The role of multi-particle correlations in light nuclei and hyperon production at RHIC

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Femtoscopy
STAR
- Light nuclei
- Hyperons

EMMI Workshop „Understanding light (anti-)nuclei production at RHIC and LHC"
8-12 April 2024
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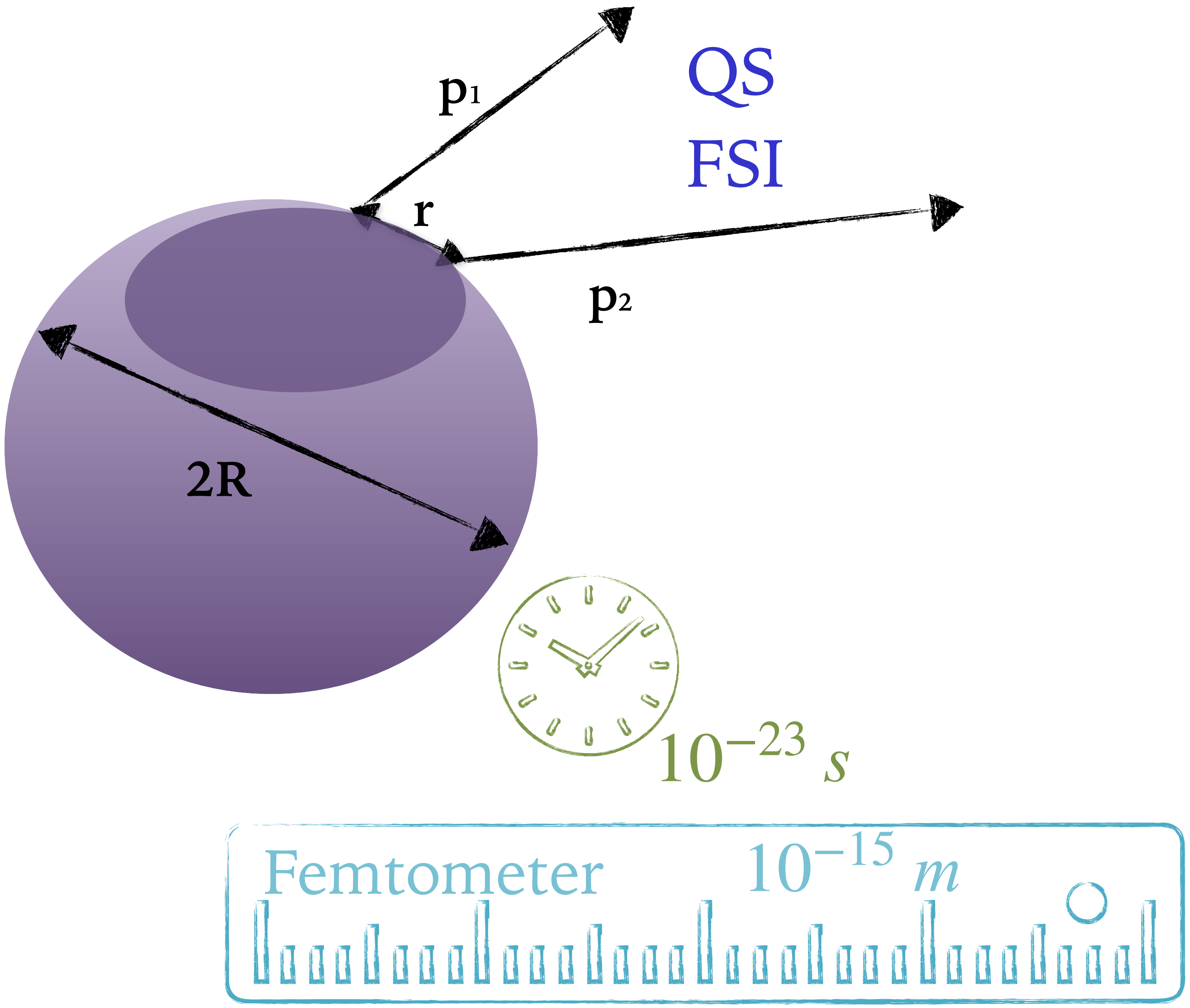
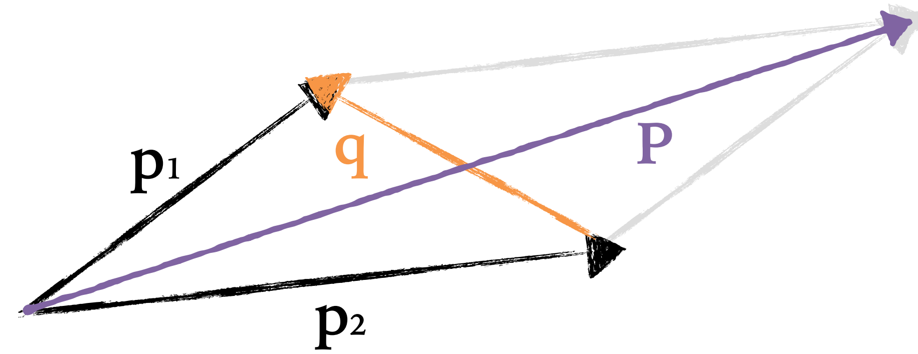
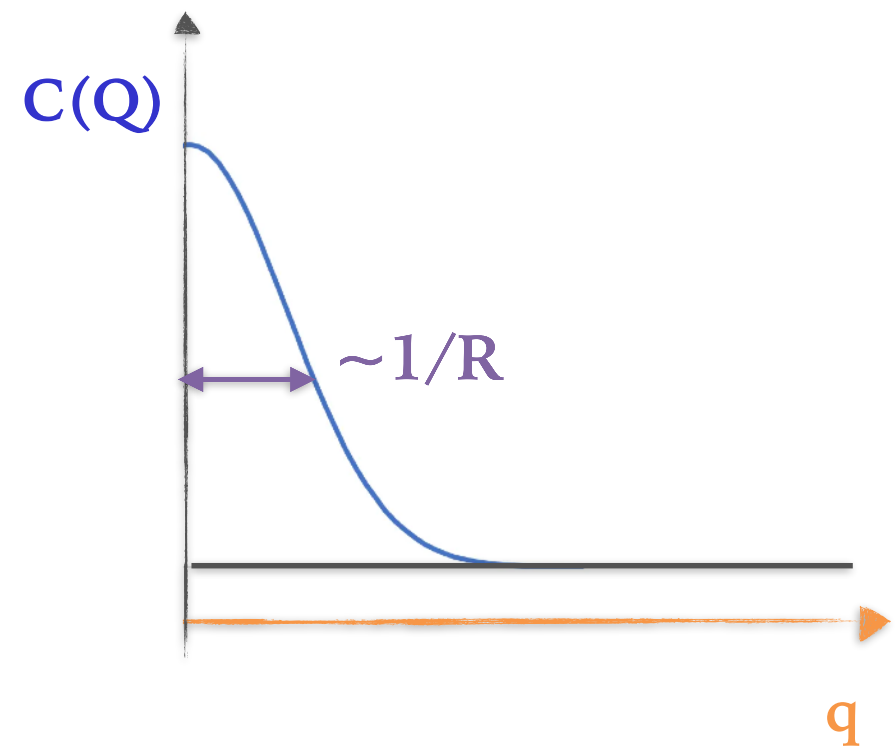






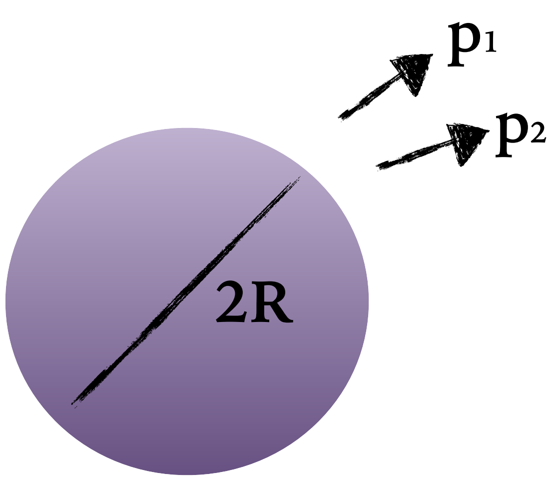
Femtoscopy

… the method to probe **geometric** and **dynamic** properties of the source (emission region, range of correlations-interactions, phase-space cloud, …)

Femtoscopy does not measure the whole source, but homogeneity length.

Classic femtoscopy

Femtoscopy (originating from HBT):
the method to probe **geometric** and **dynamic** properties of the source



Space-time properties ($10^{-15}m$, $10^{-23}s$) determined thanks to two-particle correlations:

Quantum Statistics (Fermi-Dirac, Bose-Einstein);

Final State Interactions (Coulomb, strong)

determined | assumed | measured

$$C(k^*, r^*) = \int S(r^*)\,|\Psi(k^*, r^*)|^2\, d^3r^* = \frac{Sgnl(k^*)}{Bckg(k^*)}$$

k^* - momentum of the first particle in the Pair Rest Frame reference

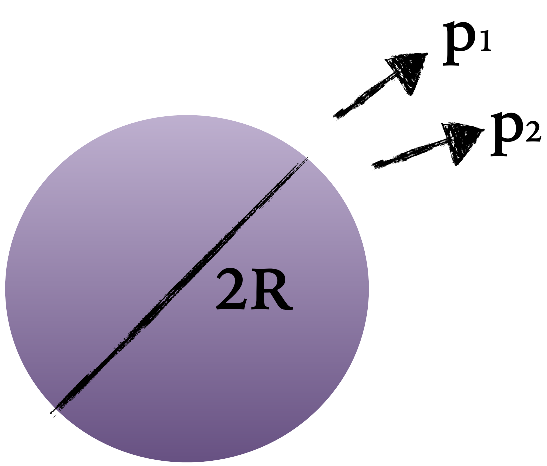
$S(r^*)$ - source function

$\Psi(k^*, r^*)$ - two-particle wave function (includes e.g. FSI interactions)

$\frac{Sgnl(k^*)}{Bckg(k^*)}$ - correlation function

Gateway to study interactions

If we assume we know the **source function**, measured **correlations** are used to determine **interactions in the final state.**



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$\frac{Sgnl(k^*)}{Bckg(k^*)}$ - correlation function

Bertsch-Pratt parametrization, 3D- and 1D-dimensional cases

→ R_{side} spatial source evolution in the transverse direction

→ R_{out} related to spatial and time components

→ R_{out}/R_{side} signature of phase transition

→ $R_{out}^2 - R_{side}^2 = \Delta\tau^2 \beta_t^2$; $\Delta\tau$ – emission time

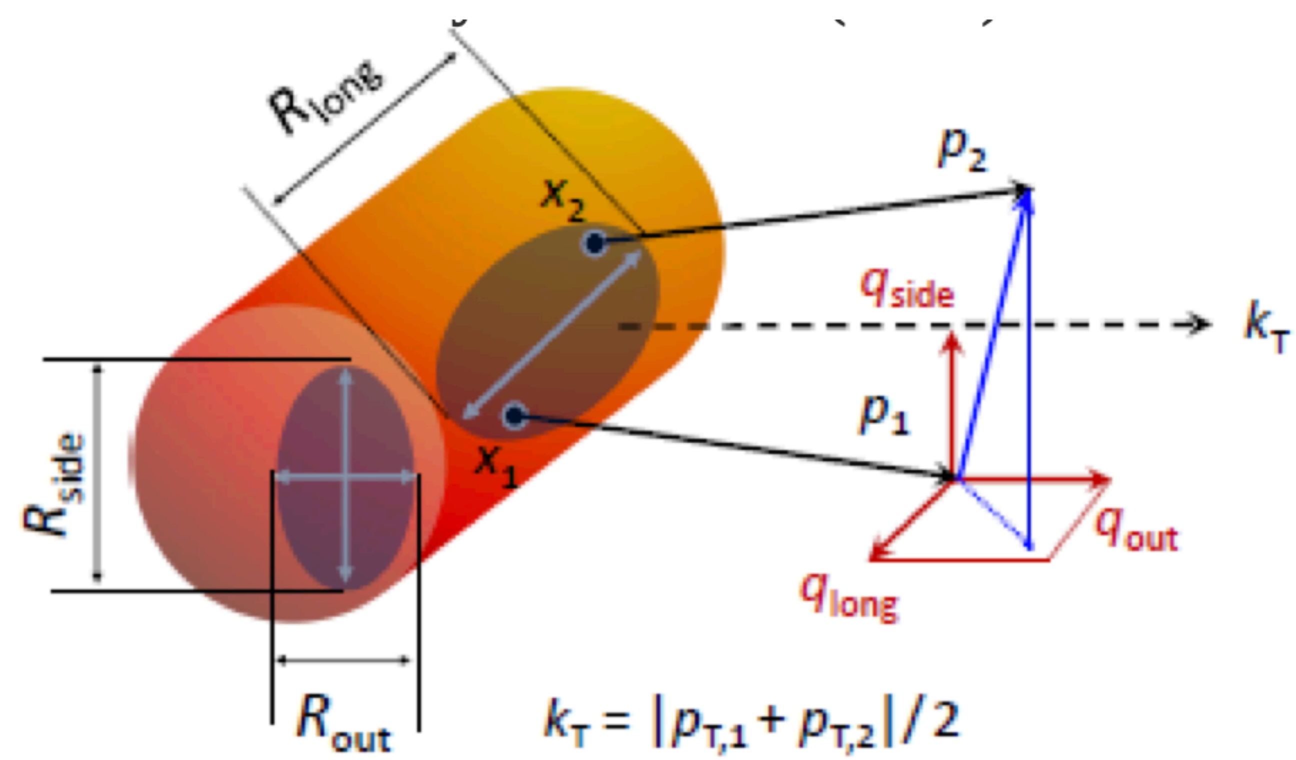
→ R_{long} temperature of kinetic freeze-out and source lifetime

