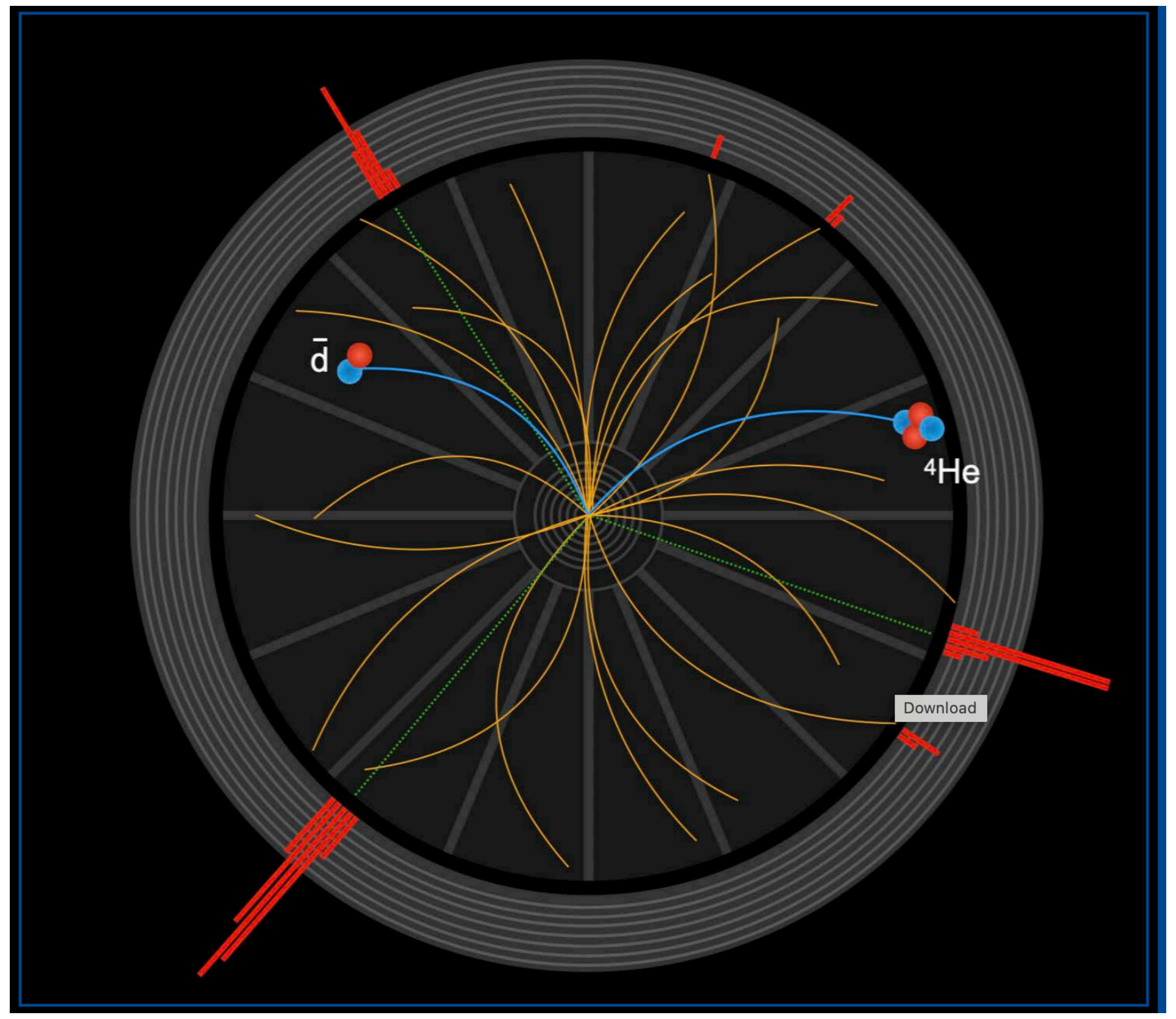
Study of hadronic interactions via momentum-space correlations at the LHC



Bhawani Singh

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EMMI Rapid Reaction Task Force

GSI Helmholtzzentrum für Schwerionenforschung GmbH

April 9, 2024, Darmstadt, Germany

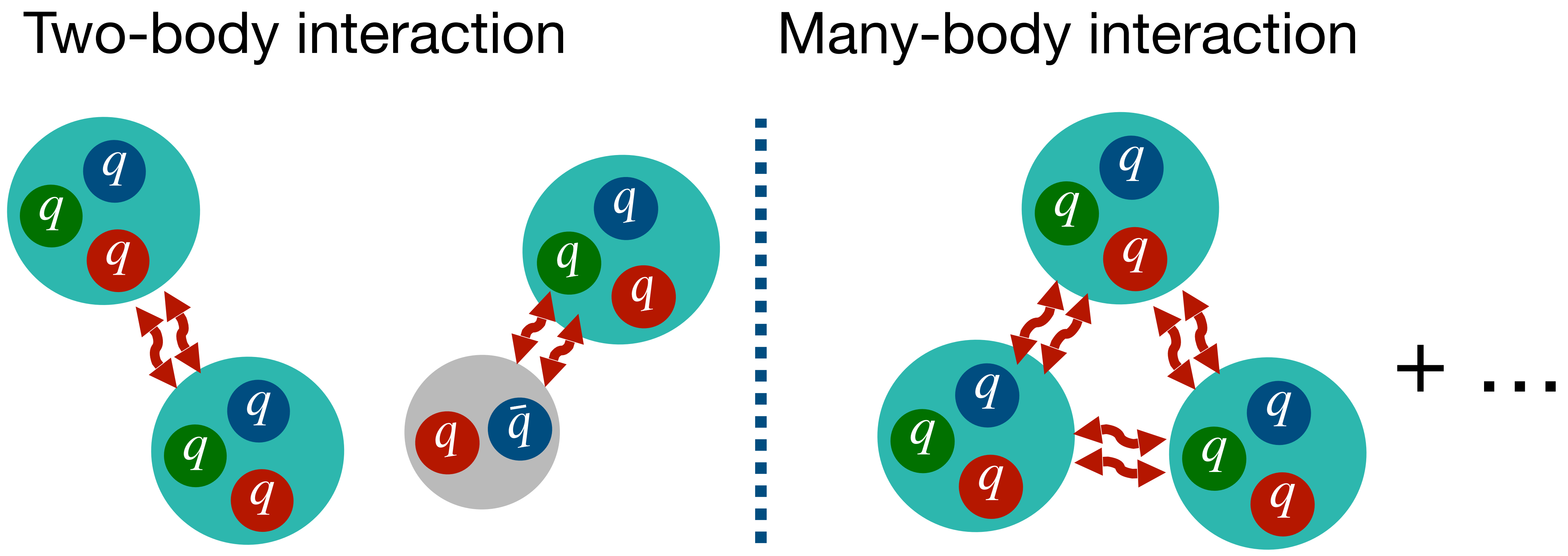
bhawani.singh@tum.de

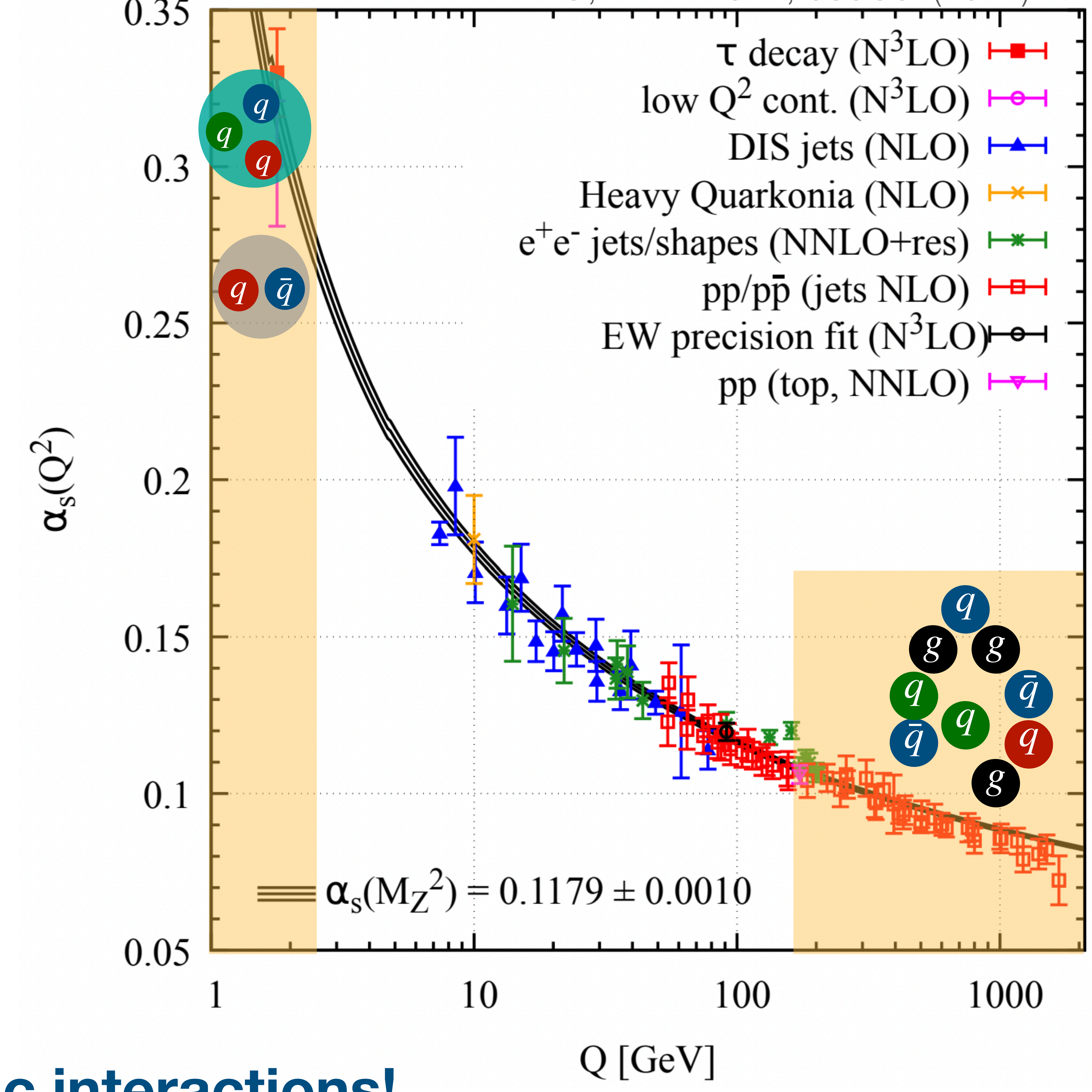
Hadronic interactions and QCD

Understanding how QCD evolves from high-energy to low-energy regime

Non-perturbative QCD $\rightarrow Q \sim 1$ GeV

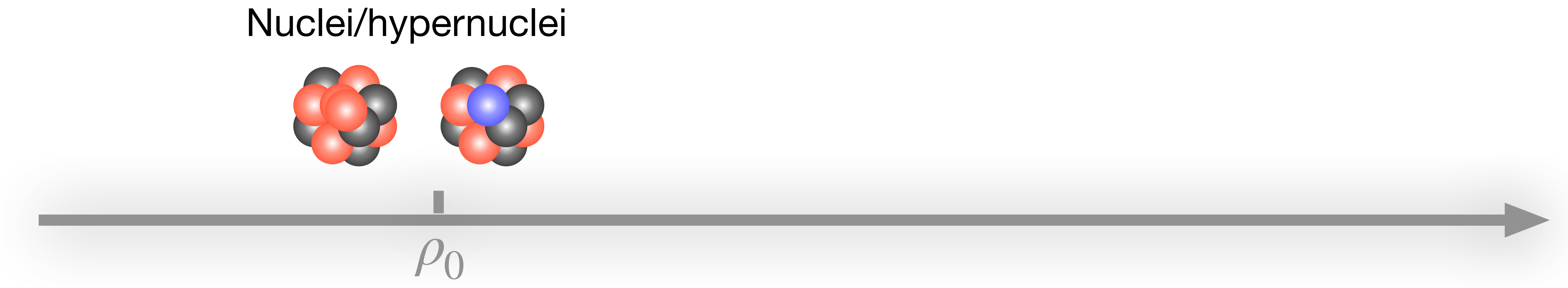
- Emergence of hadrons
- How do **hadrons** interact?



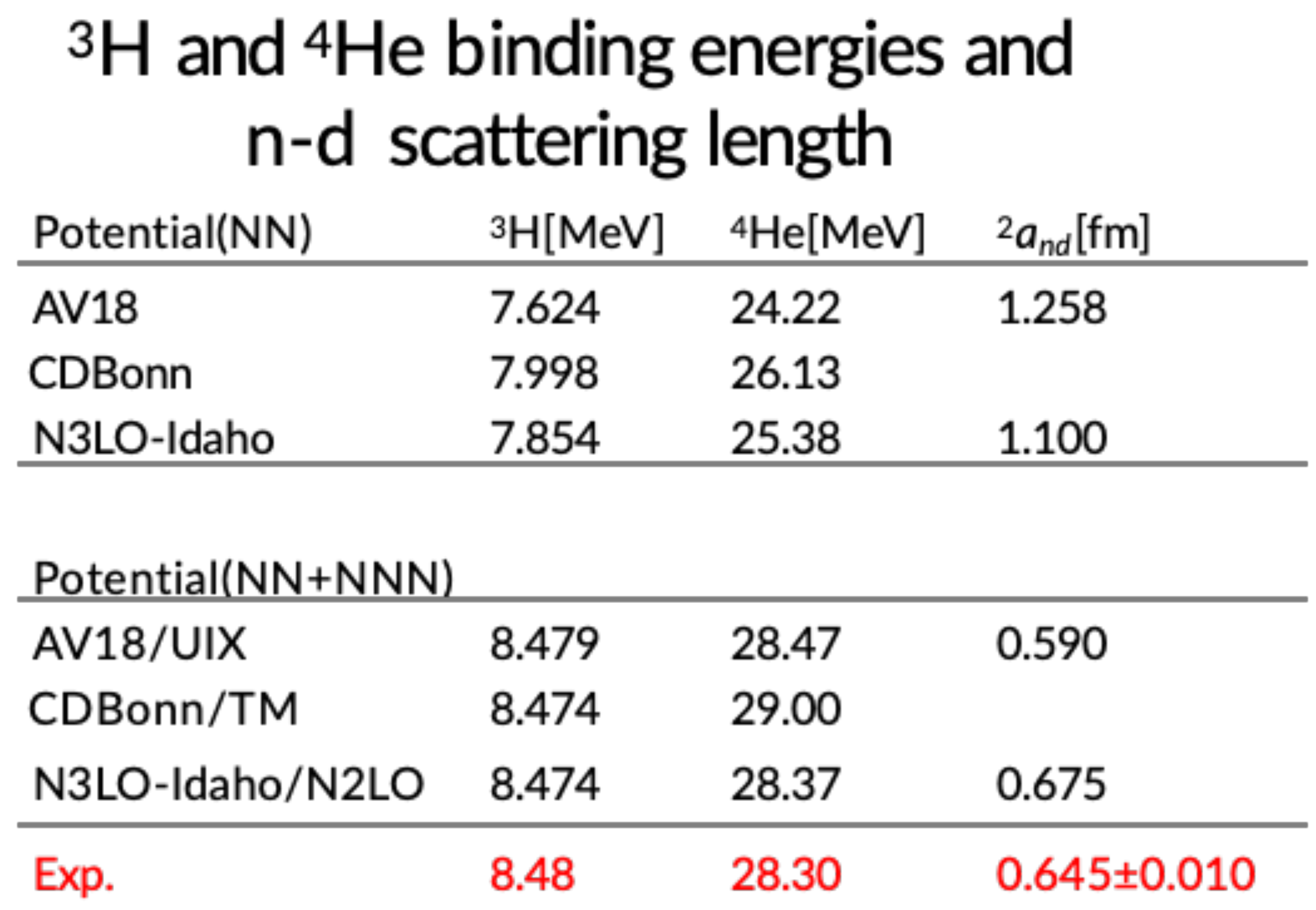

PDG, PTEP 2022, 083C01(2022)

We need data for hadronic interactions!

Three-body force in many-body systems



- Properties of nuclei and hypernuclei cannot be described satisfactorily with two-body forces only
- NNN interaction contributes **~10%** to the binding energies of light nuclei A~3 and 4
- Many-body scattering requires three-body calculations (e.g. neutron-deuteron) L. Girlanda et al., PRC 102, 064003 (2020)



^{3}H and ^{4}He binding energies and n-d scattering length

Potential(NN)	^{3}H[MeV]	^{4}He[MeV]	$^2a_{nd}$[fm]
AV18	7.624	24.22	1.258
CDBonn	7.998	26.13	
N3LO-Idaho	7.854	25.38	1.100
Potential(NN+NNN)			
AV18/UIX	8.479	28.47	0.590
CDBonn/TM	8.474	29.00	
N3LO-Idaho/N2LO	8.474	28.37	0.675
Exp.	8.48	28.30	0.645±0.010

Sekiguchi, K. Few-Body Syst 60, 56 (2019)
L.E. Marcucci et al., Front. Phys. 8:69 (2020)

Three-body force in many-body systems

- Properties of nuclei and hypernuclei cannot be described satisfactorily with two-body forces only

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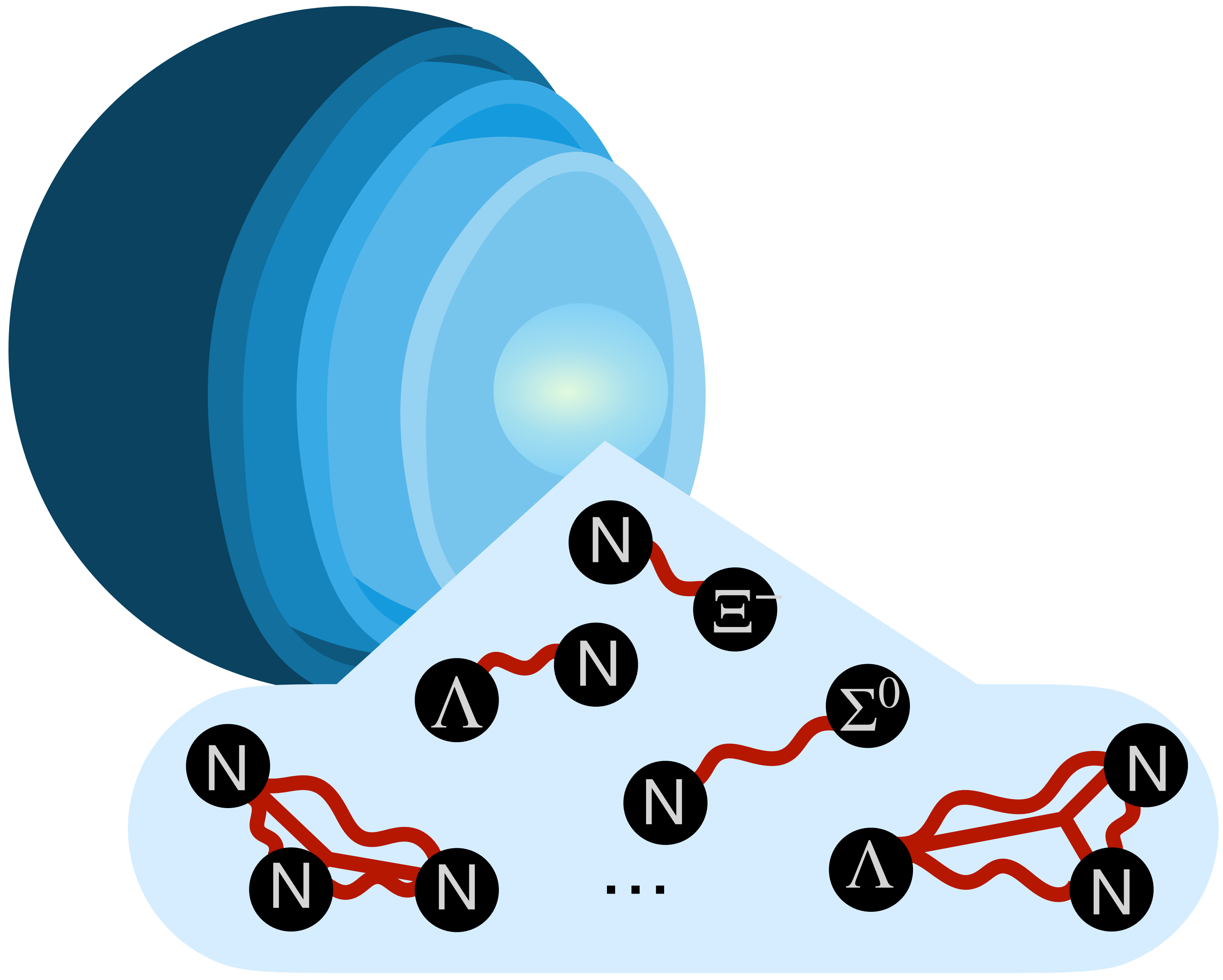
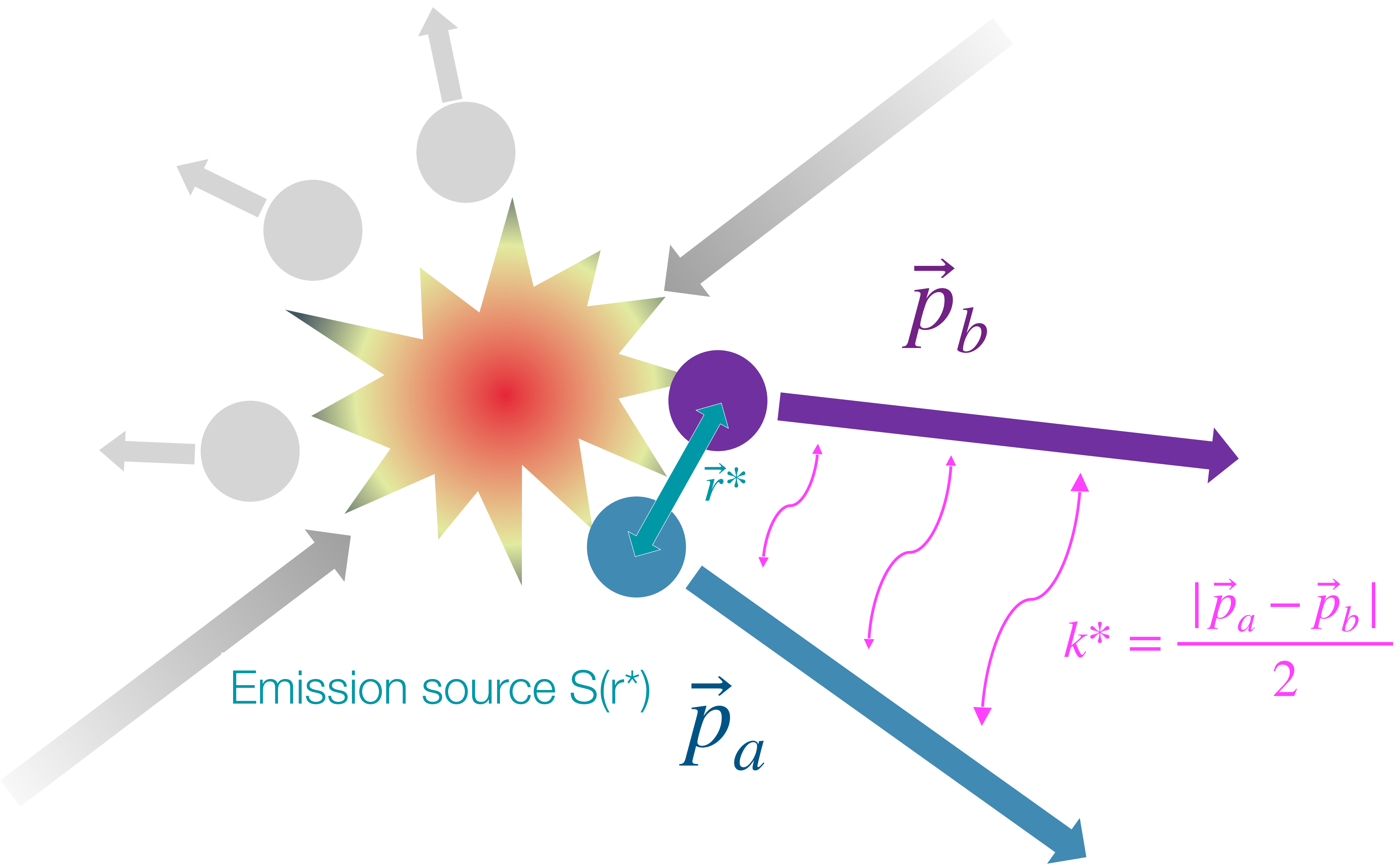
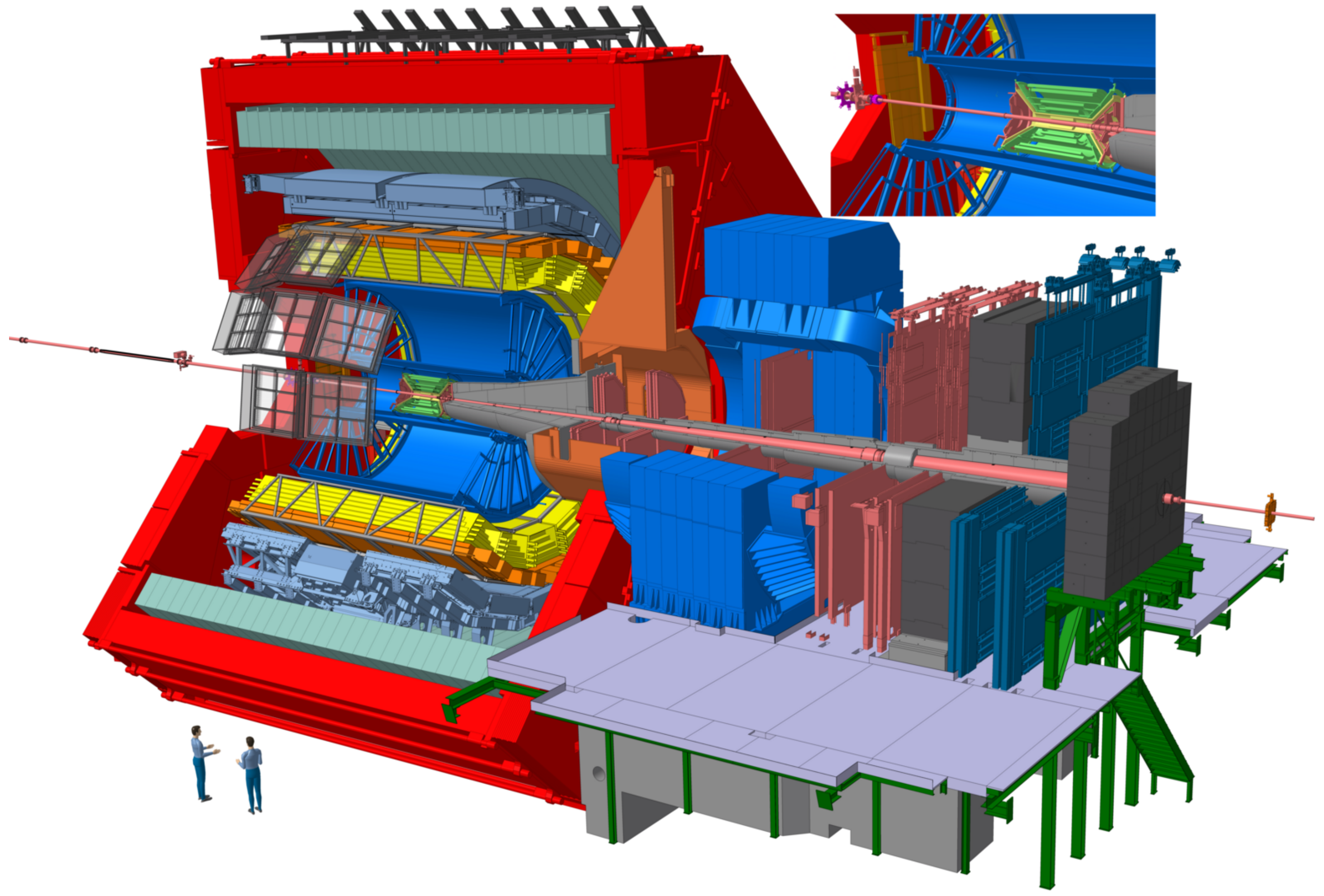
- NNN and NNΛ interactions used in the modeling of the equation of the state of neutron stars



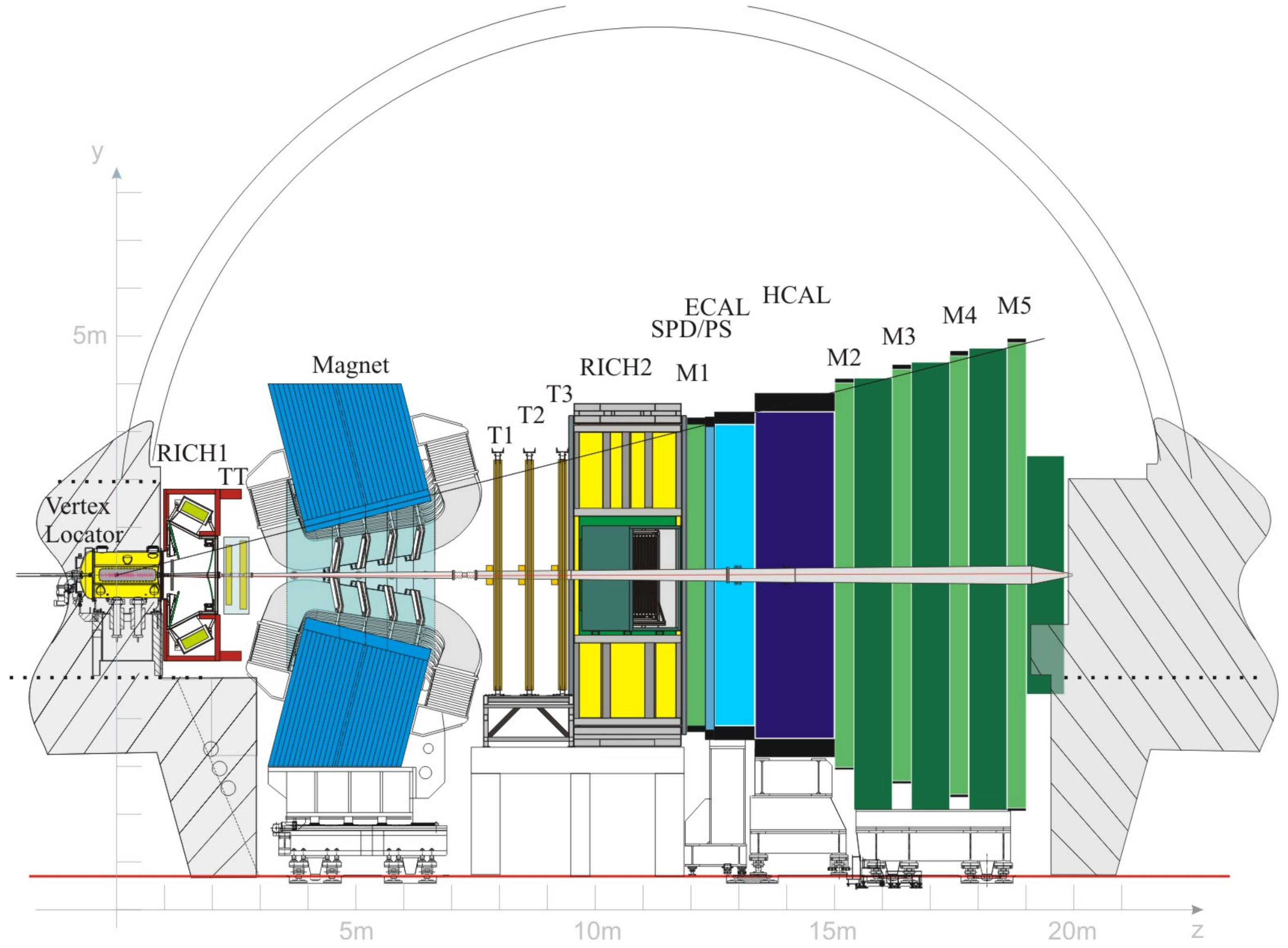
D. Lonardoni et al, PRL 114, 092301 (2015)
J. Schaffner-Bielich et al, NPA 835 (2010)

Femtoscopy at the LHC



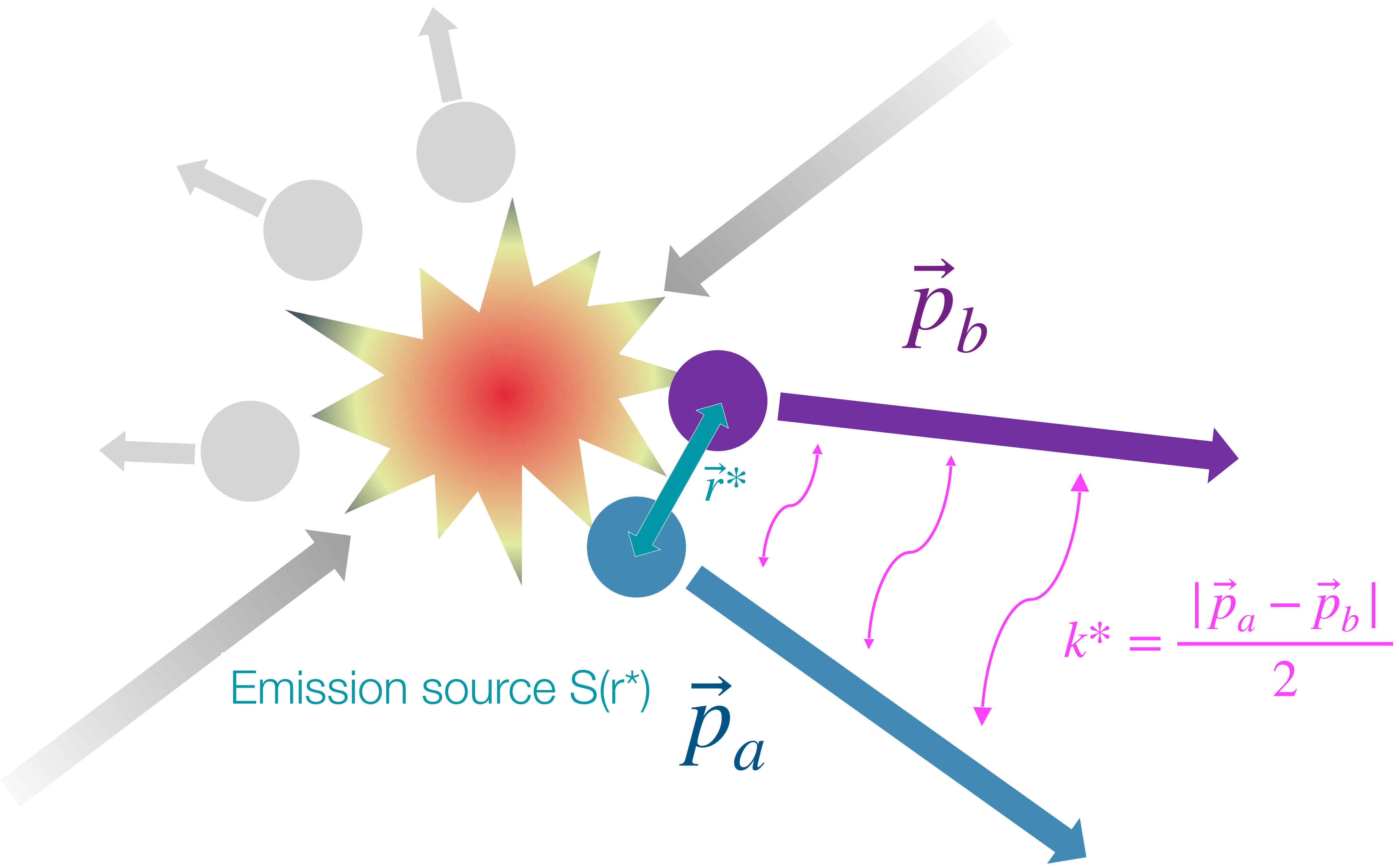


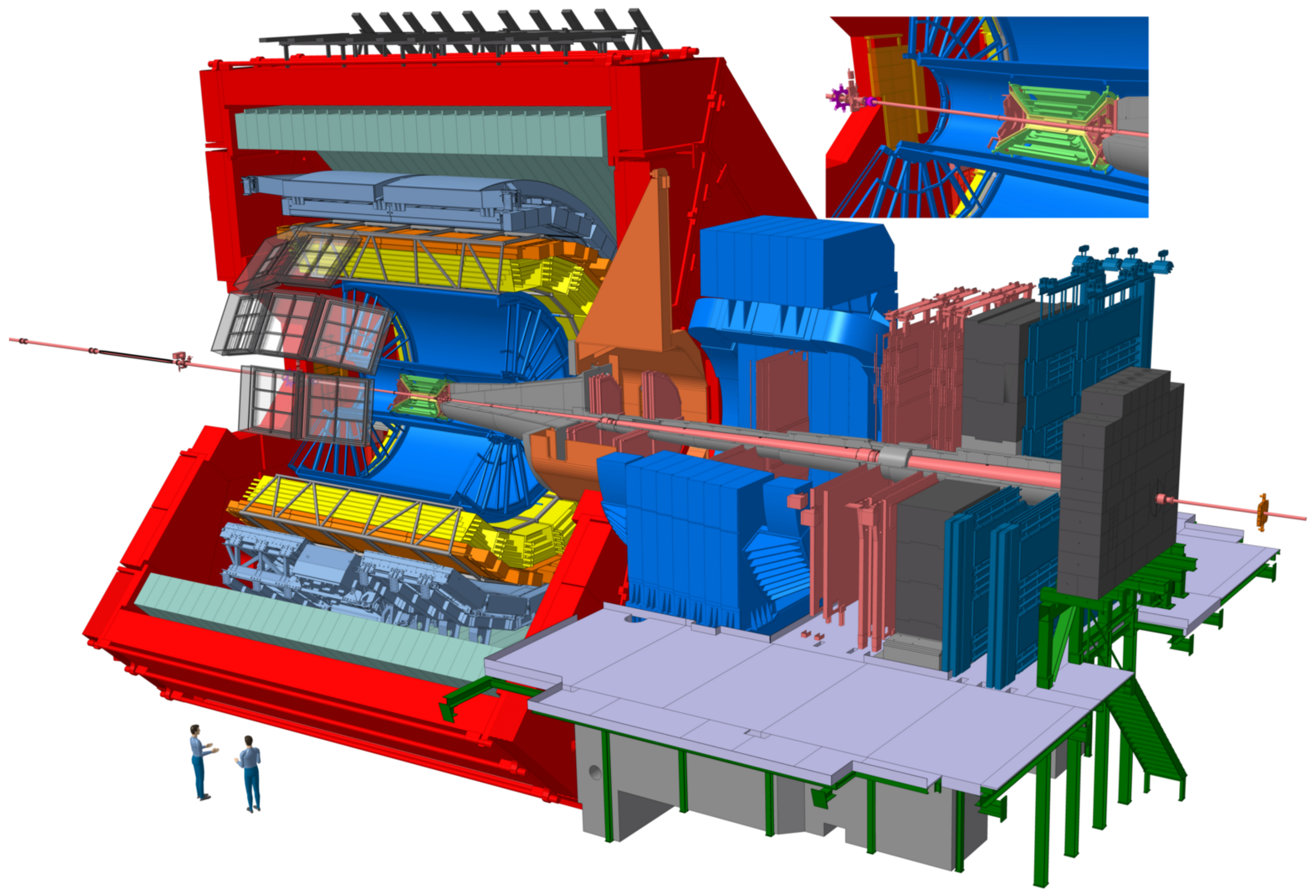
ALICE : ITS and TPC upgrades



LHCb: Sketch from 2008

Femtoscopy at the LHC





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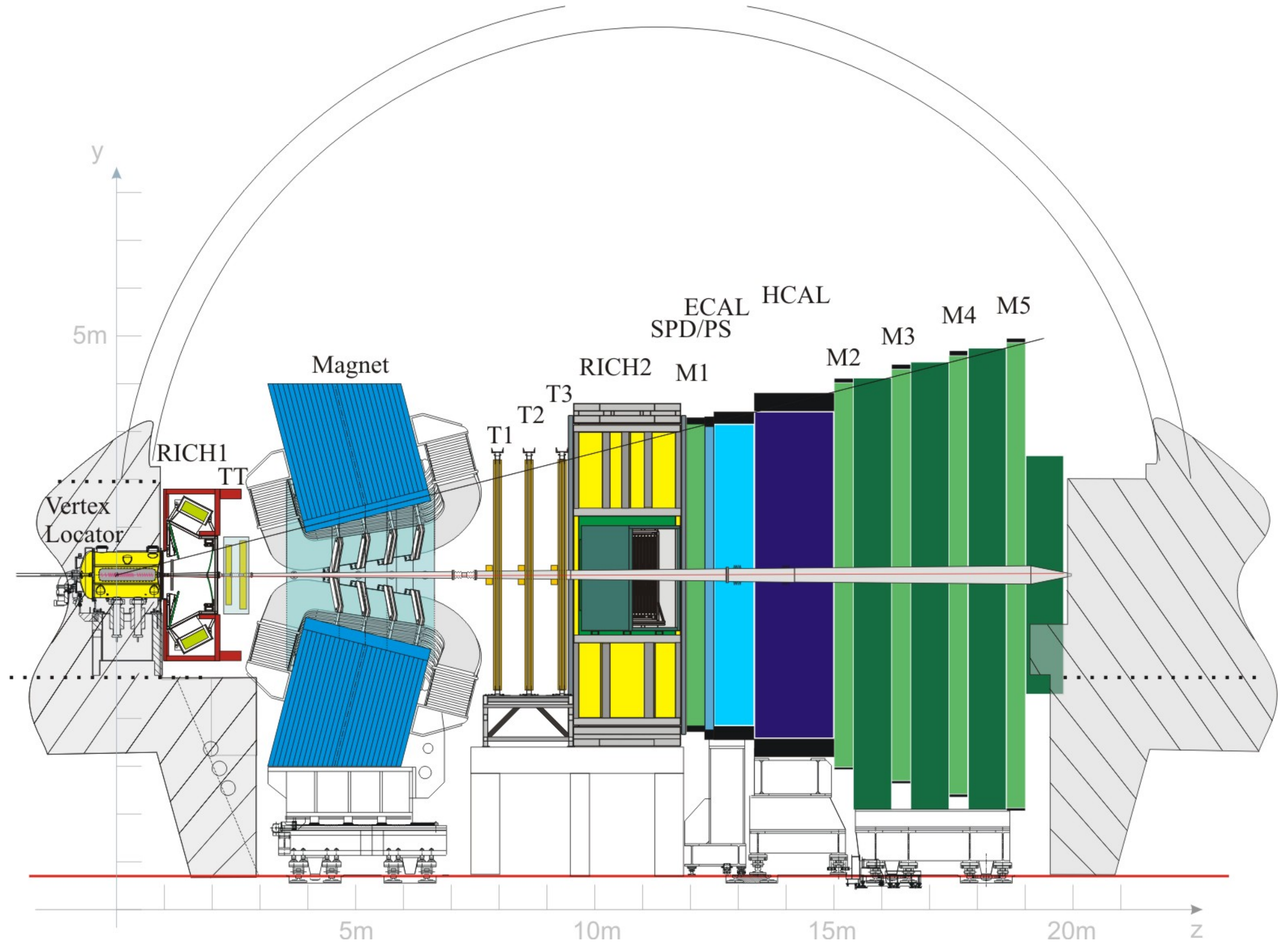
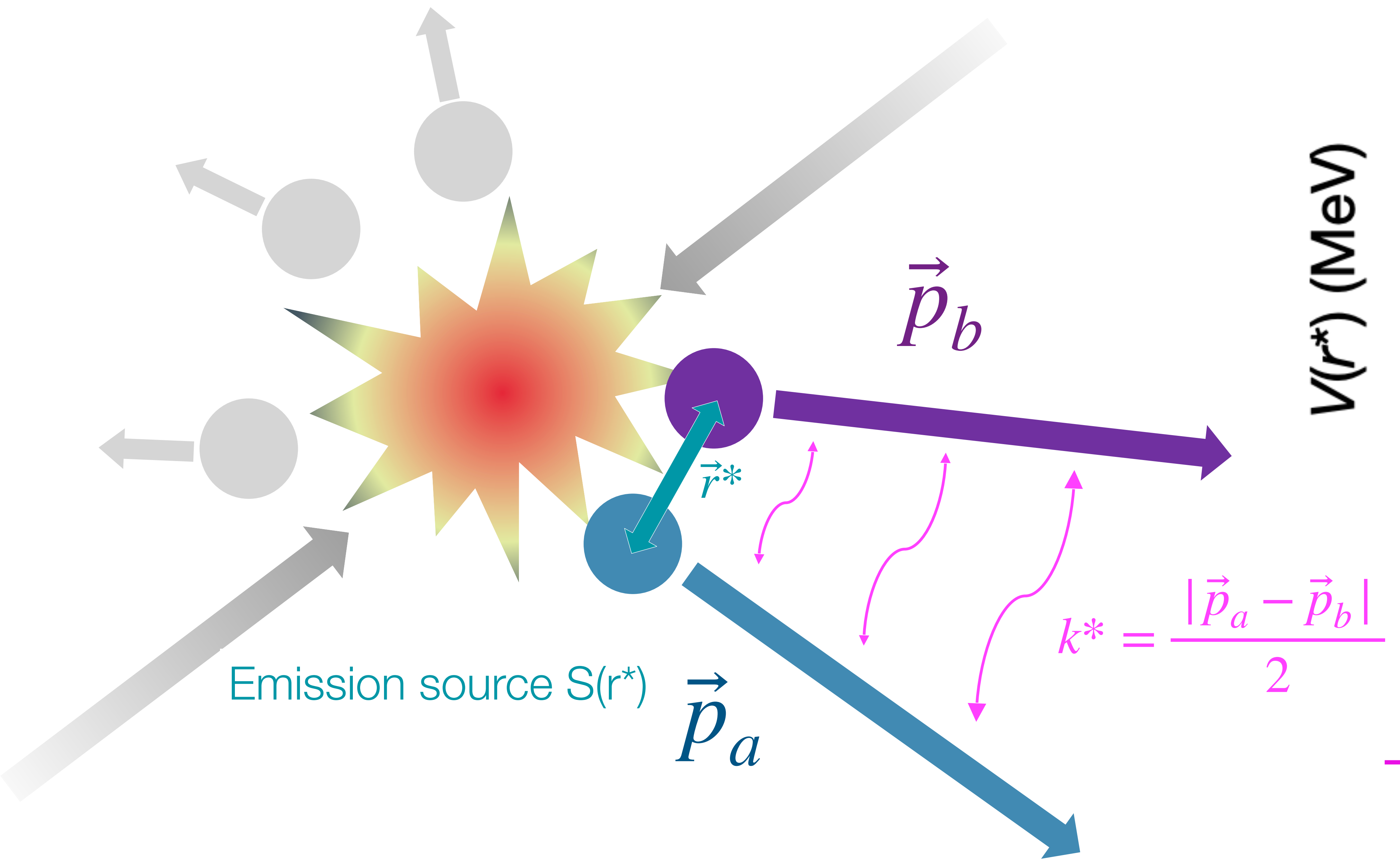


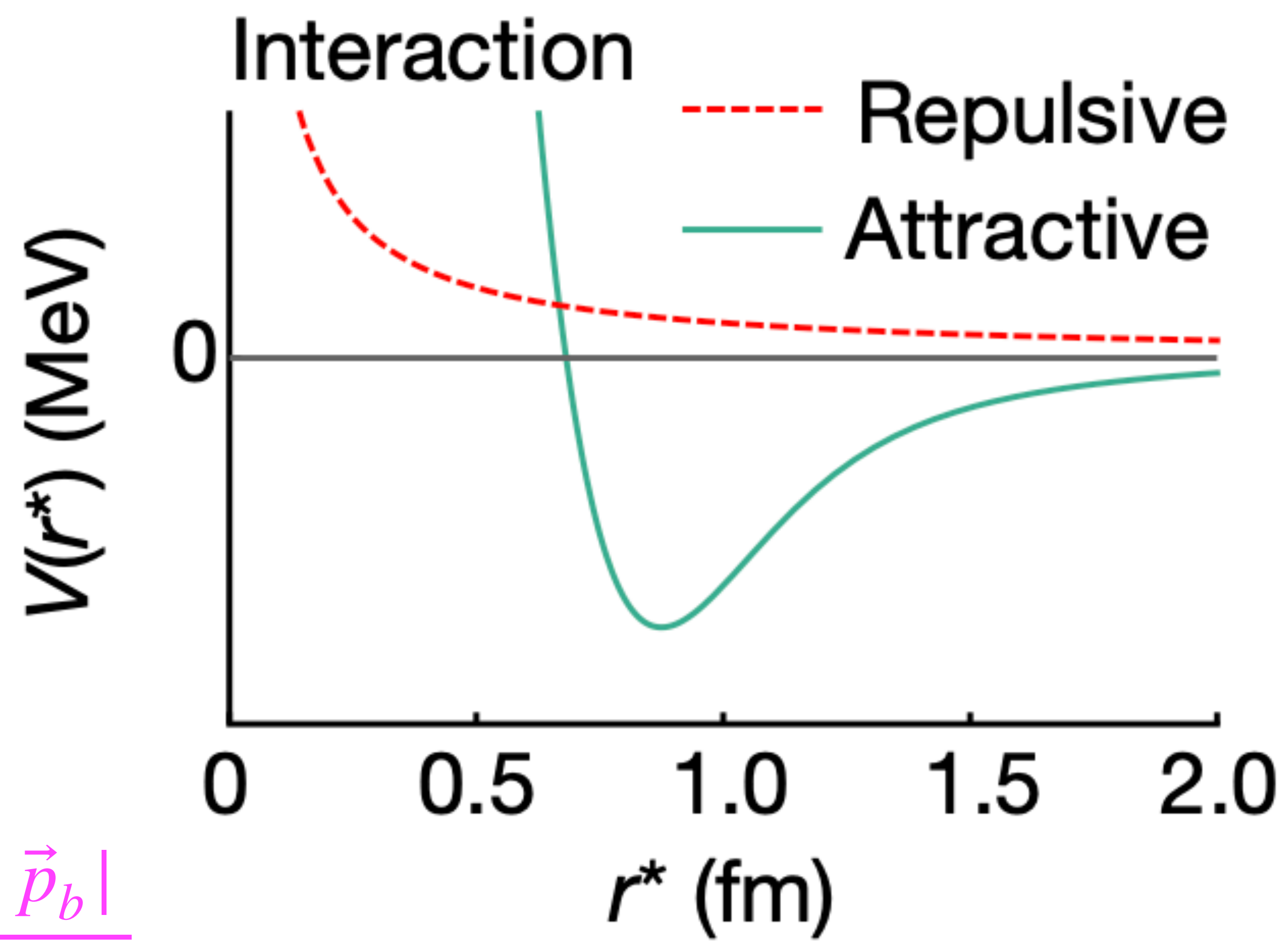
LHCb: Sketch from 2008

$$C(k^*) = \underbrace{\mathcal{N}\frac{N_{\mathrm{same}}(k^*)}{N_{\mathrm{mixed}}(k^*)}}_{\text{experimental definition}}$$

