Coalescence production of light nuclei in HIC

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- Introduction
- The coalescence model
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- Coalescence vs transport models
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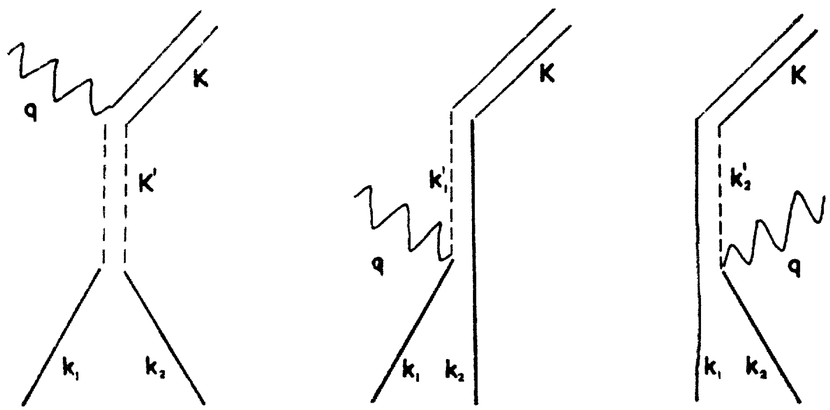
Introduction

1) Butler and Pearson, PR 129, 836 (1963): Two nucleons coalescence into a deuteron with the nuclear matter acting as a catalyzer. In second-order perturbation theory,



$$N_d(\mathbf{K}) \propto [N_p(\mathbf{K}/2)]^2$$

2) Schwalzschild and Zupancic, PR 129, 854 (1963): The deuteron-to-proton ratio is governed by the probability of finding a neutron within a small sphere of radius ρ around the proton in momentum space

$$dN_d(K)/dN_p(K) \propto \frac{4\pi\rho^3}{3}$$

Coalescence production of light nuclei at Bevalac

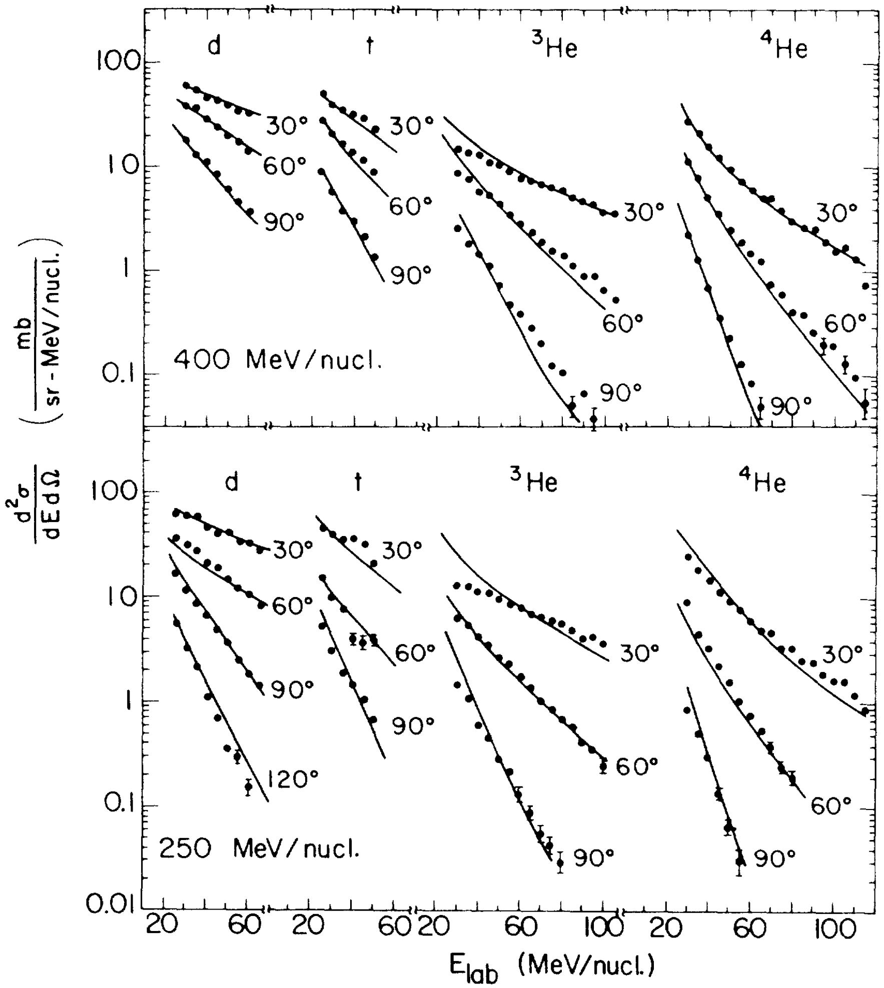


FIG. 3. Experimental points and calculated lines for the double-differential cross sections of fragments from the irradiation of uranium by ^{20}Ne ions at 250 and 400 MeV/nucleon.

Gutbrod et al., PRL 37, 667 (1976)

$$E_A \frac{d^3 N_A}{dp_A^3} = B_A \left(E_p \frac{d^3 N_p}{dp_p^3} \right)^A$$

$$B_A = \left(\frac{4\pi}{3} p_0^3 \right)^{A-1} \frac{M}{m^A}, \; p_A = A p_p$$

Coalescence radius p_0 (MeV)

Nuclei	250	400
d	126	129
t	140	129
^{3}He	135	129
^{4}He	157	142

Butler & Peterson, PR 129, 836 (1963)
Schwalzchild & Zupancic, PR 129, 854 (1963)

3) Bond, Johanson, Koonin, and Garpman, PLB 71, 43 (1977): Coalescence is formulated in the sudden approximation

4) Kapusta, PRC 21, 1301 (1980): Comparison of coalescence model, sudden approximation, and thermal model for deuteron production.

5) Sato and Yazaki, PLB 98, 153 (1981): Density matrix formalism for evaluating the coalescence parameter.

$$\text{Deuteron}: \qquad \frac{4\pi}{3}p_0^3 = \frac{3}{4}\cdot 2^{3/2}(4\pi)^{3/2}[\nu_d\nu/(\nu_d+\nu)]^{3/2}$$

$$\text{Triton}: \qquad \frac{1}{2}\left(\frac{4\pi}{3}p_0^3\right)^2 = \frac{1}{4}\cdot 2^{3/2}(4\pi)^{3}[\nu_t\nu/(\nu_t+\nu)]^{3}$$

$$\text{Alpha}: \qquad \frac{1}{4}\left(\frac{4\pi}{3}p_0^3\right)^3 = \frac{1}{16}\cdot 2^{3/2}(4\pi)^{9/2}[\nu_\alpha\nu/(\nu_\alpha+\nu)]^{9/2}$$

ν and ν_A are size parameters of emitting source and nuclei, respectively

→ B_A decreases with decreasing source size ($1/\nu$) and size ($1/\nu_A$) of nuclei

The Coalescence model

Wave functions for initial |i>=|1,2> and final |f>=|3> states

$$\langle \mathbf{r}_1, \mathbf{r}_2 | i \rangle = \phi_1(\mathbf{r}_1)\phi_2(\mathbf{r}_2)$$

$$\langle \mathbf{r}_1, \mathbf{r}_2 | f \rangle = \frac{1}{\sqrt{V}} e^{i\mathbf{K}\cdot(\mathbf{r}_1+\mathbf{r}_2)/2}\Phi(\mathbf{r}_1 - \mathbf{r}_2)$$

Probability for 1+2 -> 3 $\quad \mathcal{P} = |\langle f | i \rangle|^2$

Probability for particle 1 of momentum $\mathbf{k}_1$ and particle 2 of momentum $\mathbf{k}_2$ to coalescence to cluster 3 with momentum $\mathbf{K}$

$$\frac{dN}{d^3\mathbf{K}} = g \int d^3\mathbf{x}_1 d^3\mathbf{k}_1 d^3\mathbf{x}_2 d^3\mathbf{k}_2 W_1(\mathbf{x}_1, \mathbf{k}_1) W_2(\mathbf{x}_2, \mathbf{k}_2)$$

$$\times W(\mathbf{y}, \mathbf{k}) \delta^{(3)}(\mathbf{K} - \mathbf{k}_1 - \mathbf{k}_2), \qquad \mathbf{y} = \mathbf{x}_1 - \mathbf{x}_2, \quad \mathbf{k} = \frac{\mathbf{k}_1 - \mathbf{k}_2}{2}$$

