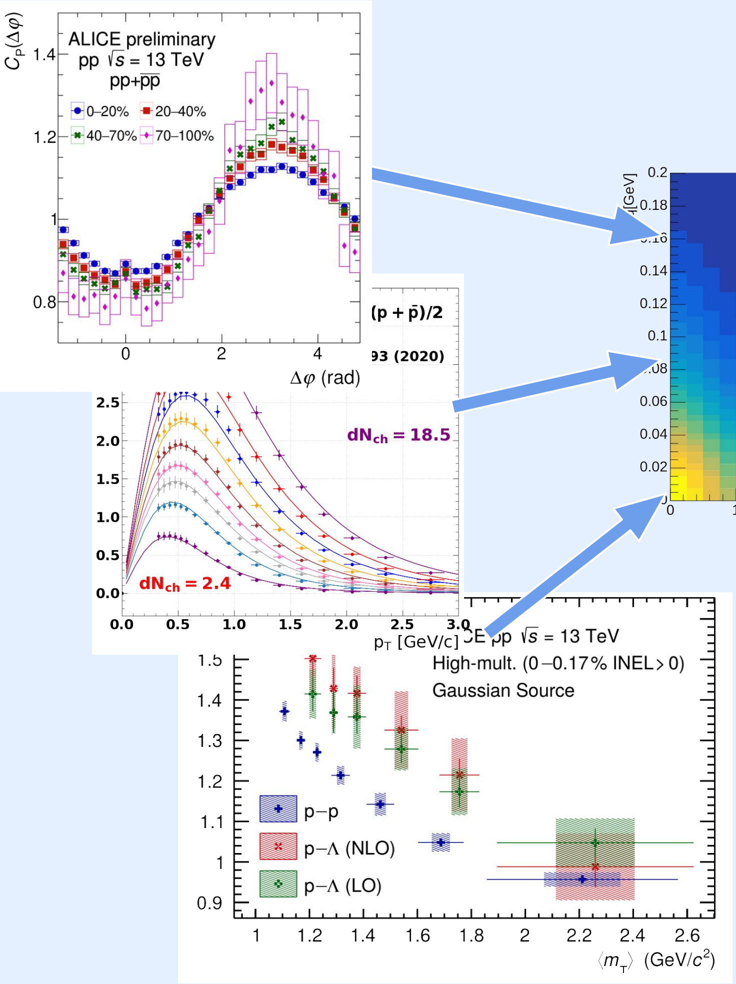

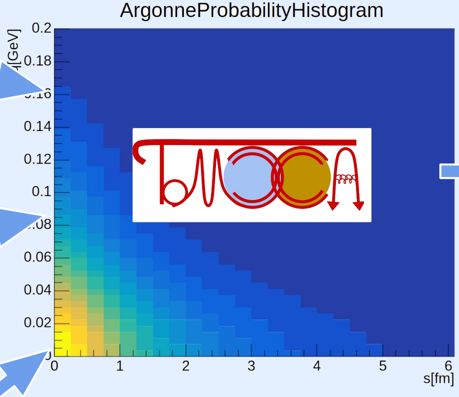

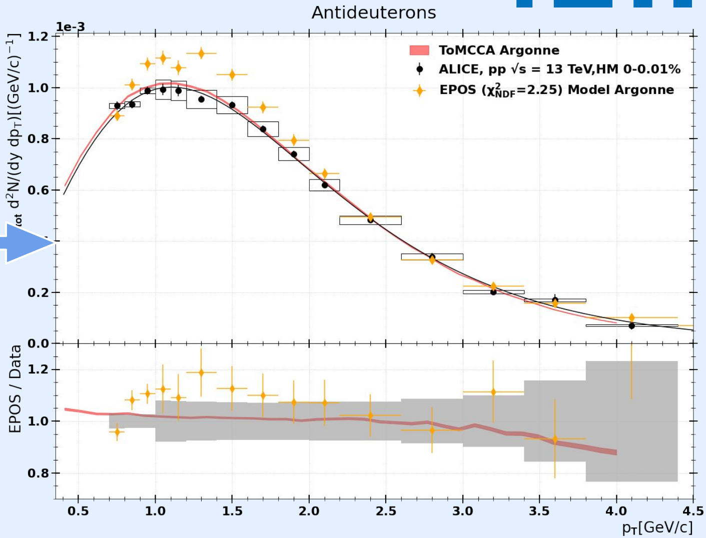

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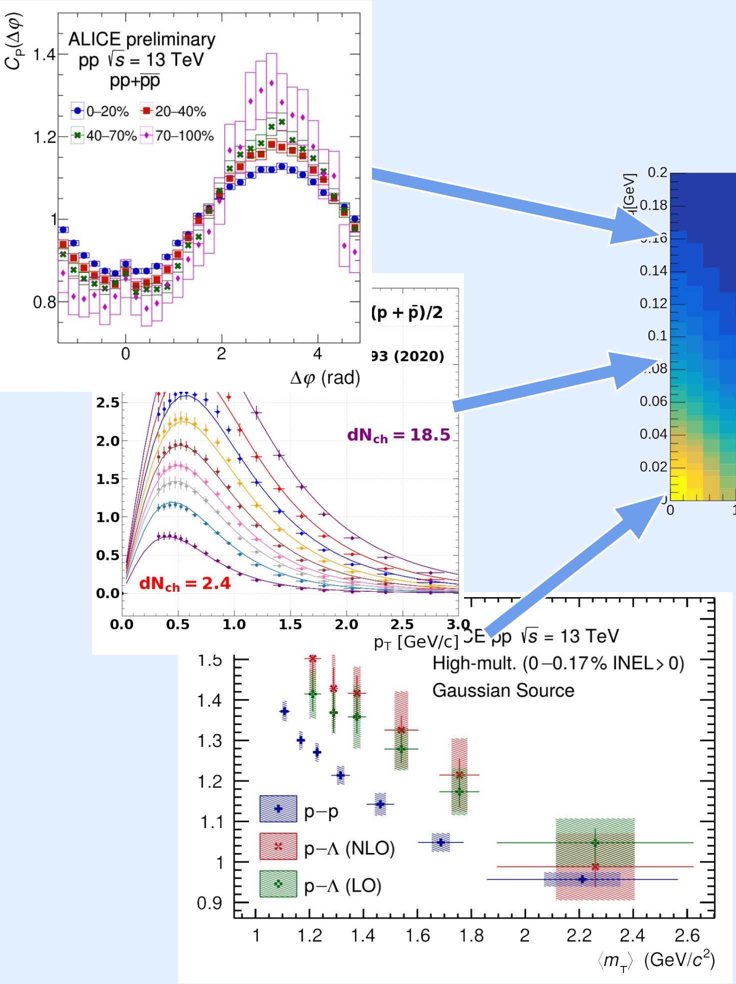
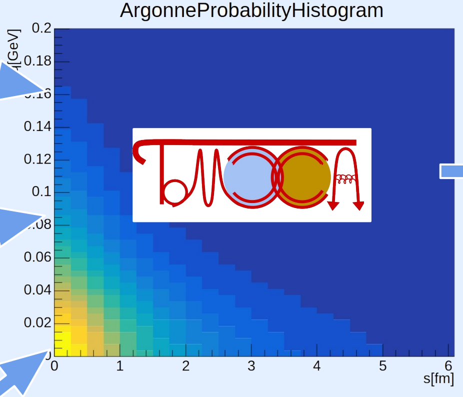
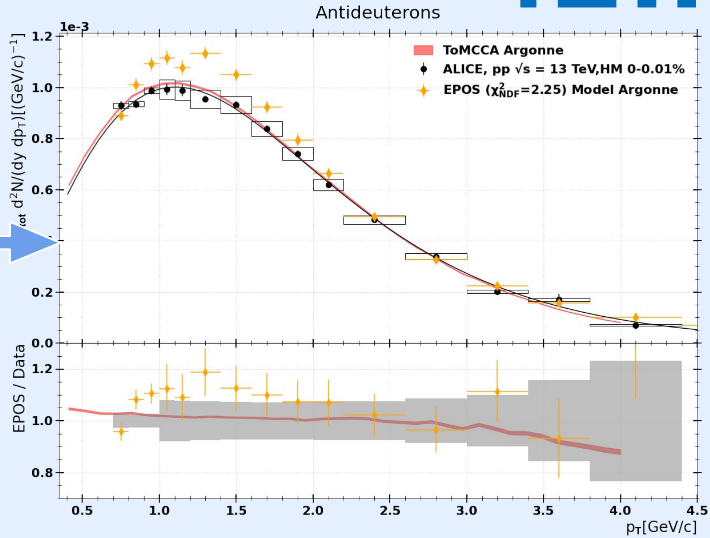