S.E. Koonin, PLB 70 43 (1977)
L. Fabbietti et al, Ann. Rev. Nucl. Part.Sci. 71 (2021) 377-402

Femtoscopy at the LHC

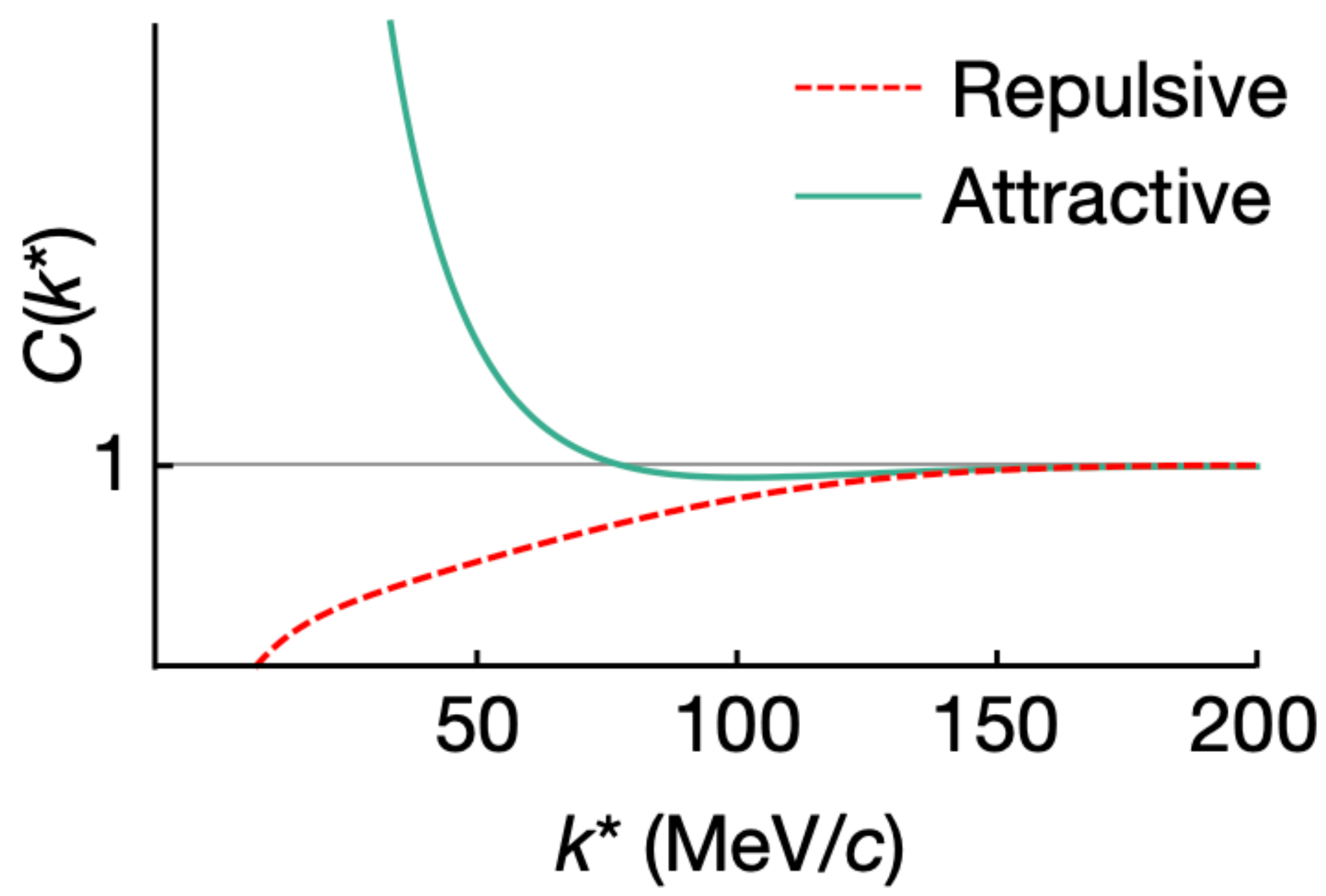




Schrödinger equation

Two-particle wave function

$\psi(\vec{k}^*, \vec{r}^*)$



Correlation function $C(k^*)$

$$C(k^*) = \underbrace{\mathcal{N}\frac{N_{\text{same}}(k^*)}{N_{\text{mixed}}(k^*)}}_{\text{experimental definition}} = \underbrace{\int S(\vec{r}^*)\left|\psi(\vec{k}^*, \vec{r}^*)\right|^2 d^3r^*}_{\text{theoretical definition}} \xrightarrow{k^*\to\infty} 1$$

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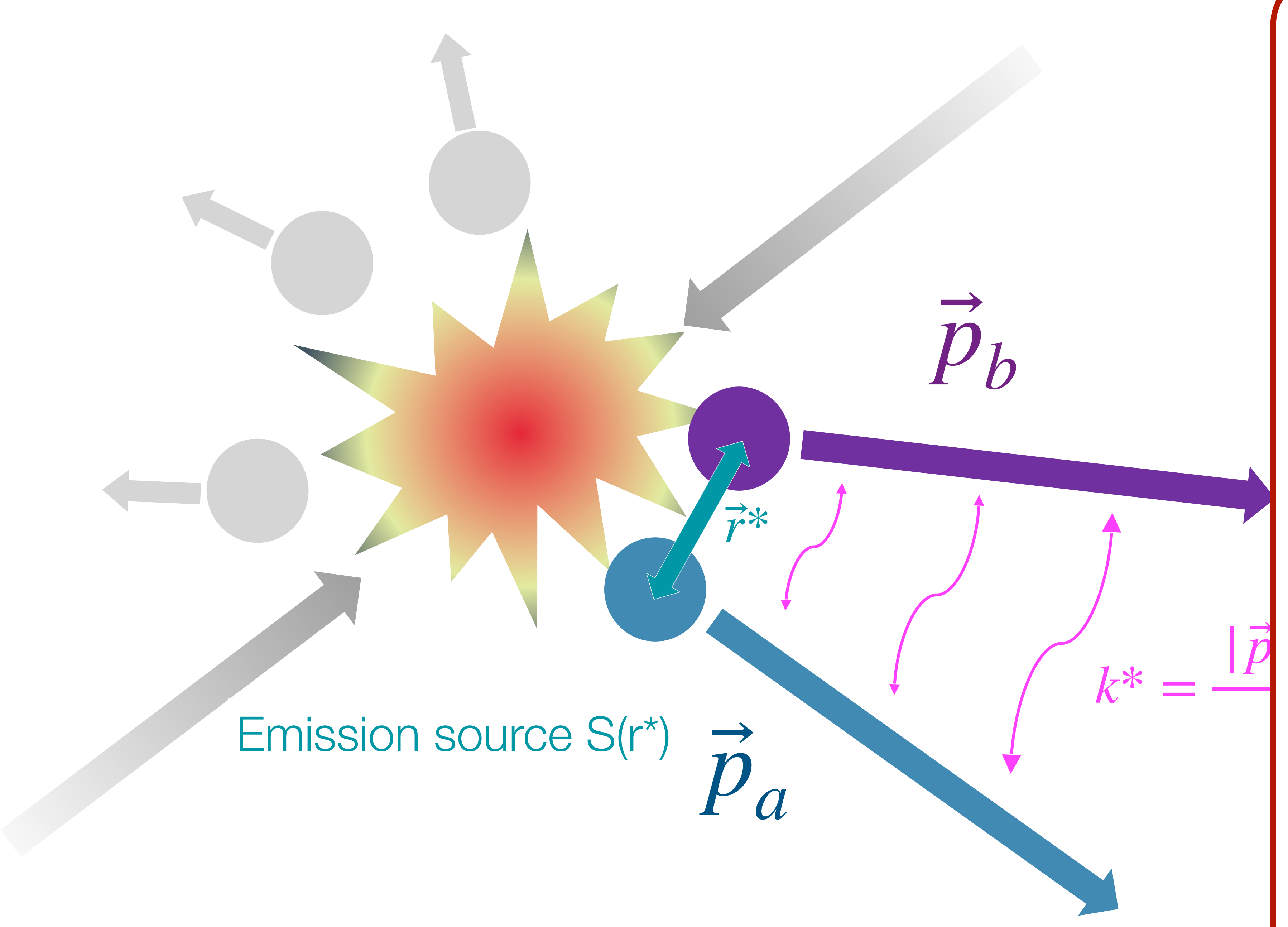
Femtoscopy at the LHC

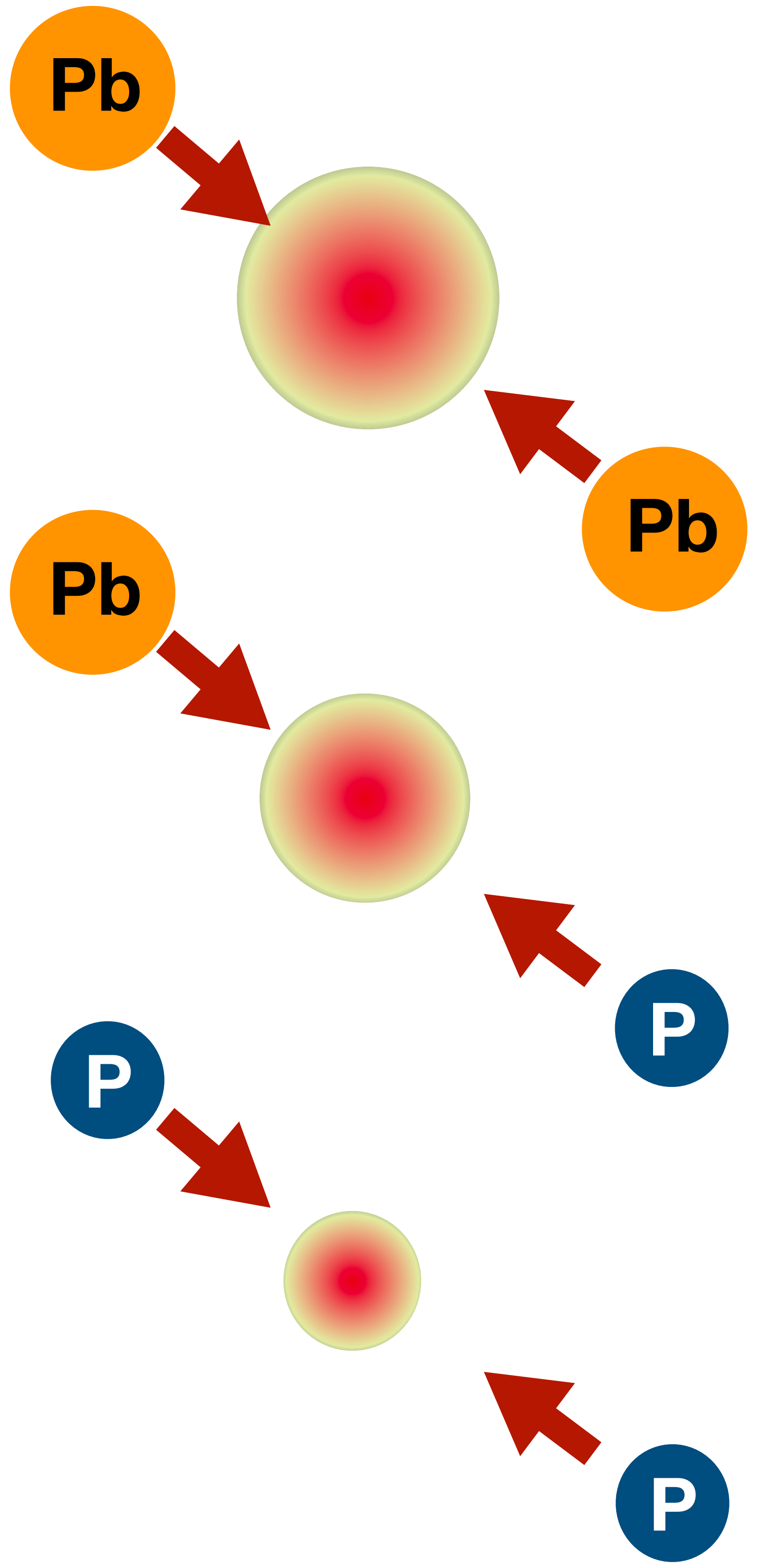


Collisions system sizes

Pb–Pb: 5–10 fm

p–Pb: 2–4 fm

pp: 1–1.5 fm



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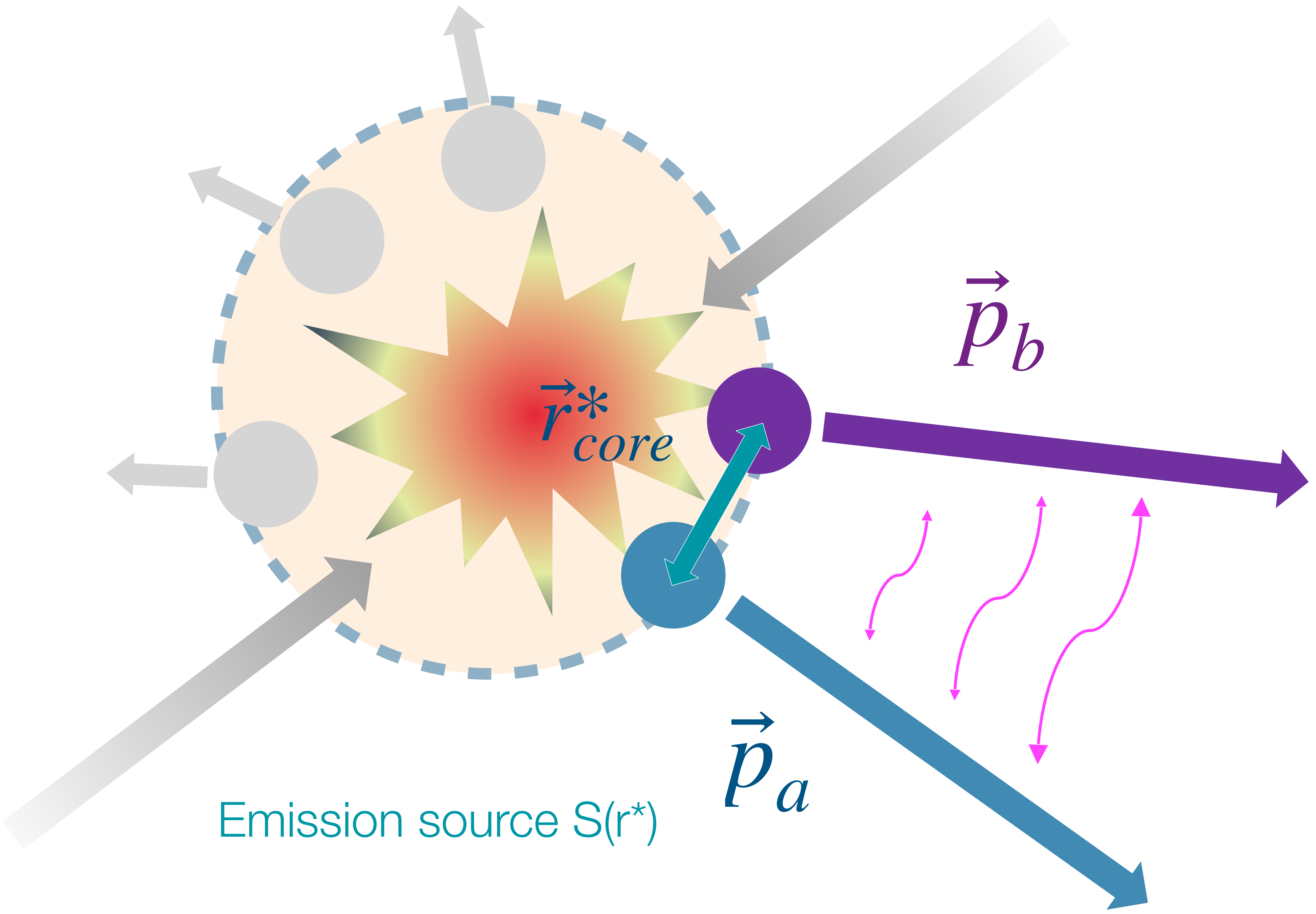
Constraining the source in pp collisions

- Common emission profile for **all hadron pairs**

$$S(r^*) = \frac{1}{\left(4\pi\, r_{core}{}^2\right)^{3/2}} exp\left(-\frac{r^{*2}}{4\, r_{core}{}^2}\right)$$



Gaussian core source

ALICE Coll, PLB 811 (2020)

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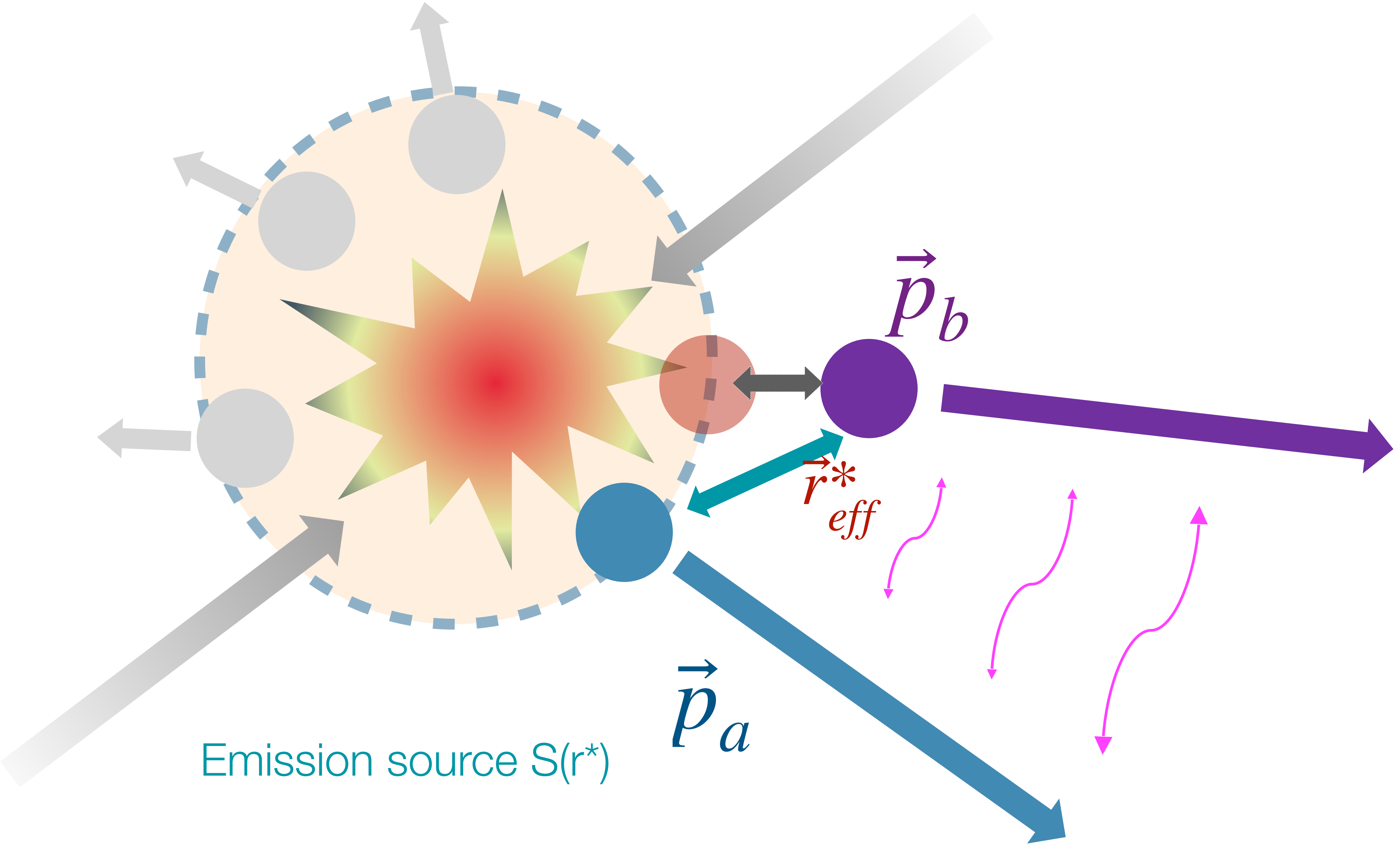
$$S(r^*) = \frac{1}{\left(4\pi\, r_{core}{}^2\right)^{3/2}} exp\left(-\frac{r^{*2}}{4\, r_{core}{}^2}\right)$$

- Include **short-lived strongly decaying** resonances
  - Lifetime

    $c\tau \sim r_{core} \sim$ 1fm (Δ^{++},N^*,Σ^*)
  - Yields are constrained using the thermal fist



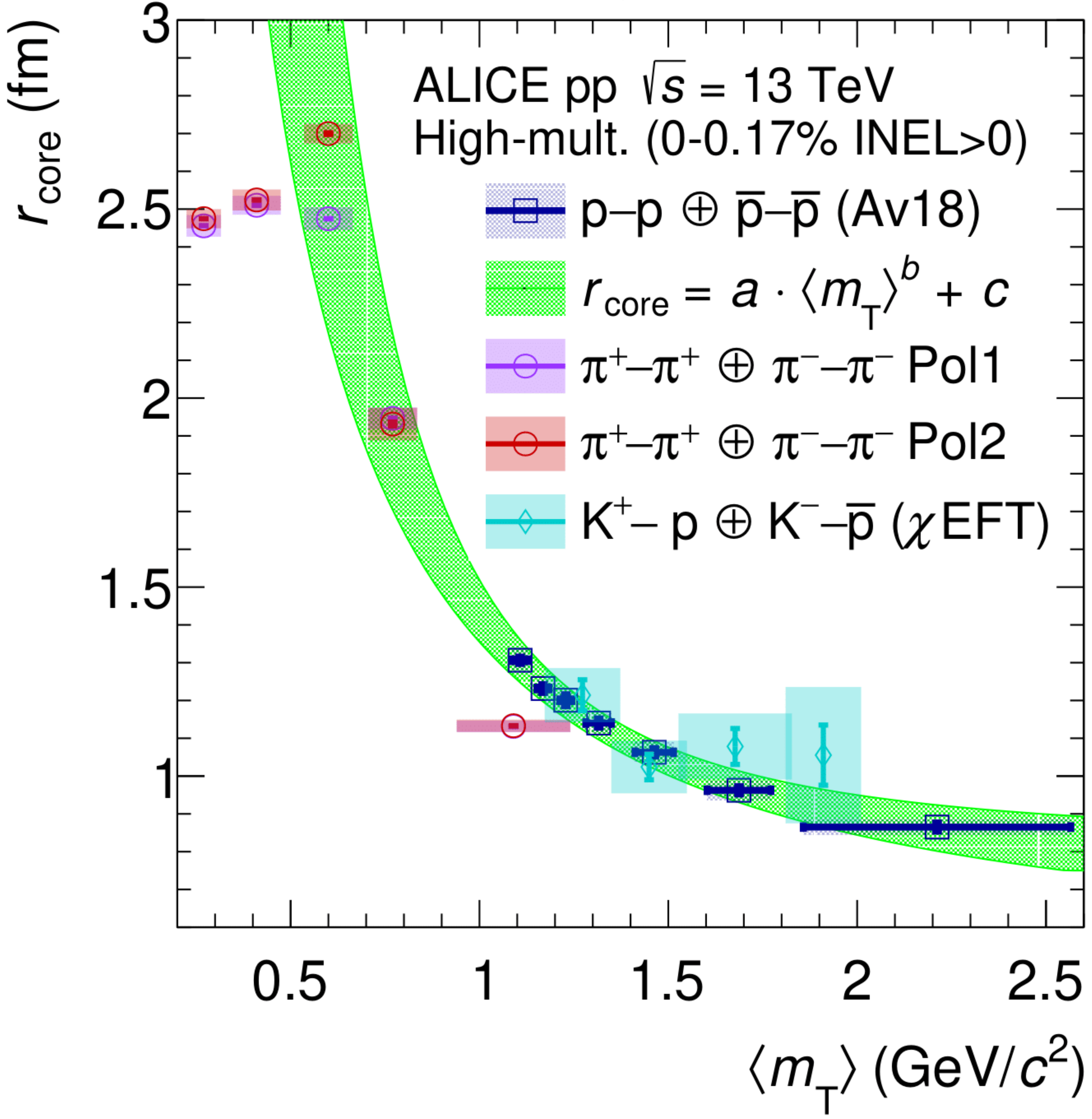
Gaussian core source +resonance contributions

ALICE Coll, PLB 811 (2020)

Constraining the source in pp collisions

- Common emission profile for **all hadron pairs**

ALICE Coll, arXiv:2311.14527



$$k_{\mathrm{T}} = \frac{1}{2}\left|\vec{p}_{\mathrm{T},1} + \vec{p}_{\mathrm{T},2}\right| \qquad m_{\mathrm{T}} = \sqrt{k_{\mathrm{T}}^2 + \langle m \rangle^2}$$

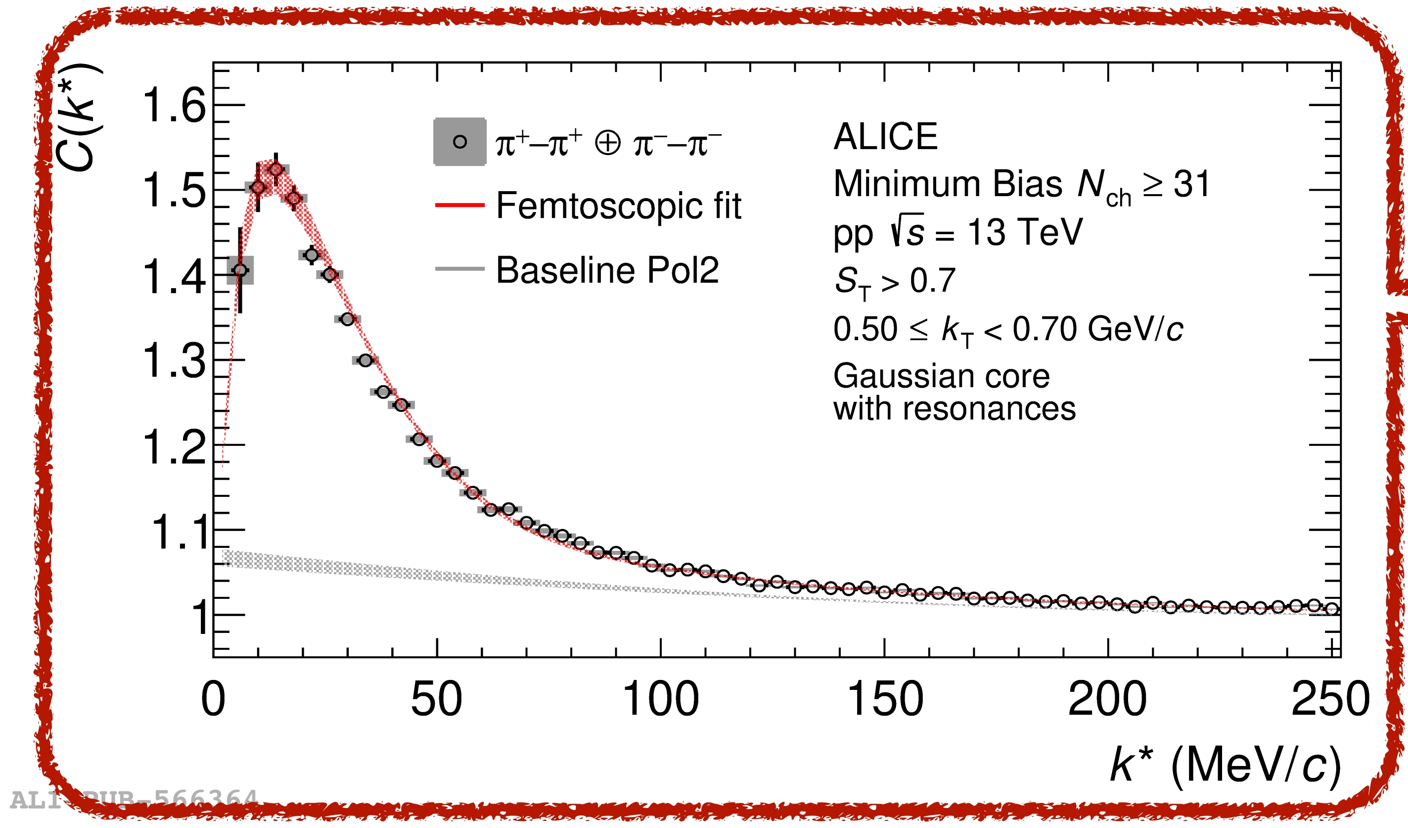
ALICE Coll, PLB 811 (2020)

Constraining the source in pp collisions

- Common emission profile for **all hadron pairs**

π-π (same charge):

Coulomb + quantum statistics



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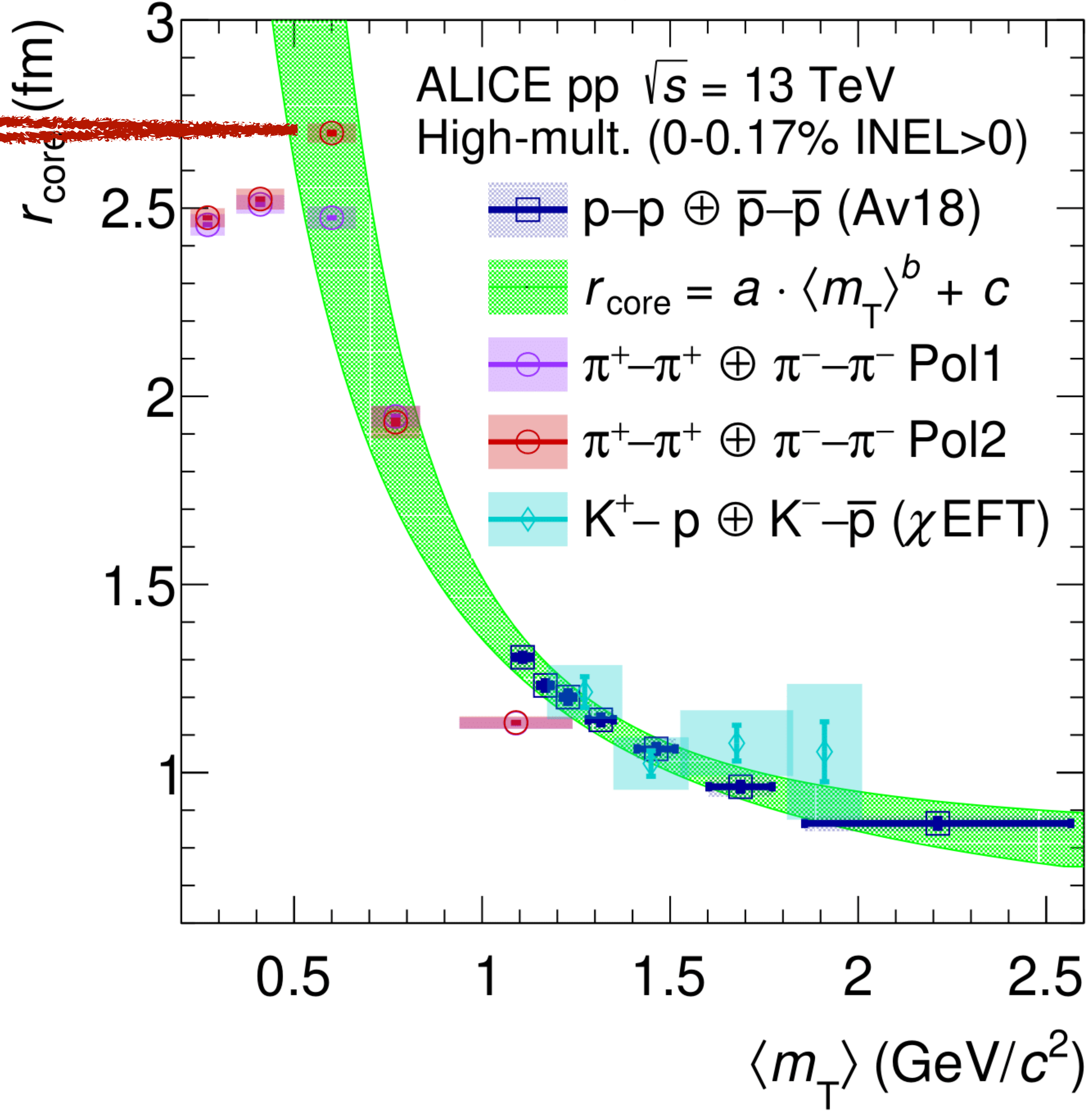

$$k_T = \frac{1}{2}\left|\vec{p}_{T,1} + \vec{p}_{T,2}\right| \qquad m_T = \sqrt{k_T^2 + \langle m \rangle^2}$$

ALICE Coll, PLB 811 (2020)

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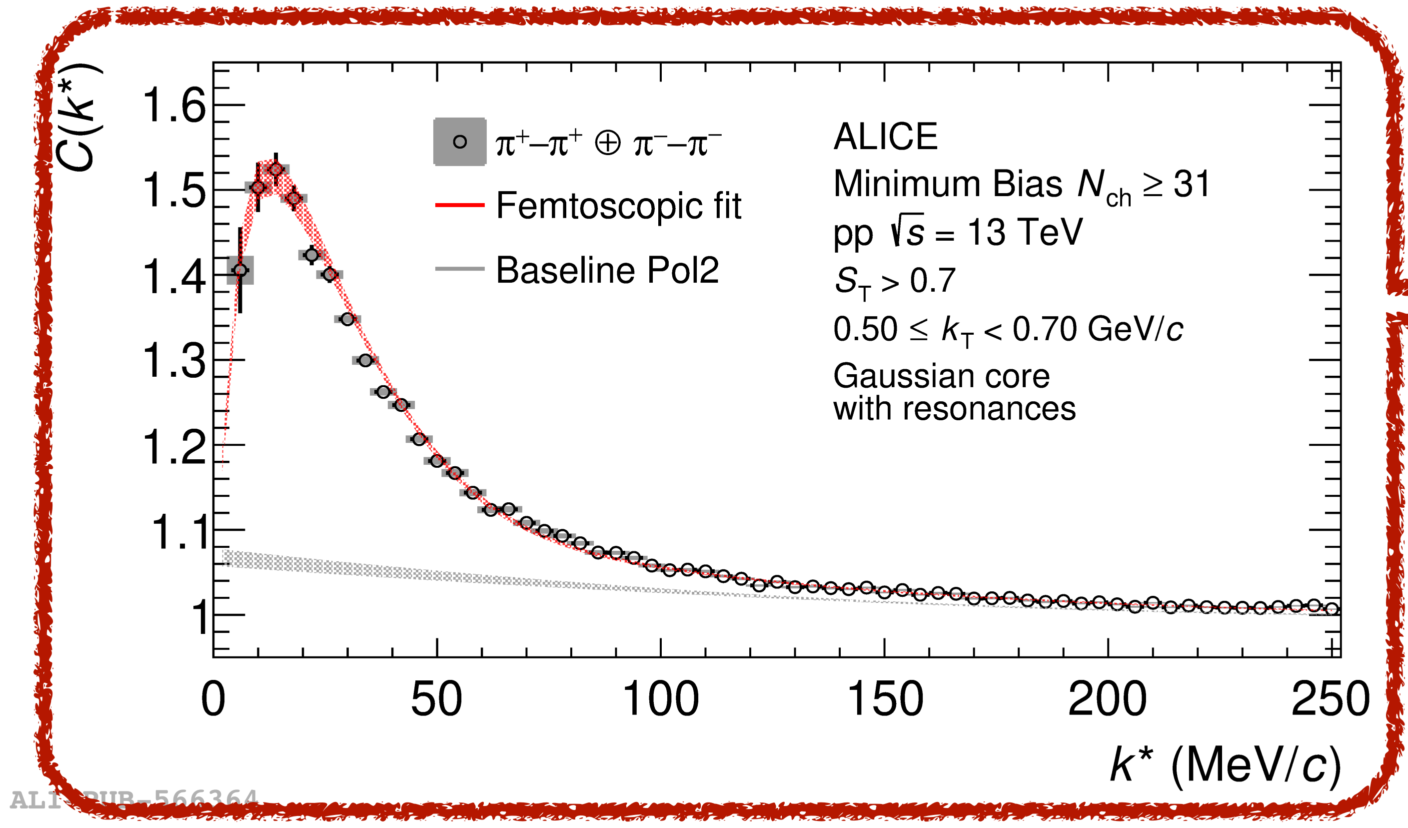

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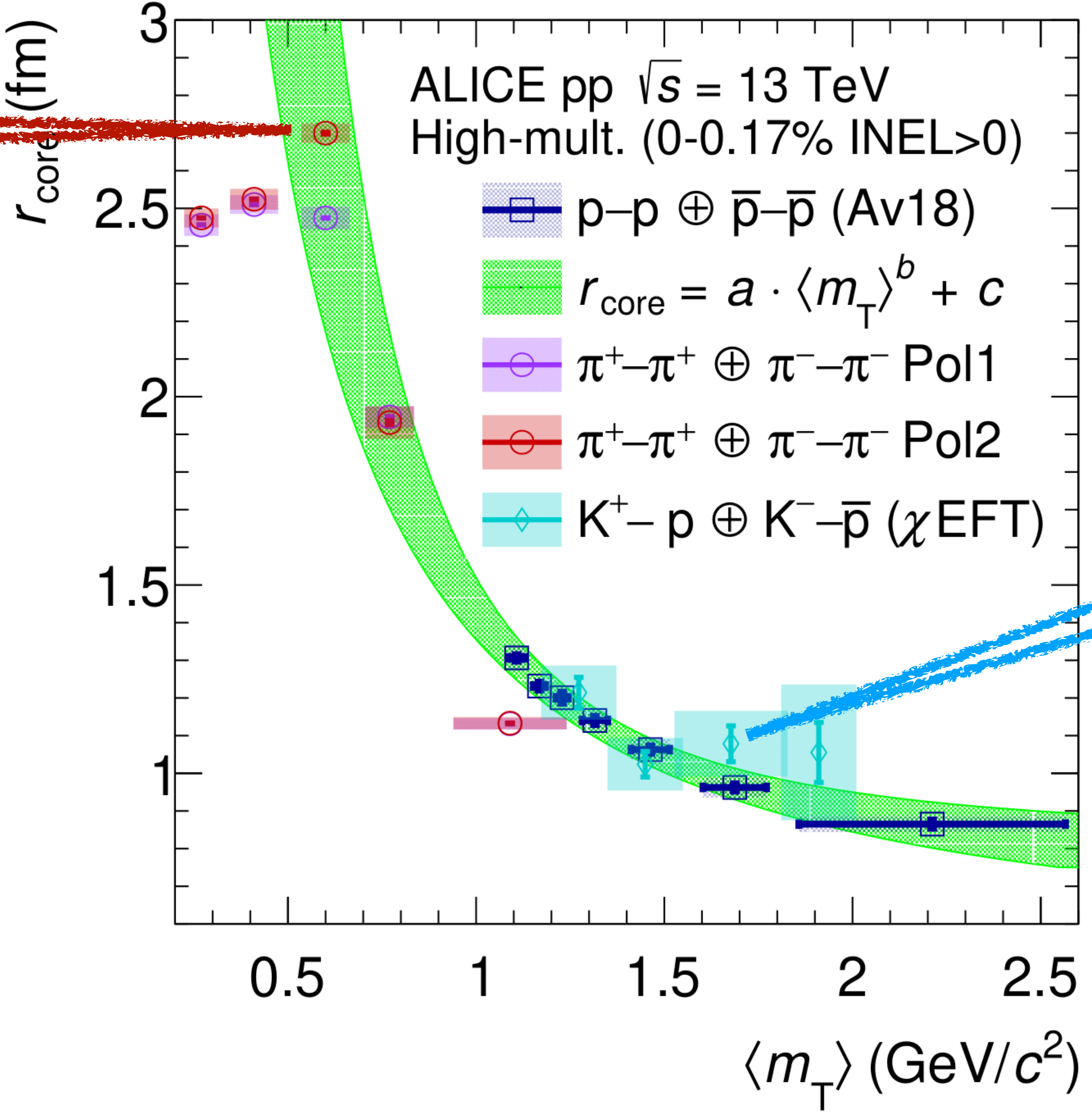
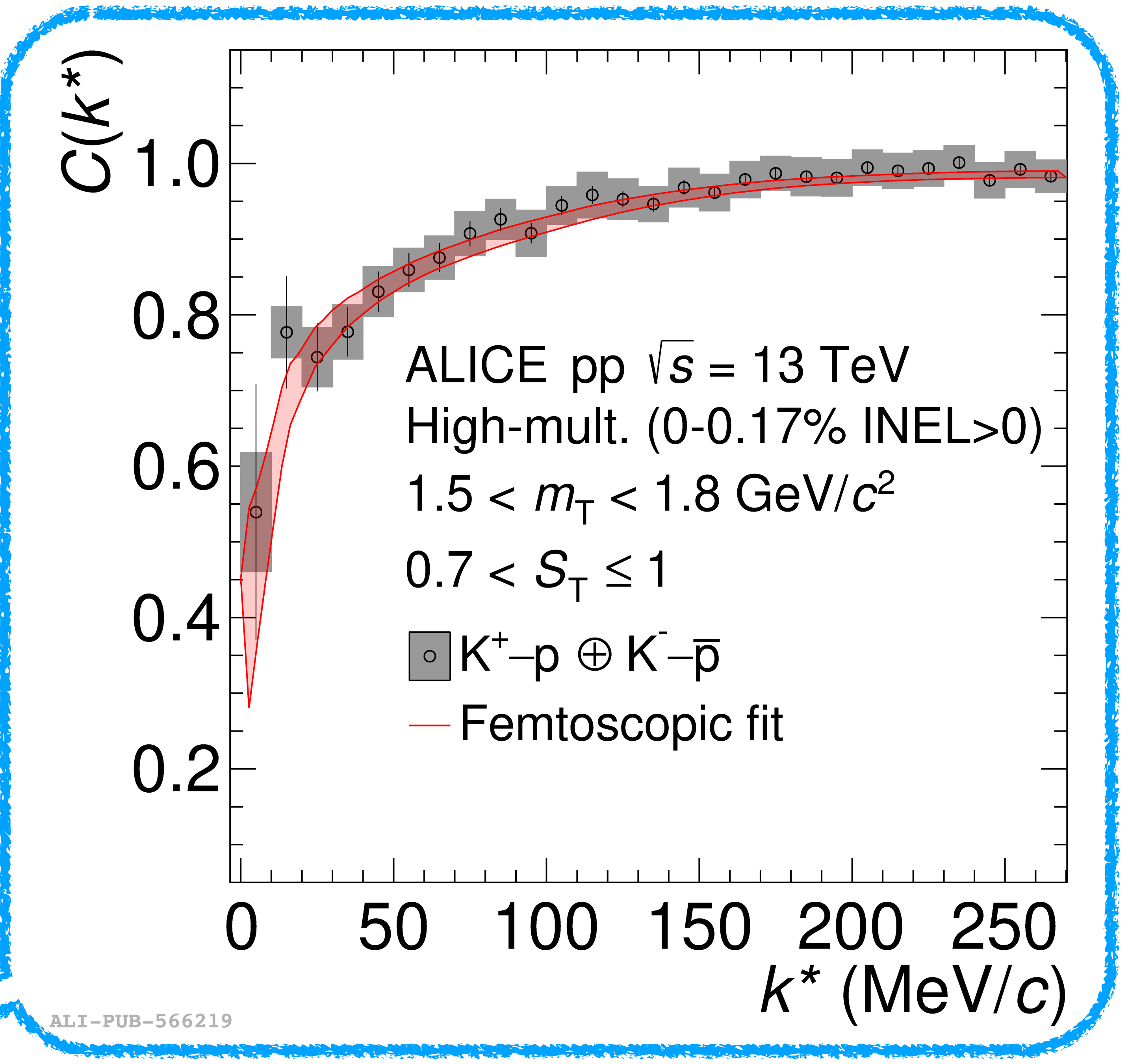


K+-p interaction:
Coulomb + KN Chiral potentials



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ALICE Coll, PLB 811 (2020)

