Multi-dimensional fit approach for particle tracking through GLAD

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Tracking fast charged particles through GLAD

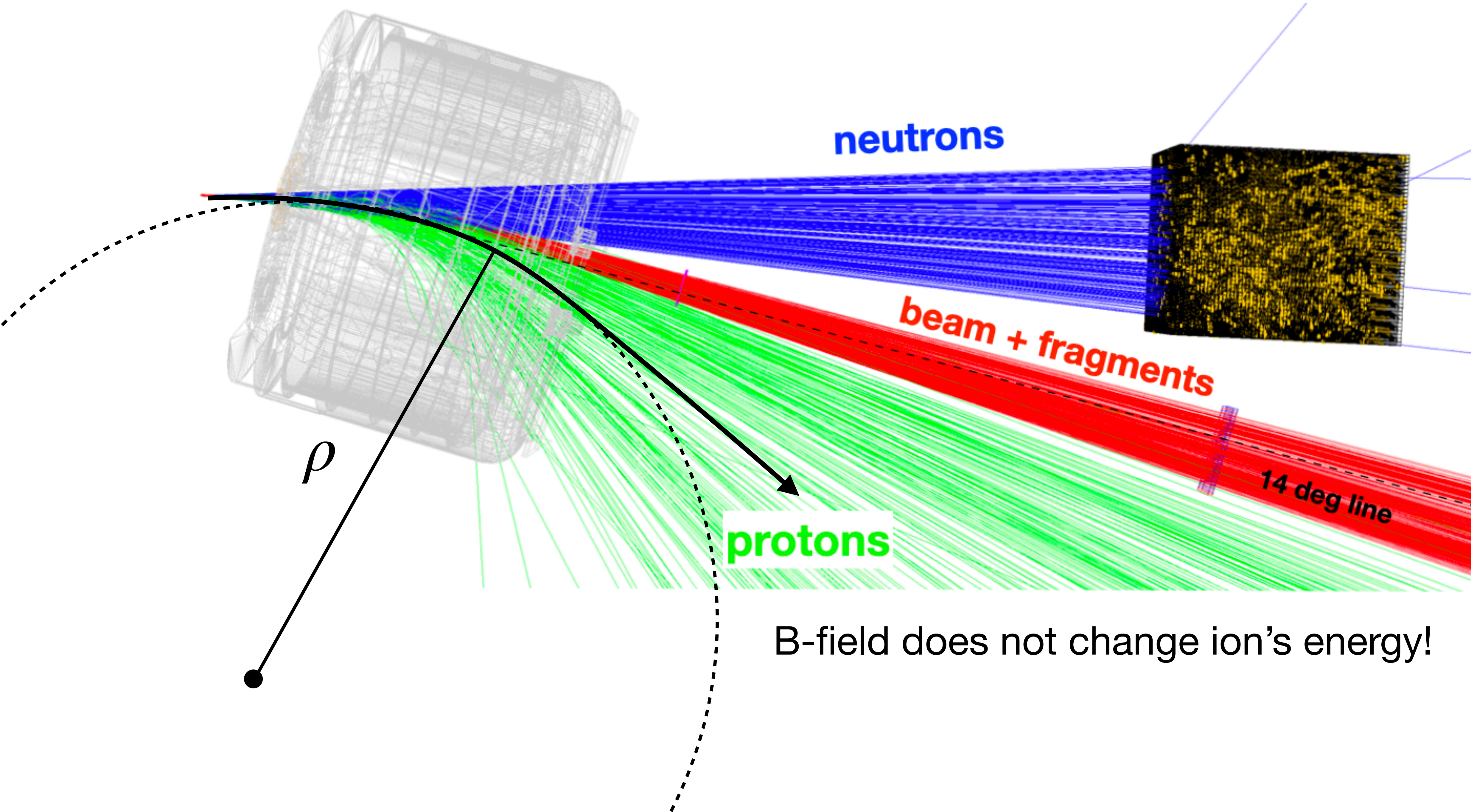
- Magnetic rigidity is the main experimental observable:

$$B\rho[\mathrm{Tm}] = 3.3356 \times \frac{P}{Q}[\mathrm{GeV/c/e}] = \frac{A}{Z}\frac{m_u \beta\gamma}{e}$$

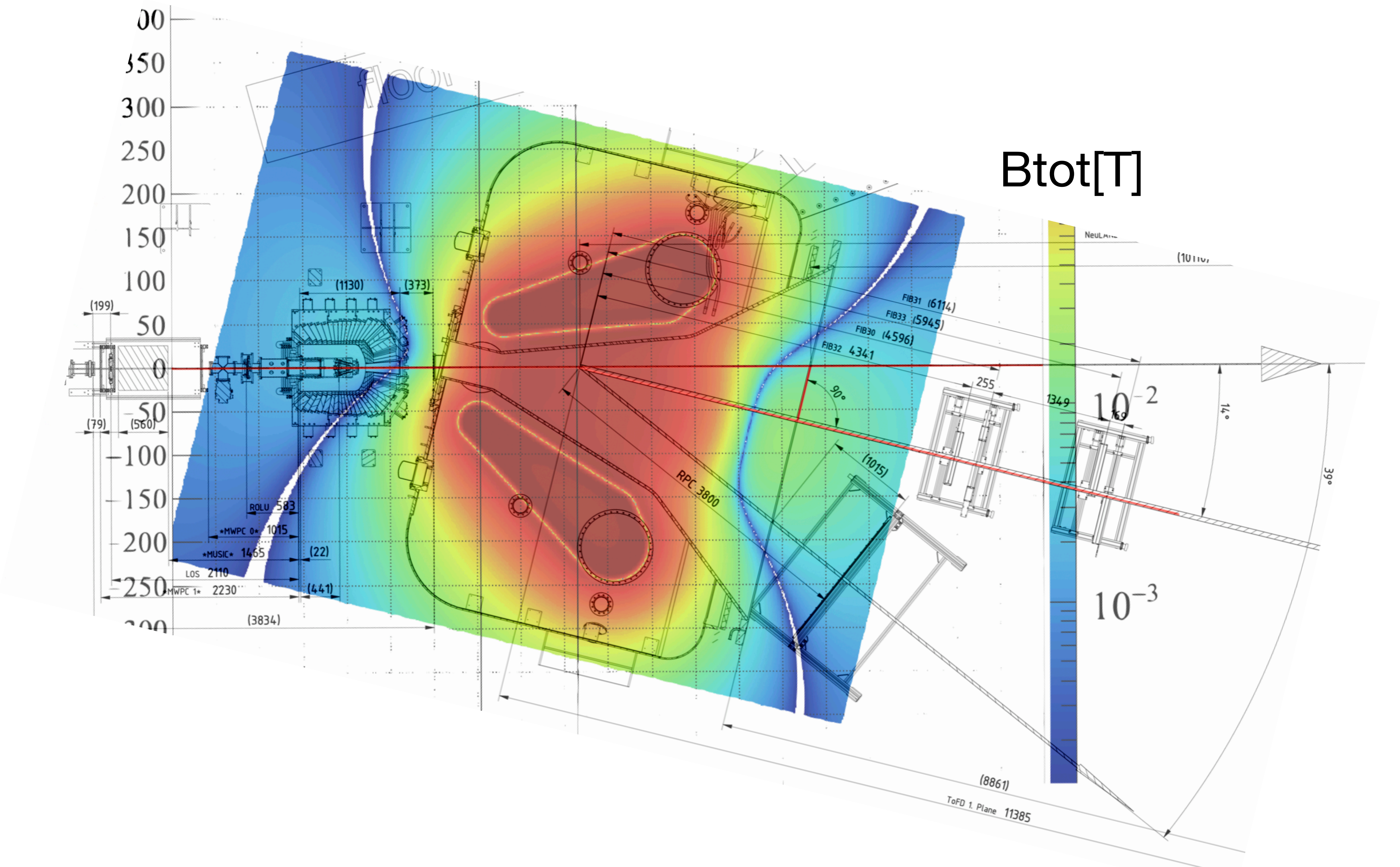
- Isotopes with different A/Z may have the same B-rho
  - ➡ Need particle velocity for complete A/Z id
    - ➡ Trajectory length + ToF

- Large acceptance spectrometer: measuring large B-rho spread
  - ➡ bending angles from 0 to ~40 deg
  - ➡ incoming angles: max ~80 mrad (4.6 deg), depending on the target position

- Real B-field is not homogeneous
  - ➡ field integrals depend on the entrance point and in/out angles



B-field does not change ion's energy!