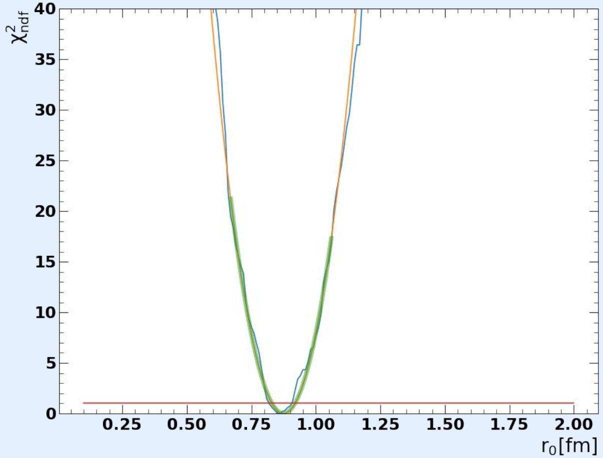




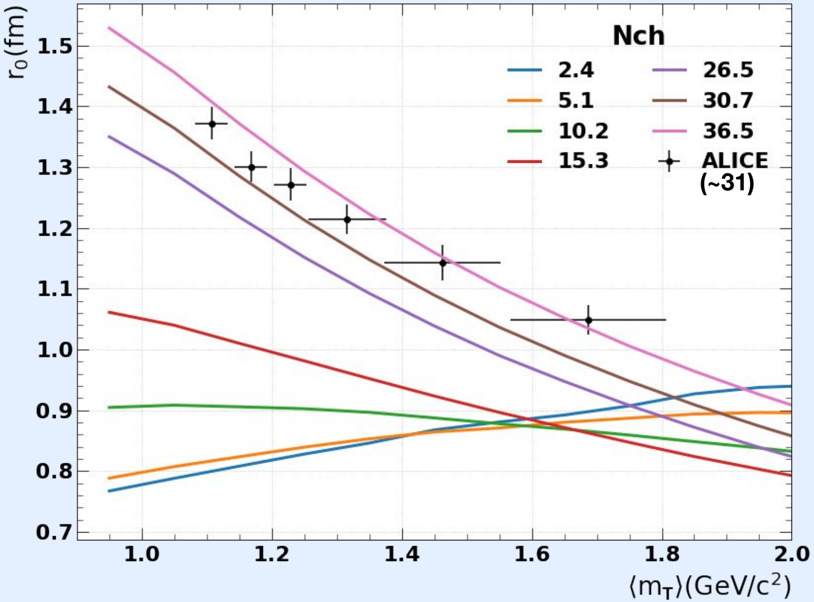
Model works if we have all the input!
But: No measurements of the source size outside of *13 TeV HM*

Fitting Procedure:

- Run ToMCCA with a fixed source size (e.g. 1.8 fm, flat in m_T)
- For the resulting deuteron spectra calculate the χ^2 for each bin and save it
- Reduce source size
- Repeat until source size is 0

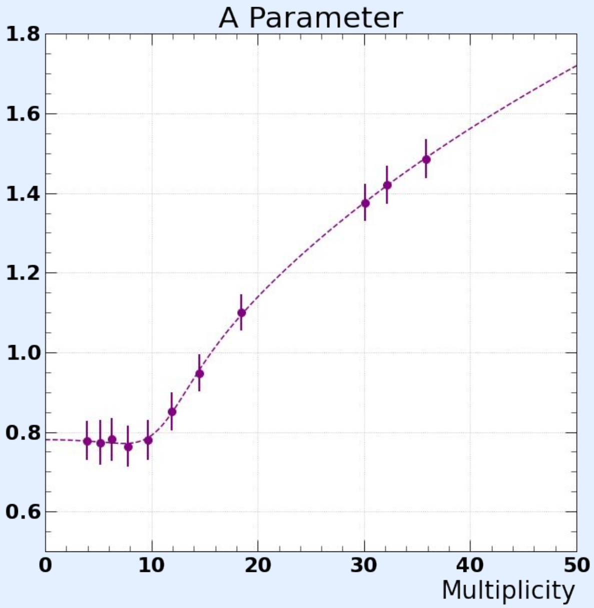


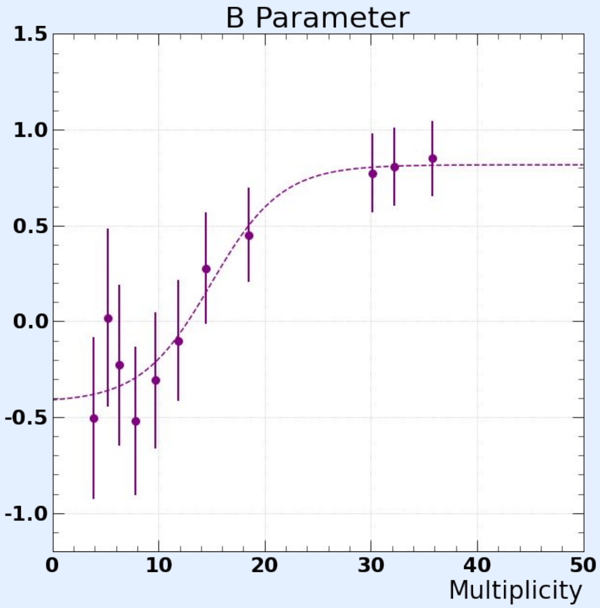
Fit to all 5 TeV + HM pp data!