long - determined by the beam direction

out - determined by the pair transverse momentum

side - perpendicular to *long* and *side*

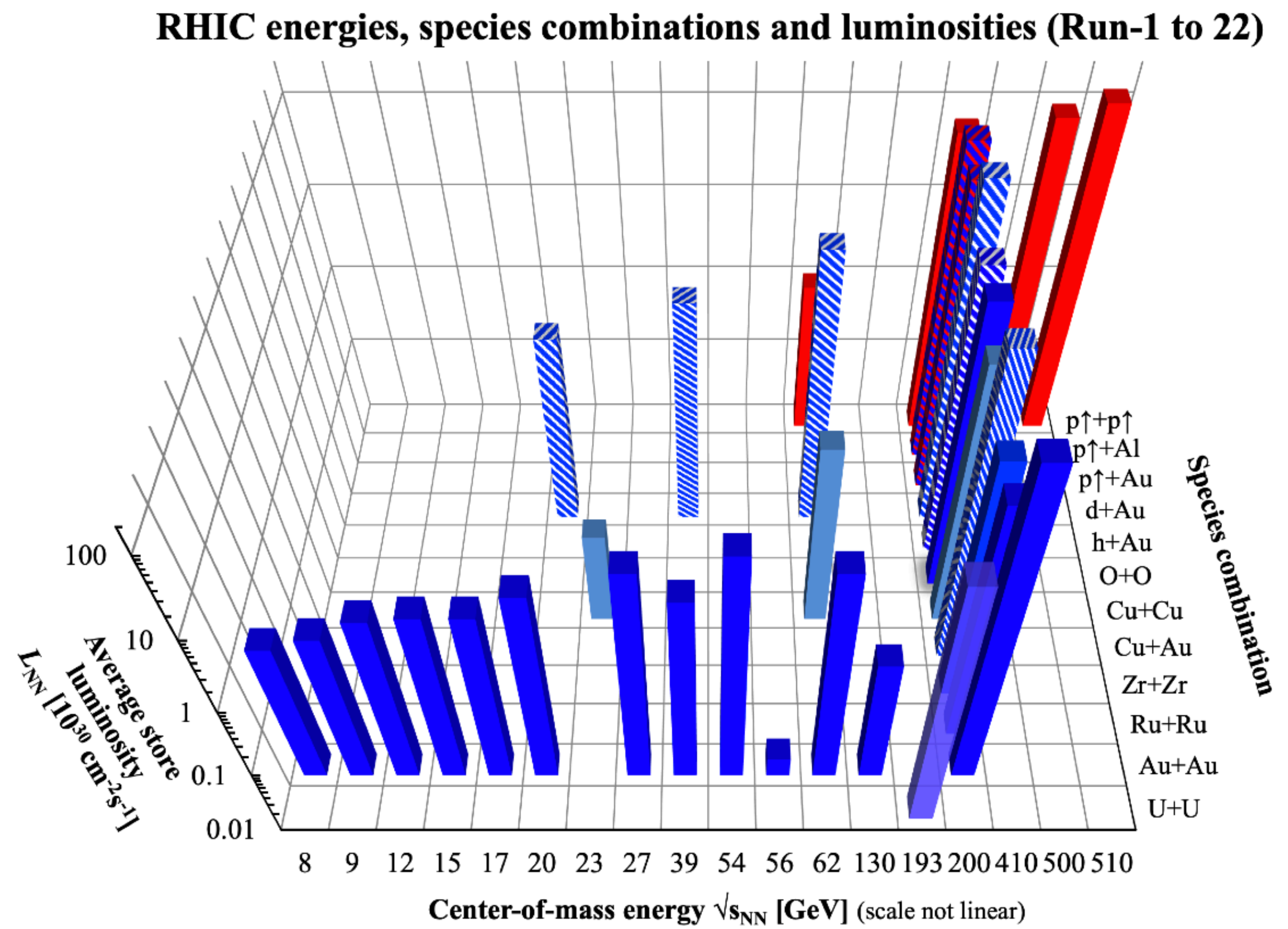
3D case is considered if statistics is enough and two-particle correlations are easy to describe (Quantum Statistics and Coulomb FSI).
It is challenging for systems interacting strongly.
1D case is considered then (assuming spherical source).



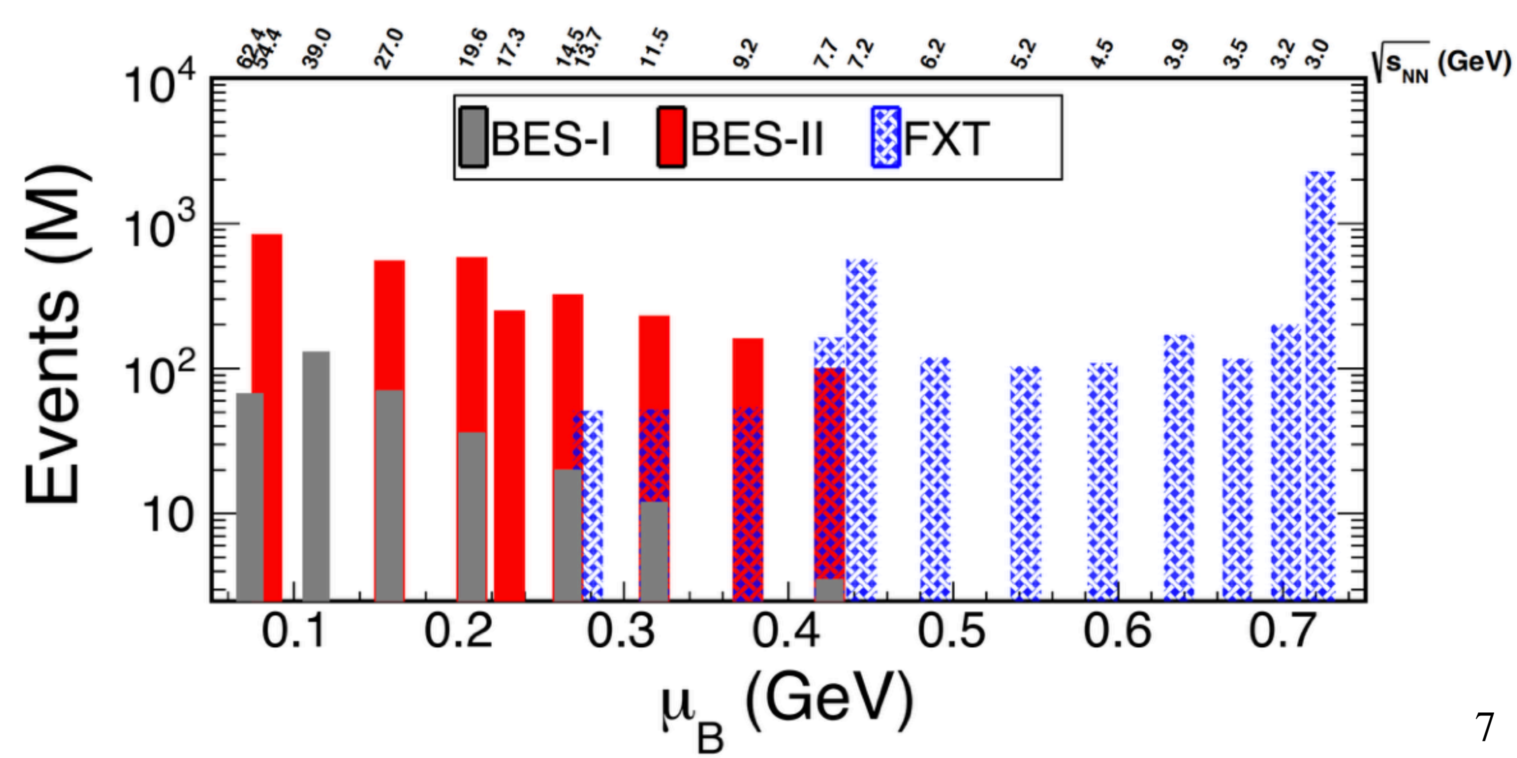
STAR

Solenoidal Tracker At RHIC

Versatile experiment + Many detector subsystems →
Varied and interesting program to better understand Quantum Chromodynamics

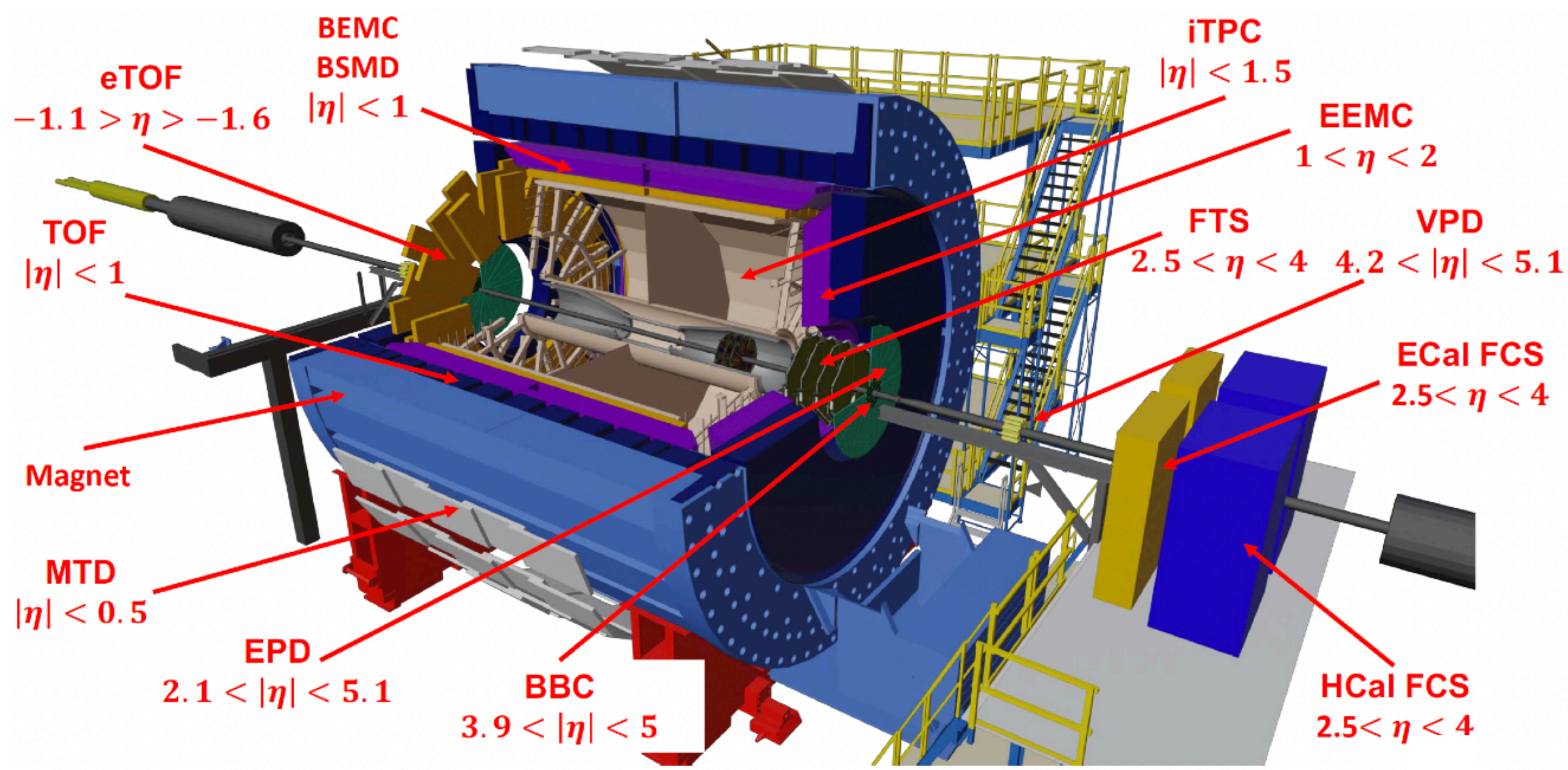


- Beam Energy Scan
- System Size at top RHIC Energy
- Exploring QGP Dynamical Structure
- Electro-Magnetic Probes
- Hard Probes
- Understanding QCD and Nucleons