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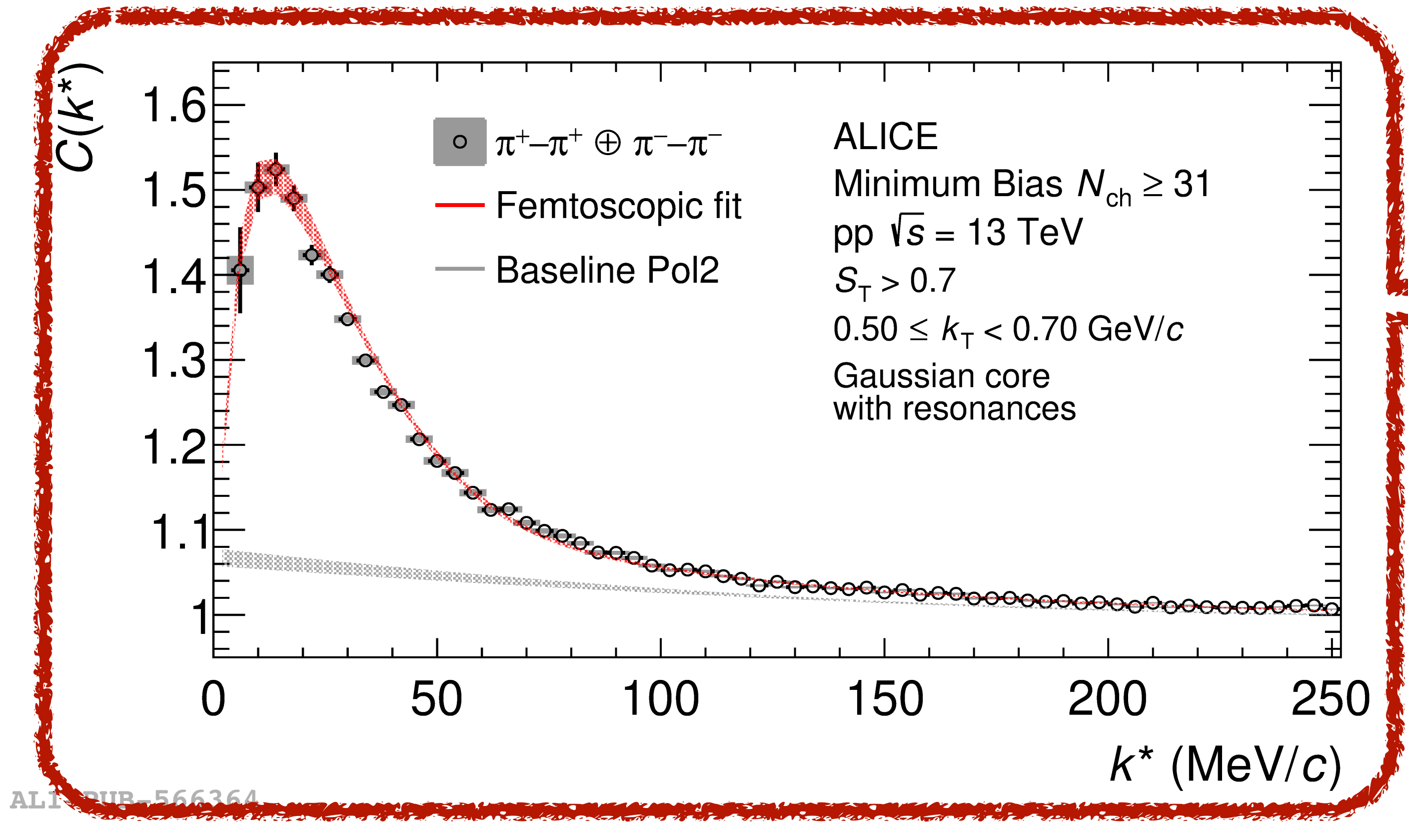
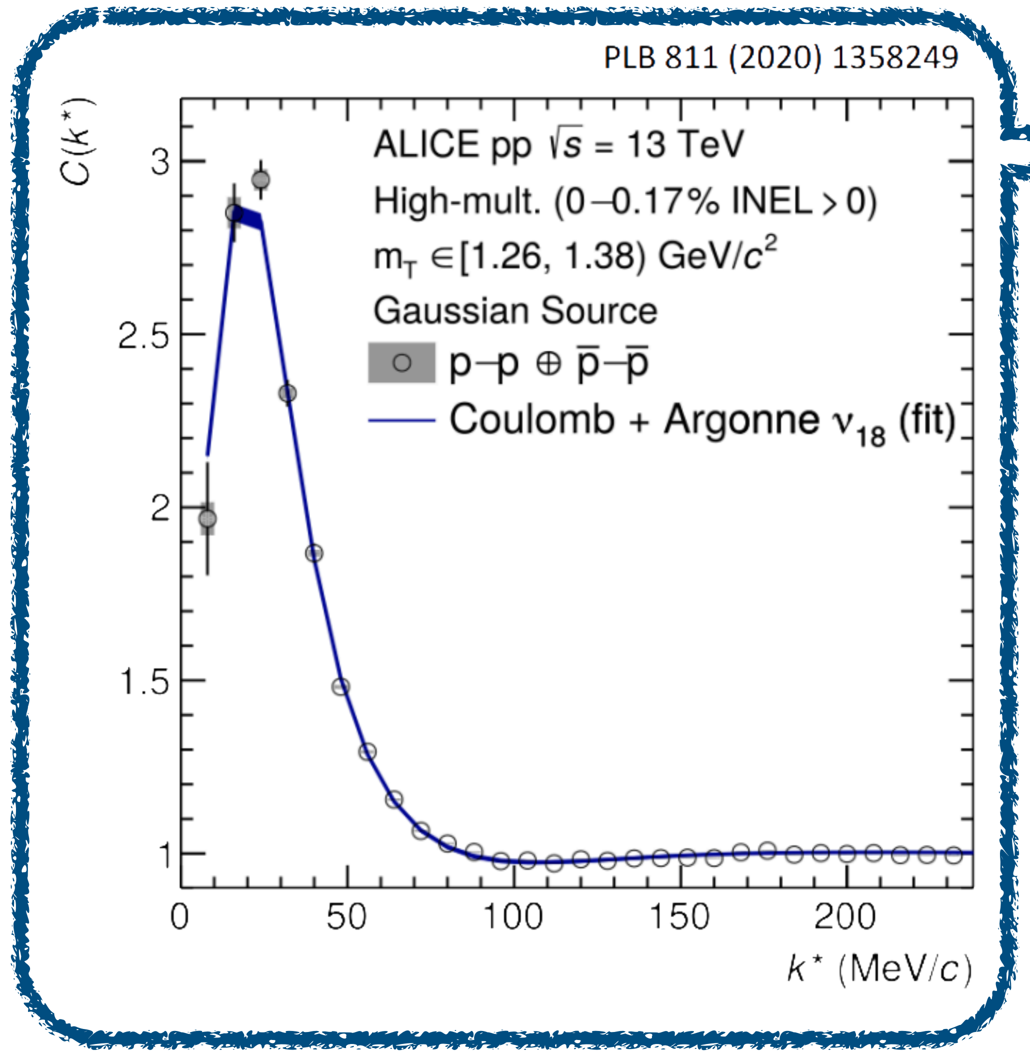
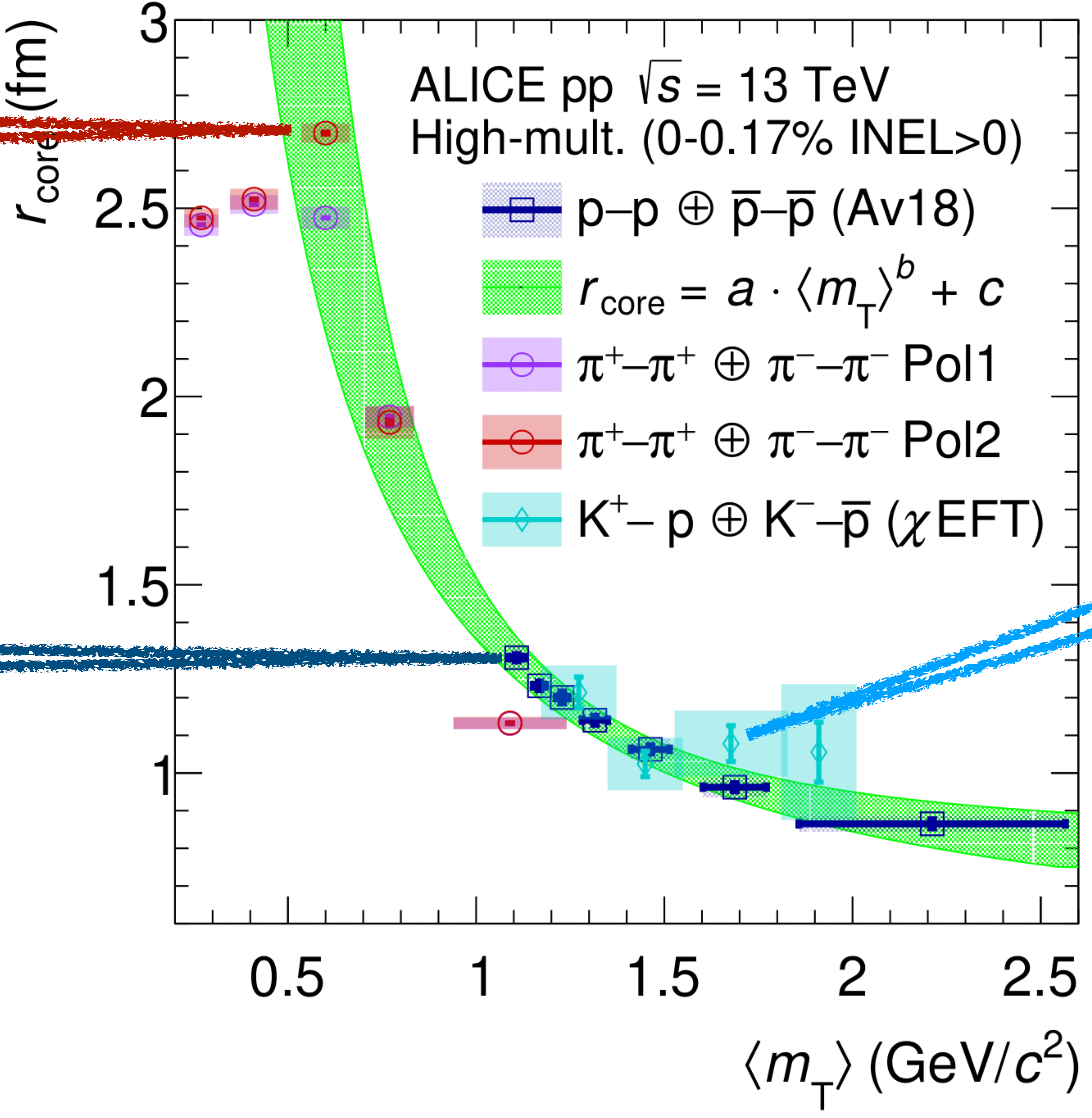


p-p: Coulomb + Argonne v18



ALICE Coll, arXiv:2311.14527



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K^+-p interaction:
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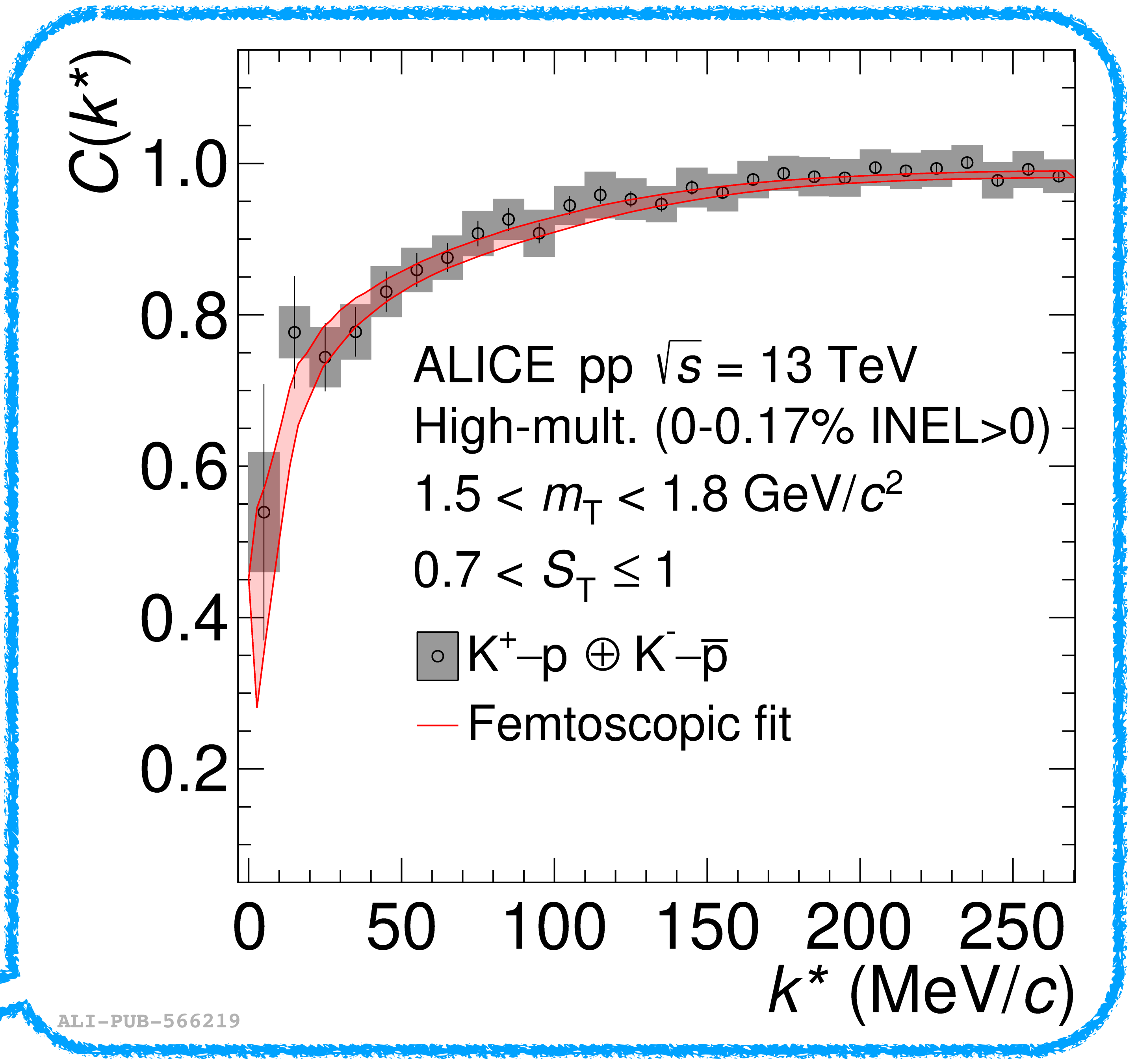
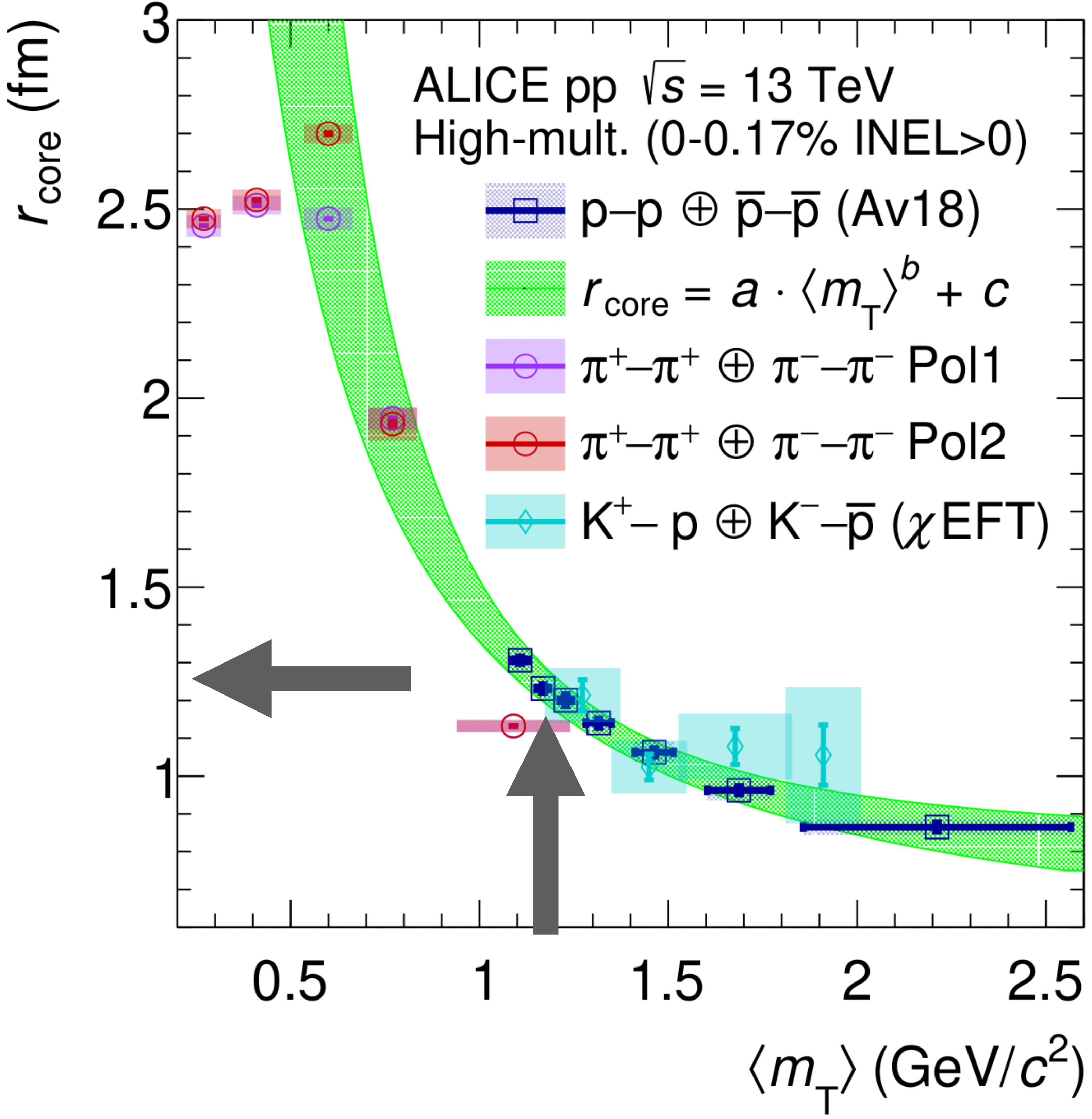


ALICE Coll, PLB 811 (2020)

Constraining the source in pp collisions

- Common emission profile for **all hadron pairs**

ALICE Coll, arXiv:2311.14527



Constrained values of r_{core} of the pair + effect of short-lived resonances

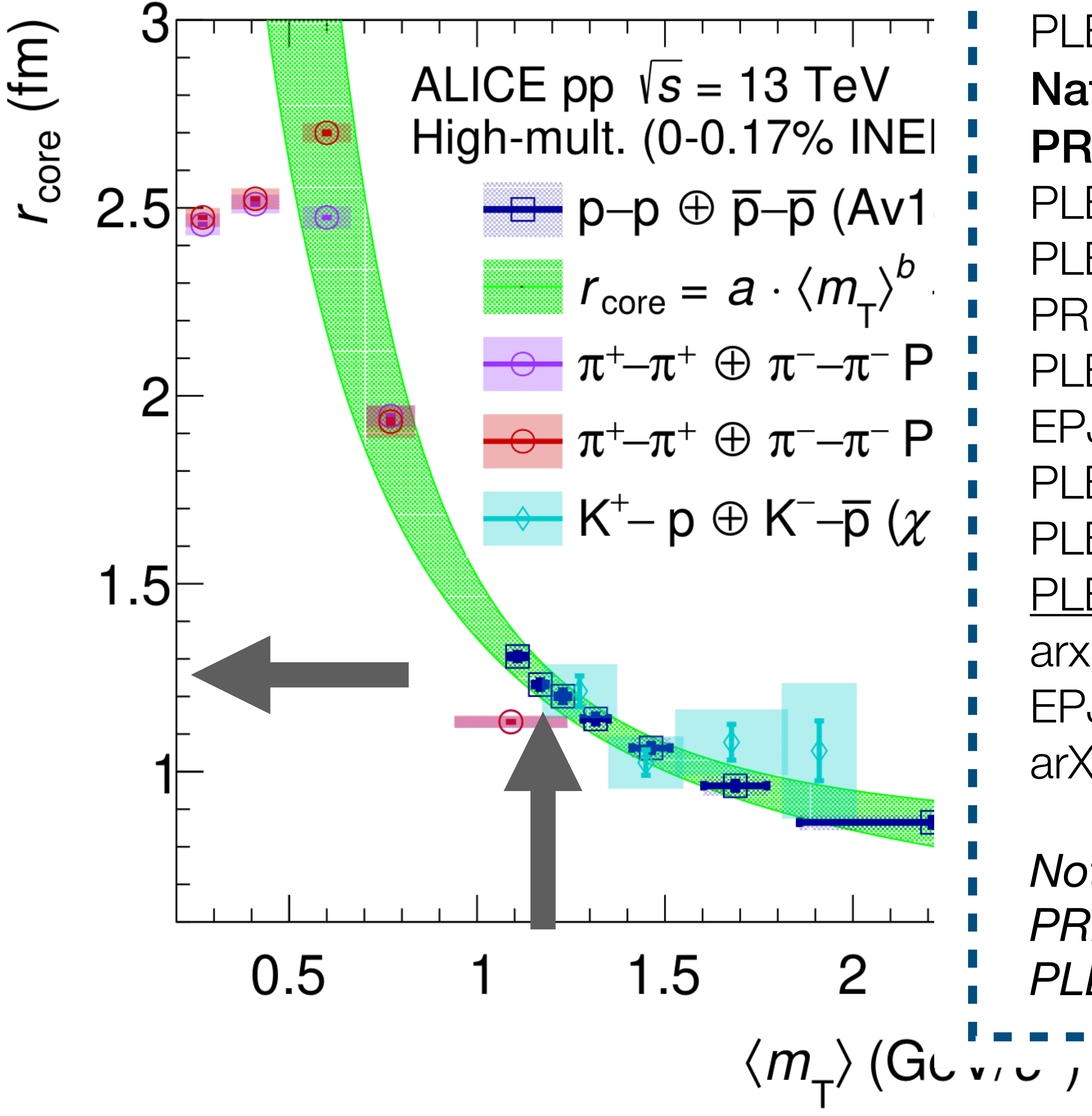
m_{T} of the pair from analysis

ALICE Coll, PLB 811 (2020)

Constraining the source in pp collisions

- Common emission profile for **all hadron pairs**

ALICE Coll, arXiv:2311.1



Constrained values of r_{core} of the pair + effect of short-lived resonances

m_T of the pair from analysis

ALICE Collaboration	System
PRC 99 (2019) 2, 024001	p–p, p–Λ
PLB 797 (2019), 134822	Λ–Λ
PRL 123 (2019), 112002	**p–Ξ**
PLB 805 (2020), 135419	p–Σ
PRL 124 (2020) 092301	**p–K**
PLB 811 (2020), 135849	p–p, p–Λ
Nature 588 (2020) 232-238	**p–Ω and p–Ξ**
PRL 127 (2021), 172301	**p–ϕ**
PLB 833 (2022), 137272	p–Λ
PLB 829 (2022), 137060	baryon–(anti)baryon
PRD 106 (2022) 5, 052010	p–D
PLB (2022), 137223	Λ–Ξ
EPJC 83 (2023) 4, 340	K–p
PLB 822 (2021) 136708	p–K
PLB 845 (2023), 138145	Λ–K
PLB 845 (2023), 138145	Λ–K
arxiv 2308.16120	p–d
EPJA 59 (2023) 7, 145	p–p–p, p–p–Λ
arXiv:2303.13448	p–p–K

Not using a universal source

PRL 124 (2020), 09230	*(anti)K–p*
PLB 822 (2021), 136708	*(anti)K–p*

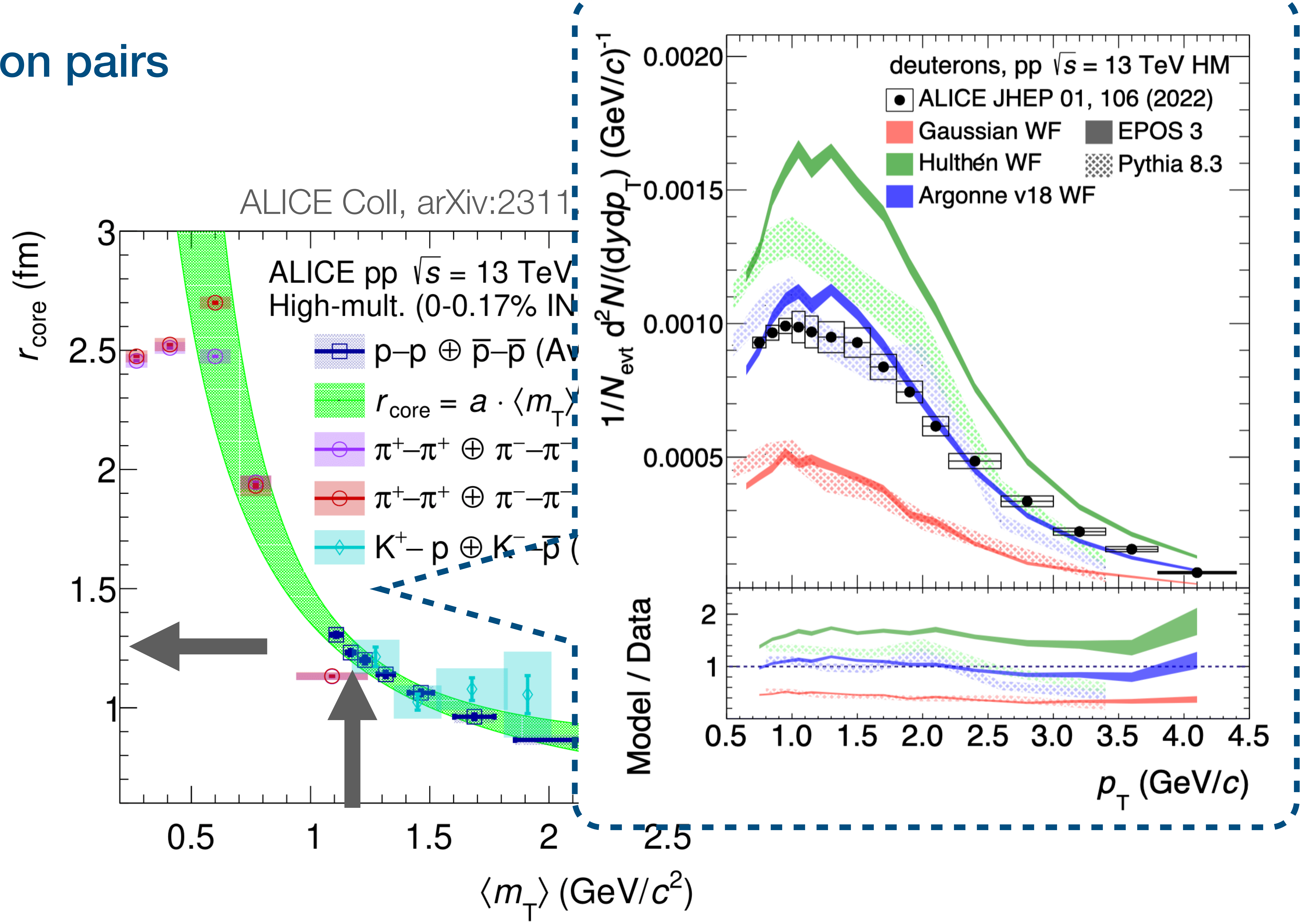
ALICE Coll, PLB 811 (2020)

Constraining the source in pp collisions

- Common emission profile for **all hadron pairs**

Talk by M. Mahlien
Wed @14:45

ALICE Coll, arXiv:2311



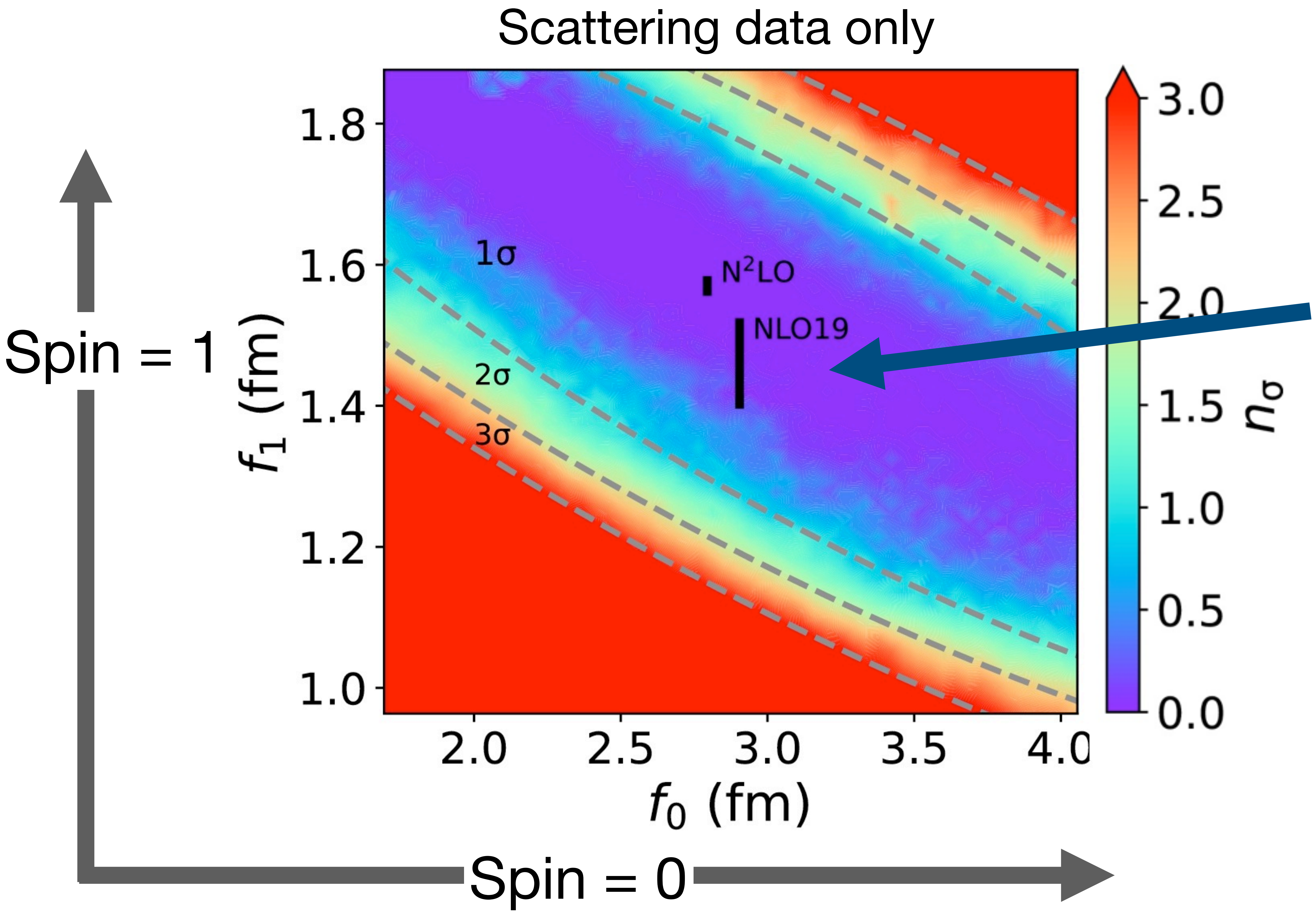
Constrained values of r_{core} of the pair + effect of short-lived resonances

m_{T} of the pair from analysis

ALICE Coll, PLB 811 (2020)

Λ-p interaction

- Important in the context of hypernuclei D. Mihaylov, J. Haidenbauer and V. Mantovani Sarti, PLB 850 (2024), 138550



Values from chiral effective field theory (χEFT) are presently constrained to the scattering data.

Λ-p interaction

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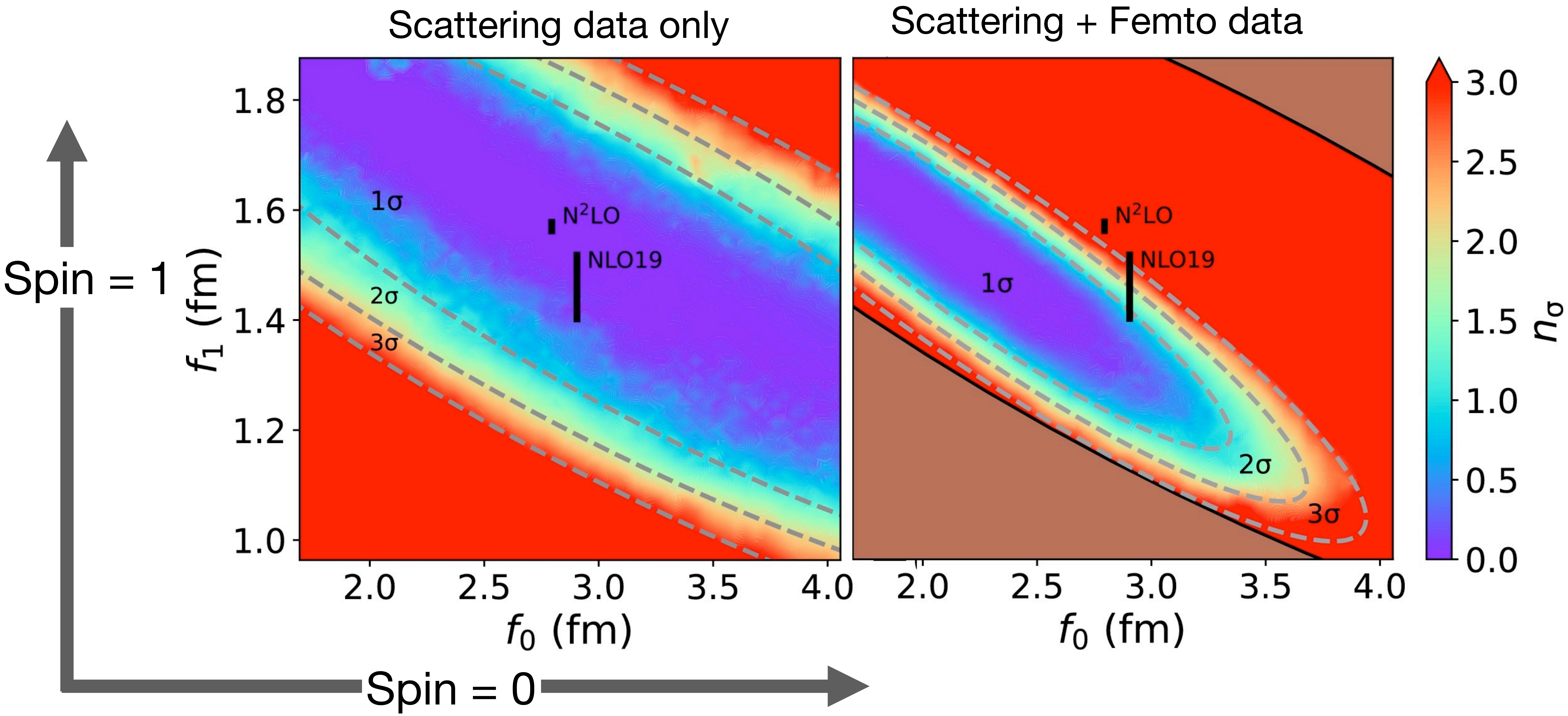

Λ-p interaction

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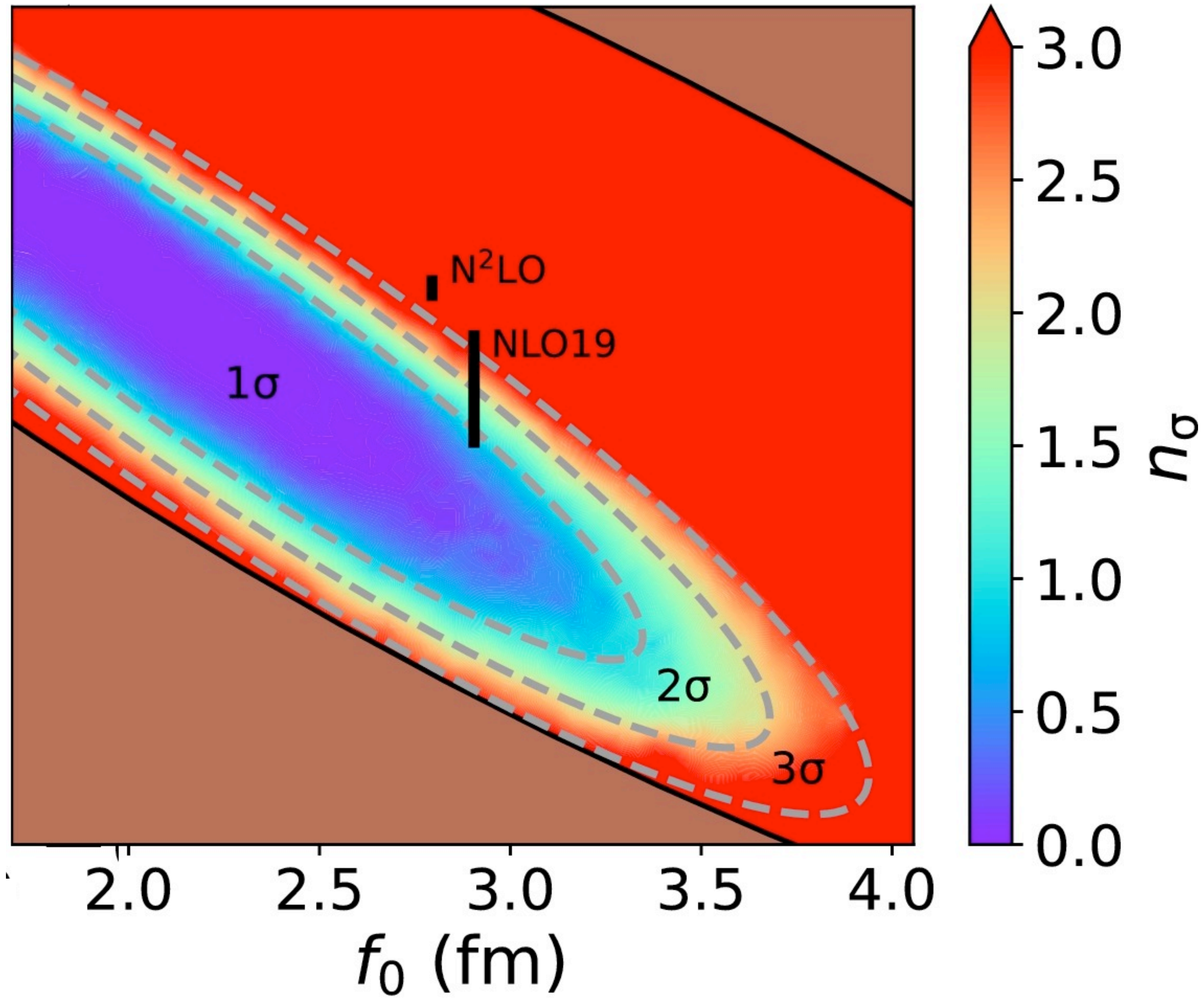
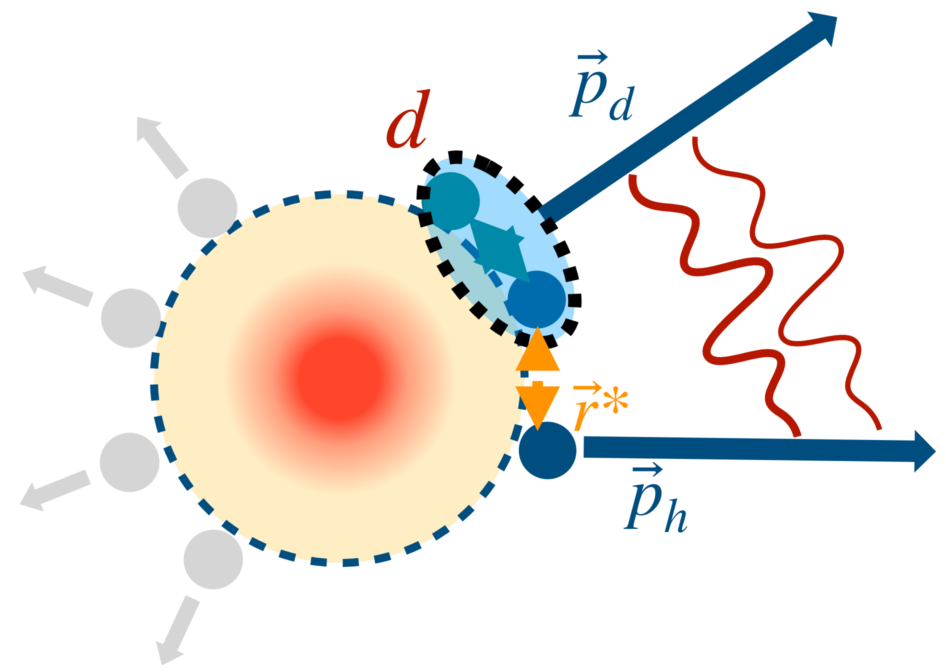
Scattering data only

Scattering + Femto data

- Spin-averaged scattering length of 1.77 ± 0.18 fm. Which is lower than current estimates
- Feed these results into χEFT (NLO19*) and constrain its low energy constants.
  *) Haidenbauer et al. Eur.Phys.J.A 56 (2020) 3, 91
- We can expect new prediction of WF for hypertriton



K$^+$–d correlation in pp collisions

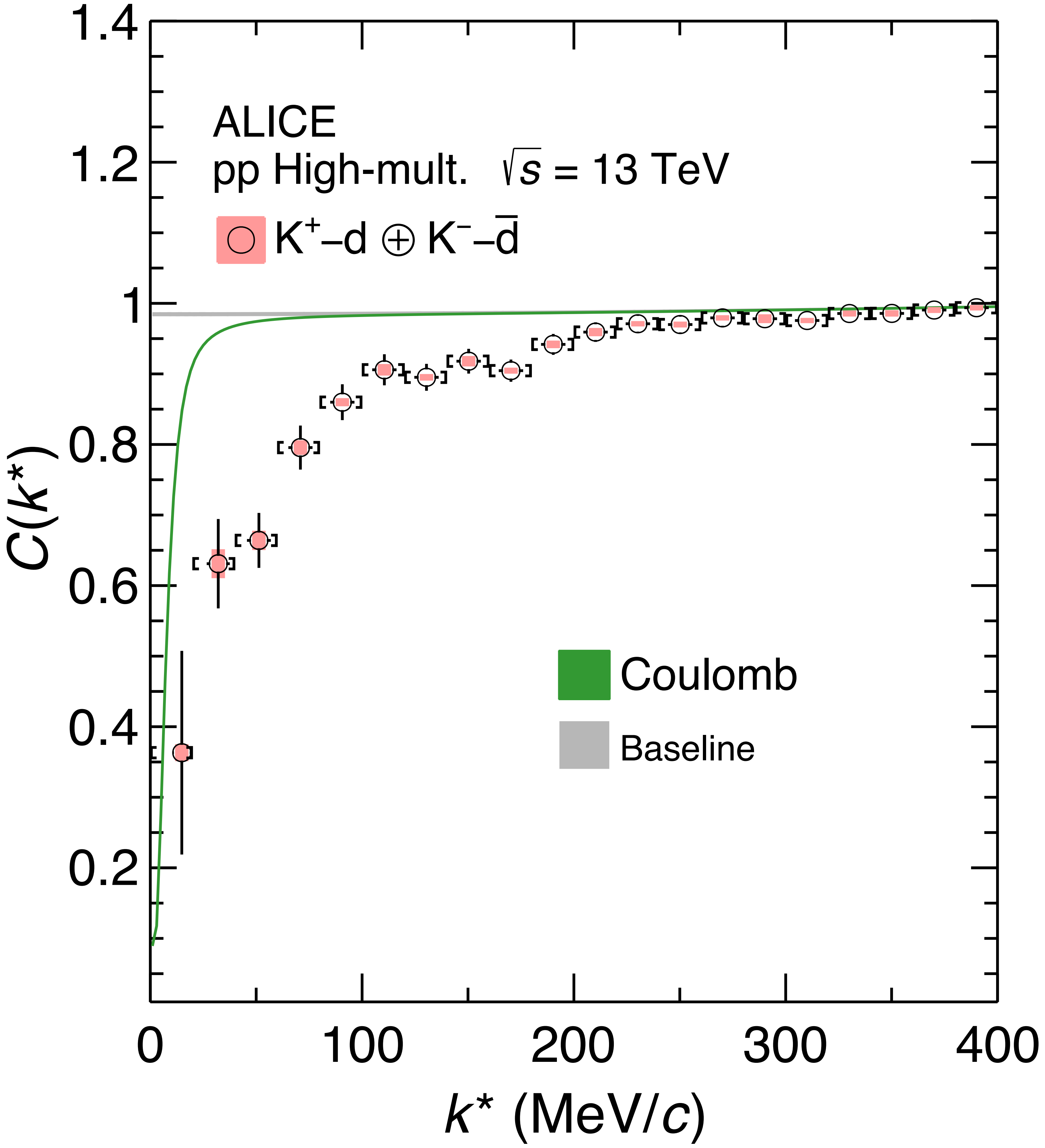


Deuterons follow hadron-hadron m$_T$-scaling?

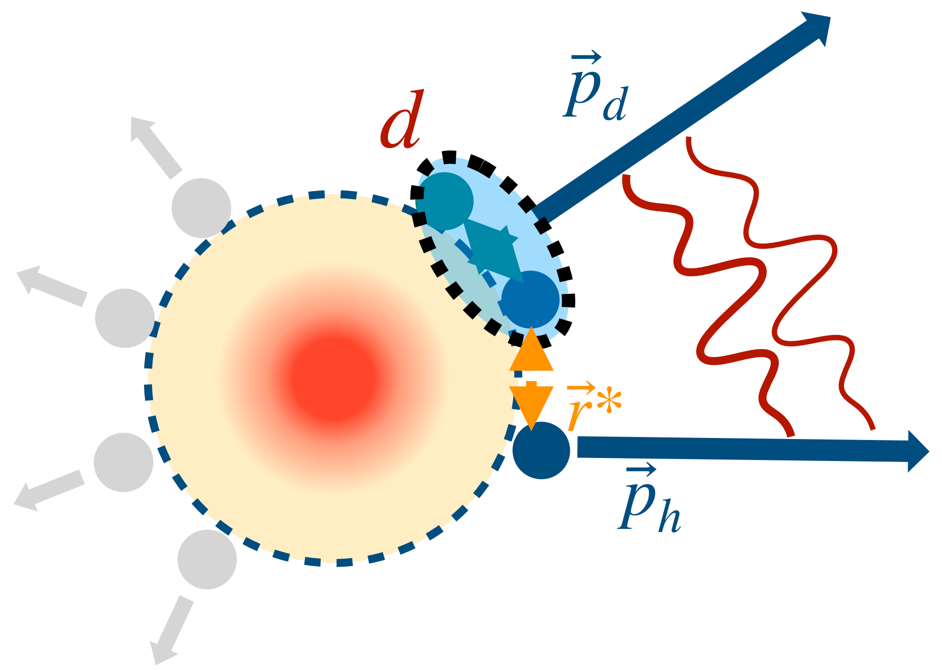
K$^+$–d source size = $1.35^{+0.04}_{-0.05}$ fm

- Coulomb potential: disagree

ALICE Coll. arXiv:2308.16120



K^+–d correlation in pp collisions



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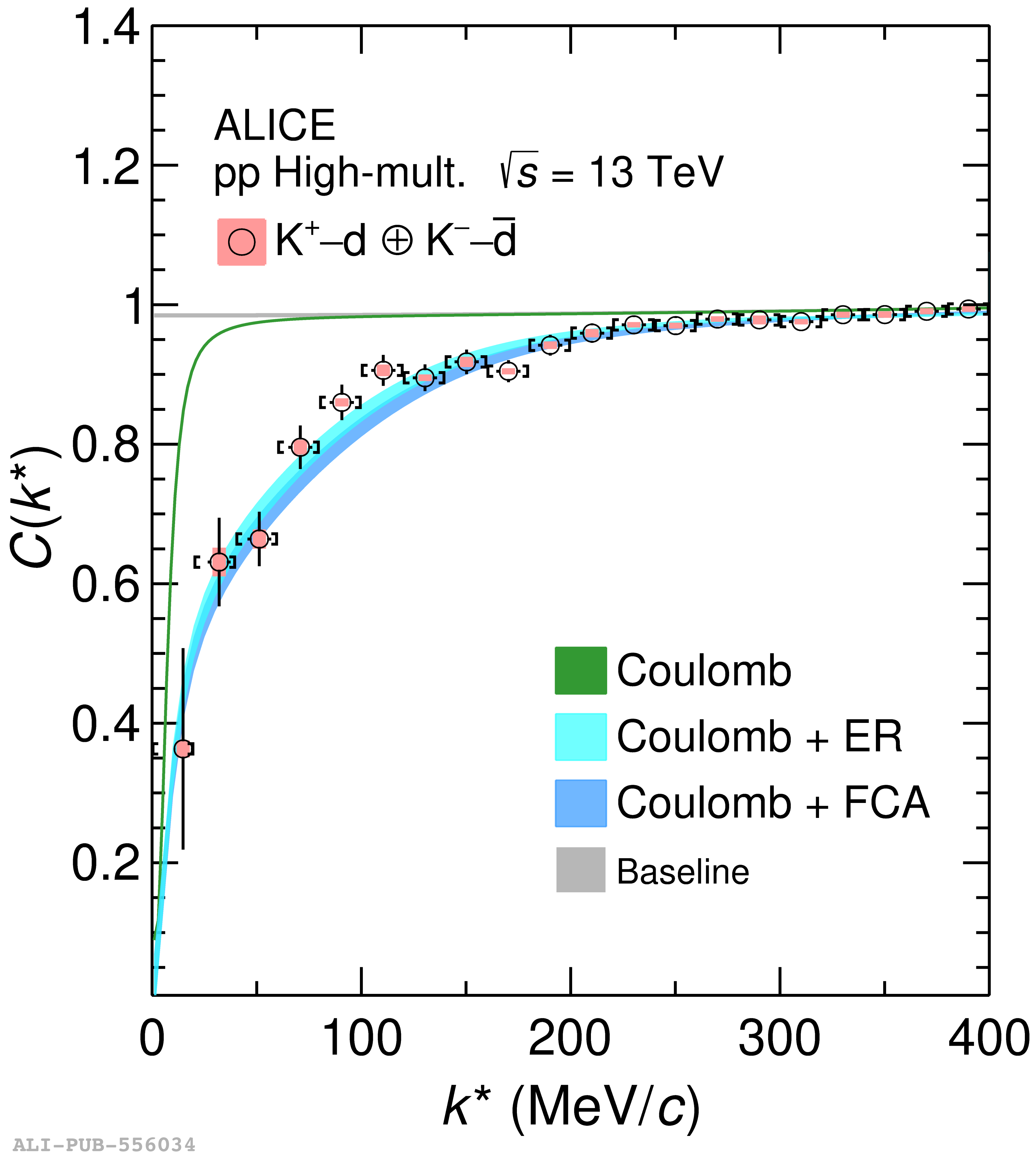


- Coulomb potential: disagree
- K^+–d as an **effective two-body** system: Lednický-Lyuboshits approach[1]
- K^+–d scattering parameters
  - Effective-Range Approximation (ER):
    $a_0 = -0.47$ fm, $d_0 = -1.75$ fm[2]
  - Fixed-center approximation (FCA):
    $a_0 = -0.54$ fm, $d_0 = 0$ fm[3]

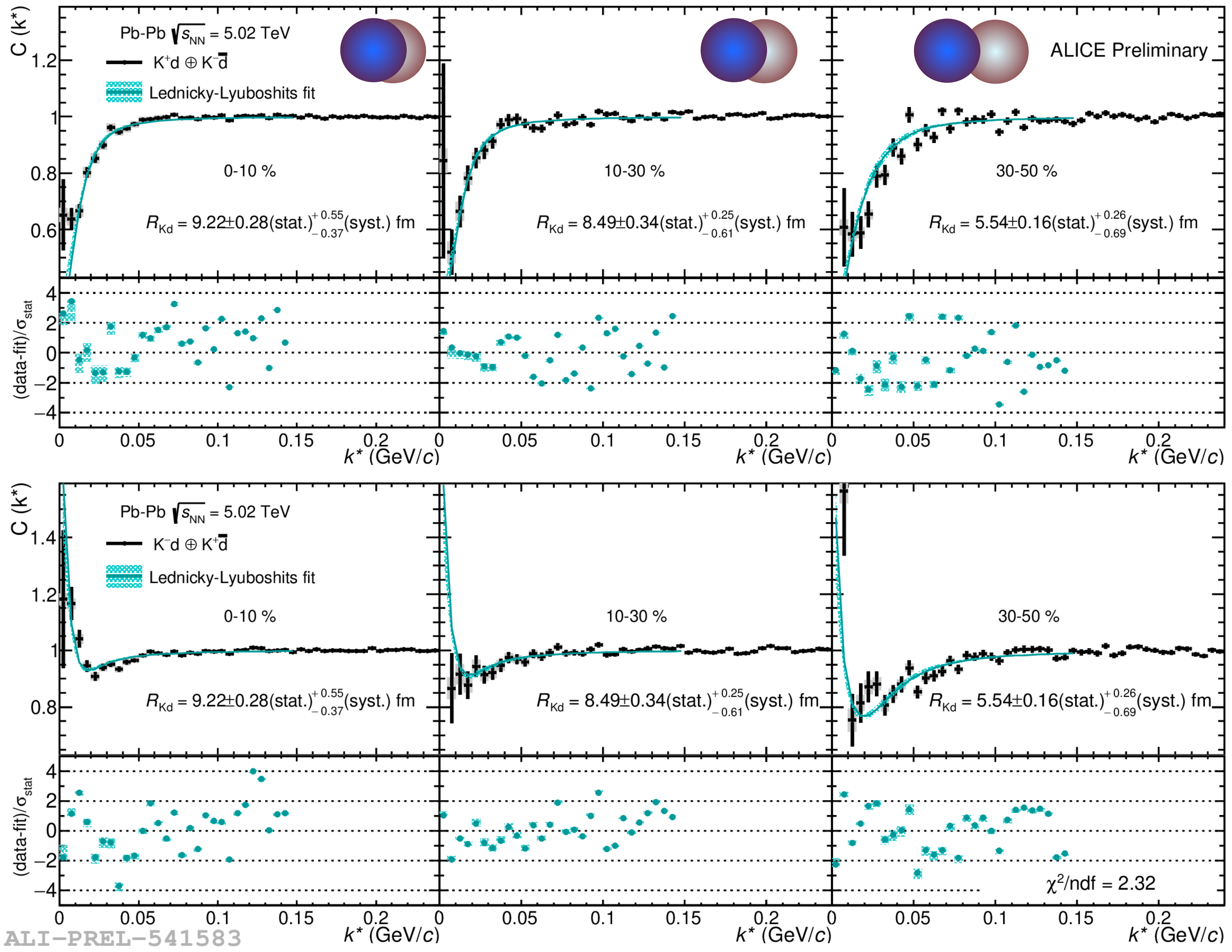
Deuterons follow the same m_T scaling as other hadrons