Wigner functions $\quad W(\mathbf{x}, \mathbf{k}) = \int d^3\mathbf{y} \phi^* \left(\mathbf{x} - \frac{\mathbf{y}}{2}\right) \phi \left(\mathbf{x} + \frac{\mathbf{y}}{2}\right) e^{-i\mathbf{k}\cdot\mathbf{y}}$

For a system of particles 1 and 2 with phase-space distributions $f_i(\mathbf{x}_i,\mathbf{k}_i)$ normalized to $\int d^3\mathbf{x_i} d^3\mathbf{k}_i f_i(\mathbf{x}_i,\mathbf{k}_i) = N_i$, the number of particle 3 produced from coalescence of N_1 of particle 1 and N_2 of particle 2

$$\begin{aligned}\frac{dN}{d^3\mathbf{K}} &\approx g\int d^3\mathbf{x}_1 d^3\mathbf{k}_1 d^3\mathbf{x}_2 d^3\mathbf{k}_2 f_1(\mathbf{x}_1,\mathbf{k}_1) f_2(\mathbf{x}_2,\mathbf{k}_2)\\ &\times \overline{W}(\mathbf{y},\mathbf{k})\delta^{(3)}(\mathbf{K}-\mathbf{k}_1-\mathbf{k}_2)\end{aligned}$$

$$\overline{W}(\mathbf{y},\mathbf{k}) = \int \frac{d^3\mathbf{x}_1' d^3\mathbf{k}_1'}{(2\pi)^3}\frac{d^3\mathbf{x}_2' d^3\mathbf{k}_2'}{(2\pi)^3} W_1(\mathbf{x}_1',\mathbf{k}_1') W_2(\mathbf{x}_2',\mathbf{k}_2') W(\mathbf{y}',\mathbf{k}')$$

Wigner function $W_i(\mathbf{x}_i',\mathbf{k}_i')$ centers around $\mathbf{x}_i$ and $\mathbf{k}_i$

$g = \frac{2J+1}{(2J_1+1)(2J_2+1)}$ Statistical factor for two particles of spin J_1 and J_2 to form a particle of spin J

The above formula can be straightforwardly generalized to multi-particle coalescence but is usually used by taking particle Wigner functions as delta functions in space and momentum.

Gyulassy, Frankel, and Remler, NPA 402, 596 (1983): Generalized coalescence model using nucleon Wigner functions that are delta functions in space and momentum, i.e., evaluating

$$\overline{W}(\mathbf{y},\mathbf{k}) = \int \frac{d^3\mathbf{x}_1' d^3\mathbf{k}_1'}{(2\pi)^3} \frac{d^3\mathbf{x}_2' d^3\mathbf{k}_2'}{(2\pi)^3} W_1(\mathbf{x}_1',\mathbf{k}_1') W_2(\mathbf{x}_2',\mathbf{k}_2') W(\mathbf{y}',\mathbf{k}')$$

with $W_i(\mathbf{x}_i',\mathbf{k}_i') = (2\pi)^3 \delta^3(\mathbf{x}_i' - \mathbf{x}_i)\delta^3(\mathbf{k}_i' - \mathbf{k}_i)$

$$\rightarrow \frac{dN}{d^3\mathbf{K}} \approx g \int d^3\mathbf{x}_1 d^3\mathbf{k}_1 d^3\mathbf{x}_2 d^3\mathbf{k}_2 f_1(\mathbf{x}_1,\mathbf{k}_1) f_2(\mathbf{x}_2,\mathbf{k}_2) \times W(\mathbf{y},\mathbf{k})\delta^{(3)}(\mathbf{K}-\mathbf{k}_1-\mathbf{k}_2)$$

It is later called by Kahana et al. the standard Wigner calculation in contrast to the general one which they called the quantum Wigner calculation.

Quantum coalescence model

Han, Fries & Ko, PRC 93, 045207 (2016)
Kordell, Fries & Ko, Ann. Phys. 443, 168960 (2022)

In quantum Wigner approach, wave functions of particles 1 and 2 are Gaussian wave packets

$$\begin{aligned}\phi_i(x'_i - x_i) &= \frac{1}{(\pi\delta^2)^{1/4}} \exp\left[-\frac{(x'_i - x_i)^2}{2\delta^2}\right] \exp(ik_i x'_i) \\ W(x'_i, k'_i) &= 2e^{-(x'_i - x_i)^2/\delta^2} e^{-\delta^2(k'_i - k_i)^2}\end{aligned}$$

Using harmonic oscillator wave functions for the wave function of formed particle 3, which gives its Wigner function as

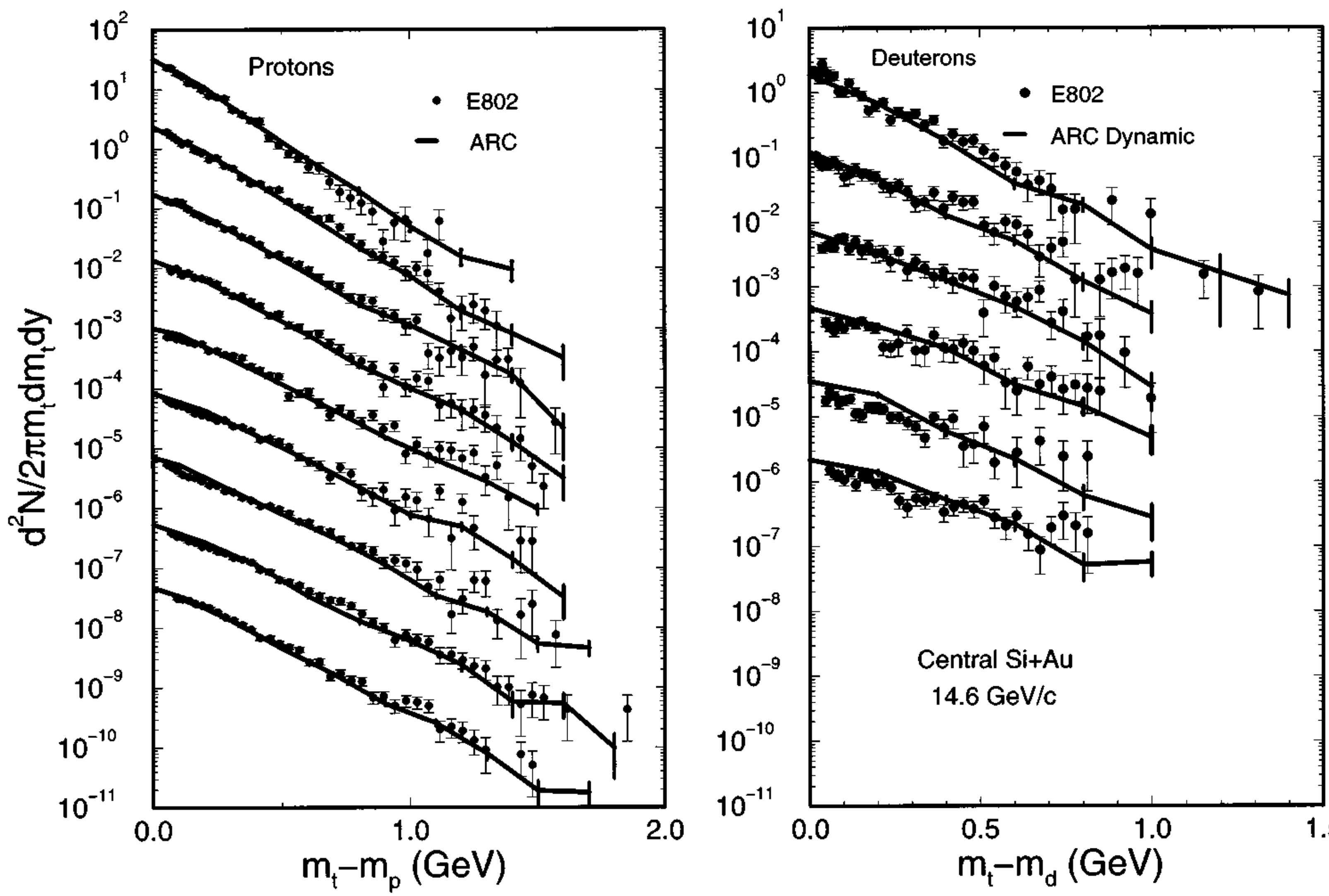
$$W_{\mathrm{d}}(\mathbf{x}, \mathbf{p}) = 8 \exp\left(-\frac{x^2}{\sigma^2} - \sigma^2 p^2\right)$$