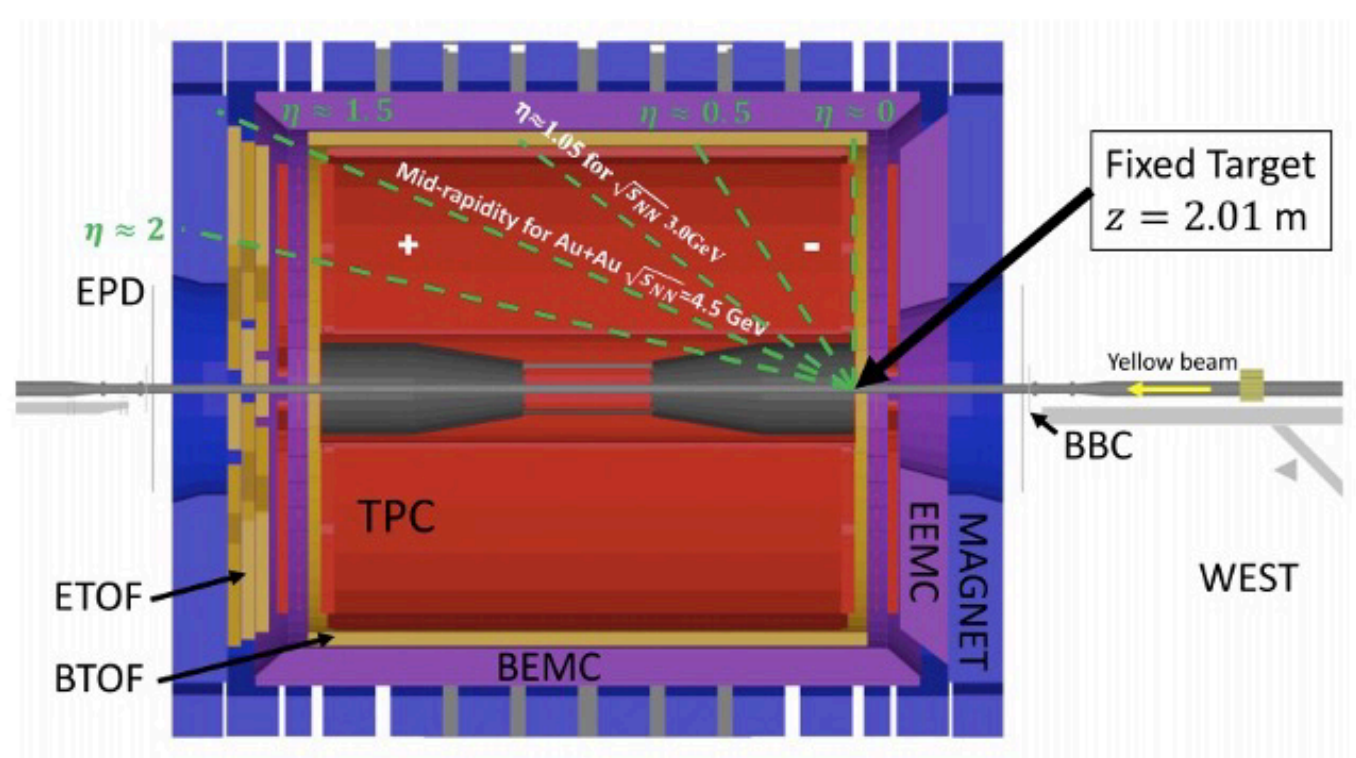


Solenoidal Tracker At RHIC

Excellent particle identification
Large, uniform acceptance at mid-rapidity

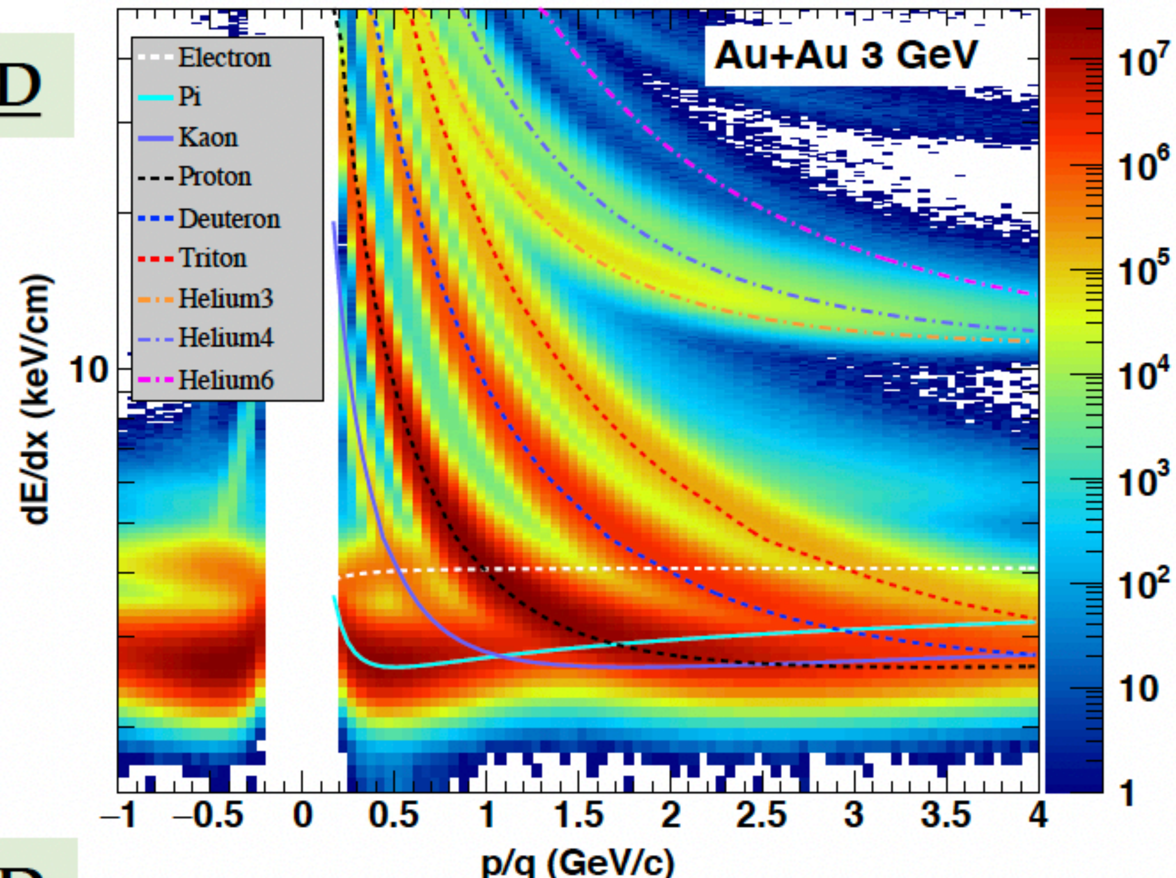


STAR fixed-target experiment setup

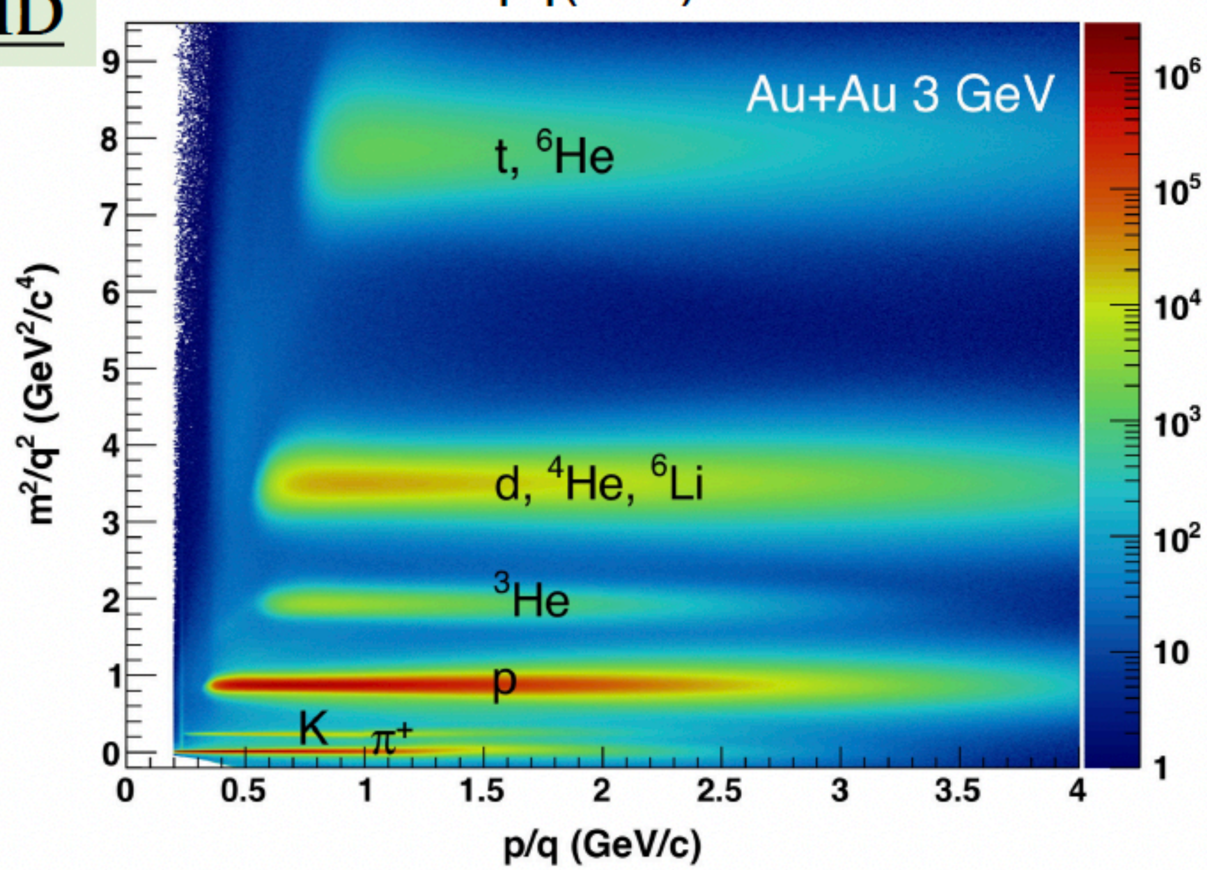


Particle identification at STAR

TPC PID



TOF PID



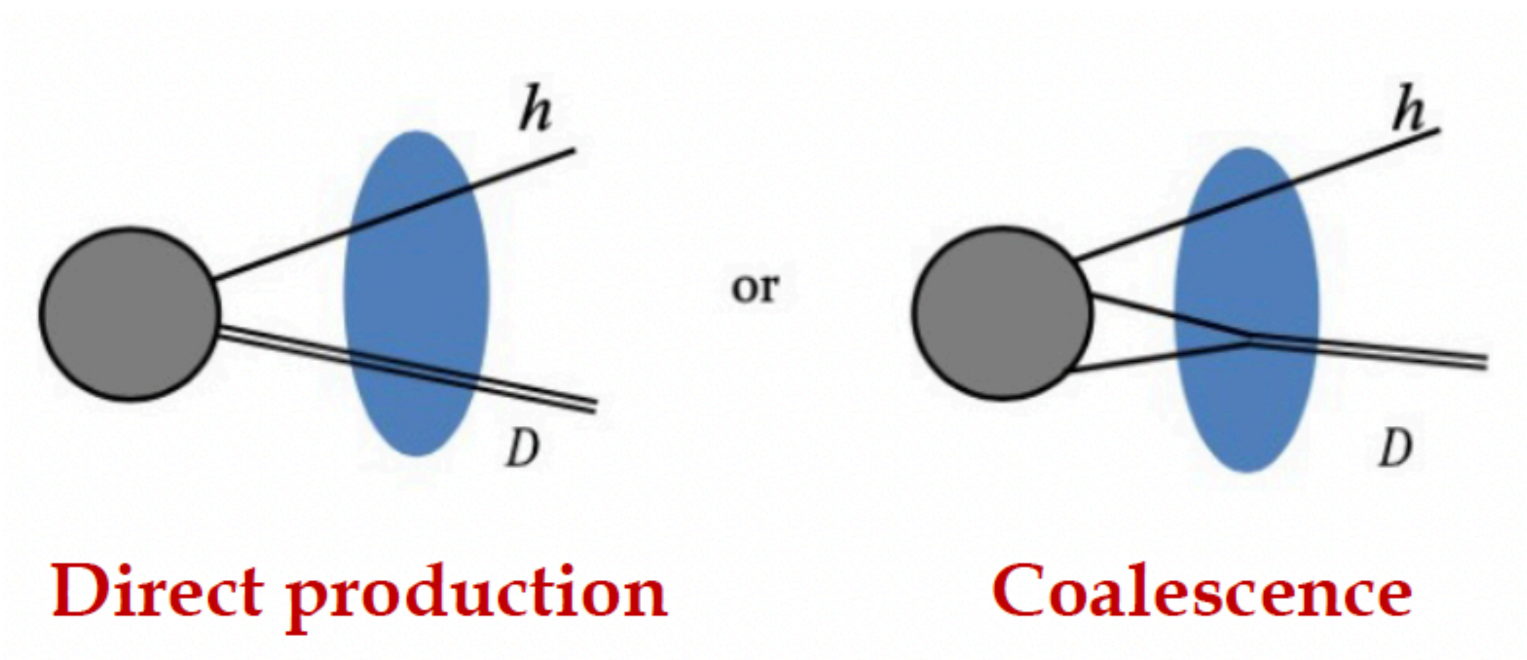
Excellent particle identification due to combined information from Time Projection Chamber and Time of Flight detectors.

Unique *mass/dEdx* separation for π, *K, p, d, t,* ^{3}He, ^{4}He, ^{6}He, ^{6}Li .

Light nuclei

Light nuclei correlation: p-d, d-d correlations

1) A systematic measurement of **p-p**, **p-d**, and **d-d** correlations may tell us whether **deuterons** are directly emitted from the fireball or formed due to final-state interactions;
2) A large amount of light nuclei produced at Au+Au collisions at $\sqrt{s_{NN}} = 3$ GeV allows one for precision measurements.



S. Mrówczyński and P. Słoń, Acta Physica Polonica B 51, 1739 (2020)
S. Mrówczyński and P. Słoń, Physical Review C 104, 024909 (2021)

Lednicky-Lyuboshitz model

The correlation function can be calculated analytically by averaging **Ψ** over the total spin S and the distribution of the relative distances **S(r*)**

Ref : Lednicky, Richard & Lyuboshits, V.L.. (1982). Sov. J. Nucl. Phys. (Engl. Transl.); (United States). 35:5.

$$C(k^*) = \int S(r^*)\,|\Psi(r^*,k^*)|^2\, d^3r$$

The normalized pair separation distribution (source function) **S(r*)** is assumed to be Gaussian,

$$S(r^*) = (2\sqrt{\pi}r_0)^{-3} e^{-\frac{r^{*2}}{4r_0^2}},$$

$$\Psi^S(r^*,k^*) = e^{-ik^*r^*} + f^S(k^*)\frac{e^{ik^*r^*}}{r^*} \qquad f^S(k^*) = \left(\frac{1}{f_0^S} + \frac{1}{2}d_0^S k^{*2} - ik^*\right)^{-1}$$

Strong

$$|\Psi^C(r^*,k^*)| = \sqrt{A_C}\, e^{-ik^*r^*} F(-i\eta, 1, i\zeta) \qquad A_C(\eta) = \frac{2\pi}{k^* a_c}\left(exp\left(\pm\frac{2\pi}{k^* a_c}\right) - 1\right)^{-1}$$

Coulomb

F- confluent hypergeometric function

f_0 and d_0 - parameters of strong interaction.

Theoretical correlation function (k*) depends on: R, f_0 and d_0.

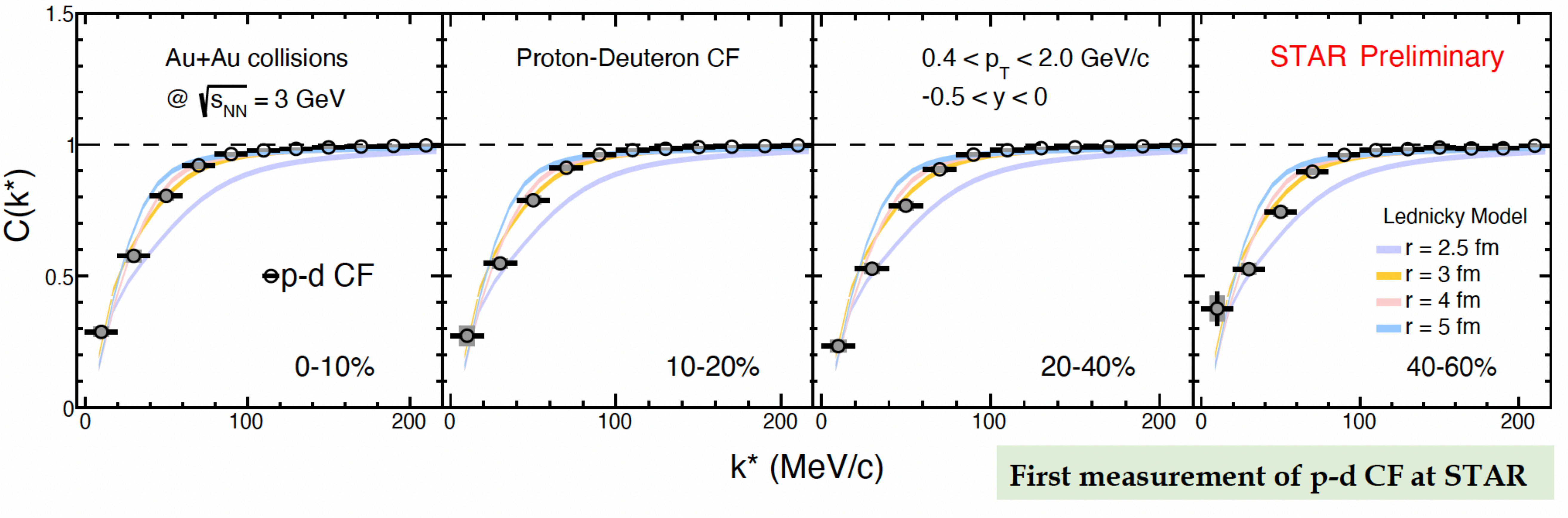
f_0 - the scattering length, determines low-energy scattering.