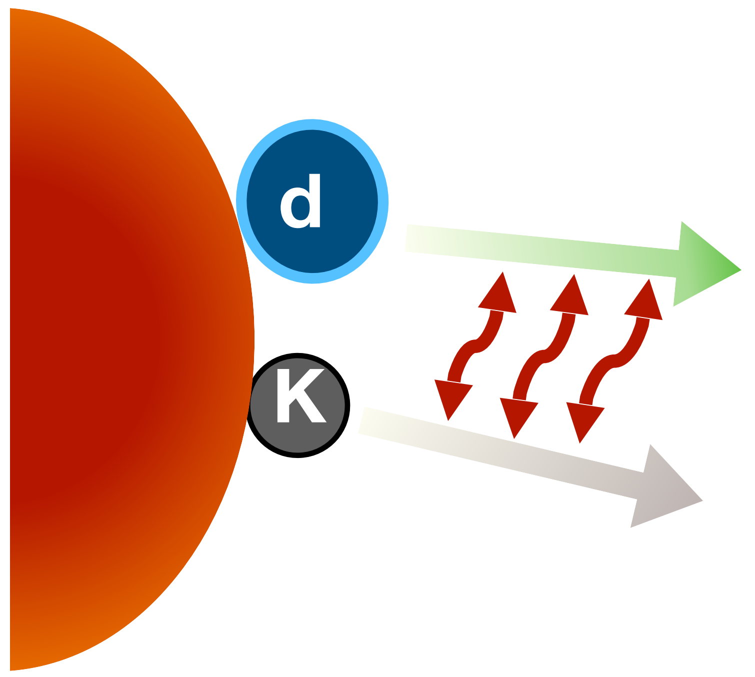
[1] R. Lednický, Phys. Part. Nuclei 40, 307–352 (2009)
[2] provided by Prof. Johann Haidenbaur
[3] provided by Prof. Tetsuo Hyodo

K± -d correlations in Pb–Pb

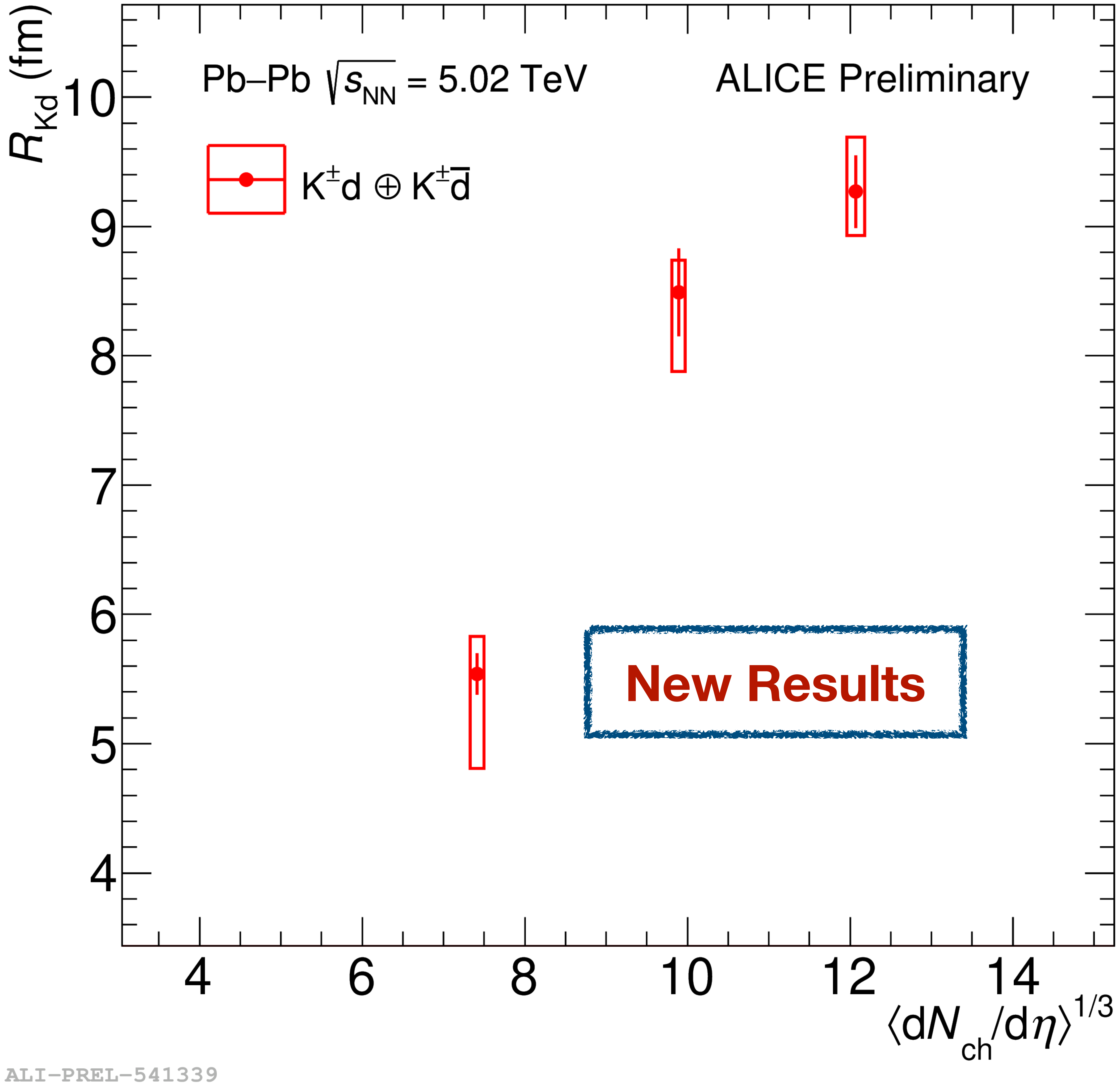
- **K±–d correlation functions** in Pb–Pb collisions at $\sqrt{s_{NN}}$ = 5.02 TeV
    - 3 centrality intervals: 0–10%, 10–30%, and 30–50%

- **Lednicky´-Lyuboshitz approach**
    - Coulomb effects + strong interaction (via scattering parameters)





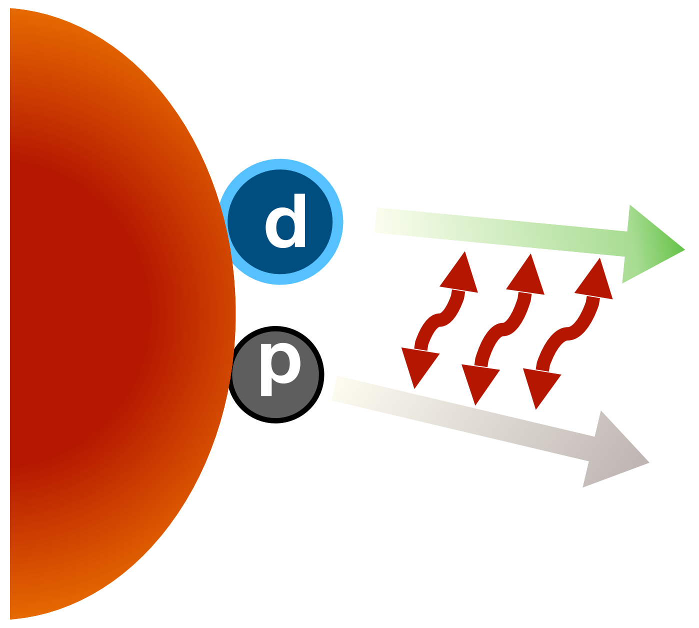
$K^{\pm}$-d source size in Pb–Pb



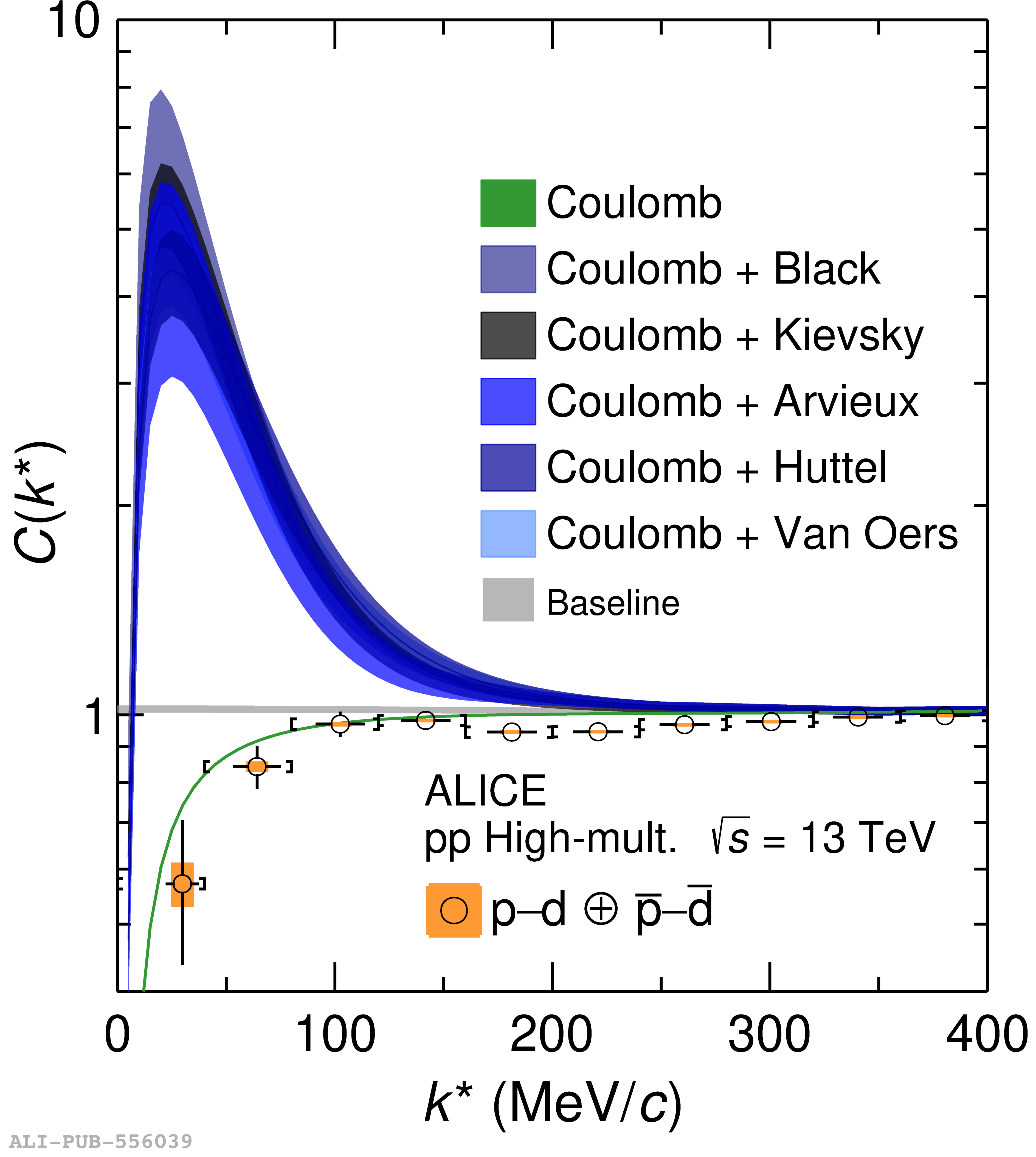
- 3 radii as a function of centrality
  - for 3 centralities (the same radius for all particle pairs)
- Source size increases with multiplicity!

p–d correlation in pp collisions

- p–d as an **effective two-body**: Lednický-Lyuboshits approach[1]
- Source size: $1.08^{+0.06}_{-0.06}$ fm
- Strong interaction: constrained from the scattering measurements[2]



ALICE Coll. arXiv:2308.16120



[1] R. Lednický, Phys. Part. Nuclei 40, 307–352 (2009)

[2] Measured scattering parameters of p-d ref in the backup

p–d correlation in pp collisions

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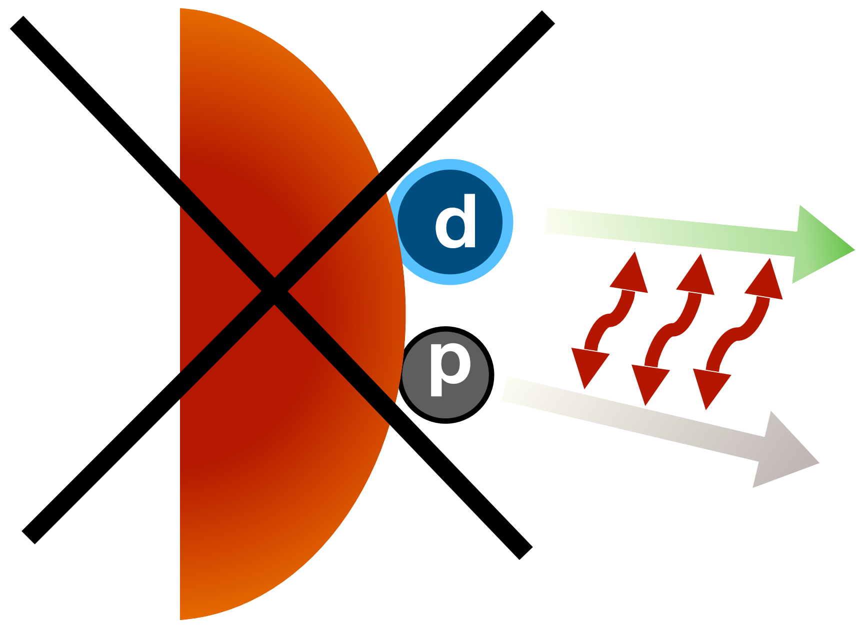
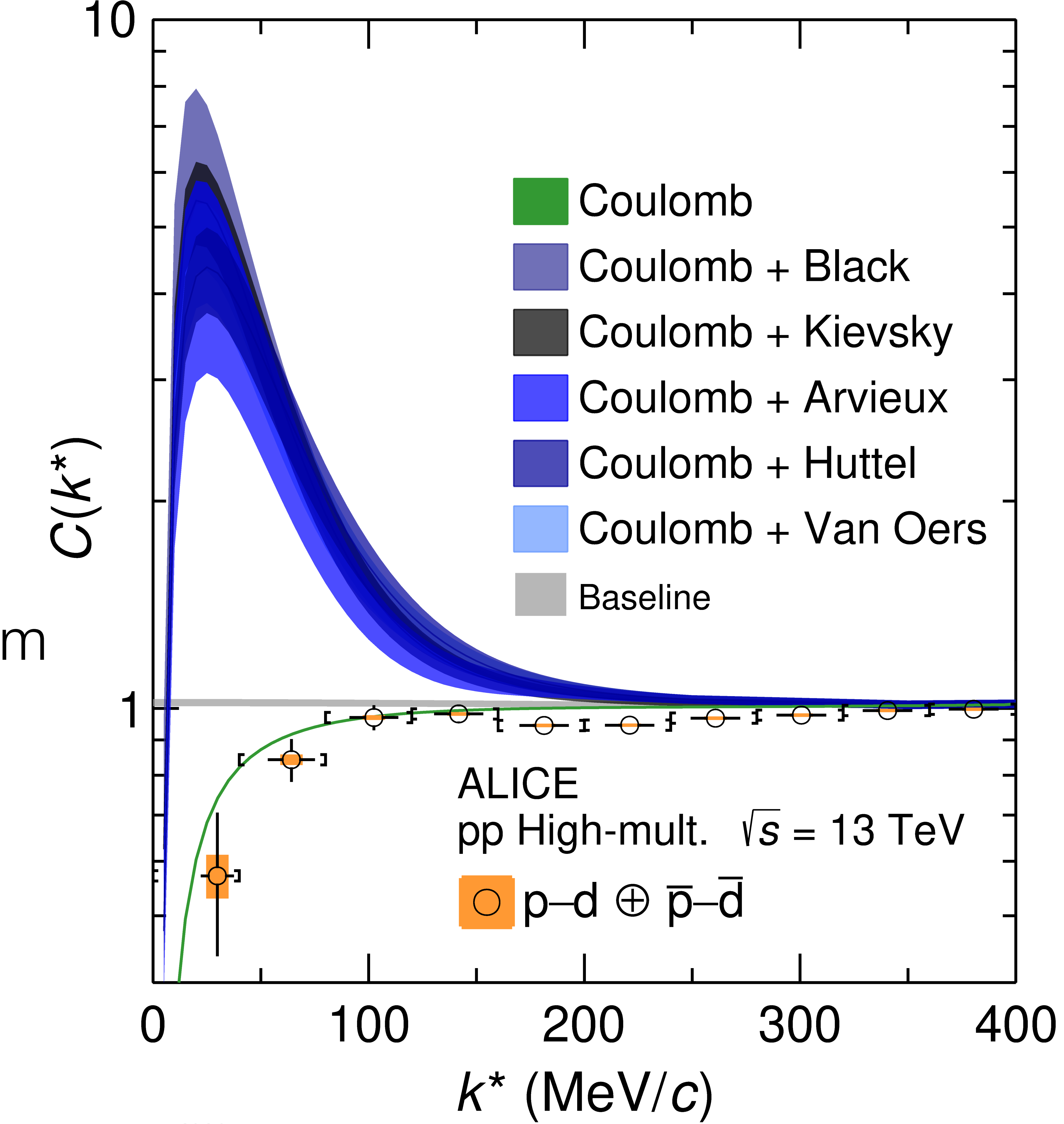


ALICE Coll. arXiv:2308.16120



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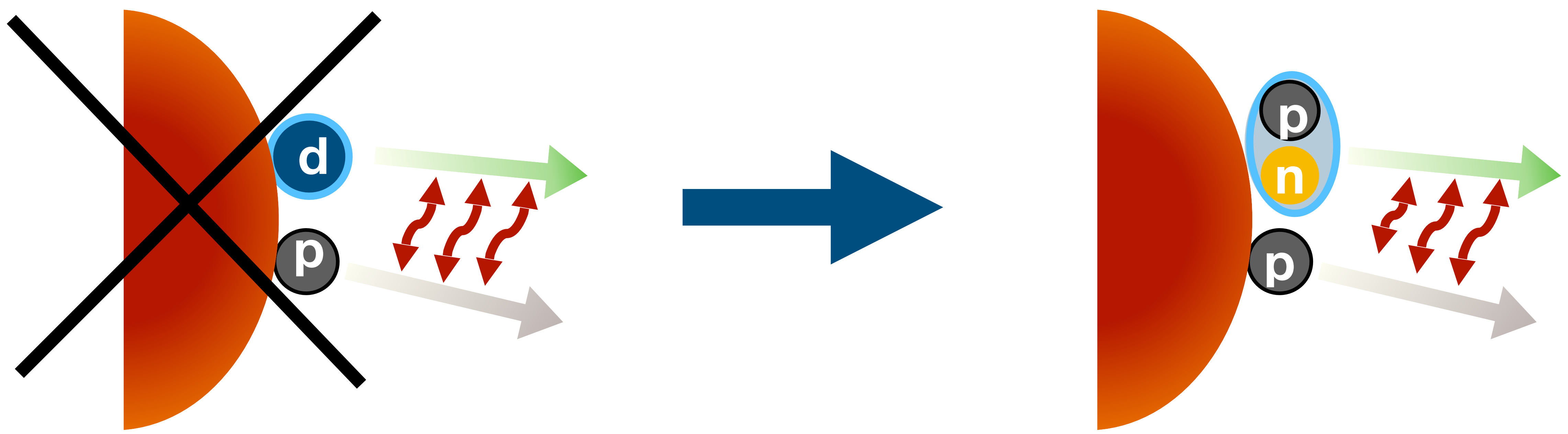

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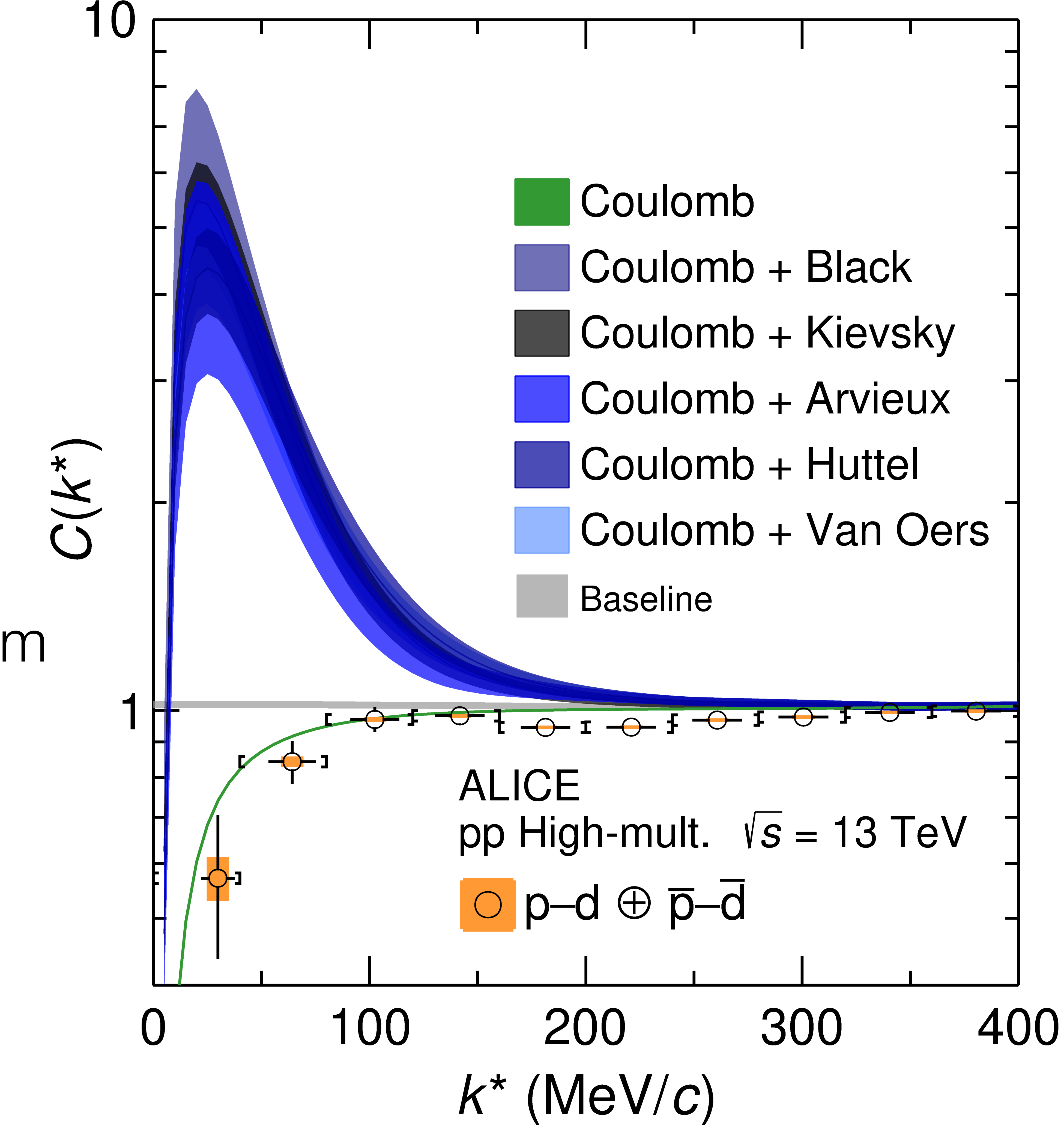

Need for three-body calculations accounting for p-(pn) dynamics

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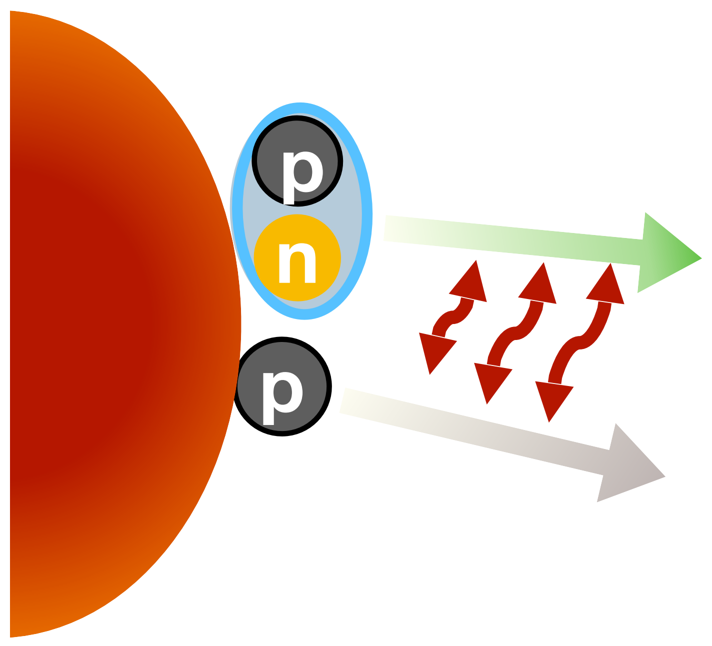
p–d as three-body system

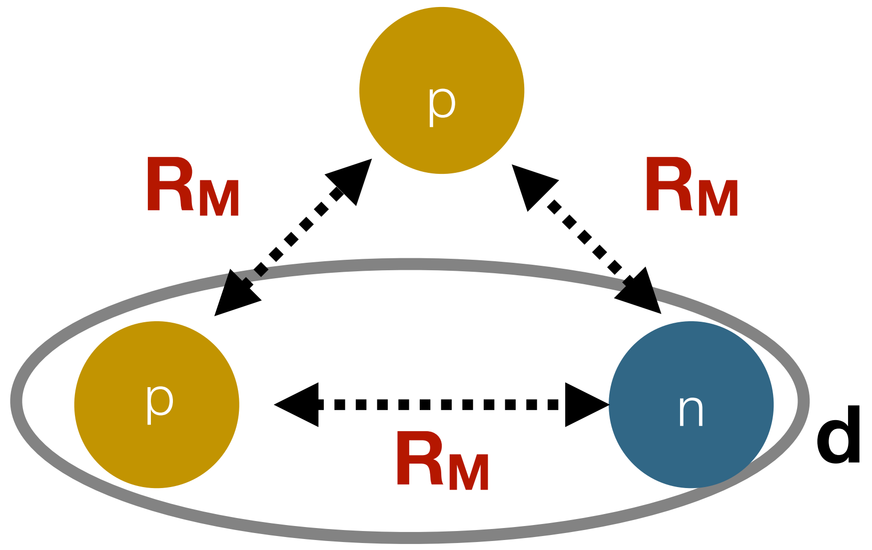
- **Start from p–(pn) system that form p–d state**:

Single-particle Gaussian emission source

$$C_{pd}(k^*) = \frac{1}{6\boxed{A_d}} \sum_{m_1,m_2} \int \boxed{S(r_1)S(r_2)S(r_3)} \left|\Psi(k^*, r_1, r_2, r_3)\right|^2 d^3r_1\, d^3r_2\, d^3r_3$$

$$= \frac{1}{6\,A_d} \sum_{m_1,m_2} \int \frac{e^{-\rho^2/4R_M^2}}{\left(4\pi R_M^2\right)^3} \left|\Psi(k^*, \rho)\right|^2 \rho^5\, d\rho\, d\Omega$$





- $\Psi(k^*, \rho)$ the three-nucleon wave function, p–(pn) to p–d state asymptotically
- $R_\mathrm{M} = 1.43 \pm 0.16\,\mathrm{fm}$ nucleon-nucleon source size in p–d (obtained from analysis)

M. Viviani, B. Singh et al. Phys. Rev. C 108, 064002 (2023)

Calculations: theory collaborators

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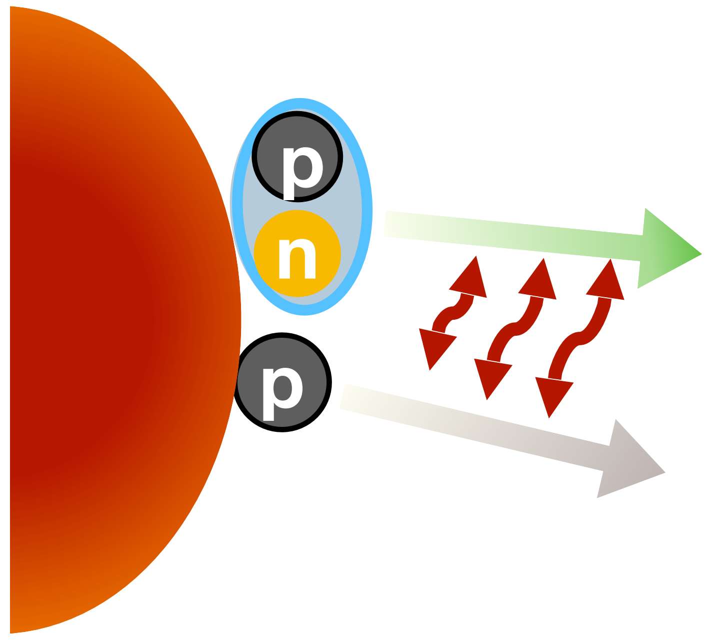

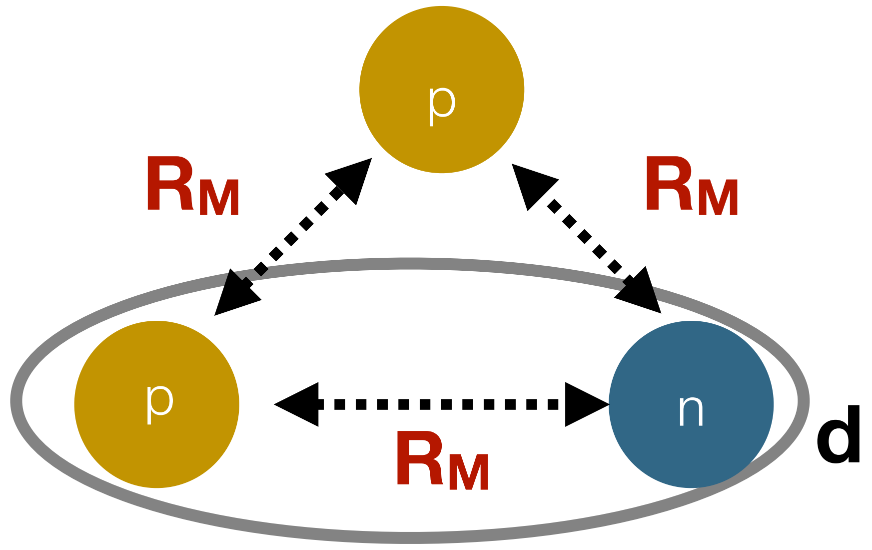

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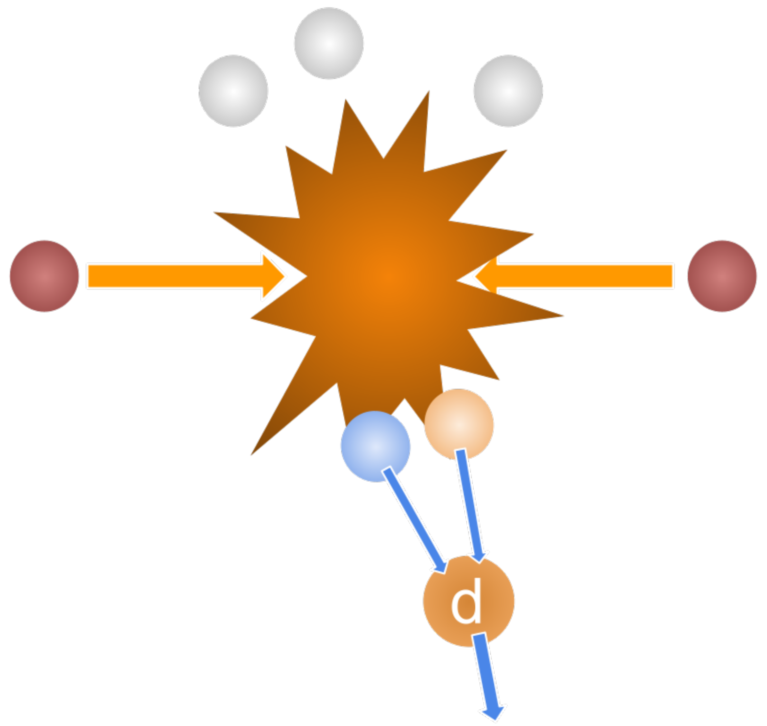

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- Argonne v18(2N) + Urbana IX (**genuine three-body force**) potentials[1,2]
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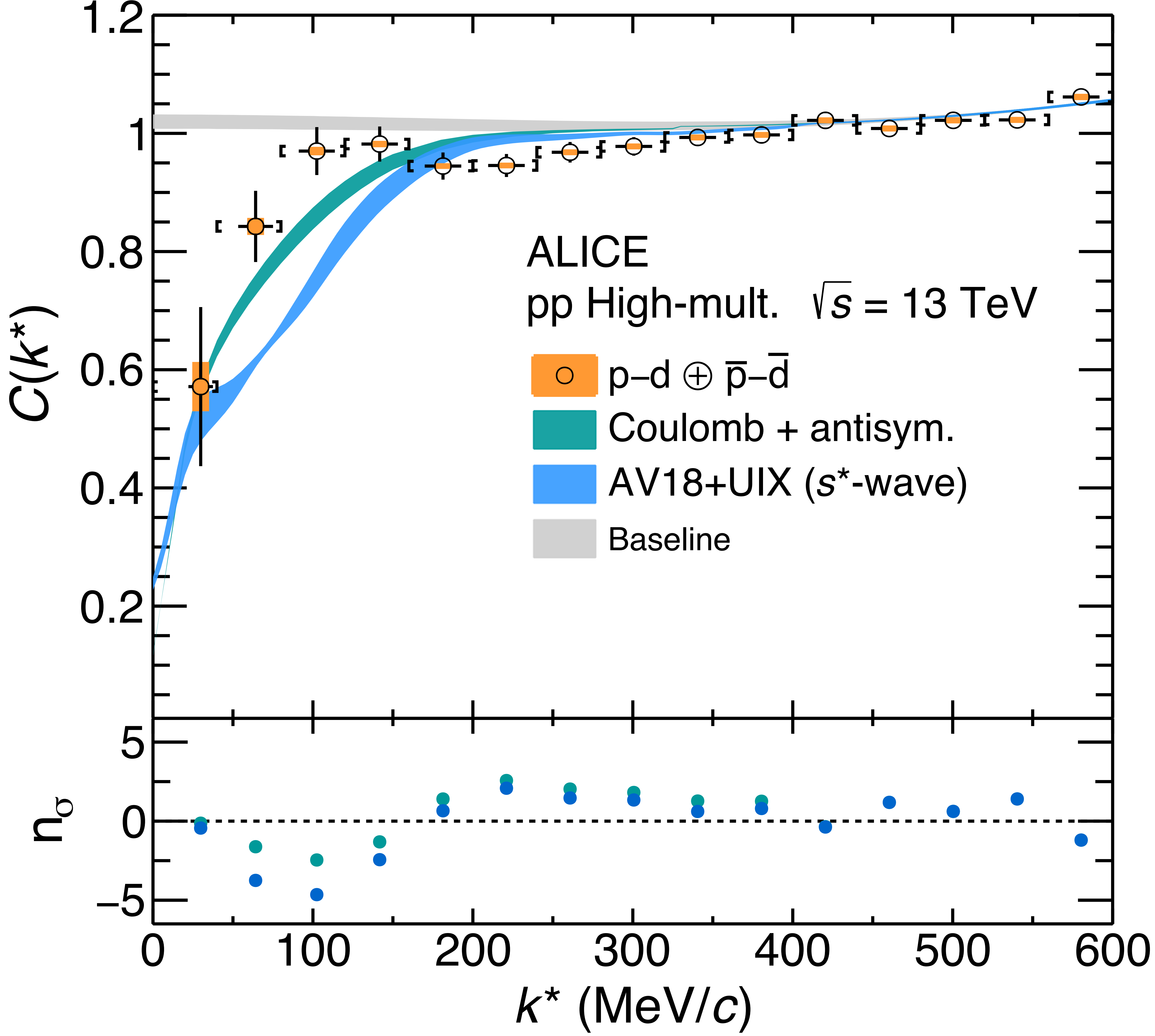

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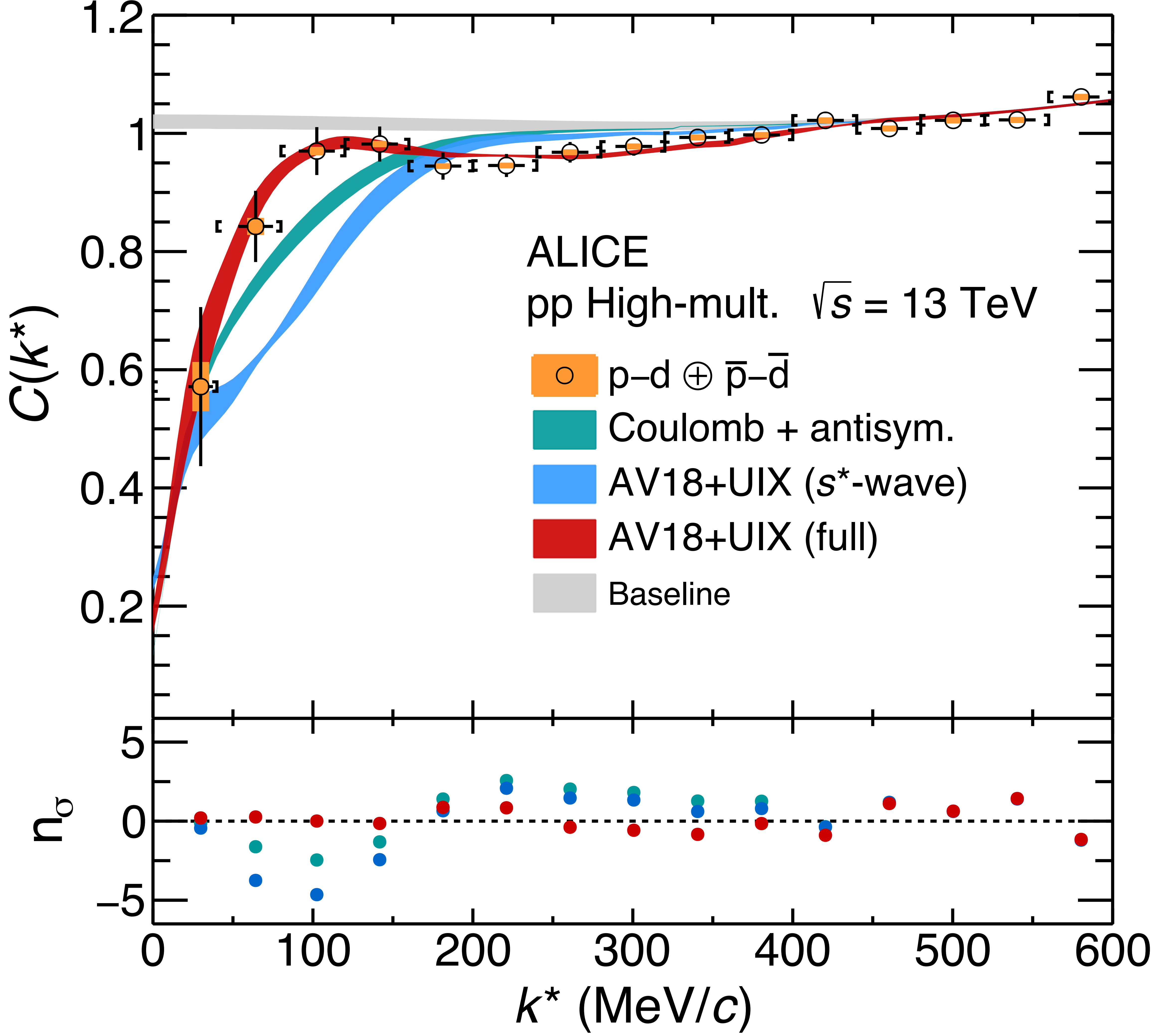

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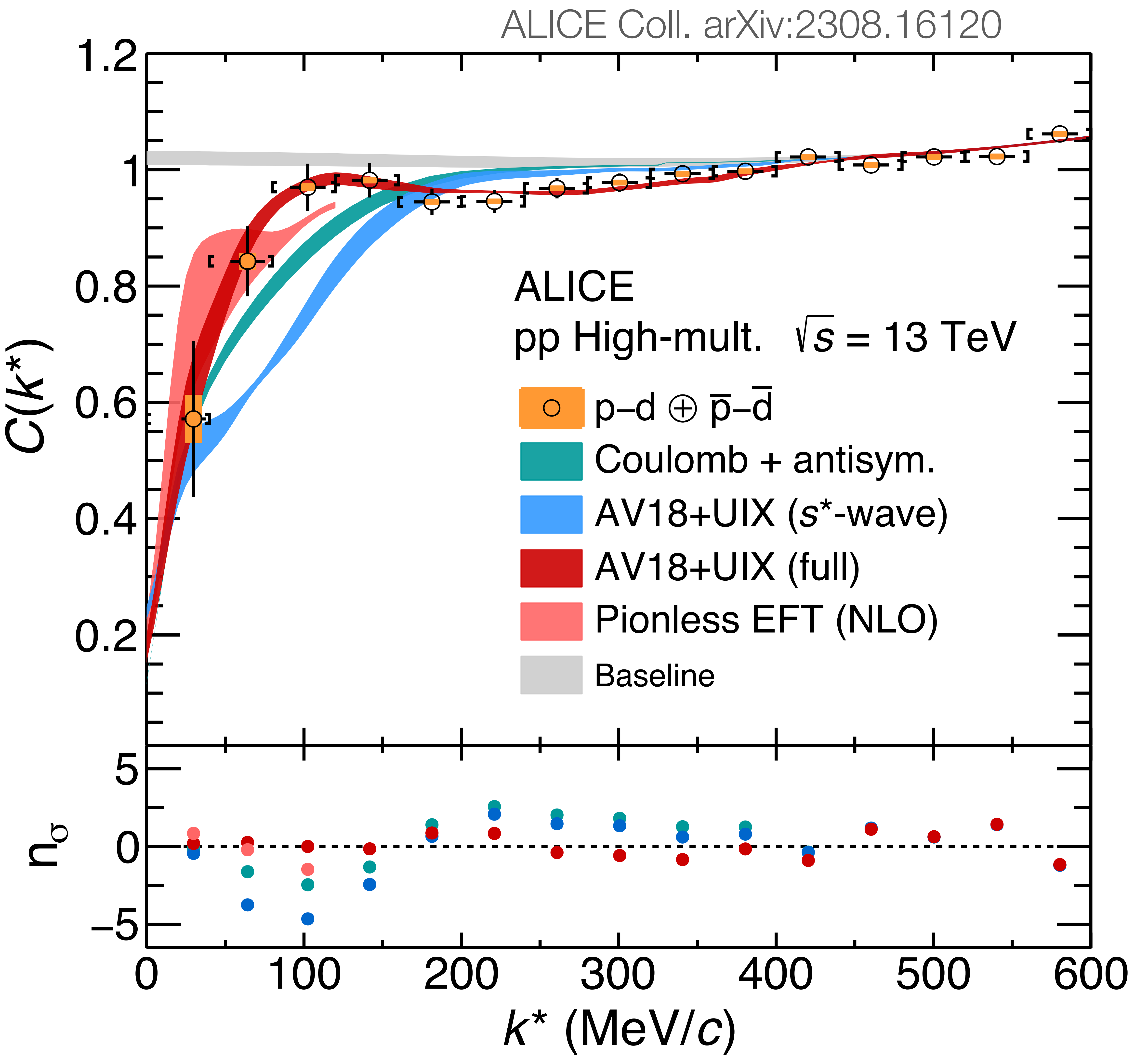
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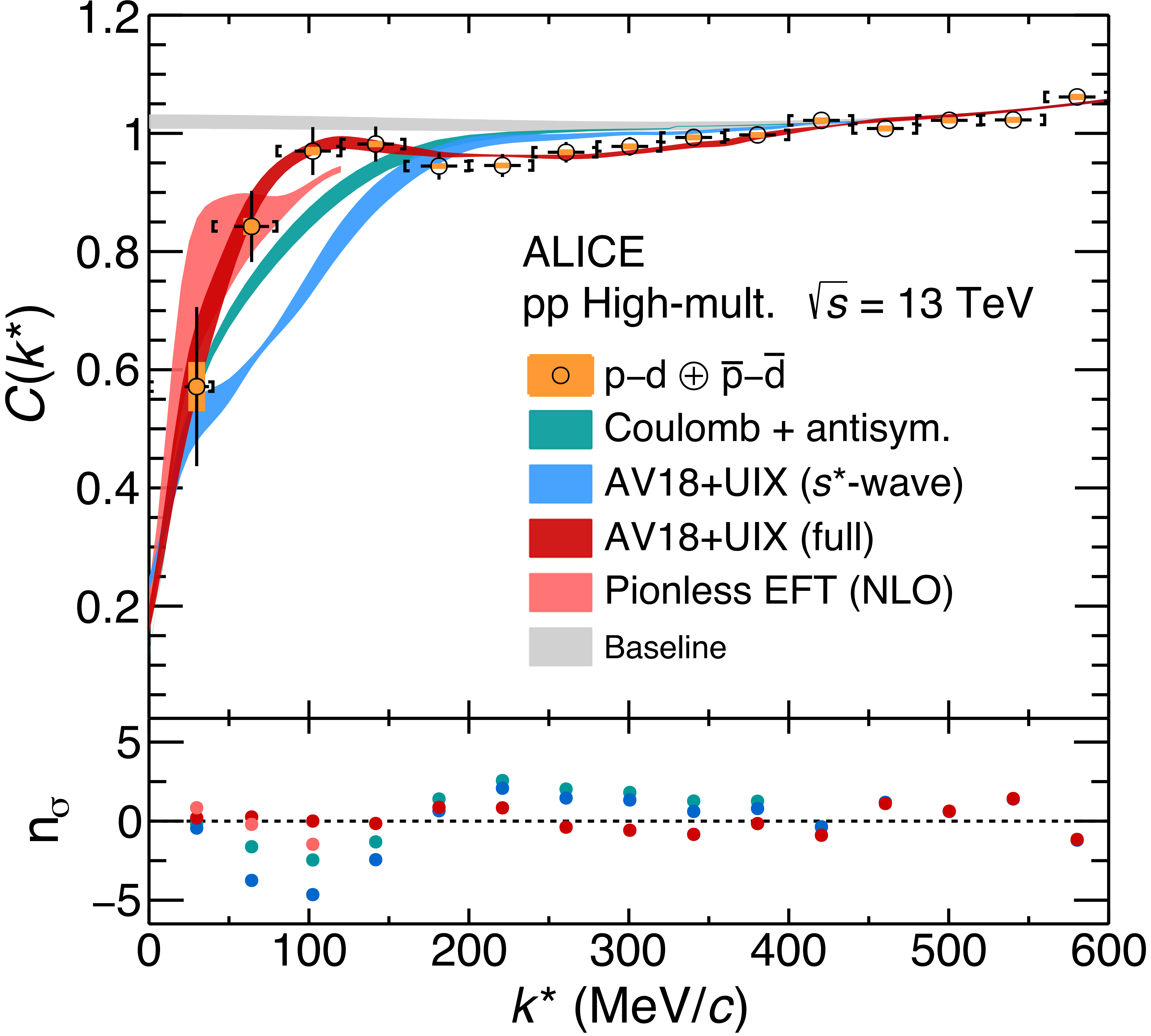
Dynamics of the p–(pn) triplet and higher partial waves at short distances!

Avenue for the study of hadron–deuteron systems including charm and strange hadrons!