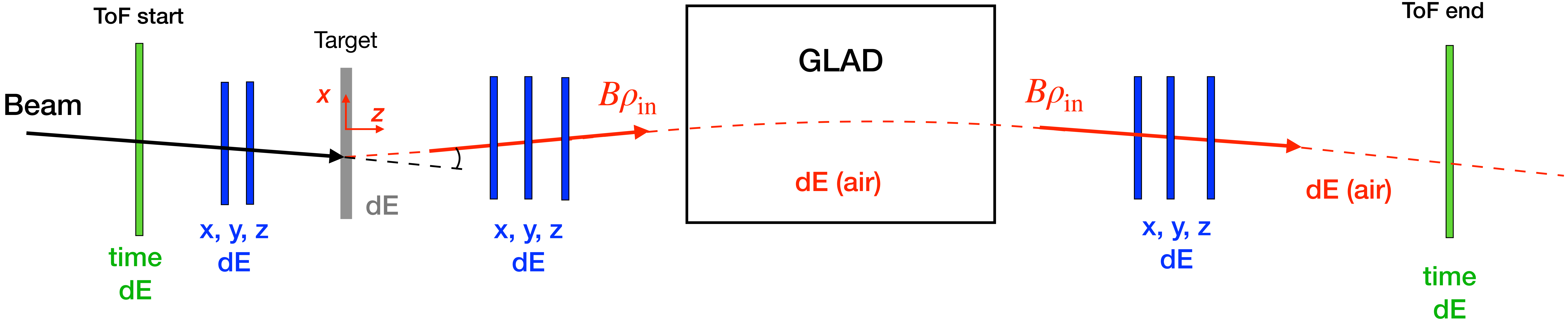
Tracking schematics in a typical R3B experiment

Missing mass: incoming and outgoing angles

$$S_p = \beta\gamma(\vec{p}_1 + \vec{p}_2)_{//} - (\gamma - 1)m_p - \gamma(T_1 + T_2) - \frac{q^2}{2M_{A-1}}$$

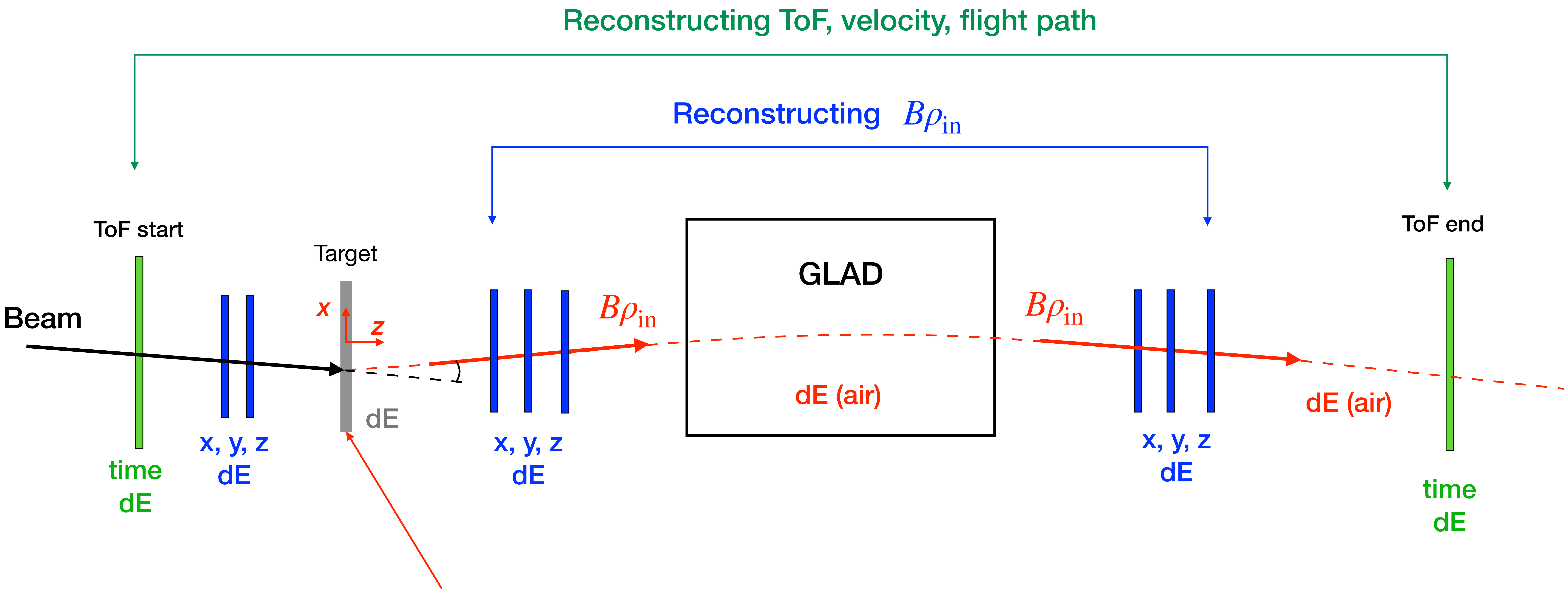
Invariant mass: only outgoing angles

$$E_{rel} = \left(\sqrt{\sum_i m_i^2 + \sum_{i\neq j} \gamma_i\gamma_j m_i m_j (1 - \beta_i\beta_j \cos\sphericalangle[i,j])} - \sum_i m_i\right) \cdot c^2$$



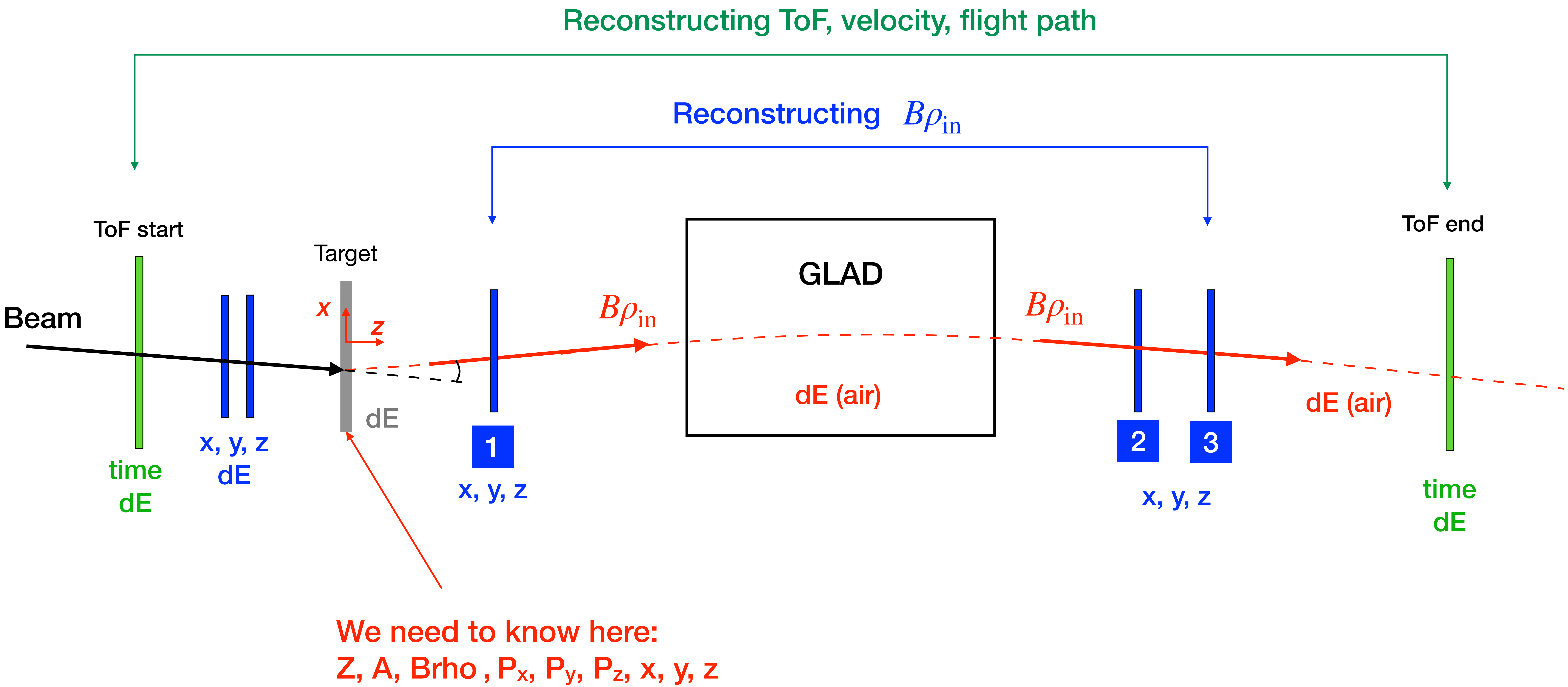
Every reaction has it’s own reference frame!