The elastic cross section, σ_e, (at low energies) determined by the scattering length, $\lim_{k\to 0}\sigma_e = 4\pi f_0^2$

d_0 - the effective range, corresponds to the range of the potential (simplified scenario - the square well potential.

For identical systems one has to include QS (Fermi-Dirac / Bose-Einstein) as well.

Proton-deuteron correlations



First measurement of p-d CF at STAR

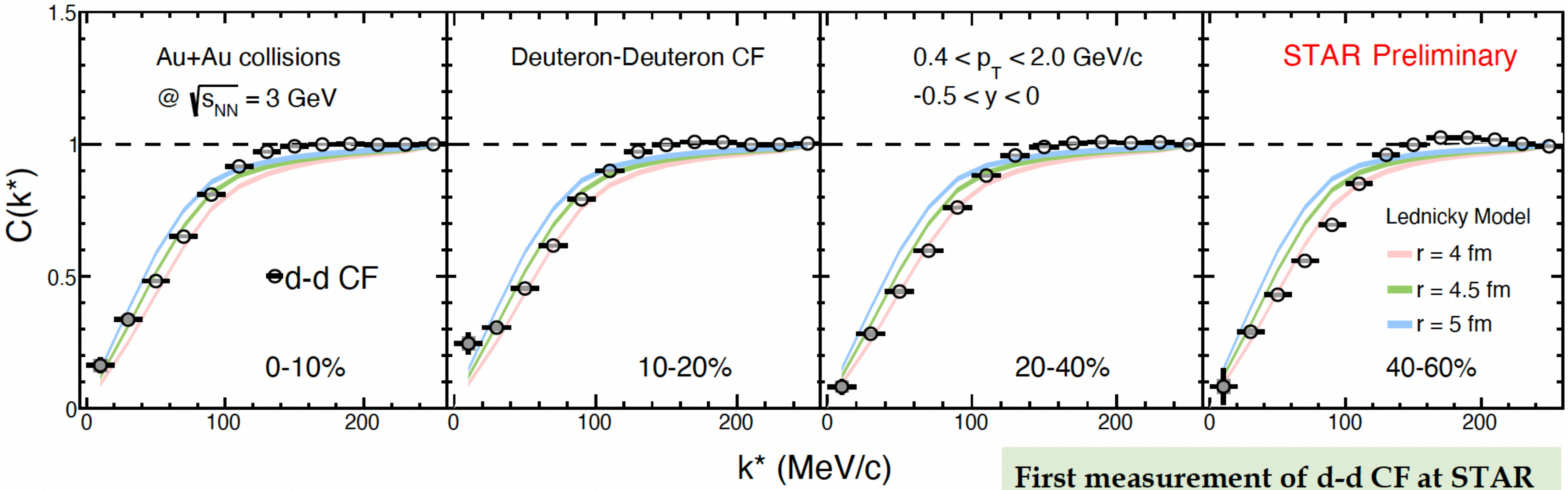
1. Clear depletion at small k* range seen in data due to interactions in the final states;

2. Data compared with Lednicky - Lyuboshitz model;

3. A spherical source size with r = 3 - 4 fm is consistent with data.

1.Lednický, R,Lyuboshitz, V; Sov.J.Nucl.Phys.35:770(1982)
2.J. Arvieux, Nucl. Phys. A 221 (1974) 253–268

Deuteron-deuteron correlations



First measurement of d-d CF at STAR

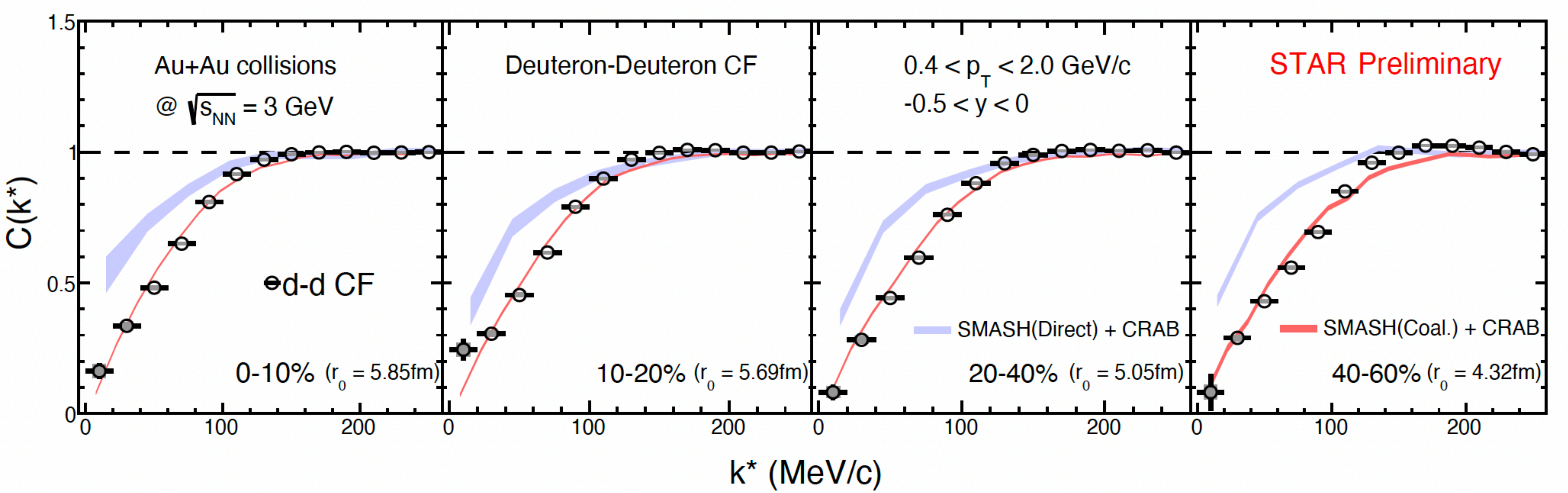
1. Clear depletion at small k* range seen in data;

2. Data compared with Lednicky - Lyuboshitz model;

3. A spherical source size with r = 4 - 5 fm is consistent with data, larger than in p-d case.

1.Lednický, R,Lyuboshitz, V; Sov.J.Nucl.Phys.35:770(1982)
2.J. Arvieux, Nucl. Phys. A 221 (1974) 253–268

Deuteron-deuteron correlations



1. Compared with SMASH model;

2. Correlations calculated with coalescence of deuterons is in better agreement with data; Support the deuteron formation at $\sqrt{s_{NN}} =$ 3 GeV is dominated by coalescence;

3. SMASH source size: 4.3 - 5.9 fm from peripheral to central collisions.

SMASH: J. Weil et al. Phys.Rev.C 94 (2016) 5, 054905
Coalescence: W.Zhao et al. Phys. Rev. C.98 (2018) 5,054905

Wrap-up-1: deuteron correlations

- First measurement of **p-d** and **d-d** correlation functions from STAR;
- **p-d** and correlations qualitatively described by Lednicky - Lyuboshitz model: **d-d** has larger emission source size than **p-d;**
- **d-d** correlations described better by the model including **coalescence**;
- Light nuclei are likely to be formed via coalescence;
- In the BES-II, STAR has collected 10 - 20 times more data in Au+Au collisions at the energy range $\sqrt{s_{NN}} =$ 3 - 19.6 GeV (higher precision femtoscopy analysis possible);

Hyperons

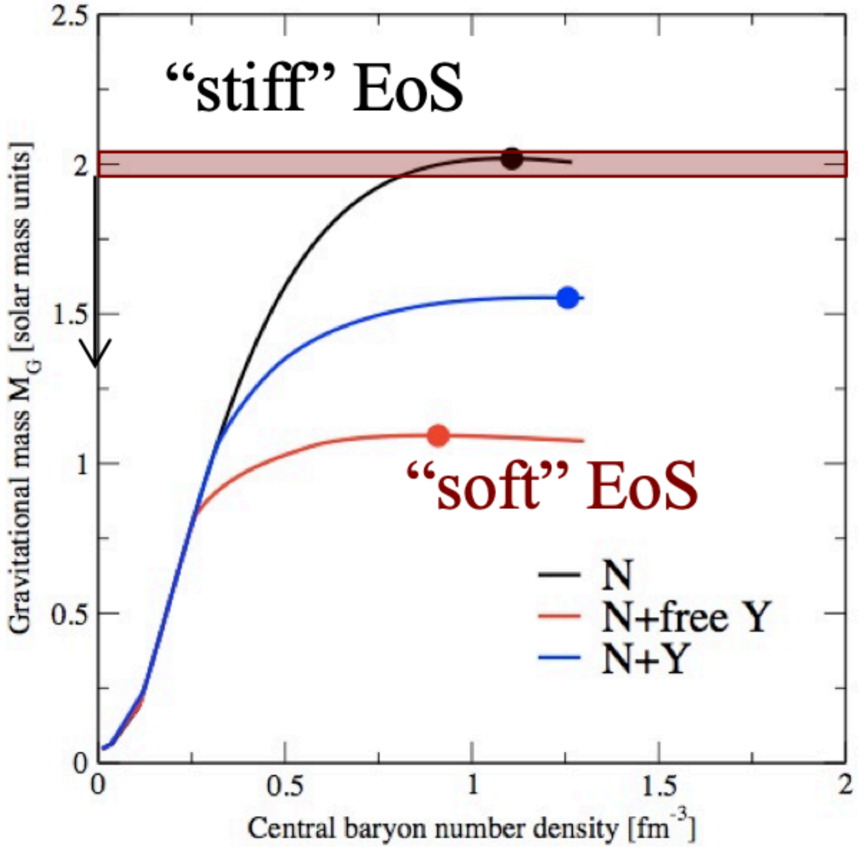
Neutron star puzzle

- **Hyperons**: expected in the core of neutron stars; **conversion of N into Y energetically favorable.**

- Appearance of Y: The relieve of Fermi pressure → **softer EoS → mass reduction (incompatible with observation).**

$M_{NS} \approx 1 \div 2\ M_{\odot}$
$R \approx 10\text{-}12$ km
$\rho \approx 3 \div 5\ \rho_0$

The solution requires a mechanism that could provide the **additional pressure** at high densities needed to make the EoS stiffer.



A few possible mechanisms, one of them:

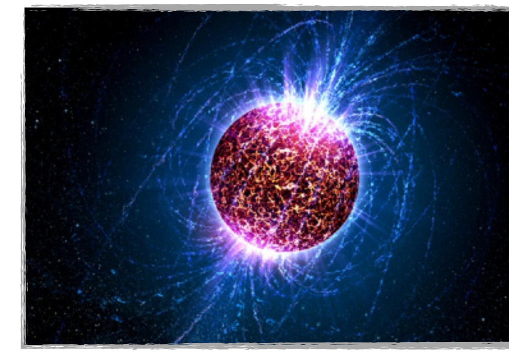
- **Two-body YN & YY interactions**

A lot of experimental and theoretical effort to understand:

- The **KN** interaction, governed by the presence of **Λ(1405)**
- The nature of **Λ(1405)**, the consequences of **KNN** formation
- **K** and $\bar{K}$ investigated to understand kaon condensation

