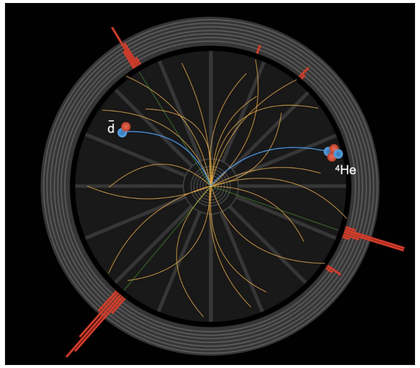
Thermal production of light (anti)(hyper)nuclei

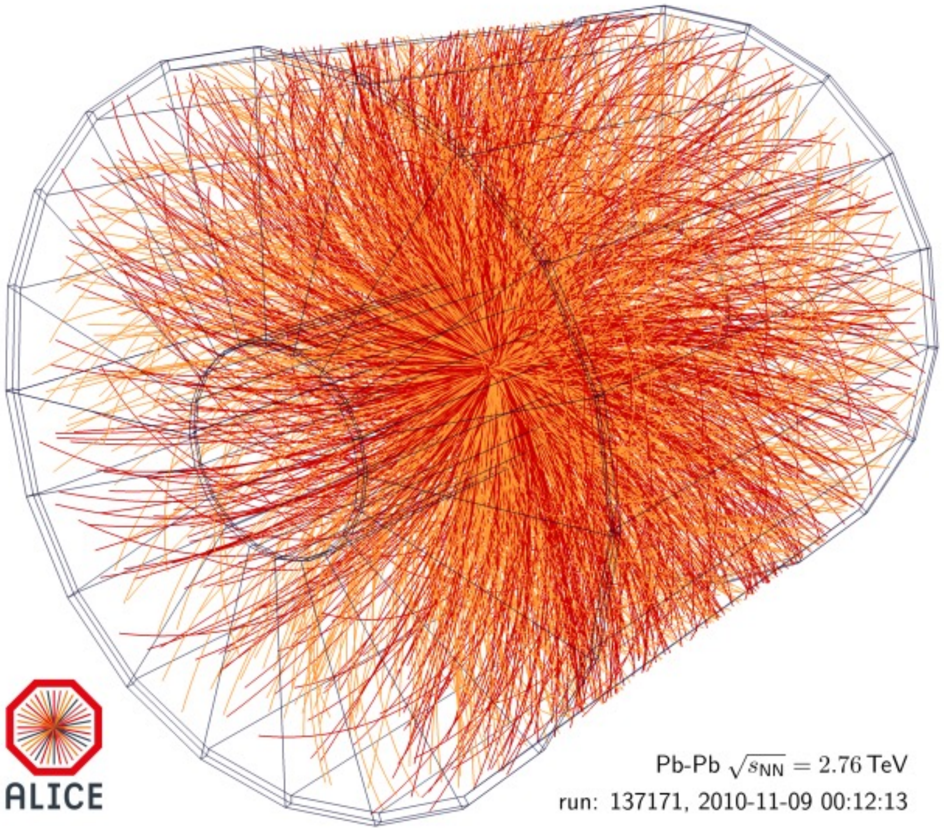
Volodymyr Vovchenko (University of Houston)



EMMI RRTF Open Symposium "Understanding light (anti-)nuclei production at RHIC and LHC"
GSI, Darmstadt, Germany

April 8, 2024

Particle production in heavy-ion collisions



Event display of a Pb-Pb collision in ALICE at the LHC

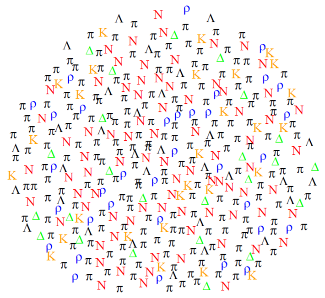
Thousands of particles created in relativistic heavy-ion collisions

↓

Apply concepts of statistical mechanics

Hadron resonance gas (HRG) model

- **HRG model:** free gas of known hadrons and resonances



$$p(T,\mu_B) = \underbrace{T\,\phi_M(T)}_{\text{mesons}} + \underbrace{2\,T\,\phi_B(T)}_{\text{baryons}} \cosh(\mu_B/T)$$

$$\phi_{M(B)}(T) = \sum_{i\in M(B)} \frac{d_i}{2\pi^2} \int dk\, k^2 \exp\left(-\frac{\sqrt{m_i^2+k^2}}{T}\right)$$

  - Hadronic interactions dominated by resonance formation*
  - Leading order in relativistic virial expansion
  - Matches well with lattice QCD below T_{pc}
  - Non-resonant interactions incorporated in extended descriptions

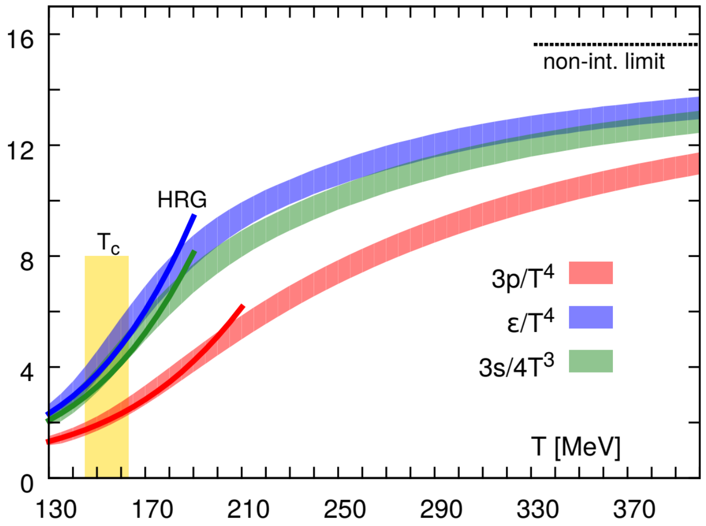


Figure from HotQCD Collaboration, PRD '14

HRG model and heavy-ion collisions:

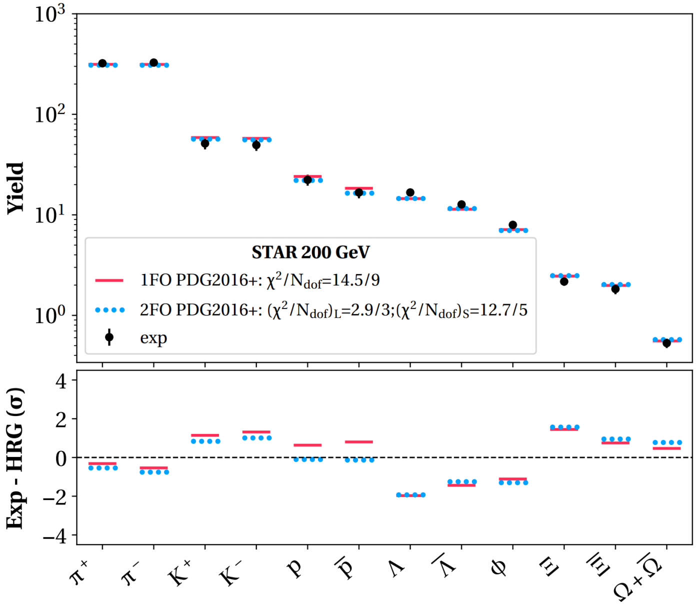
- Basis for the thermal model of particle production

All bells and whistles implemented in open-source codes, e.g. Thermal-FIST [VV, Stoecker, Comput. Phys. Commun. 244, 295 (2019)]

* *Dashen, Ma, Bernstein, "S-matrix formulation of statistical mechanics", Phys. Rev. (1969); Prakash, Venugopalan, Nucl. Phys. A (1992)*

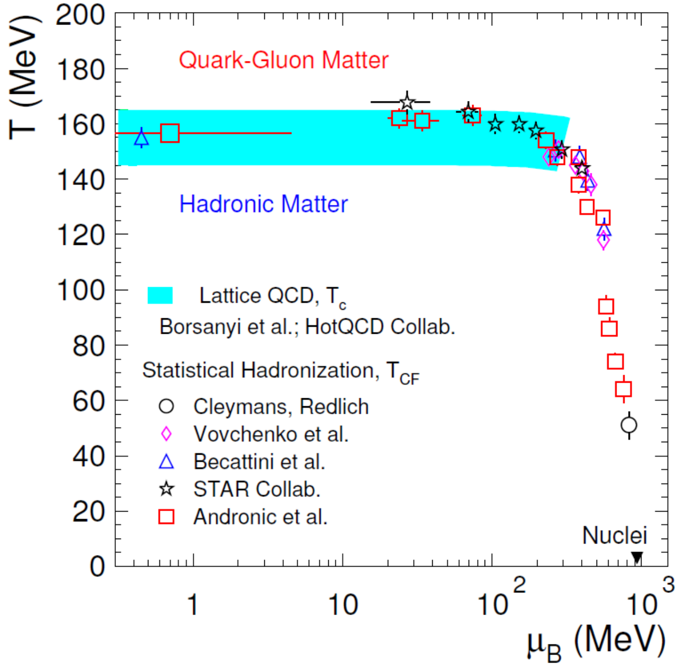
Mapping heavy-ion collisions onto the QCD phase diagram

Yield: $$\underbrace{\langle N_i \rangle}_{\textit{total}} = \underbrace{\langle N_i^* \rangle}_{\textit{primordial}} + \underbrace{\sum_R \langle n_i \rangle_R \langle N_R^* \rangle}_{\textit{decays}}$$



P. Alba et al. (UH group), Phys. Rev. C 101, 054905 (2020)

$$\xrightarrow{\qquad} \boxed{\begin{matrix} T(\sqrt{s_{NN}}) \\ \mu_B(\sqrt{s_{NN}}) \end{matrix}} \xrightarrow{\qquad}$$

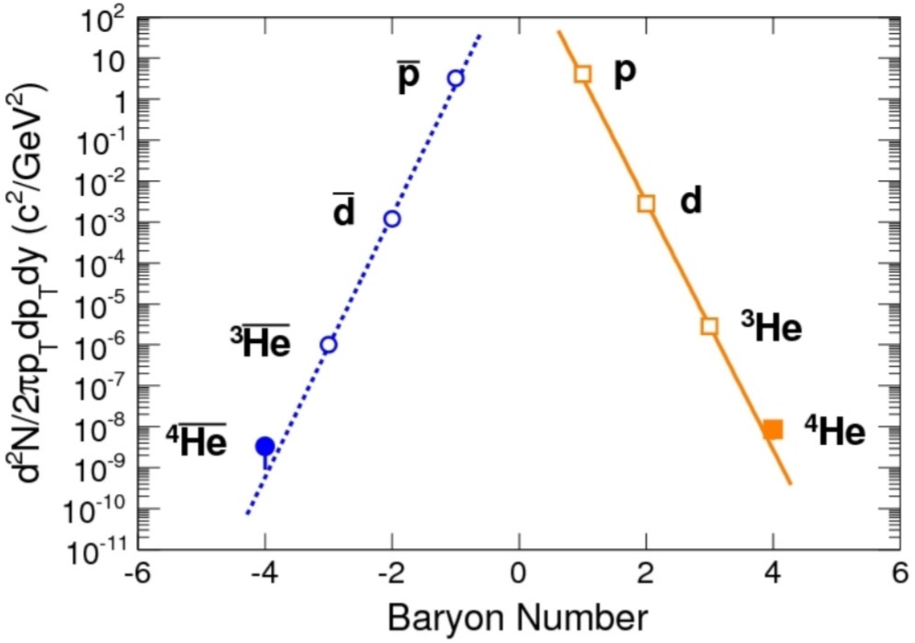


A. Andronic et al., Nature 561, 321 (2018)

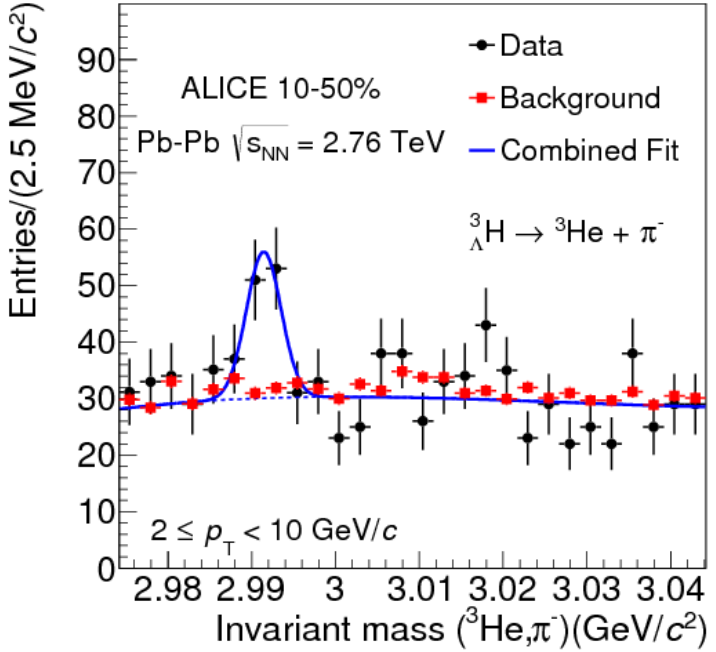
Extensions:

- Flavor-dependent freeze-out F. Flor, G. Olinger, R. Bellwied, Phys. Lett. B 814, 136098 (2021)
- Partial chemical equilibrium A. Motornenko, VV, C. Greiner, H. Stoecker, Phys. Rev. C 102, 024909 (2020)
- Charm A. Andronic et al., JHEP 07, 035 (2021)
- **Light nuclei** A. Andronic, P. Braun-Munzinger, J. Stachel, H. Stoecker, Phys. Lett. B 697, 203 (2011)

Loosely-bound objects in heavy-ion collisions



[STAR collaboration, Nature 473, 353 (2011)]



[ALICE Collaboration, PLB 754, 360 (2016)]

binding energies: ^{2}H, ^{3}He, ^{4}He, $^3_\Lambda$H: 2.22, 7.72, 28.3, 0.130 MeV $\ll T \sim 150$ MeV

"snowballs in hell"

The production mechanism is not established. Common approaches:

- **thermal** nuclei emission together with hadrons [Andronic et al., PLB '11;...]
- final-state **coalescence** of nucleons close in phase-space [Butler, Pearson, PRL '61; Scheibl, Heinz, PRC '99;...]
- **kinetic/transport** approaches [Danieliewicz, Bertsch, NPA '91; Oh, Ko, PRC '07; Oliinychenko et al., PRC '18;...]

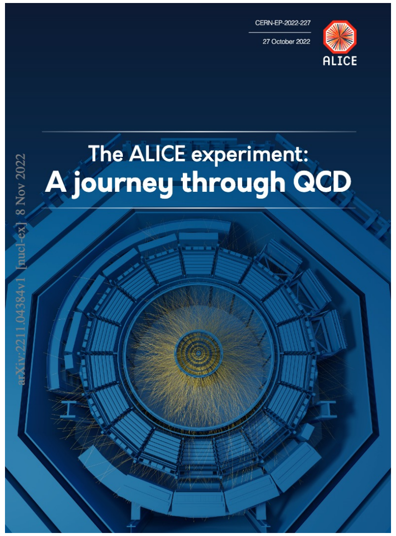
Grand-canonical thermal model description of AA collisions

Light nuclei in thermal model

Add as separate degrees of freedom into the partition function

$$p(T,\mu_B) \mathrel{+}= \sum_{A\in d,t,\ldots} \frac{d_A m_A^2 T^2}{2\pi^2} K_2(m_A/T)\, e^{A\mu_B/T}$$

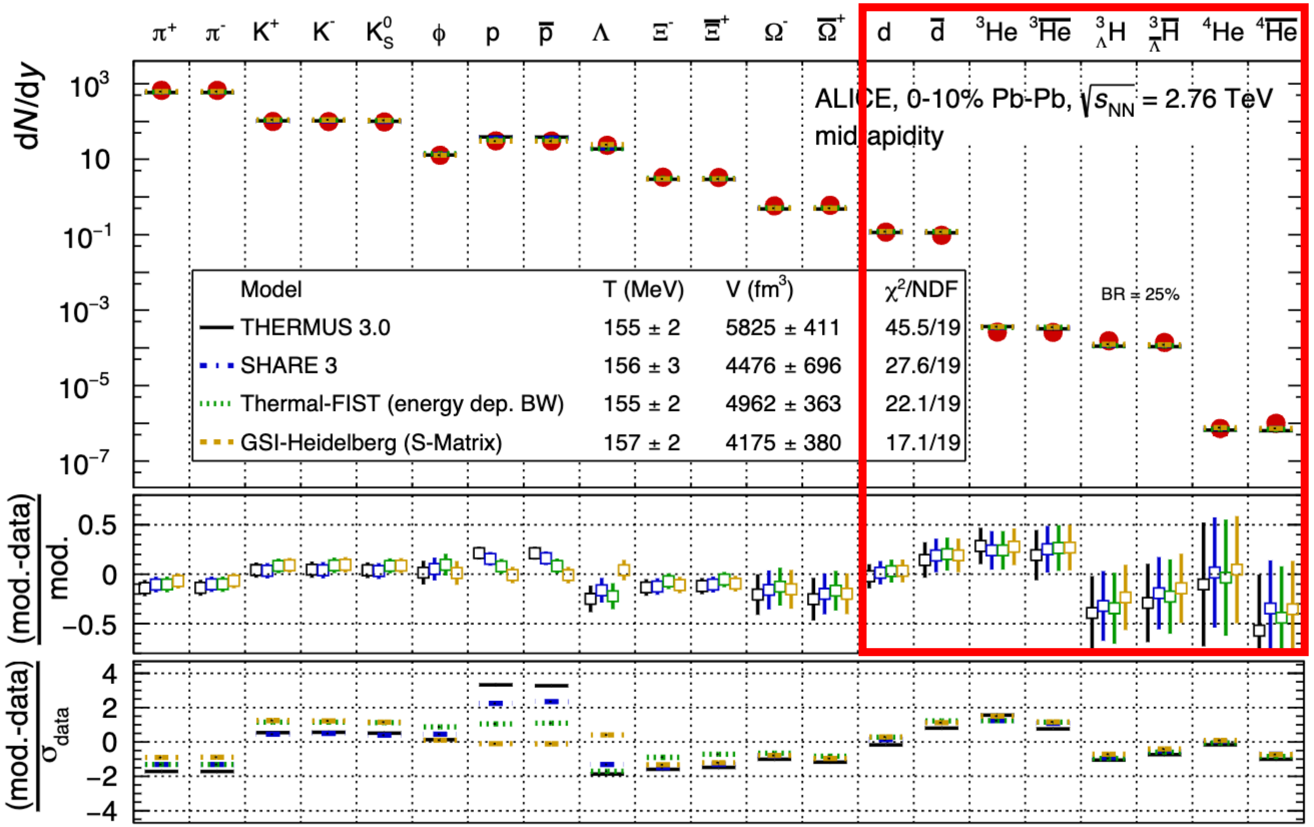
Essentially parameter-free prediction



CERN-EP-2022-227
27 October 2022
ALICE

The ALICE experiment:
A journey through QCD

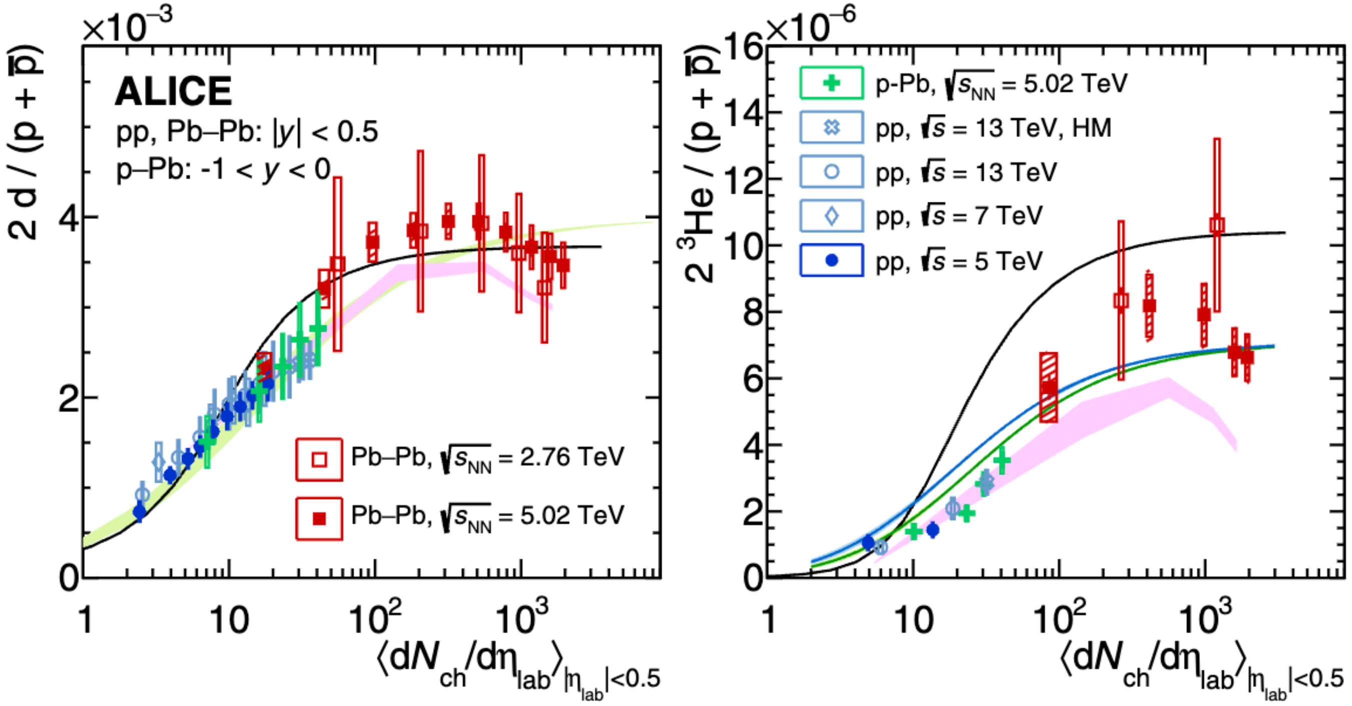
arXiv:2211.04384v1 [nucl-ex] 8 Nov 2022



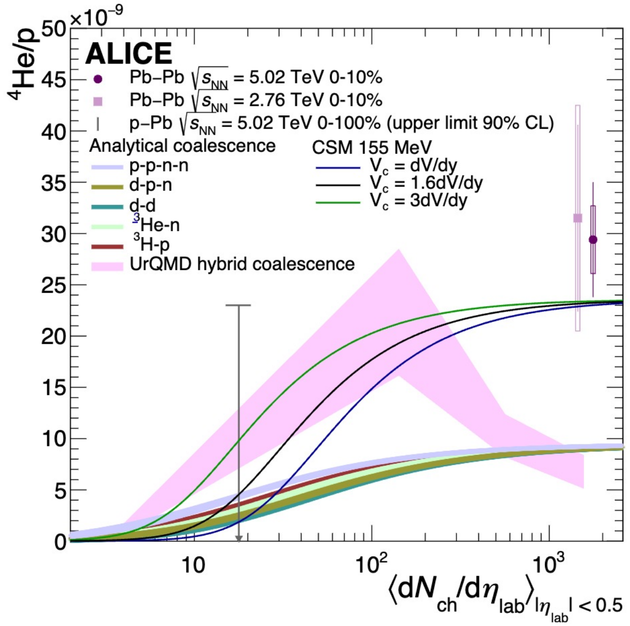
Model	T (MeV)	V (fm^3)	χ^2/NDF
THERMUS 3.0	155 ± 2	5825 ± 411	45.5/19
SHARE 3	156 ± 3	4476 ± 696	27.6/19
Thermal-FIST (energy dep. BW)	155 ± 2	4962 ± 363	22.1/19
GSI-Heidelberg (S-Matrix)	157 ± 2	4175 ± 380	17.1/19