ALICE Coll. arXiv:2308.16120



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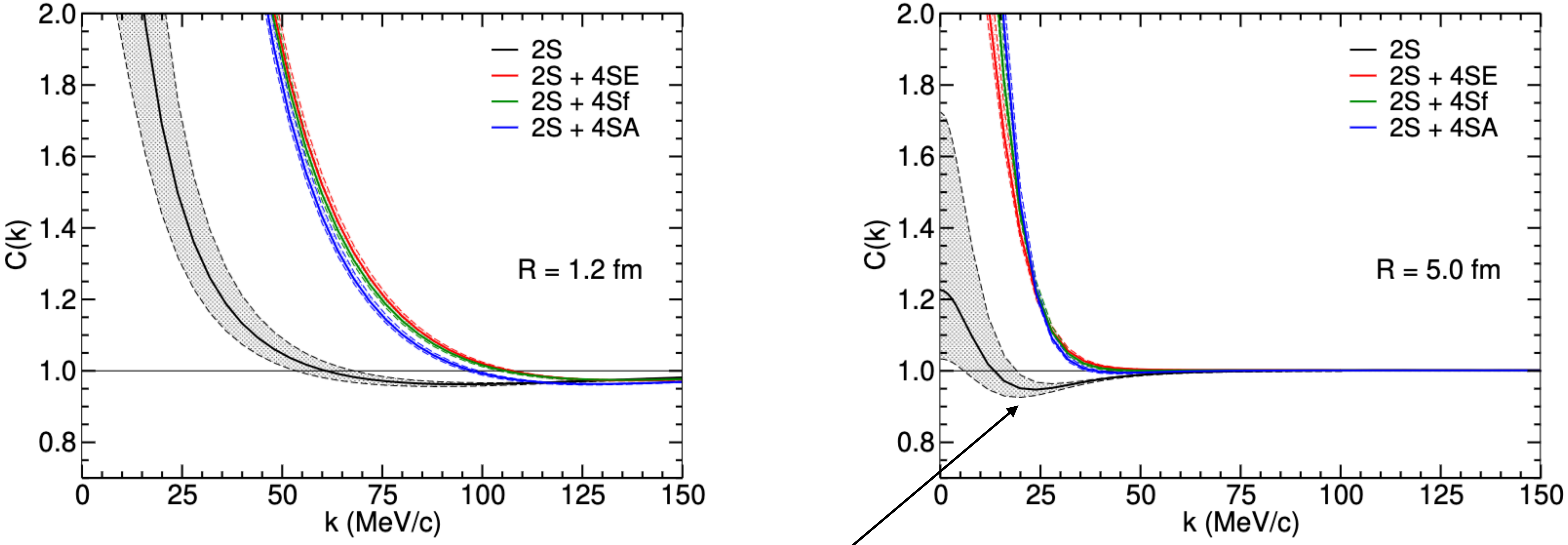
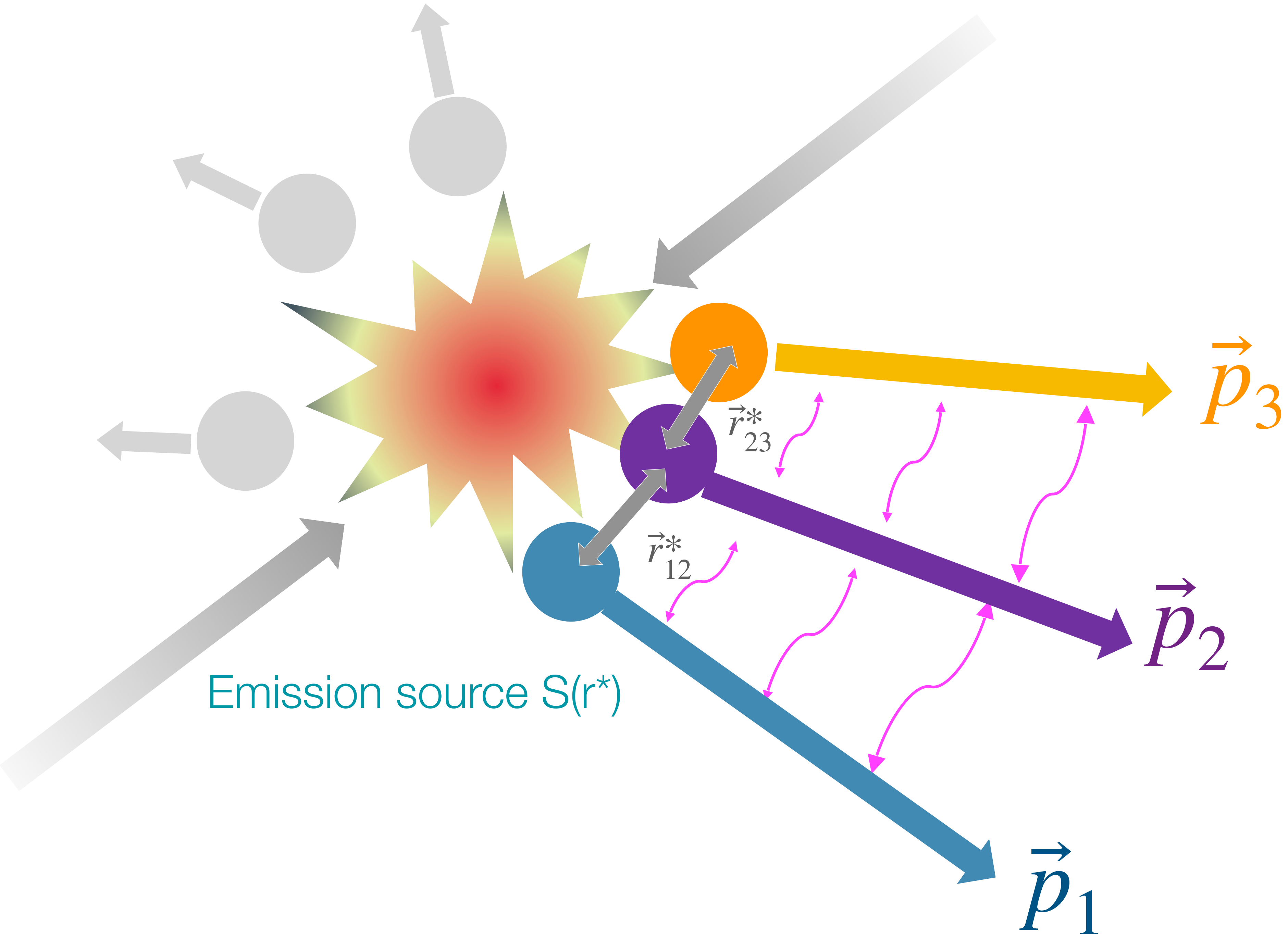
Λ–d correlation

- Measurement in LHC Run2 was not investigated in detail due to the lack of statistics
- LHC Run 3 ~2 orders of magnitude increase in pair statistics: **p-d and Λ–d (results very soon)**
- Theoretical predictions: based on LL model by J. Haidenbauer, Phys. Rev. C 102, 034001 (2020)

Only s-wave contribution: in doublet S= 1/2 and quartet (S= 3/2)



Possibility to see $^{\Lambda}_{3}\mathrm{He}$



Femtoscopy opens the door for the study of interactions in unbound system of three hadron (3 to 3 scattering process)

Three-body femtoscopy with ALICE

- Extending femtoscopy to three-particle correlations: p-p-p and p-p-Λ[1]
- Study interaction in hadron-triplets

$$C(Q_3) = N \frac{N_{\text{same}}(Q_3)}{N_{\text{mixed}}(Q_3)} \qquad Q_3 = \sqrt{q_{12}^2 + q_{23}^2 + q_{13}^2}$$

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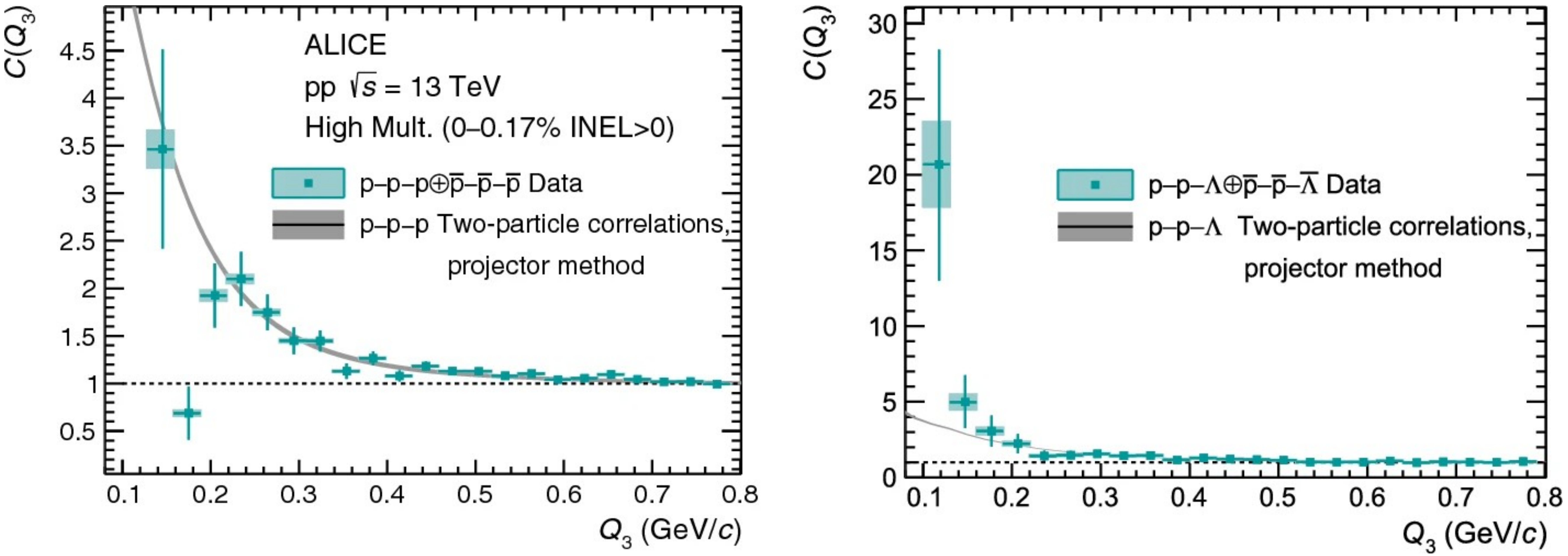


- Effects beyond two-body contributions[2]

[1] ALICE Coll, EPJA 59, 145 (2023)

[2] Del Grande et al, EPJC 82, 244 (2022)

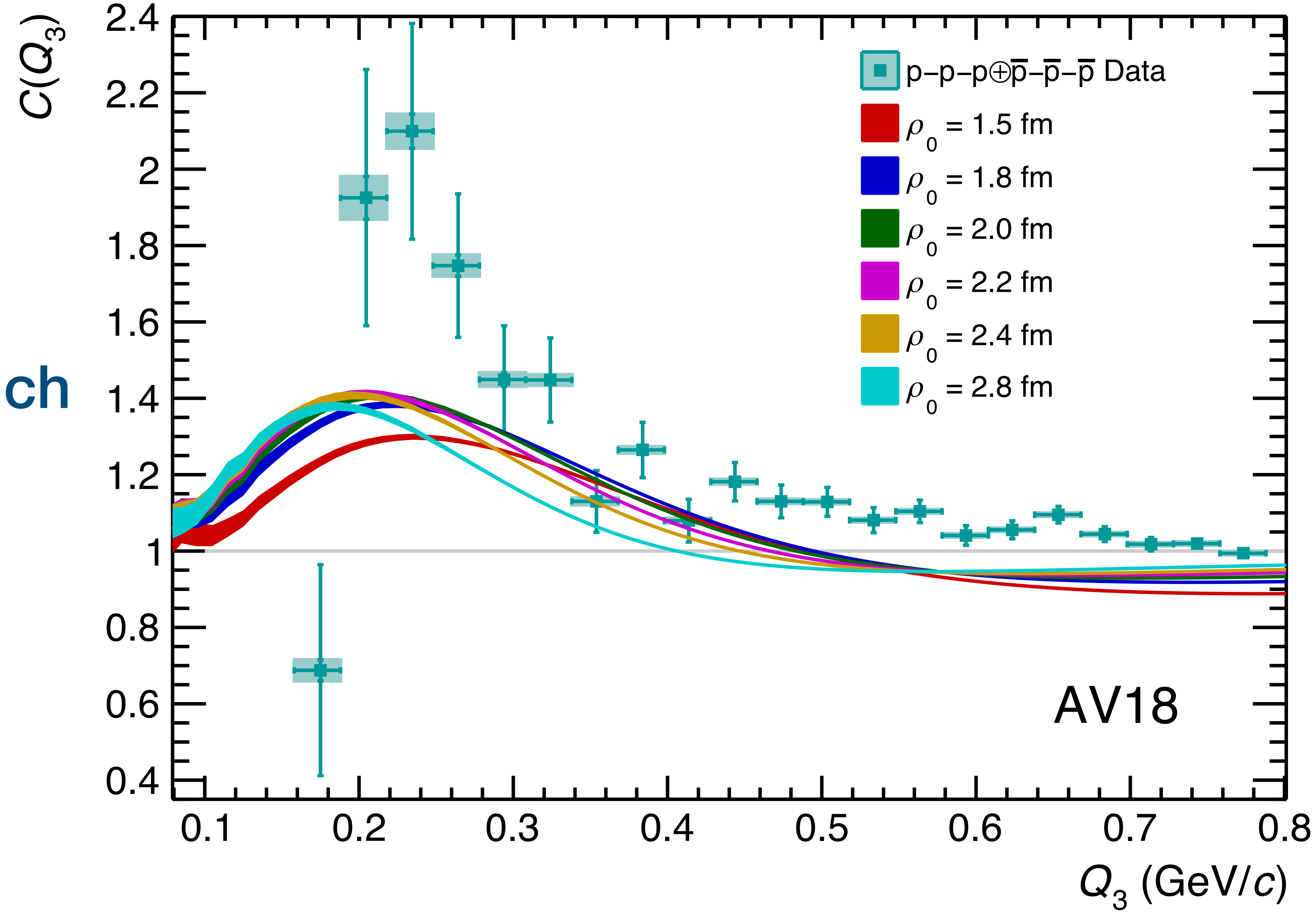
p-p-p correlation using AV18 potential

- Three-body correlation function with HH approach[1]

$$C\left(Q_3\right) = \int S(\rho)\left|\psi\left(Q_3, \rho\right)\right|^2 \rho^5 \, d\rho$$

Work of Laura Šerkšnyte, and Raffaele Del Grande (**Munich group**) in collaboration with **INFN PISA group**

- $\Psi(Q_3, \rho)$ computed using pp AV18 strong interaction, Coulomb corrections, and quantum statistics
- Attractive AV18 interaction: results peak
- Pauli-blocking: depletion in C(Q3)

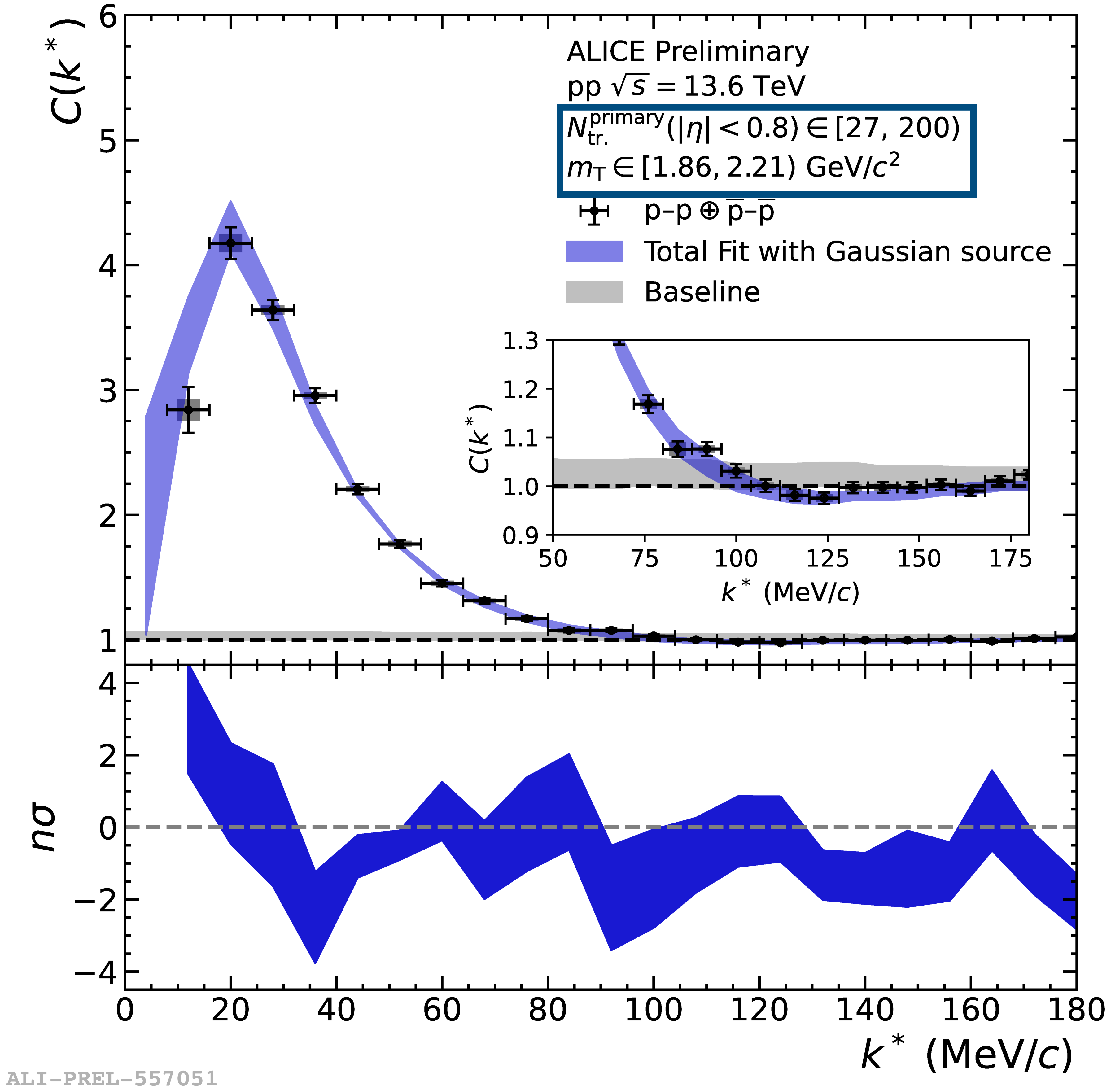


p-p-Λ: theoretical work in progress

[1] A Kievsky et al, arXiv:2310.10428 (accepted by PRC)

Femtoscopy towards LHC Run 3

- LHC Run 3 pp collisions at 13.6 TeV: 2 orders of magintude increased p–p pair statistics
- Fixed source for all interaction studies using femtoscopy



p–p correlation function measured in m_{T} and multiplicity differential

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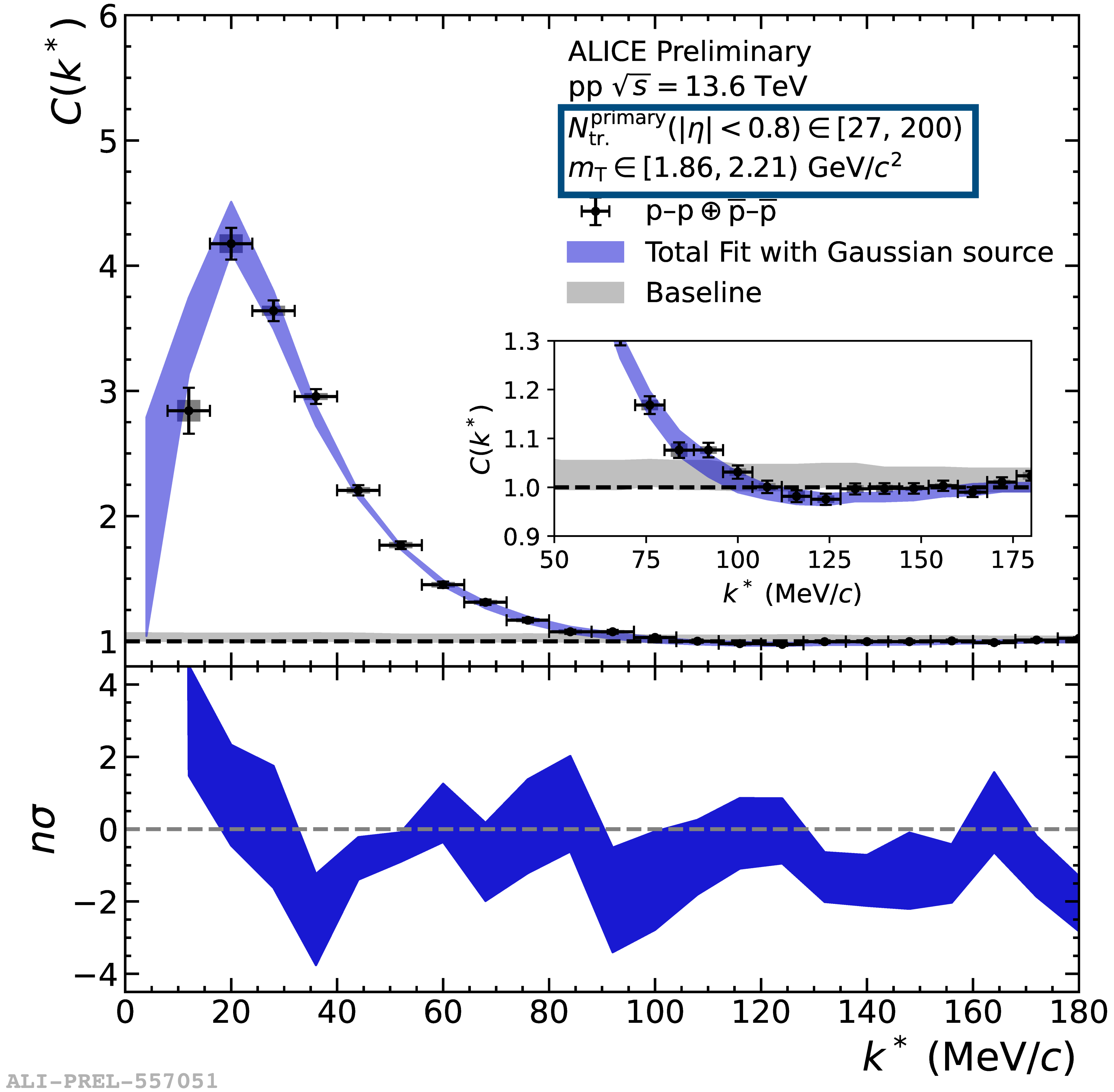

ALI-PREL-557051

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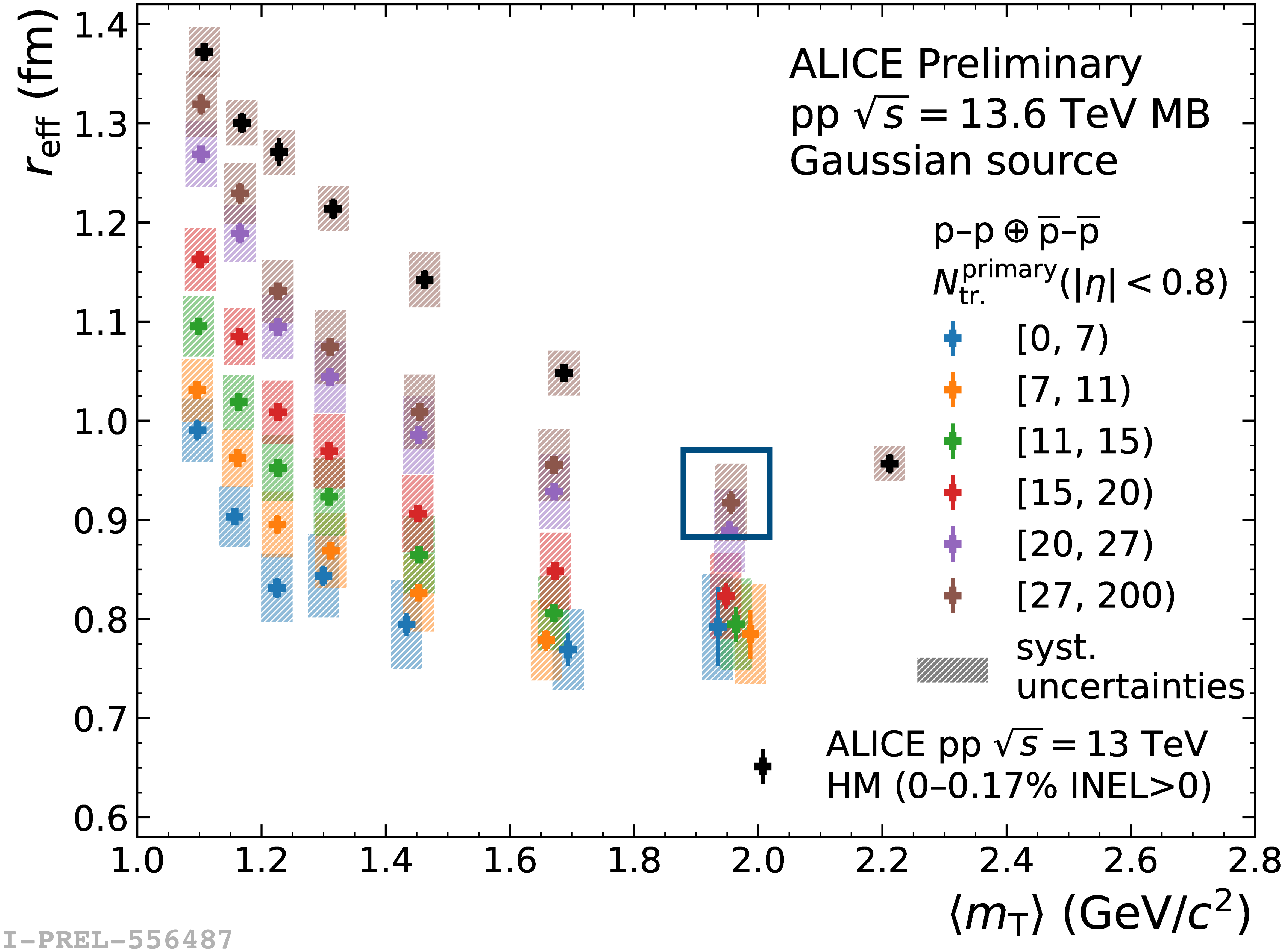

ALI-PREL-556487

m_{T}-scaling of the effective source size for p–p pairs in different multiplicity classes

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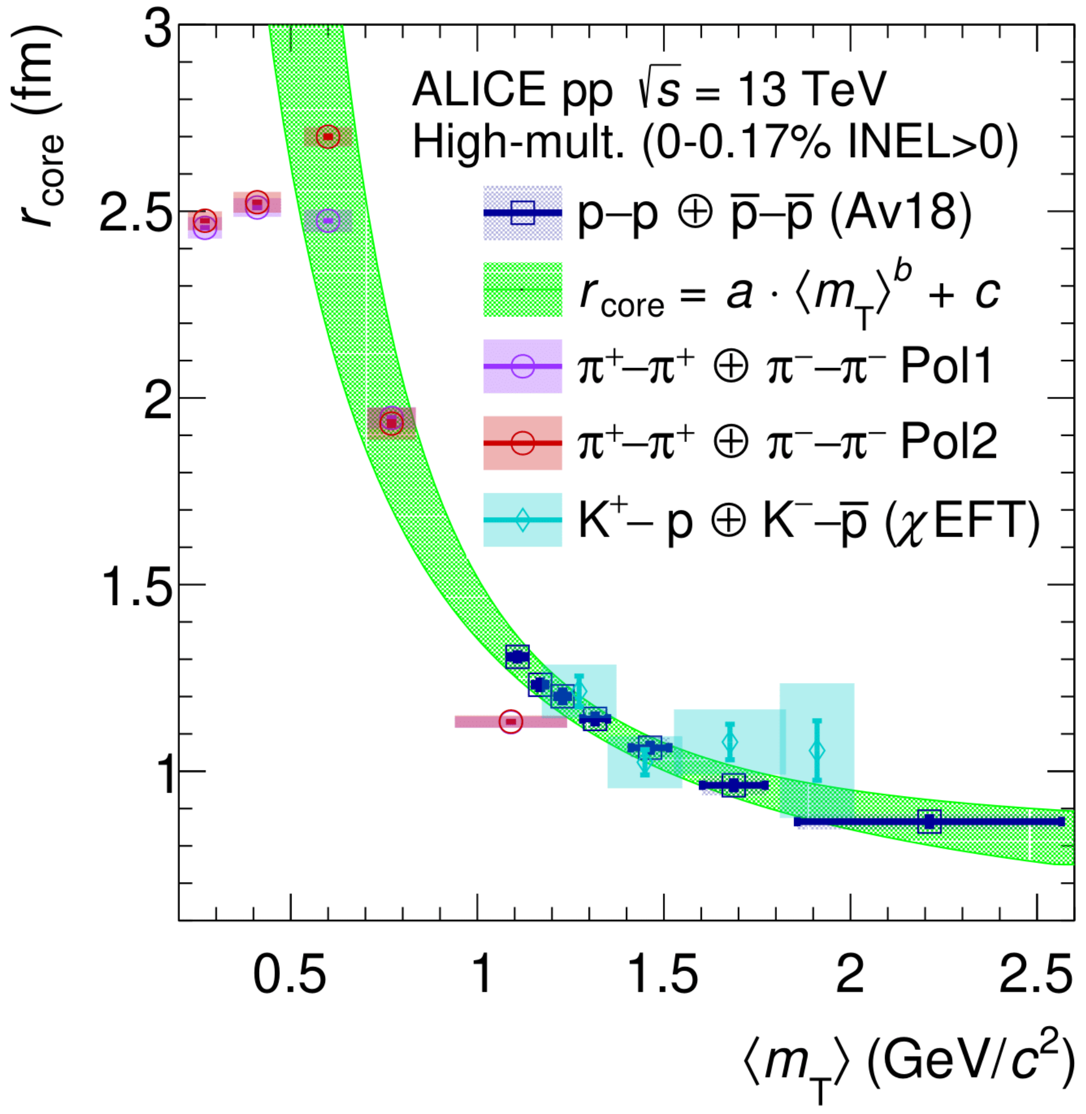
  - Nuclei production studies via coalescence



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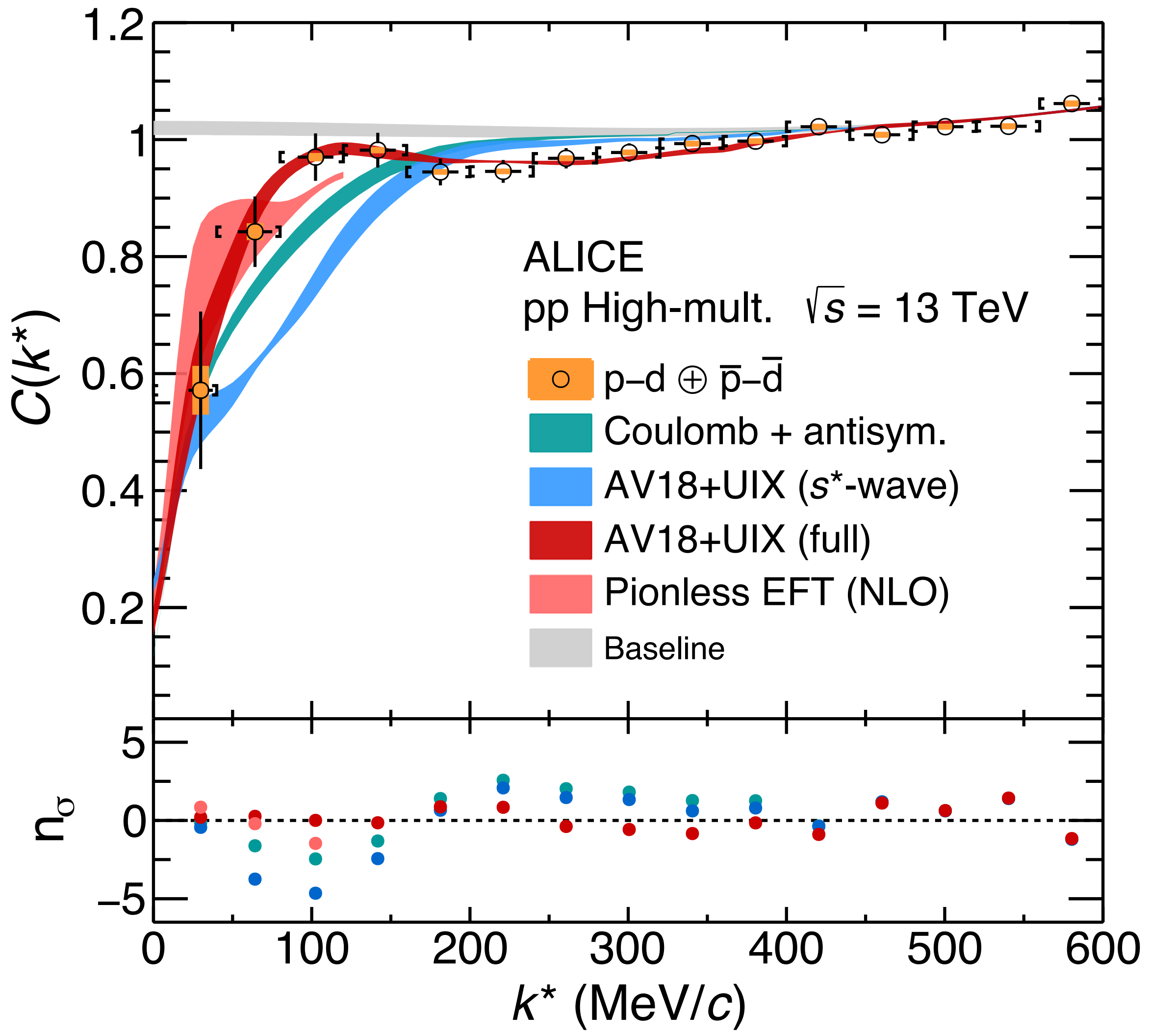

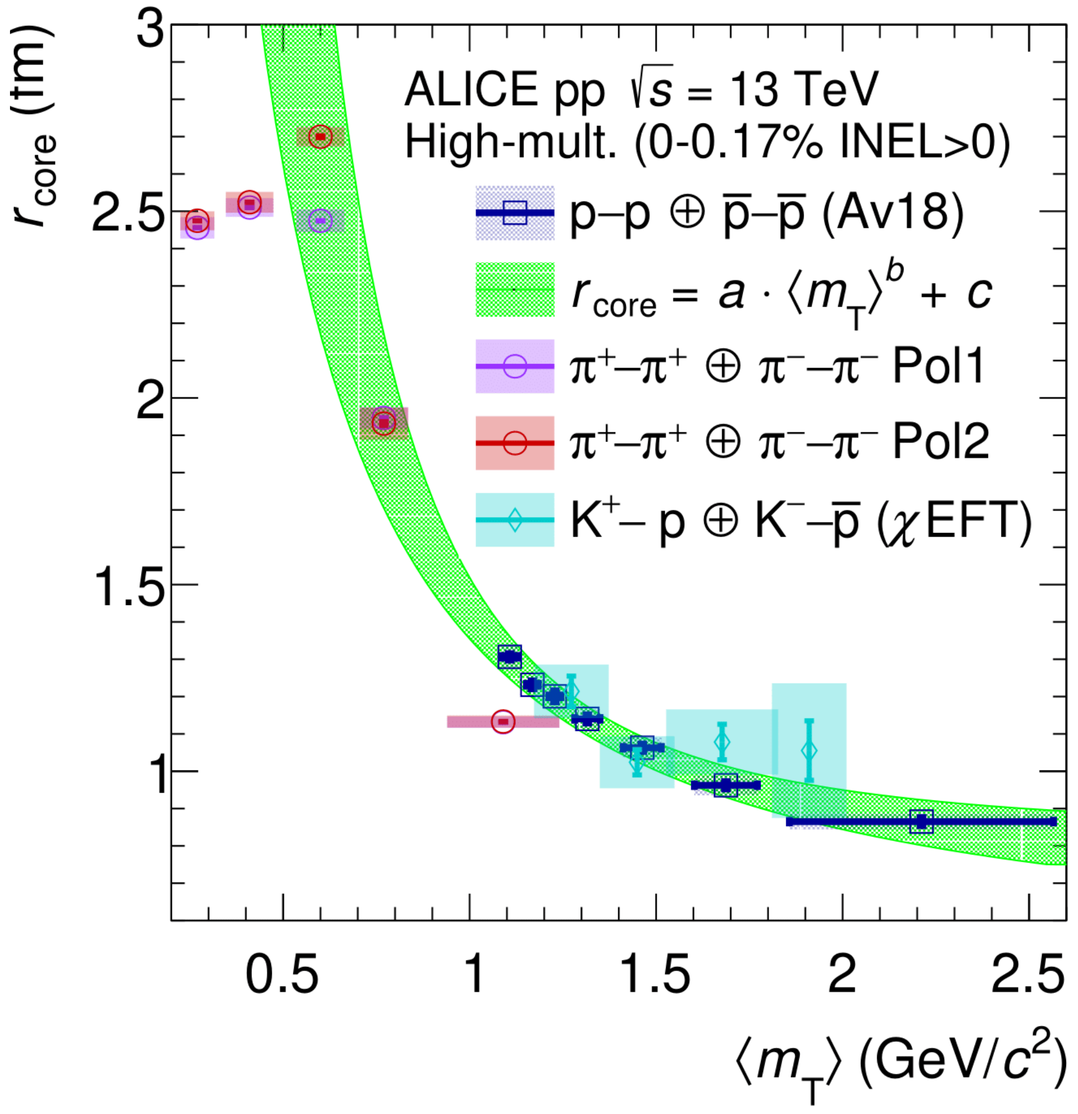

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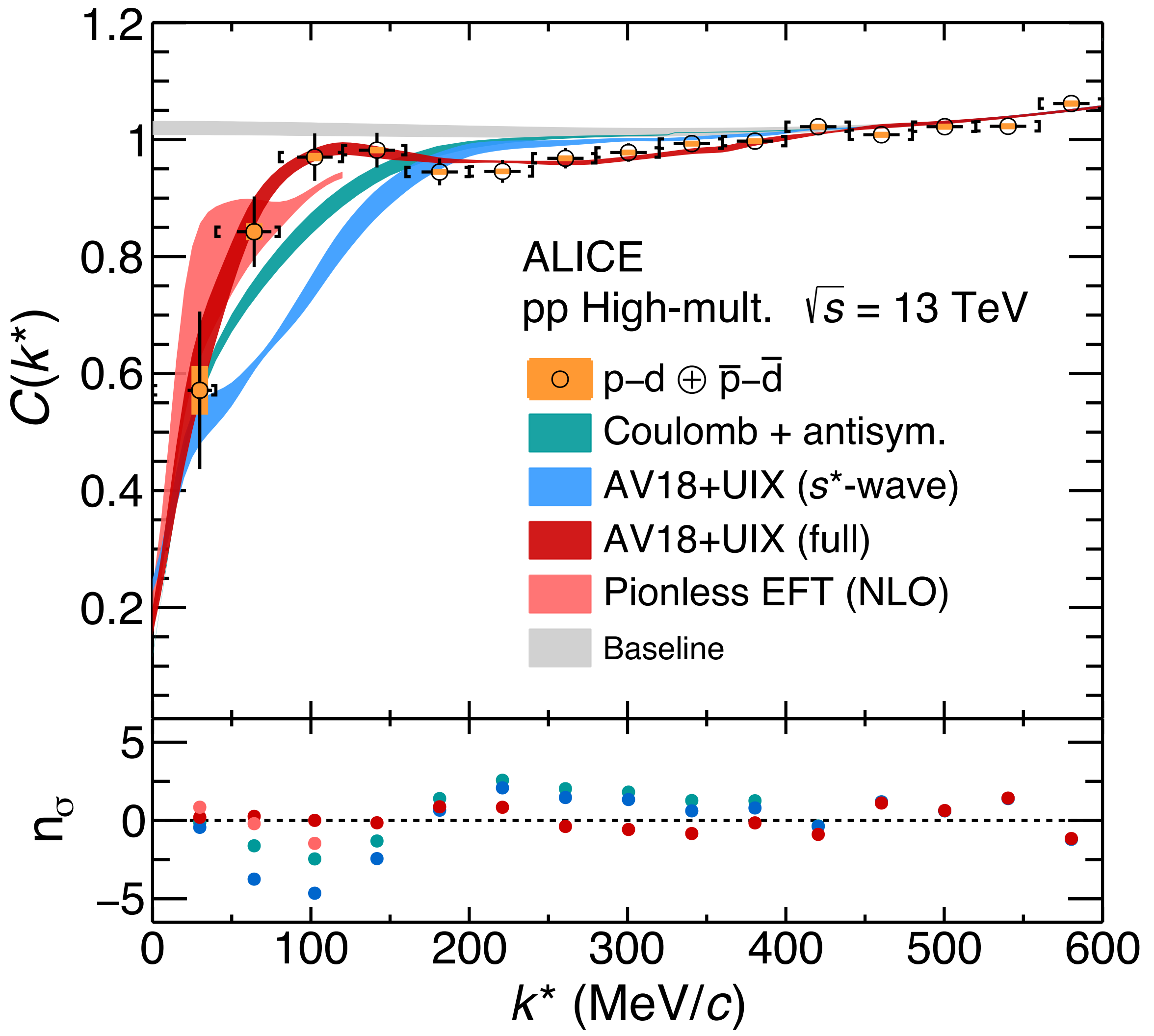

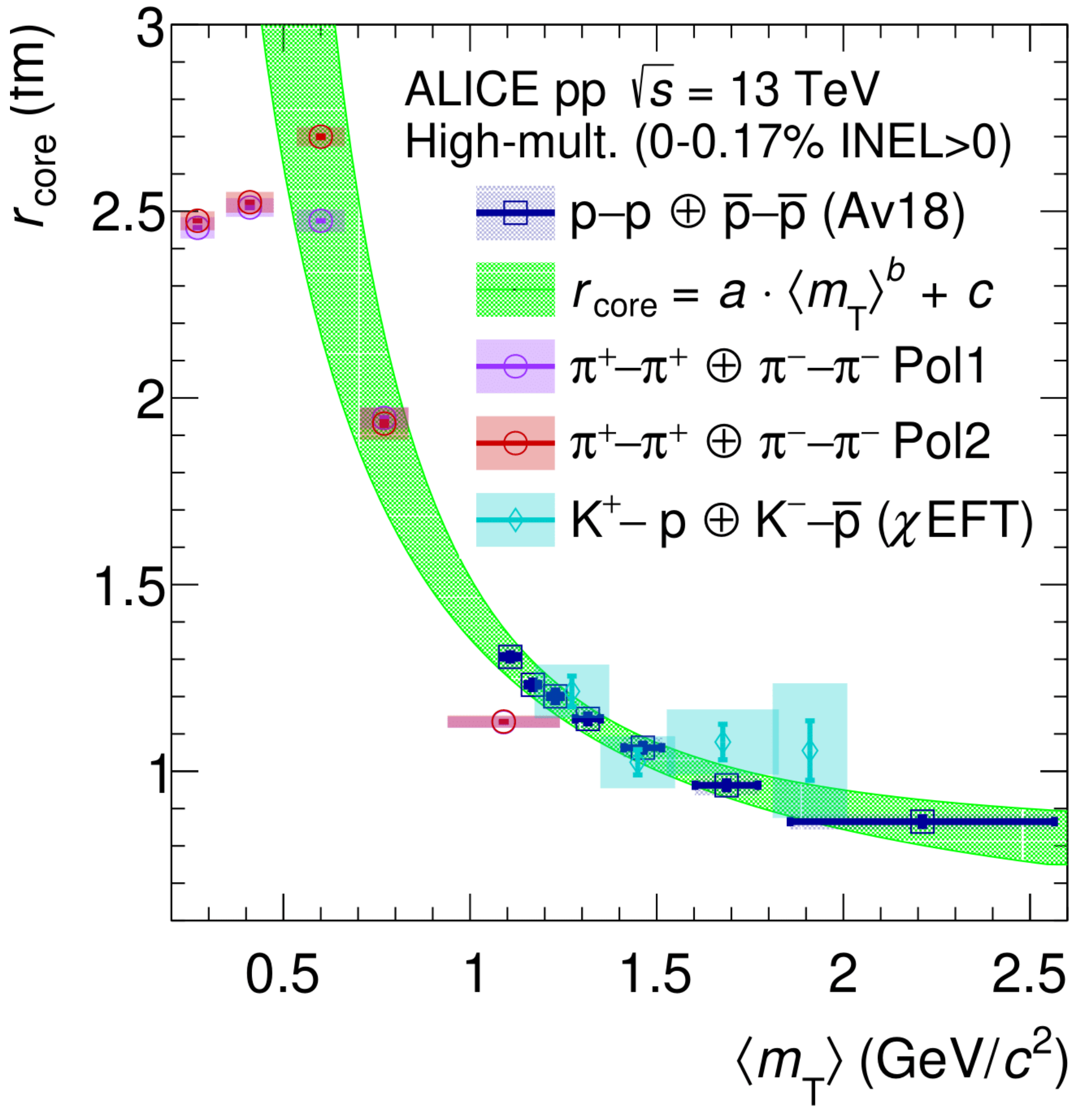

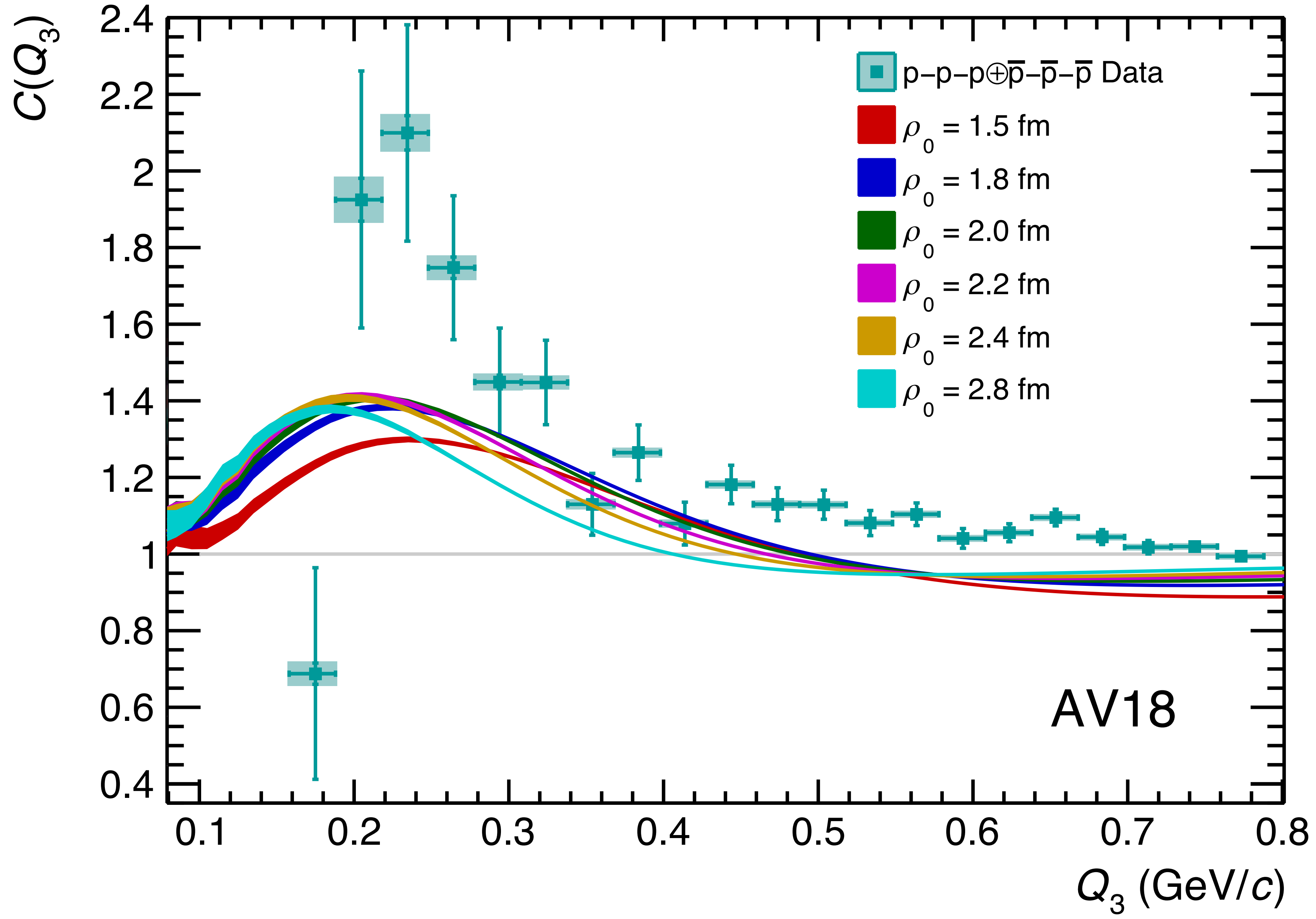

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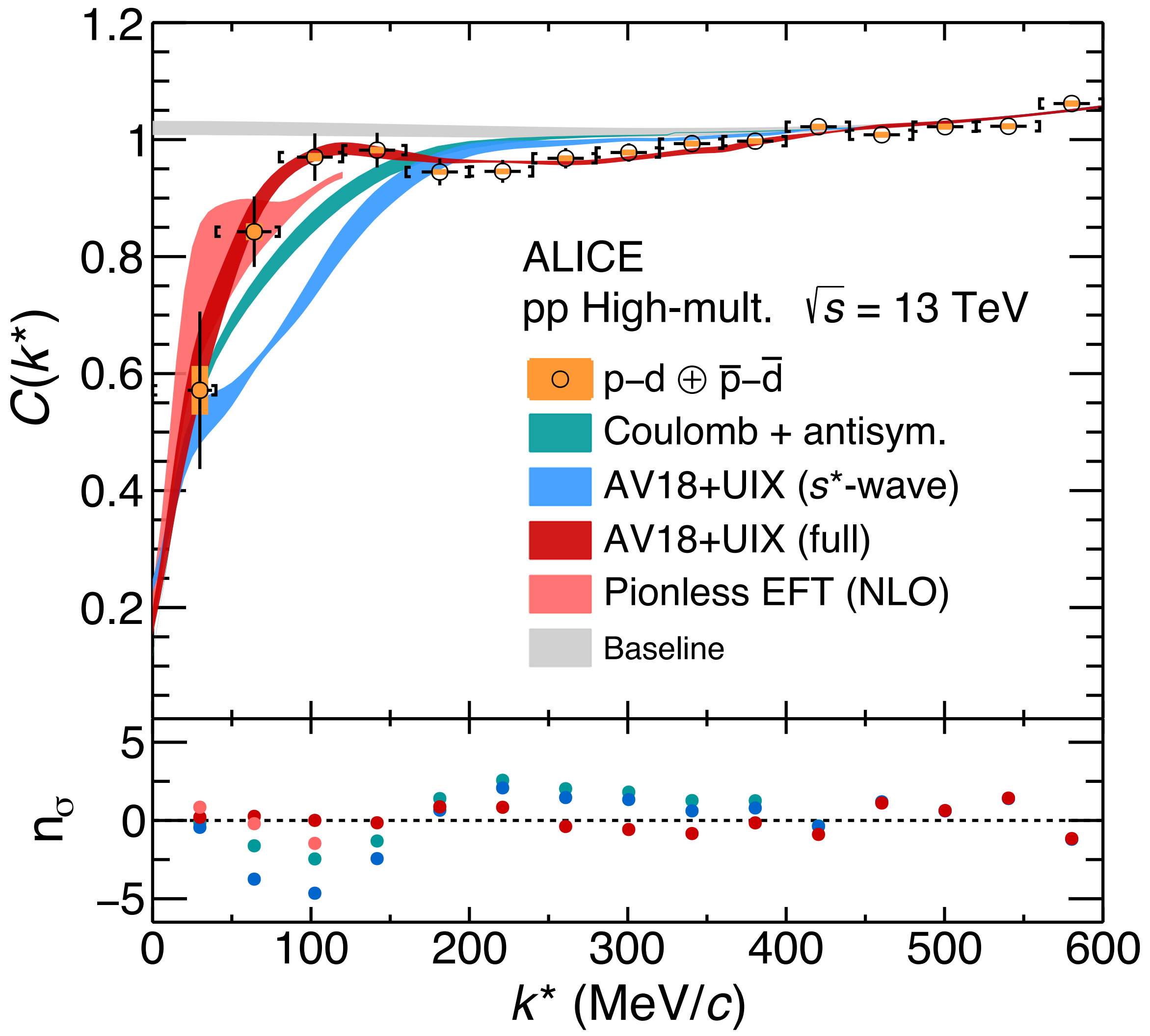