For $\delta = \sigma$, one has

$$\overline{W}(\mathbf{x}, \mathbf{p}) = 8e^{-\frac{1}{2}\left(\frac{\mathbf{x}^2}{\sigma^2} + \mathbf{p}^2\sigma^2\right)}$$

Coalescence production of light nuclei at AGS

Kahana et al., PRC 54, 388 (1996)

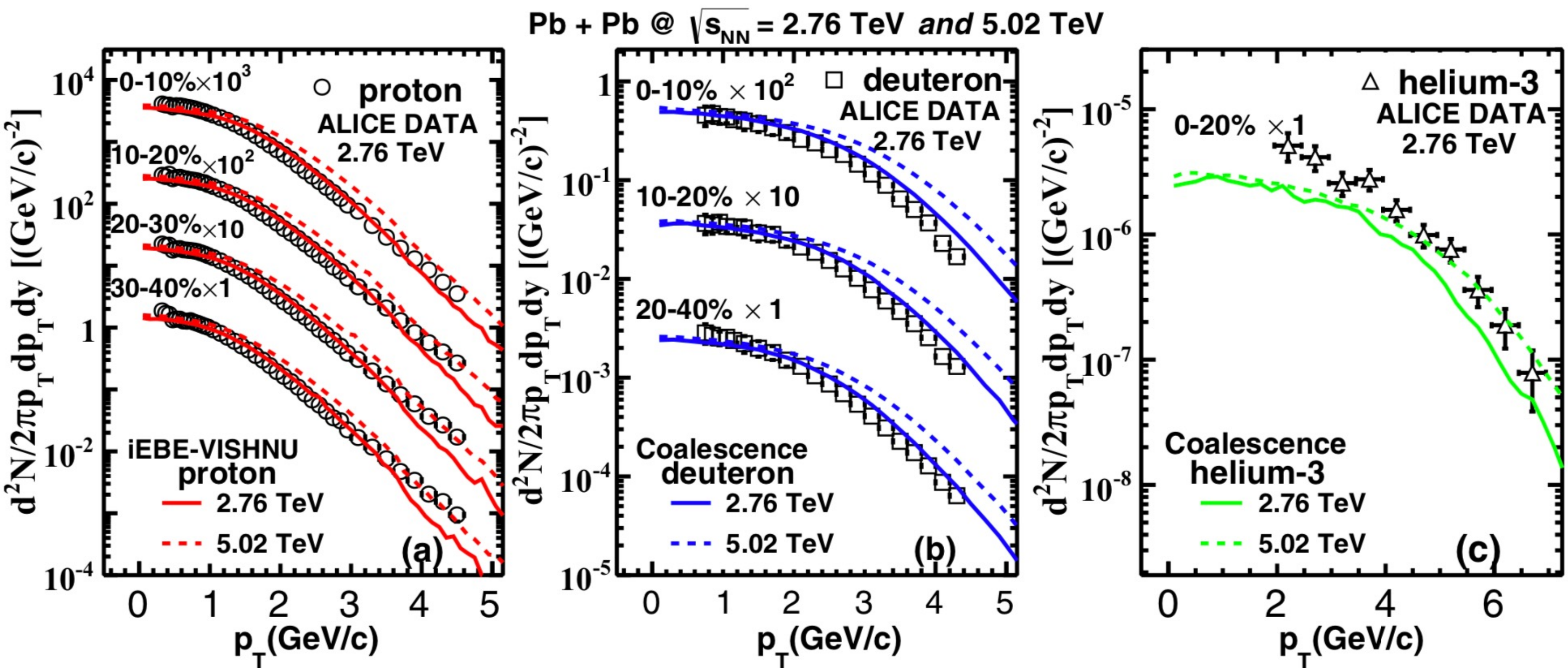


PHYSICAL REVIEW C **98**, 054905 (2018)

Spectra and flow of light nuclei in relativistic heavy ion collisions at energies available at the BNL Relativistic Heavy Ion Collider and at the CERN Large Hadron Collider

Wenbin Zhao,[1,2] Lilin Zhu,[3] Hua Zheng,[4,5] Che Ming Ko,[6] and Huichao Song[1,2,7]

IEBE-VISHNU hybrid model with AMPT initial conditions



Elliptic flow of deuteron measured by ALICE is also satisfactorily described.

Particle yields in thermal model

Braun-Munzinger and Donigus, NPA 987, 144 (2019)

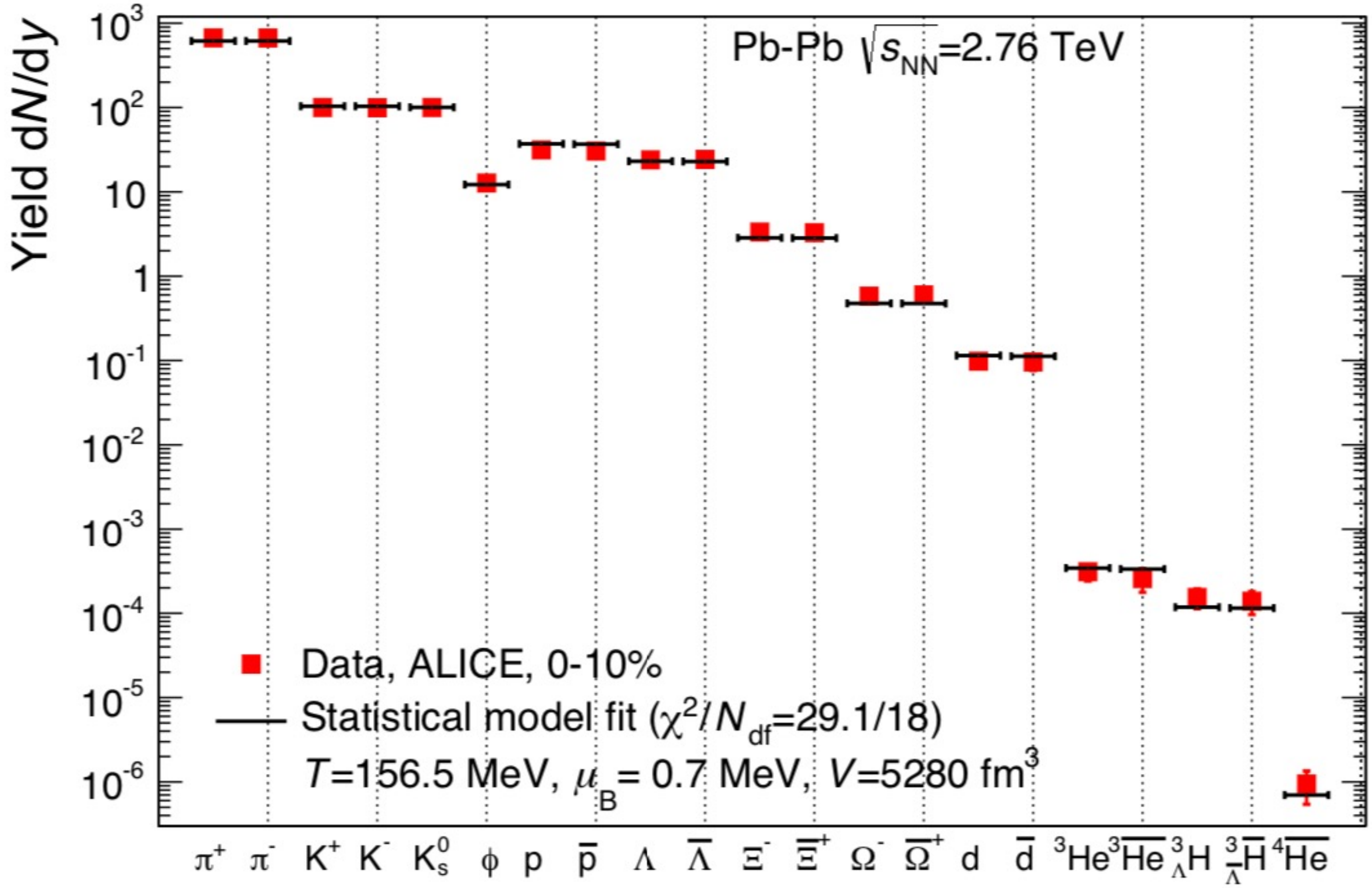


Figure 11: Thermal model description of the production yields (rapidity density) of different particle species in heavy-ion collisions at the LHC for a chemical freeze-out temperature of 156.5 MeV (from [59], where more details can be found, see also [60]).