Quantitative tests with 5 TeV data

ALICE Collaboration, PRC 107, 064904 (2023)



ALICE Collaboration, arXiv:2311.11758

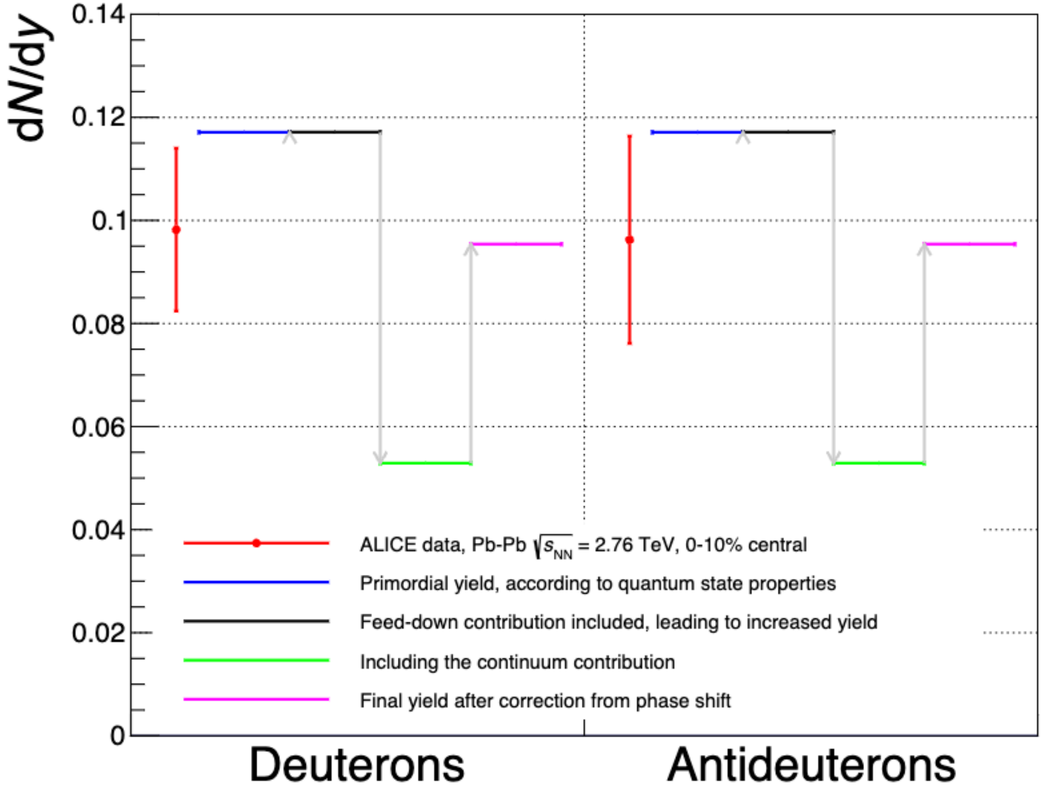


	2.76 TeV	**5.02 TeV**
Thermal fit to 0-10% (d, ^{3}He, ^{4}He) yields:	$T = 159 \pm 5$ MeV	$T = 147 \pm 2$ MeV
	$\chi_2/dof = 1.2/3$	$\chi_2/dof = 12.9/3$

At T = 155 MeV, the thermal model overestimates ^{3}He by factor ~1.6

Interactions

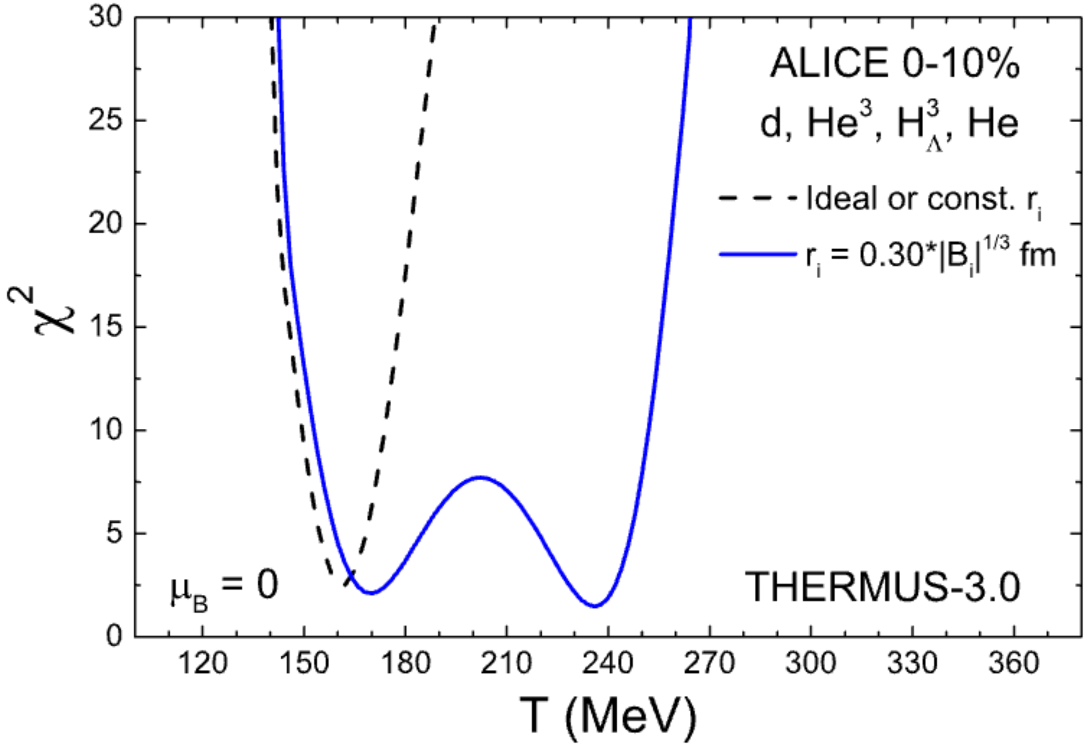
Pion-deuteron phase shifts



Donigus, Ropke, Blaschke, PRC 106, 044908 (2022)

Excluded volume

$r_p = r_d = r_t = r_a = 0.3$ fm vs $r_A \sim A^{1/3}$

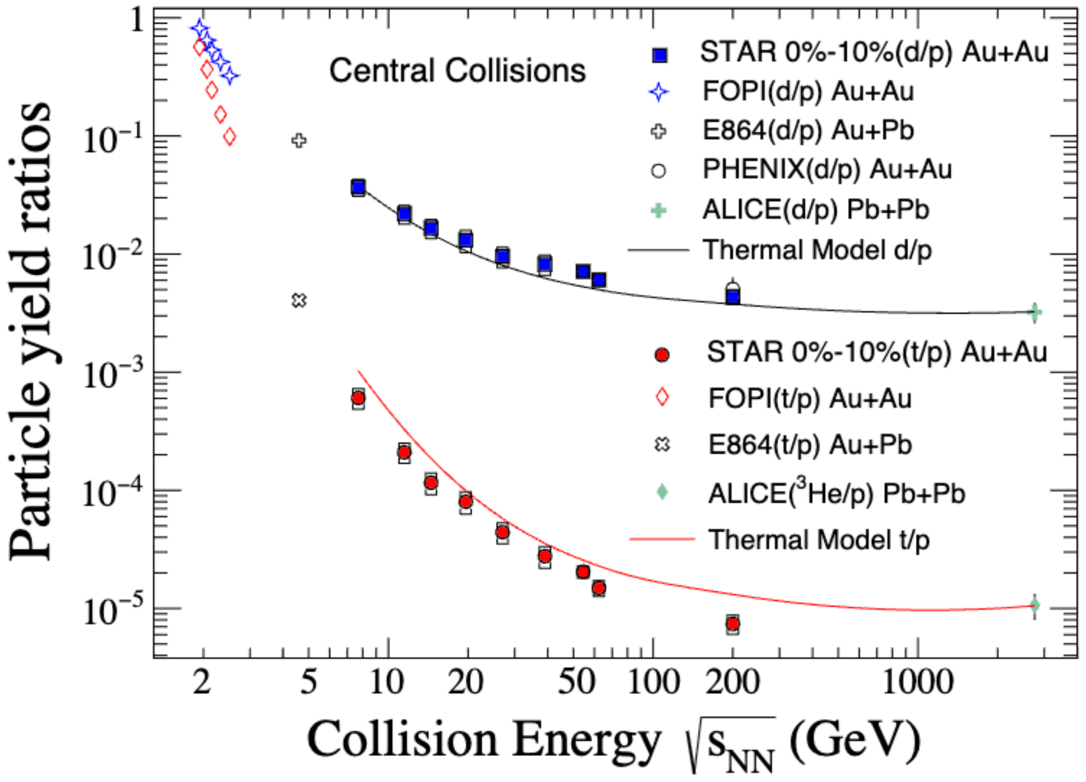


Vovchenko, Stoecker, JPG 44, 055103 (2017)

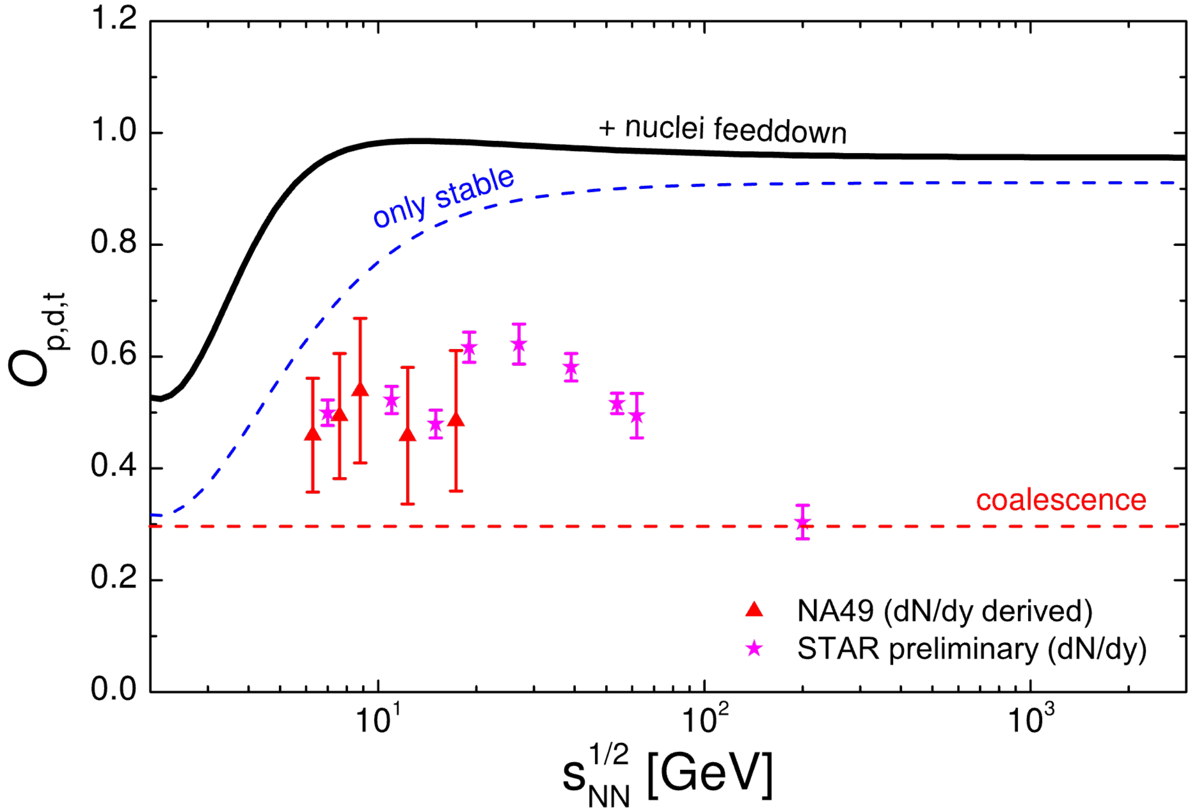
Thermal model at RHIC beam energy scan

$O_{t,p,d} = N_t\, N_p/(N_d)^2$ cancels out chemical potentials

but has feeddown $O_{p,d,t} = 1/(2\sqrt{3}) \times (1 + Res \to p)$



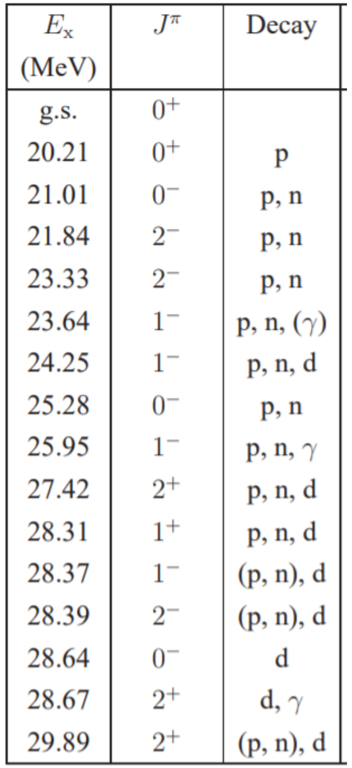
STAR Collaboration, PRL 130, 202301 (2023)



Excited nuclei

^{4}He, ^{4}H, ^{4}Li etc. have many excited states feeding down the final yield
[Hahn, Stöcker, NPA '88; Torres-Rincon, Shuryak, PRC '20]

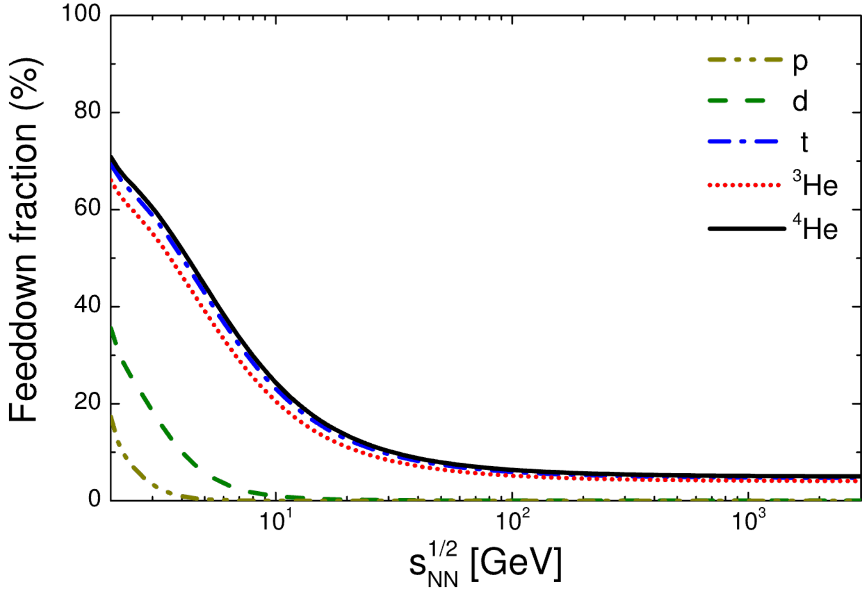
^{4}He



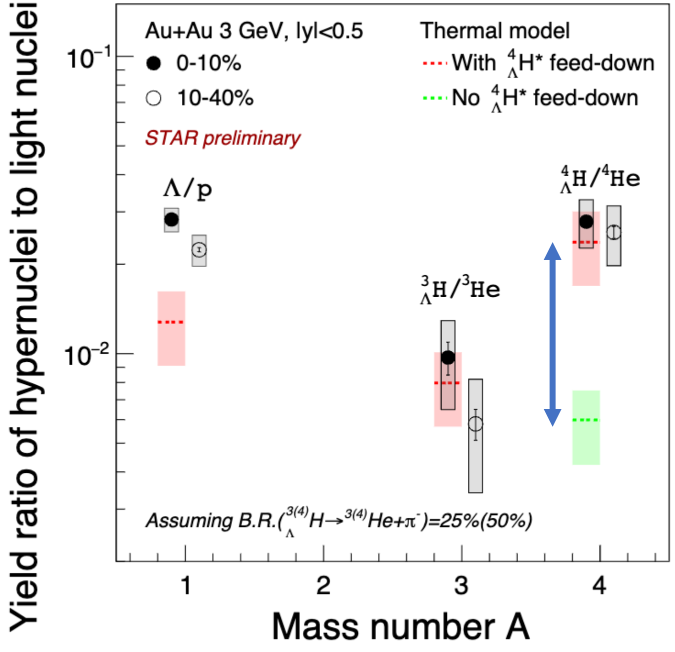
E_x (MeV)	J^π	Decay
g.s.	0^+	
20.21	0^+	p
21.01	0^-	p, n
21.84	2^-	p, n
23.33	2^-	p, n
23.64	1^-	p, n, (γ)
24.25	1^-	p, n, d
25.28	0^-	p, n
25.95	1^-	p, n, γ
27.42	2^+	p, n, d
28.31	1^+	p, n, d
28.37	1^-	(p, n), d
28.39	2^-	(p, n), d
28.64	0^-	d
28.67	2^+	d, γ
29.89	2^+	(p, n), d

[Tilley, Weller, Hale, NPA '92]

See also https://www.nndc.bnl.gov/



VV, Dönigus, Kardan, Lorenz, Stoecker, PLB 809, 135746 (2020)



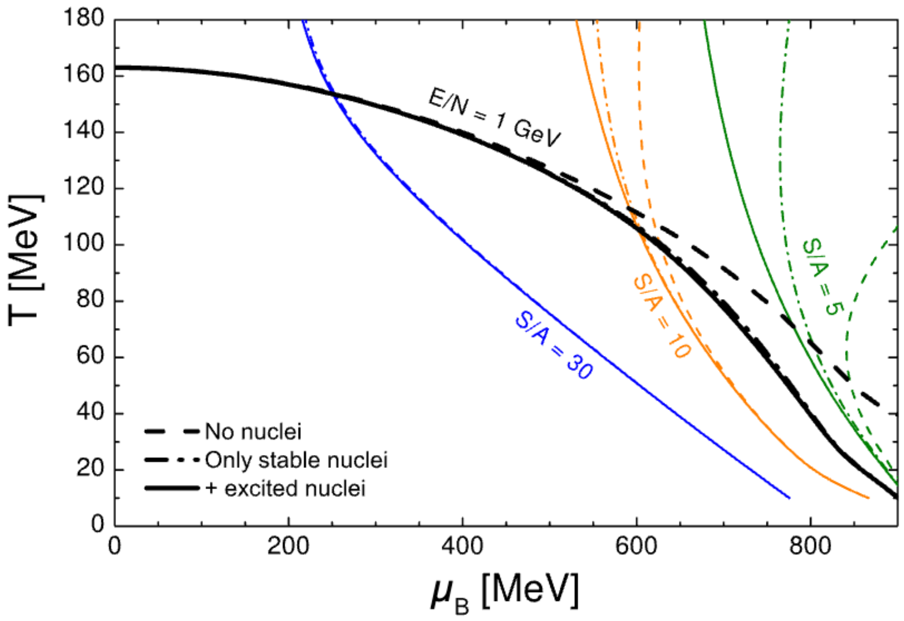
Y.H. Leung (STAR), QM2022

Strongest effects at lower energies

Baryon-rich matter

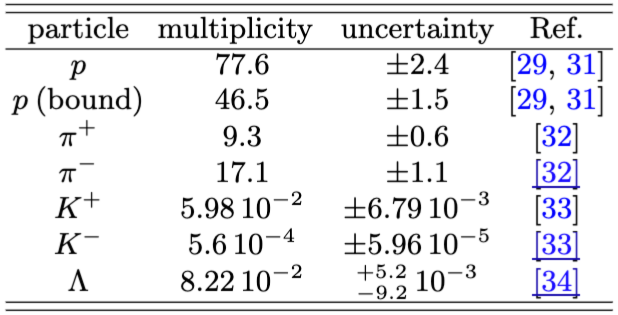
At low energies nuclei dominate the baryon number