Parameterization

- m_T scaling can be described by a power law $A \cdot m_T^{-B}$
- A: scaling with $N_{ch}^{1/3}$ at high multiplicity and saturation to minimum size ~Proton radius
- B: Sigmoid function, motivated by the observance of constant B in Heavy Ion [1]





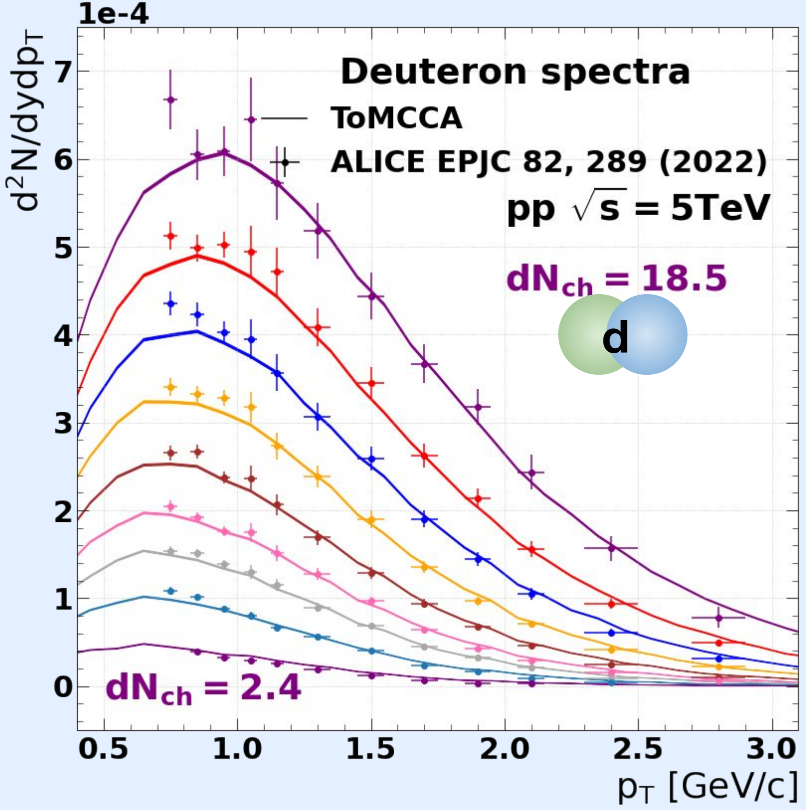
Discuss!

Source size predictions are very model dependent!

[1] arXiv:1409.4571

Putting the source size back into ToMCCA:

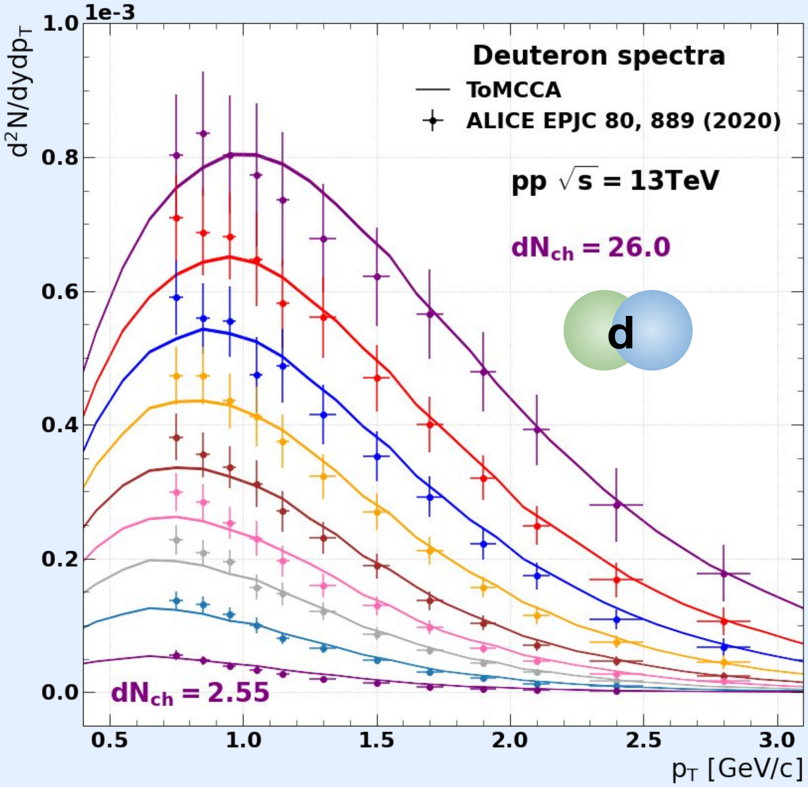
- 5 TeV data reproduced (as expected)



Multiplicity dependent results

Putting the source size back into ToMCCA:

- 5 TeV data reproduced (as expected)
- 13 TeV data also reproduced →not part of the fitting procedure!



ToMCCA

Multiplicity dependent results

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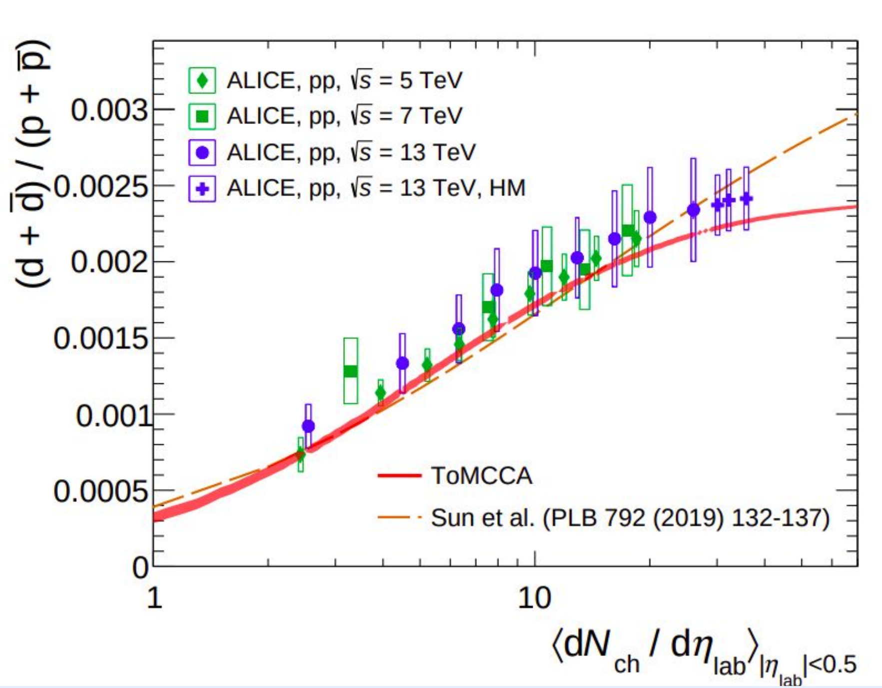
- d/p and B_2 reproduced



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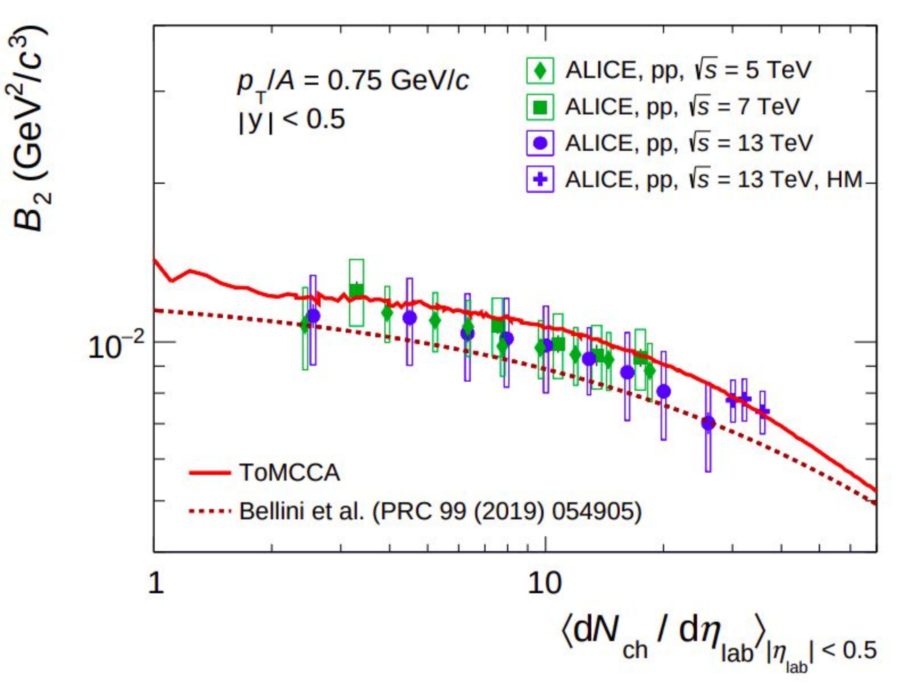