Coalescence vs statistical production of deuteron

With N_p protons and N_n neutrons of temperature T uniformly distributed in V, the deuteron number in coalescence model with Gaussian Wigner function of width parameter σ for deuteron is

$$N_d^{\text{coal}} = \frac{3}{2^{1/2}}\left(\frac{2\pi}{mT}\right)^{3/2}\frac{1}{\left(1+\frac{1}{mT\sigma^2}\right)^{3/2}}\frac{N_pN_n}{V}$$

while that in the thermal model is

$$N_d^{\text{thermal}} = \frac{3}{2^{1/2}}\left(\frac{2\pi}{mT}\right)^{3/2}\frac{N_pN_n}{V}e^{B_d/T},$$

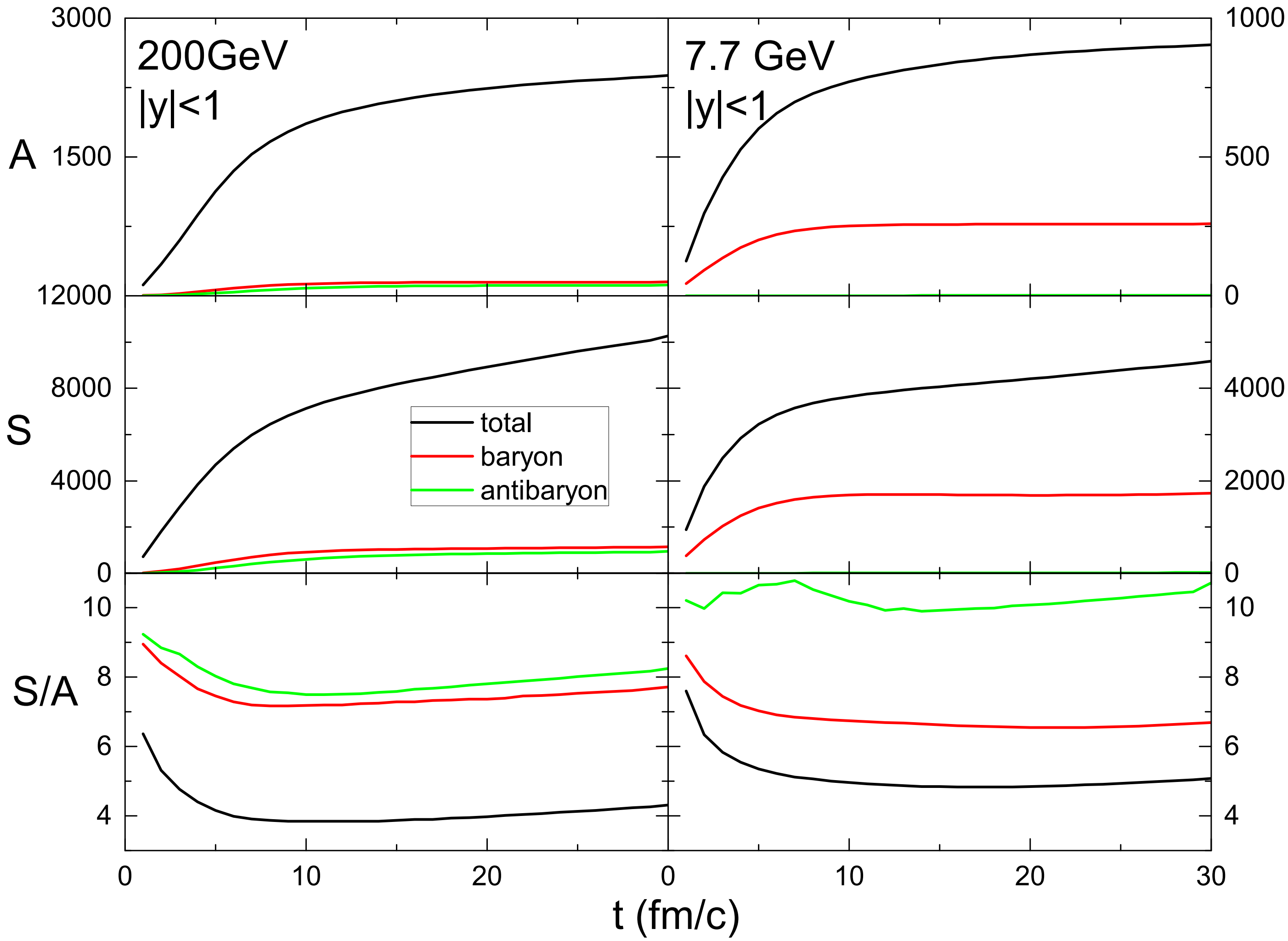
where B_d is deuteron binding energy. So

$$N_d^{\text{coal}} = N_d^{\text{thermal}} \quad \text{if } T >> B_d \text{ and } mT >> 1/\sigma^2,$$

i.e., temperature of nucleons is much larger than deuteron binding energy and nucleon thermal wavelength is much smaller than deuteron size.

Why $N_d^{\text{thermal}}(T_C) = N_d^{\text{thermal}}(T_K)$?

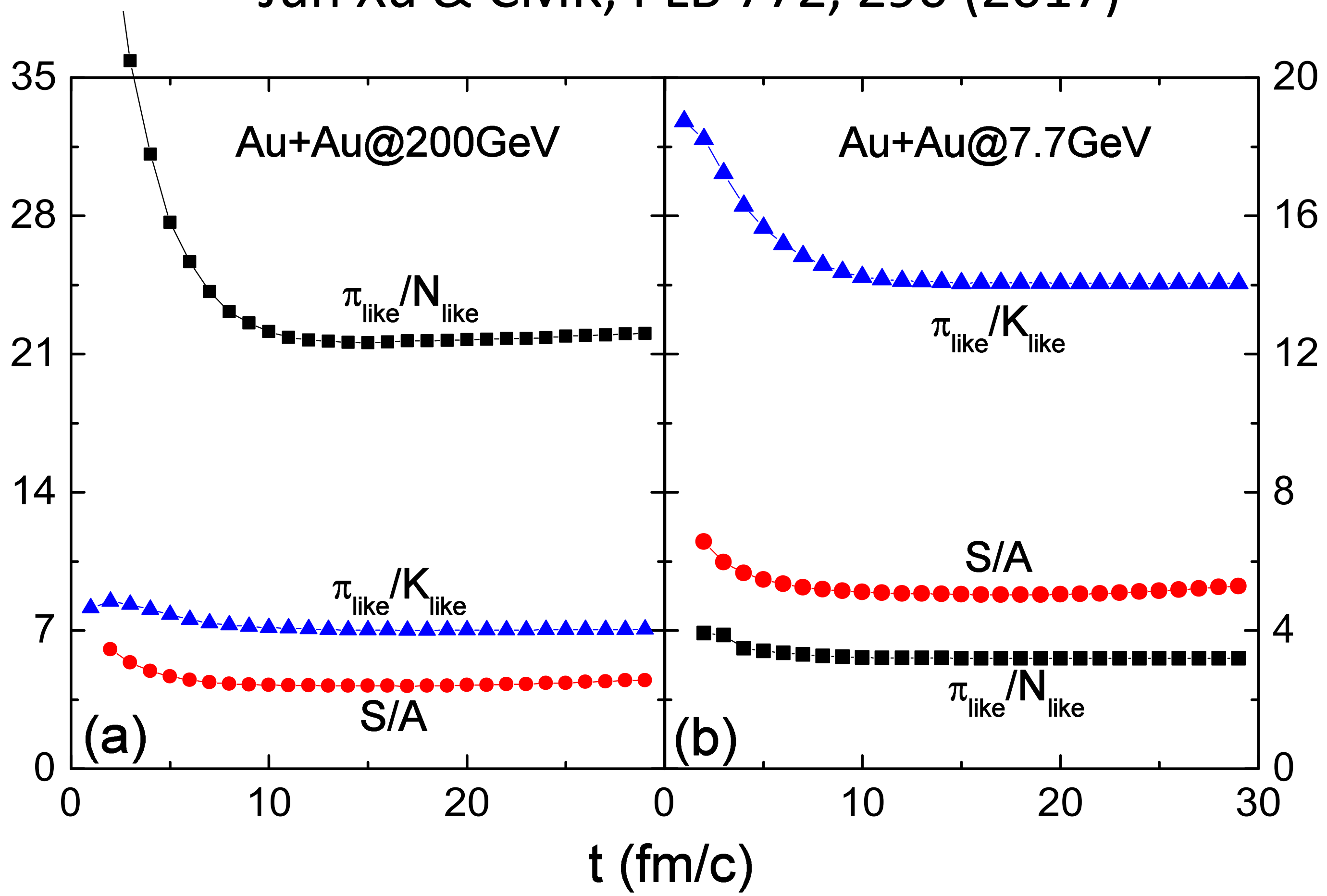
Time evolution of baryon entropy in relativistic heavy ion collisions from the hadronic phase of AMPT



- Baryon entropy per baryon remains essentially constant during hadronic evolution, thus similar d/p ratio at T_C and T_K.