Whether nuclei are included into HRG or not has strong effect on its thermodynamics and thermal fits



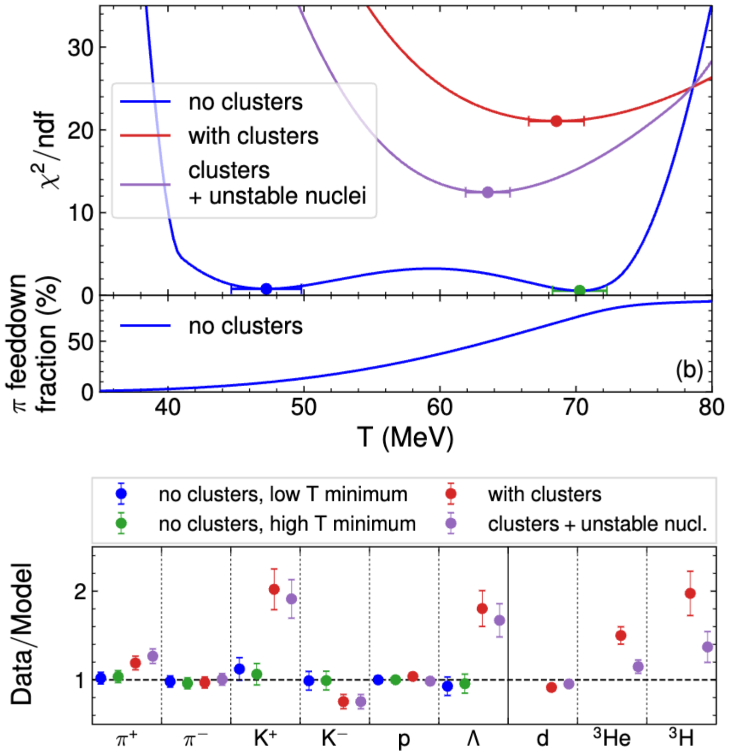
VV, Dönigus, Kardan, Lorenz, Stoecker, PLB 809, 135746 (2020)

HADES:



particle	multiplicity	uncertainty	Ref.
p	77.6	± 2.4	[29, 31]
p (bound)	46.5	± 1.5	[29, 31]
π^+	9.3	± 0.6	[32]
π^-	17.1	± 1.1	[32]
K^+	$5.98\,10^{-2}$	$\pm 6.79\,10^{-3}$	[33]
K^-	$5.6\,10^{-4}$	$\pm 5.96\,10^{-5}$	[33]
Λ	$8.22\,10^{-2}$	${}^{+5.2}_{-9.2}\,10^{-3}$	[34]

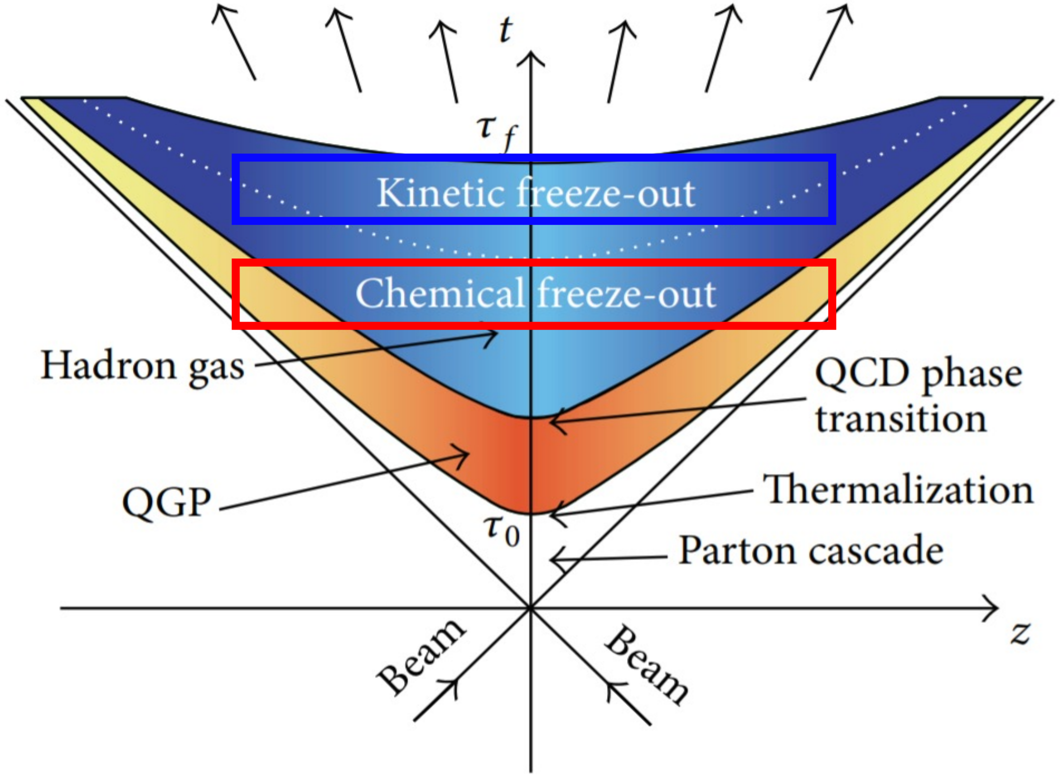
Harabasz et al., PRC 102, 054903 (2020)



Motornenko et al., PLB 822, 136703 (2021)

Hadronic phase and Saha equation

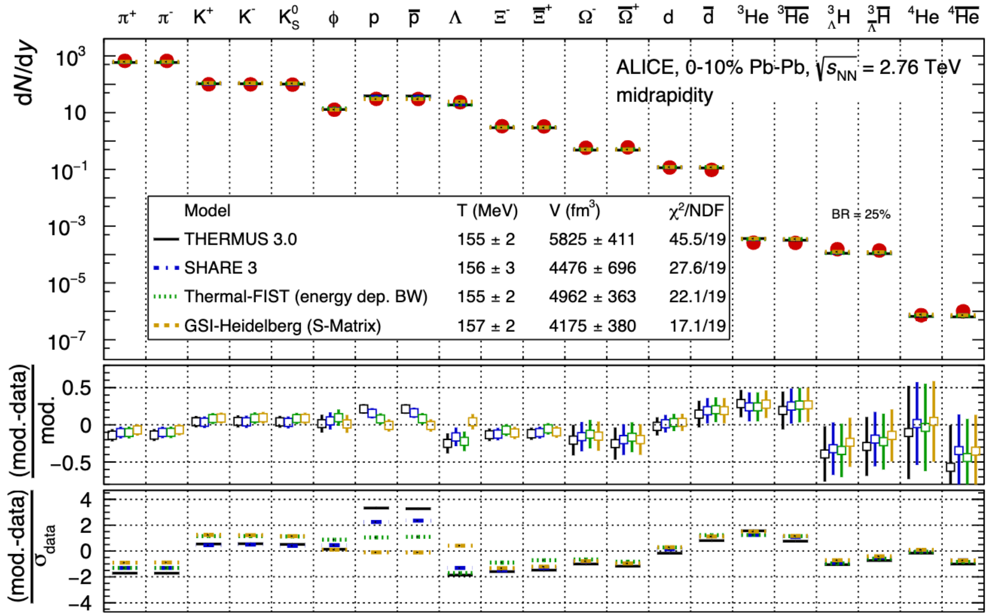
Hadronic phase in heavy-ion collisions



- At $T_{ch} \approx 150 - 160$ MeV inelastic collisions cease, yields of *stable* hadrons frozen
- Kinetic equilibrium maintained down to $T_{kin} \approx 100 - 120$ MeV through (pseudo)elastic scatterings
- Do nuclei interact in the hadronic phase?

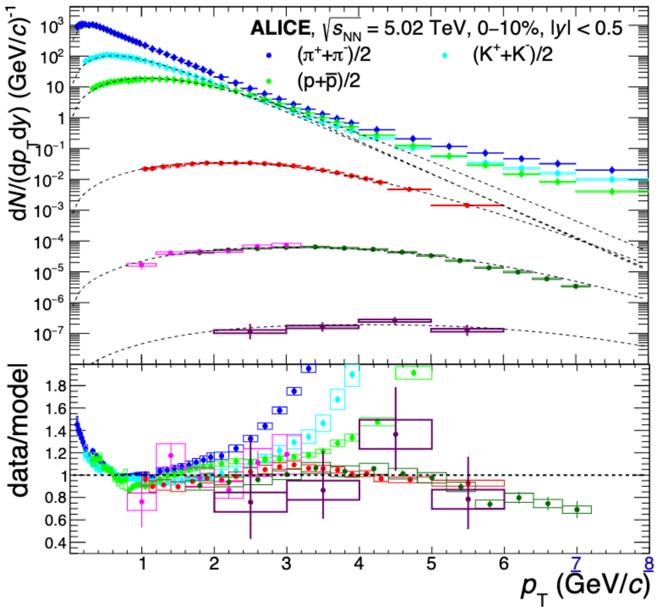
Chemical and kinetic freeze-outs at the LHC

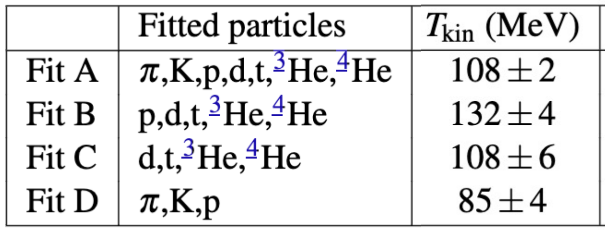
1. Measured hadron/nuclei yields are described by thermal model at $T_{ch} \approx 155$ MeV



ALICE Collaboration, arXiv:2211.04384

2. Momentum spectra described by the blast-wave model at $T_{kin} \approx 100-120$ MeV





	Fitted particles	T_{kin} (MeV)
Fit A	π,K,p,d,t,^{3}He,^{4}He	108 ± 2
Fit B	p,d,t,^{3}He,^{4}He	132 ± 4
Fit C	d,t,^{3}He,^{4}He	108 ± 6
Fit D	π,K,p	85 ± 4

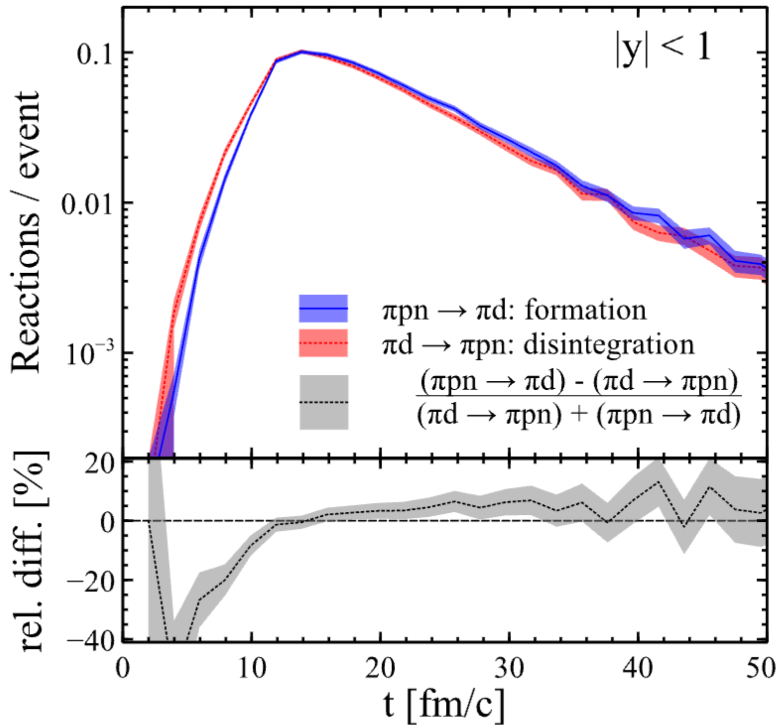
ALICE Collaboration, arXiv:2311.11758

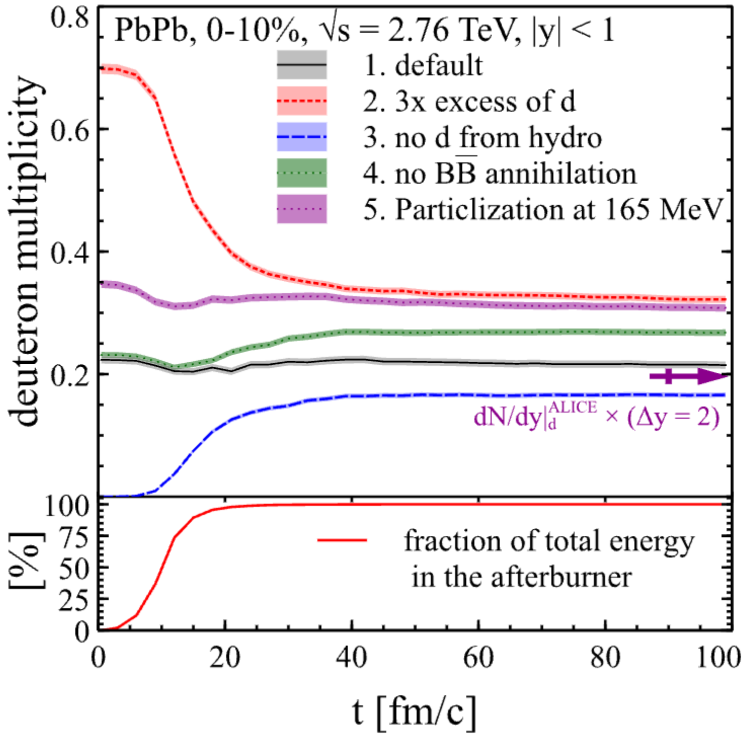
If T_{kin} = 100-120 MeV holds for nuclei, they must scatter in the hadronic phase.
How do they survive it?

Deuterons in hadronic transport

Hadronic afterburner SMASH with pion-deuteron reactions, $\pi d \leftrightarrow \pi p n$

Oliinychenko, Pang, Elfner, Koch, PRC 99, 044907 (2019)





Detailed balance of deuteron creation and destruction

Explains why thermal model yields does not change dramatically in hadronic phase...
and implies that the measured deuterons are not created at hadronization

Hadronic phase thermodynamics: Partial chemical equilibrium

Expansion of hadron resonance gas in partial chemical equilibrium at $T < T_{ch}$

[H. Bebie, P. Gerber, J.L. Goity, H. Leutwyler, Nucl. Phys. B '92; C.M. Hung, E. Shuryak, PRC '98]

Chemical composition of stable hadrons is fixed, kinetic equilibrium maintained through pseudo-elastic resonance reactions $\pi\pi \leftrightarrow \rho$, $\pi \mathrm{K} \leftrightarrow \mathrm{K}^*, \pi N \leftrightarrow \Delta$, etc.

E.g.: $\pi + 2\rho + 3\omega + \cdots = const$, $\mathrm{K} + \mathrm{K}^* + \cdots = const$, $N + \Delta + N^* + \cdots = const$,

Effective chemical potentials:

$$\tilde{\mu}_j = \sum_{i \in \text{stable}} \langle n_i \rangle_j \, \mu_i ,$$

$\langle n_i \rangle_j$ – mean number of hadron i from decays of hadron j, $j \in$ HRG

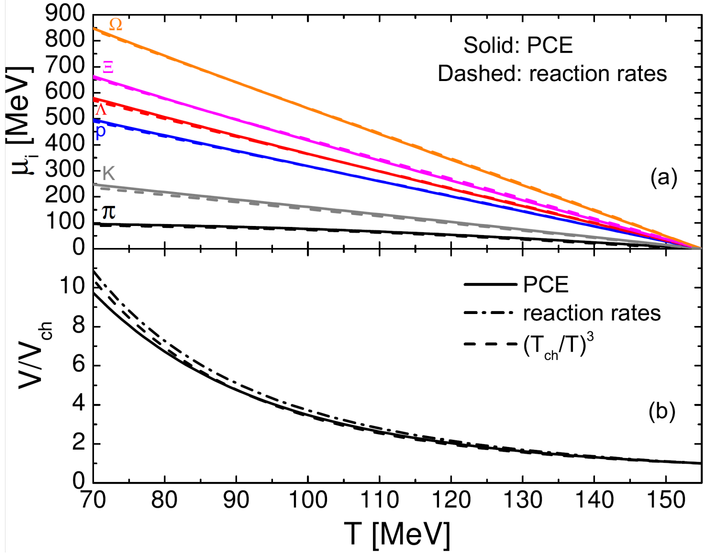
Conservation laws:

$$\sum_{j \in \text{hrg}} \langle n_i \rangle_j \, n_j(T, \tilde{\mu}_j) \, V = N_i(T_{\text{ch}}), \quad i \in \text{stable}$$

$$\sum_{j \in \text{hrg}} s_j(T, \tilde{\mu}_j) \, V = S(T_{\text{ch}})$$

numerical solution

$$\{\mu_i(T)\}, V(T)$$



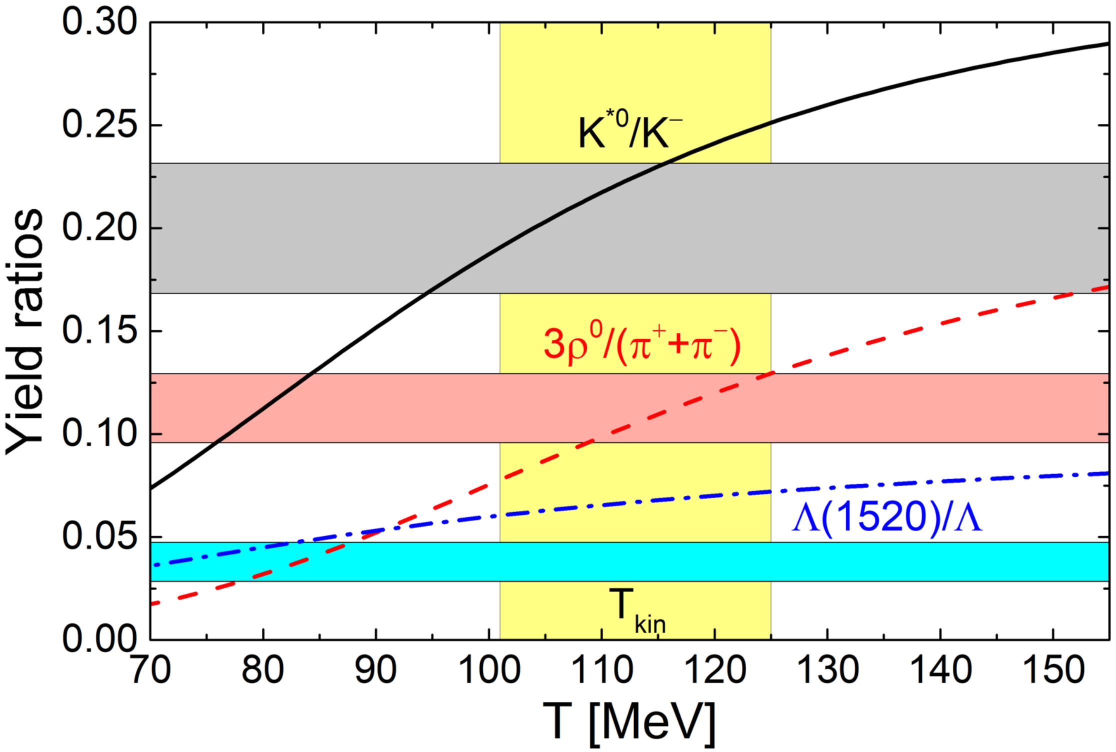
Numerical implementation within **Thermal-FIST** package (since v1.3)

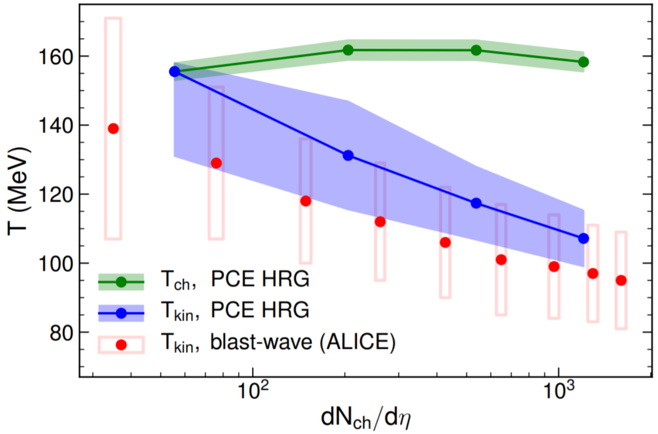
VV, H. Stoecker, *Comput. Phys. Commun. 244, 295 (2019);* Motornenko et al., PRC 102, 024909 (2020)

Resonance suppression in hadronic phase

Yields of resonances are *not* conserved in partial chemical equilibrium

Use the sensitivity of short-lived resonance yields to T_{kin} extract the kinetic freeze-out temperature





A. Motornenko, VV, C. Greiner, H. Stoecker, PRC 102, 024909 (2020)

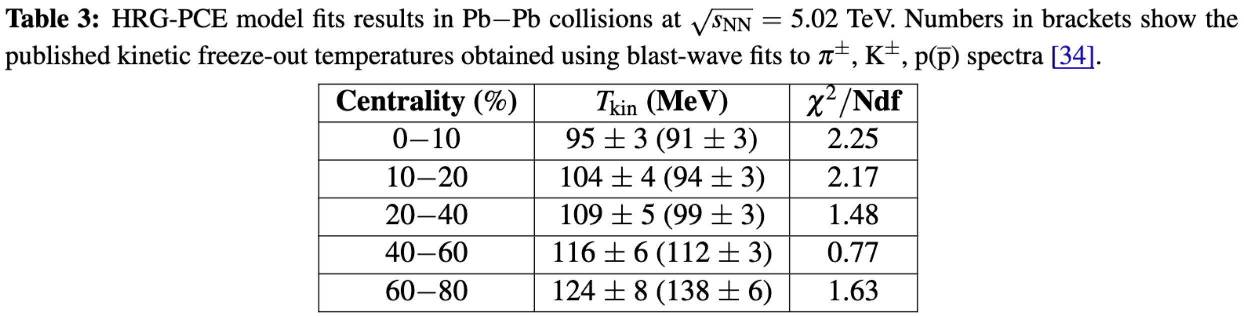


Table 3: HRG-PCE model fits results in Pb−Pb collisions at $\sqrt{s_{NN}} = 5.02$ TeV. Numbers in brackets show the published kinetic freeze-out temperatures obtained using blast-wave fits to $\pi^\pm$, $K^\pm$, $p(\bar{p})$ spectra [34].