Tracking schematics in a typical R3B experiment

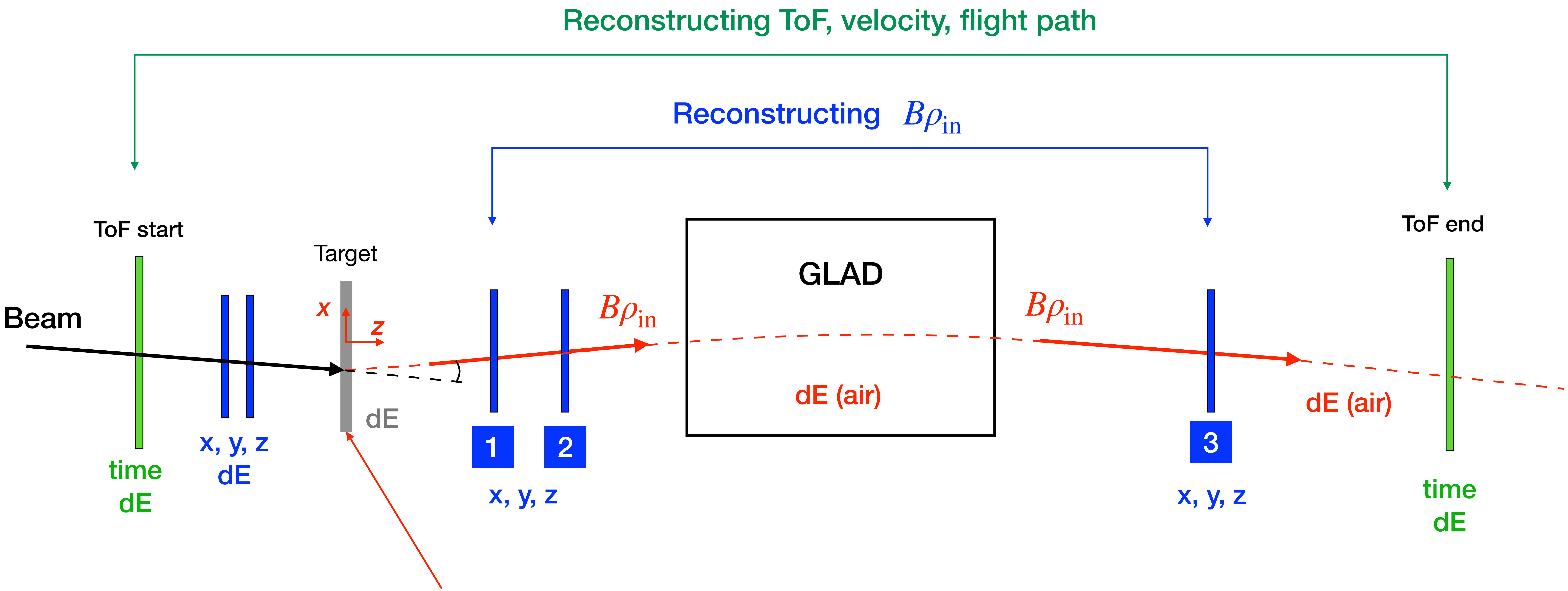


We need to know here:
Z, A, Brho , P_x, P_y, P_z, x, y, z

Tracking schematics in a typical R3B experiment



Tracking schematics in a typical R3B experiment



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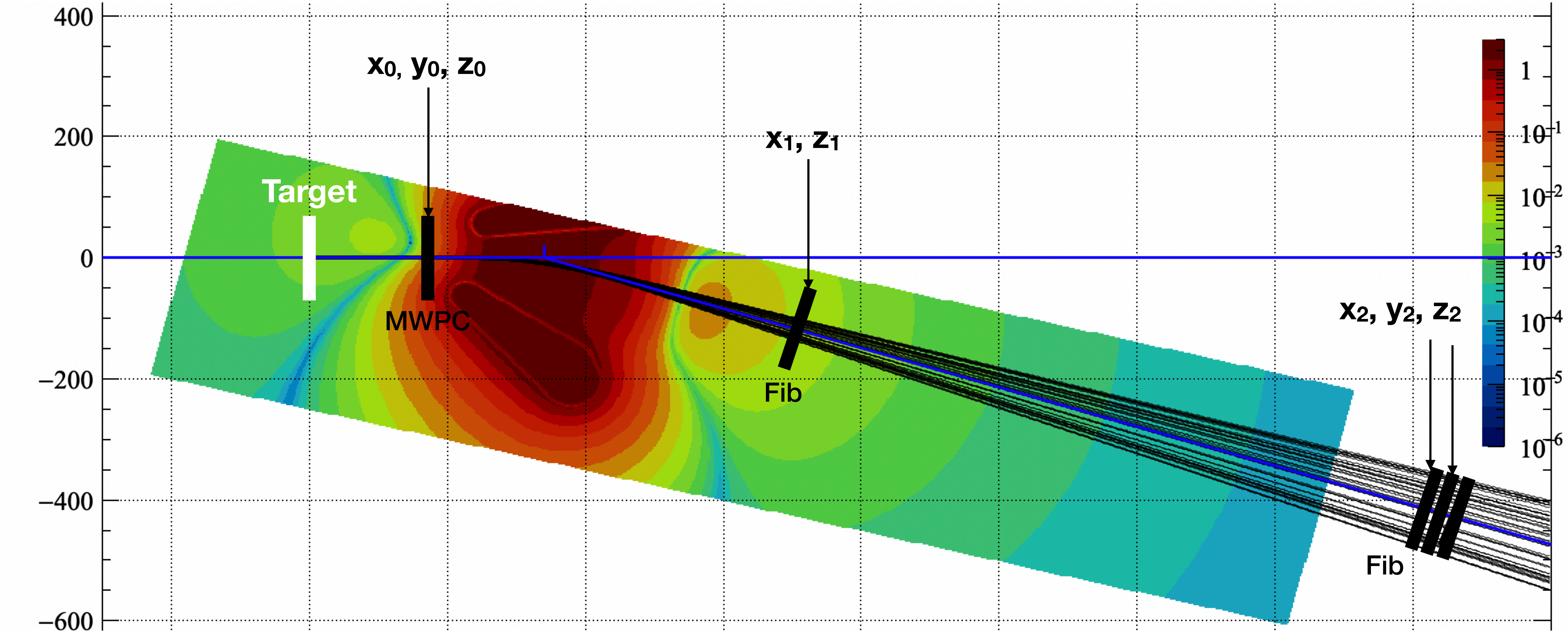
Trajectories of ^{124}Sn isotopes ~900MeV/u (s515)



P_o +_15 %

- Calculating trajectories based on one-step Runge-Kuta propagator from FairRKPropagator class
  - ➡ Implementation in R3BTPropagator class of R3BRoot
- Sampling (partial) information of the ion trajectory at in a few positions (detectors)
- Track variable (Brho, flight path, angles etc.) is expected to be function of the measured coordinates, e.g.:

$$B\rho = f(x_0, y_0, z_0, x_1, y_1, x_2, y_2, z_2)$$

General concept of the multi-dimensional fitting

- *P* is a known quantity of interest (e.g. Brho, P/Z, trajectory length, angle, etc.)
- *P* depends on N observables $(x_1, \ldots, x_N)$ *e. g.*

- Use R3BTPropagator to obtain $(x_1, \ldots, x_N)$ for a particle with given A, Z, P, theta, phi, etc.
- Collect a training sample of several (thousands) trajectories and form tuples *(xj, Pj, Ej)* varying initial particle's energy/angles
  - $x_j = (x_{1,j}, \ldots, x_{N,j})$ - are N observables (detector hits) in the event j
  - P_j - known value in the event j
  - E_j - known error of P_j in the event j

- Use TMultiDimFit class from ROOT to find the parameterization:

$$P_p(\mathbf{x}) = \sum_{l=1}^{L} c_l \prod_{i=1}^{N} p_{li}(x_i) = \sum_{l=1}^{L} c_l F_l(\mathbf{x})$$

such that

$$S = \sum_{j=1}^{M} (P_j - P_p(\mathbf{x}_j))^2$$

is minimal

$p_{li}(x_i)$ - Monomials, Legendre or Chebyshev polynomials of x_i
c_l - coefficients determined by the fit

- Optional usage of the Principle Component Analysis (PCA) to find the parameterisation in the orthogonal basis
- The resulting MDF function is stored as txt file which can be read by R3BMDFWrapper to perform the tracking of real data

Example of MDF paramterization for P/Q in s515

Parameterization for P/Q using Chebyshev polynomials:

Normalised variables:

y_0 = 1 + 2 * (x_0 - 7.627) / (7.627 +7.648)
y_1 = 1 + 2 * (x_1 - 7.935) / (7.935 +7.601)
y_2 = 1 + 2 * (x_2 - 169.8) / (169.8 - 159.8)
y_3 = 1 + 2 * (x_3 +79.15) / (-79.15 +154.4)
y_4 = 1 + 2 * (x_4 - 720.0) / (720.0 - 688.1)
y_5 = 1 + 2 * (x_5 + 305.8) / (-305.8 +523.4)
y_6 = 1 + 2 * (x_6 - 1660.) / (1660. - 1584.)
y_7 = 1 + 2 * (x_7 - 0.03134) / (0.03134 +0.02279)