Chemical freeze-out in relativistic heavy ion collisions

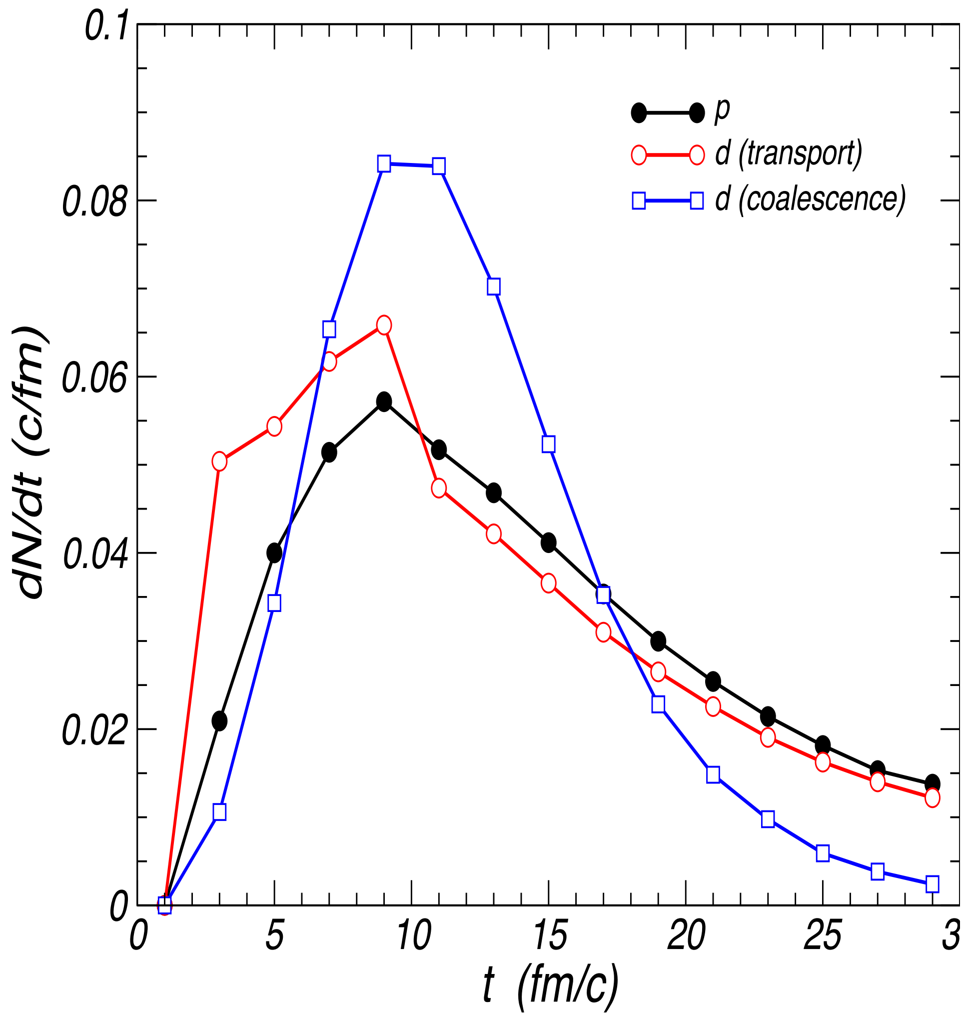
Jun Xu & CMK, PLB 772, 290 (2017)



- Both ratio of effective particle numbers and entropy per particle remain essentially constant from chemical to kinetic freeze-out.

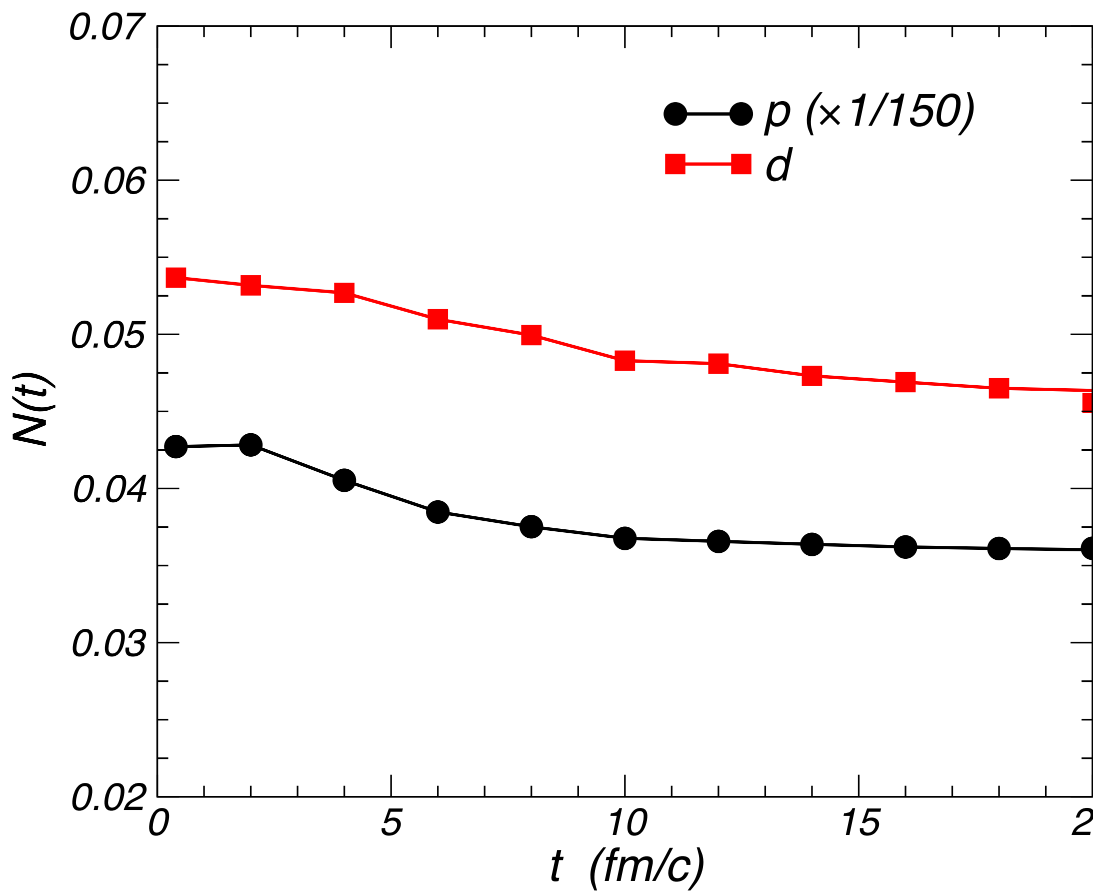
Deuteron production from an extended ART model

Oh & Ko, PRC 76, 054910 (2007); Oh, Lin & Ko, PRC 80, 064902 (2009)



- Include deuteron production ($n+p \rightarrow d+\pi$) and annihilation ($d+\pi \rightarrow n+p$) as well as its elastic scattering
- Similar emission time distributions for protons and deuterons in coalescence model
- Slightly different deuteron emission time distribution in transport and coalescence models