ToMCCA

Multiplicity dependent results

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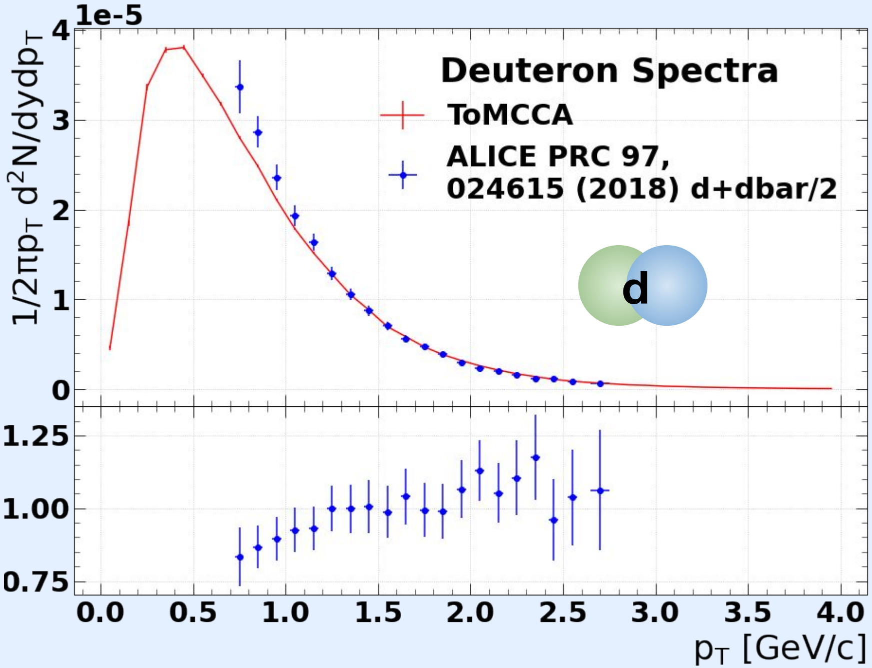

ToMCCA

Multiplicity dependent results

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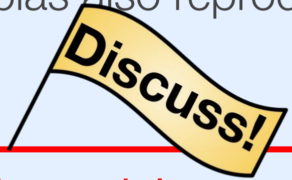
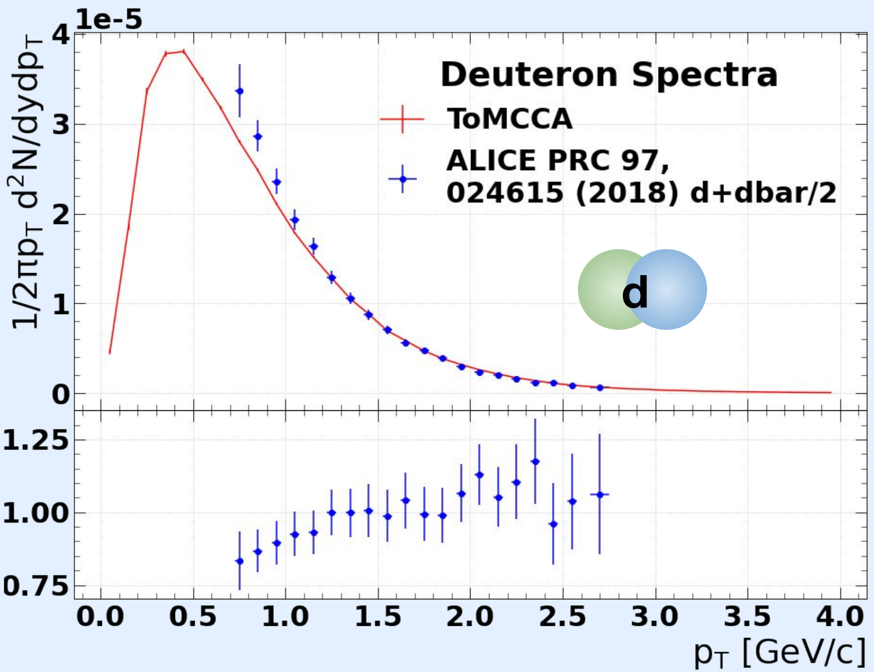


Use this model to extrapolate to arbitrary energies (only depending on the multiplicity!)



ToMCCA

Sneak Peak: PbPb and Hypertriton

2-body coalescence ($d+\Lambda \rightarrow {}^{3}_{\Lambda}H$)

$$\mathcal{P}(q,\sigma,b) = \frac{8\,b^3\,S}{(b^2+2\sigma^2)^{3/2}}\,e^{-b^2q^2}$$

$b=\sqrt{2/9}\times 10\text{fm}\sim 4.7\text{ fm}$

3-body coalescence ($p+n+\Lambda \rightarrow {}^{3}_{\Lambda}H$)

$$\mathcal{P}(k_1,q_1,\sigma,b) = \frac{64\,b^6\,S_{^3\mathrm{He}}}{(b^2+2\sigma^2)^3\,3\sqrt{3}}\,e^{-b^2(k_1^2+q_1^2)}$$

$b=4.9\text{fm}$

$$S_3 = ({}^{3}_{\Lambda}\mathrm{H}/{}^{3}\mathrm{He})/(\Lambda/p)$$

^{3}He from Sun et al.