Centrality (%)	T_{kin} (MeV)	χ^2/Ndf
0−10	95 ± 3 (91 ± 3)	2.25
10−20	104 ± 4 (94 ± 3)	2.17
20−40	109 ± 5 (99 ± 3)	1.48
40−60	116 ± 6 (112 ± 3)	0.77
60−80	124 ± 8 (138 ± 6)	1.63

- Consistency with blast-wave fits
- Solves the T_{kin}-vs-$\langle\beta_T\rangle$ anticorrelation problem of BW fits

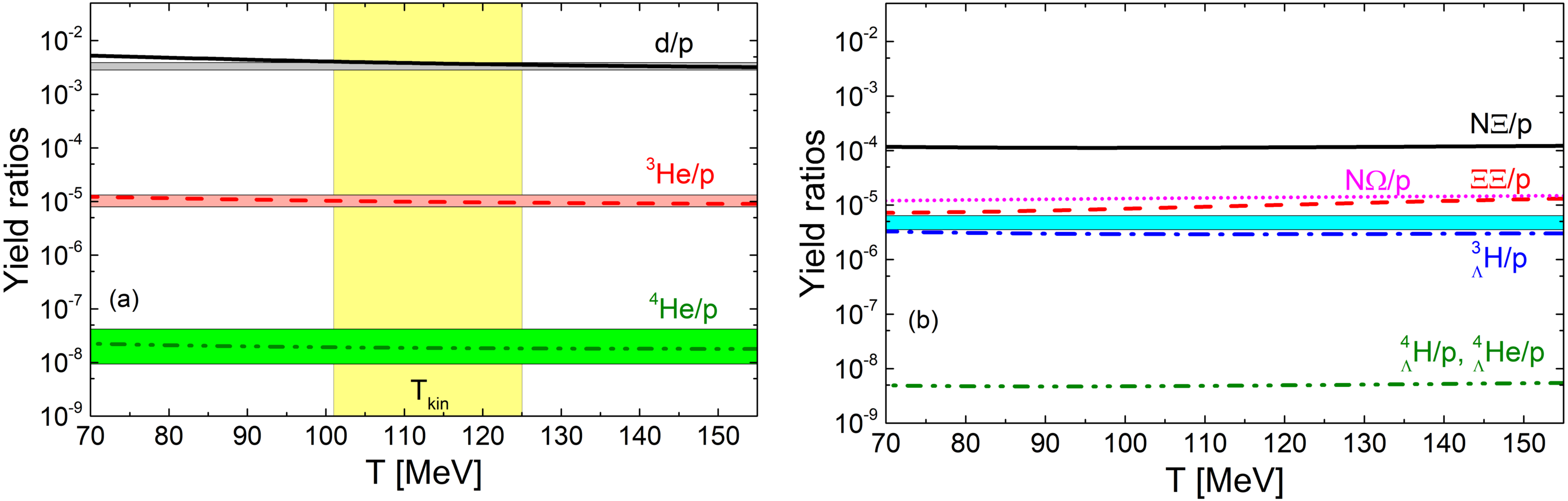
ALICE Collaboration, PRC 109, 044902 (2024)

PCE and light (anti-)(hyper-)nuclei: Saha equation

VV, Gallmeister, Schaffner-Bielich, Greiner, PLB 800, 135131 (2020)

Saha equation: Detailed balance for nuclear reactions

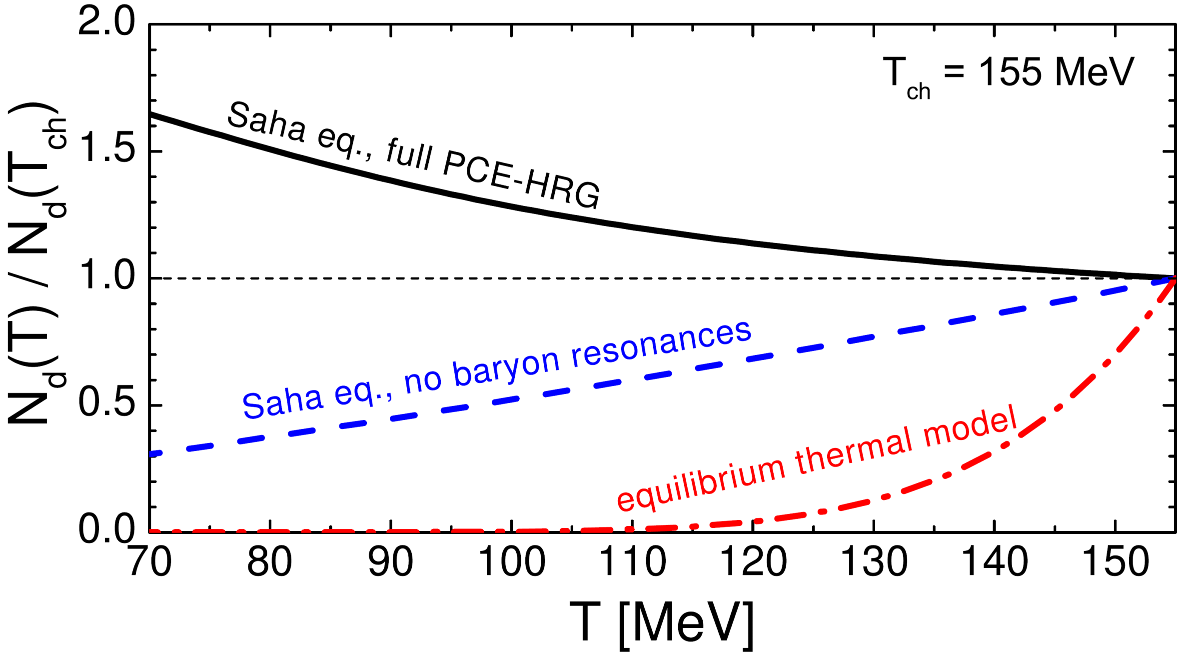
$$\frac{n_A}{\prod_i n_{A_i}} = \frac{n_A^{\text{eq}}}{\prod_i n_{A_i}^{\text{eq}}}, \quad \Leftrightarrow \quad \mu_A = \sum_i \mu_{A_i}, \qquad \text{e.g.} \quad \mu_d = \mu_p + \mu_n, \; \mu_{^3\text{He}} = 2\mu_p + \mu_n, \; \ldots$$



Data permit freeze-out of light (anti-)(hyper-)nuclei at any $T<T_{ch}$ in the hadronic phase!

Similar results obtained by using rate equations T. Neidig et al., Phys. Lett. B 827, 136891 (2022)

Closer look at the yields



Saha equation (no resonances):

$$\frac{N_A(T)}{N_A(T_{\rm ch})} \simeq \left(\frac{T}{T_{\rm ch}}\right)^{\frac{3}{2}(A-1)} \exp\left[B_A\left(\frac{1}{T}-\frac{1}{T_{\rm ch}}\right)\right]$$

$$\updownarrow$$

$B_A \ll T$

Thermal model:

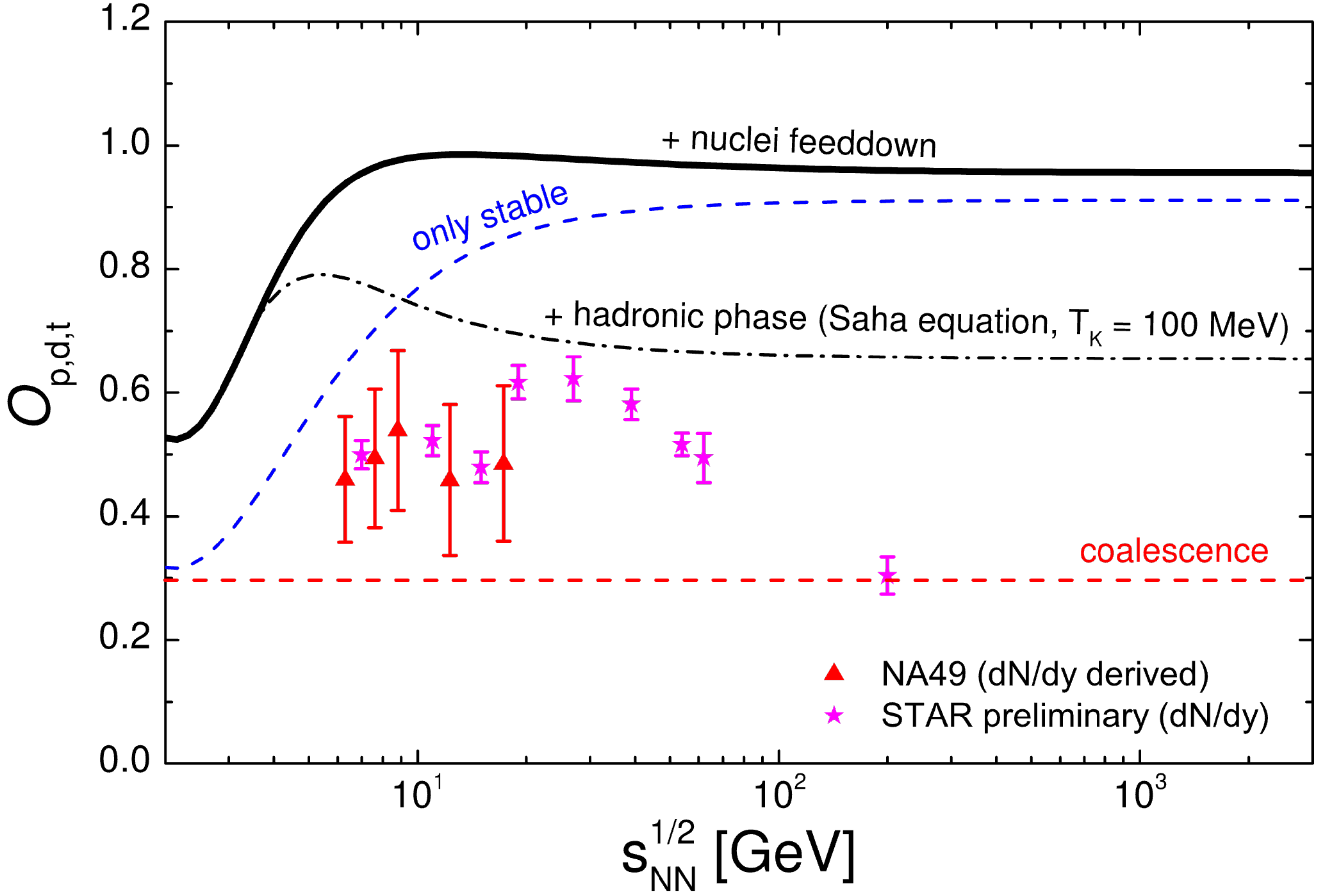
$$\left[\frac{N_A(T)}{N_A(T_{\rm ch})}\right]_{\rm eq.} \simeq \left(\frac{T}{T_{\rm ch}}\right)^{-\frac{3}{2}} \exp\left[-m_A\left(\frac{1}{T}-\frac{1}{T_{\rm ch}}\right)\right]$$

$m_A \gg T$

Nuclei yields are *not constant* in the Saha equation approach but the strong exponential dependence on the temperature is eliminated

Quantitative outcome is sensitive to the feeding from baryonic resonances

Saha equation at RHIC-BES



- Can obtain non-monotonic collision energy dependence
- The results get closer to coalescence due to diminished decay feeddown at 100 MeV

Formation times and heavy-ion time scales

Heisenberg: Formation time $t_{form} \sim \frac{1}{B_A} \sim 100$ fm/c $\gg$ heavy-ion time scales

How can one have nuclei degrees of freedom in thermal model/transport if it is not yet formed?

Study time-dependent dynamics with open quantum systems

Schroedinger equation

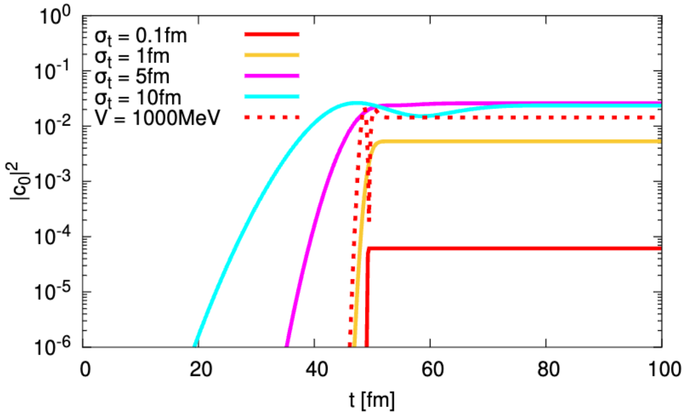
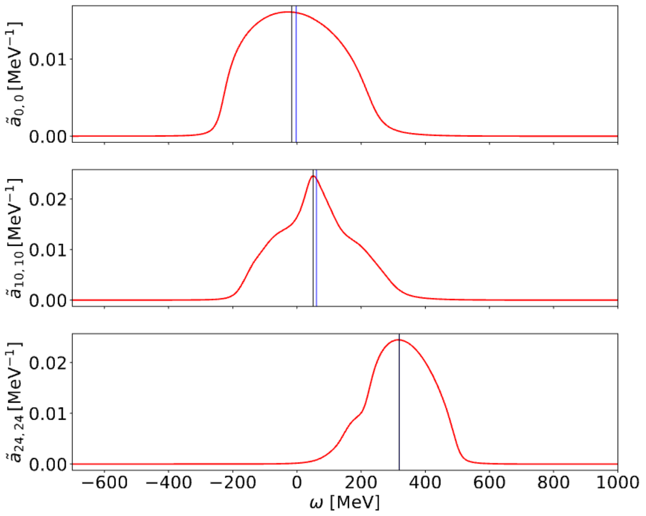


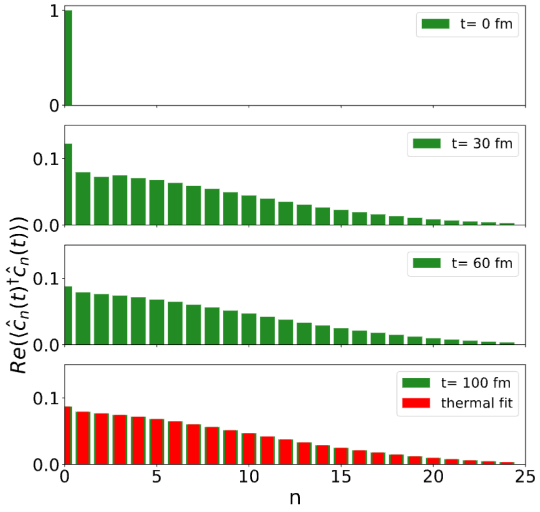
FIG. 20. Bound state formation of $|c_{50}(t=0)|^2 = 1$ for different time length (solid lines) at $V = 100$ MeV and a comparison of a strong potential of 1000 MeV with $\sigma_t = 1$ fm (dashed line), one pulse.

Rais, Hees, Greiner, PRC 106, 064004 (2022)

Kadanoff-Baym



Spectral functions



Occupation numbers

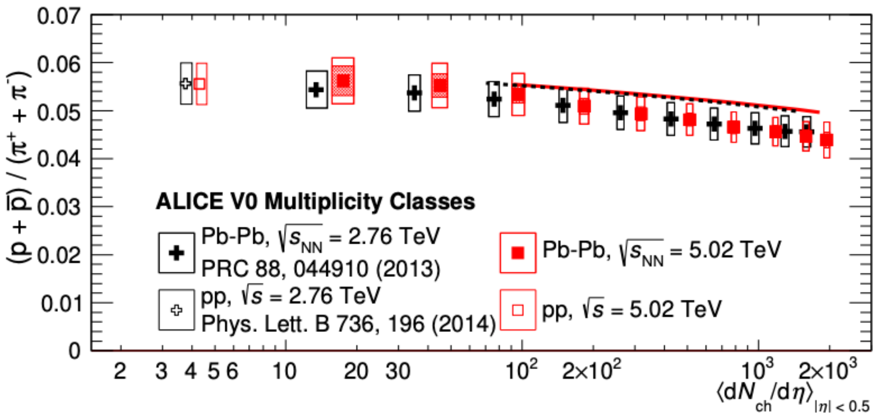
Neidig, Rais, Bleicher, Hees, Greiner, PLB 851, 138589 (2024)

Occupation numbers redistribute on time scales of energy transfer rather than binding energy

Baryon annihilation

Proton yields at the LHC: 5 TeV data

ALICE Collaboration, Phys. Rev. C 101 (2020) 044907



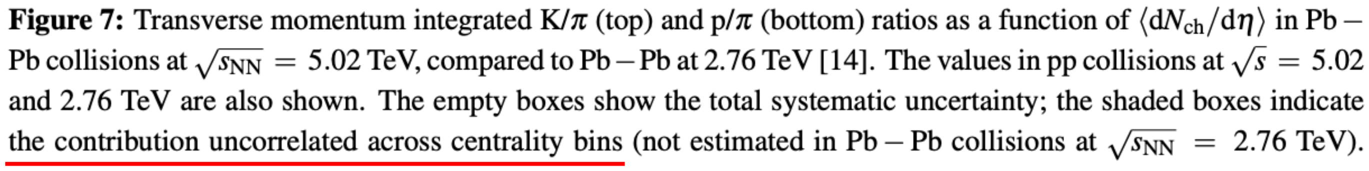
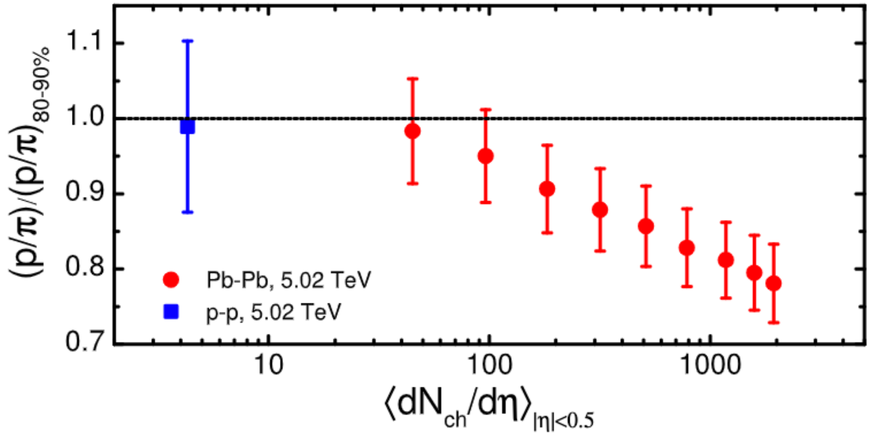


Figure 7: Transverse momentum integrated K/π (top) and p/π (bottom) ratios as a function of $\langle dN_{ch}/d\eta\rangle$ in Pb – Pb collisions at $\sqrt{s_{NN}}$ = 5.02 TeV, compared to Pb – Pb at 2.76 TeV [14]. The values in pp collisions at $\sqrt{s}$ = 5.02 and 2.76 TeV are also shown. The empty boxes show the total systematic uncertainty; the shaded boxes indicate the contribution uncorrelated across centrality bins (not estimated in Pb – Pb collisions at $\sqrt{s_{NN}}$ = 2.76 TeV).