$\rho_0 \approx 2.8 \times 10^{14}$ g/cm^3

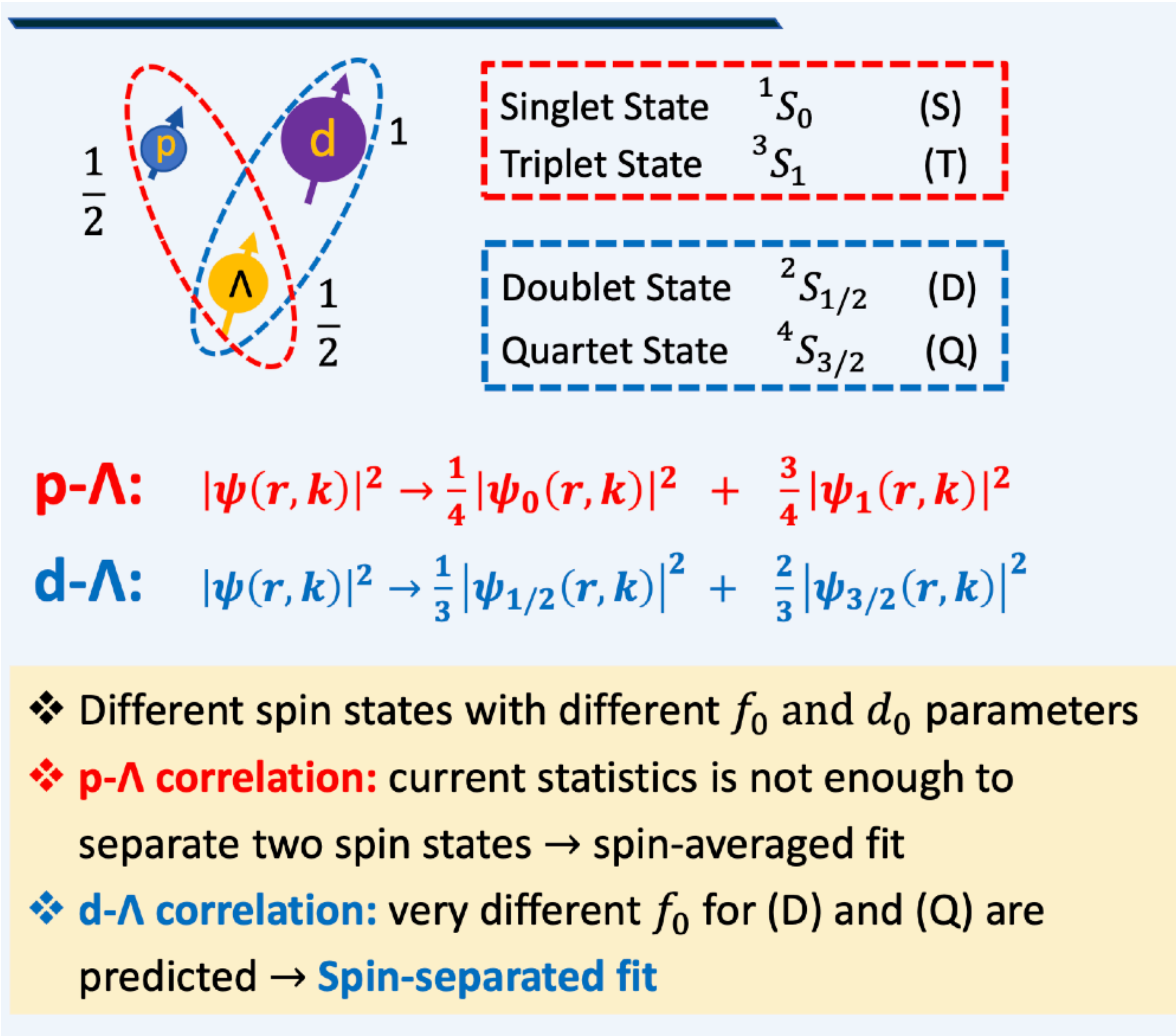
YN and YY interactions

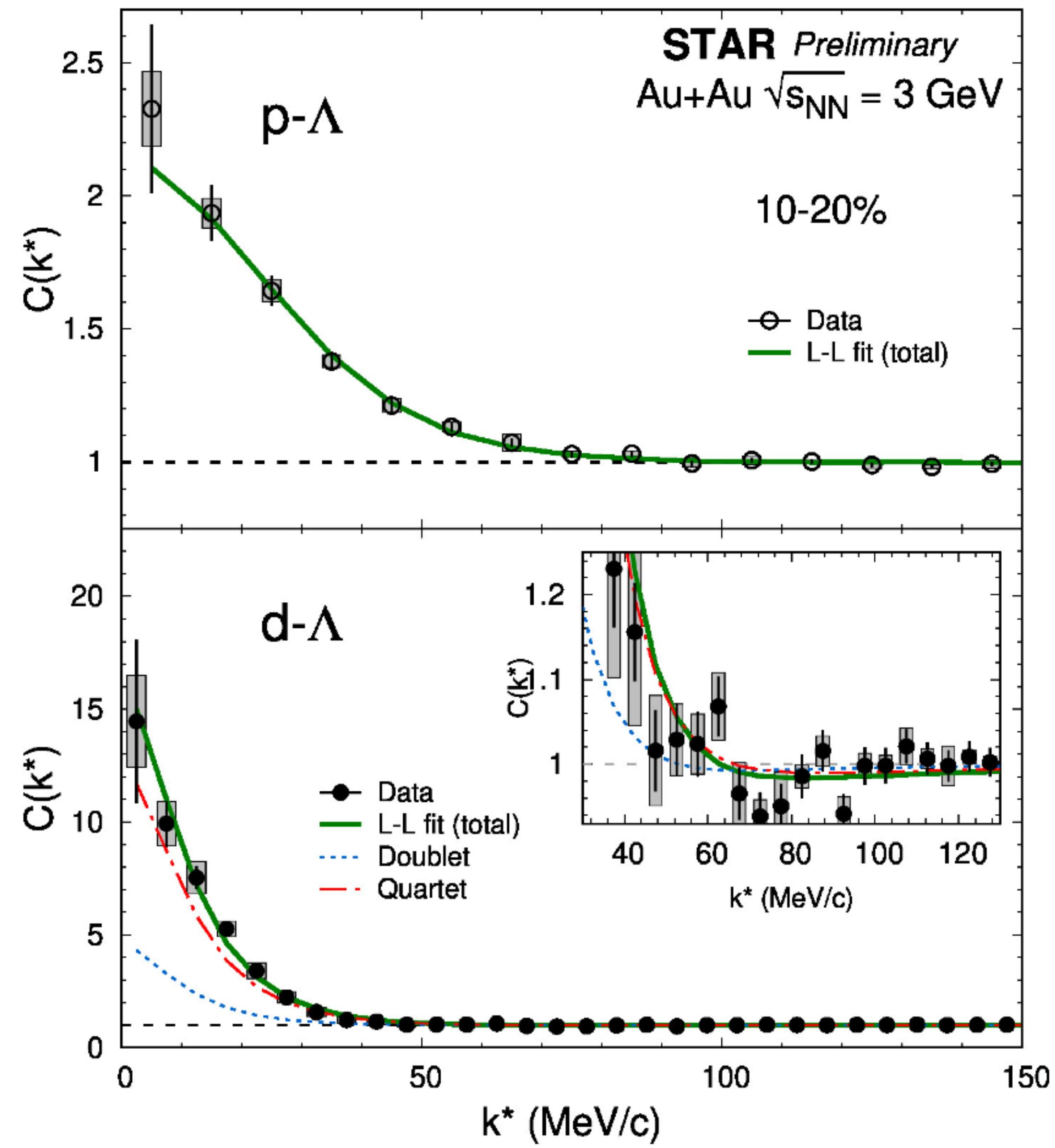


- **Experiment**: More … and more!
  interest about YN and YY interactions!

- **Theory**: Major steps forward have been taken (Lattice QCD).
- **Numerous theoretical predictions** exist, **many experimental searches look for** evidence for **bound states**.
- The existence of **hypernuclei** (confirmed by attractive YN interaction) → indicates the possibility to bind Y to N.
- The measurement of the YN and YY interactions leads to important implications for the possible formation of **YN** or **YY bound states**.
- A precise knowledge of these interactions help to explore unknown structure of neutron stars.

YN correlations at STAR



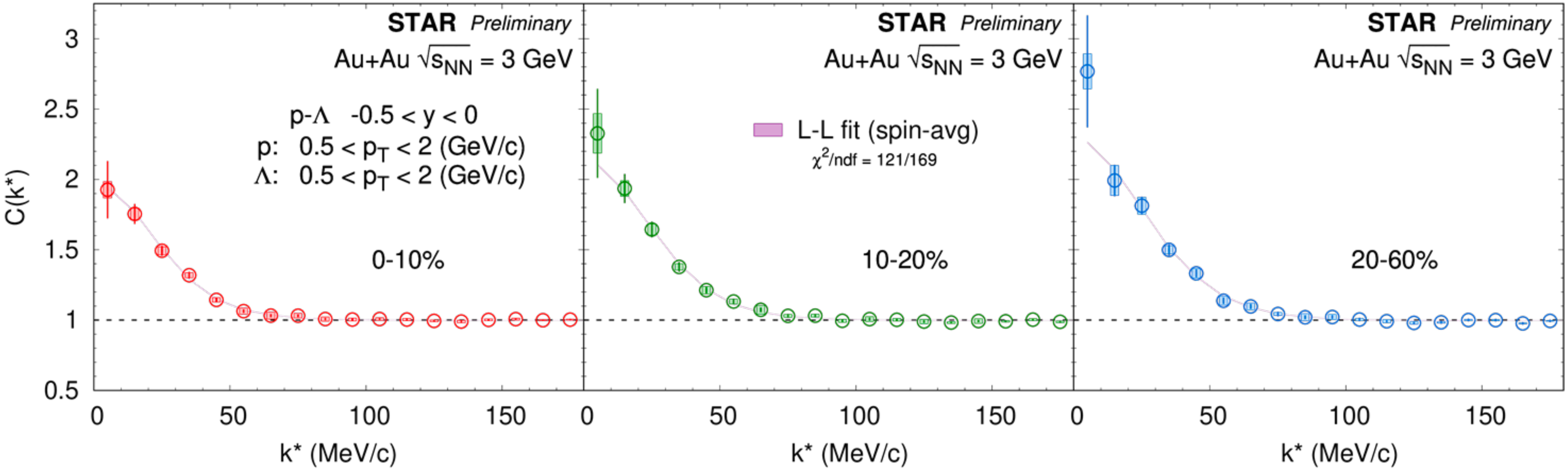


Different spin states with different FSI parameters

p-Λ correlation: currently spin-averaged fit

d-Λ correlation: spin-separated fit

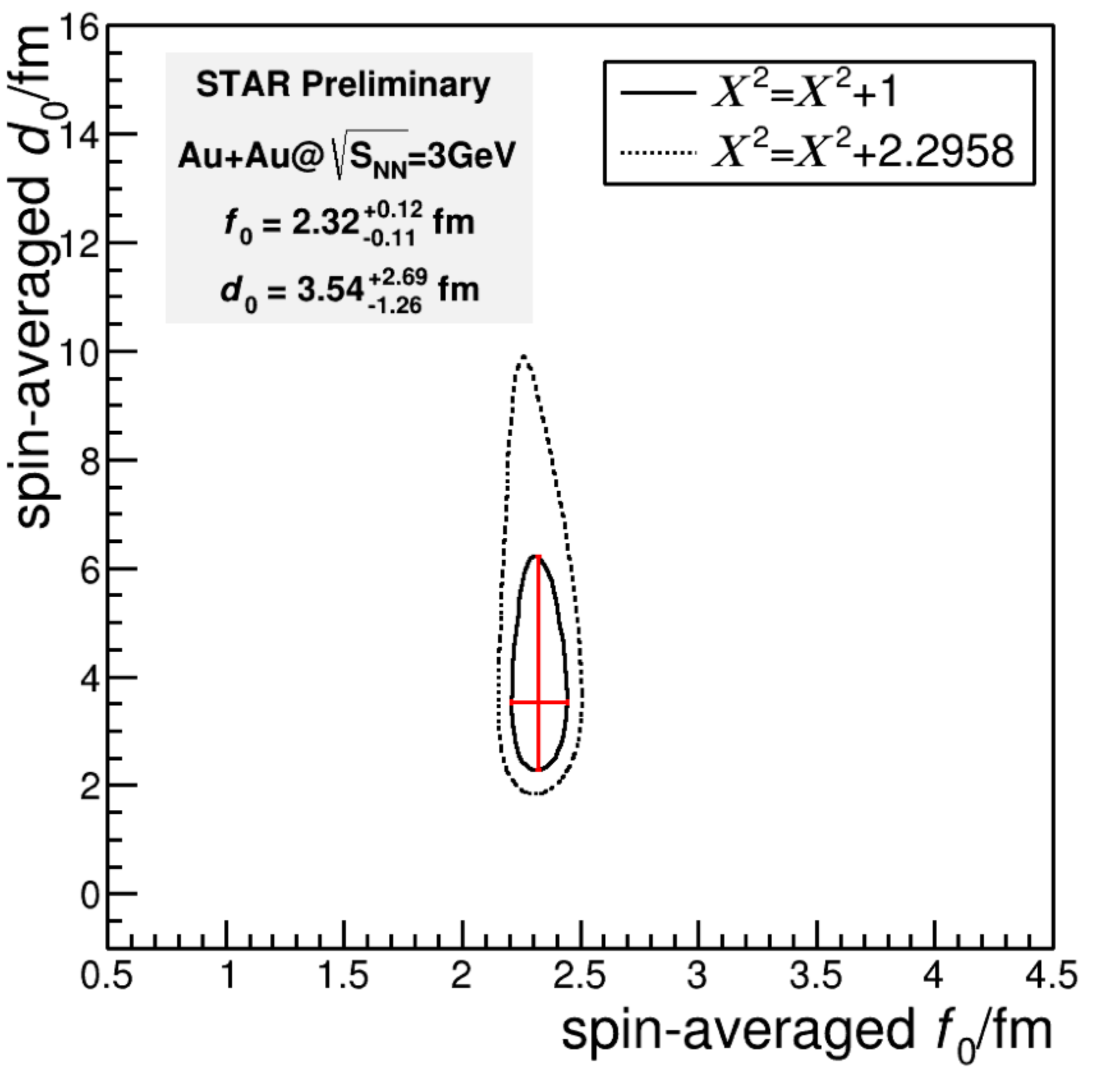
YN ($p - \Lambda$) correlations at STAR



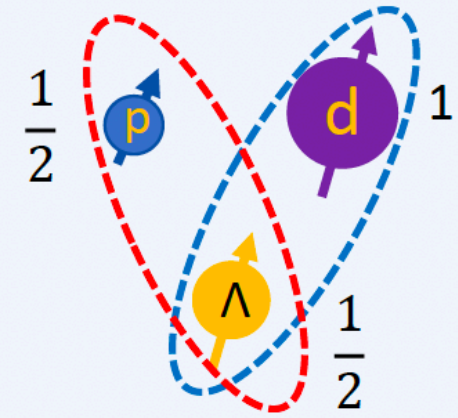
- Simultaneous fit to data in different centralities and rapidities;
- Source size R and parameters of SI: f_0 and d_0 with Lednicky-Lyuboshitz approach;
- Spin-averaged scattering length and effective range: $f_0 = 2.32^{+0.12}_{-0.11}\, fm$ and $d_0 = 3.5^{+2.7}_{-1.3}\, fm$.

YN ($p-\Lambda$) interactions at STAR

X^2 contour of spin-averaged d_0 and f_0 for $p\Lambda$ (-1<y<0)



H. W. Hammer, Nucl. Phys. A 705 (2002) 173
A. Cobis, et al. J. Phys. G 23 (1997) 401
J. Haidenbauer, Phys.Rev.C 102 (2020) 3, 034001
M. Schäfer, et al. Phys.Lett.B 808 (2020) 135614
G. Alexander, et al. Phys. Rev. 173 (1968) 1452
J. Haidenbauer, et al. Nucl. Phys. A 915 (2013) 24
F. Wang, et al. Phys.Rev.Lett. 83 (1999) 3138



$$\frac{1}{f(k)} \approx \frac{1}{f_0} + \frac{d_0 k^2}{2} - ik$$

- ❖ The constraint of the effective range (d_0) is weaker

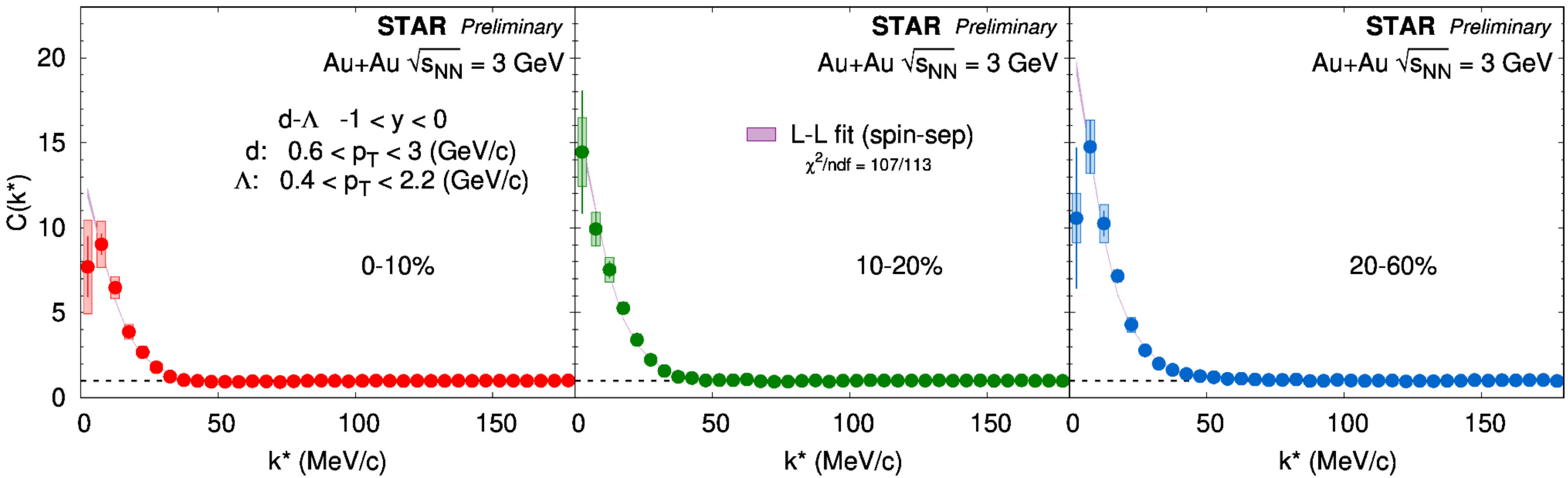
- ❖ The measurement is done at freeze-out
- ❖ Spin-avg for f_0 & d_0 p-Λ system