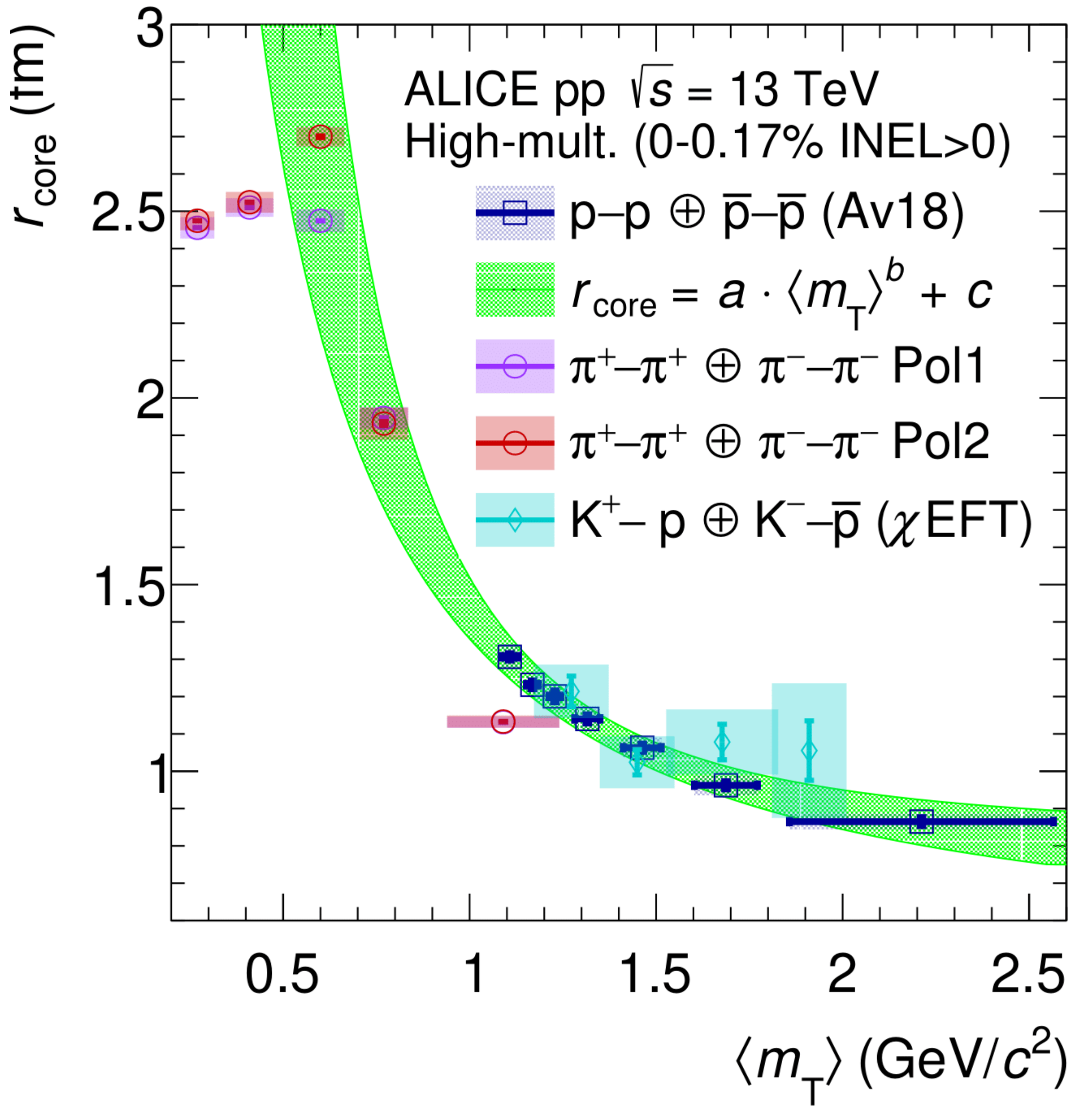

Outlook: Large statistics of LHC run 3 and run 4

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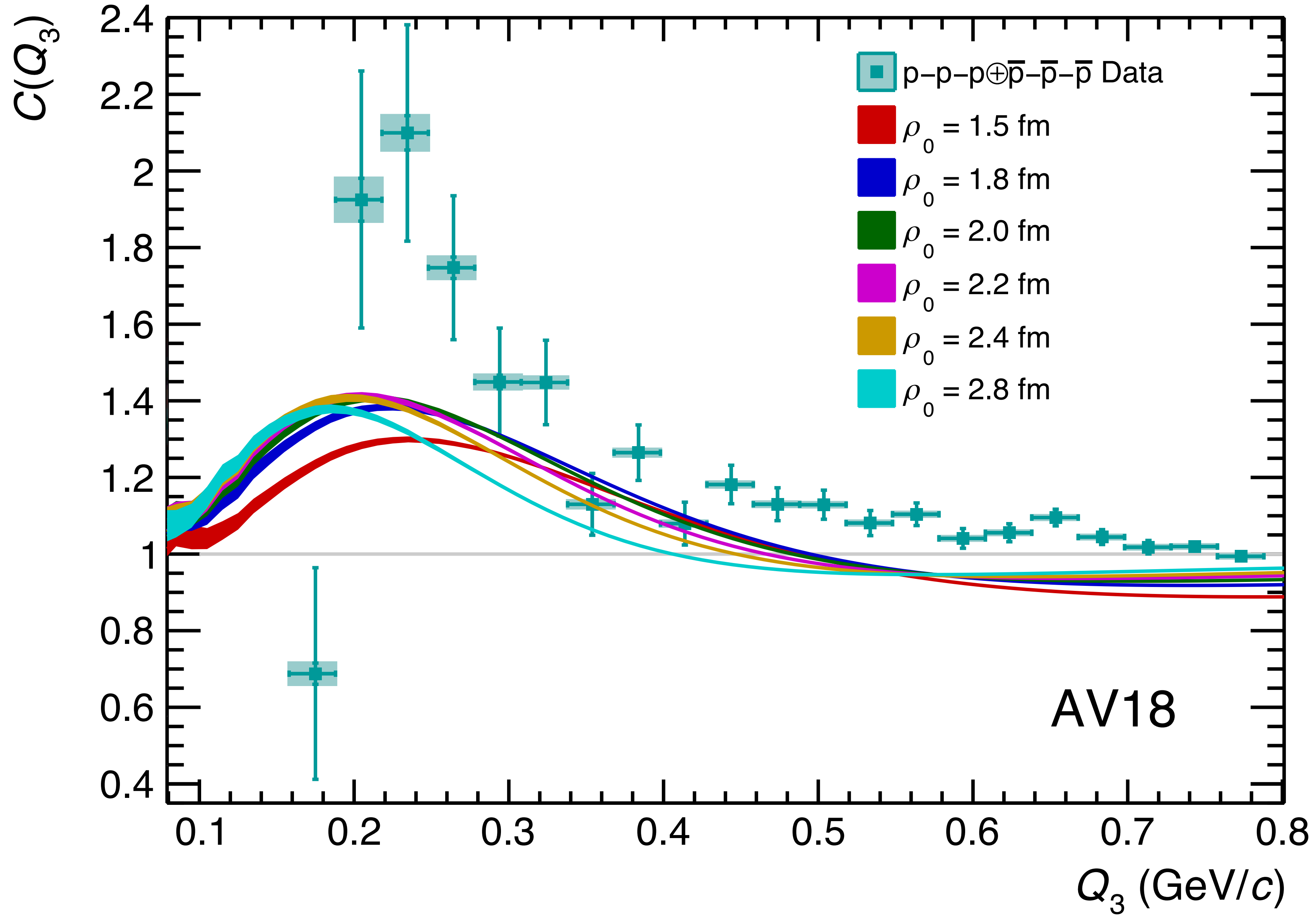

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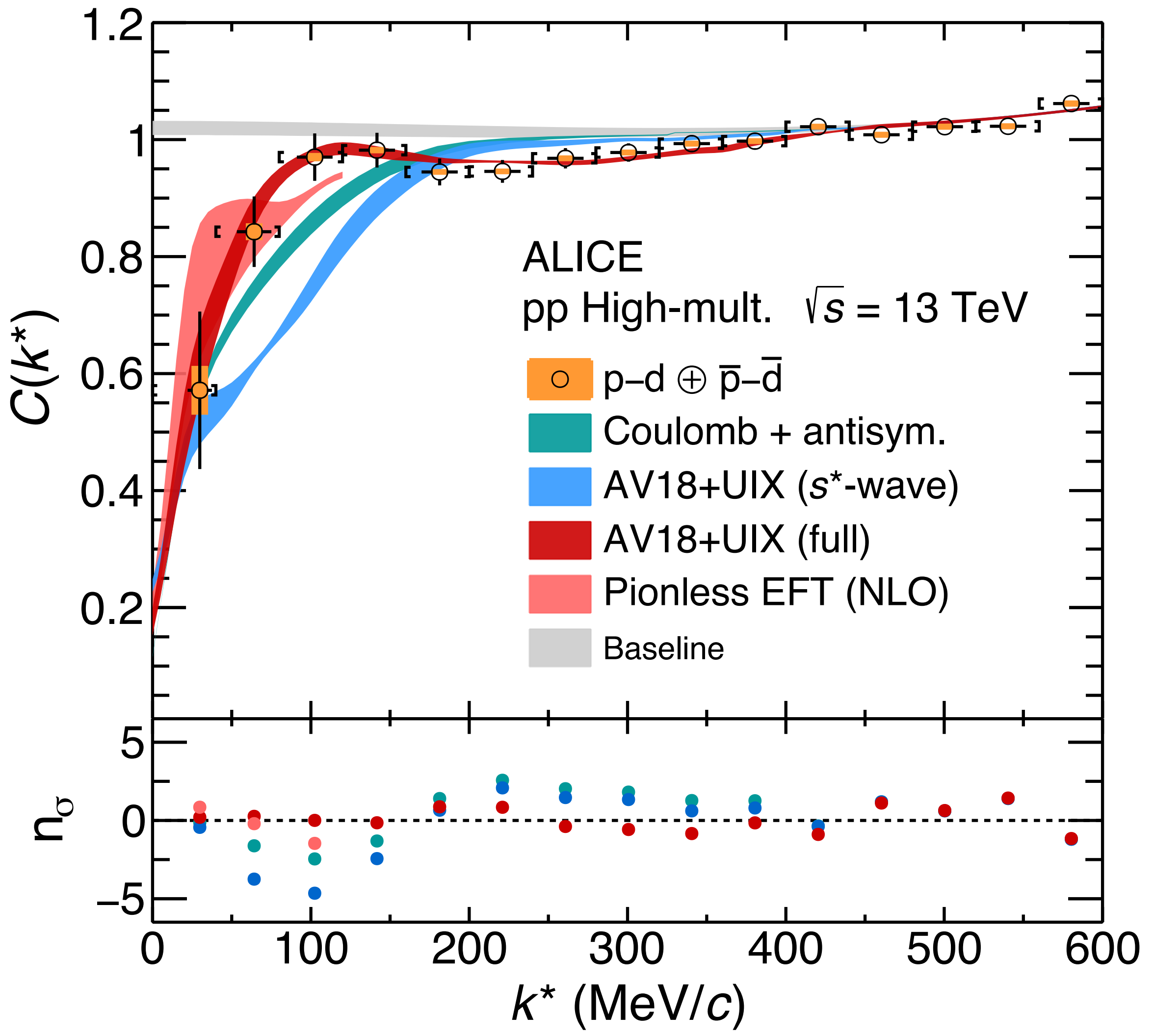

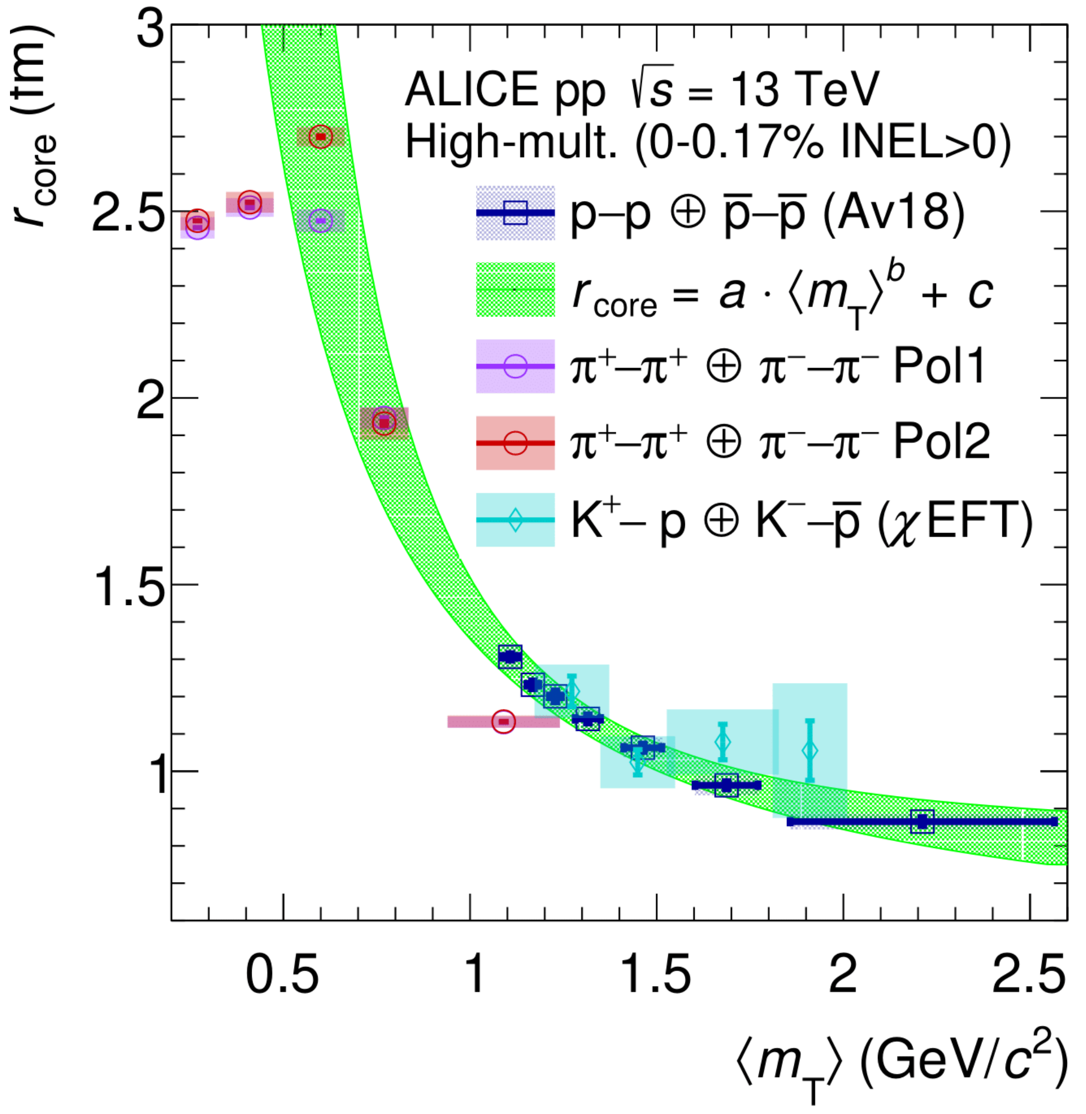

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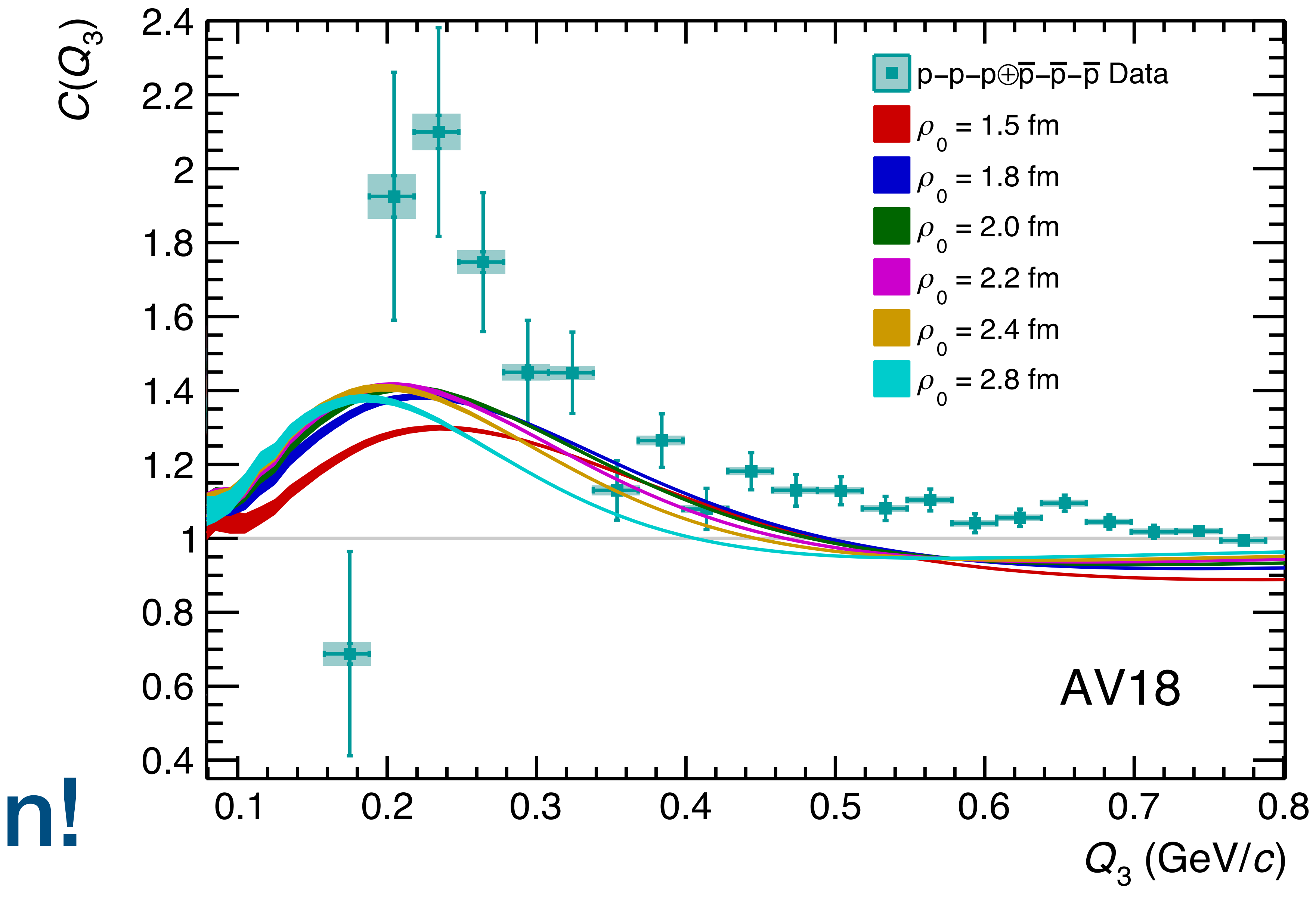
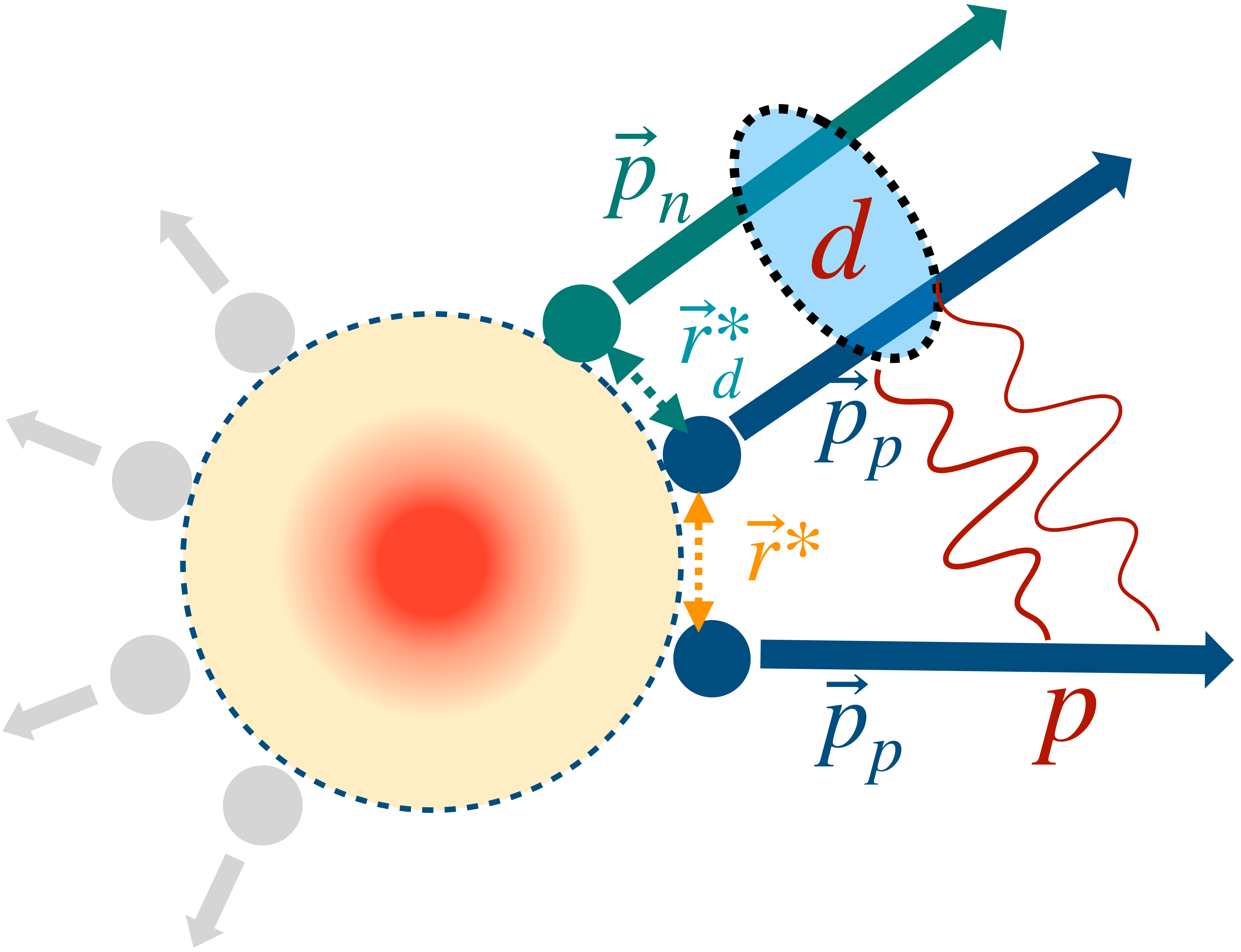


Thank you for your attention!

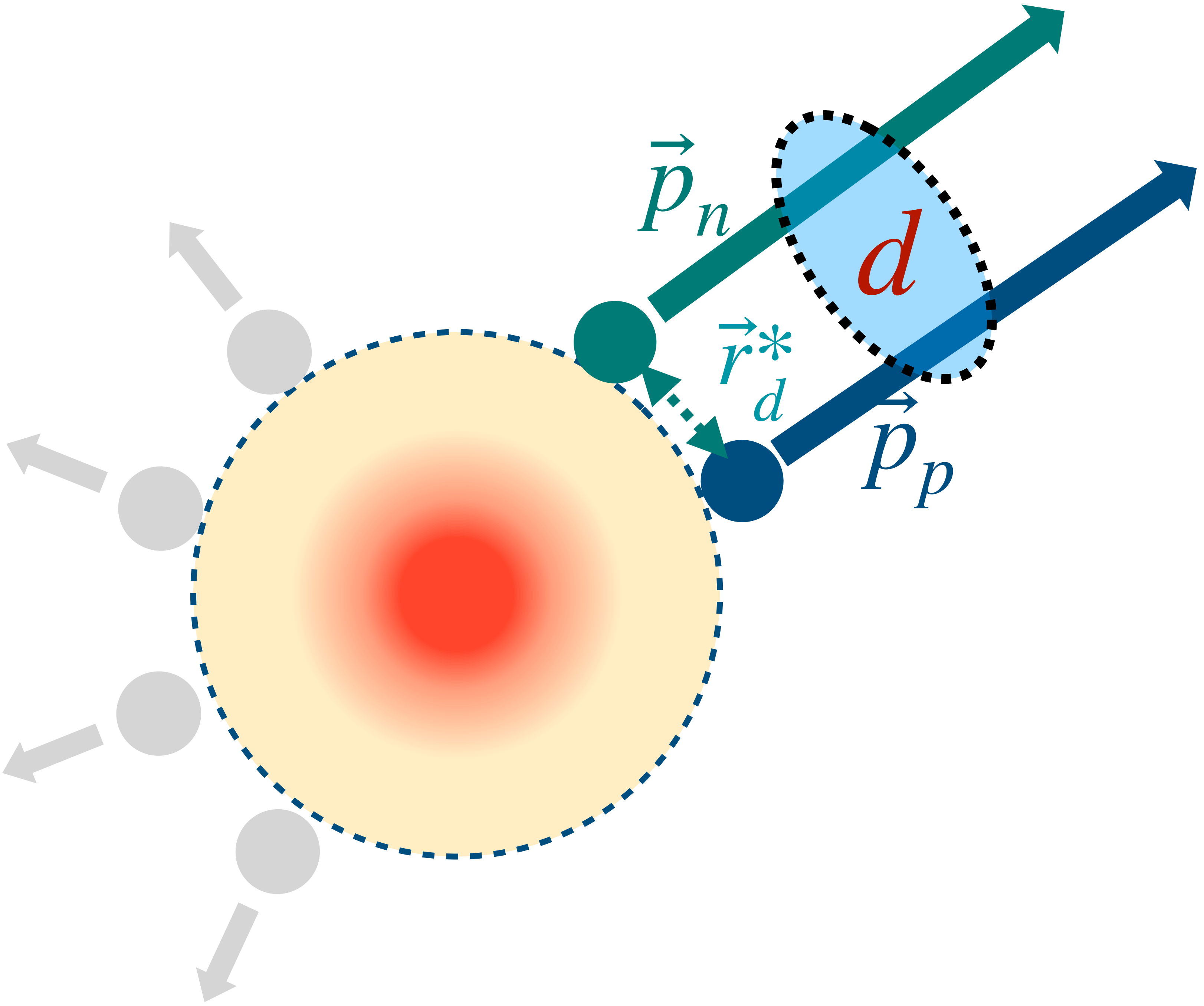
Backup

From (anti)d production to strong interaction

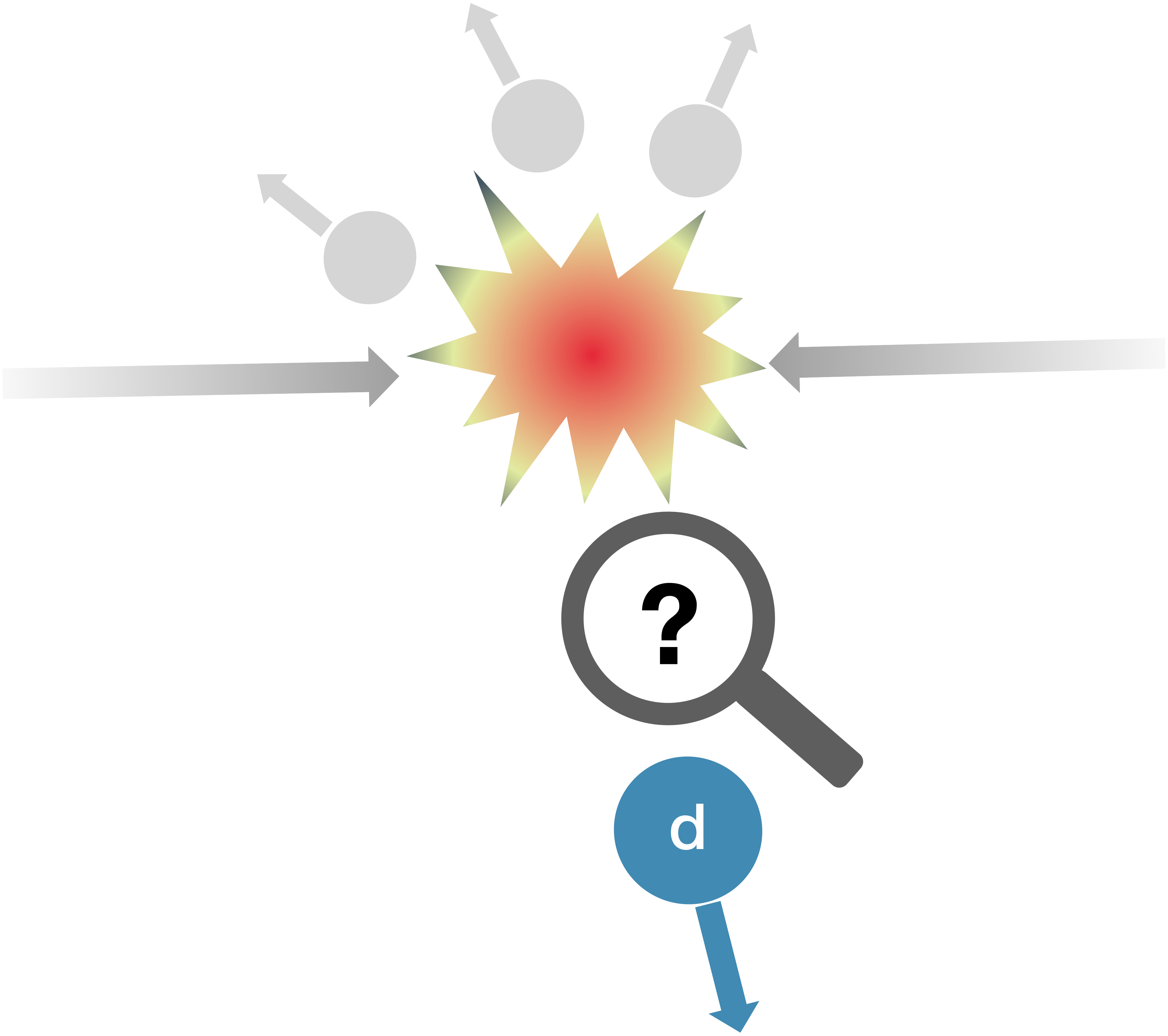
Strong interaction in proton-deuteron system



Microscopic description of (anti)deuteron production



(Anti)nuclei production models



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Production described by two models

- **Statistical hadronization model**
  - Particle yields (including nuclei) described by filling the available phase-space after the collision
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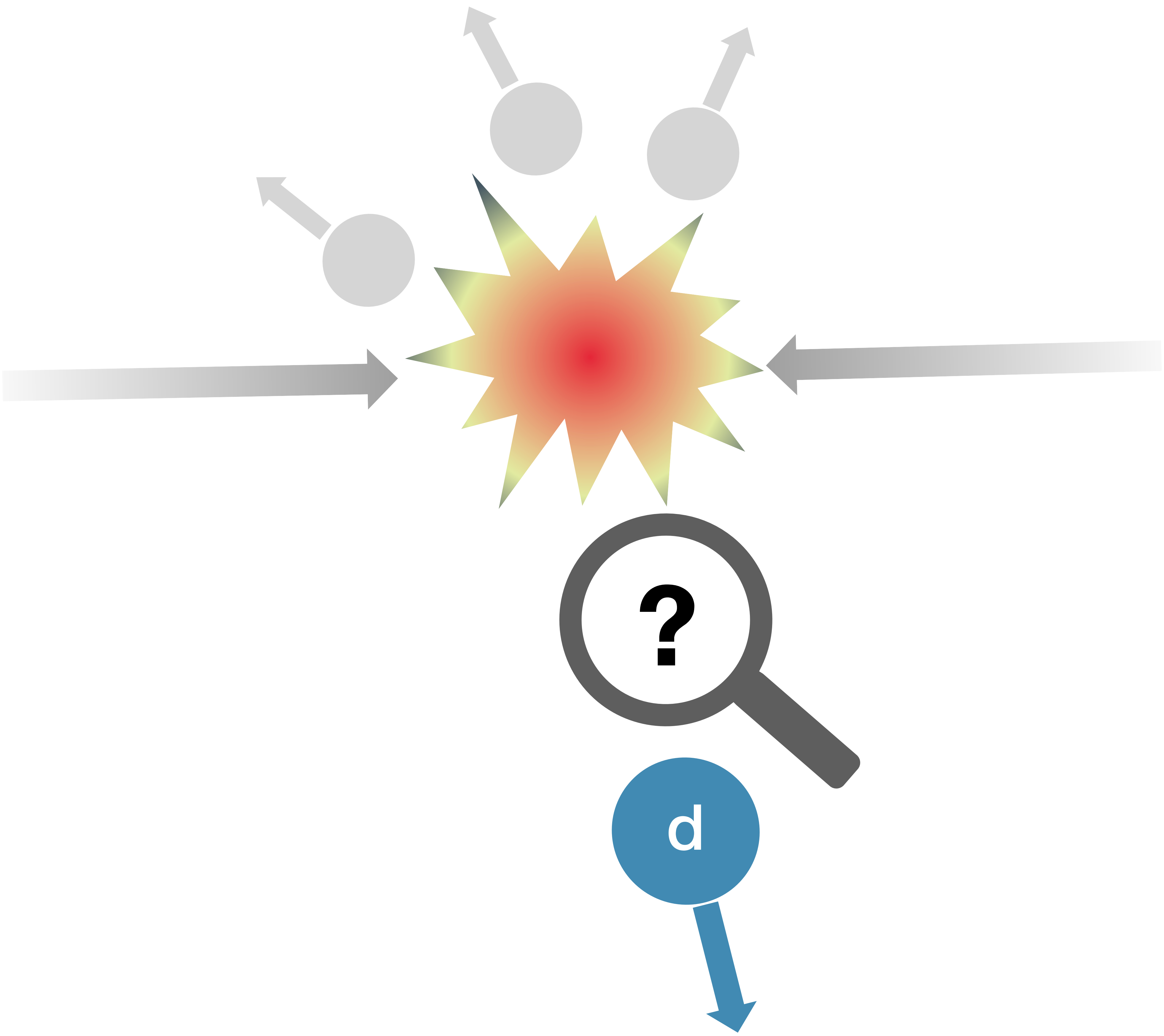

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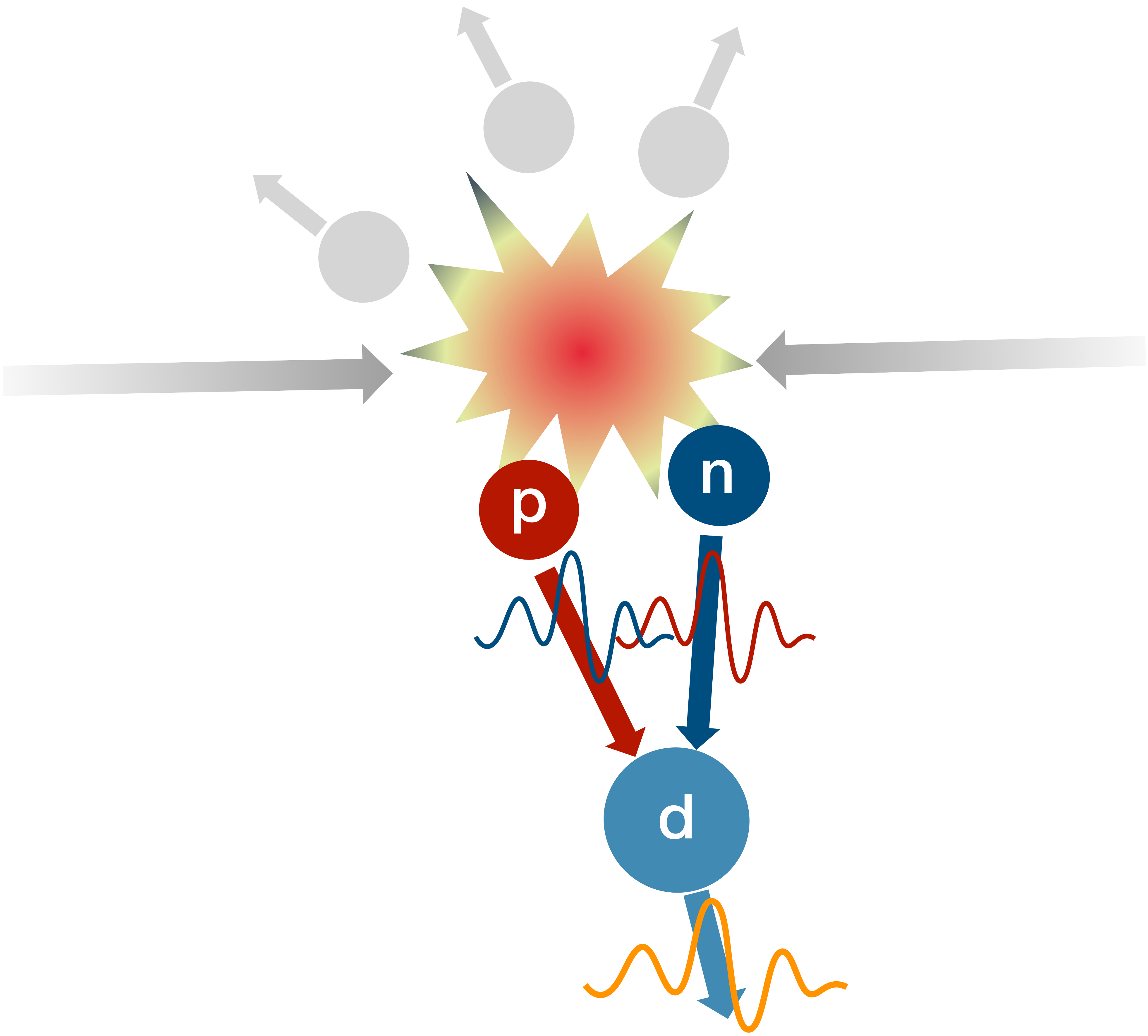

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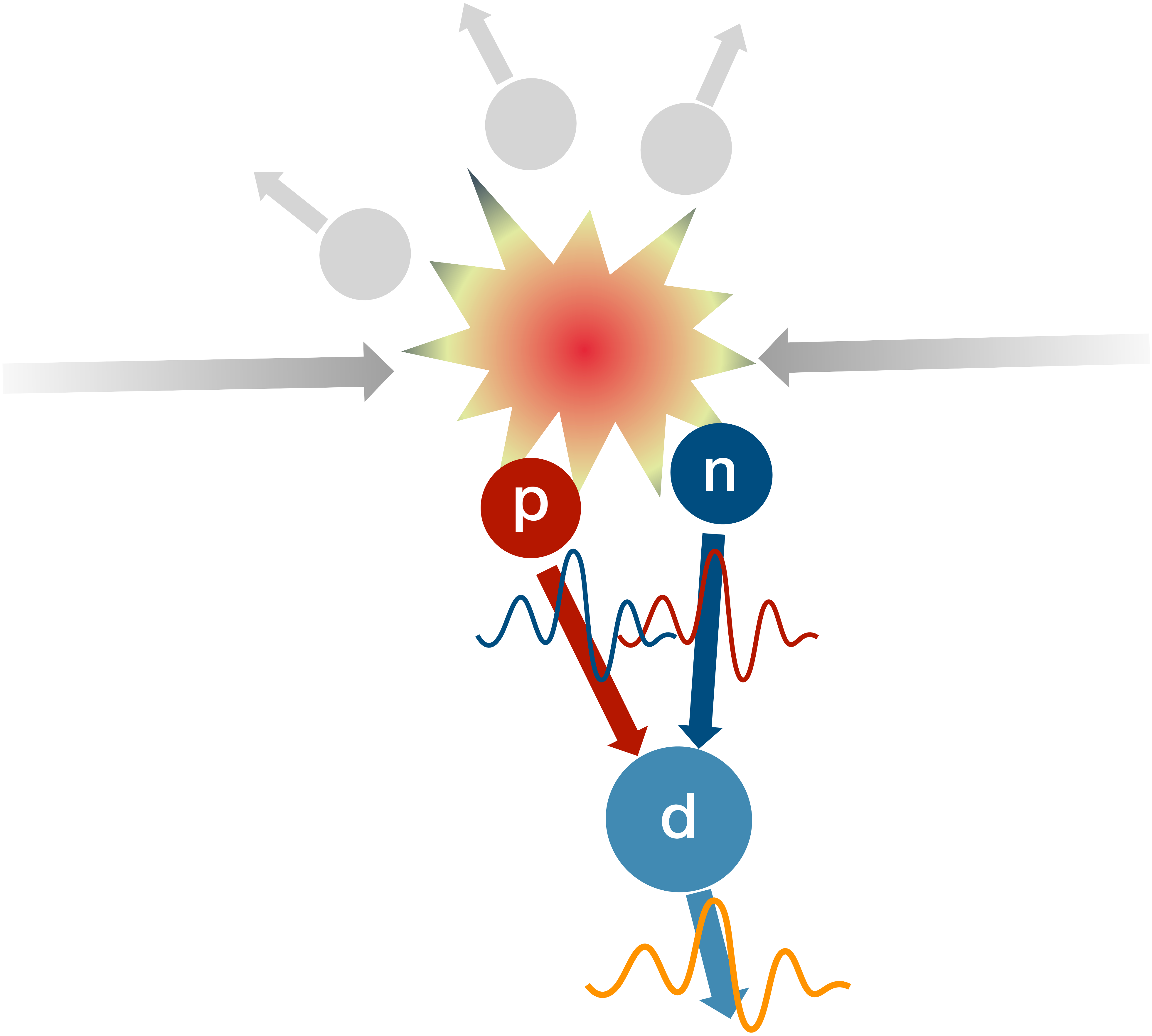


Deuteron spectra in coalescence

- Deuteron spectra instead of B_2[1]

Nucleon momenta distributions

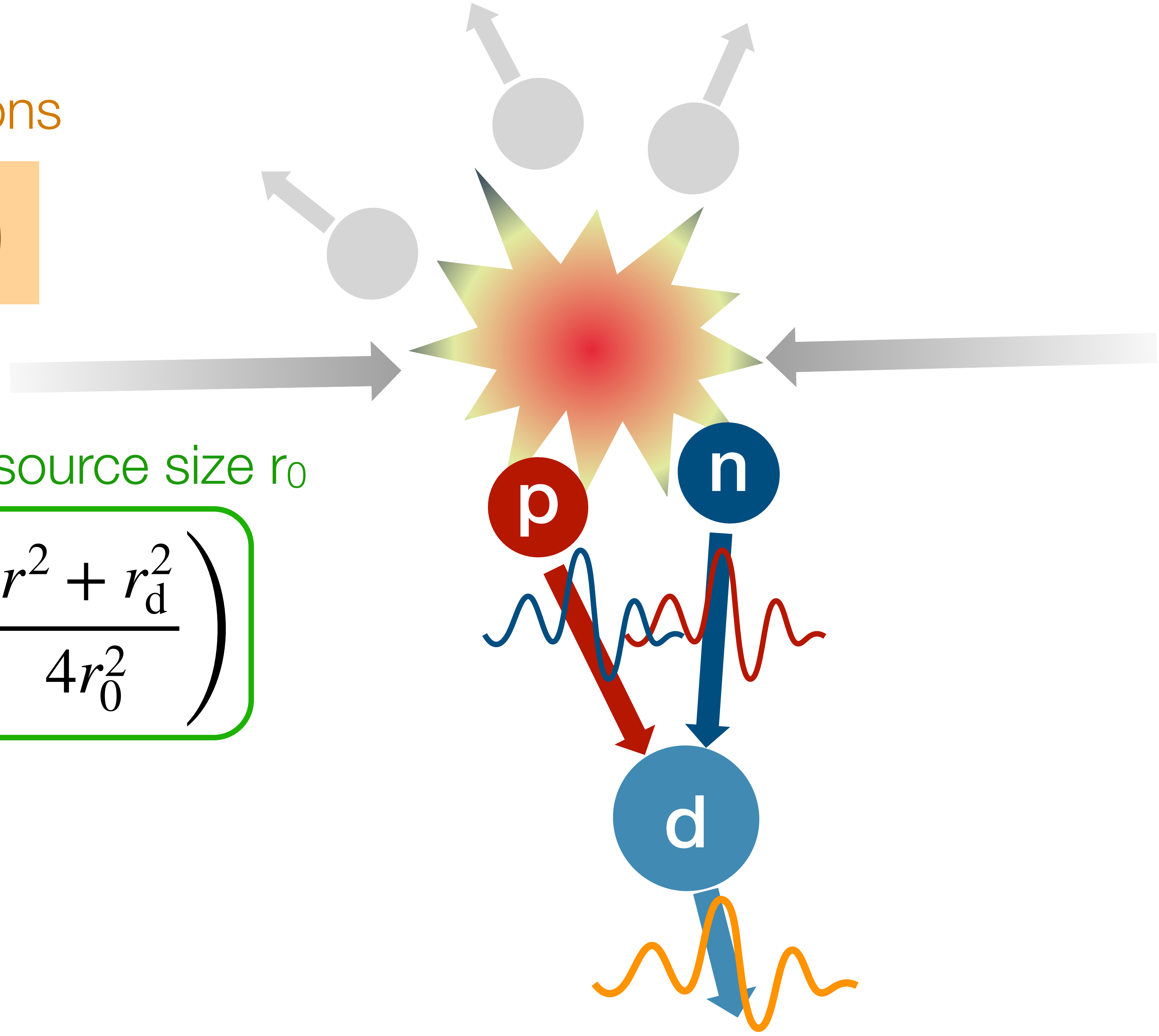
$$\frac{d^3N}{dP^3} = \frac{S_\mathrm{d}}{(2\pi)^6} \int d^3k \int d^3r_\mathrm{n} \int d^3r_\mathrm{p}\, \mathscr{D}(\vec{k},\vec{r})\, H_{np}\left(\vec{r}_n,\vec{r}_\mathrm{p}\right) G_{np}\left(\vec{P}/2+\vec{k},\vec{P}/2-\vec{k}\right)$$

Deuteron Wigner density

$$\mathscr{D}(\vec{k},\vec{r}) = \int \mathrm{d}^3\xi e^{i\vec{k}\cdot\vec{\xi}} \varphi_\mathrm{d}\left(\vec{r}+\frac{\vec{\xi}}{2}\right) \varphi^*_\mathrm{d}\left(\vec{r}-\frac{\vec{\xi}}{2}\right)$$

Emission Gaussian Ansatz, with source size r_0

$$H_{np}\left(\vec{r}_n,\vec{r}_\mathrm{p};r_0\right) = \frac{1}{(2\pi r_0)^3}\exp\left(-\frac{r^2+r_\mathrm{d}^2}{4r_0^2}\right)$$



[1] Kachelriess et al. EPJA 57 (5) 167, 2021

Deuteron spectra in coalescence

- Deuteron spectra instead of B_2[1]

Nucleon momenta distributions

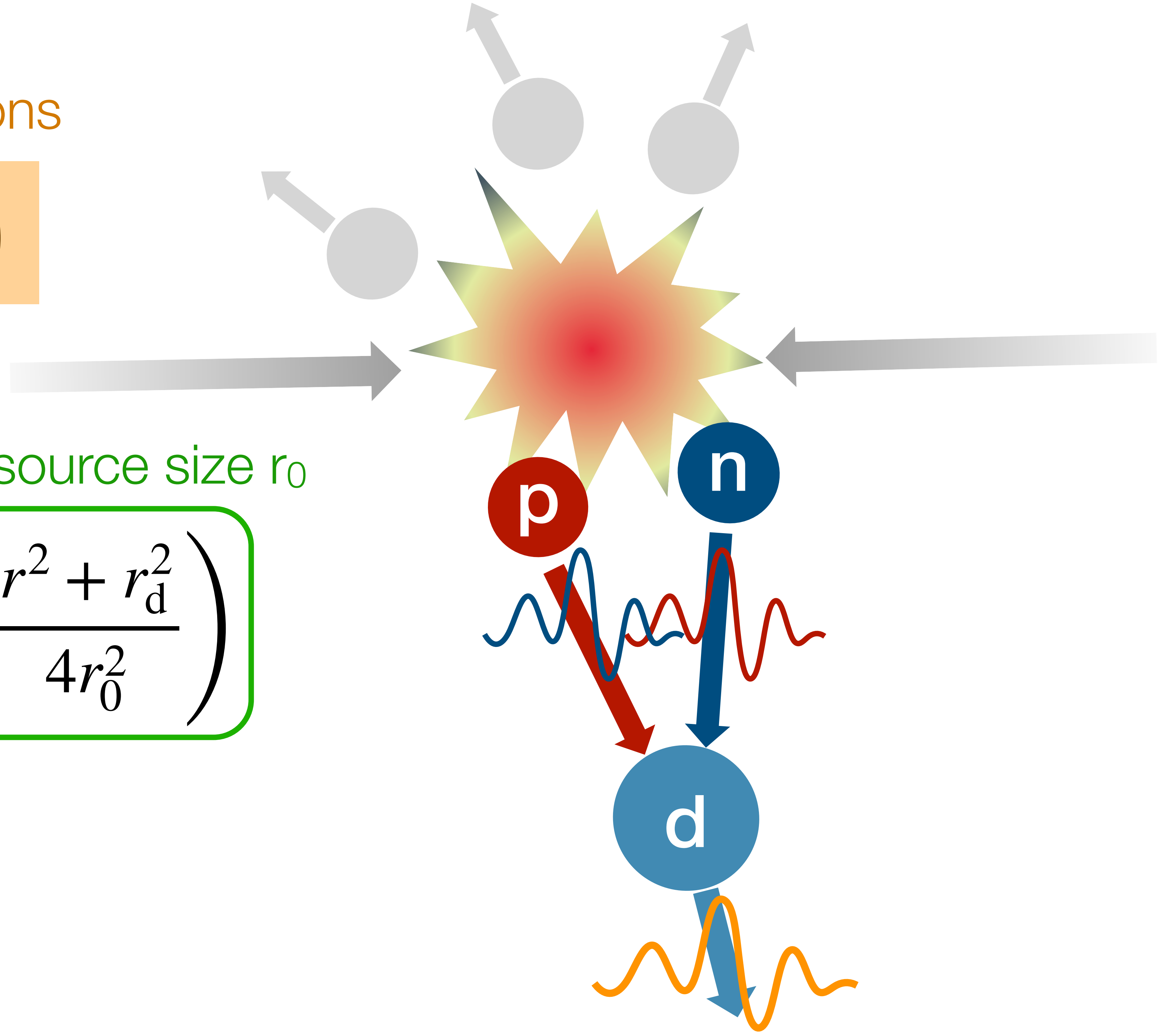
$$\frac{d^3N}{dP^3} = \frac{S_\mathrm{d}}{(2\pi)^6}\int d^3k \int d^3r_\mathrm{n} \int d^3r_\mathrm{p}\, \mathscr{D}(\vec{k},\vec{r})\, H_{np}\left(\vec{r}_n,\vec{r}_\mathrm{p}\right) G_{np}\left(\vec{P}/2+\vec{k}, \vec{P}/2-\vec{k}\right)$$

Deuteron Wigner density

$$\boxed{\mathscr{D}(\vec{k},\vec{r}) = \int \mathrm{d}^3\xi e^{i\vec{k}\cdot\vec{\xi}} \varphi_\mathrm{d}\left(\vec{r}+\frac{\vec{\xi}}{2}\right)\varphi_\mathrm{d}^*\left(\vec{r}-\frac{\vec{\xi}}{2}\right)}$$

Emission Gaussian Ansatz, with source size r_0

$$\boxed{H_{np}\left(\vec{r}_n,\vec{r}_\mathrm{p};r_0\right) = \frac{1}{(2\pi r_0)^3}\exp\left(-\frac{r^2+r_\mathrm{d}^2}{4r_0^2}\right)}$$



- Deuteron formation probability

$$\mathscr{P}\left(r_0,k\right) = \int d^3r_d \int d^3r\, H_{np}\left(\vec{r},\vec{r}_d;r_0\right)\, \mathscr{D}(\vec{k},\vec{r})$$

[1] Kachelriess et al. EPJA 57 (5) 167, 2021

Deuteron spectra

- Deuteron production spectra in event generators

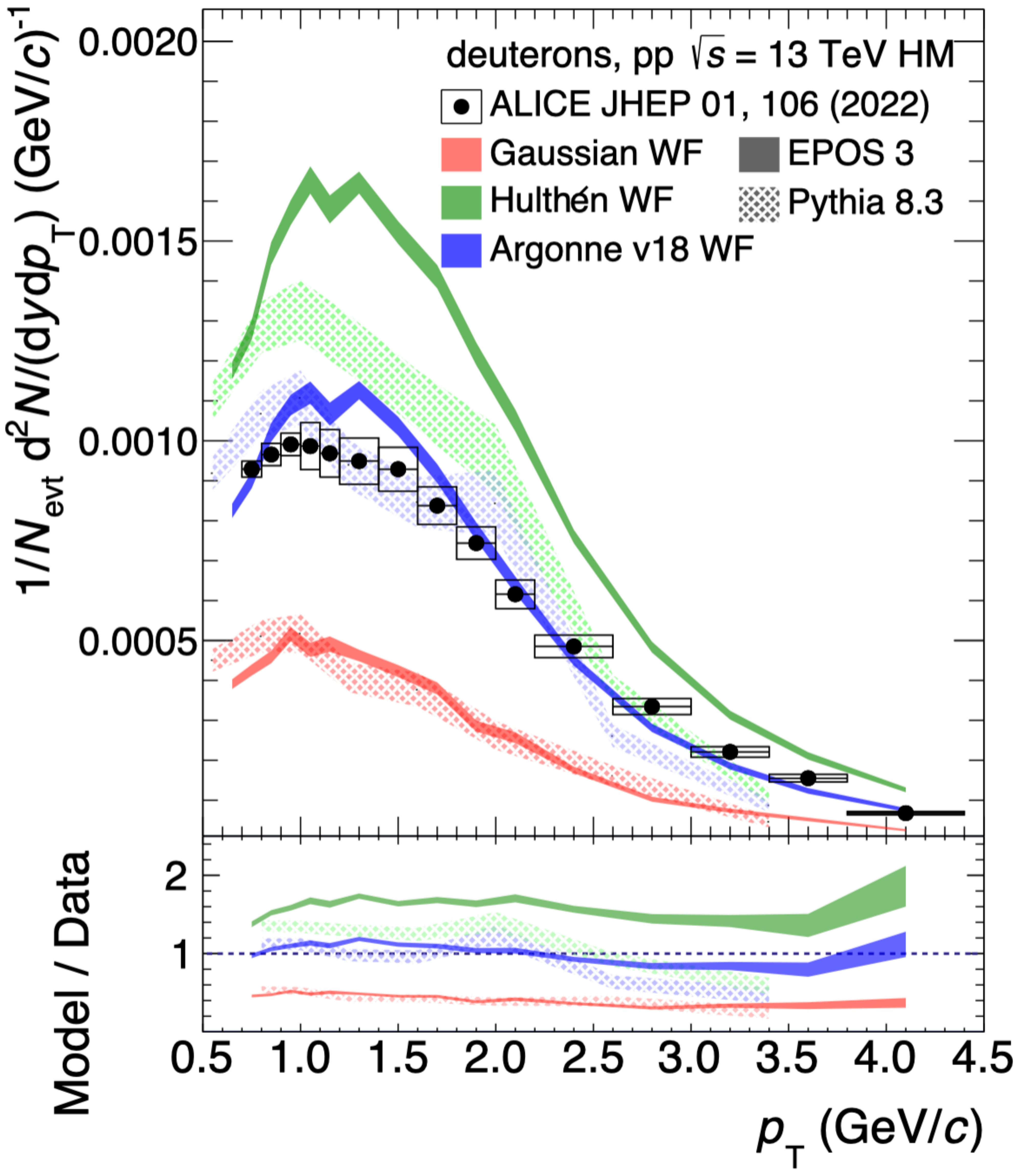
✓ **Event generators**

$$\frac{d^3N_d}{dP_d^3} = \frac{S_d}{(2\pi)^6}\int d^3k \mathscr{P}(r_0,k)\, G_{np}\left(\vec{P}_d/2+\vec{k}, \vec{P}_d/2-\vec{k}\right)$$

- Two event generators, PYTHIA 8.3 and EPOS
  - Tuned to reproduce proton spectra and p-p source size measurements with ALICE in pp collisions
  - **Parameter-free prediction of deuteron yield!**

Calculations with Argonne v_{18} and Chiral EFT (not shown for visibility) shows the best agreement with measurements!