- Evidence for suppression of p/pi ratio in central collisions (~20%, $>4\sigma$ level)
- Baryon annihilation in hadronic phase?

Partial chemical equilibrium with baryon annihilation

Add nucleon annihilations $N\bar{N} \leftrightarrow 5\pi$ into the PCE framework

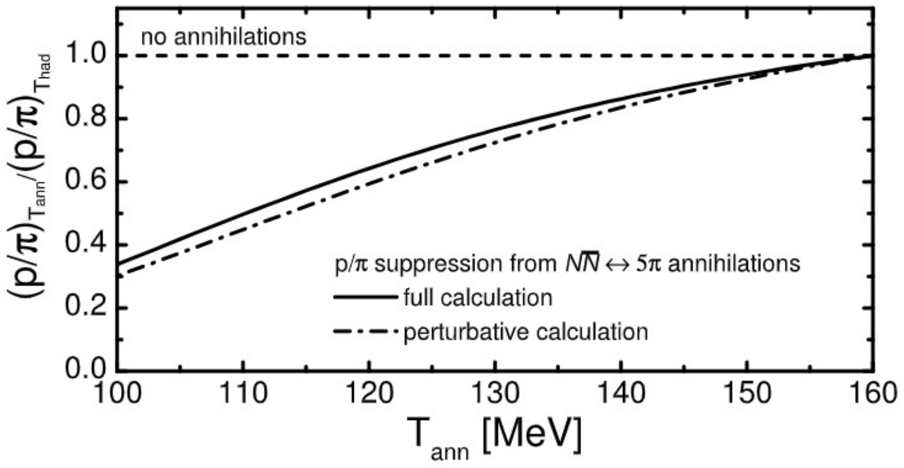
(Anti)nucleon and pions numbers no longer conserved, $N_N, N_{\bar{N}}, N_\pi \neq$ const. but

$$\frac{N_N + N_{\bar{N}}}{2} + \frac{N_\pi}{5} = \text{const}$$

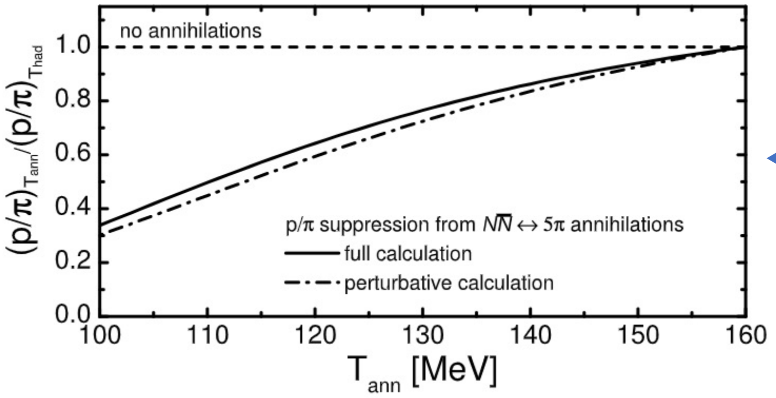
If $N\bar{N} \leftrightarrow 5\pi$ proceeds in relative equilibrium, $\boxed{\mu_N = \mu_{\bar{N}} = \frac{5}{2}\mu_\pi}$

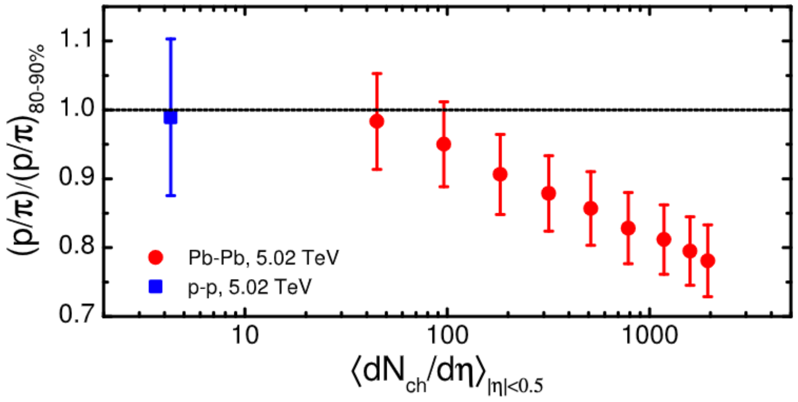
Also, $\pi N \leftrightarrow \Delta$ equilibrium implies $\Delta\bar{N} \leftrightarrow 6\pi$ and $\Delta\bar{\Delta} \leftrightarrow 7\pi$,
i.e. baryon resonances annihilate as well

p/ π ratio is suppressed during the cooling in the hadronic phase

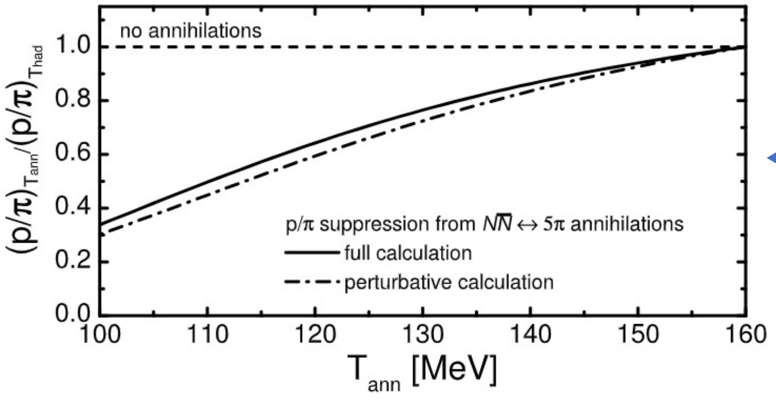


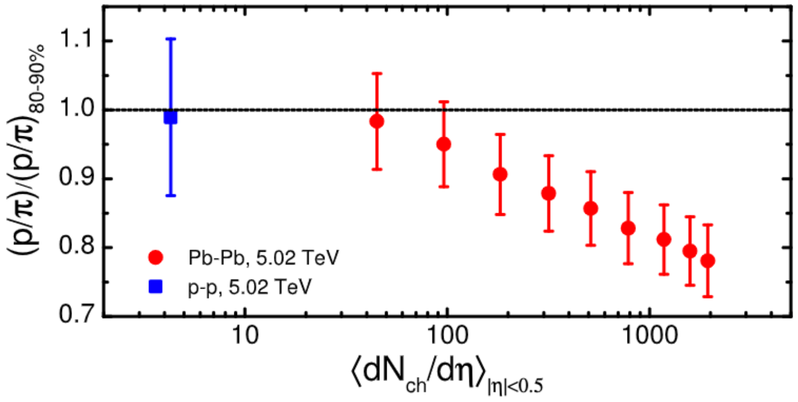
Baryon annihilation freeze-out temperature

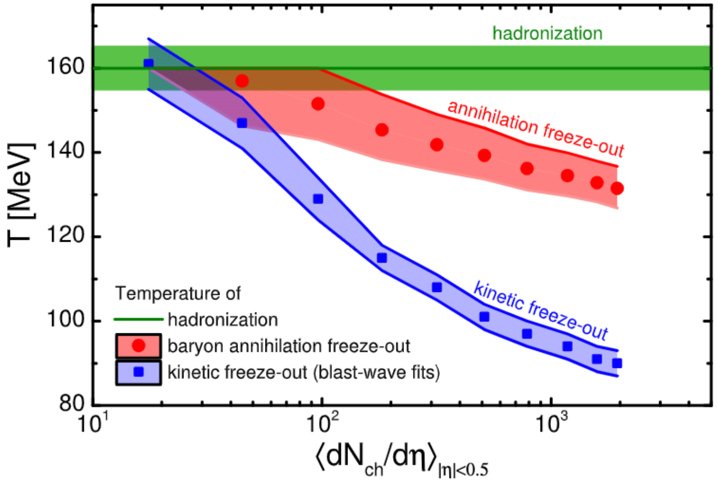




Baryon annihilation freeze-out temperature





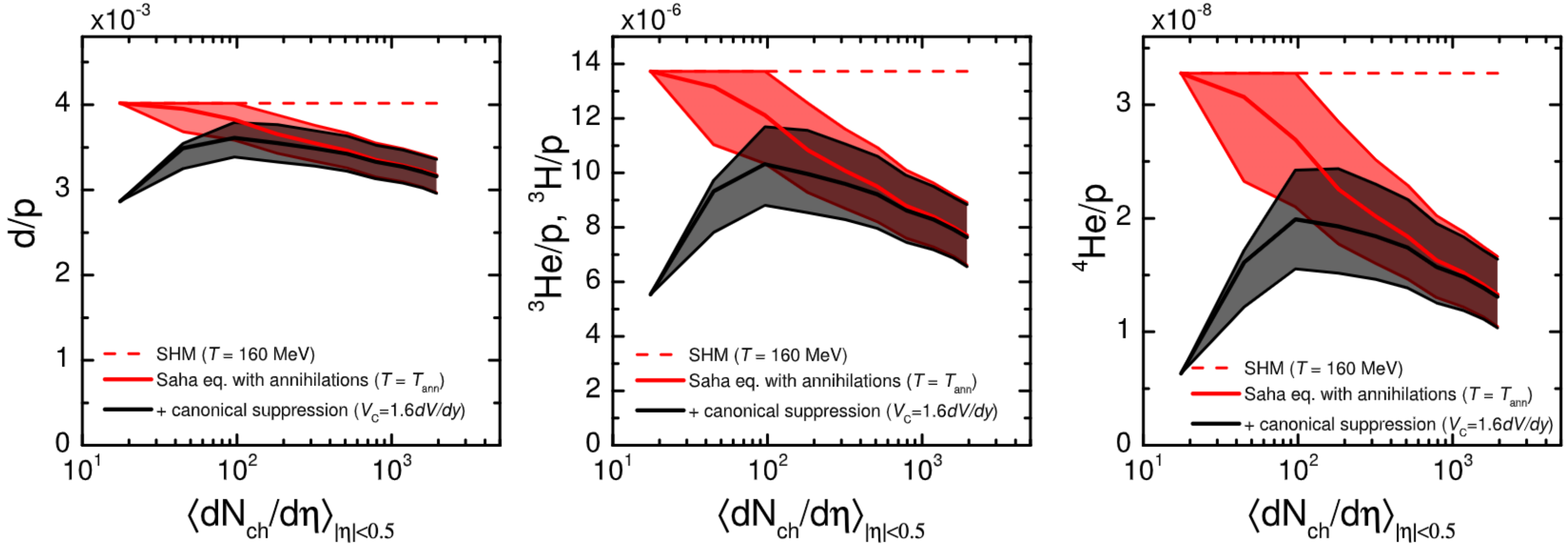


Centrality	$\langle dN_{ch}/d\eta \rangle$	T_{ann} [MeV]
0–5%	1943 ± 56	132 ± 5
5–10%	1587 ± 47	133 ± 5
10–20%	1180 ± 31	135 ± 5
20–30%	786 ± 20	136 ± 6
30–40%	512 ± 15	139 ± 6
40–50%	318 ± 12	142 ± 7
50–60%	183 ± 8	145 ± 8
60–70%	96.3 ± 5.8	152 ± 8
70–80%	44.9 ± 3.4	157^{+3}_{-11}
80–90%	17.5 ± 1.8	160

Baryon annihilation remains relevant in the initial stage of the hadronic phase but freezes out earlier than (pseudo-)elastic hadron scatterings

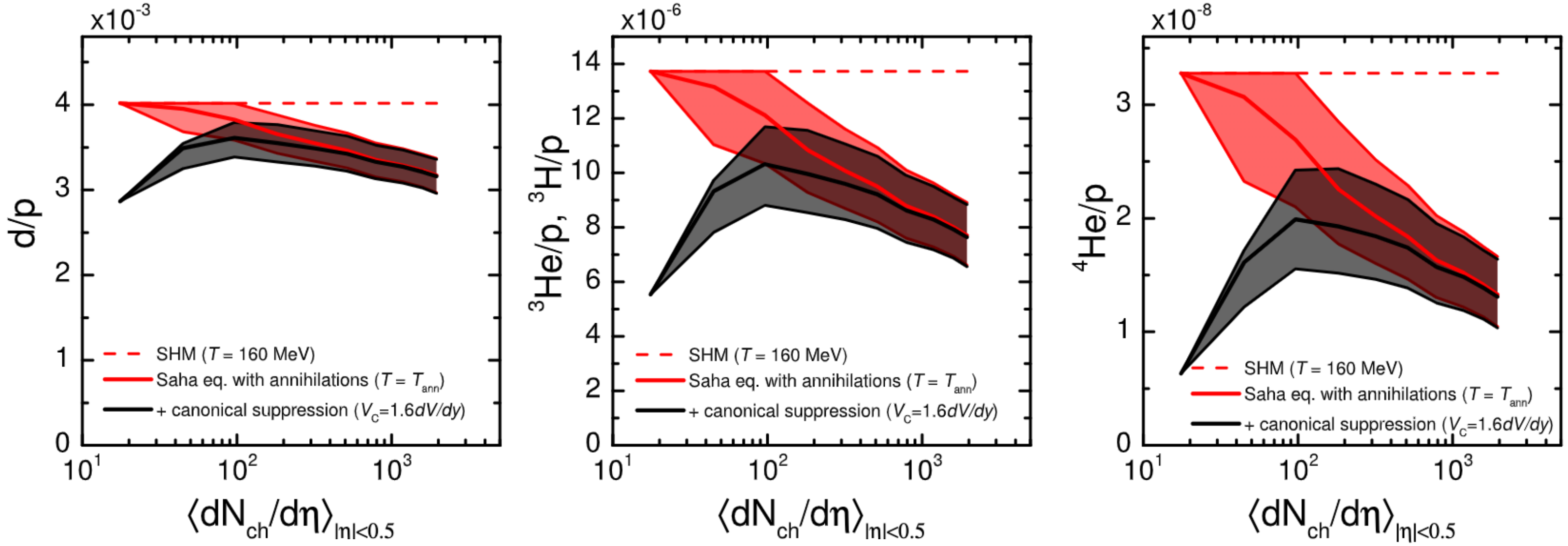
Baryon annihilation and light nuclei

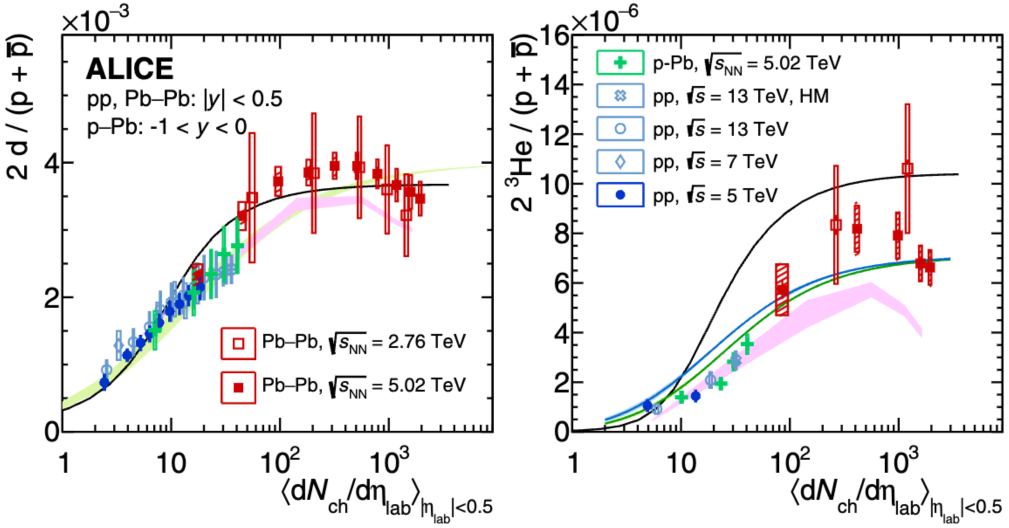
Saha equation with baryon annihilation, at $T = T_{\mathrm{ann}}$

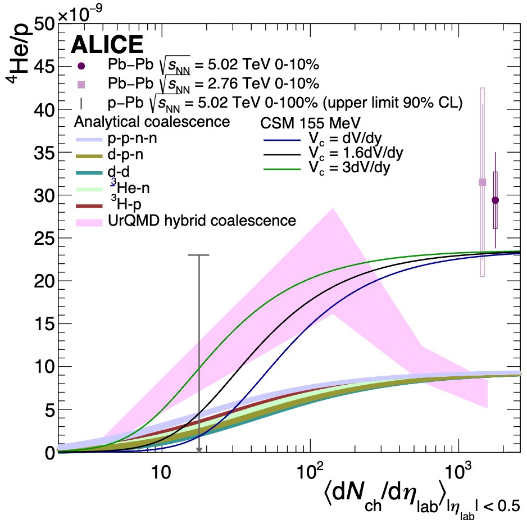


Baryon annihilation and light nuclei

Saha equation with baryon annihilation, at $T = T_{ann}$



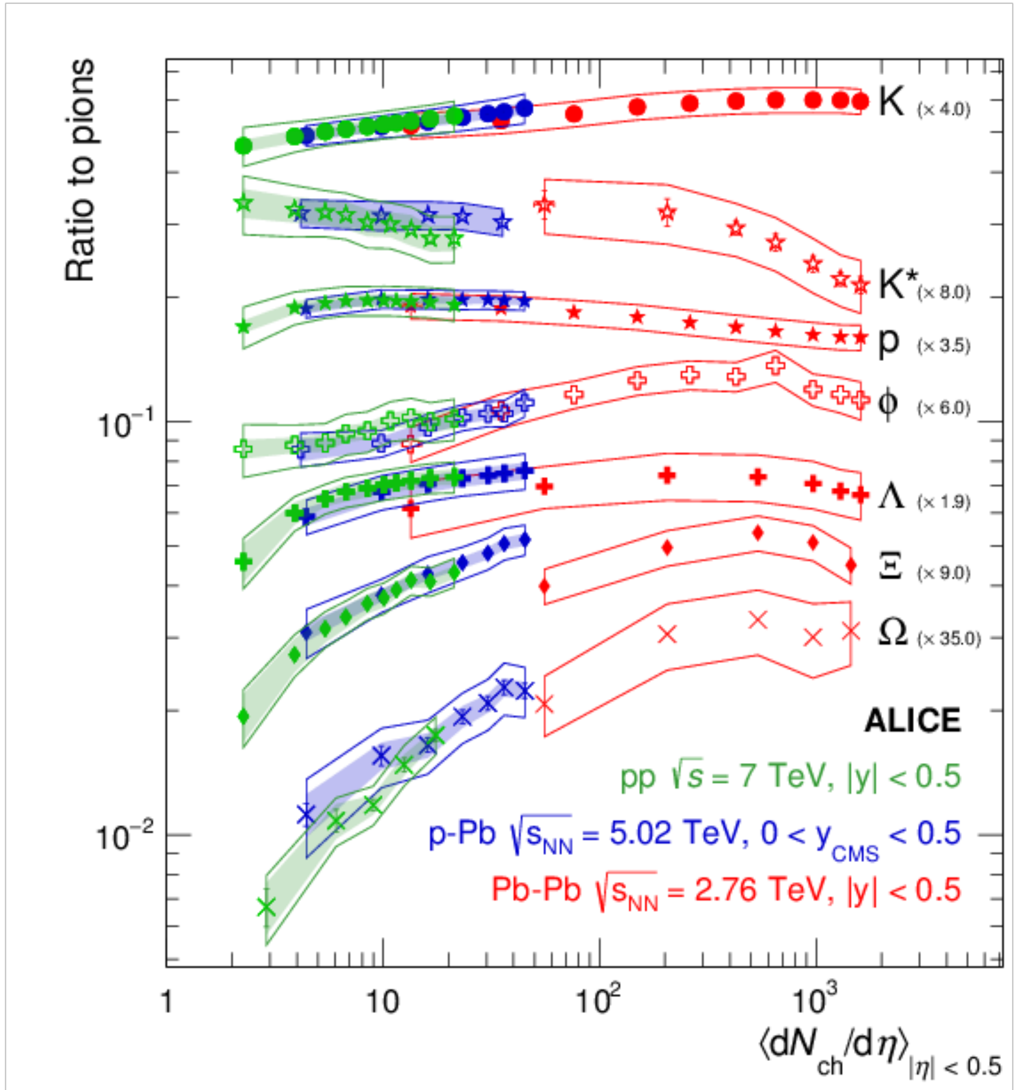




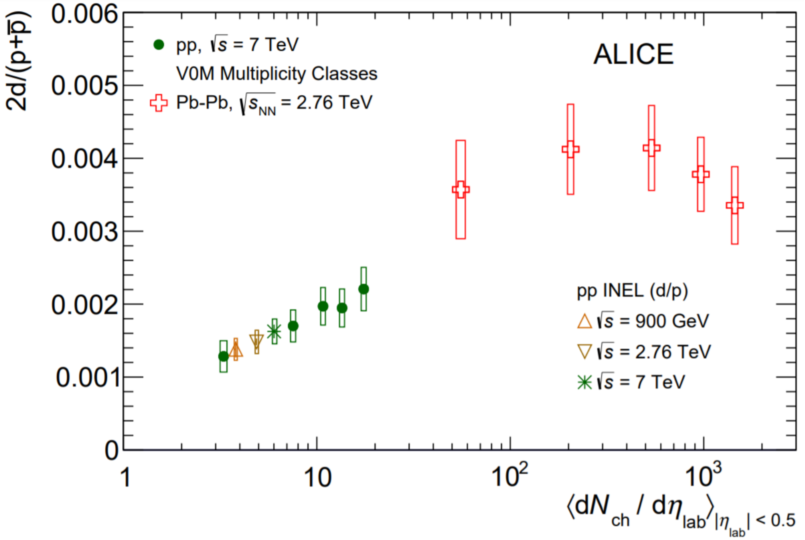
- Better description of d & ^{3}He
- Makes ^{4}He worse

Small systems and canonical suppression of light nuclei

Multiplicity dependence of hadrochemistry



[ALICE collaboration, 1807.11321]



[ALICE collaboration, 1902.09290]

- Grand-canonical thermal picture predicts no multiplicity dependence
- Apply canonical statistical model?