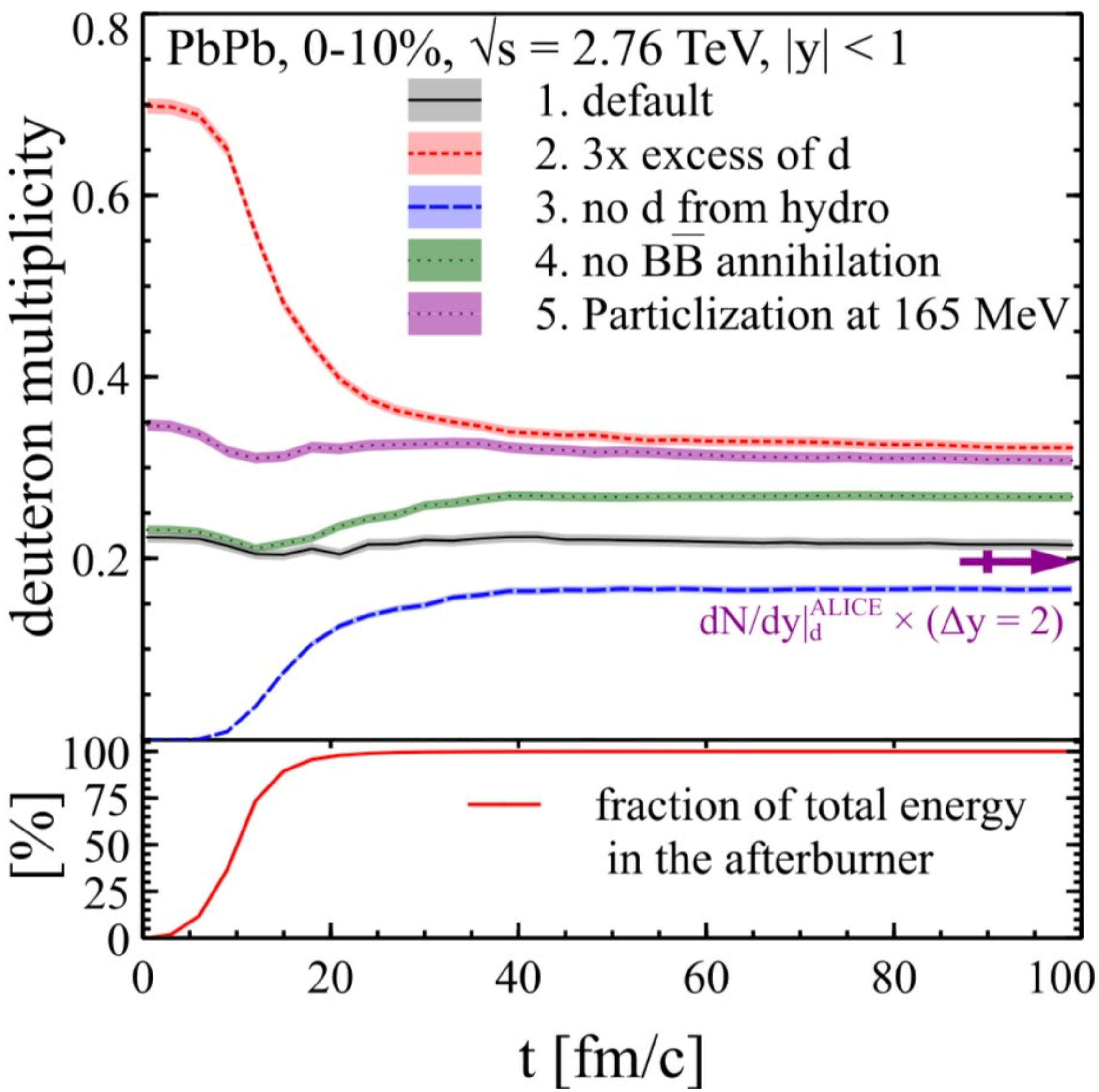
Time evolution of proton and deuteron numbers



- Both proton and deuteron numbers decrease only slightly with time $\longrightarrow$ early chemical equilibration

Deuteron production in SMASH

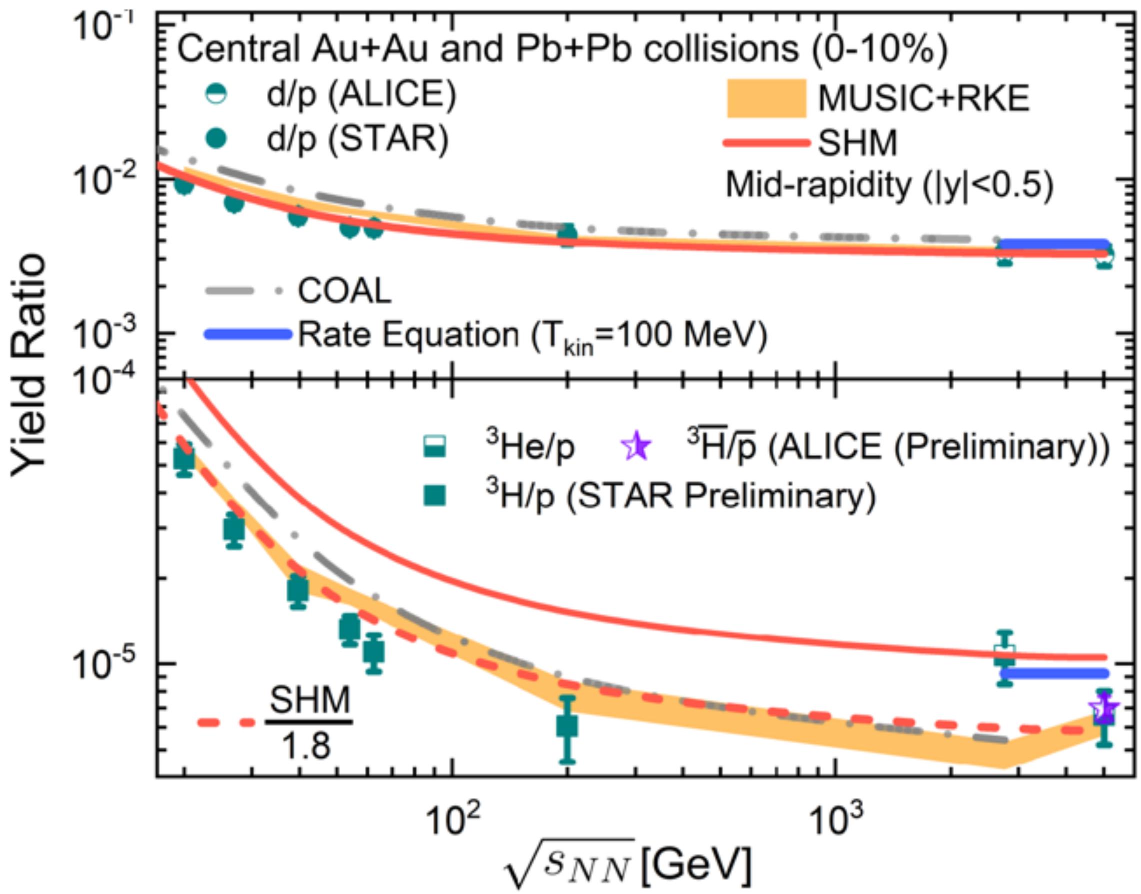
Oliinychekov, Pang, Elfner & Koch, PRC 99, 044907 (2019)



- Using a large $\pi NN \leftrightarrow \pi d$ cross section of about 100 mb.
- Deuteron number essentially remains unchanged during hadronic evolution

Hadronic rescattering effects on light nuclei production

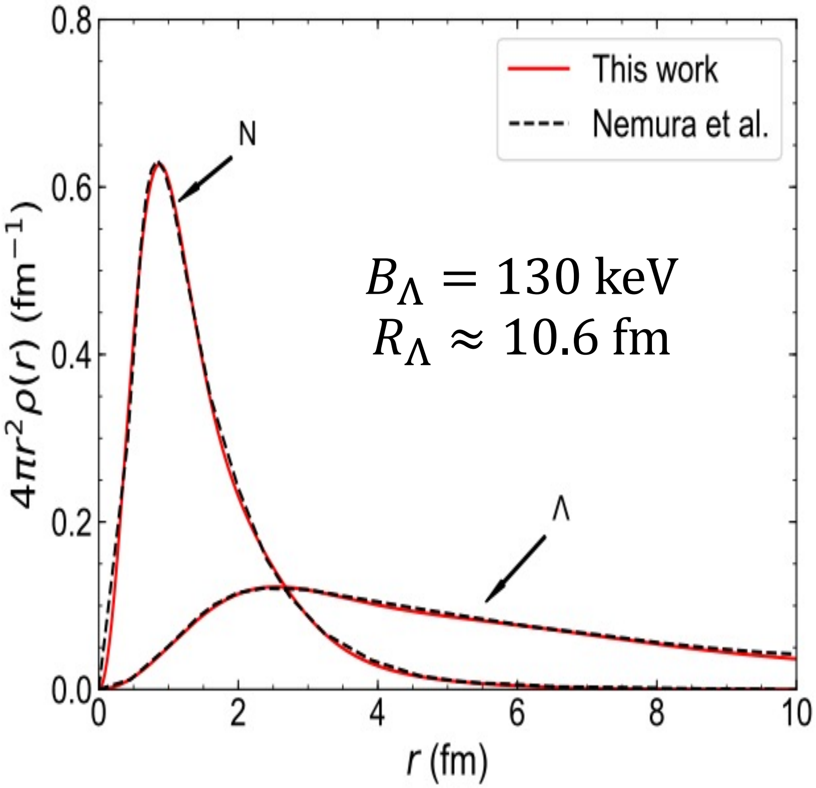
Sun et al., arXiv:2207.12532 [nucl-th]