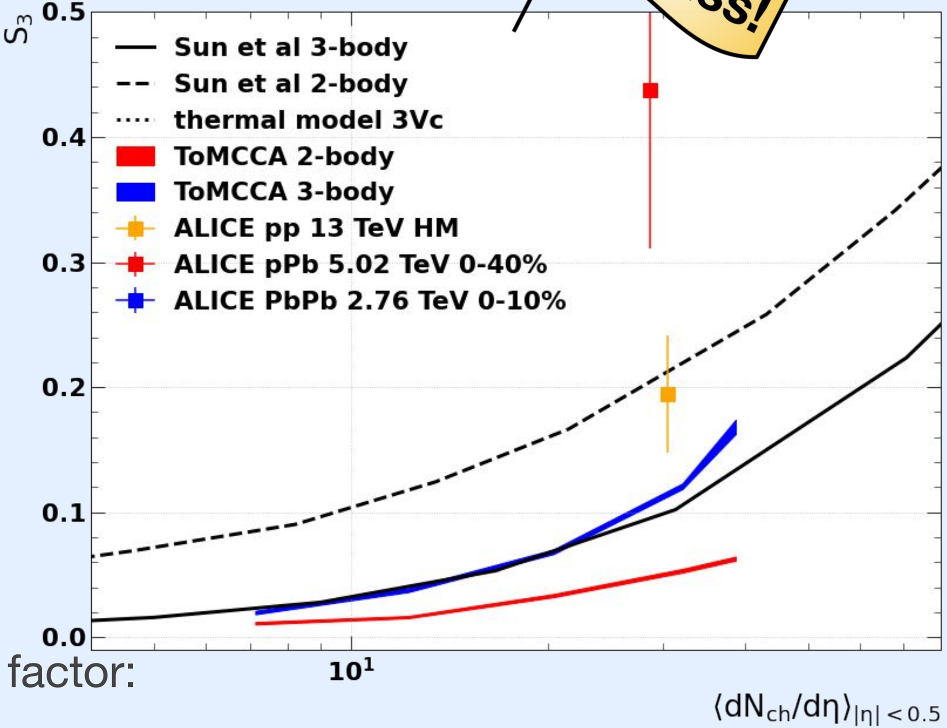
$\Lambda/p=0.84$ (const.)

Spin-Isospin factor:

-2/9 (3body)

-3/8 × 1/3 (2 body)

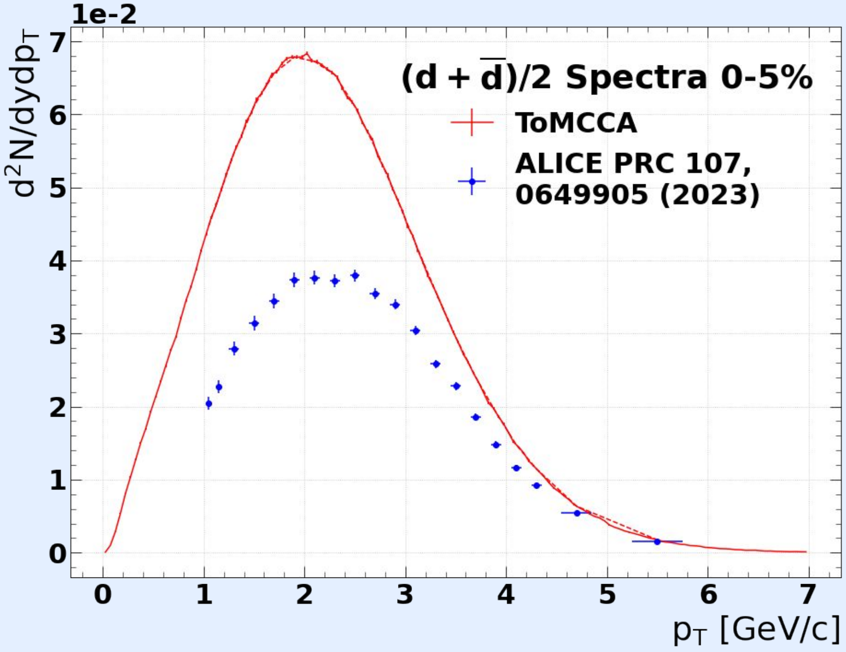
Discuss!



Sneak Peak: PbPb and Hypertriton

Extend Model to PbPb:

- Source from ongoing ALICE analysis
- Protons tuned in similar fashion as pp
- Ignore angular correlations for now (much weaker in PbPb)
- Model fails to reproduce data

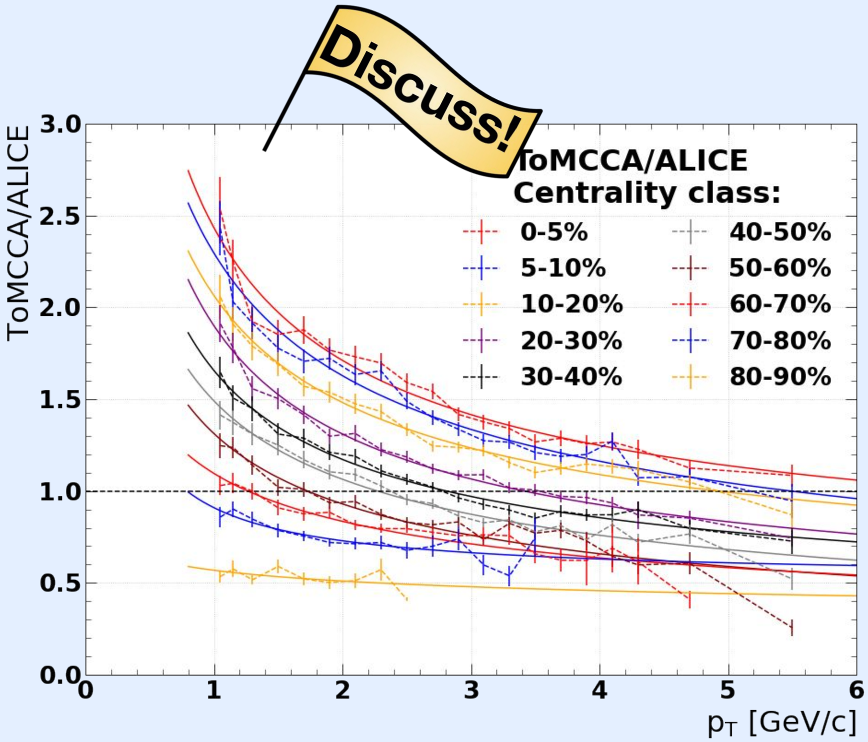


ToMCCA

Sneak Peak: PbPb and Hypertriton

Extend Model to PbPb:

- Source from ongoing ALICE analysis
- Protons tuned in similar fashion as pp
- Ignore angular correlations for now (much weaker in PbPb)
- Model fails to reproduce data
- Rescattering/Regeneration in the hadronic phase?
- Proton-neutron ρ_{pn} correlations?