Canonical statistical model (CSM)

Exact conservation of B, Q, S in a correlation volume V_C

[Rafelski, Danos, et al., PLB '80; Hagedorn, Redlich, ZPC '85]

$$\mathcal{Z}(B,Q,S) = \int_{-\pi}^{\pi} \frac{d\phi_B}{2\pi} \int_{-\pi}^{\pi} \frac{d\phi_Q}{2\pi} \int_{-\pi}^{\pi} \frac{d\phi_S}{2\pi} \, e^{-i\,(B\phi_B + Q\phi_Q + S\phi_S)} \, \exp\left[\sum_j z_j^1 \, e^{i\,(B_j\phi_B + Q_j\phi_Q + S_j\phi_S)}\right]$$

$$z_j^1 = V_c \int dm \, \rho_j(m) \, d_j \frac{m^2 T}{2\pi^2} K_2(m/T) \qquad \langle N_j^{\rm prim} \rangle^{\rm ce} = \frac{Z(B-B_j, Q-Q_j, S-S_j)}{Z(B,Q,S)} \langle N_j^{\rm prim} \rangle^{\rm gce}$$

[Becattini et al., ZPC '95, ZPC '97]

Implemented in Thermal-FIST for full HRG

Exact conservation across k units of rapidity, giving correlation volume

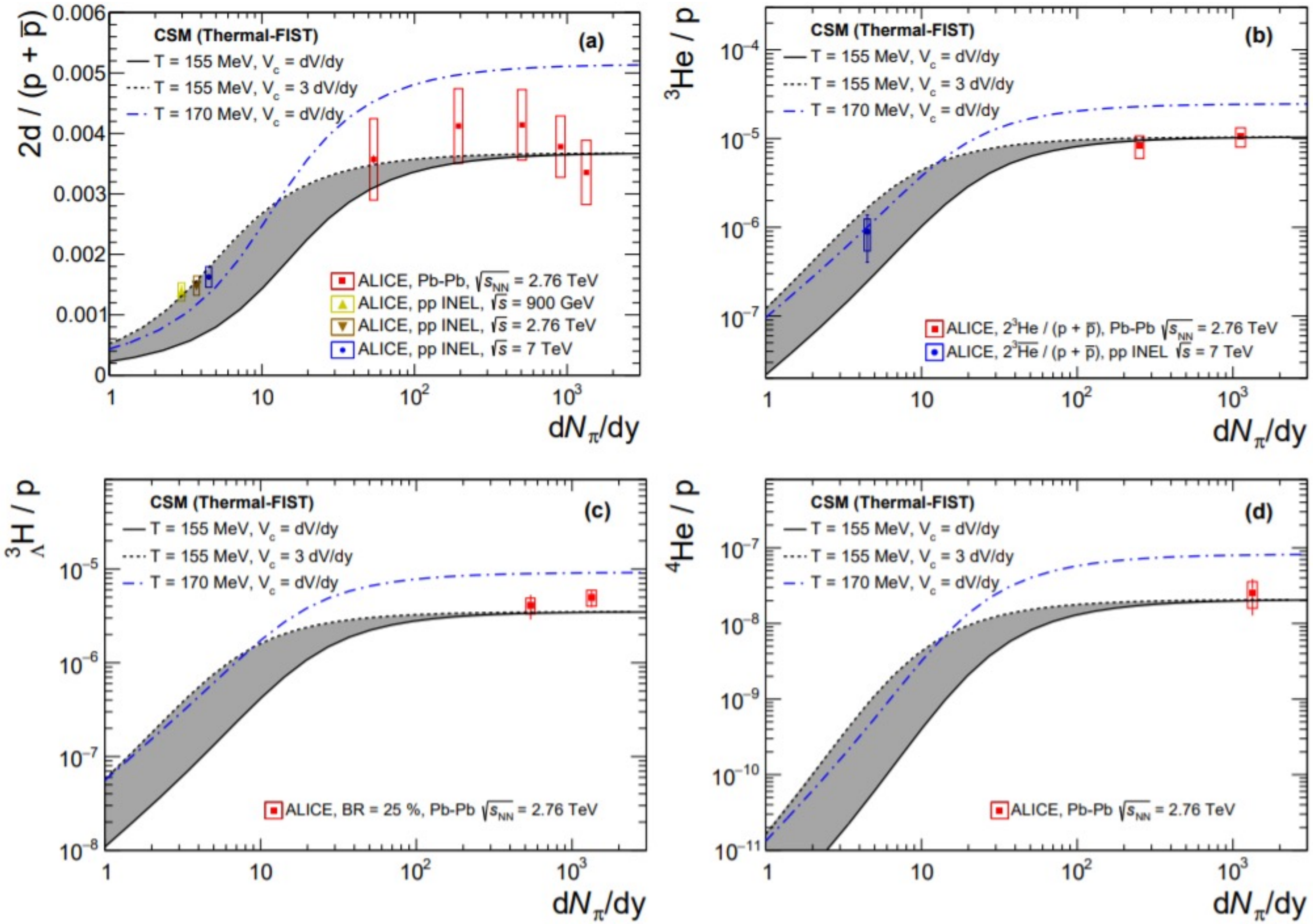
$$V_C = k dV/dy$$

- Hadron yield fits (pp, pPb, PbPb): $k \sim 3$ [VV, Dönigus, Stoecker, 1906.03145, PRC '19]
- Proton fluctuations (PbPb): $k \geq 3$ [ALICE Collaboration, PLB 807, 135564 (2020)]
- Xi-Kaon correlations (PbPb): $k \approx 3$ [M. Ciacco (ALICE), QM2023]
- Proton-deuteron correlations (PbPb): $k \sim 1.6$ [ALICE Collaboration, PRL 131, 041901 (2022)]
- Hadron/deuteron yield fits (pp, pPb): $k < 4$ [Sharma, Kumar, Lo, Redlich, PRC 107, 054903 (2023)]

“Vanilla” CSM

$T_{ch} = 155$ MeV, $V_C = 3dV/dy$, multiplicity dependence driven by V_C only

[VV, Dönigus, Stoecker, 1808.05245, PLB ‘18]



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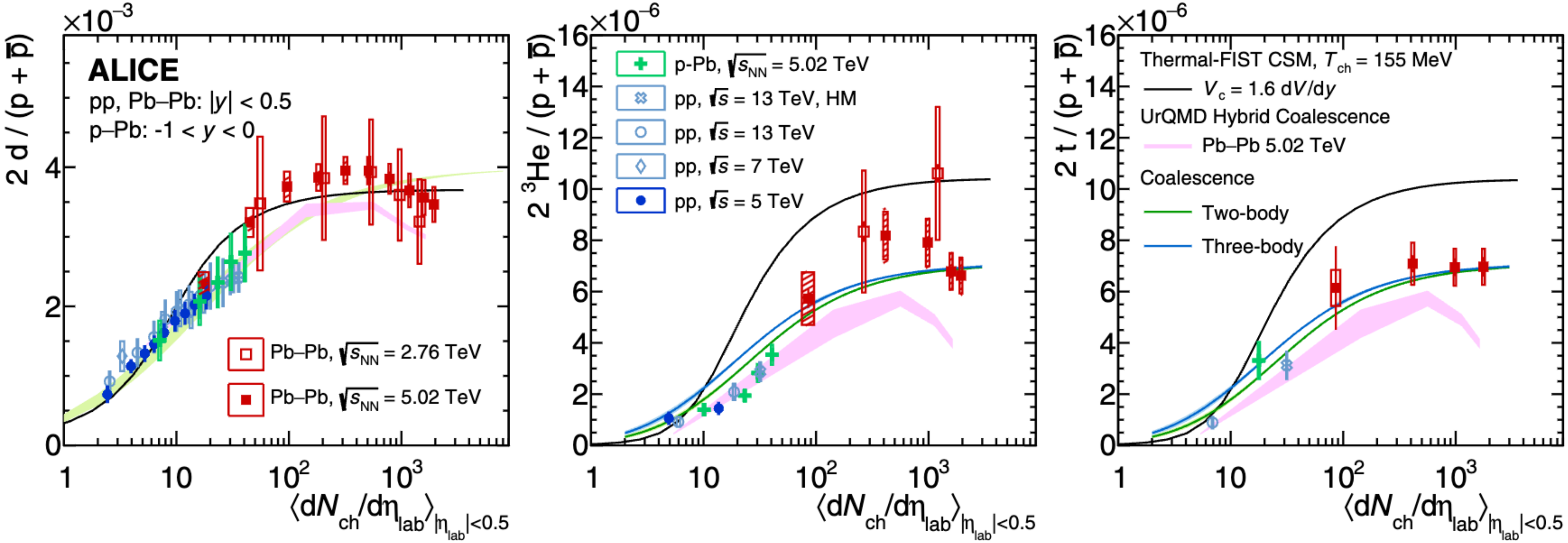


ALICE Collaboration, PRC 107, 064904 (2023)

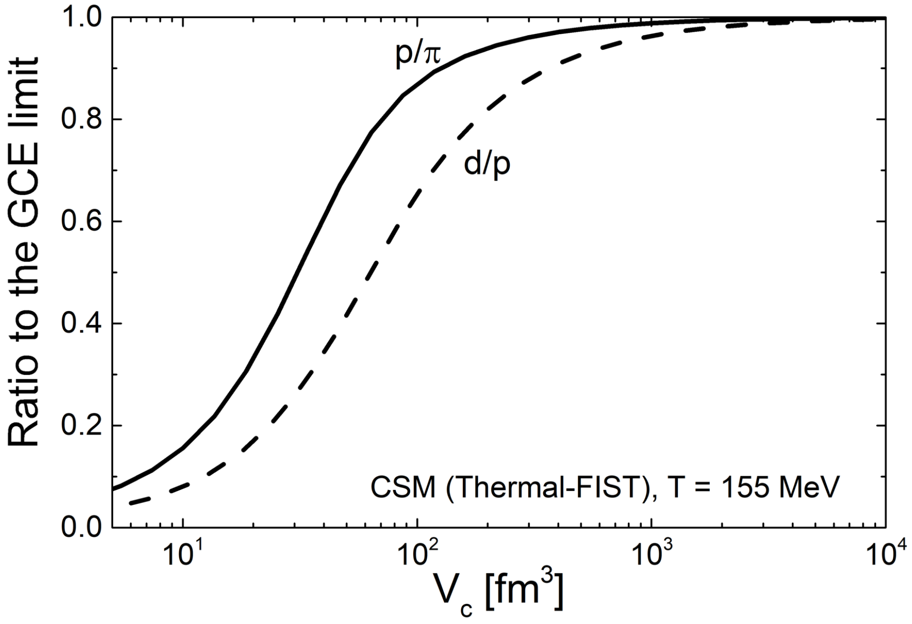
Basic CSM appears to capture trends seen in light nuclei production data

Yields of t and ^{3}He overestimated

“Vanilla” CSM: nuclei vs p/π ratio

Canonical suppression affects not only nuclei, but also the p/π ratio

The effect for p/π is generally milder than d/p, but not insignificant



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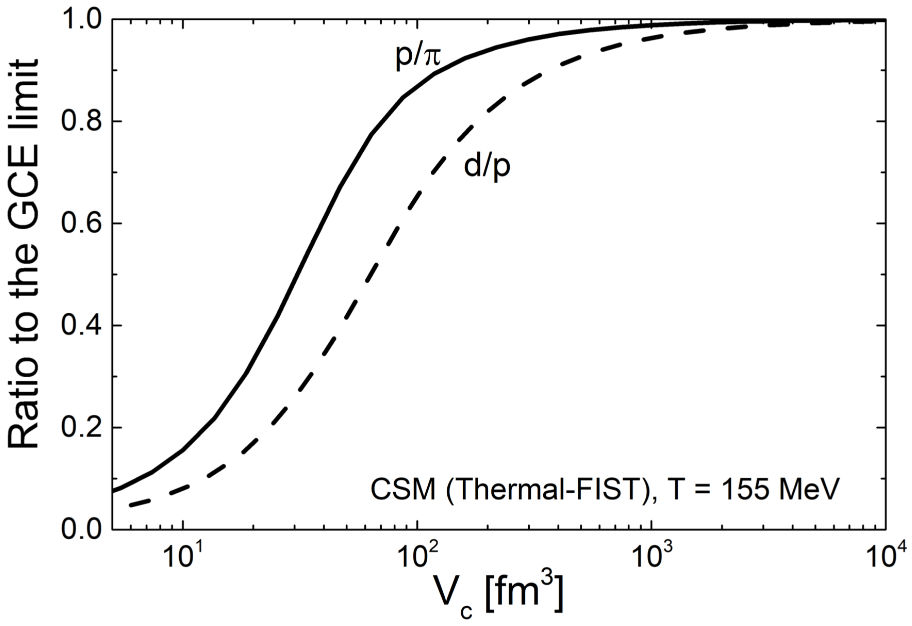
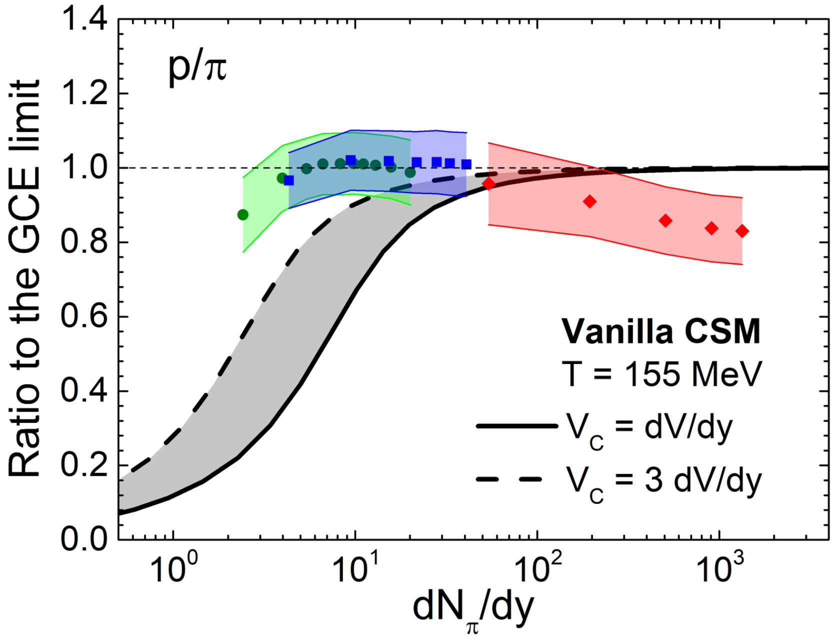


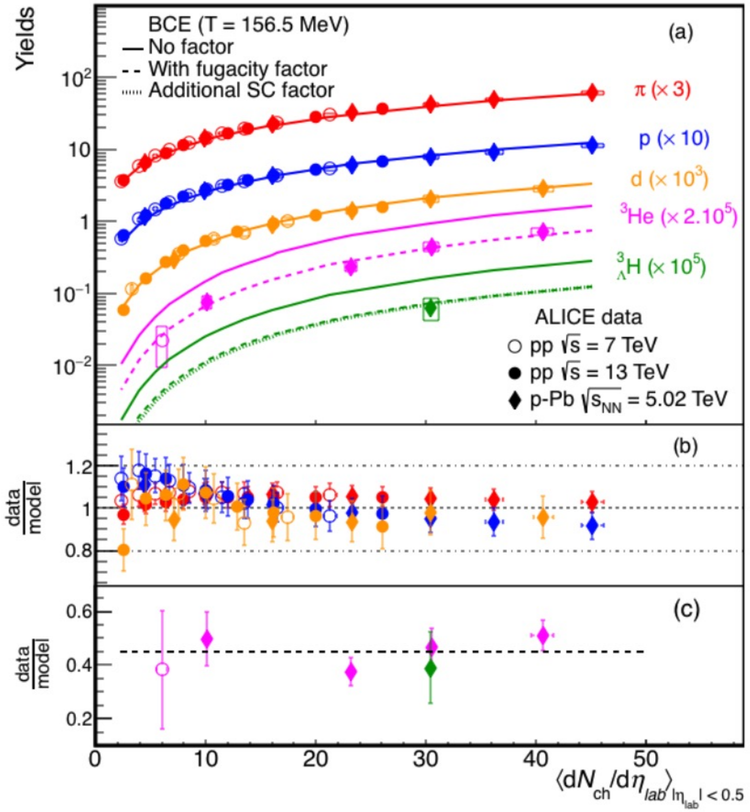


p/π suppression predicted by vanilla CSM not supported by the data

Challenging to describe light nuclei and p/π ratios simultaneously

Multiplicity dependent correlation volume

- Baryon-canonical ensemble THERMUS fits to p-p and p-Pb yields of pions, protons, and deuterons
- Correlation volume $V_C = k dV/dy$ is a free fit parameter at each multiplicity

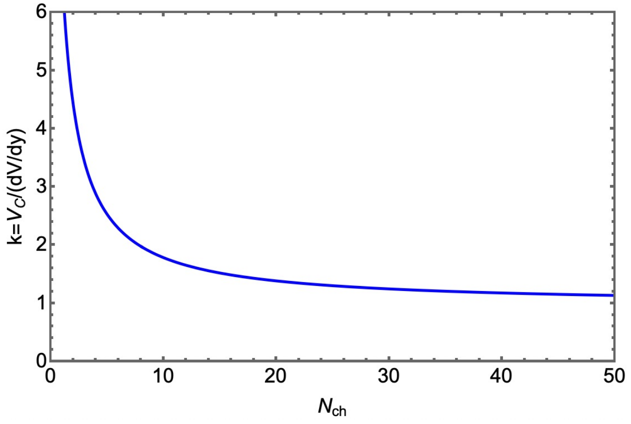


Sharma, Kumar, Lo, Redlich, PRC 107, 054903 (2023)

- V_C and dV/dy can be parameterized as

$$V_C \simeq 27.3 + 2.9 \times \frac{dN_{ch}}{d\eta} \qquad dV/dy \simeq 1.55 + 3.0 \times \frac{dN_{ch}}{d\eta}$$

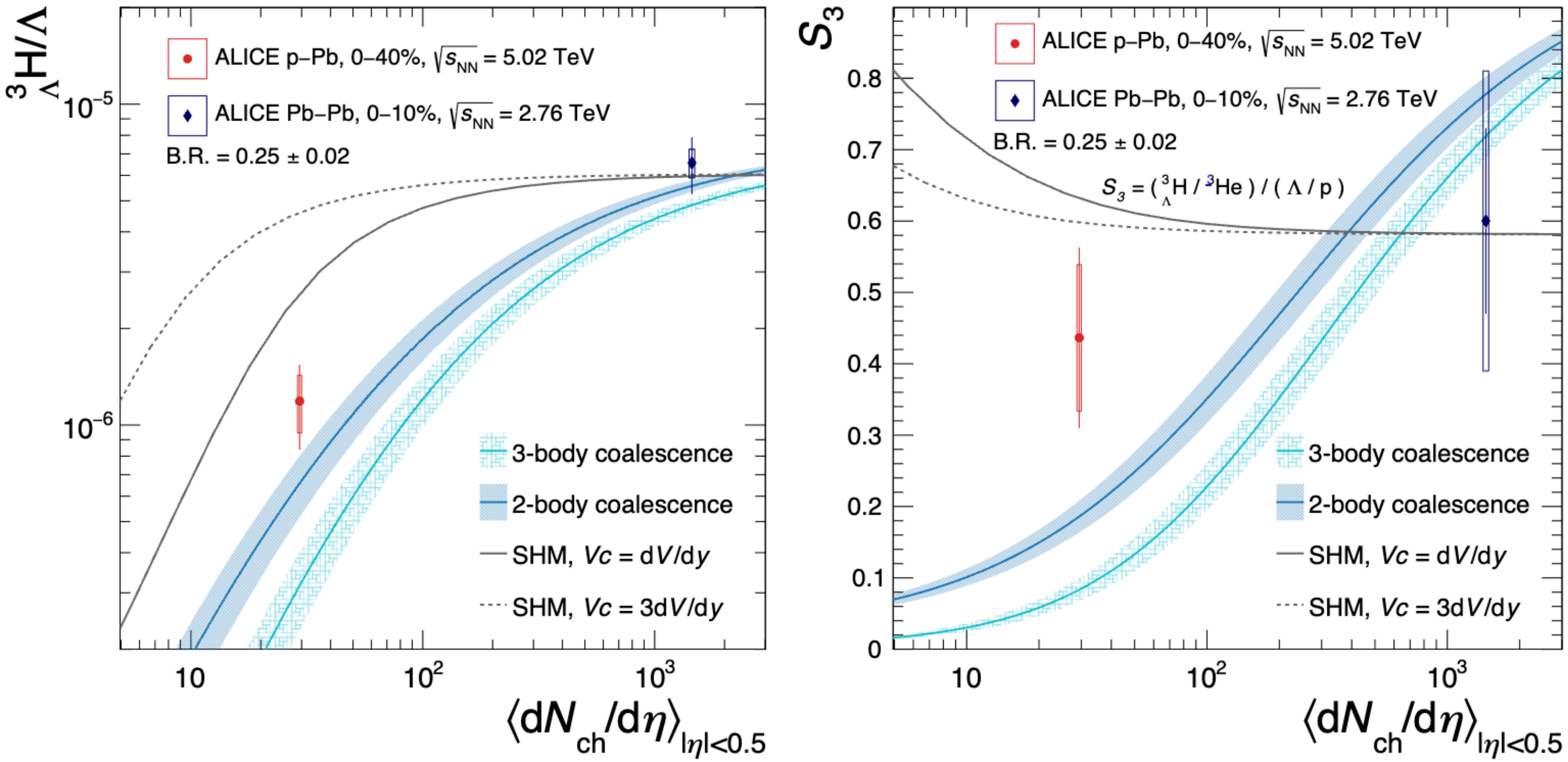
- Giving multiplicity-dependent correlation length