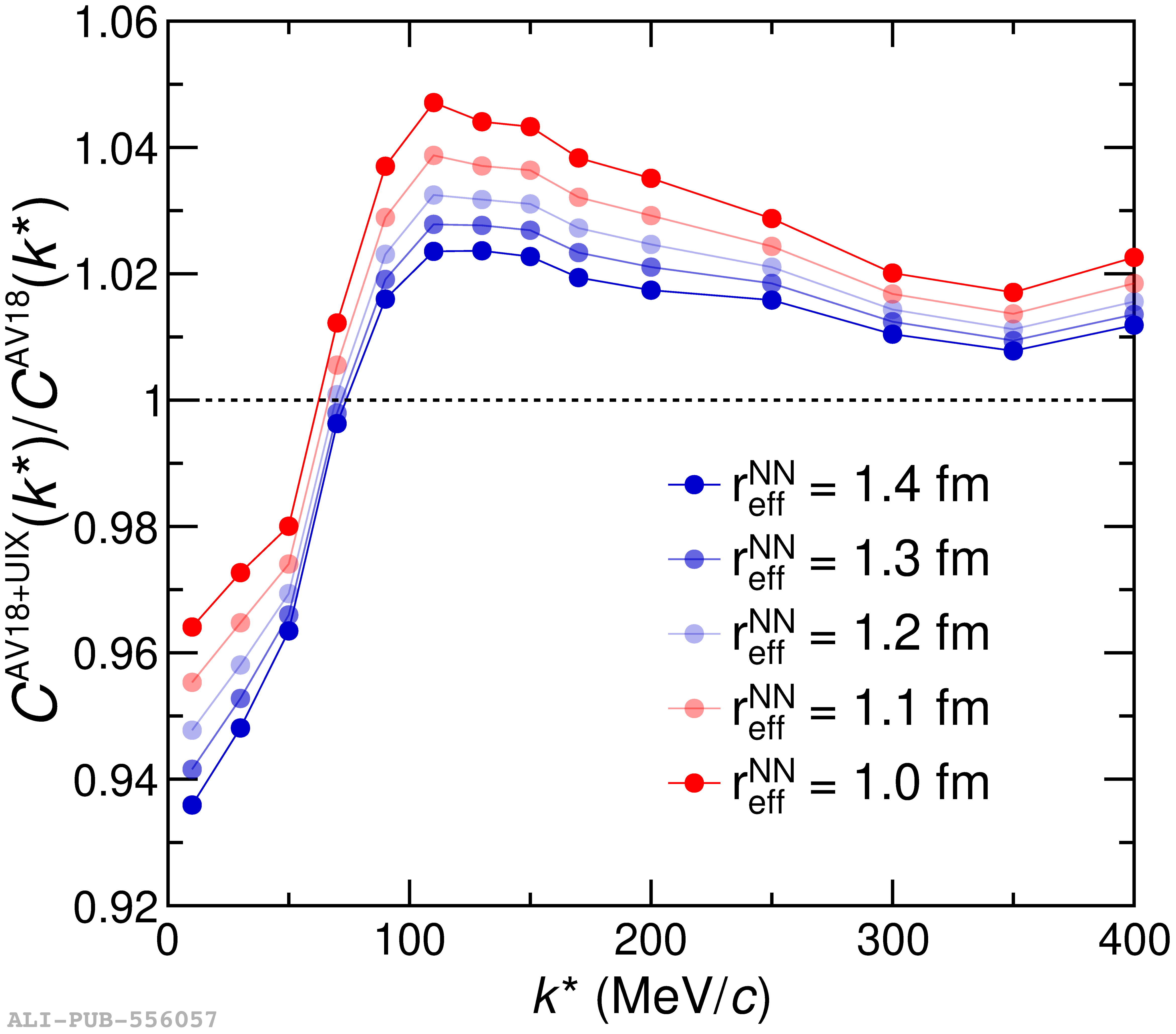
M. Mahlein, B. Singh et al. Eur. Phys. J. C 83, 804 (2023)



Sensitivity to genuine three-body force

- **Computed correlation function with and without genuine three-body force**
  - Up to 5% effect of genuine three-body interaction
  - Run 2: limited statistics does not allow for resolution to see the effect of three-body force
- **LHC Run 3:** ~2 orders of magnitude increase in pair statistics
  - Possibility to perform m_T differential analysis

ALICE Coll. arXiv:2308.16120 [submitted to **PRX**]



Avenue for the study of hadron–deuteron systems, including charm and strange hadrons!

Coalescence model

- Observable: coalescence probability **B$_A$** for A nucleons to coalesce and form a nuclei

$$\mathrm{B}_A\left(p_\mathrm{T}^\mathrm{p}\right) = \underbrace{\frac{1}{2\pi p_\mathrm{T}^\mathrm{A}} \frac{\mathrm{d}^2 N_\mathrm{A}}{\mathrm{d}y\ \mathrm{d}p_\mathrm{T}^\mathrm{A}}}_{\text{Nuclei yield}} \Bigg/ \underbrace{\left(\frac{1}{2\pi p_\mathrm{T}^\mathrm{p}} \frac{\mathrm{d}^2 N_\mathrm{p}}{\mathrm{d}y\ \mathrm{d}p_\mathrm{T}^\mathrm{p}}\right)^A}_{\text{Nucleon yield}}$$

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- Theoretical description of B_2 (A = 2 for deuteron)[1]

$$\boxed{\mathrm{B}_2(R) = \frac{3}{2m}\int d^3k\, e^{-R^2k^2}\,\mathrm{D}(\vec{k})}$$

[1] Blum et al, PRC 99 (2019) 044913

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Emission source size for pair of nucleons

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Emission source size for pair of nucleons

Deuteron Wigner density $D(k) = \int d^3r\, e^{-i\vec{k}\cdot\vec{r}}\left|\phi_\mathrm{d}(\vec{r})\right|^2$

Deuteron wavefunction (nucleons interact)

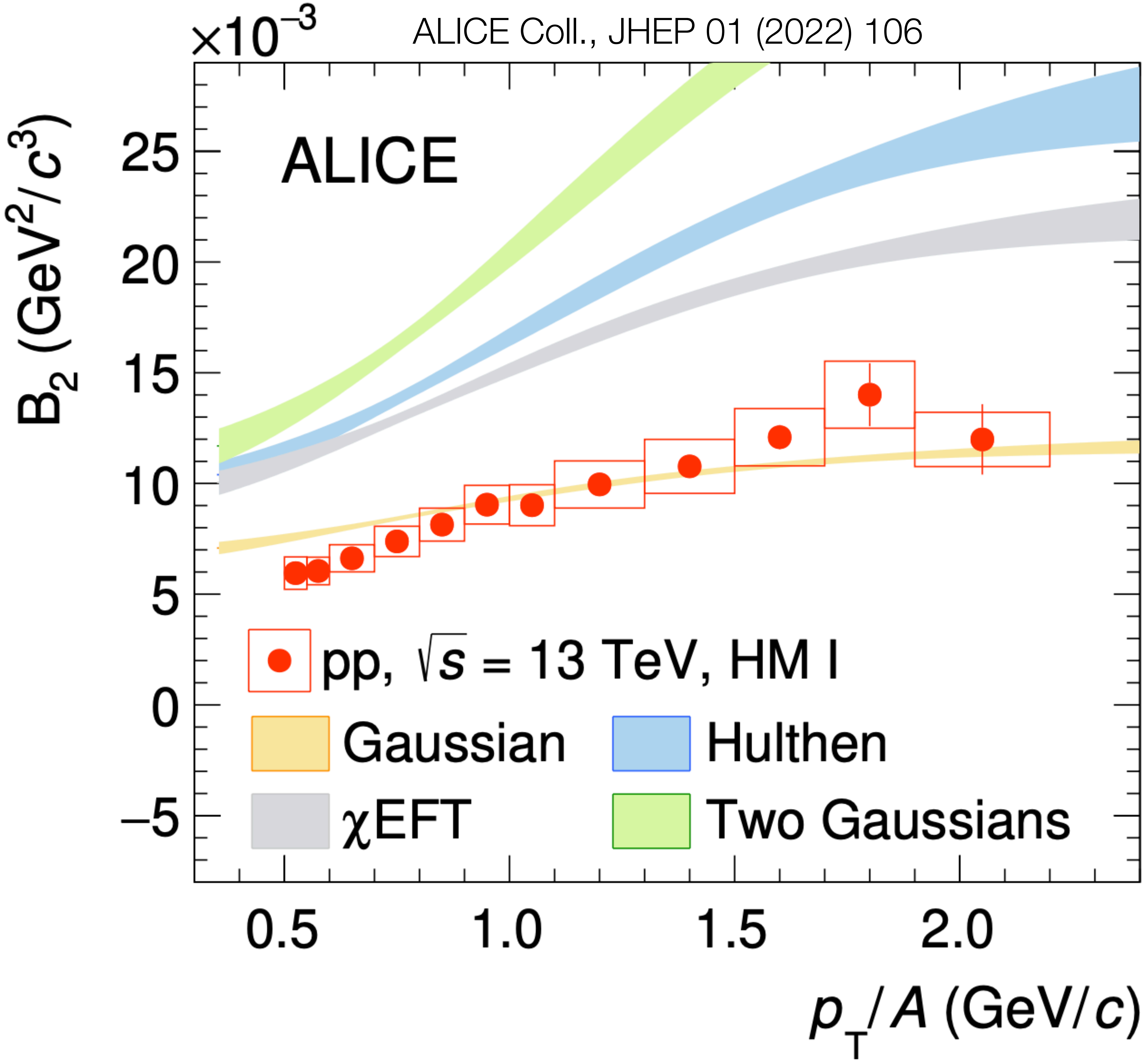
[1] Blum et al, PRC 99 (2019) 044913

B_2 vs p_T/A in pp collisions

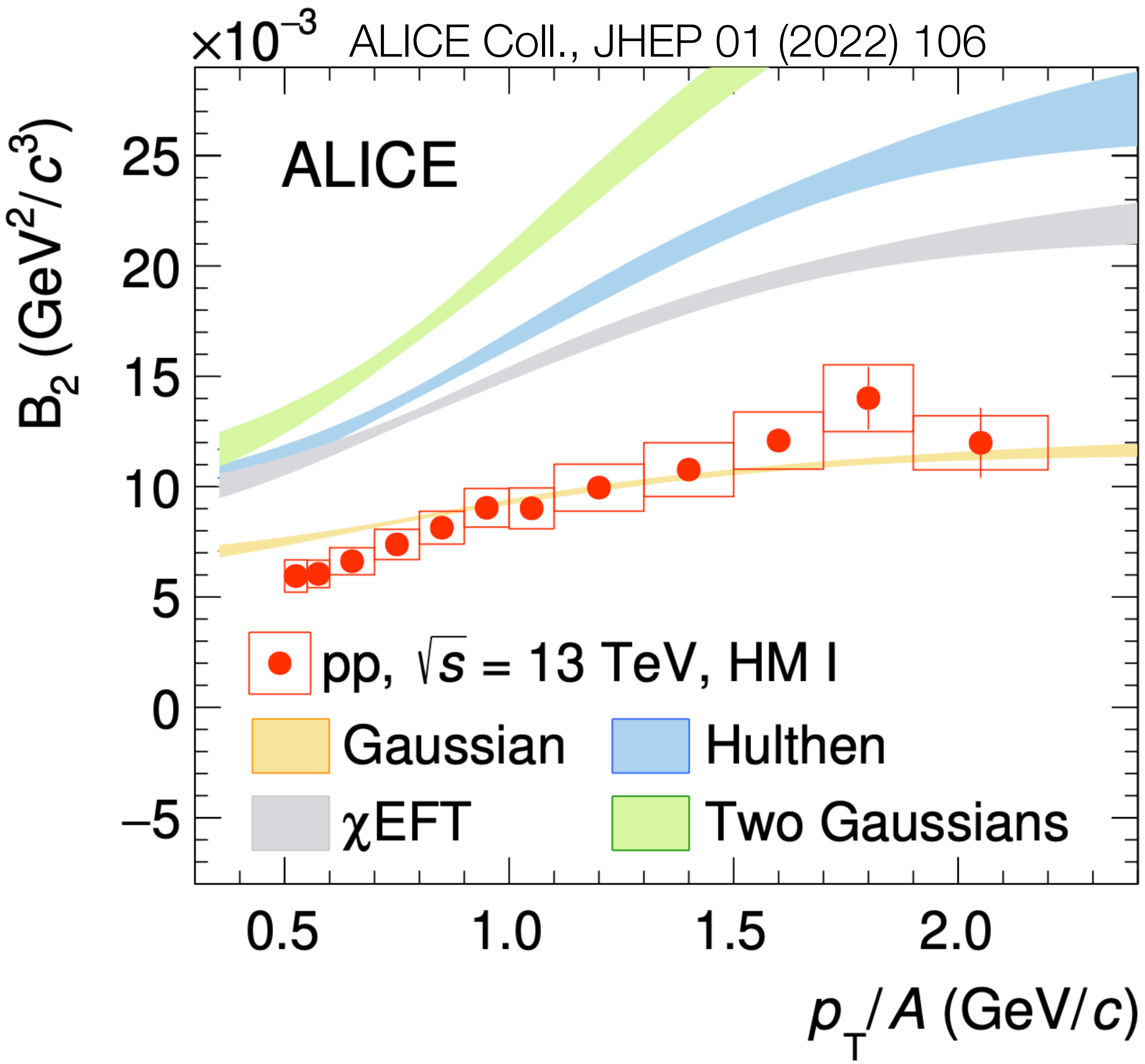
- First p_T-differential study of B_2 using p-p source size in pp collisions

- B_2 calculations using **χEFT** wavefunction differ by a factor of ~2

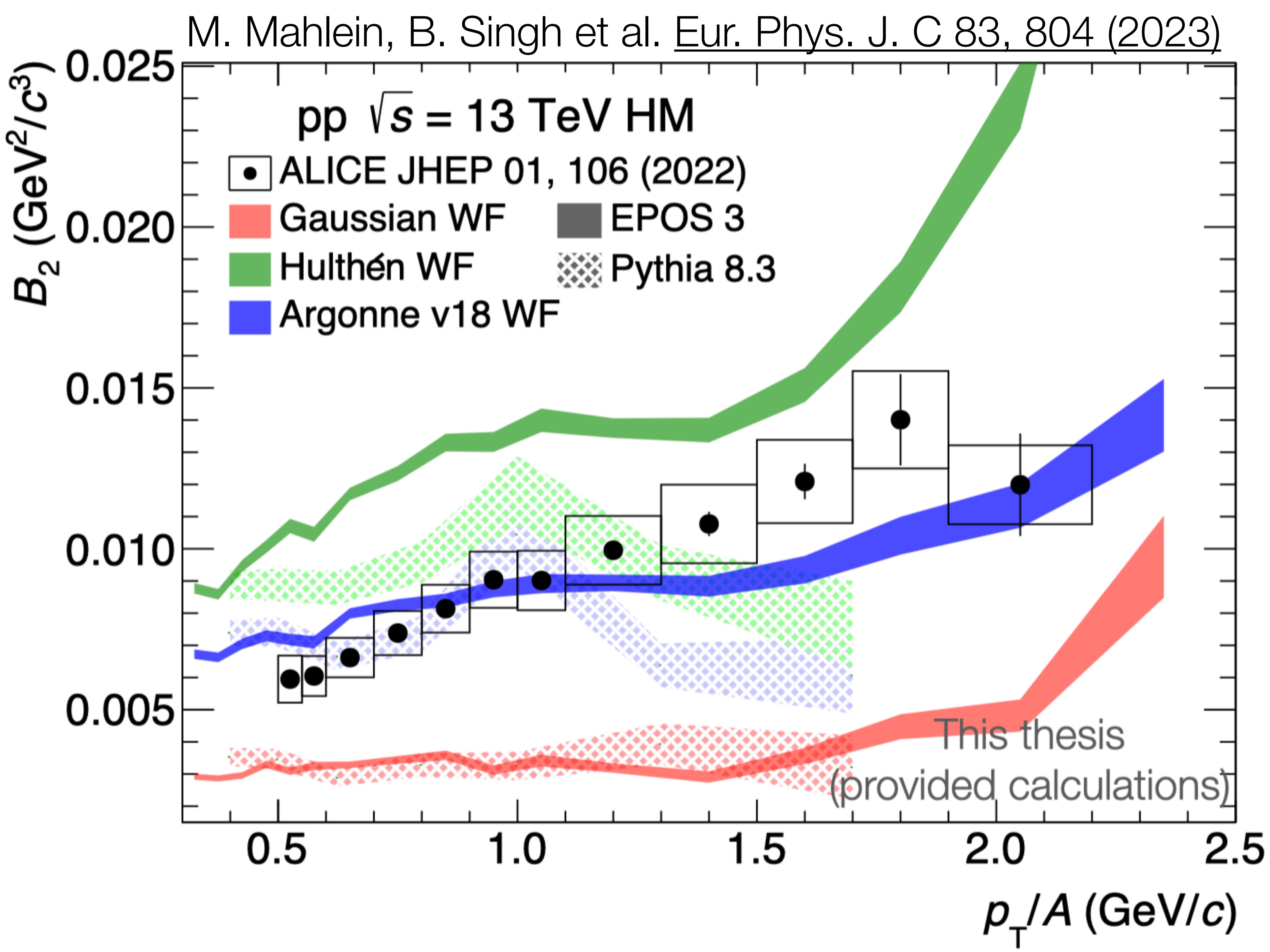
- Further improvements in the theoretical framework for B_2 within coalescence model are required



Coalescence parameters B_2



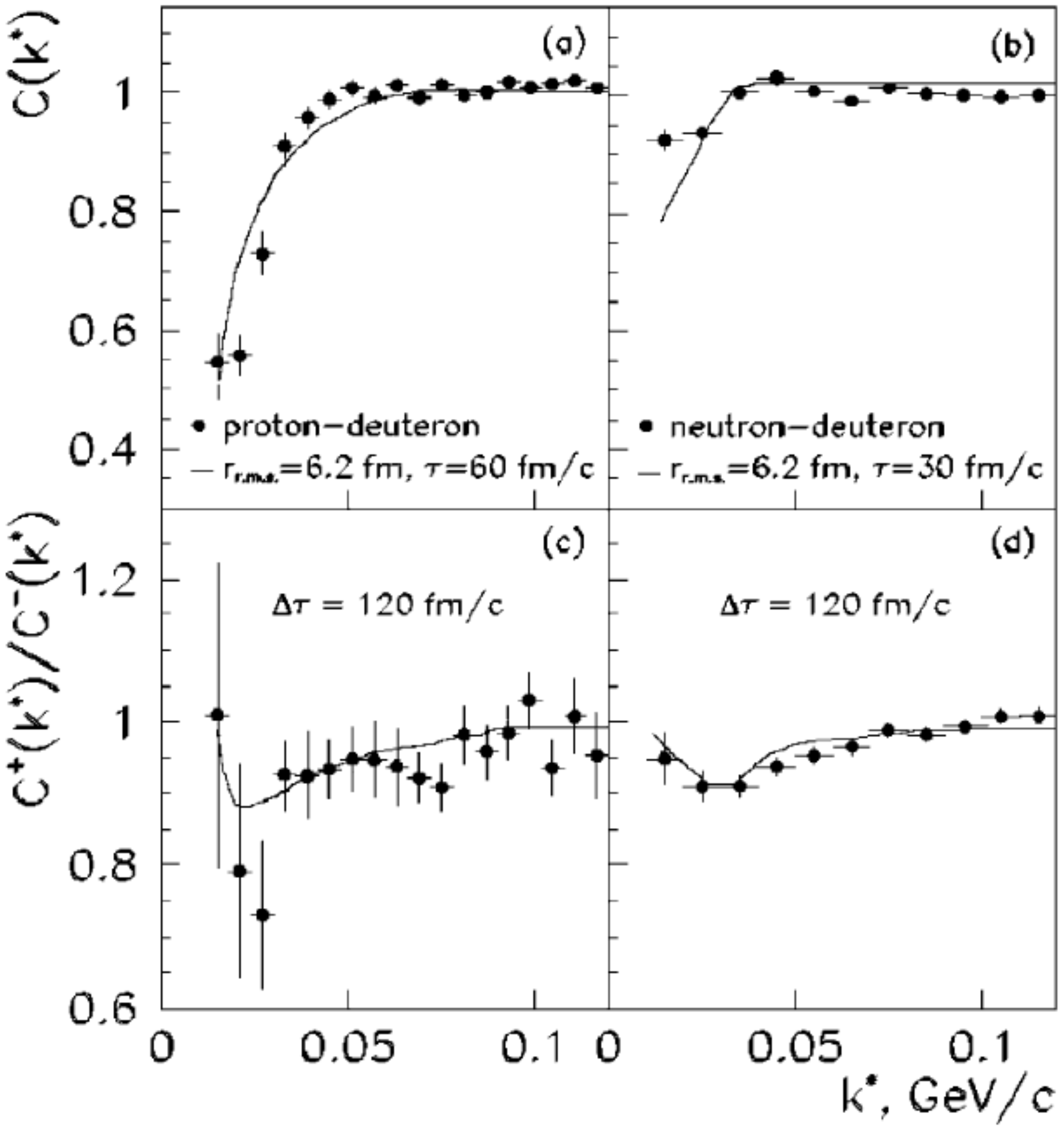
VS



- The use of an event generator in the Wigner approach preserves correlation in $\vec{k}$ and $\vec{r}$
  - B_2 calculated using AV18 (Chiral EFT) WF agrees with the measurement!

p-d correlation in the past

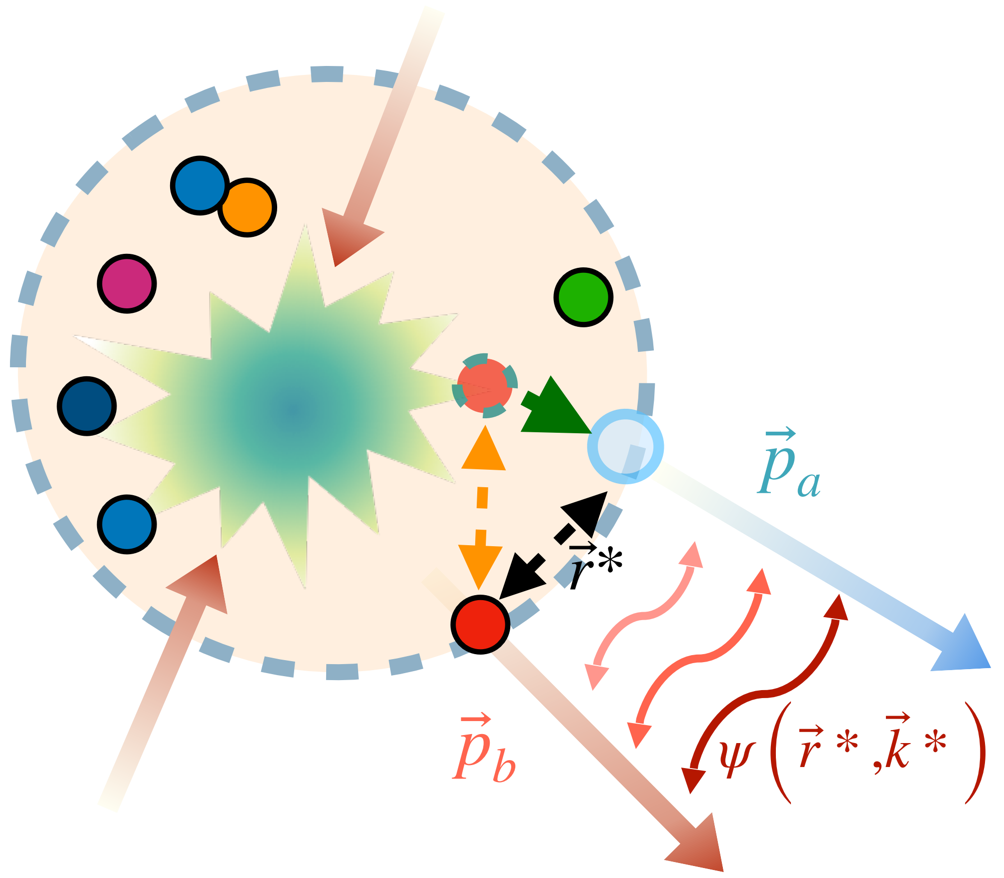
- Interpreted using the LL approach
- Measurement performed at AGS



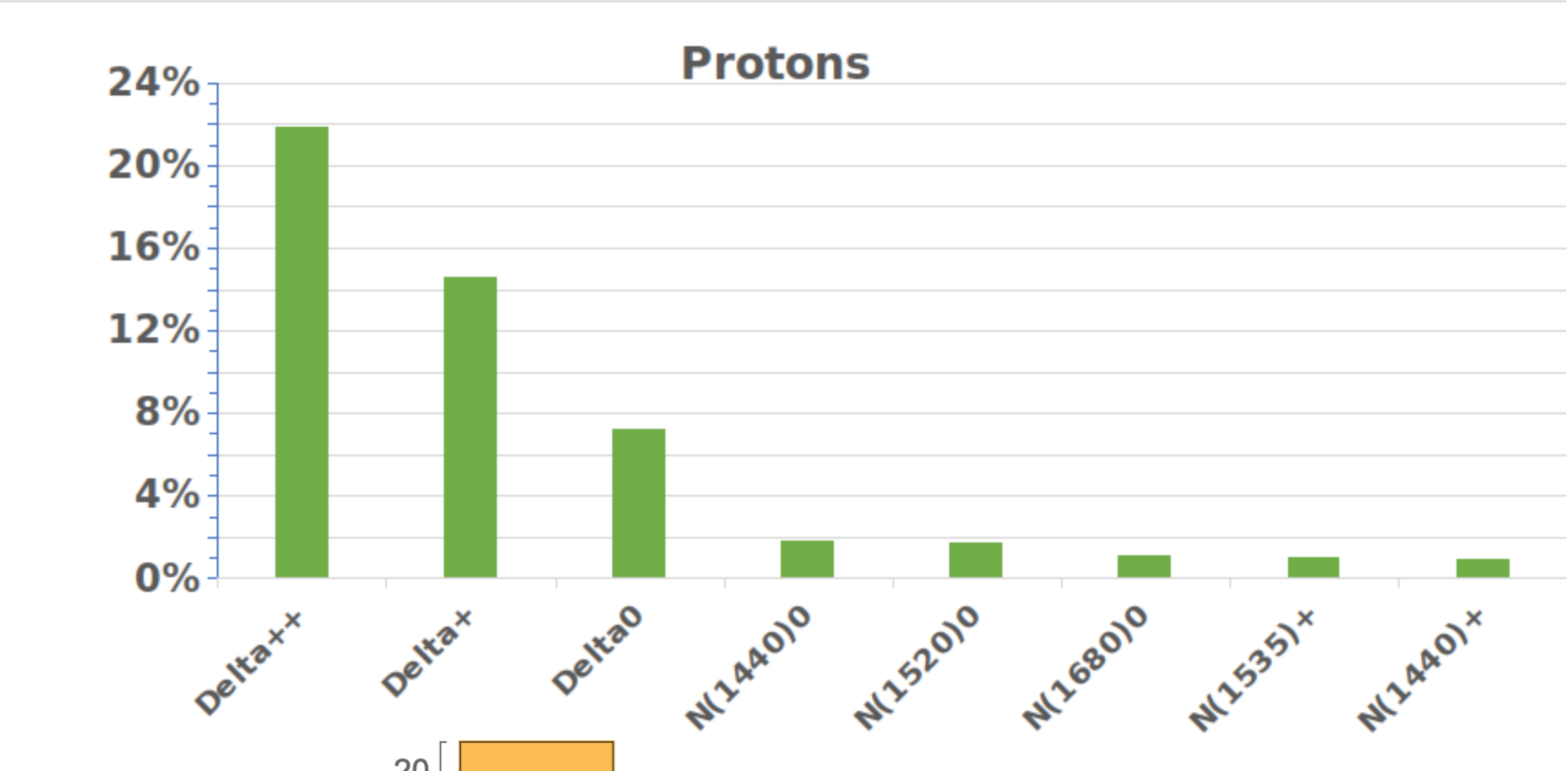
[1] Wosińska, K., Pluta, J., Hanappe, F. *et al. Eur. Phys. J. A* 32, 55–59 (2007)

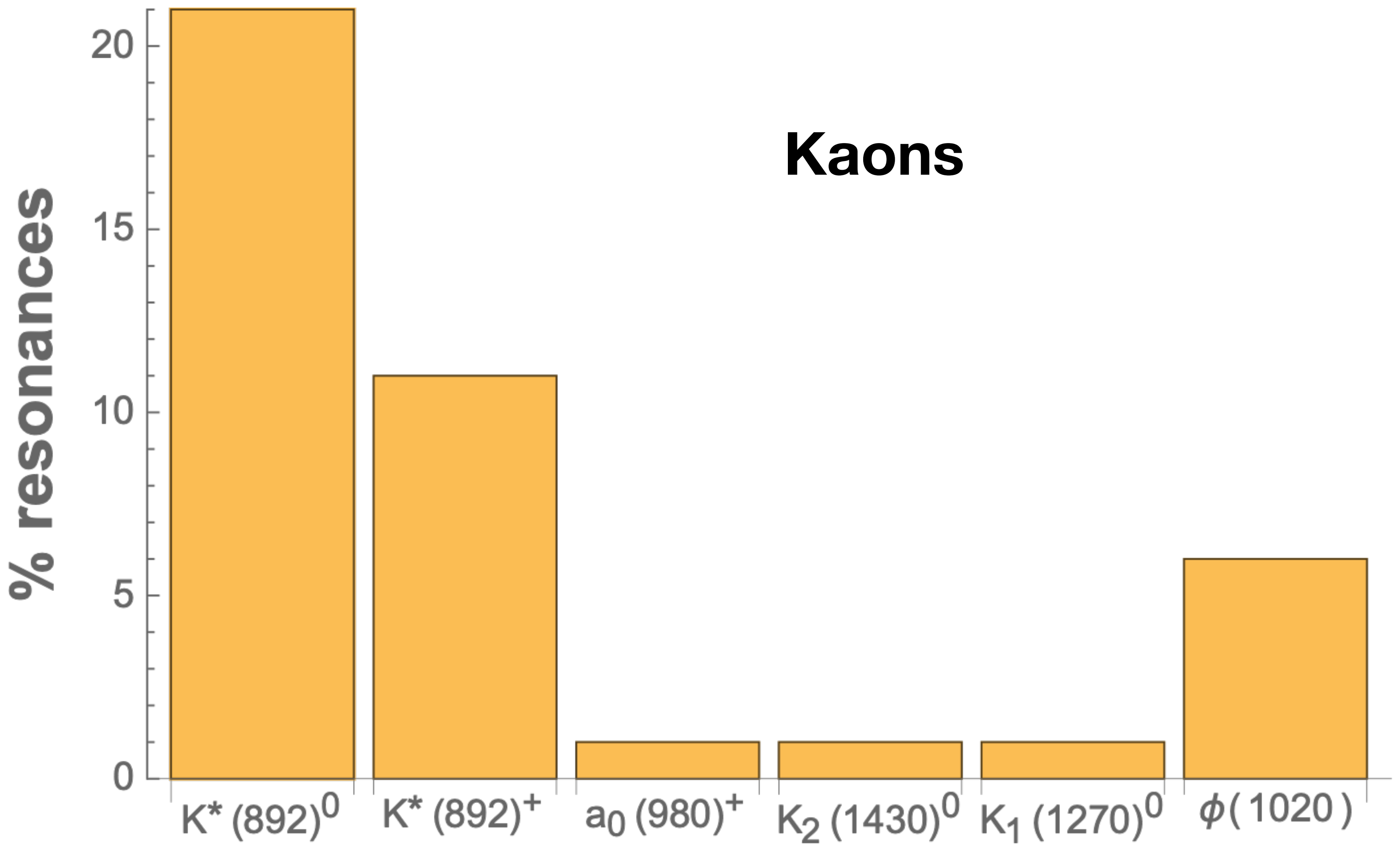
Source size for p–d and K⁺–d pairs

- The source radius is effectively increased by **short-lived strongly decaying resonances (cτ ≈ r_{core})** e.g. Δ-resonances in case of protons



Source size	mean value:p–d	mean value:K⁺–d
r_{core}	0.99±0.05 fm	1.04±0.04 fm
r_{eff}	1.08±0.06 fm	$1.35^{+0.04}_{-0.05}$ fm





(1)

Hadron-deuteron pairs are created at very small distances in pp collisions at the LHC!

(1) ϕ(1020) corrected as feed-down

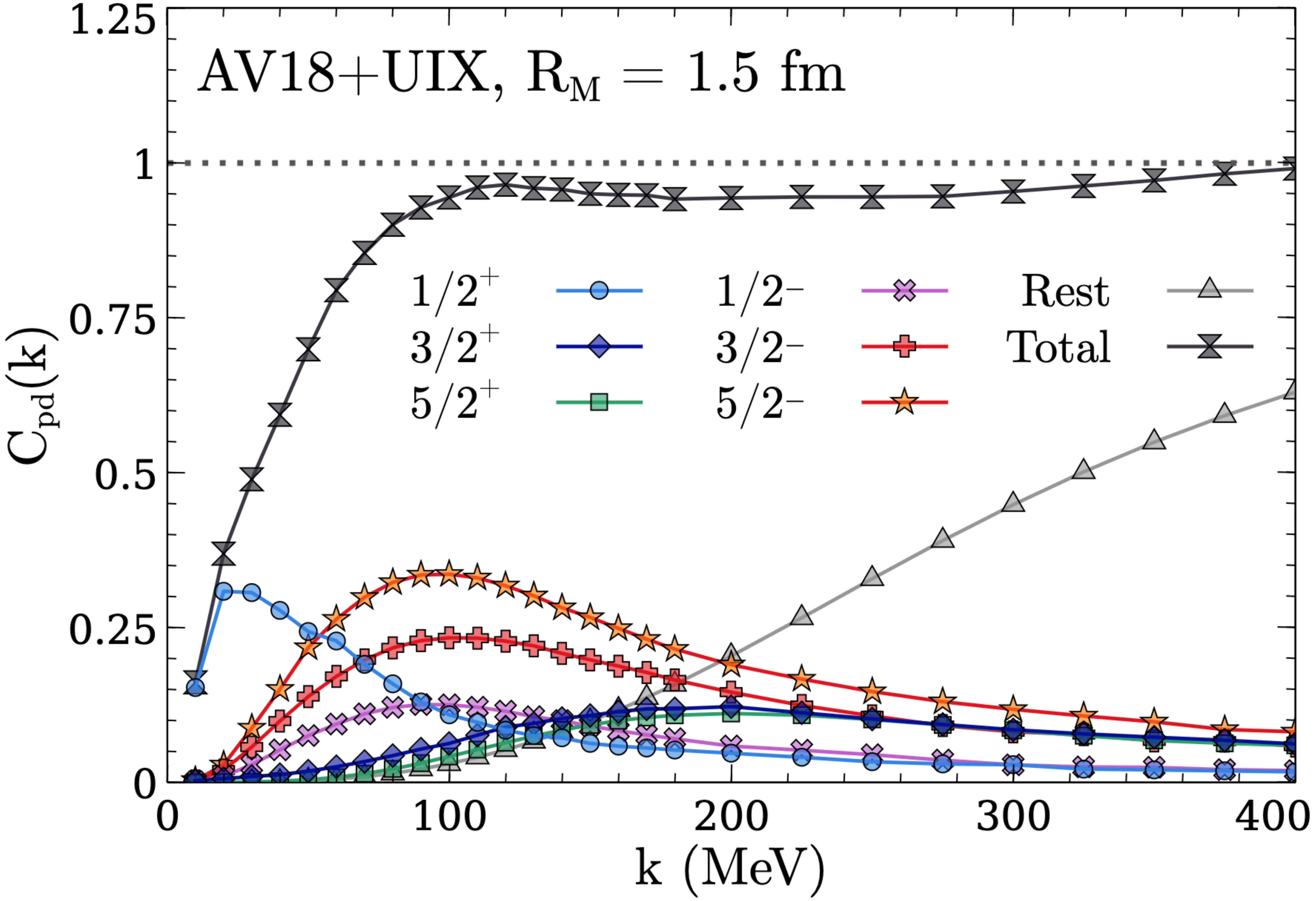
Total wavefunction for p-d system

$$
\begin{aligned}
\Psi_{LSJJ_z} &= \sum_{n,\alpha} \frac{u_{n,\alpha}(\rho)}{\rho^{5/2}} \mathcal{Y}_{n,\alpha}(\Omega) \\
&+ \frac{1}{\sqrt{3}} \sum_{\ell}^{\text{even perm.}} \left\{ Y_L(\hat{\boldsymbol{y}}_\ell) \left[\varphi^d(i,j)\chi(\ell) \right]_S \right\}_{JJ_z} \frac{F_L(\eta, k y_\ell)}{k y_\ell} \\
&+ \sum_{L'S'} T^J_{LS,L'S'} \frac{1}{\sqrt{3}} \sum_{\ell}^{\text{even perm.}} \left\{ Y_{L'}(\hat{\boldsymbol{y}}_\ell) \left[\varphi^d(i,j)\chi(\ell) \right]_{S'} \right\}_{JJ_z} \\
&\times \frac{\overline{G}_{L'}(\eta, k y_\ell) + i F_{L'}(\eta, k y_\ell)}{k y_\ell} \; .
\end{aligned}
$$

Partial wave decomposition of p-d

- Precise calcualtion using AV18+UIX as well NVIa3/3N chiral potentials

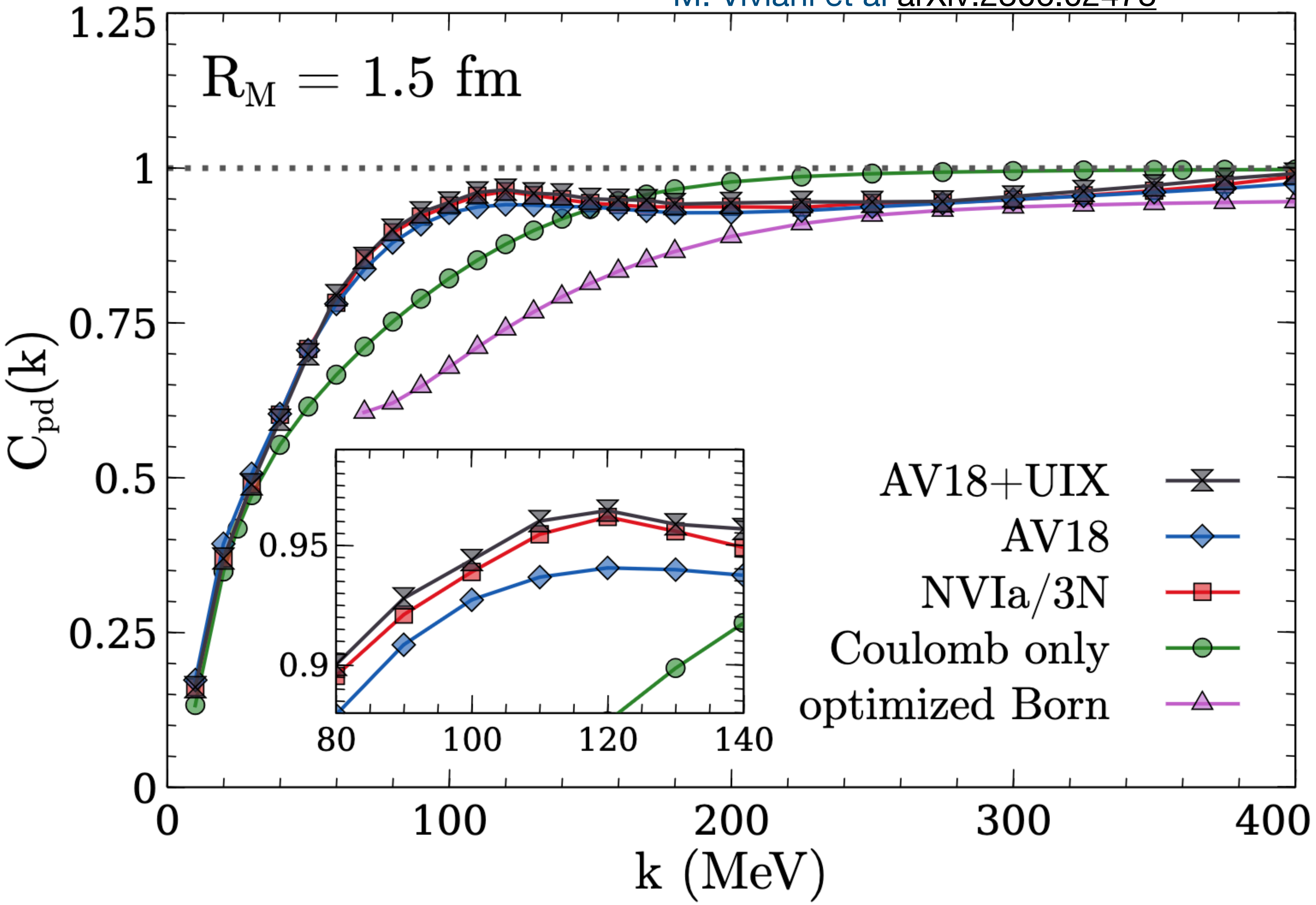
M. Viviani et al arXiv:2306.02478



AV18+UIX vs NVIa3 3N Chiral potentials

- Precise calcualtion using AV18+UIX as well NVIa3/3N chiral potentials

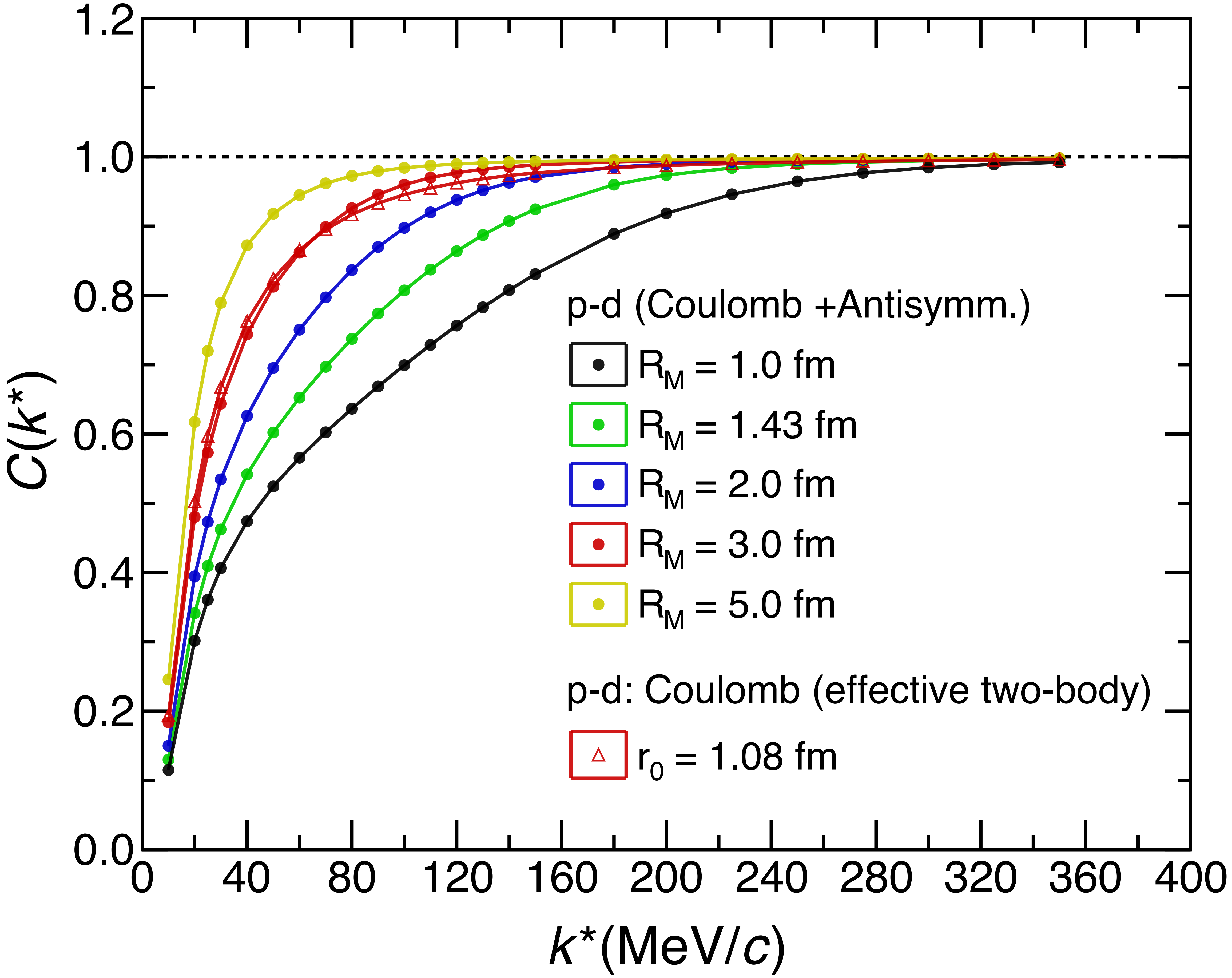
M. Viviani et al arXiv:2306.02478



Coulomb only

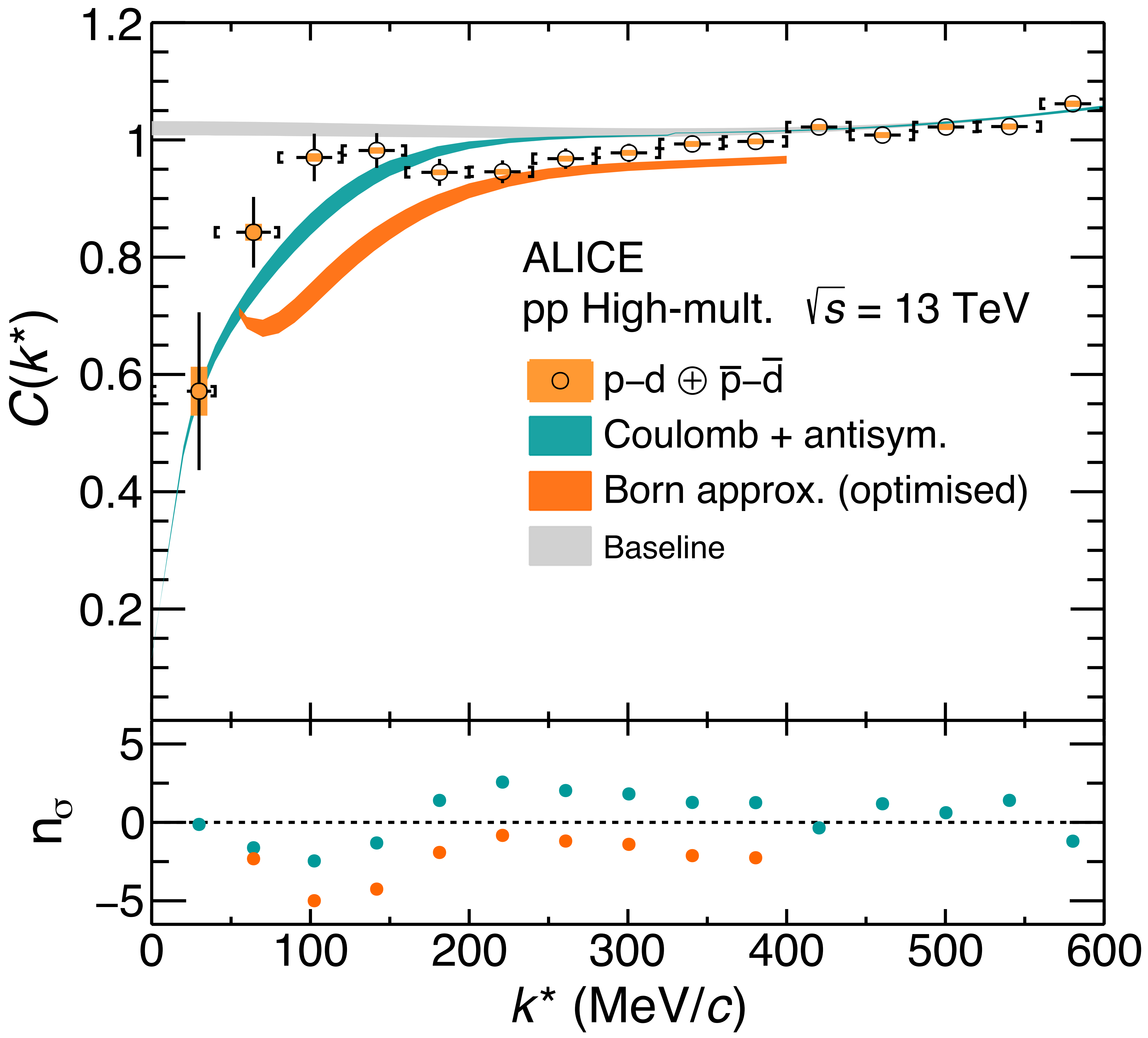
- Complete p–pn dynamics, but the strong interaction is **absent** at very short-range!
  - $\mathbf{r^{NN}_{eff}} = \mathbf{1.43 \pm 0.16}$ fm (nucleon-nucleon distance)
- In the case of the two-body picture Coulomb-only interaction differs from the one using the p-(pn) dynamics
  - $\mathbf{r^{pd}_{eff}} = \mathbf{1.08 \pm 0.06}$ fm (proton-deuteron distance)
  - More repulsion due to the Pauli-blocking