Discussion

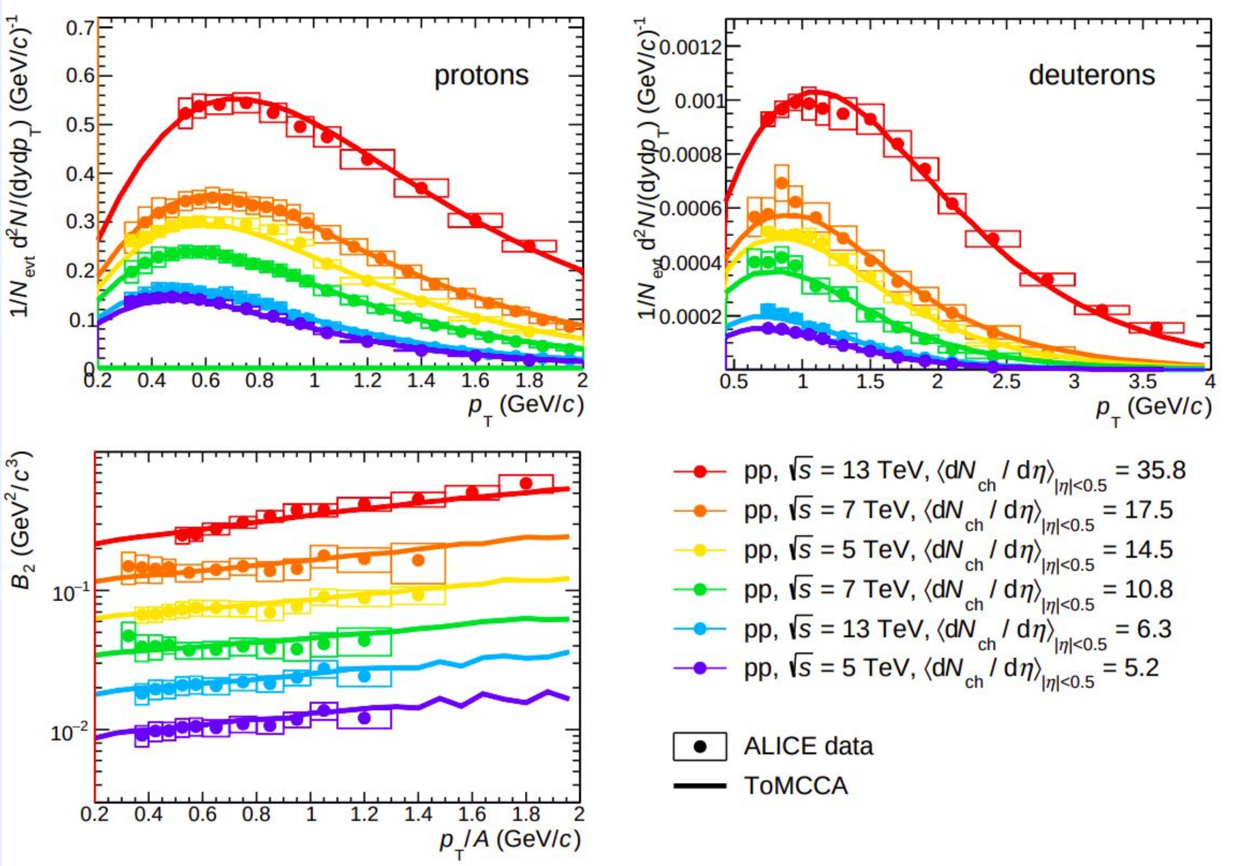
Summary of discussion points

1. Pearson Number Correlation

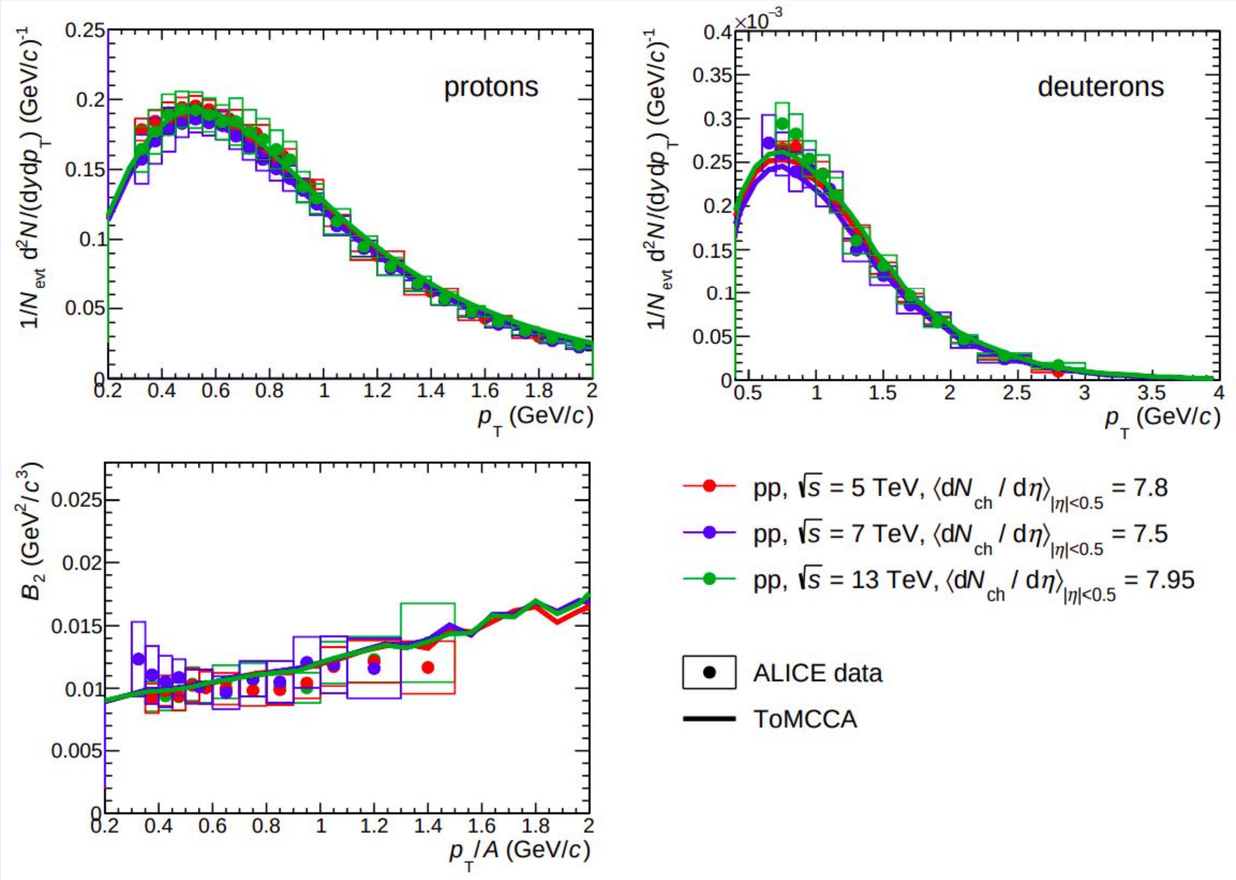
	Uncorrelated Emission	String Fragmentation	Quark recombination	Tuned Emission
ρ_{pn}	0	-0.052	-0.058	-0.024

2. $\Delta\phi$(Nch) and $\Delta\phi$(pT), pp➔pΛ scaling
3. Source size predictions
4. Extrapolation of this model to lower energies
5. Hypertriton size parameters
6. Failure in PbPb