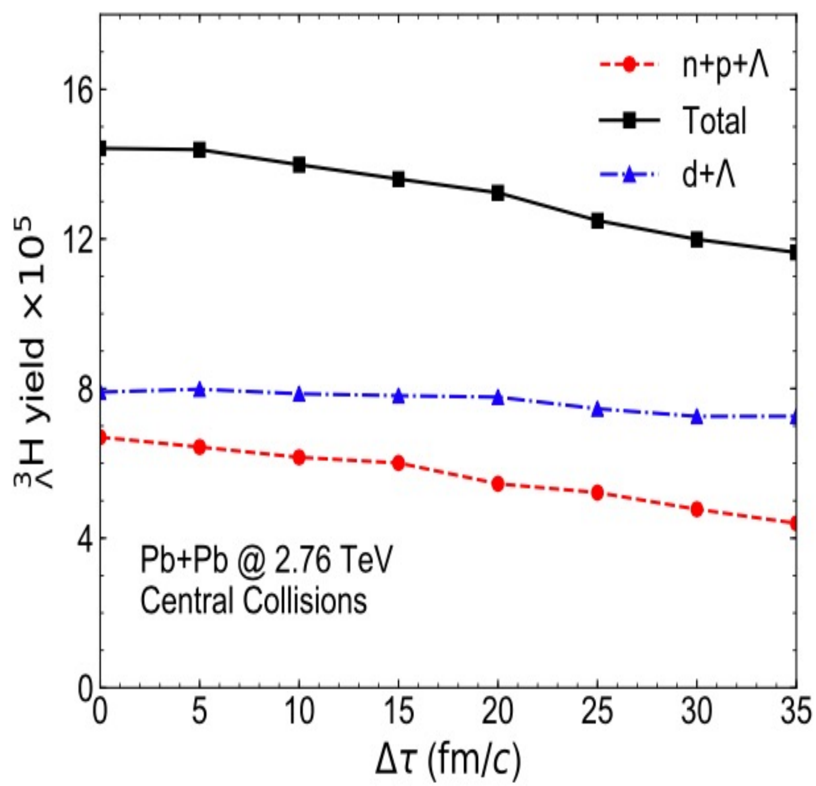


- *d/p* and *t/p* ratios are similar in kinetic approach and coalescence model.
- Hadronic re-scatterings reduce the triton yields by about a factor of 2 as a result of constant $tp/d^2 = 1/2\sqrt{3}$ and decreasing d/p due to decay of baryon resonances as the hadronic matter expands and cools.

Hypertriton production in coalescence model

Zhang & Ko, PLB 780, 191 (2018)

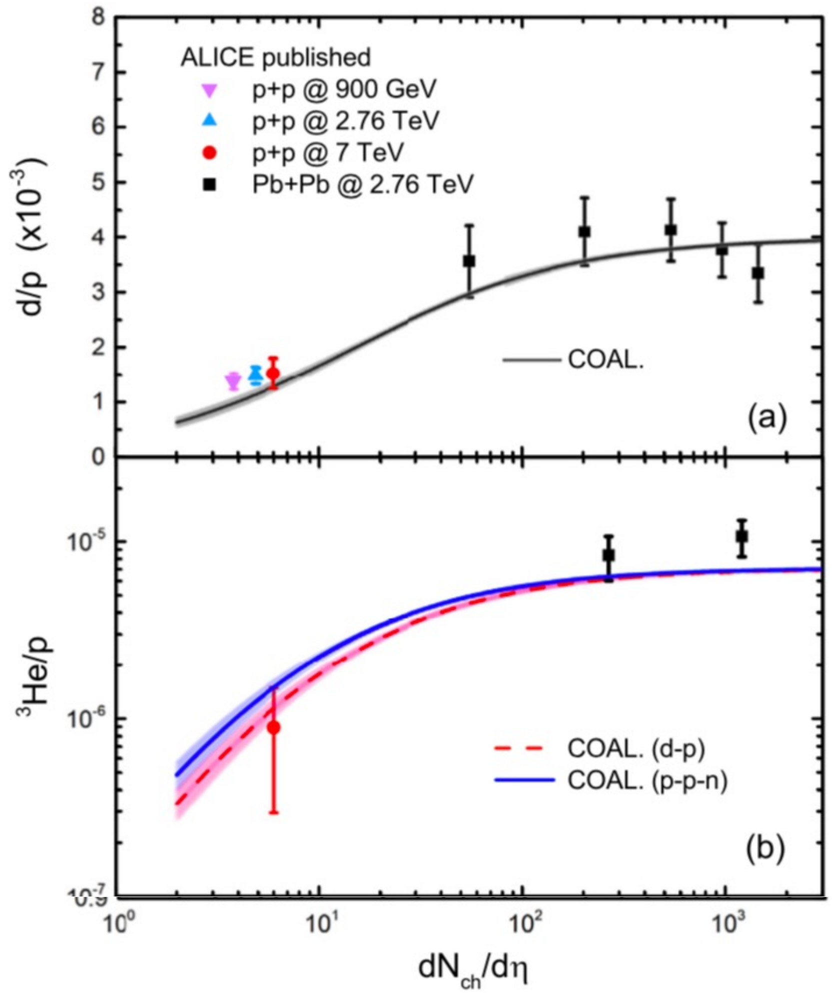




- Because of its large size, hypertriton yield changes little after long free streaming of kinetic freeze out nucleons if produced from their coalescence.

System size dependence of light nuclei yield

Sun, Ko & Doenigus, PLB 792, 132 (2019)



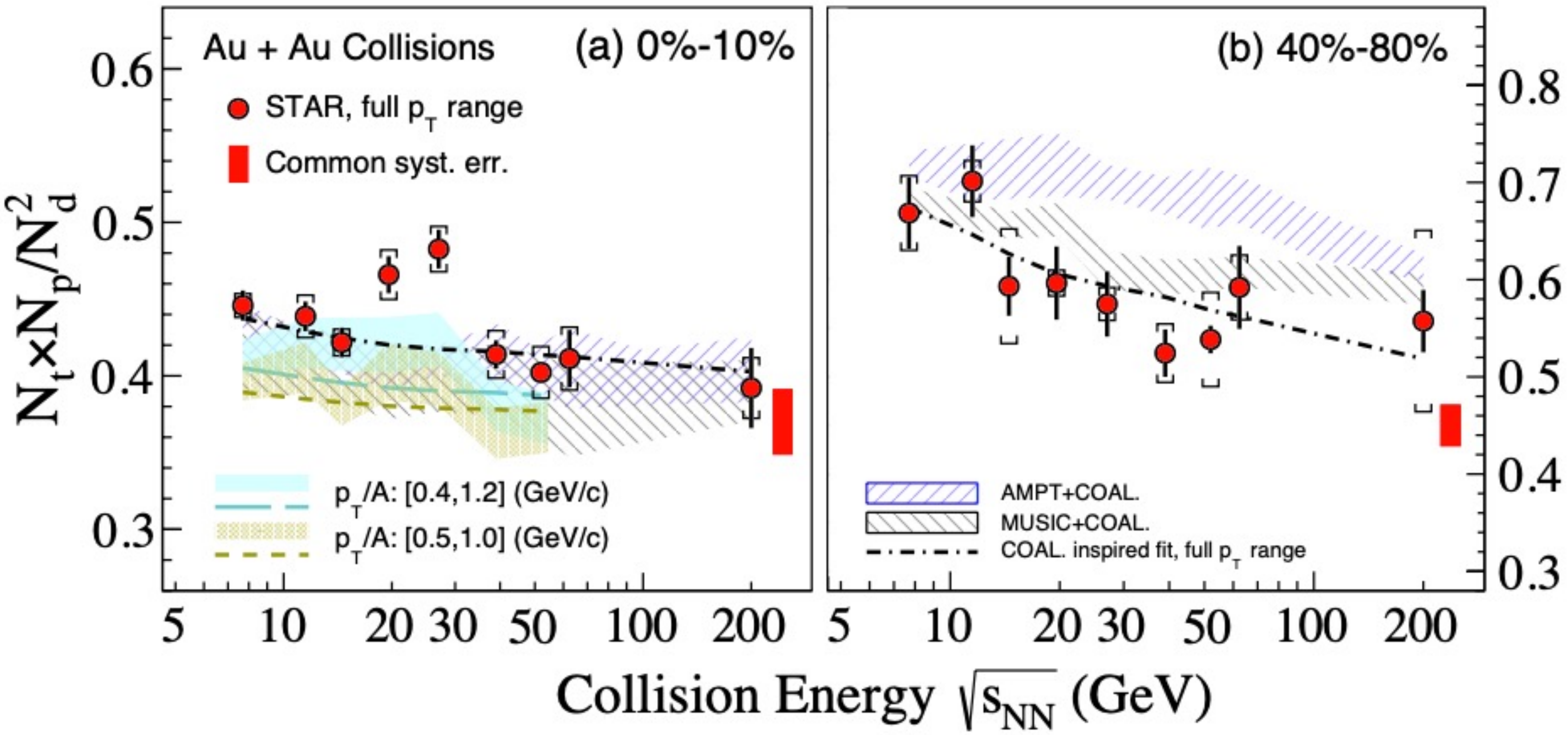
$$\frac{N_d}{N_p} \approx \frac{3N_n}{4(mT_K R^2)^{3/2}} \frac{1}{1+\frac{2r_d^2}{3R^2}}$$