Main set of tracking variables

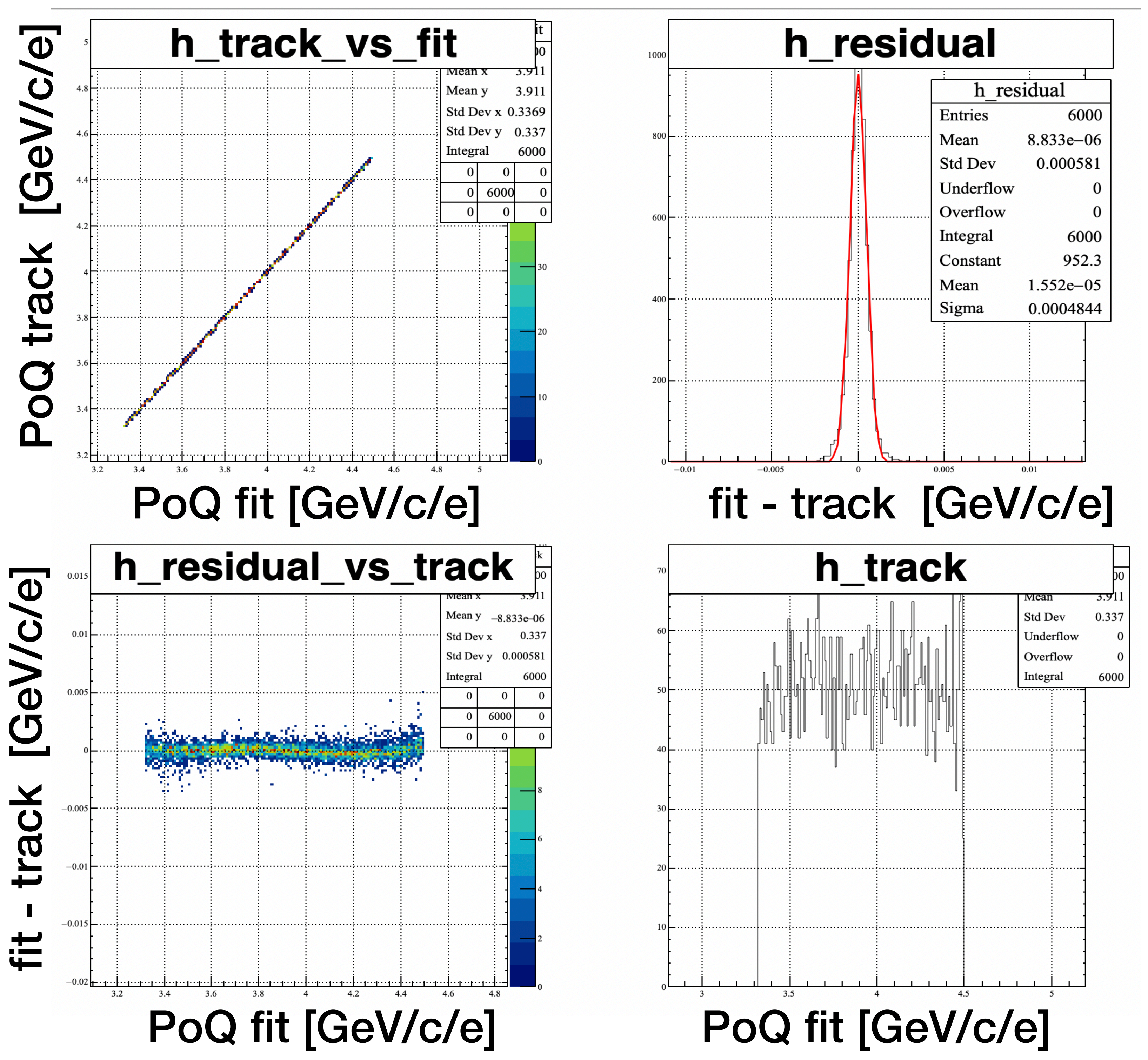
x_0 = X_{mwpc}
x_1 = Y_{mwpc}
x_2 = Z_{mwpc}
x_3 = X_{fib10}
x_4 = Z_{fib10}
x_5 = X_{fib11}
x_6 = X_{fib11}
x_7 = $(Y_{fib12} - Y_{mwpc})/(Z_{fib12} - Z_{mwpc})$

F(y_0, y_1, y_2, y_3, y_4, y_5, y_6, y_7) = 4.610 + 3.026 * y_6+ 0.1485 * y_0+ 4.035 * y_5 - 4.036 * y_3- 0.004515 * C_3(y_4)+ 0.01324 * y_3 * C_2(y_4)- 0.5629 * y_4+ 0.4459 * C_2(y_6)+ 0.1344 * y_0 * y_6+ 2.404 * C_2(y_5)- 8.498 * y_3 * y_5 + 0.07895 * y_4 * C_2(y_5) * y_6 - 0.01227 * C_4(y_5) …. —> many more terms

Relative precision obtained: 0.0001723
Maximum powers used: **4 2 1 4 4 4 4 2**

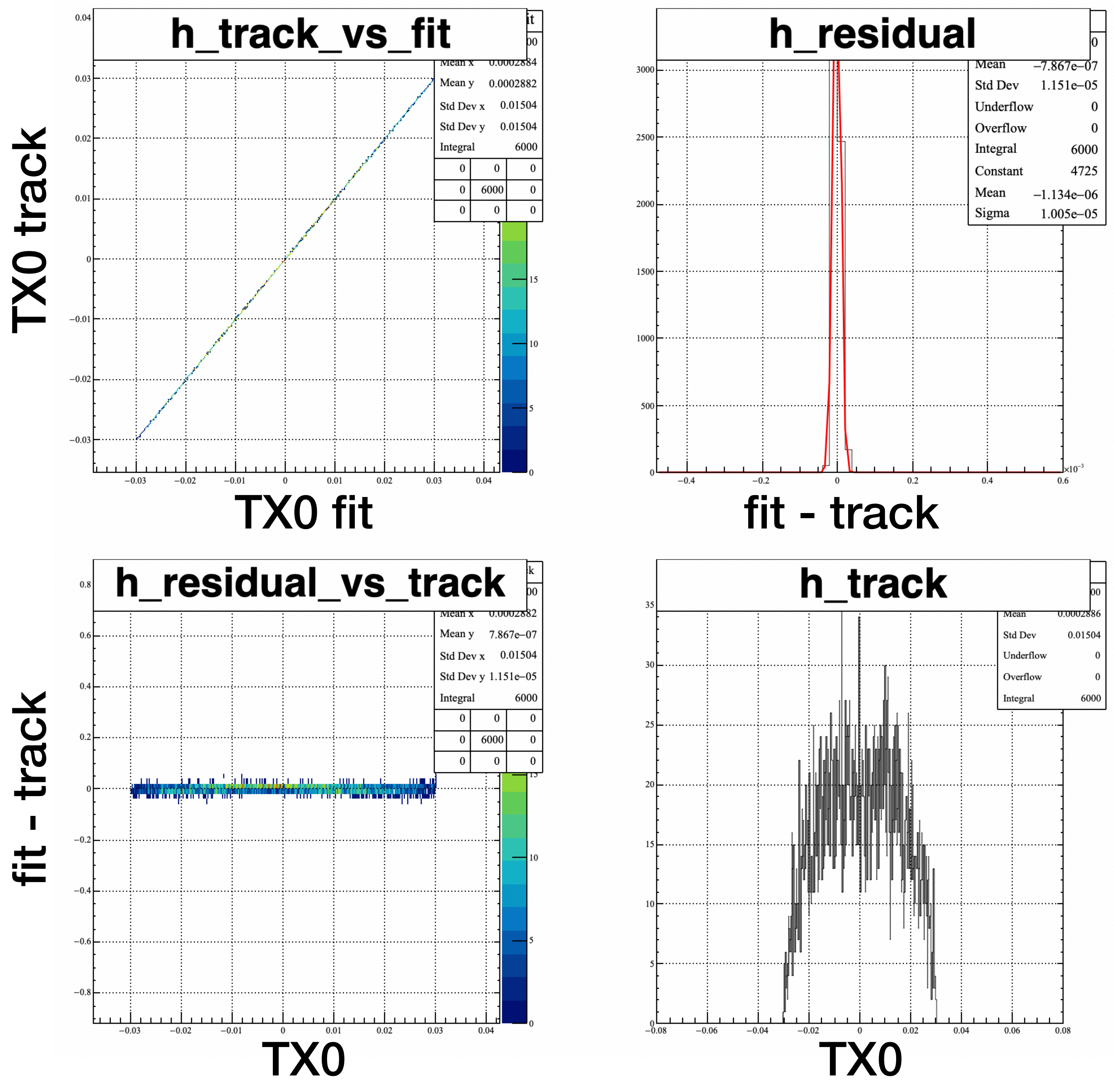
Example of MDF training results for S515