- Deuterons described while ^{3}He & hypertriton overpredicted by factor 2

Hypertriton

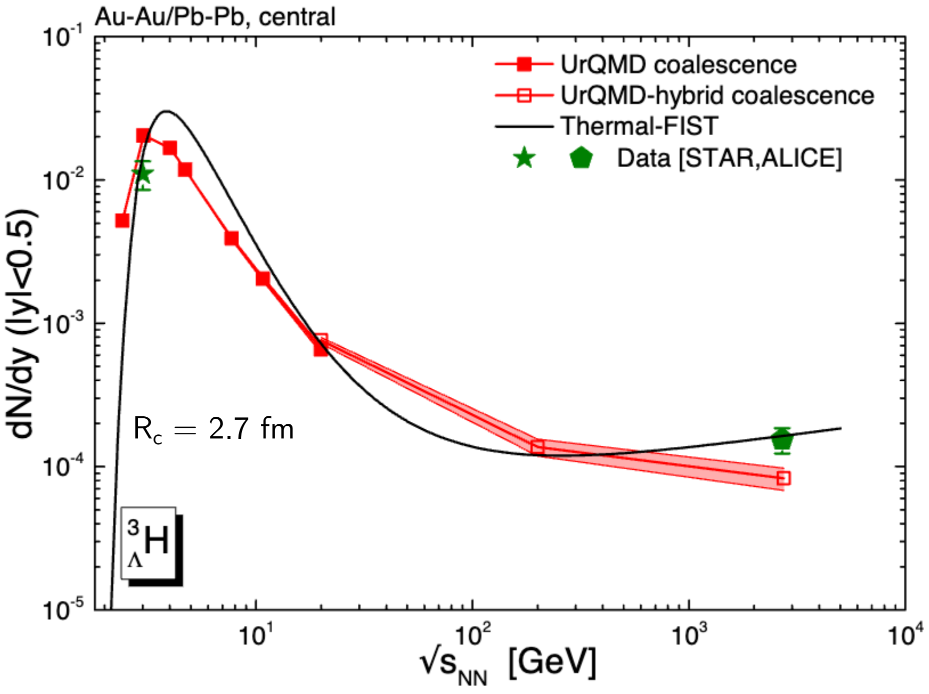
$$S_3 = ({}^{3}_{\Lambda}\mathrm{H}/{}^{3}\mathrm{He}) / (\Lambda/\mathrm{p})$$



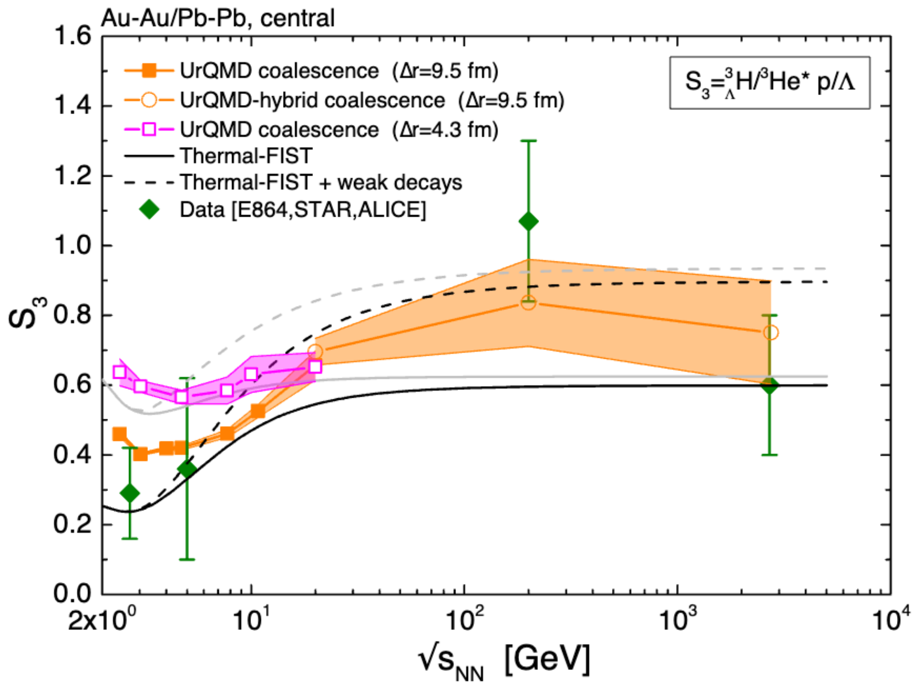
ALICE Collaboration, PRL 128, 252003 (2022)

- Canonical effects essentially cancel out in $S_{3,}$ making it good probe to test production mechanisms

Hypertriton



$$S_3 = ({}^{3}_{\Lambda}\mathrm{H}/{}^{3}\mathrm{He}) / (\Lambda/p)$$



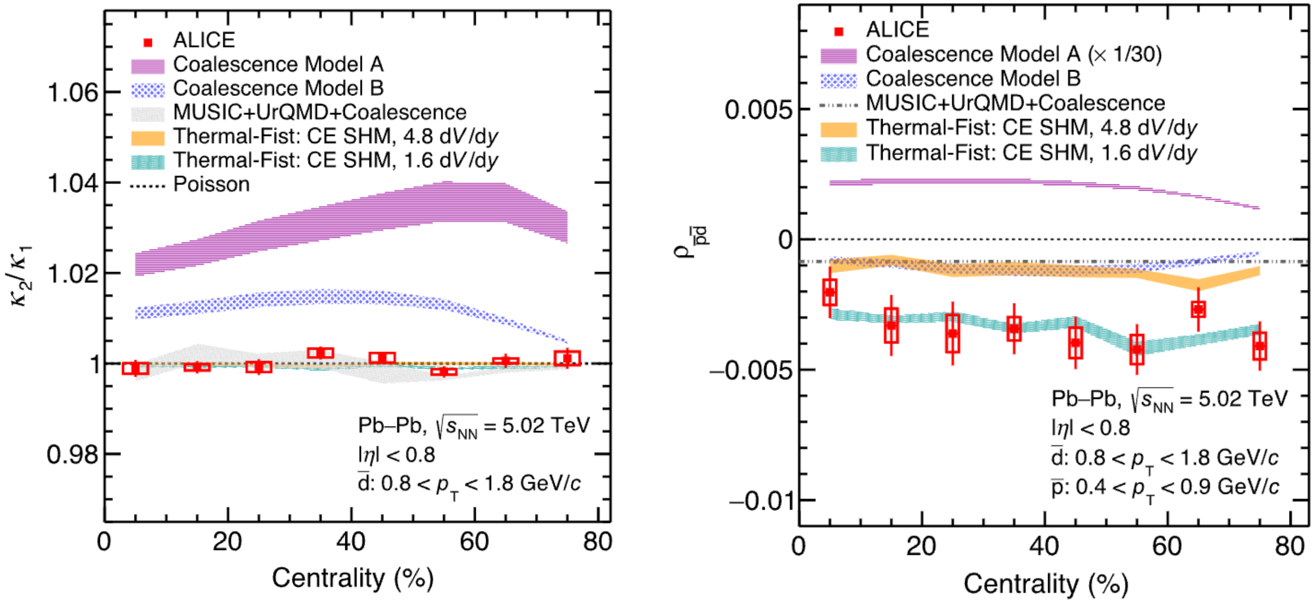
T. Reichert et al., PRC 107, 014912 (2023)

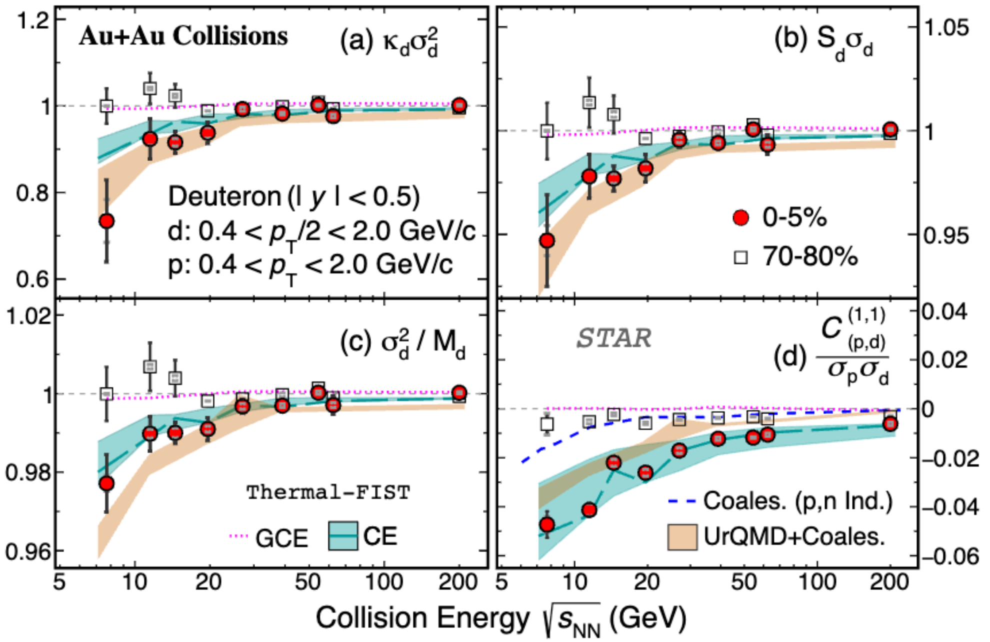
At small energies hypertriton is affected by strangeness canonical suppression

Fluctuations and correlations

Correlations and fluctuations involving nuclei serve as additional probe of production mechanism

Evaluated in given acceptance using the Monte Carlo counterpart of the thermal model (FIST sampler)





ALICE Collaboration, PRL 131, 041901 (2022)

STAR Collaboration, 2304.10993

$V_c = 1.6\, dV/dy$

$V_c = 2 - 4\, dV/dy$

Summary

- Thermal model is a powerful tool with very few parameters that describes yields of (anti)(hyper)nuclei within an order of magnitude
    - Generally good quantitative description of the deuteron yield
    - Overpredicts t & ^{3}He by factor x1.5-2 at RHIC and LHC
    - Apparent survival in hot medium can be attributed to balance between destruction and regeneration reactions that approximately maintain the thermal yield
- Extended descriptions
    - Saha equation: nuclei can freeze out at any point in the hadronic phase
    - Baryon annihilation suppresses the yields in central collisions, helps with t & ^{3}He but not ^{4}He
- Canonical effects
    - Suppression of nuclei yields in small systems and affects fluctuations and correlations
    - Different observables give somewhat conflicting constraints on the rapidity extent of correlation volume

Thanks for your attention!

Backup slides

Light nuclei: Saha equation

Detailed balance for nuclear reactions

$$\frac{n_A}{\prod_i n_{A_i}} = \frac{n_A^{\mathrm{eq}}}{\prod_i n_{A_i}^{\mathrm{eq}}}, \quad \underset{\textit{Saha equation}}{\Leftrightarrow} \quad \mu_A = \sum_i \mu_{A_i}, \qquad \text{e.g.} \quad \mu_d = \mu_p + \mu_n, \ \mu_{^3\mathrm{He}} = 2\mu_p + \mu_n, \ \ldots$$

Kinetic theory example: deuteron number evolution through $p + n + X \leftrightarrow d + X$ reactions

$$\underset{\textbf{\textit{small}}}{\frac{dN_d}{d\tau}} = \overset{\textbf{gain}}{\underset{\textbf{\textit{big}}}{\langle \sigma_{dX} v_{rel} \rangle \, N_d^0 \, n_x^0 \, e^{\mu_p/T} \, e^{\mu_n/T} \, e^{\mu_X/T}}} - \overset{\textbf{loss}}{\underset{\textbf{\textit{big}}}{\langle \sigma_{dX} v_{rel} \rangle \, N_d^0 \, n_x^0 \, e^{\mu_d/T} \, e^{\mu_X/T}}}$$

$$\text{gain} \approx \text{loss} \qquad \rightarrow \qquad \mu_d \approx \mu_p + \mu_n$$

Saha equation

= detailed balance

= law of mass action

Early Universe: $X_A = d_A \left[\zeta(3)^{A-1} \pi^{\frac{1-A}{2}} 2^{\frac{3A-5}{2}} \right] A^{\frac{5}{2}} \left(\frac{T}{m_N} \right)^{\frac{3}{2}(A-1)} \eta^{A-1} X_p^Z X_n^{A-Z} \exp\left(\frac{B_A}{T} \right)$

E. Kolb, M. Turner, “The Early Universe” (1990)

LHC nucleosynthesis: simplified setup

- Chemical equilibrium lost at $T_{ch} = 155$ MeV, abundances of nucleons are frozen and acquire effective fugacity factors: $n_i = n_i^{eq} e^{\mu_N/T}$
- Isentropic expansion driven by effectively massless mesonic d.o.f.

$$\frac{V}{V_{\rm ch}} = \left(\frac{T_{\rm ch}}{T}\right)^3, \qquad \mu_N \simeq \frac{3}{2} T \ln\left(\frac{T}{T_{\rm ch}}\right) + m_N \left(1 - \frac{T}{T_{\rm ch}}\right)$$

- Detailed balance for nuclear reactions, $X + A \leftrightarrow X + \sum_i A_i$, X is e.g. a pion

$$\underbrace{\frac{n_A}{\prod_i n_{A_i}} = \frac{n_A^{\rm eq}}{\prod_i n_{A_i}^{\rm eq}}}_{\text{Saha equation}}, \quad \Leftrightarrow \quad \mu_A = \sum_i \mu_{A_i}, \qquad \text{e.g. } \mu_d = \mu_p + \mu_n, \; \mu_{^3\rm He} = 2\mu_p + \mu_n, \; \ldots$$

$$\Downarrow$$

$$\boxed{X_A = d_A \left[(d_M)^{A-1} \zeta(3)^{A-1} \pi^{\frac{1-A}{2}} 2^{-\frac{3+A}{2}}\right] A^{5/2} \left(\frac{T}{m_N}\right)^{\frac{3}{2}(A-1)} \eta_B^{A-1} \exp\left(\frac{B_A}{T}\right)}$$

$$d_M \sim 11 - 13, \quad \eta_B \simeq 0.03$$

BBN: $X_A = d_A \left[\zeta(3)^{A-1} \pi^{\frac{1-A}{2}} 2^{\frac{3A-5}{2}}\right] A^{\frac{5}{2}} \left(\frac{T}{m_N}\right)^{\frac{3}{2}(A-1)} \eta^{A-1} X_p^Z X_n^{A-Z} \exp\left(\frac{B_A}{T}\right)$

[E. Kolb, M. Turner, "The Early Universe" (1990)]

Light nuclei production with rate equations

Catalyzed light nuclei reactions. Destruction through $AX \to \sum_i A_i X$ and creation through $\sum_i A_i X \to AX$. Detailed balance principle respected but *relative chemical equilibrium not enforced*

$$\frac{dN_A}{d\tau} = \langle \sigma^{\rm in}_{AX} v_{\rm rel} \rangle \, n_X \, (N_A^{\rm saha} - N_A)$$

Static fireball: $n_X \,, N_A^{\rm saha}, \langle \sigma^{\rm in}_{AX} v_{rel} \rangle = const$

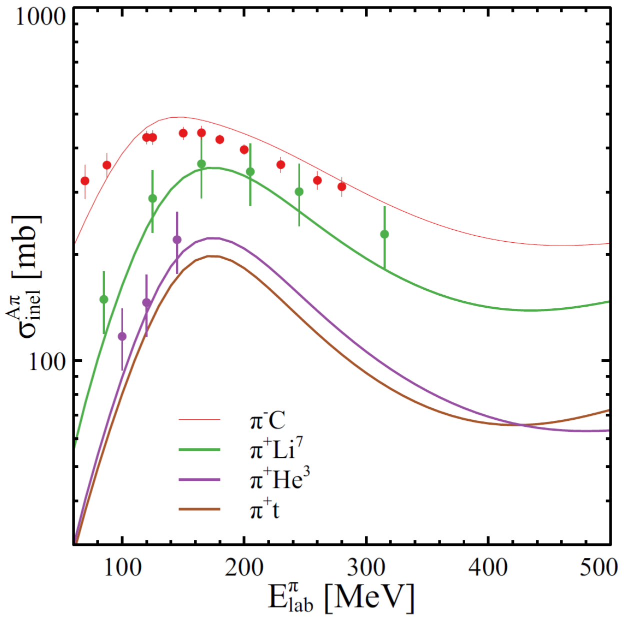
$$N_A(\tau) = N_A^{\rm saha} + (N_A(\tau_0) - N_A^{\rm saha})\, e^{-\frac{\tau-\tau_0}{\tau_{\rm eq}}} \,, \qquad \tau_{\rm eq} = \frac{1}{\langle \sigma^{\rm in}_{AX} v_{\rm rel} \rangle \, n_X}$$

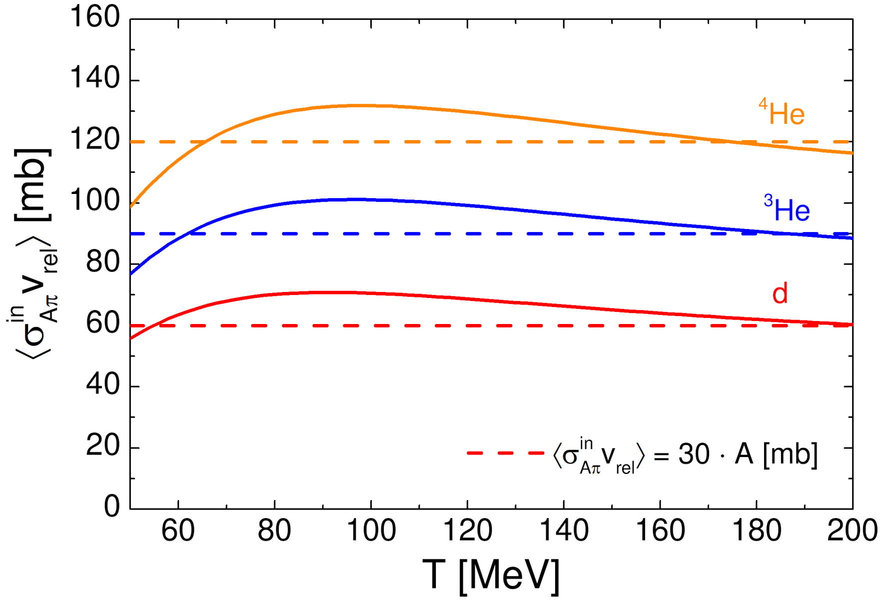
Saha limit: $\tau_{eq} \to 0 \; (\sigma^{\rm in}_{AX} \to \infty)$

Model input

- **Rates:** Use guidance from kinetic theory

Optical model for $\sigma_{A\pi}^{\text{in}}$ [J. Eisenberg, D.S. Koltun, '80]





Being implemented in SMASH transport [D. Oliinychenko's talk]