$f_0 = 2.32^{+0.12}_{-0.11}$ fm $\quad$ $d_0 = 3.5^{+2.7}_{-1.3}$ fm

Source size extracted from the source assuming Gaussian shape;

Separation of emission source from the parameters of the final state interaction;

YN ($d - \Lambda$) correlations at STAR

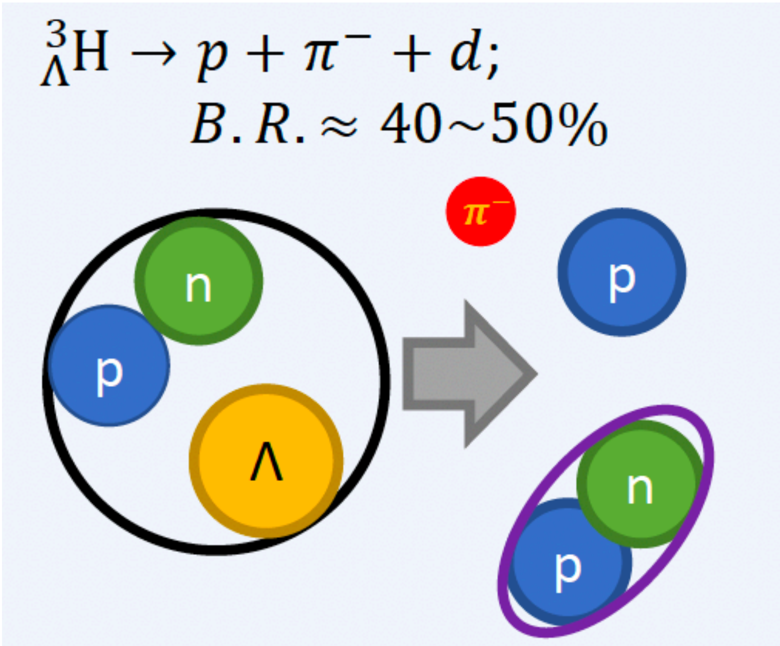


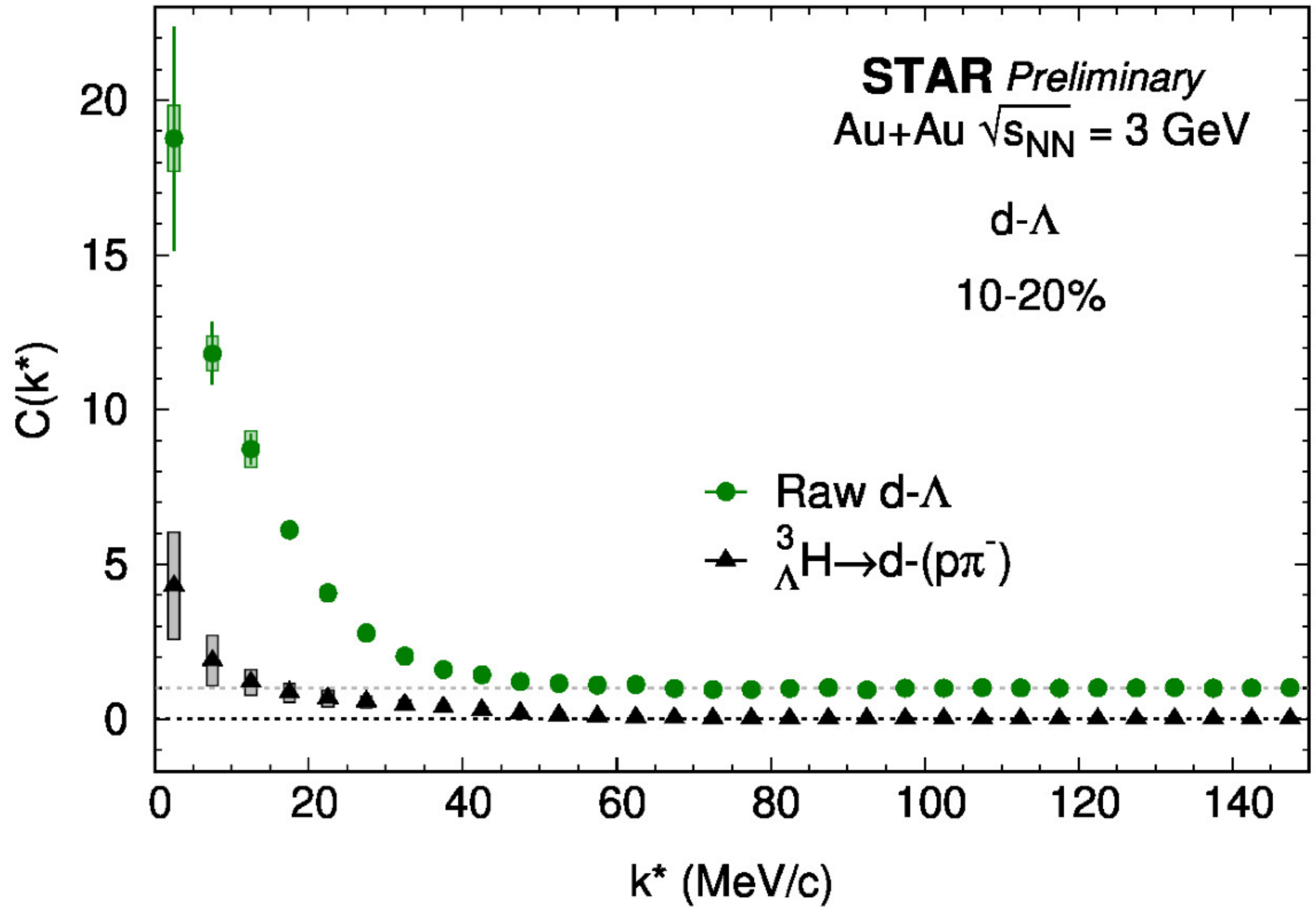
- Simultaneous fit to data in different centralities and rapidities;
- Source size R and parameters of SI: f_0 and d_0 with Lednicky-Lyuboshitz approach;
- Spin-separated scattering length and effective range:

$f_0(D) = 20^{+3}_{-3}\,fm;\ d_0(D) = 3^{+2}_{-1}\,fm;$

$f_0(Q) = 16^{+2}_{-1}\,fm;\ d_0(Q) = 2^{+1}_{-1}\,fm.$

YN ($d - \Lambda$) correlations at STAR

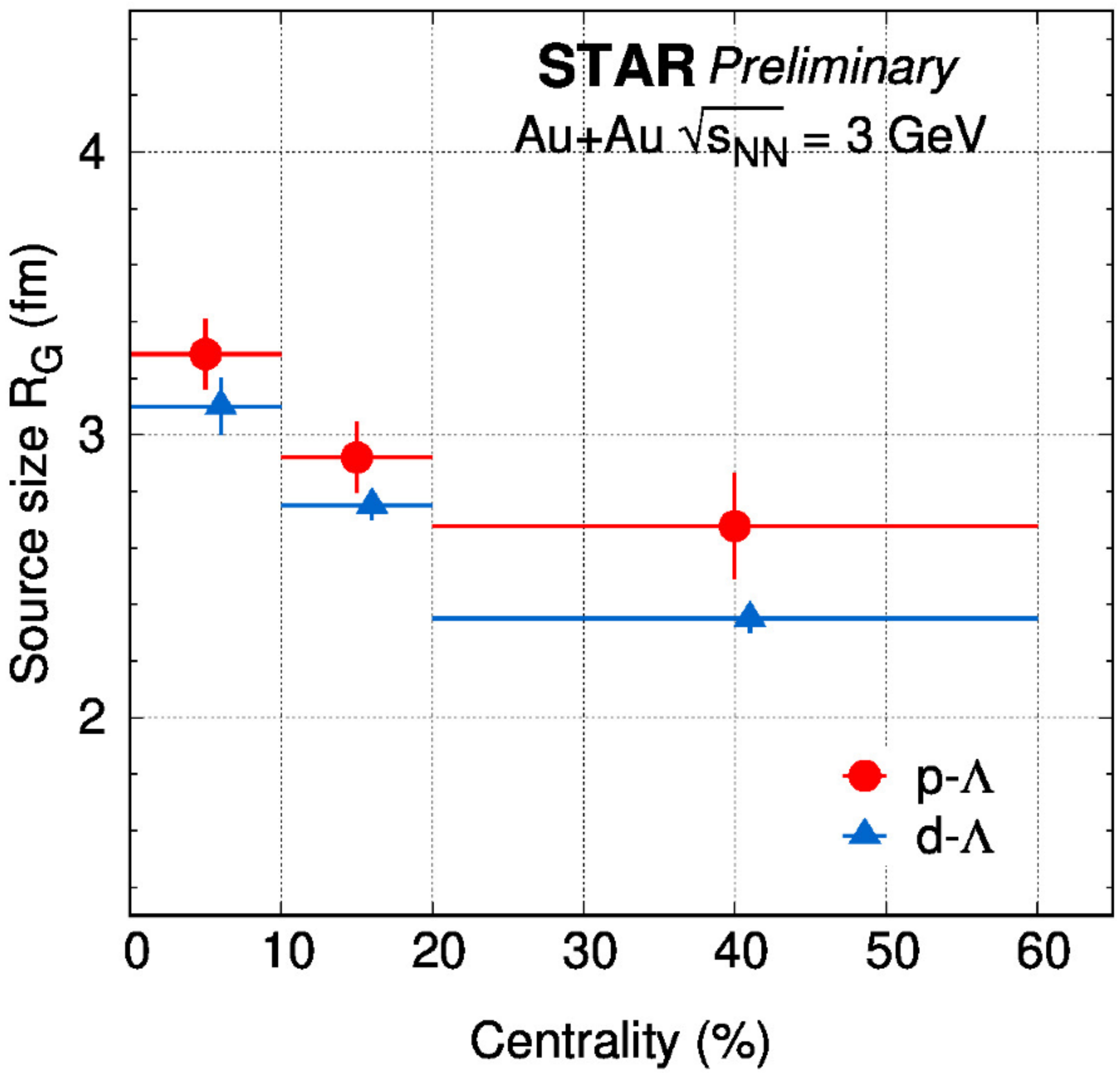




Simulation based on STAR $^{3}_{\Lambda}H$ yield measurement:
4 - 8% of $d - \Lambda$ entries come from $^{3}_{\Lambda}H$ decay for low k*;
Contamination subtracted from inclusive $d - \Lambda$ correlations;

Correlations of $^{3}_{\Lambda}H$ from $d - \Lambda$ and $d - (p - \pi^{-})$
are **not** experimentally **distinguishable.**

YN correlations at STAR



Source size extracted from the source assuming Gaussian shape;

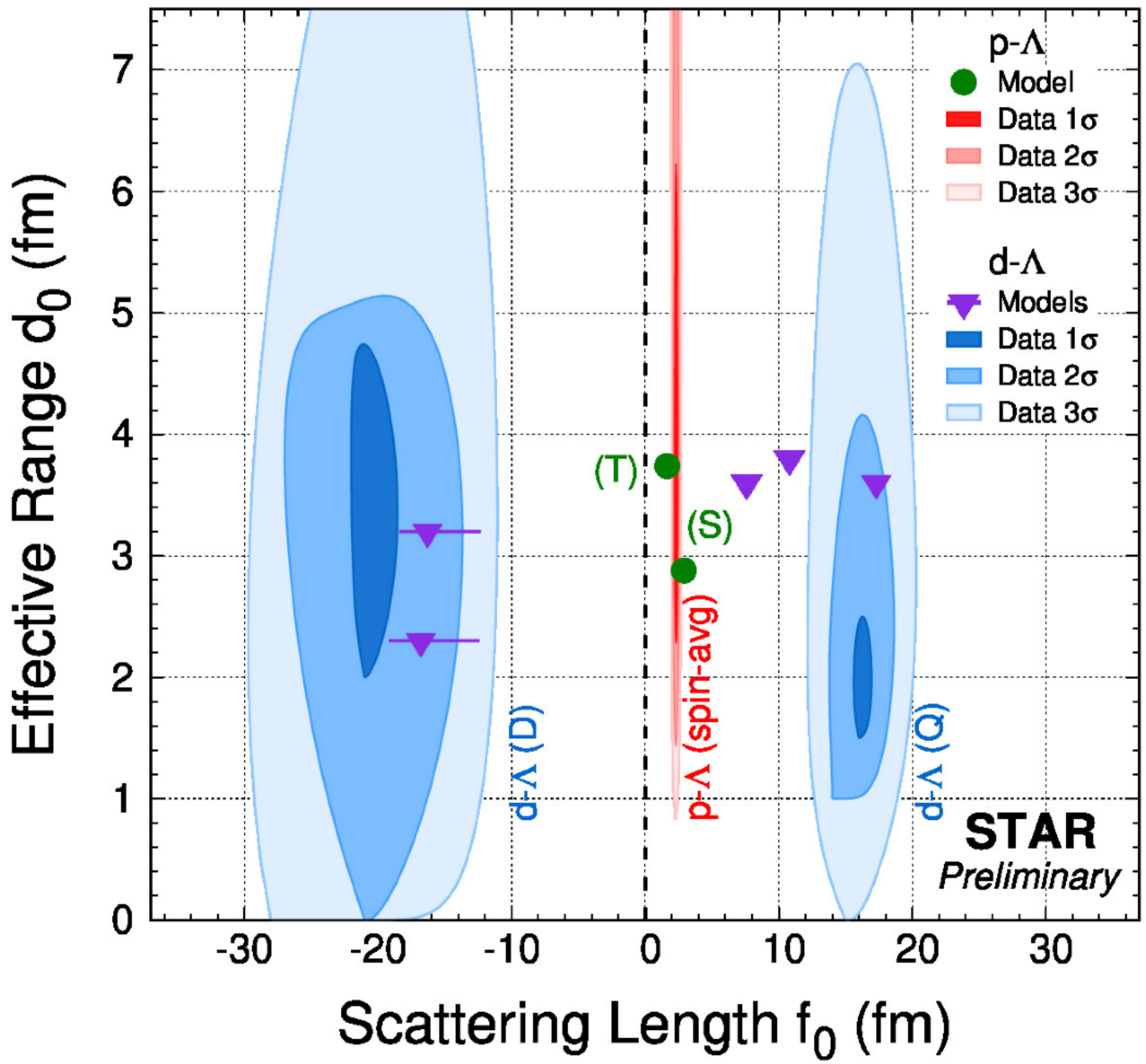
Separation of emission source from the parameters of the final state interaction;