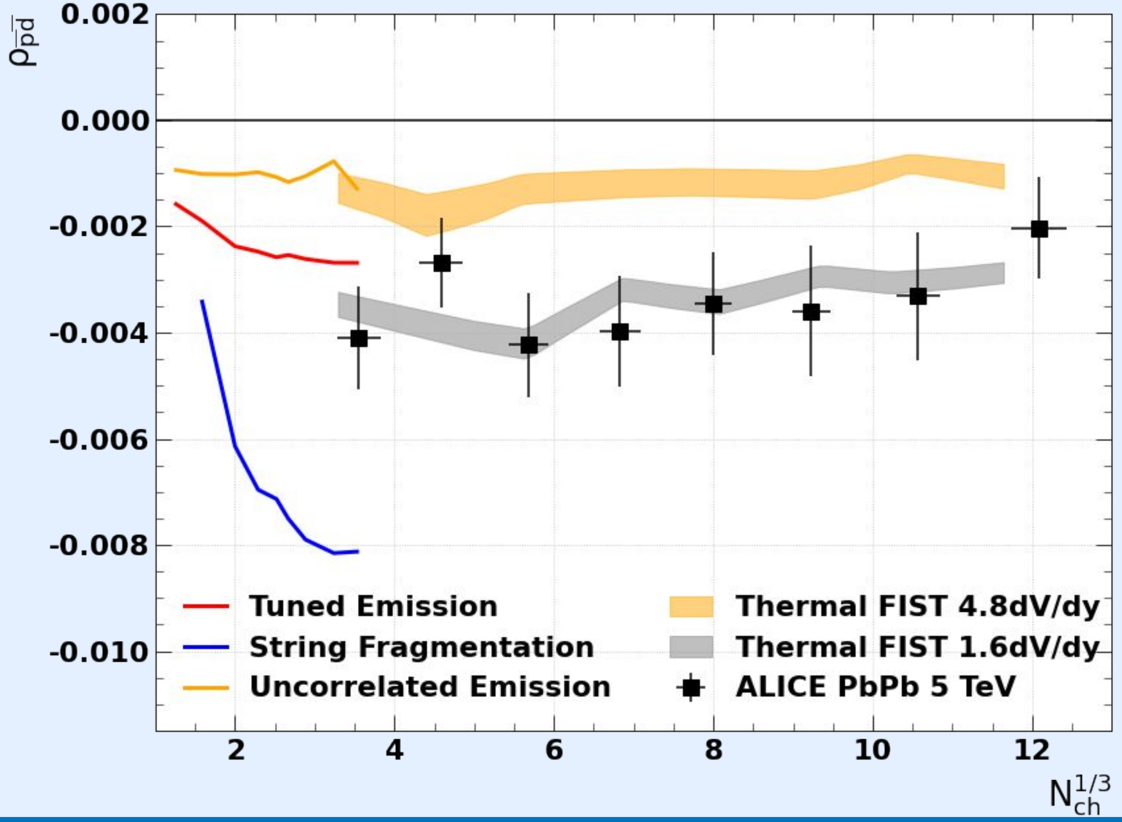
Additional Slides



protons
deuterons
$1/N_{evt}\ d^2N/(dydp_T)\ (GeV/c)^{-1}$
p_T (GeV/c)
B_2 (GeV^2/c^3)
p_T/A (GeV/c)
pp, $\sqrt{s}$ = 13 TeV, $\langle dN_{ch}/d\eta \rangle_{|\eta|<0.5}$ = 35.8
pp, $\sqrt{s}$ = 7 TeV, $\langle dN_{ch}/d\eta \rangle_{|\eta|<0.5}$ = 17.5
pp, $\sqrt{s}$ = 5 TeV, $\langle dN_{ch}/d\eta \rangle_{|\eta|<0.5}$ = 14.5
pp, $\sqrt{s}$ = 7 TeV, $\langle dN_{ch}/d\eta \rangle_{|\eta|<0.5}$ = 10.8
pp, $\sqrt{s}$ = 13 TeV, $\langle dN_{ch}/d\eta \rangle_{|\eta|<0.5}$ = 6.3
pp, $\sqrt{s}$ = 5 TeV, $\langle dN_{ch}/d\eta \rangle_{|\eta|<0.5}$ = 5.2
ALICE data
ToMCCA

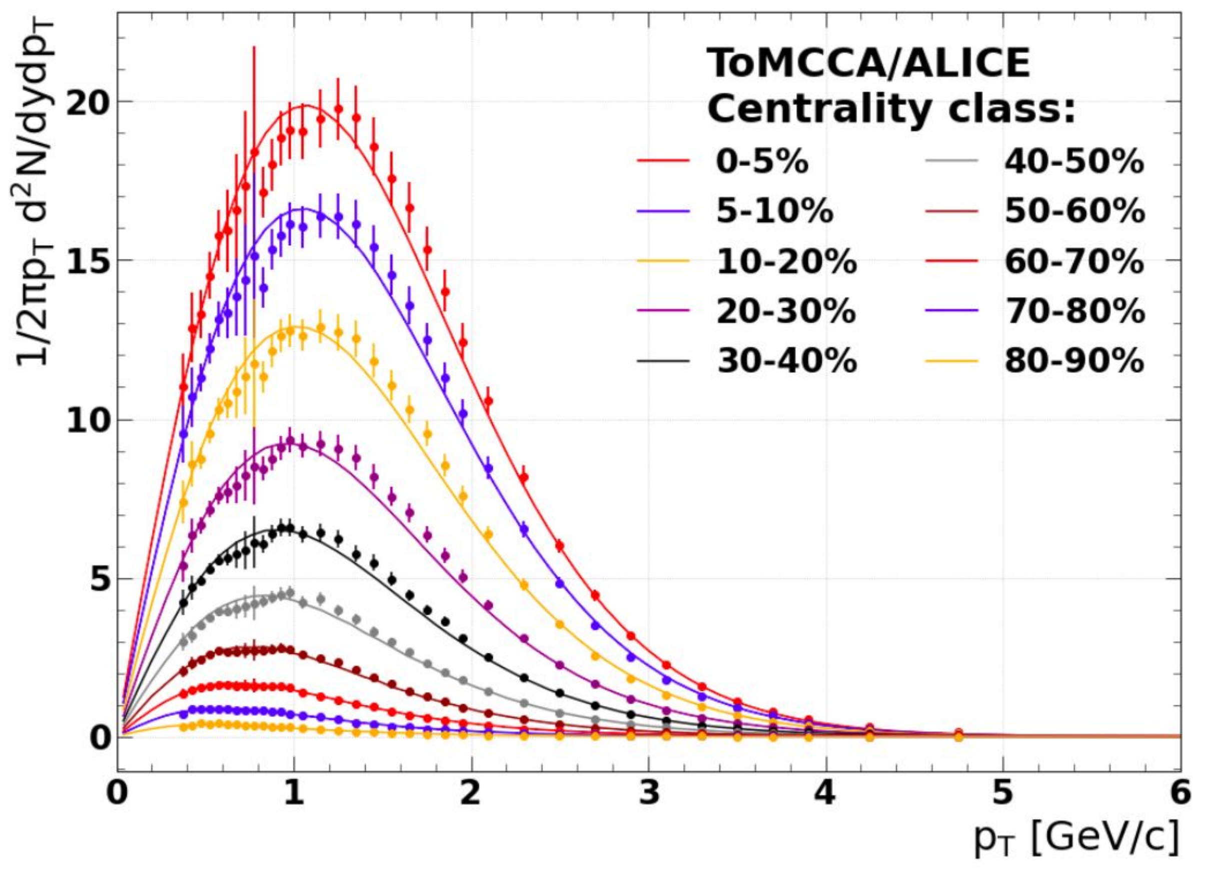

protons
deuterons
$1/N_{evt}\, d^2N/(dy dp_T)$ (GeV/c)$^{-1}$
p_T (GeV/c)
$\times 10^{-3}$
B_2 (GeV^2/c^3)
p_T/A (GeV/c)
pp, $\sqrt{s}$ = 5 TeV, $\langle dN_{ch} / d\eta \rangle_{|\eta|<0.5}$ = 7.8
pp, $\sqrt{s}$ = 7 TeV, $\langle dN_{ch} / d\eta \rangle_{|\eta|<0.5}$ = 7.5
pp, $\sqrt{s}$ = 13 TeV, $\langle dN_{ch} / d\eta \rangle_{|\eta|<0.5}$ = 7.95
ALICE data
ToMCCA