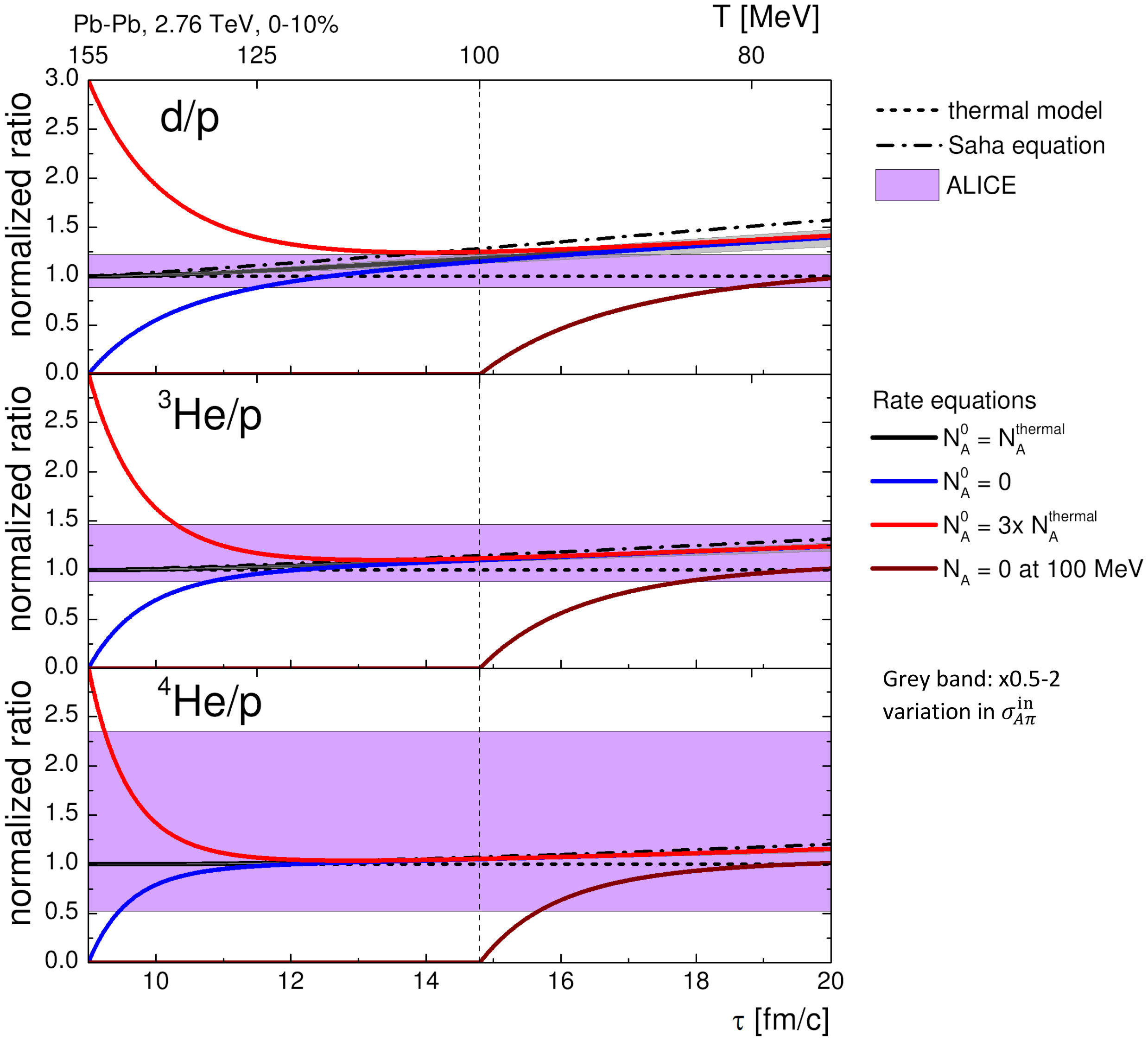
In practice $\langle \sigma_{A\pi}^{\text{in}} v_{rel} \rangle \gtrsim 30 \cdot A$ [mb]

- **Expansion** (both transverse and longitudinal)

$$\frac{V}{V_{\text{ch}}} = \frac{\tau}{\tau_{\text{ch}}} \frac{\tau_\perp^2 + \tau^2}{\tau_\perp^2 + \tau_{\text{ch}}^2}, \qquad \tau_{\text{ch}} = 9 \text{ fm}, \qquad \tau_\perp = 6.5 \text{ fm}$$

[Y. Pan, S. Pratt, PRC 89, 044911 (2014)]

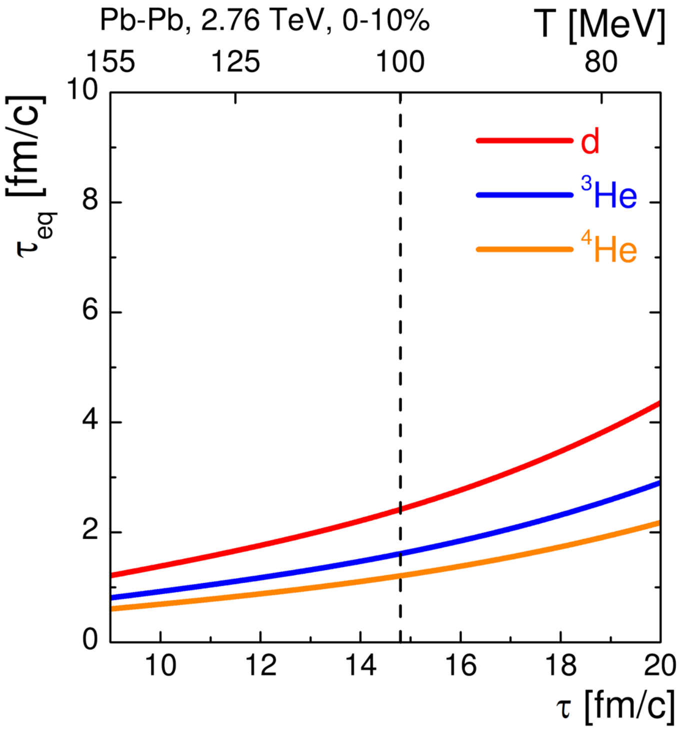
Rate equations at LHC



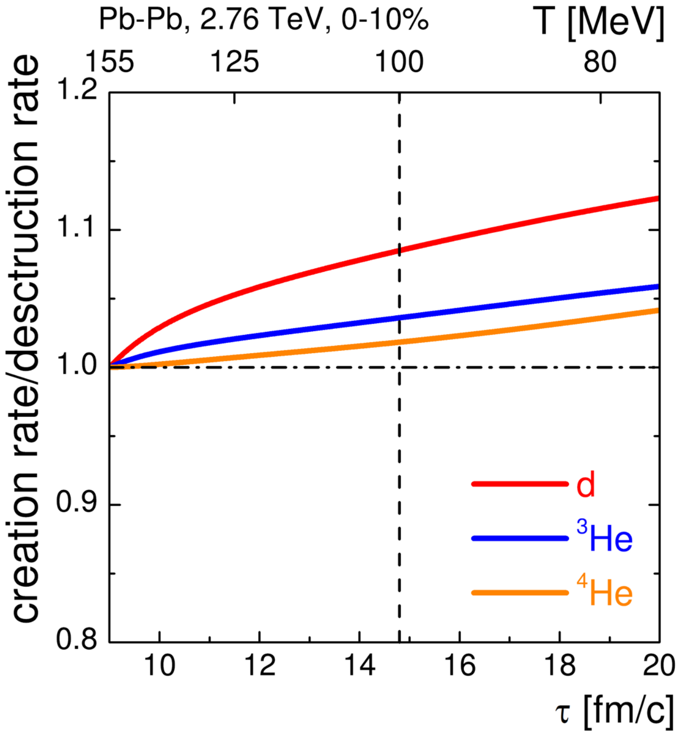
Grey band: x0.5-2 variation in $\sigma_{A\pi}^{\text{in}}$

Rate equations at LHC

$$\tau_{\text{eq}}^{-1} = \langle \sigma_{A\pi}^{\text{in}} v_{rel} \rangle \, n_{\pi}^{\text{pce}}$$



gain vs loss

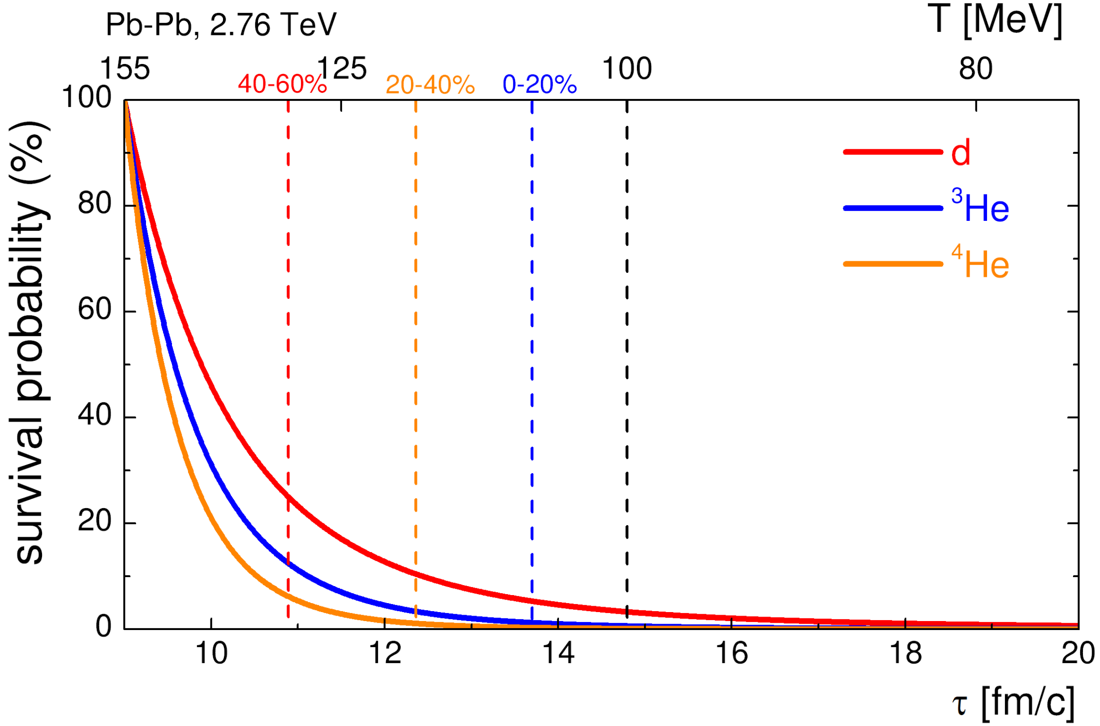


- Local equilibration times remain small
- $\tau_A^{\text{eq}} \ll B_A^{-1}$ meaning light nuclei are not fully formed
- (**gain** + **loss**) ≫ |**gain** – **loss**| → **Saha equation at work**

Can snowballs survive hell?

Count only the nuclei produced at "QGP hadronization" (at T_{ch})

$$\frac{dN_A^{qgp}}{d\tau} = -\langle \sigma_{A\pi} v_{rel} \rangle n_\pi^{pce} N_A^{qgp}, \qquad \text{survival probability} = N_A^{qgp}(\tau)/N_A^{qgp}(\tau_{ch})$$



The observed nuclei are unlikely to be (pre-)formed at the "QCD phase boundary" even the "thermal" production mechanism is correct

Rate equation for nuclei and resonances

Treat both the nuclear reactions and resonances decays and regenerations with rate equations, i.e. partial chemical equilibrium is not enforced

Rate equations

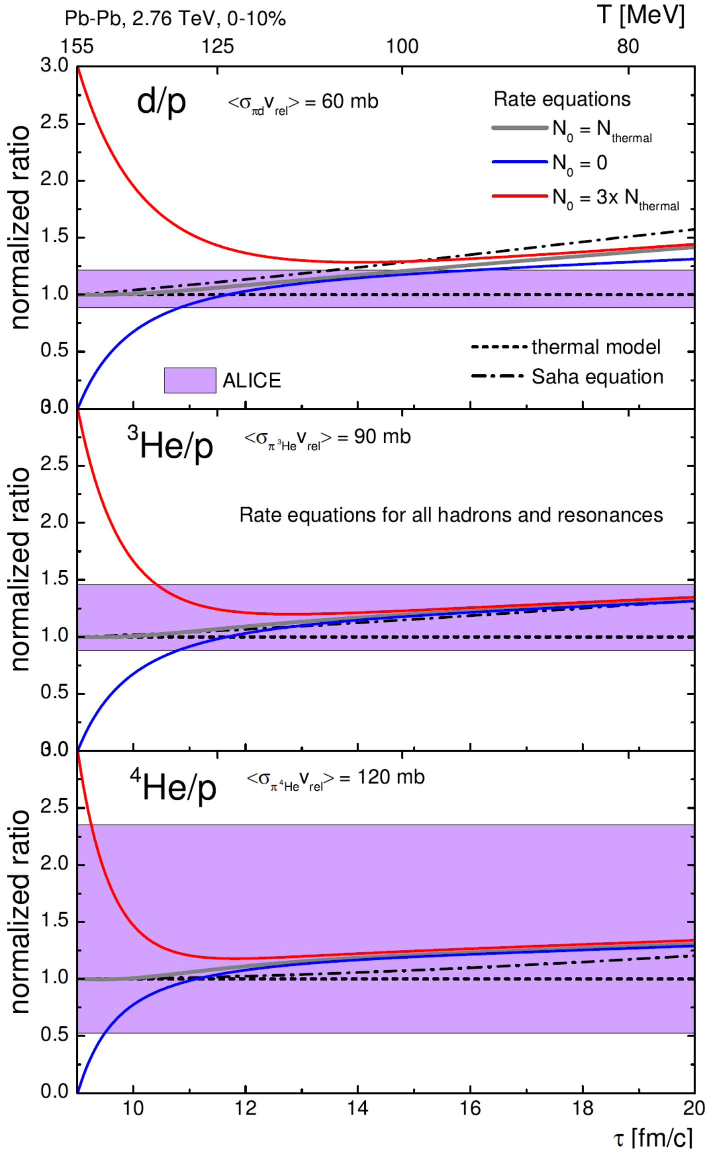
nuclei $$\frac{dN_A}{d\tau} = \langle \sigma_{A\pi} v_{rel} \rangle N_\pi \, n_A^{\mathrm{eq}} (e^{A\mu_N/T} - e^{\mu_A/T})$$

resonances $$\frac{dN_R}{d\tau} = \langle \Gamma_{R \to \sum_i a_i} \rangle N_R^{\mathrm{eq}} (e^{\sum_{i \in R} \mu_i/T} - e^{\mu_R/T})$$

Entropy production:

Increases by 0.6% between $T_{\mathrm{ch}} = 155$ MeV and $T_{\mathrm{kin}} = 100$ MeV

Results remain very close to the Saha equation



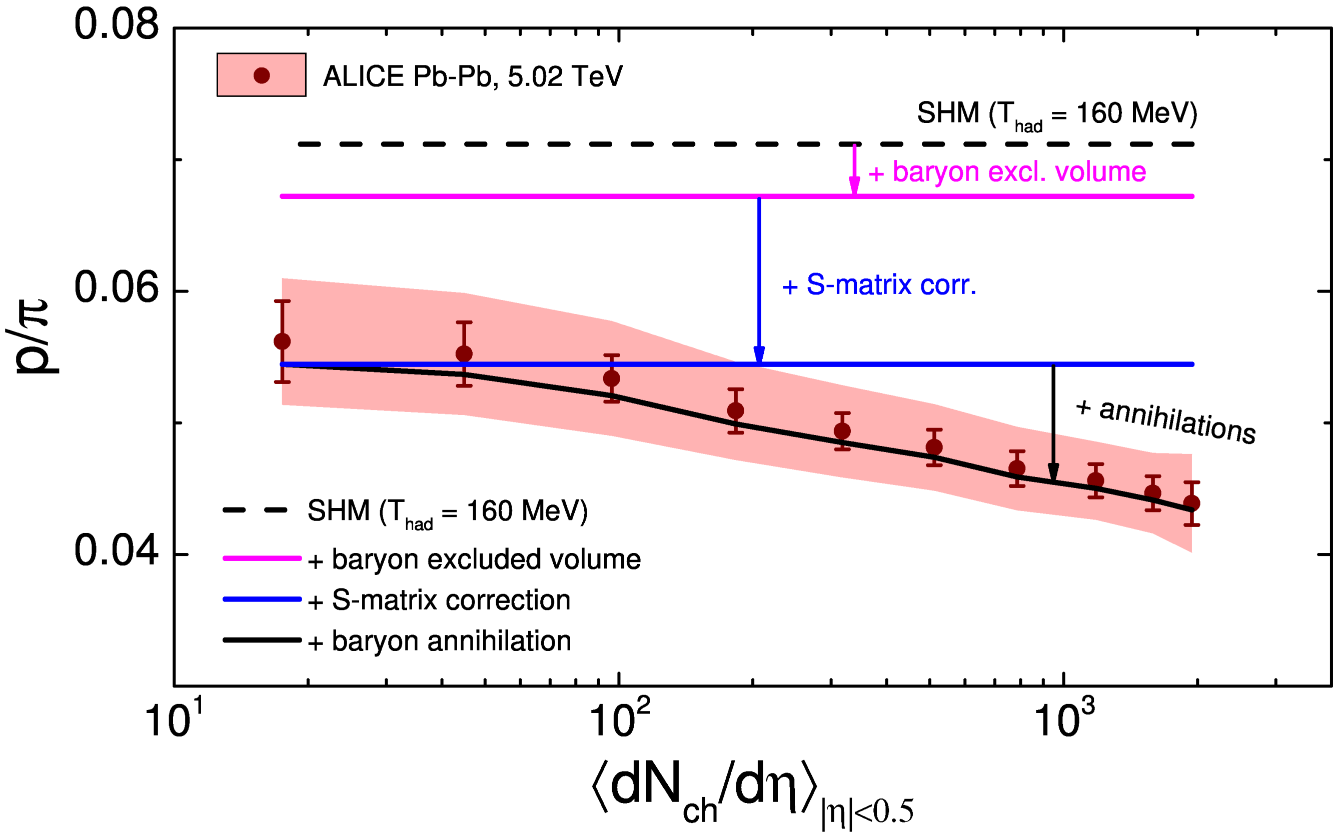
Annihilation vs other mechanisms affecting the p/π ratio

SHM: Thermal-FIST
[VV, Stoecker, Comput.Phys.Commun. 244 (2019) 295]

Baryon excl. volume
(baryon-baryon int.)
[VV et al., PLB 775 (2017) 71]

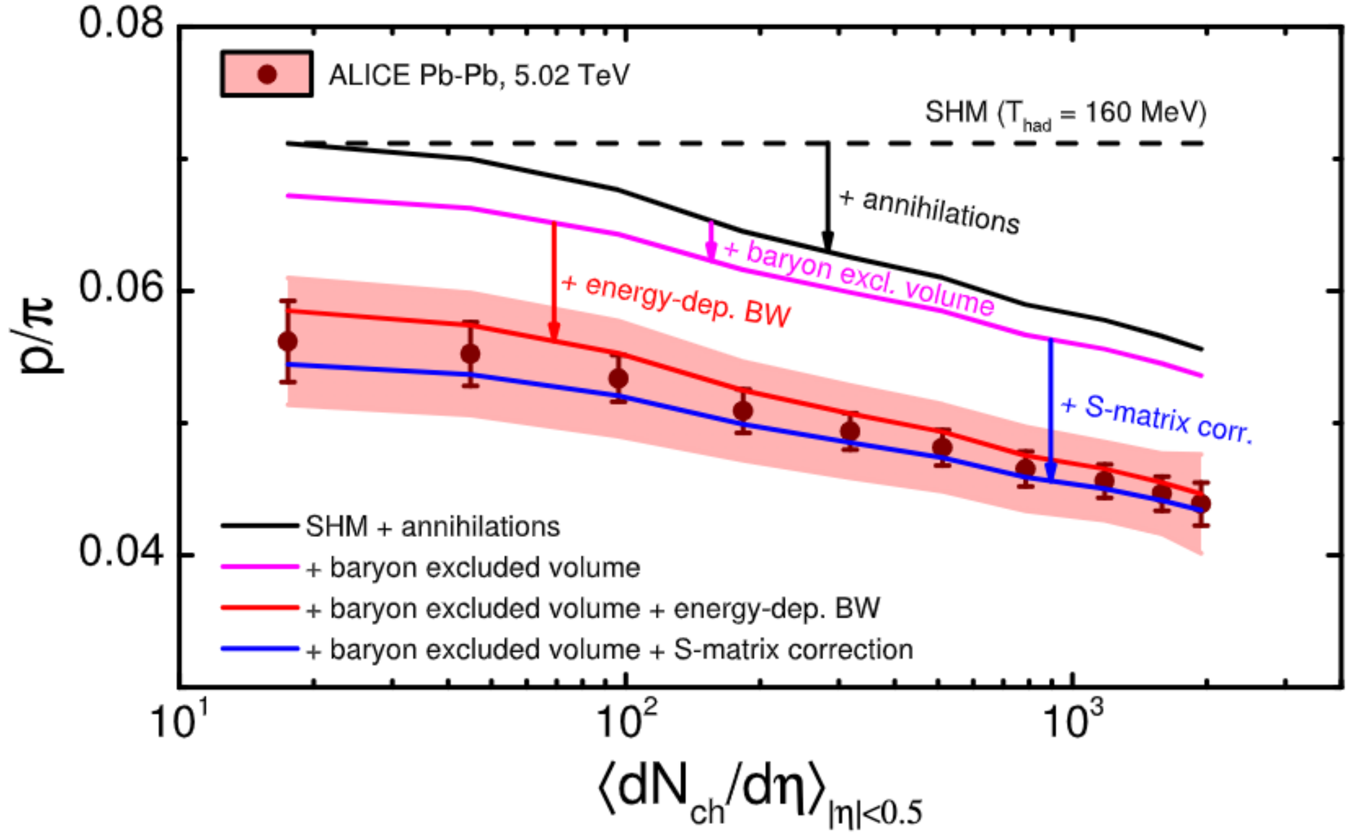
S-matrix correction
(meson-baryon int.)
[Andronic et al., PLB 792 (2019) 304]

Baryon annihilation
(baryon-antibaryon int.)
[VV, Koch, PLB 835 (2022) 137577]



Baryon annihilation and other mechanisms are complementary

Another way to look at it



Baryon annihilation and other mechanisms are complementary