Sensitivity to the dynamics of the three-body p–(pn) system even for Coulomb case



Born approximation effective two-body

- Complete p–pn dynamics, but the strong interaction is **absent** at very short-range!
  - **r^{NN}_{eff} =1.43±0.16** fm (nucleon-nucleon distance)
  - Coulomb-only interaction coincidently appears in the data (despite the large scattering lengths)
  - Coulomb+strong interaction using **Born approximation (neglecting short-range strong interaction)** and proper p–pn dynamics



Sensitivity to the dynamics of the three-body p–(pn) system at short distance

Chiral potentials calculation: NVIa+3N

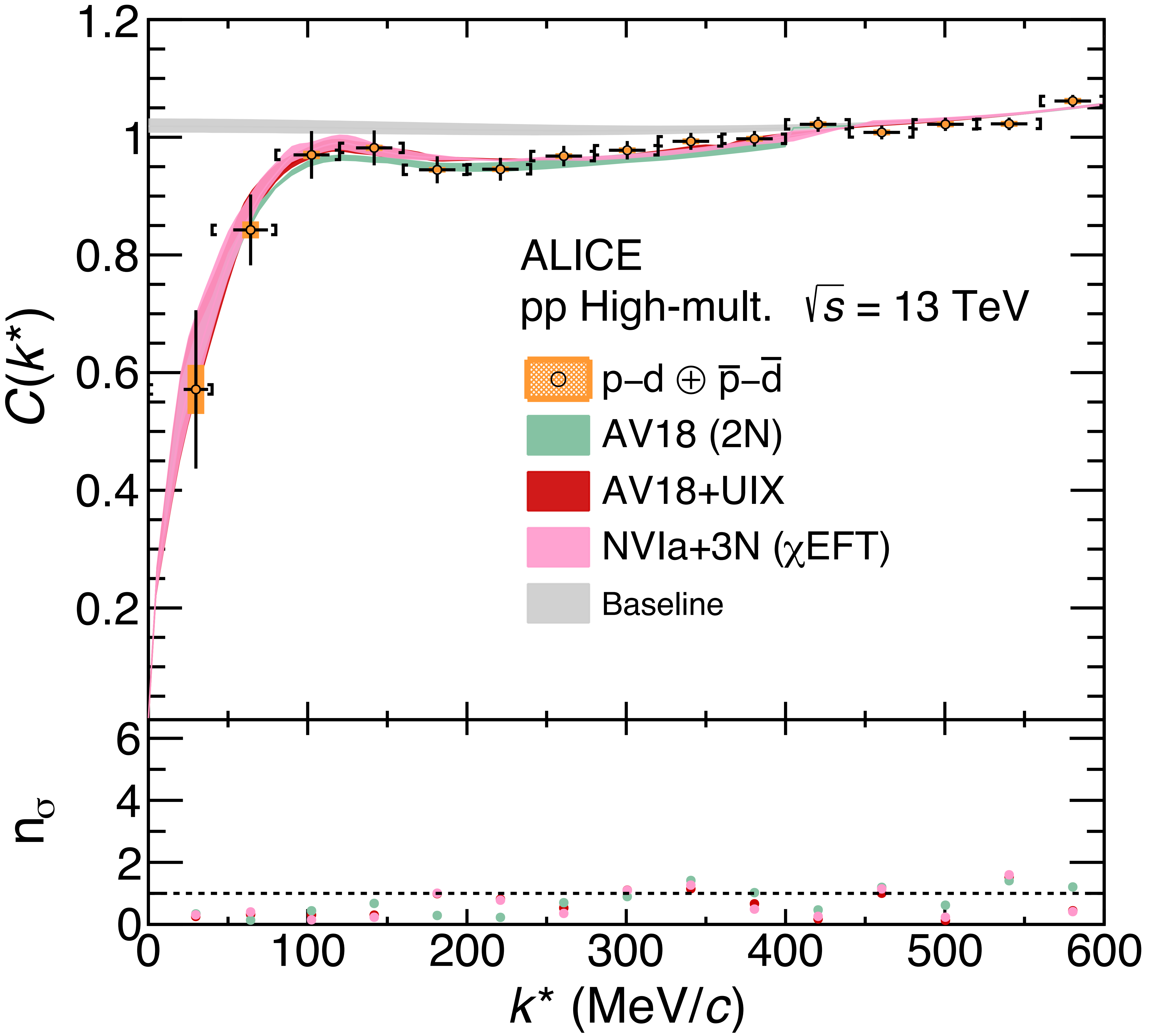
- Comparisition with Chiral potentials (**Full three-body dynamics)**[1]

- Argonne v18+Urbana IX interaction[2,3]
  - **All partial waves upto d-waves:** describes data within $n_\sigma \sim 1$ for k^* up to 400 MeV/c

- Calculations using chiral potential from NVIa+3N
  - Very good agreement with AV18+UIX

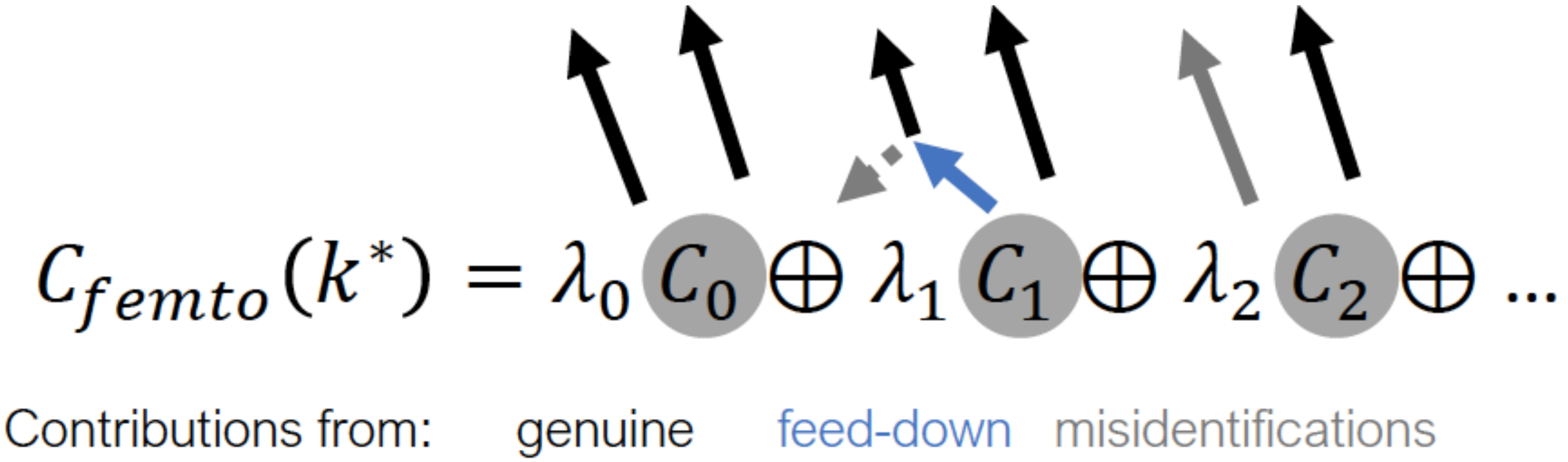
- AV18 alone: just two-body NN interaction
  - Current data cannot resolve the effect of three-body force

Both AV18+UIX and NVIa+3N calculations provide an excellent agreement with the measurement



Femtoscopic correlation

- The femtoscopic correlation may have background/contributions from
  - Particles from weak decays
  - Particles from material knock-outs
  - Misidentifications

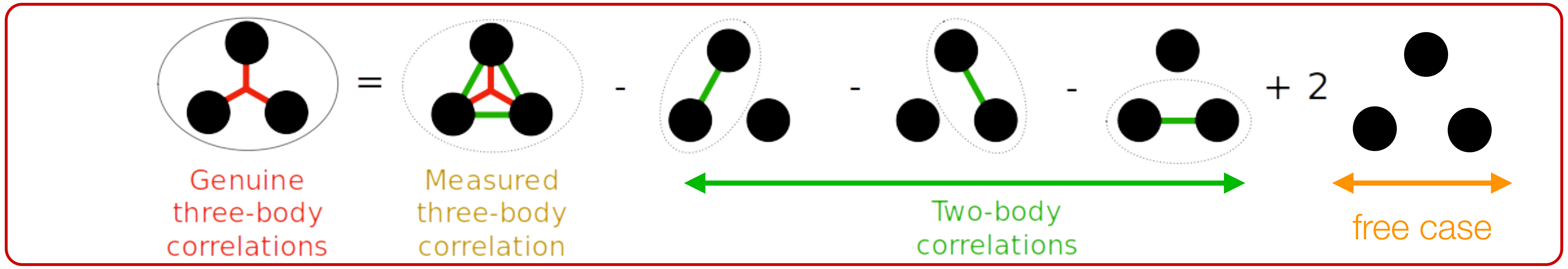


$$C_{femto}(k^*) = \lambda_0 C_0 \oplus \lambda_1 C_1 \oplus \lambda_2 C_2 \oplus \ldots$$

- Quantification of the contributions to the pairs done by the lambda parameters $\lambda_{ij} = \mathscr{P}_i . f_i \times \mathscr{P}_j . f_j$
  - Purity of the individual particles ($\mathscr{P}_i$)
  - Feed-down fractions (f_i)

Cumulant: measure for three-body effects

$c_3\ (Q_3)$



Genuine three-body correlations

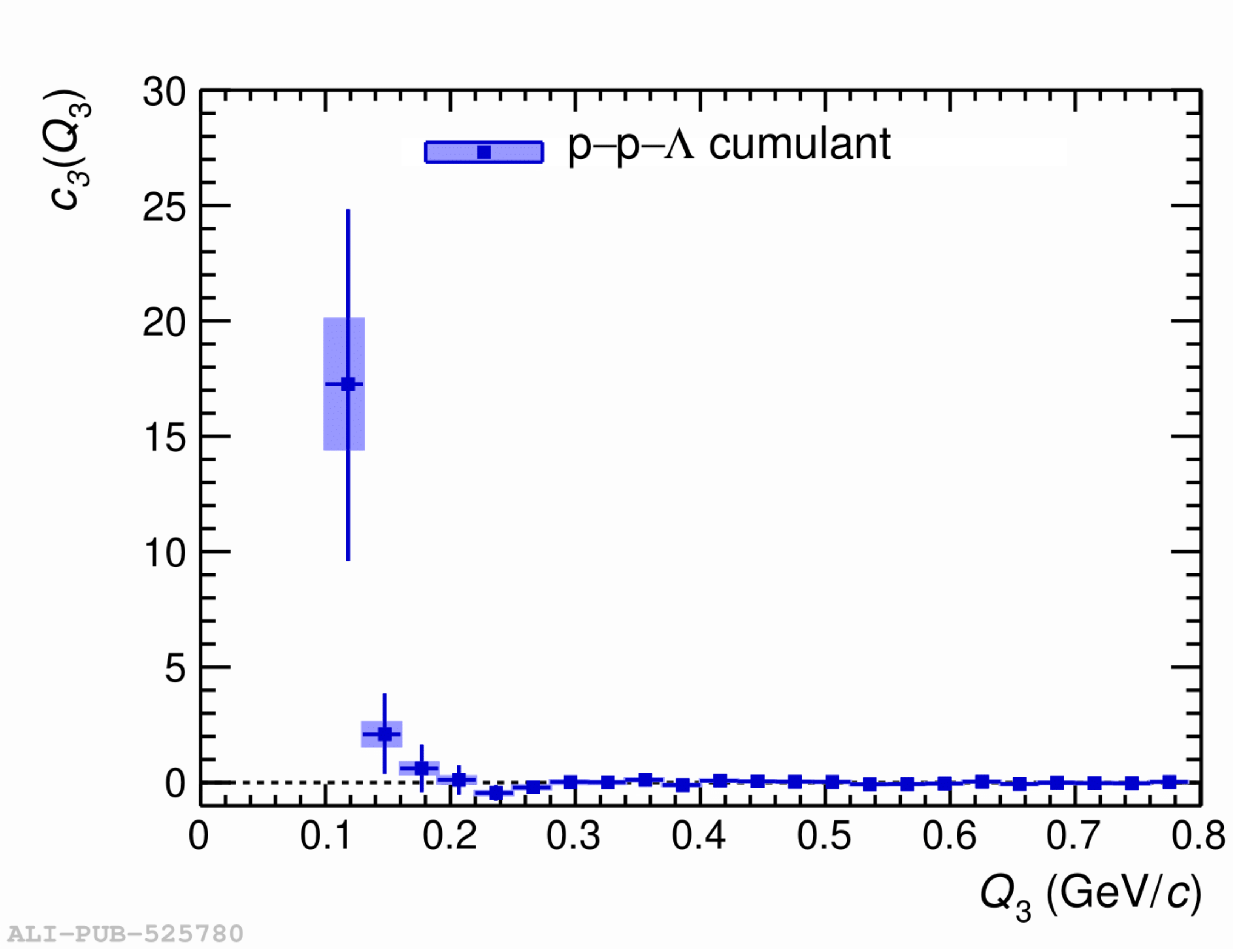
Measured three-body correlation

Two-body correlations

free case

Kubo, J. Phys. Soc. Jpn. 177 (1962)

$c_3\ (Q_3)$ allows to isolate effects associated with the genuine three-body interactions



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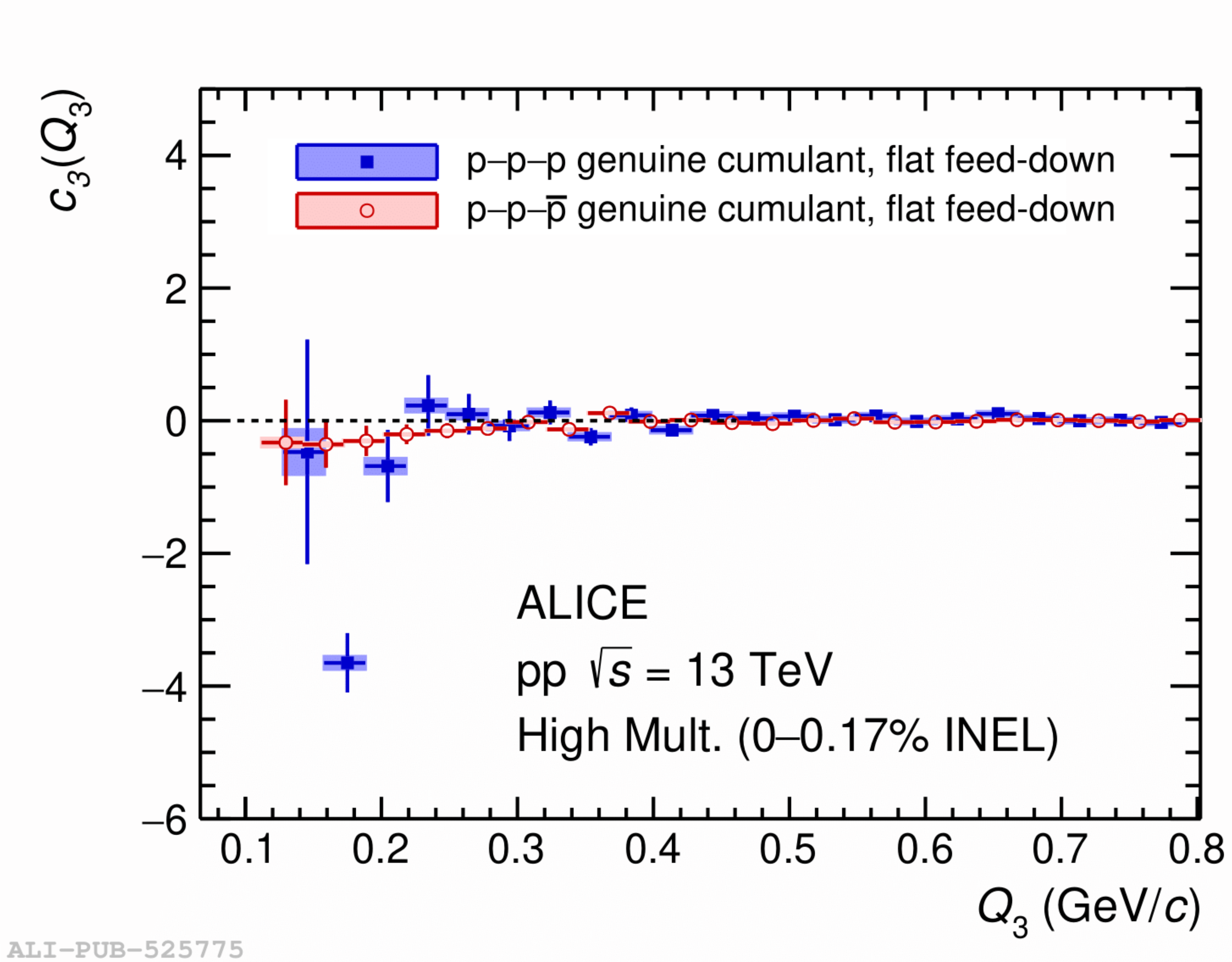


ALI-PUB-525775

Cumulants (Run 2)

p-p-p and p-p-p̄: nonzero

- Hint of a genuine three-body effect

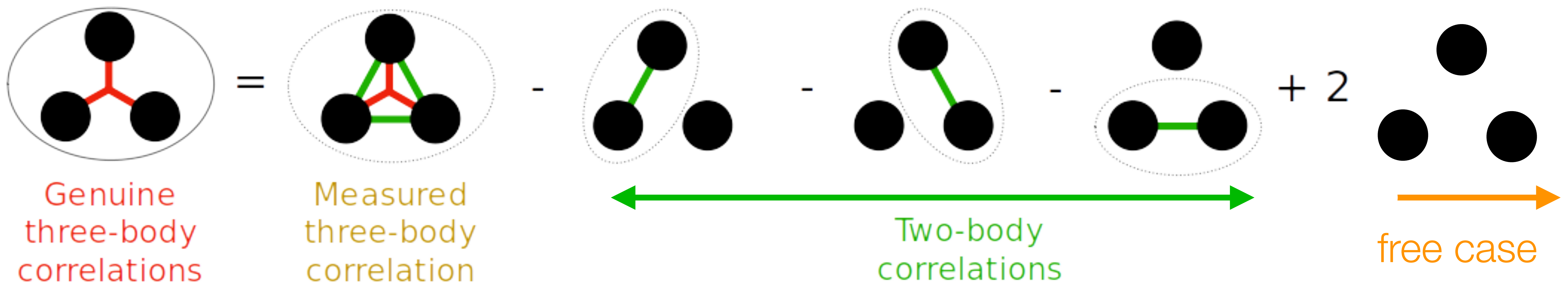
p-p-Λ: compatible with zero

- Strong rise but inclusive due to lack of statistics

Need for large statistics to precisely measure the three-body effects=> Run 3 of LHC

p-p-Λ cumulant

Kubo's cumulant approach[1]

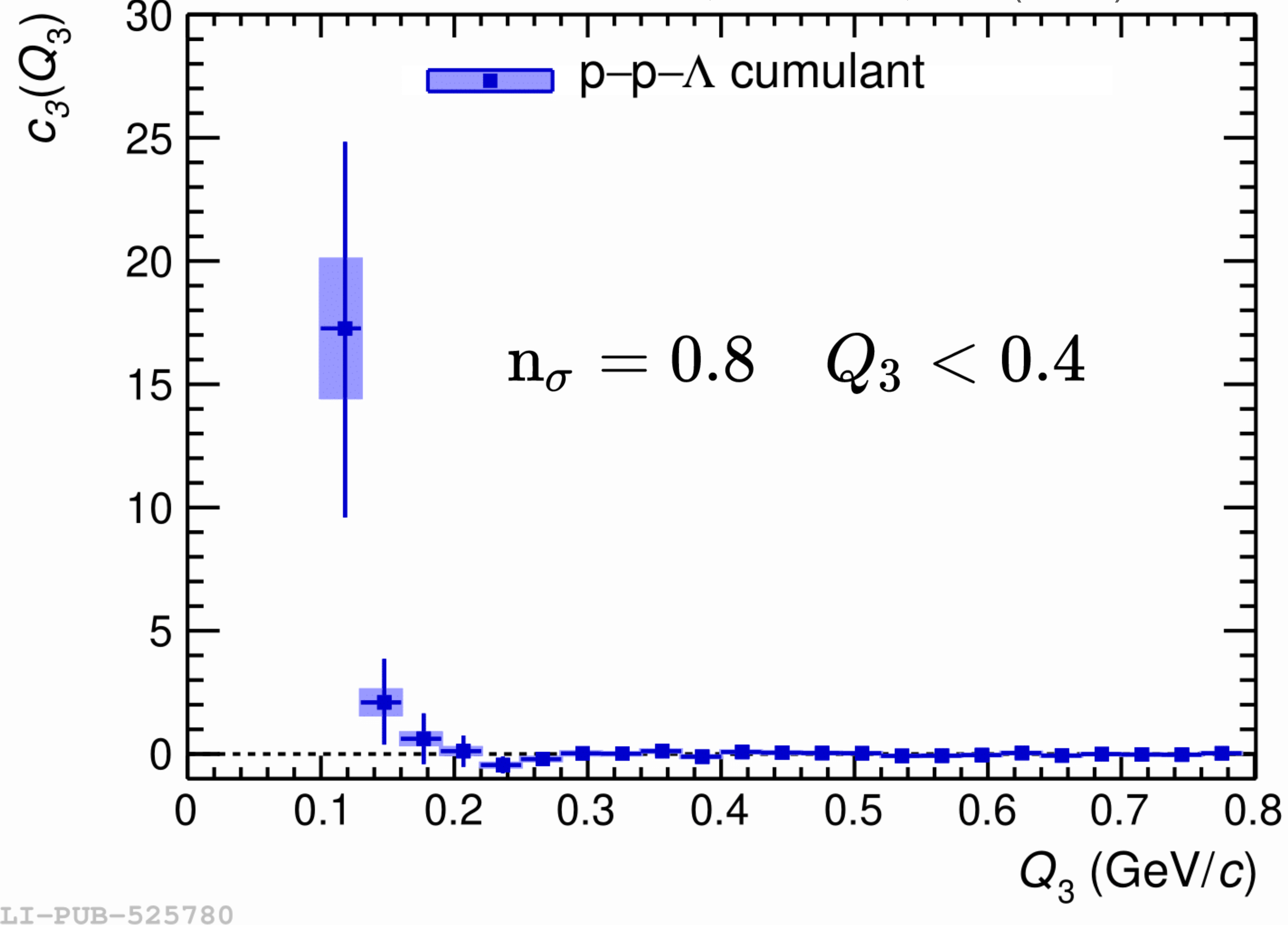


c_3 (Q_3) allows to isolate effects associated with the genuine three-body interactions

- Positive p-p-Λ cumulant
  - Two identical particle and charged particle
  - Expected dominant contribution from strong interaction
- Statistical significance:

nσ = 0.8 for Q_3 < 0.4 GeV/c

ALICE Coll., EPJ A 59, 145 (2023)



In Run 3, two orders of magnitude gain in statistics expected!

Lednicky Model

- For distinguishable particles
  - Starting from the scattering parameters ⇒ define the s-wave two-particle relative wave function
  - Considers Coulomb effects

- Coulomb-corrected wave function for final-state interactions in s wave:

$$\psi_{-k^*}(r^*) = e^{i\delta_c}\sqrt{A_c(\eta)}\left[e^{-ik^*r^*}F\left(-i\eta, 1, i\zeta\right) + f_c(k^*)\frac{\tilde{G}(\rho,\eta)}{r^*}\right]$$

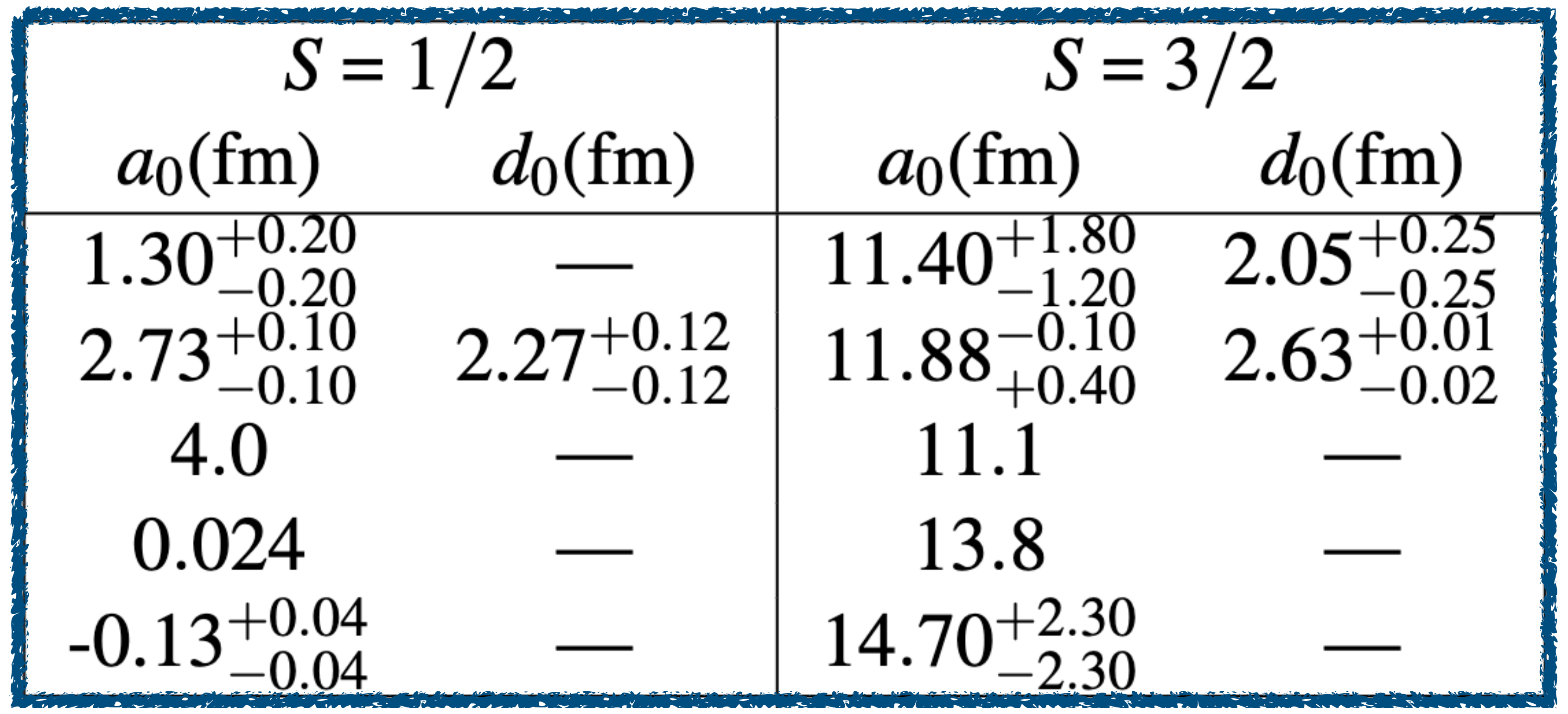
- f_c : Coulomb normalized scattering amplitude for strong interaction
- $F(-i\eta, 1, i\zeta)$: confluent hypergeometric function
- $\tilde{G}(\rho,\eta)$: combination of singular and regular Coulomb function, describes asymptotic behavior of wavefunction

⇒ to obtain two-particle correlation: apply Koonin-Pratt formula

Interaction model

- **For distinguishable pointlike particles: Lednicky approach**[1]
  - Starting from the scattering parameters ⇒ define the s-wave two-particle relative wave function
  - Considers Coulomb effects + strong interaction (via scattering parameters)

- **p–d scattering parameters** from constrained to the p-d scattering data



$S = 1/2$		$S = 3/2$	
a_0(fm)	d_0(fm)	a_0(fm)	d_0(fm)
$1.30^{+0.20}_{-0.20}$	—	$11.40^{+1.80}_{-1.20}$	$2.05^{+0.25}_{-0.25}$
$2.73^{+0.10}_{-0.10}$	$2.27^{+0.12}_{-0.12}$	$11.88^{-0.10}_{+0.40}$	$2.63^{+0.01}_{-0.02}$
4.0	—	11.1	—
0.024	—	13.8	—
$-0.13^{+0.04}_{-0.04}$	—	$14.70^{+2.30}_{-2.30}$	—

Van Oers,Brockmann et al. Nucl. Phys. A 561-583 (1967)
J.Arvieux et al. Nucl. Phys. A 221 253-268 (1973)
E.Huttel et al. Nucl. Phys. A 406 443-455 (1983)
A.Kievsky et al. PLB 406 292-296 (1997)
T.C.Black et al. PLB 471 103-107 (1999)

- **K^+–d scattering parameters**
  - ER (effective-range approximation): a_0= −0.47 fm, d_0= −1.75 fm, calculated by Prof. Johann Haidenbaur, based on potential describing K^+d low-energy cross-sections[2]
  - FCA (fixed-center approximation): a_0= −0.54 fm, d_0= 0.0 fm, calculated by Prof. Tetsuo Hyodo starting from Chiral model KN scattering lengths[3]

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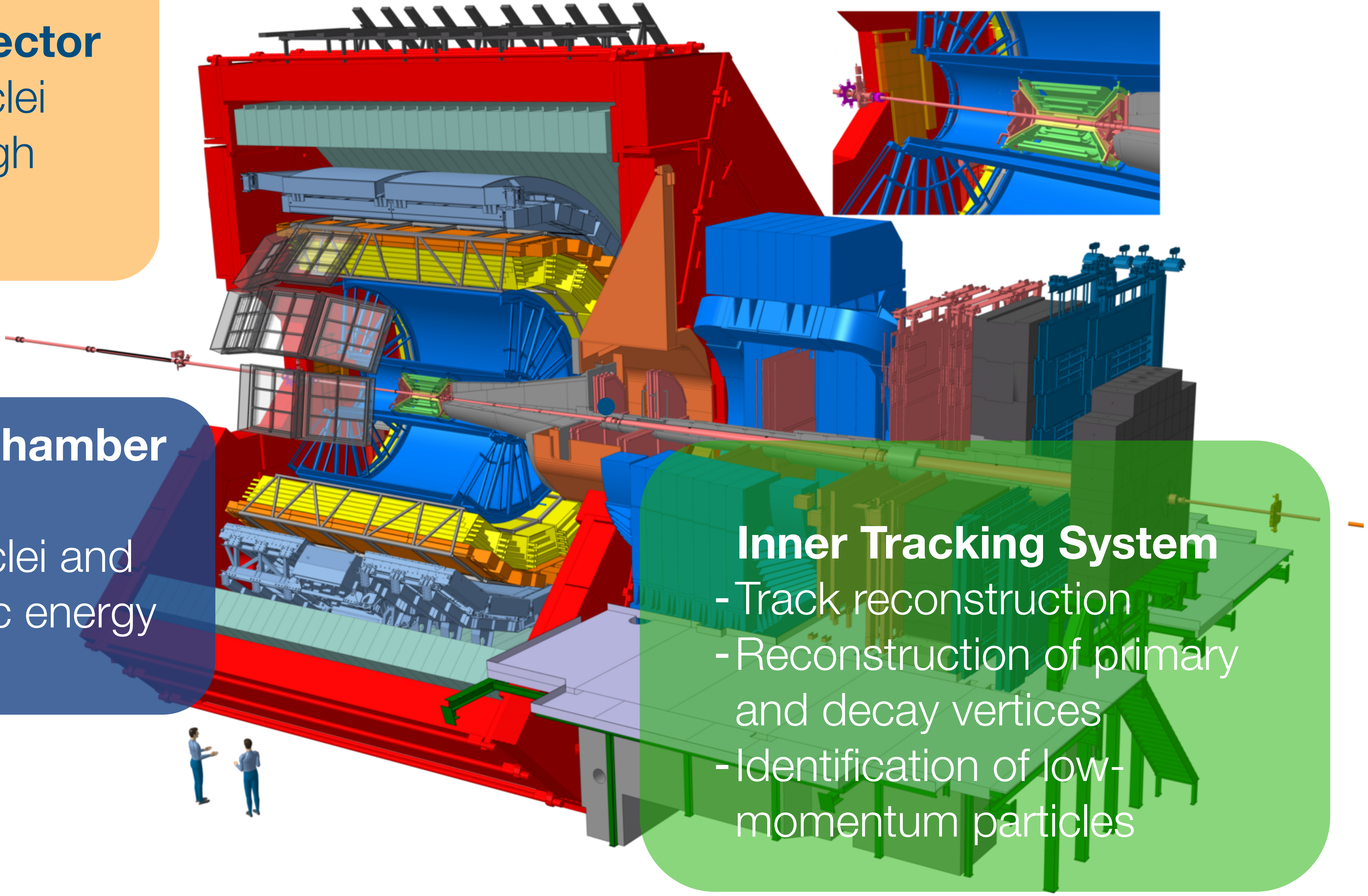
[1] R. Lednicky, Phys. Part. Nuclei 40, 307–352 (2009)
[2] T. Takaki PRC 81, 055204 (2010)]
[3] K. Aoki and D. Jido, PTEP 2019, 013D01 (2019)

ALICE detector: Run 2



Time-Of-Flight detector

- Identification of nuclei and hadrons through their time-of-flight

Time Projection Chamber

- Tracking
- Identification of nuclei and hadrons via specific energy loss

Inner Tracking System

- Track reconstruction
- Reconstruction of primary and decay vertices
- Identification of low-momentum particles

ALICE : ITS and TPC upgrades

Another calculation at hand

- Hadron-Deuteron Correlations and Production of Light Nuclei in Relativistic Heavy-Ion Collisions: arxiv.org/abs/1904.08320
  - hadron-deuteron correlation function which carries information about the source of the deuterons
  - Allows one to determine whether a deuteron is directly emitted from the fireball or if it is formed afterwards
  - Conclusion:
    - The theoretical p-d correlation function is strongly dependent on the source size

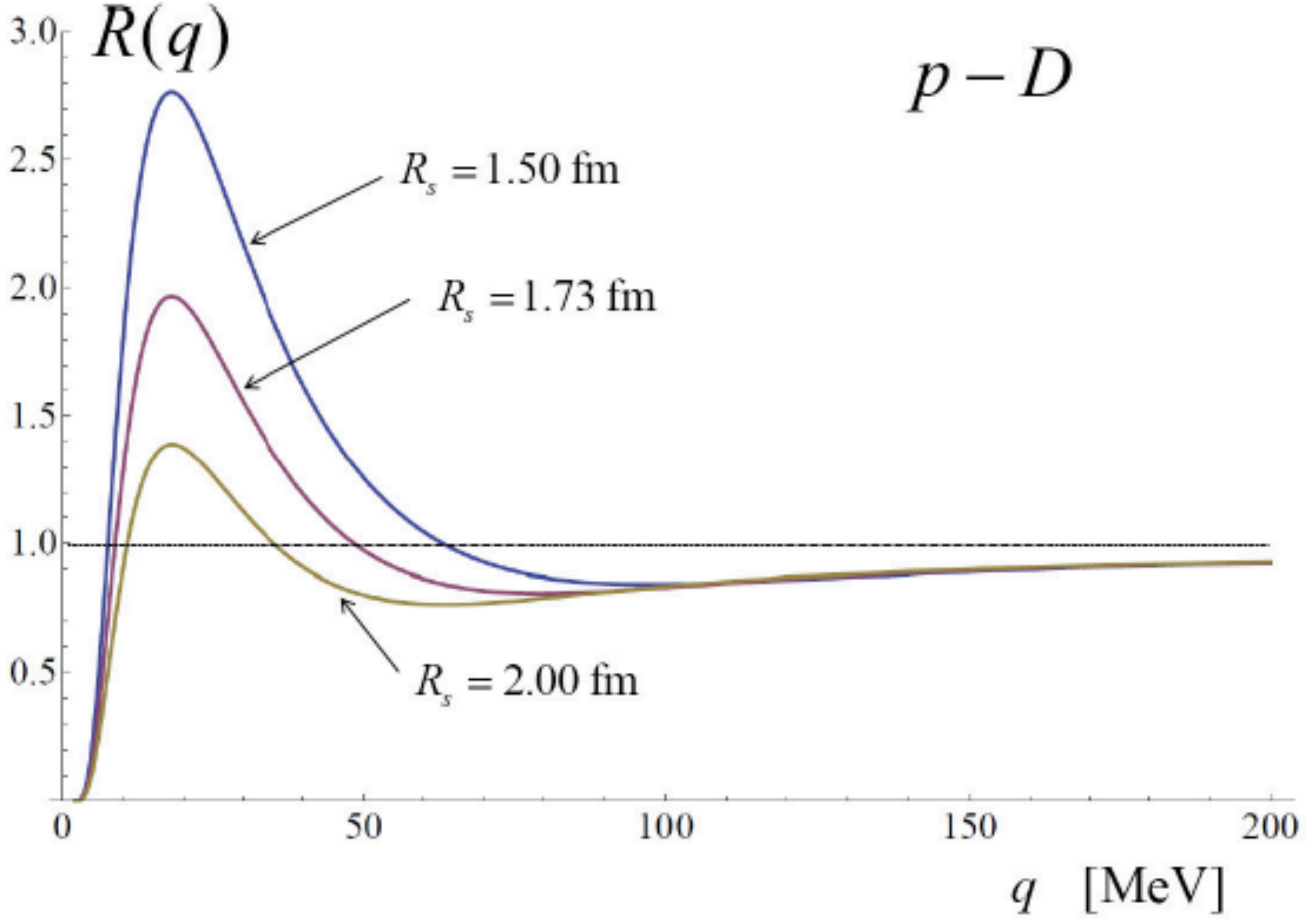
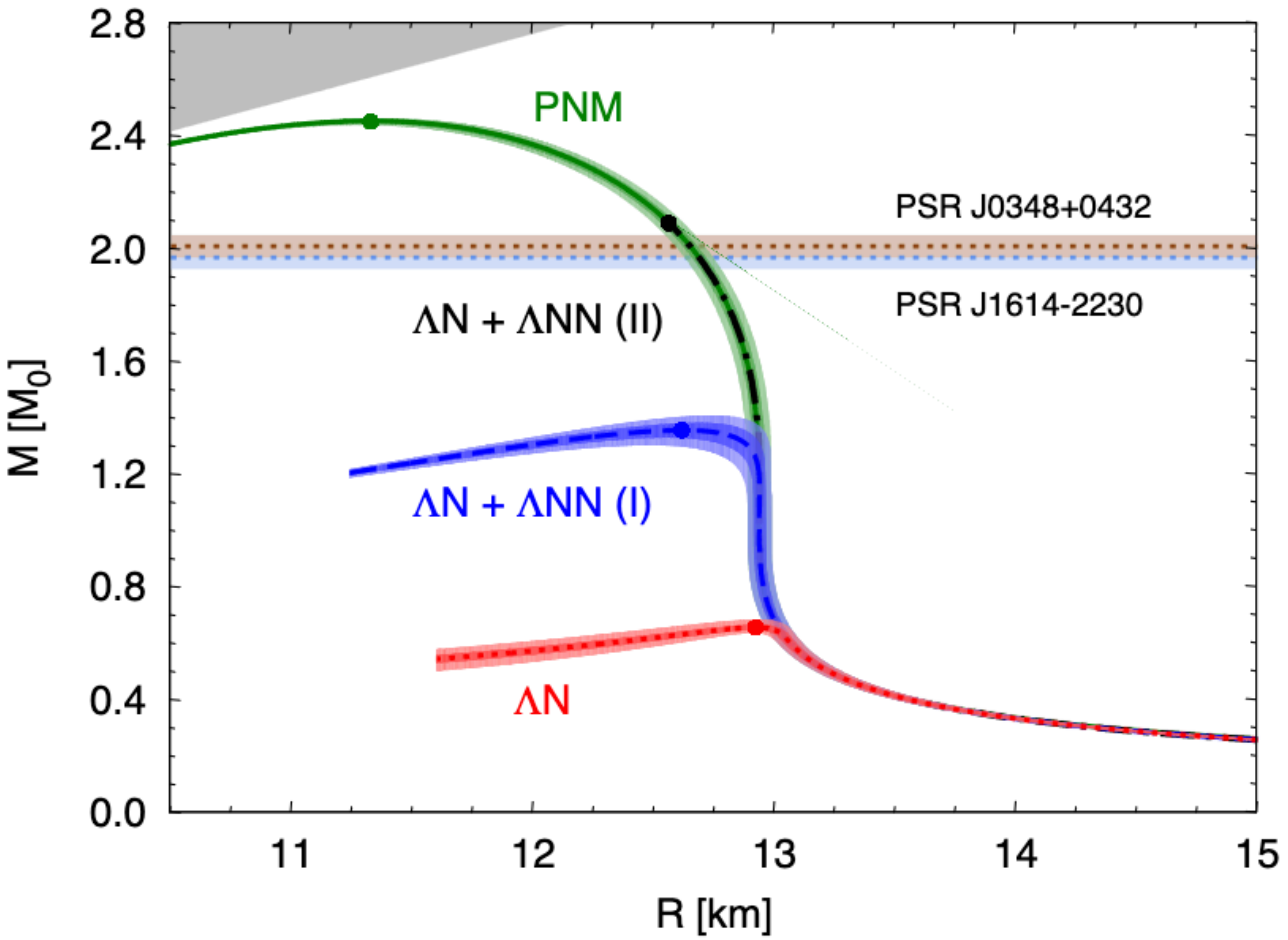
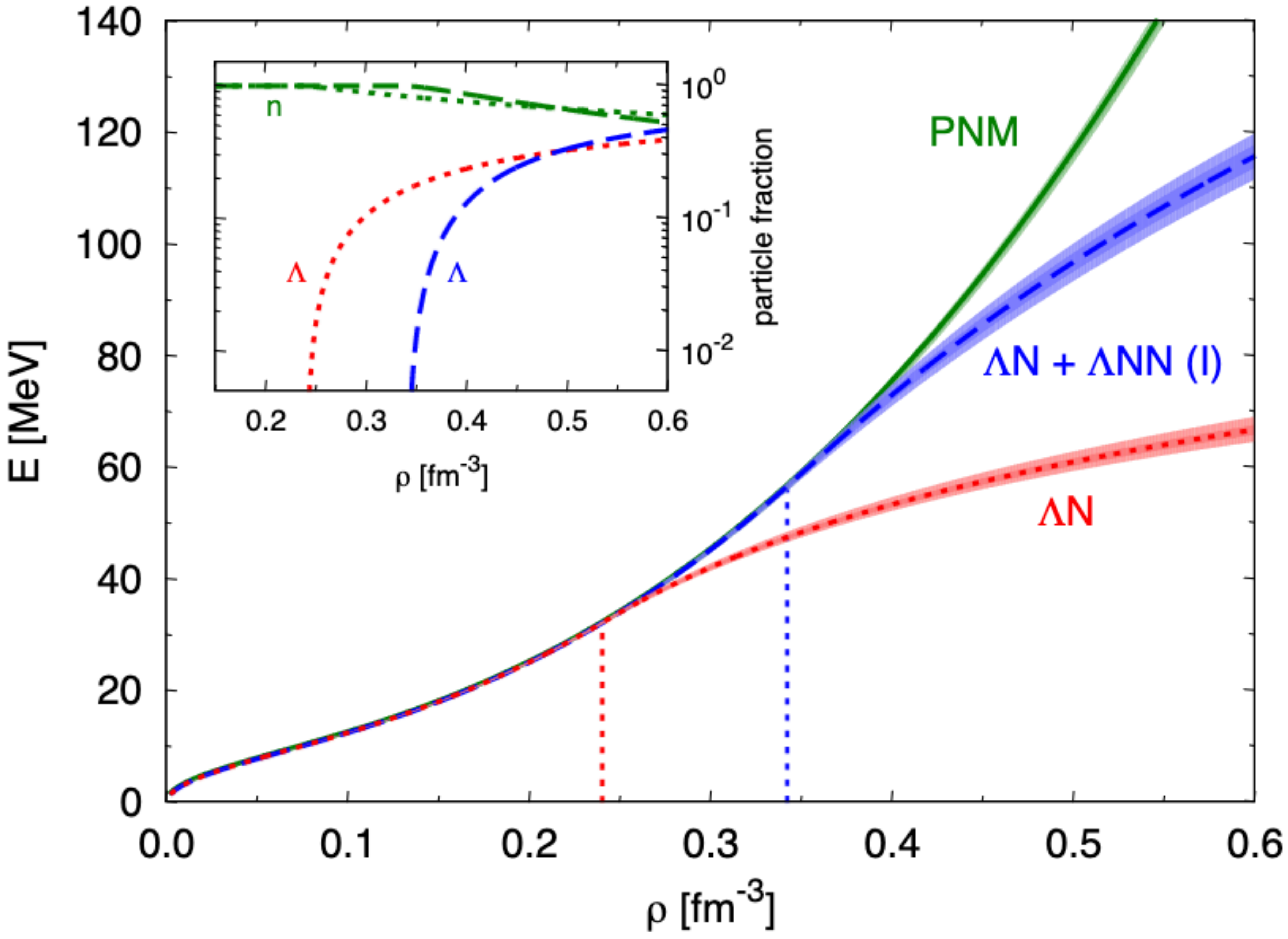


Fig. 2. $p-D$ correlation function

NS





D. Lonardoni et al., PRL 114, 092301 (2015)