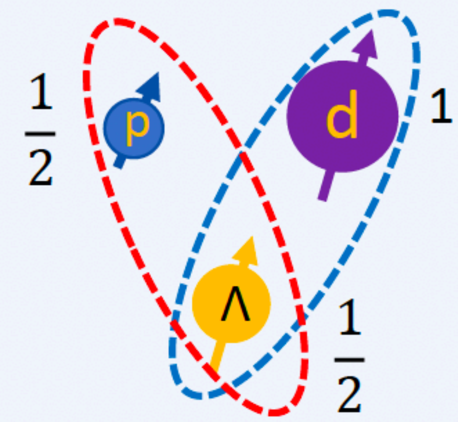
Collision dynamics as expected:

$R_c > R_p$;

$R_{p-\Lambda} > R_{d-\Lambda}$

YN ($d-\Lambda$) interactions at STAR





$$\frac{1}{f(k)} \approx \frac{1}{f_0} + \frac{d_0 k^2}{2} - ik$$

- ❖ The constraint of the effective range (d_0) is weaker

- ❖ The measurement is done at freeze-out

- ❖ Successfully separate two spin states in d-Λ

$f_0(\mathrm{D}) = -20^{+3}_{-3}$ fm $\quad d_0(\mathrm{D}) = 3^{+2}_{-1}$ fm

$f_0(\mathrm{Q}) = 16^{+2}_{-1}$ fm $\quad d_0(\mathrm{Q}) = 2^{+1}_{-1}$ fm

H. W. Hammer, Nucl. Phys. A 705 (2002) 173
A. Cobis, et al. J. Phys. G 23 (1997) 401
J. Haidenbauer, Phys.Rev.C 102 (2020) 3, 034001
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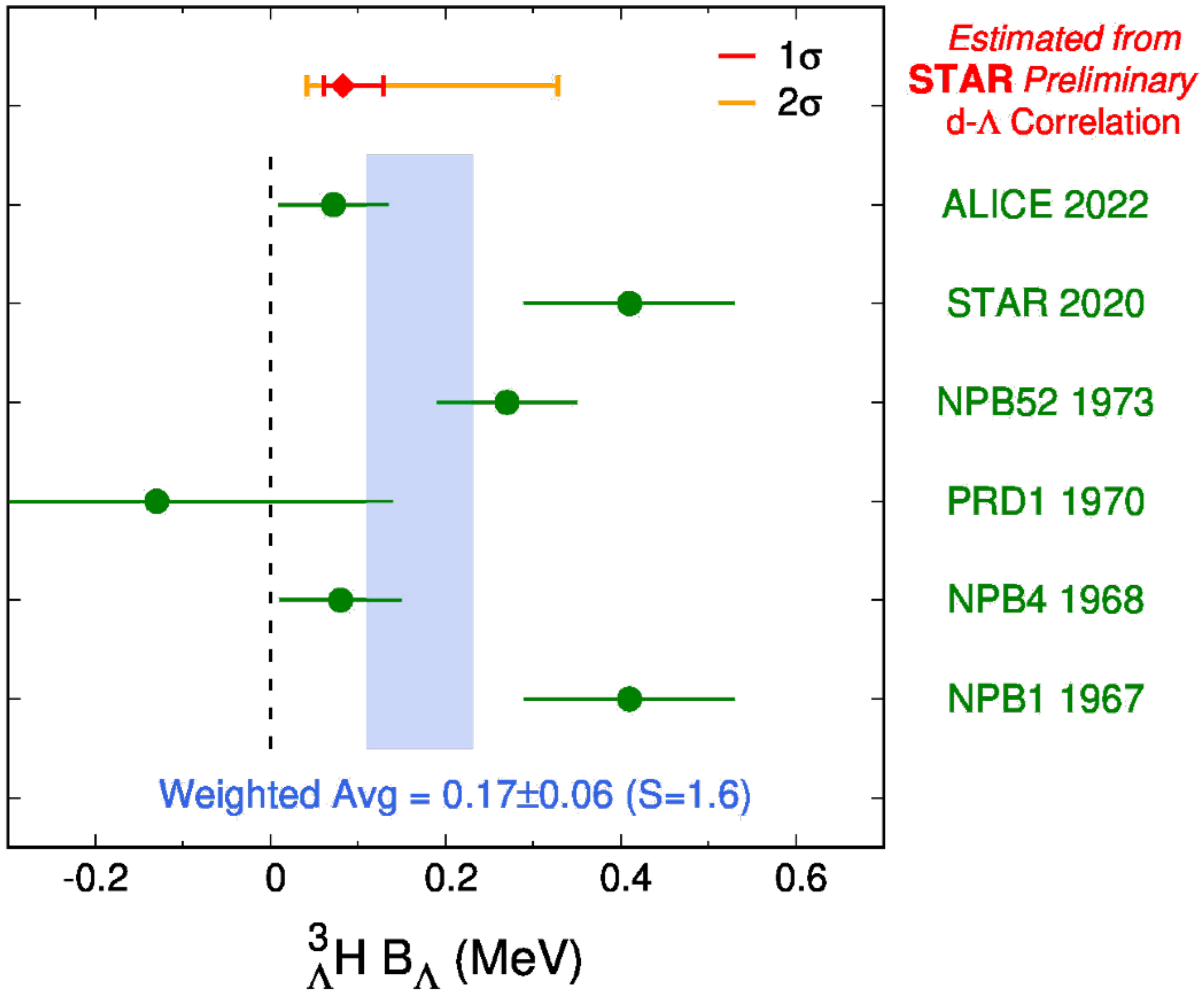
F. Wang, et al. Phys.Rev.Lett. 83 (1999) 3138

Source size extracted from the source assuming Gaussian shape;

Separation of emission source from the parameters of the final state interaction;

Binding energy



Bethe formula from Effective Range Expansion (ERE) parameters $f_0(D)$ and $d_0(D)$.

$$\frac{1}{-f_0} = \gamma - \frac{1}{2} d_0 \gamma^2$$

- $B_\Lambda = \frac{\gamma^2}{2\mu_{d\Lambda}}$
- $\mu_{d\Lambda}$: reduced mass
- γ: binding momentum

- $^{3}_{\Lambda}$H B_Λ = [0.04,0.33] (MeV) @ 95% CL
  Consistent with the world average
- A new way to constrain the $^{3}_{\Lambda}$H structure

Wrap-up-2: YN interactions at STAR

- The first **d-Λ** correlation measurements in heavy-ion collisions;
- New **p-Λ** correlation measurements with $\sqrt{s_{NN}}$ = 3 GeV Au+Au collisions ;
- Successfully separated emission source size from final state interactions in p-Λ and d-Λ correlations;

- Collision dynamics as expected: $R_c > R_p$ and $R_{p-\Lambda} > R_{d-\Lambda}$
- ***p-Λ*** correlation spin-averaged: $f_0 = 2.32^{+0.12}_{-0.11}\, fm;\ d_0 = 3.5^{+2.7}_{-1.3}\, fm;$

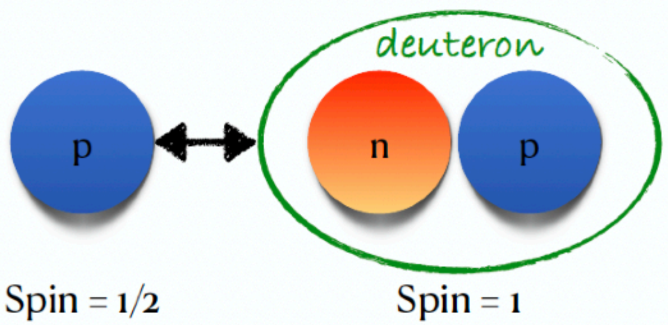
- ***d-Λ*** correlation spin-separated: $f_0(D) = 20^{+3}_{-3}\, fm;\ d_0(D) = 3^{+2}_{-1}\, fm;$
  $f_0(Q) = 16^{+2}_{-1}\, fm;\ d_0(Q) = 2^{+1}_{-1}\, fm.$

- ${}^{3}_{\Lambda}H$ $B\Lambda$=[0.04, 0.33] MeV; 95% CL from $d - \Lambda(D)$ correlation

Thank you

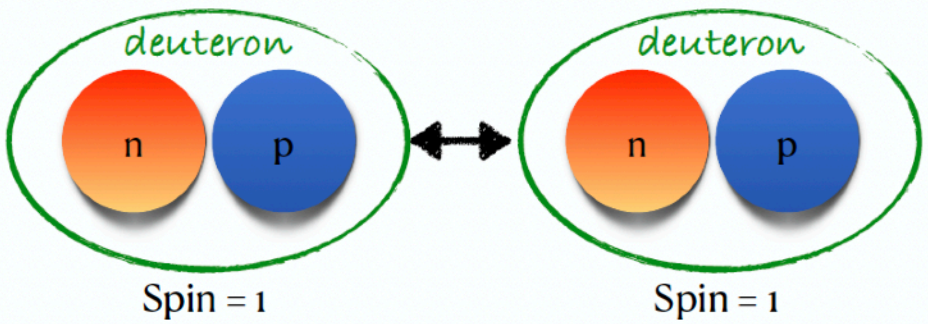
Light nuclei correlation: p-d, d-d correlations

Proton-Deuteron Pair: (SI, Coul)



^{3}He **bound** → $C_{pd} = \boxed{\frac{1}{3}C_{doublet,S=\frac{1}{2}}} + \frac{2}{3}C_{quartet,S=\frac{3}{2}}$

Deuteron-Deuteron Pair: (SI, Coul, QS)



^{4}He **bound** → $C_{dd} = \boxed{\frac{1}{9}C_{singlet,S=0}} + \frac{3}{9}C_{triplet,S=1} + \frac{5}{9}C_{quintet,S=2}$

Triplet: Do not contribute to SI

Doublet spin state $^{2}S_{1/2}$		Quartet spin state $^{4}S_{3/2}$		Ref
Scattering Length	**Effective Range**	**Scattering Length**	**Effective Range**	
1.30 +/- 0.2 fm	-	11.40 +/- 1.5 fm	2.05 +/ 0.25 fm	Oers, Brockmann et al, Nucl.Phys.A 561-583
2.73 +/- 0.1 fm	2.27 +/- 0.12 fm	11.88 +/- 0.25 fm	2.63 +- 0.02 fm	J. Arvieux, Nucl.Phys.A 221 253-268 (1973)
4.0 fm	-	11.1 fm	-	E. Huttel et al, Nucl.Phys.A 406 443-455
0.024 fm	-	13.7 fm	-	A. Kievsky et al, PLB 406 292-296 (1997)
-0.13 +/- 0.04 fm	-	14.70 +/- 2.30 fm	-	T. C. Black et al, PLB 471 103-107 (1999)

⇒ Triplet spin (S=1) : irrelevant for s-wave

⇒ Modify the component used in L-L model

$$C_{dd} = \frac{1}{6}C_{singlet,S=0} + \frac{5}{6}C_{quintet,S=2}$$

Lednicky-Lyuboshitz model

The normalized pair separation distribution (source function) **S(r*)** is assumed to be Gaussian,

$$S(r^*) = (2\sqrt{\pi}r_0)^{-3} e^{-\frac{r^{*2}}{4r_0^2}},$$

Ref : Lednicky, Richard & Lyuboshits, V.L.. (1982). Sov. J. Nucl. Phys. (Engl. Transl.); (United States). 35:5.

The correlated function can be calculated analytically by averaging Ψ^s over the total spin S and the distribution of the relative distances **S(r*)**

$$C(k^*) = 1 + \sum_S \rho_s \left[\frac{1}{2}\left|\frac{f^S(k^*)}{r_0}\right|^2 \left(1 - \frac{d_0^S}{2\sqrt{\pi}r_0}\right) + \frac{2\mathbb{R}f^S(k^*)}{\sqrt{\pi}r_0} F_1(Qr_0) - \frac{\mathfrak{I}f^S(k^*)}{r_0} F_2(Qr_0)\right]$$

$$with\ F_1(z) = \int_0^z dx e^{x^2 - z^2}/z\ and\ F_2(z) = (1 - e^{-z^2})/z$$

Decomposition for spin channels ($p - \Lambda$)

$$C(k^*) = \frac{1}{4}(1 + \lambda C(k^*, s=0)) + \frac{3}{4}(1 + \lambda C(k^*, s=1))$$

f_0 and d_0 - parameters of strong interaction.

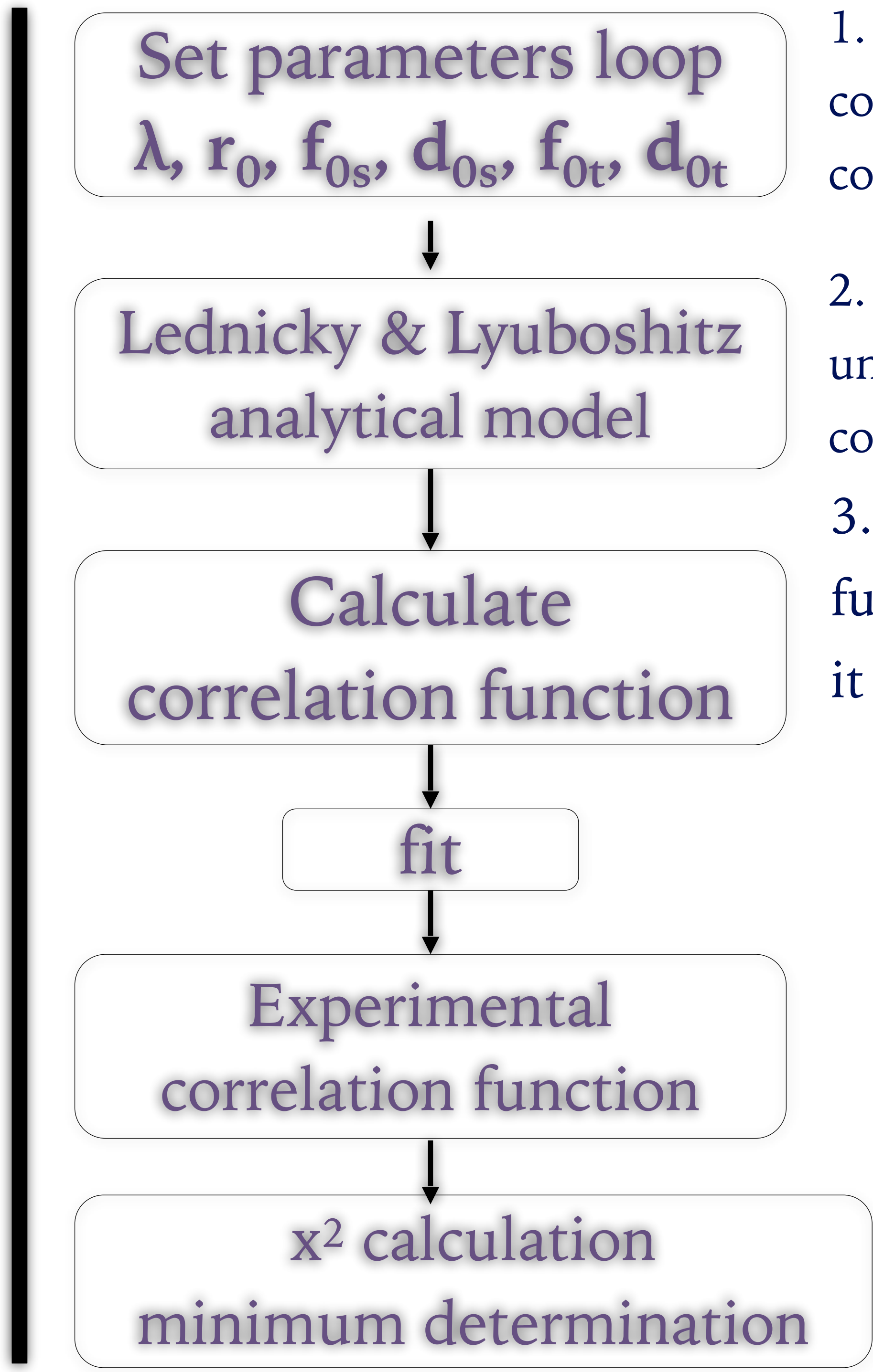
Theoretical correlation function (k*) depends on: R, f_0 and d_0.

f_0 - the scattering length, determines low-energy scattering.