P/Q



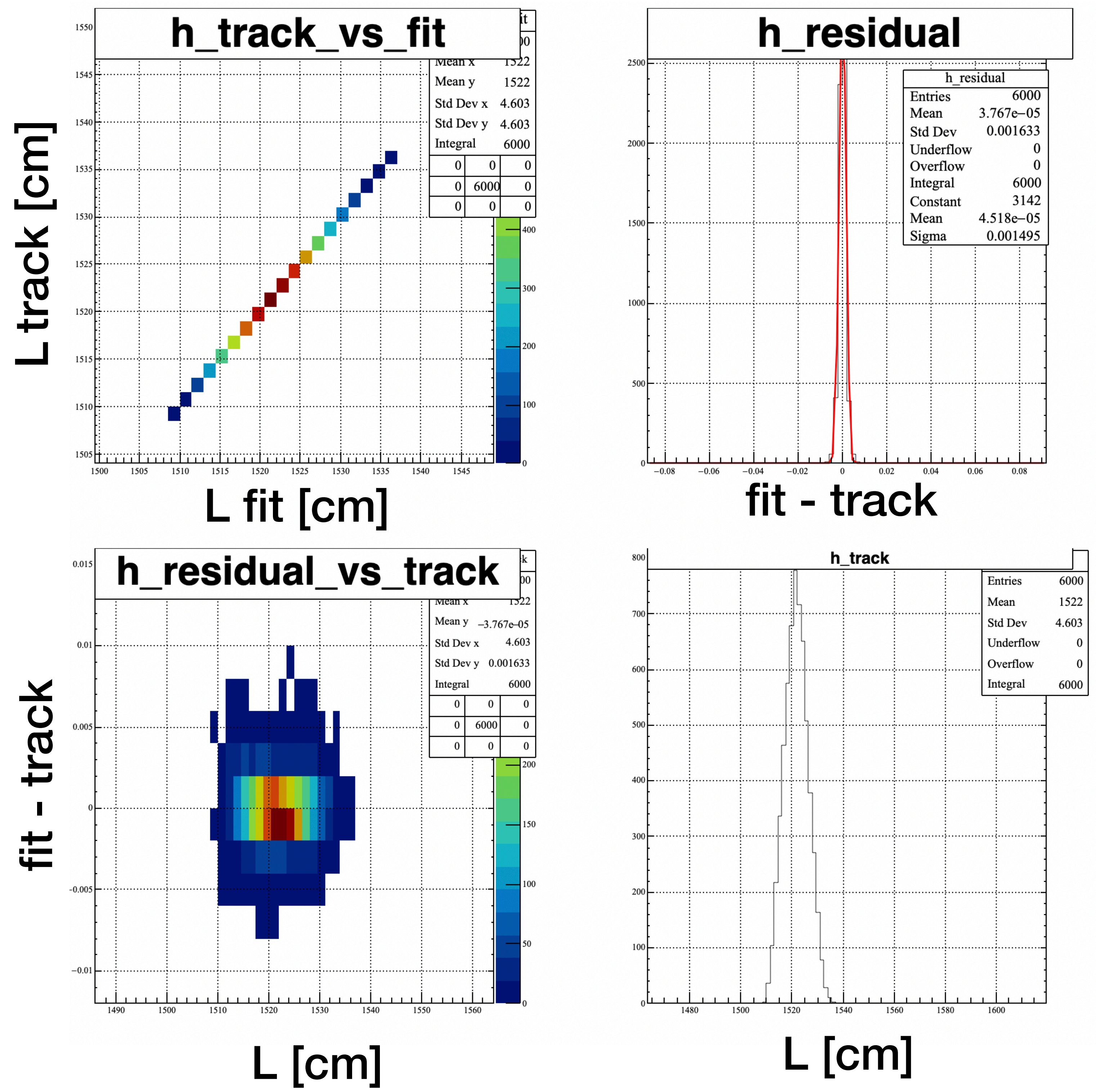
dP/P ~3 x10-6

$TX_0 = P_x/P_z$



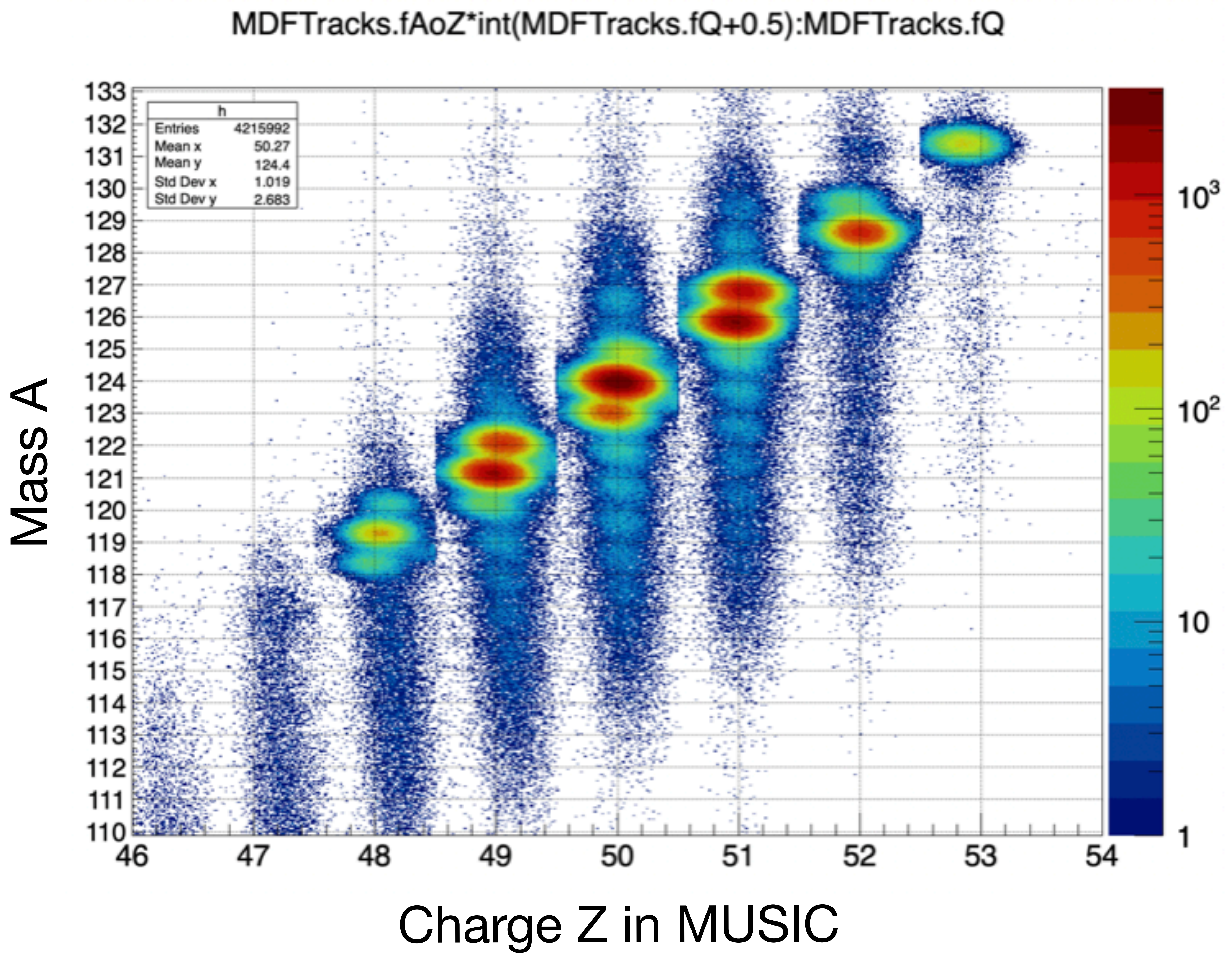
$dTX_0 \sim 10^{-2}$ mrad

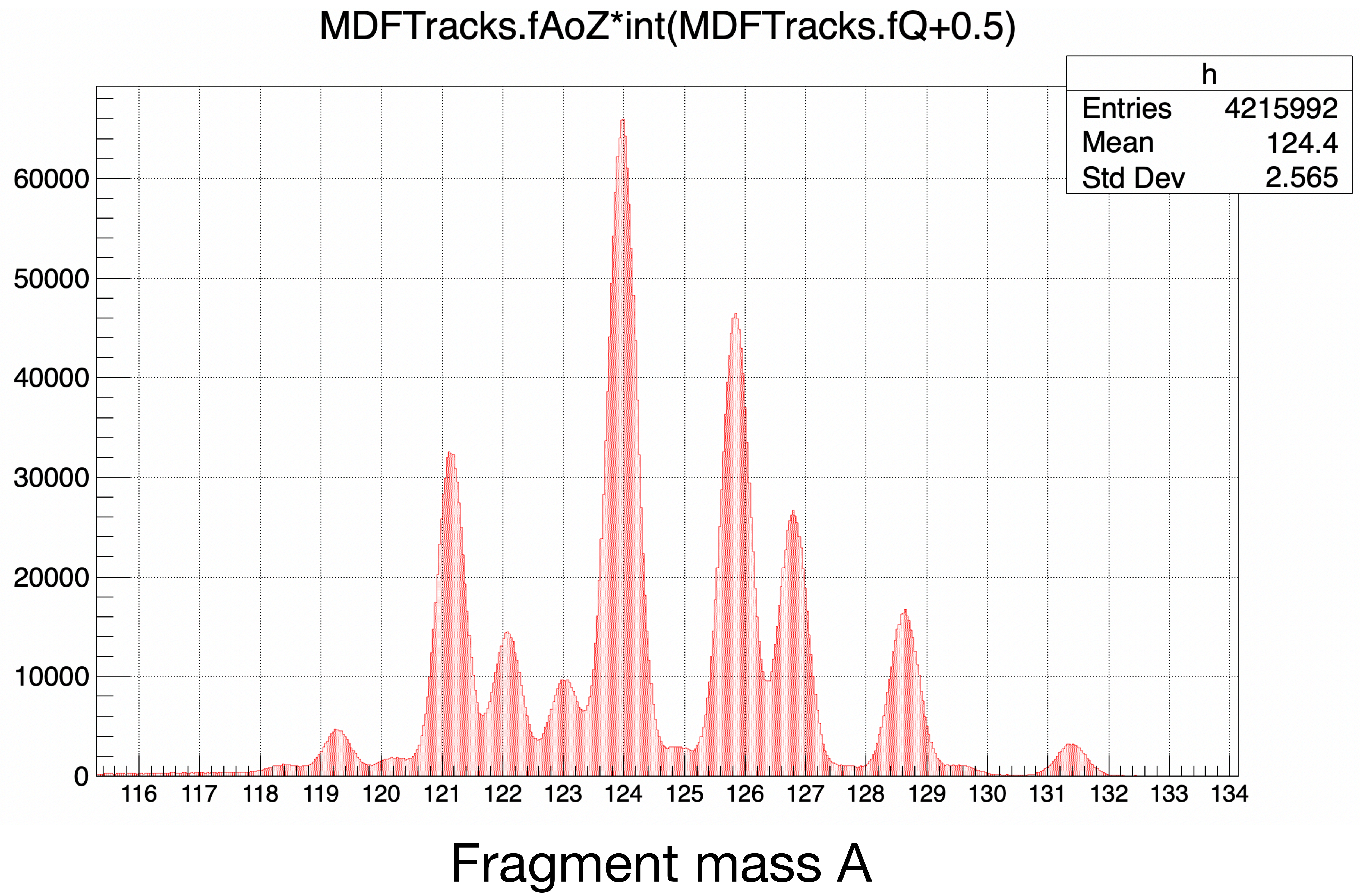
Flight path



dL ~ 15 um