Rho_pd

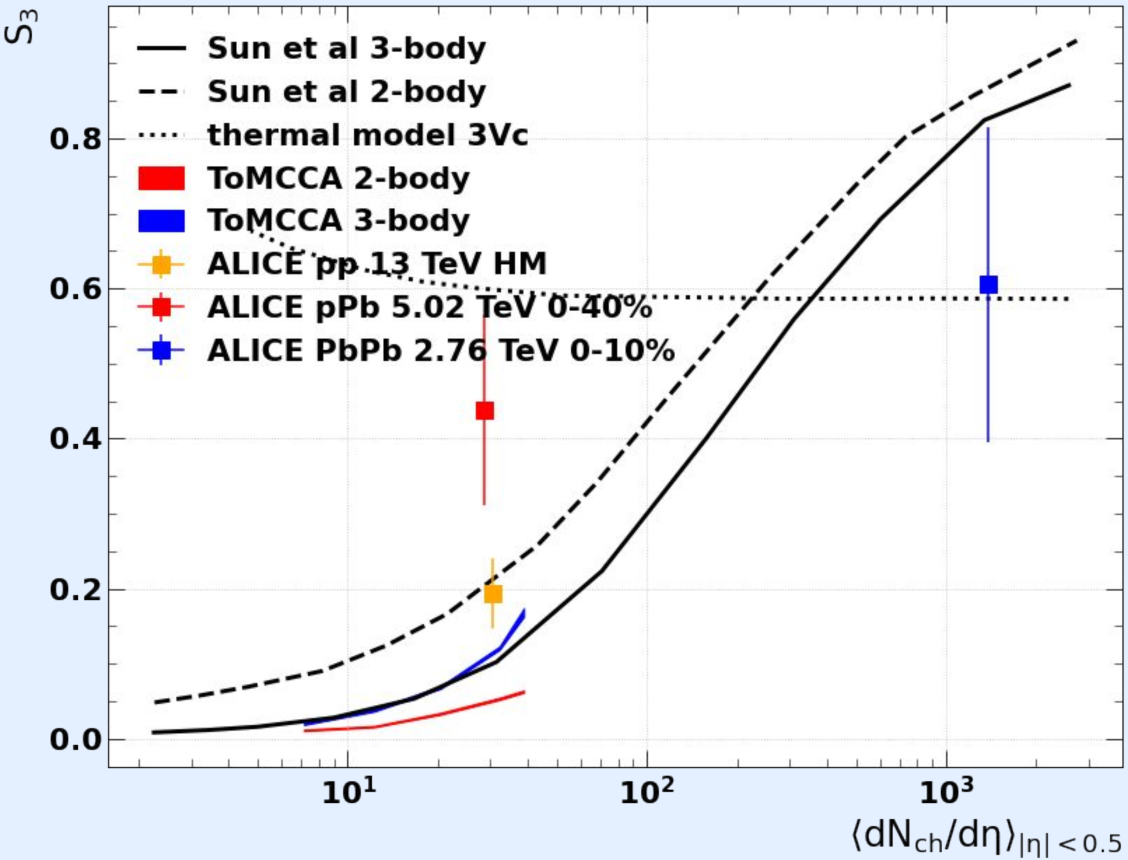


$\rho_{\bar{p}\bar{d}}$
0.002
0.000
-0.002
-0.004
-0.006
-0.008
-0.010
2
4
6
8
10
12
$N_{ch}^{1/3}$
Tuned Emission
String Fragmentation
Uncorrelated Emission
Thermal FIST 4.8dV/dy
Thermal FIST 1.6dV/dy
ALICE PbPb 5 TeV

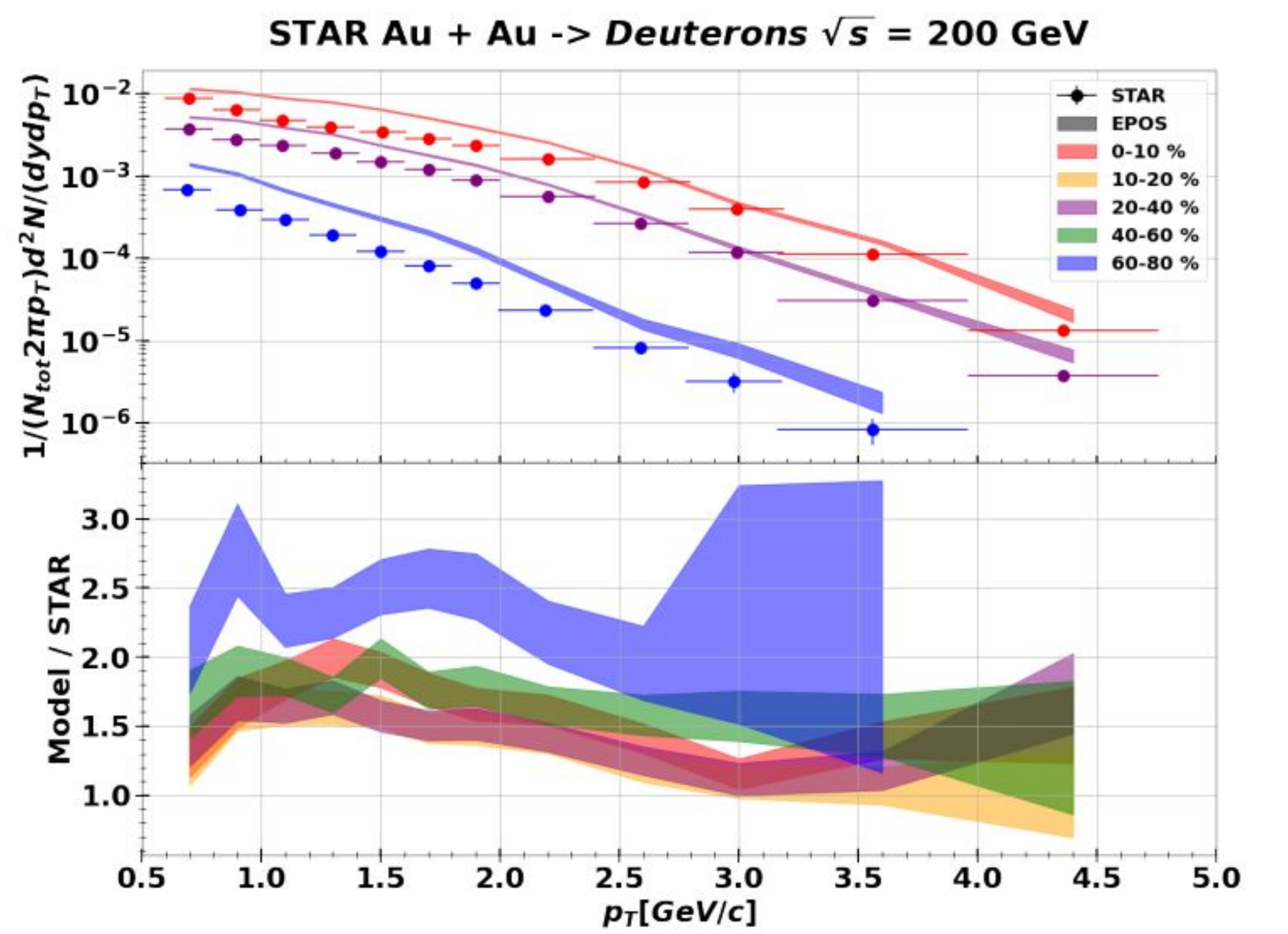
PbPb Protons



S_3 full range



S_3 full range



S_3 full range