The elastic cross section, σ_e, (at low energies) determined by the scattering length, $\lim_{k \to 0} \sigma_e = 4\pi f_0^2$

d_0 - the effective range, corresponds to the range of the potential (simplified scenario - the square well potential.

Lednicky-Lyuboshitz model



1. The Lednicky-Luboshitz semi-analytical model (utilized in CorrfitCumac codes) provides an immediate correlation function value but may be computationally intensive due to integral calculations.

2. The first fitter employs ROOT minimizers, offering precise statistical uncertainty estimation, but it operates on "continuous" maps with limited control over parameter steps.

3. The second fitter, Hal:Minimizer, accommodates "non-continuous" functions, allowing parameters to change in discrete steps. However, it provides only approximate uncertainty estimates.

```
for( int λ = 0.6; λ < 0.8; λ+=0.1 )
    for( int r0 = 1.0; r0 < 4.0; r0+=0.1 )
        for( int f0 = 0.01; f0 < 5.0; f0+=0.1 )
            for( int d0 = 0.01; d0 <5.0; d0+=0.1)
                for( int ft = 0.01; ft < 5.0 ; ft += 0.1 )
                    for( int dt =0.01; dt < 5.0; dt+=0.1 )

                     Calculate Lednicky-Luboshitz
                        correlation function : fit data
                     χ2 : value is extracted : minimizer
```

Coordinate system

Femtoscopy in Relativistic Heavy Ion Collisions: Two Decades of Progress; Mike Lisa, Scott Pratt, Ron Soltz, Urs Wiedemann

Ann.Rev.Nucl.Part.Sci.55:357-402,2005

Relative distance function:

$$S_P(r') \sim \exp\{-\frac{[r_{out} - \bar{X}_{out}]^2}{4\gamma_T^2 R_{out}^2} - \frac{r_{side}^2}{4R_{side}^2} - \frac{r_{long}^2}{4R_{long}^2}\}$$

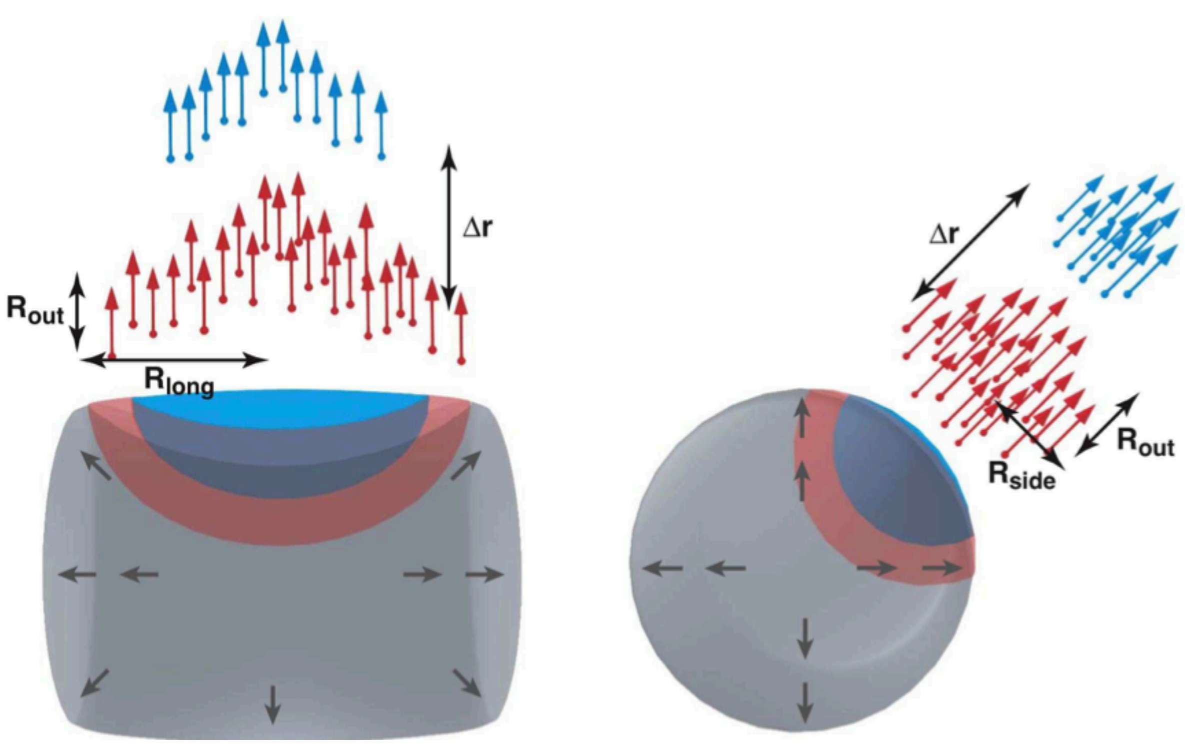
$\gamma_T = (1 - V_T)^{-1/2}$; V_T - velocity of the pair in LCMS frame.

$$R_{out}^2 = \frac{1}{2}[R_{a,out}^2 + R_{b,out}^2 + (V_{s,a-V_T})^2(\Delta\tau_a)^2 + (V_{s,b} - V_T)^2(\Delta\tau_b)^2]$$

$$R_{side}^2 = \frac{1}{2}[R_{a,side}^2 + R_{b,side}^2]$$

$$R_{long}^2 = \frac{1}{2}[R_{a,long}^2 + R_{b,long}^2]$$

$$\bar{X}_{out} = \bar{x}'_{a,out} - \bar{x}'_{b,out}$$



long - determined by the beam direction

out - determined by the pair transverse momentum

side - parallel to *long* and *side*

Coordinate system

Sensitivity to lifetime (spread of the emission time, small $\Delta\tau$ means particles were emitted rapidly, not early)

If $R^2_{a,out} + R^2_{b,out} \simeq R^2_{a,side} + R^2_{b,side}$

Then $V_{a,s} = V_{b,s}$ and $\Delta\tau_a = \Delta\tau_b = \Delta\tau$

$(V_T - V_s)^2(\Delta\tau)^2 \simeq R^2_{out} - R^2_{side}$

If emission comes from sources moving over large range of rapidity (a boost-invariant expansion), the dimension along the beam axis for the source emitting zero-rapidity particles is determined by the distance one can move before the collective velocity overwhelms the thermal velocity.

$$R_{long} \simeq \frac{V_{them}}{dv/dz} = V_{therm} < t >$$

R_{out}/R_{side} gives information about suddenness of emission

R_{long} provides the insight in to the mean time at which emission occurs as estimate of the thermal velocity.

Coordinate system

For a thermal source, the thermal velocity is determined by the temperature and transverse mass $m_T = \sqrt{m^2 + p_T^2}$

$V_{therm} = \sqrt{T/m_T}$;

assumes particles are emitted with the same Bjorken time $\tau_B = \sqrt{t^2 - z^2}$

$$\frac{1}{R_{long}^2} \sim \frac{1}{V_{therm}^2 \tau_B^2} + \frac{1}{\eta_G^2 \tau_B^2}$$

η_G is the range of rapidity over which the source is distributed

From Mike Lisa (STAR paper):

$R_{long} = \tau \sqrt{\frac{T}{m_T} \frac{K_2(m_T/T)}{K_1(m_T/T)}}$; includes modifies Bessel functions

Coordinate system

The correlation function depends on three-dimensional momenta: **P**, **q**.

$$P = p_a + p_b$$

$$q^\mu = \frac{(p_a - p_b)^\mu}{2} - \frac{(p_a - p_b)P}{2P^2}P^\mu$$

For high-energy collisions, usually LCMS system is used (moving along the longitudinal direction, $P_z = 0$)

long - determined by the beam direction

out - determined by the pair transverse momentum

side - parallel to *long* and *side*

Any four-vector V can be expressed in the coordinate system in the four-momentum P to project out the components:

$$V_{long} = (P_0 V_z - P_z V_0)/M_T$$

$$V_{out} = (P_x V_x - P_x V_y)/P_T$$

$$V_{side} = (P_x V_y - P_y V_x)/P_T$$

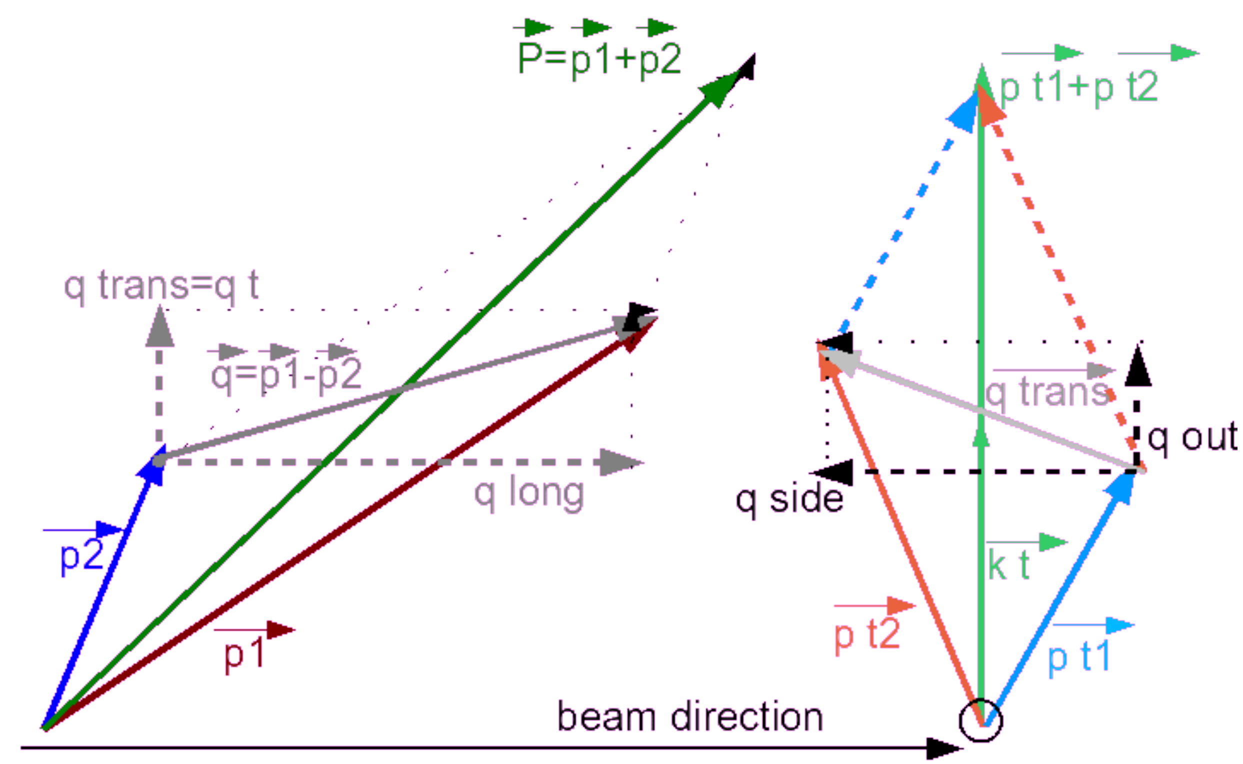
$$M_T^2 = P_0^2 - P_z^2$$

$$P_T^2 = P_x^2 + P_y^2$$

Second boost to the Pair Rest Frame ($P_T = 0$):

$$V'_{out} = \frac{M_{inv}}{M_T}\frac{(P_x V_x - P_x V_y)}{P_T} - \frac{P_T}{M_T M_{inv}}PV$$

$$M_{inv}^2 = P^2$$



Coordinate system

To gain a physical understanding of the three-dimensional spatio-temporal source distributions, it is useful to summarize its size and shape with a few parameters. This motivates to study of Gaussian parametrization for the source and two-particle correlator. Relativistic sources deviate from Gaussians. In practice, Gaussian parametrization provide the standard minimal description of experimental data. Using the reflection symmetries for mid-rapidity sources in a symmetric central Collisions, a Gaussian parametrization for the emission function for particle species:

$$s_a(p,x) \sim \exp\{-\frac{(x_{out} - \bar{x}_{a,out} - V_{s,a}(t - t_a))^2}{2R_{a,out}^2} - \frac{x_{side}^2}{2R_{a,side}^2} - \frac{x_{long}^2}{2R_{a,long}^2} - \frac{(t - \bar{t}_a)^2}{2(\Delta\tau_a)^2}\}$$

The cross term indicates that source can move in outward direction with a velocity V_S. The correlation function is determined by the phase space density of the final state, so the phase space density is

$$f_a(p,r,t) \sim \exp\{-\frac{[x_{out} - \bar{X}_a(t)]^2}{2[R_{a,out}^2 + (V_{s,a} - V_T)^2(\Delta\tau_a)^2]} - \frac{x_{side}^2}{2R_{a,side}^2} - \frac{x_{long}^2}{2R_{a,long}^2}\}$$

$$\bar{X}_a(t) = \bar{x}_{a,out} + V_T(t - t_a)$$

Relative distance function:

$$S_P(r') \sim \exp\{-\frac{[r_{out} - \bar{X}_{out}]^2}{4\gamma_T^2 R_{out}^2} - \frac{r_{side}^2}{4R_{side}^2} - \frac{r_{long}^2}{4R_{long}^2}\}$$

$$\gamma_T = (1 - V_T)^{-1/2}$$