Example of MDF mass reconstruction in S515





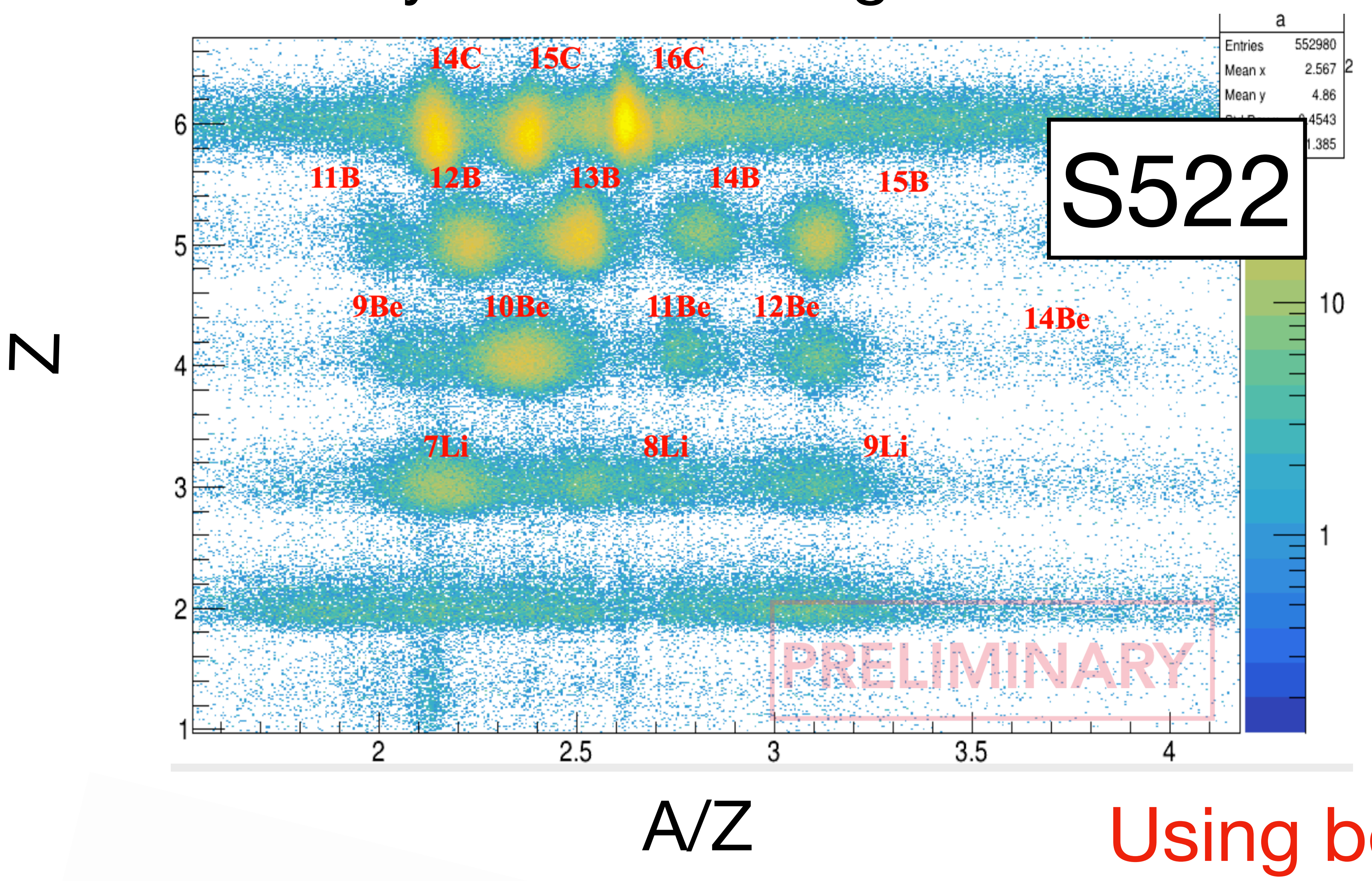
Using beta value from FRS

Mass resolution (sigma) ≈ 0.24 mass units

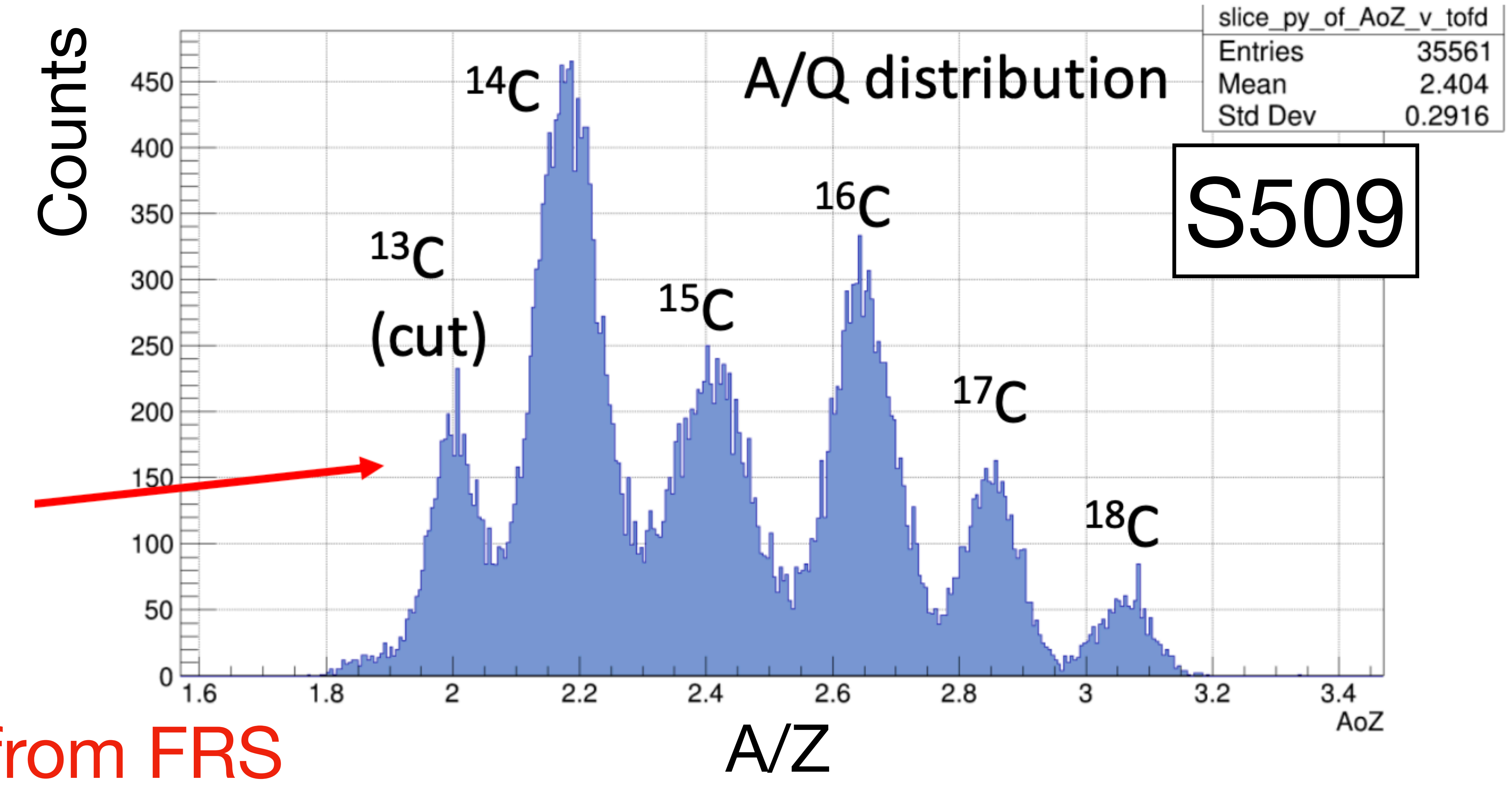
dA/A ≈ 0.2% (expecting ~0.1% in further analysis)