$$\frac{N_{^3He}}{N_p} \approx \frac{N_n N_p}{4(mT_K R^2)^{3/2}} \frac{1}{1+\frac{r_{^3He}^2}{2R^2}}$$

- Coalescence model gives a natural explanation for the suppressed production of light nuclei in small collision systems.
- Thermal model requires an unrealistically large canonical correlation volume for charge conservation. [Vovchenko, Doenigus & Stoecker, PLB 785, 171 (2018)]

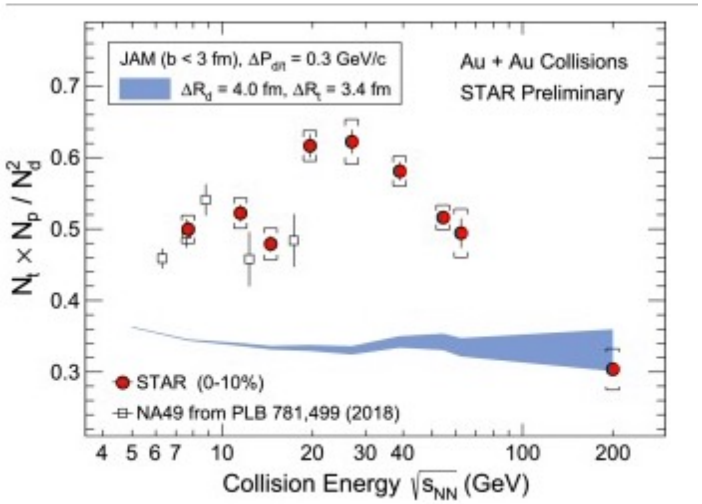
Yield ratio of $N_t N_p / N_d^2$ in Au+Au collisions at RHIC

STAR Collaboration, arXiv:2209.08058 [nucl-ex]

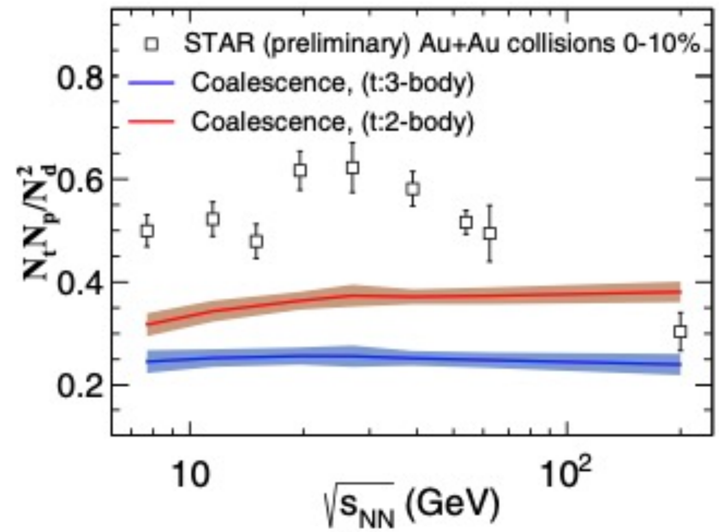


- Enhanced yield ratio of $N_t N_p / N_d^2$ at $\sqrt{s_{NN}} \approx 25$ GeV in central Au+Au collisions, compared to non-central collisions.
- Is the enhancement due to neutron density fluctuation? Both coalescence and statistical hadronizstion models predict neutron density fluctuations lead to $tp/d^2 \approx \frac{1}{2\sqrt{3}}(1 + \Delta\rho_n)$. Sun et al., EPJA 57, 313 (2021)

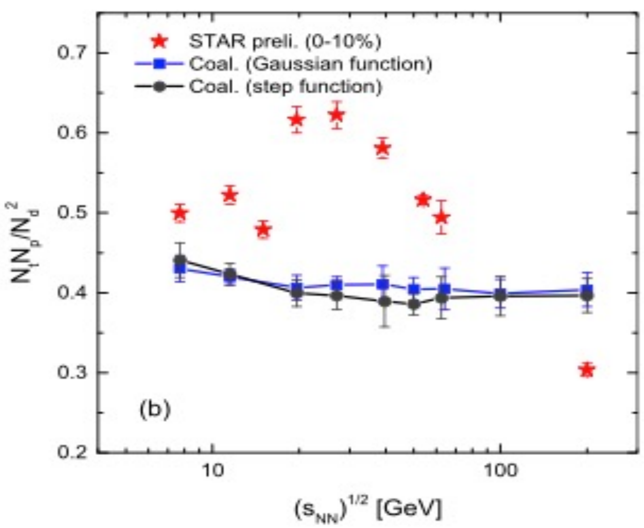
Beam-energy depdence of $N_t N_p / N_d^2$ from theoretical models



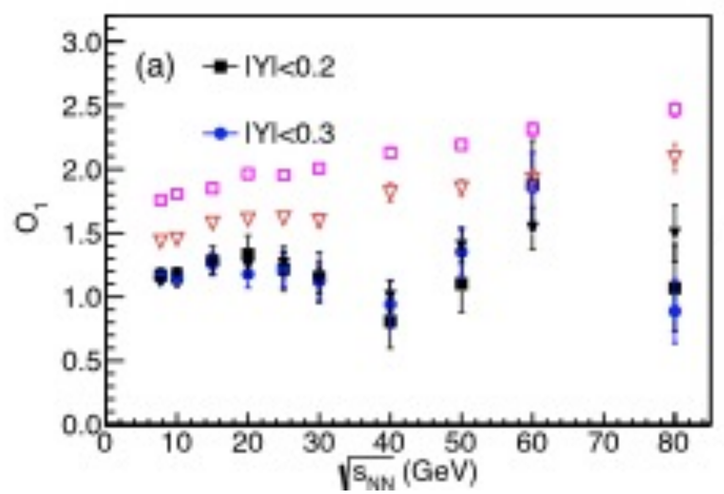
Liu, Zhang, He, Sun, Yu, Luo, PLB 805, 135452 (2020): JAM



Zhao, Shen, Ko, Liu & Song, PRC 102, 044912 (2020). IEBE+MUSIC+UrQMD



Sun, Ko & Lin, PRC 103, 064909 (2021); AMPT



Deng & Ma, PLB 808, 135668 (2020): UrQMD

Summary

- Coalescence model gives similar light nuclei yields in HIC as the thermal model if their binding energies are small compared to the temperature of the hadronic matter and their thermal wave lengths are much smaller than their sizes. Both results are similar to that from transport model studies in which deuterons are assumed to remain bounded and can be produced and dissociated.
- Hypertriton is expected to be produced significantly later after nucleons and lambda have frozen out because of its large size and small binding.
- Coalescence model can naturally explain the suppressed production of light nuclei in collisions of small systems.
- Nucleon density fluctuations enhance the production of light nuclei, providing a possible explanation for the nonmonotonic collisions energy dependence observed in the RHIC beam energy scan experiments.