Example of MDF reconstruction in other experiments

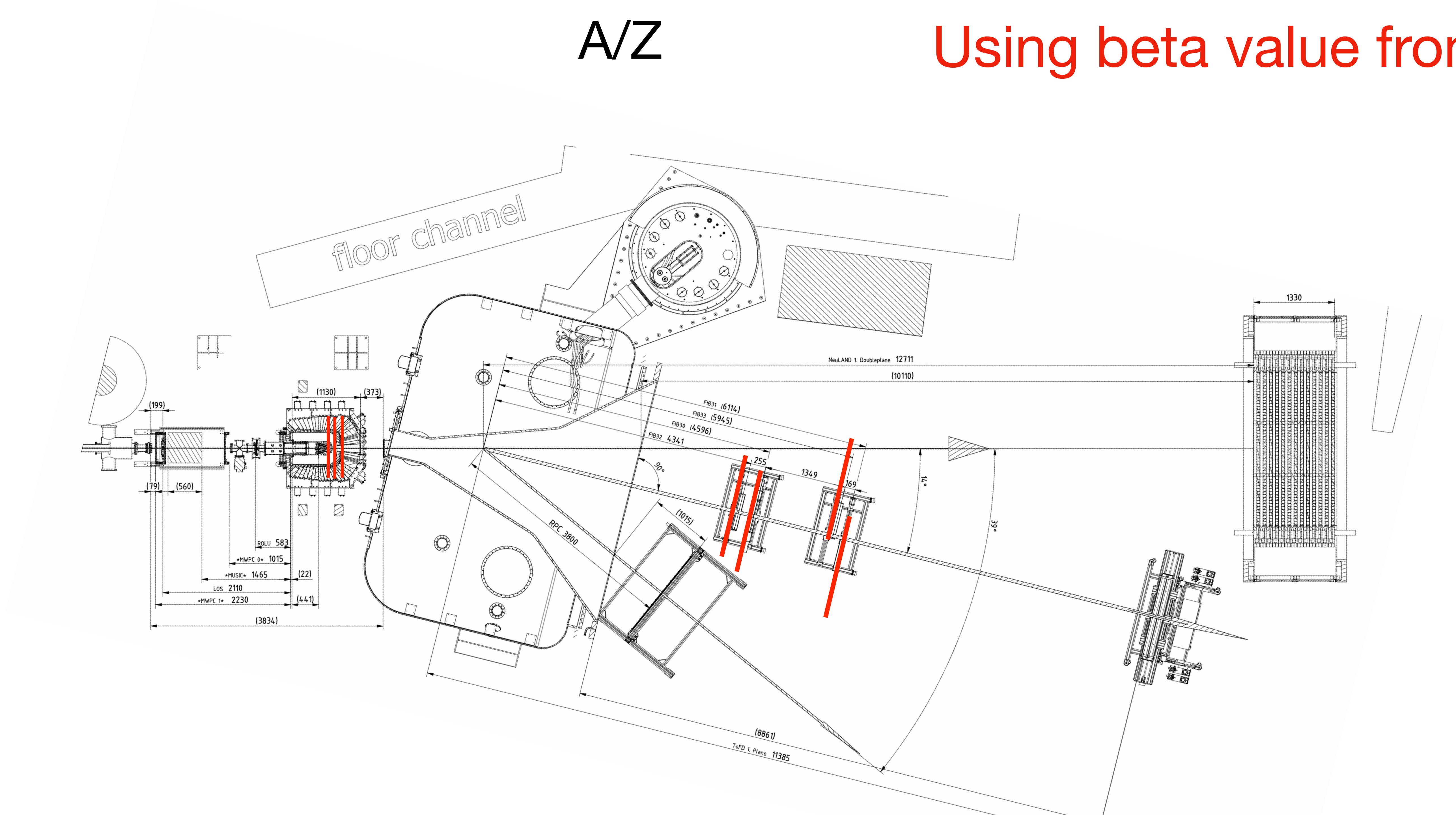
Courtesy of Andrea Lagni

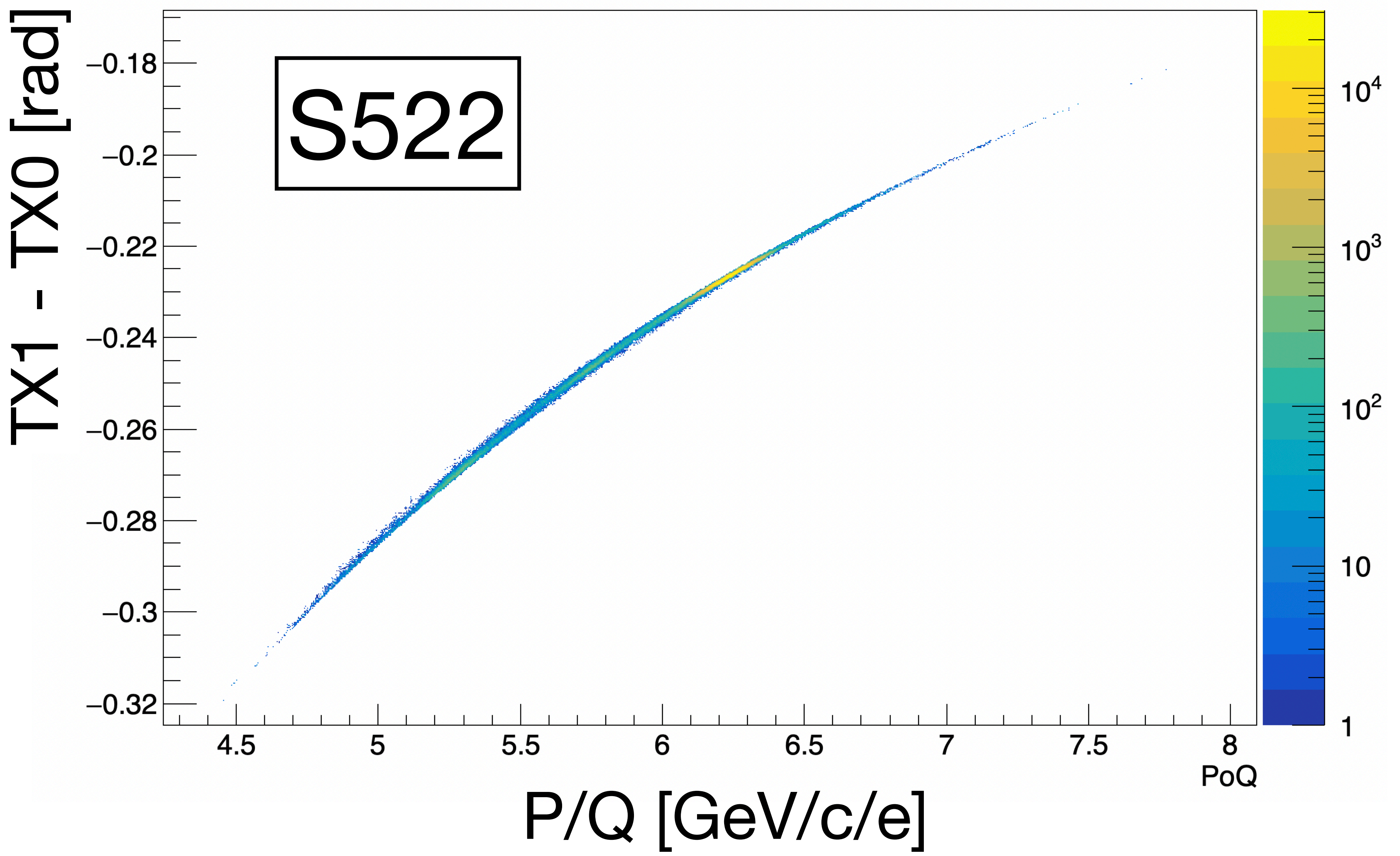


Courtesy of Antoine Barrière

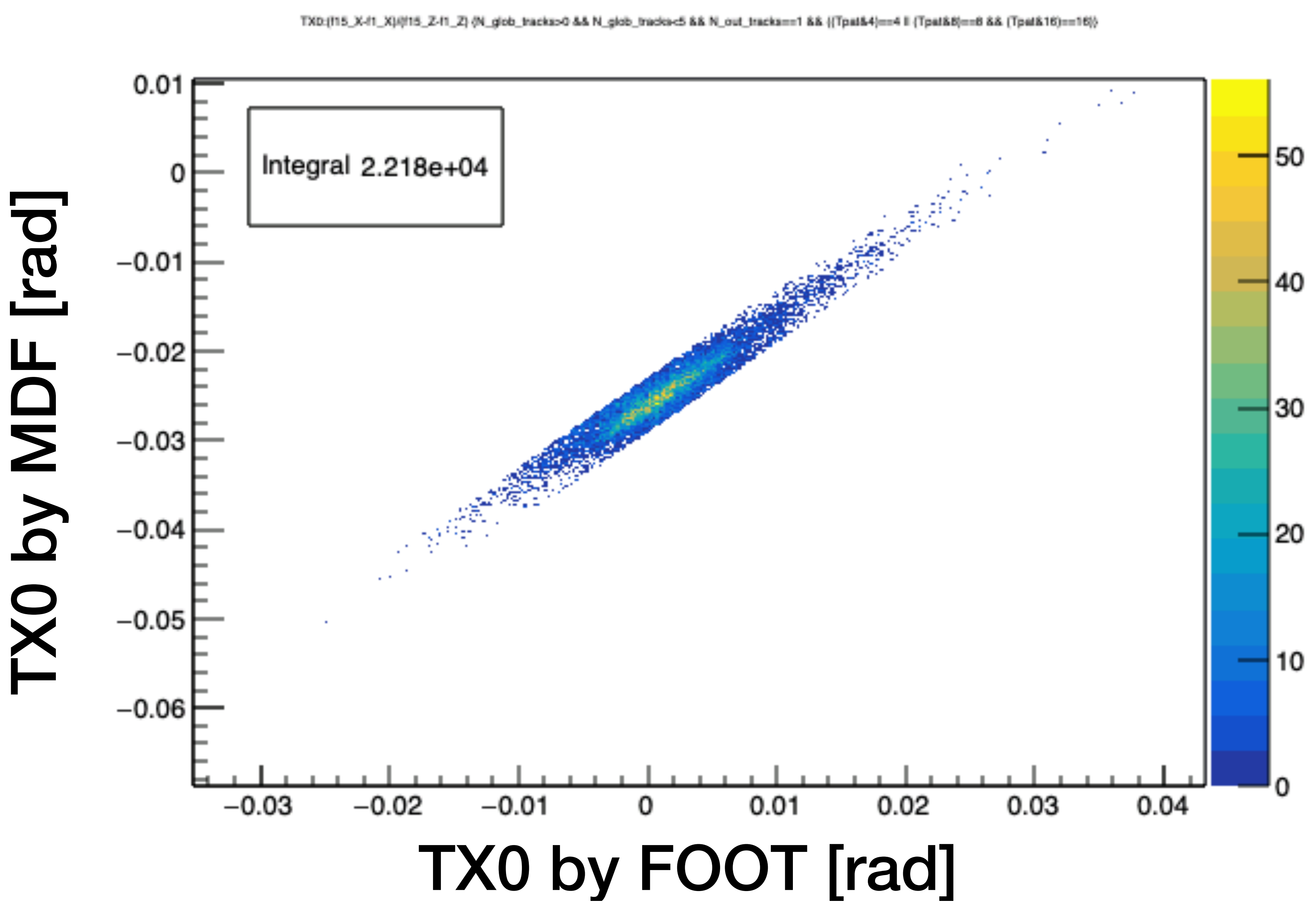


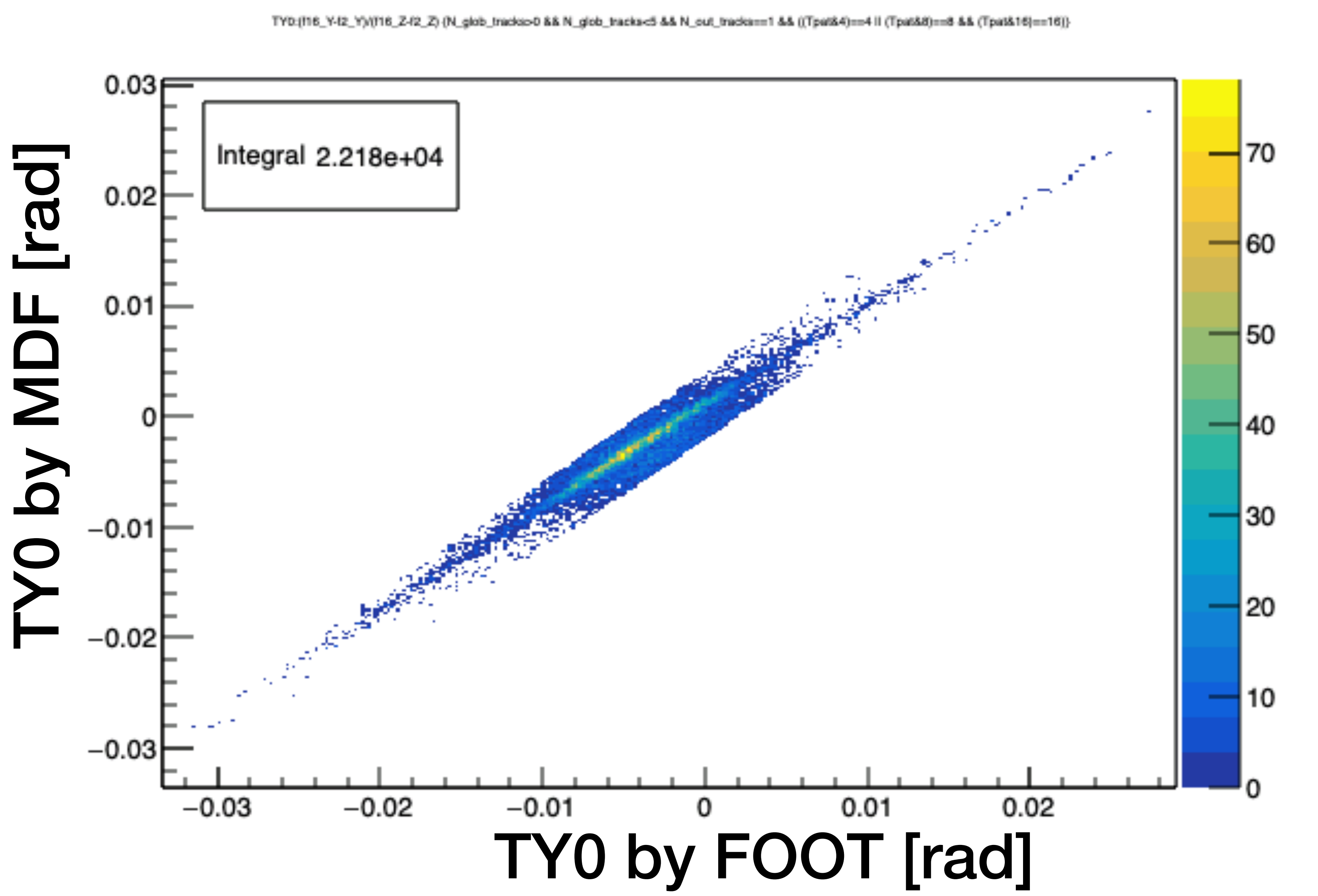
Using beta value from FRS

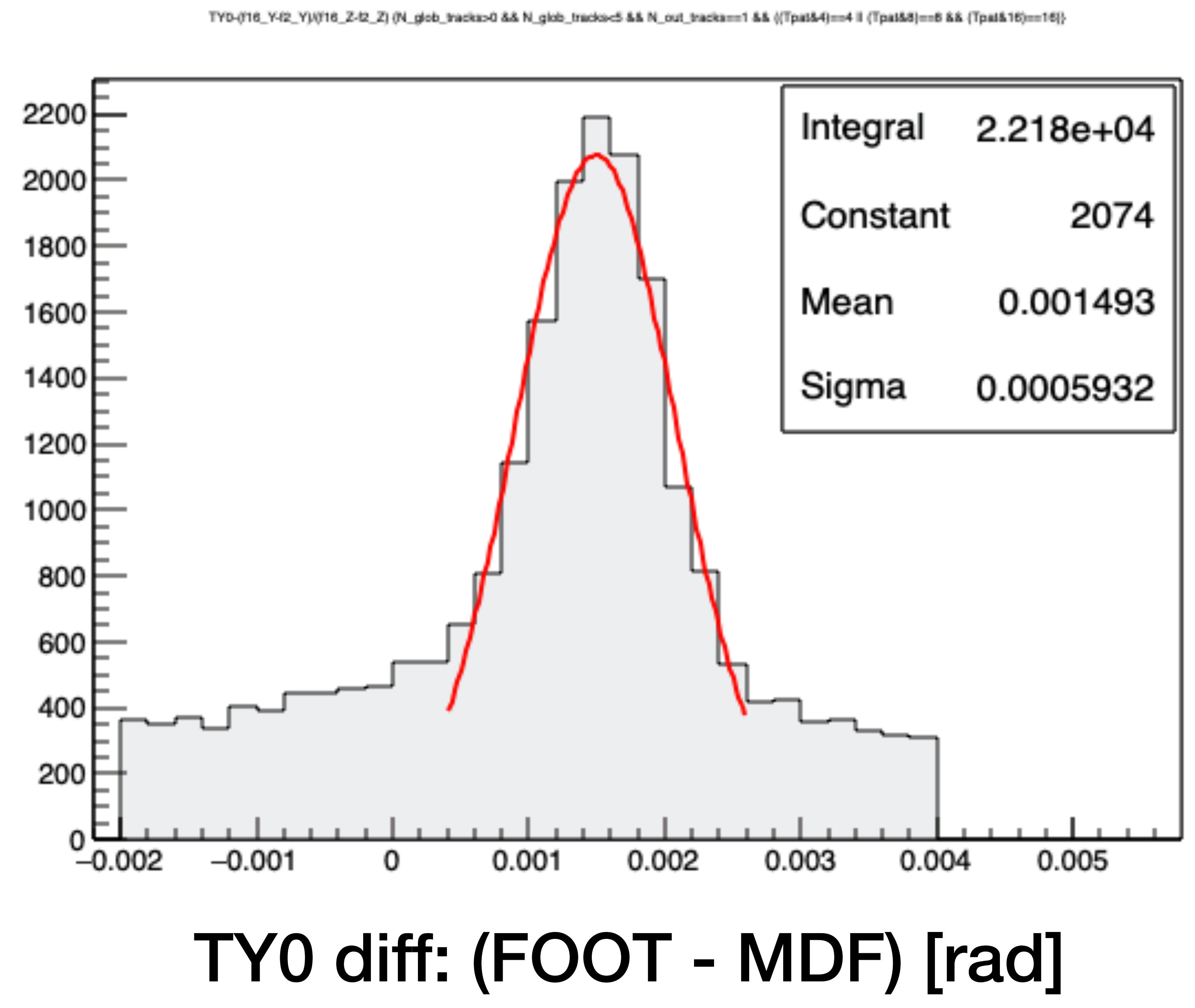


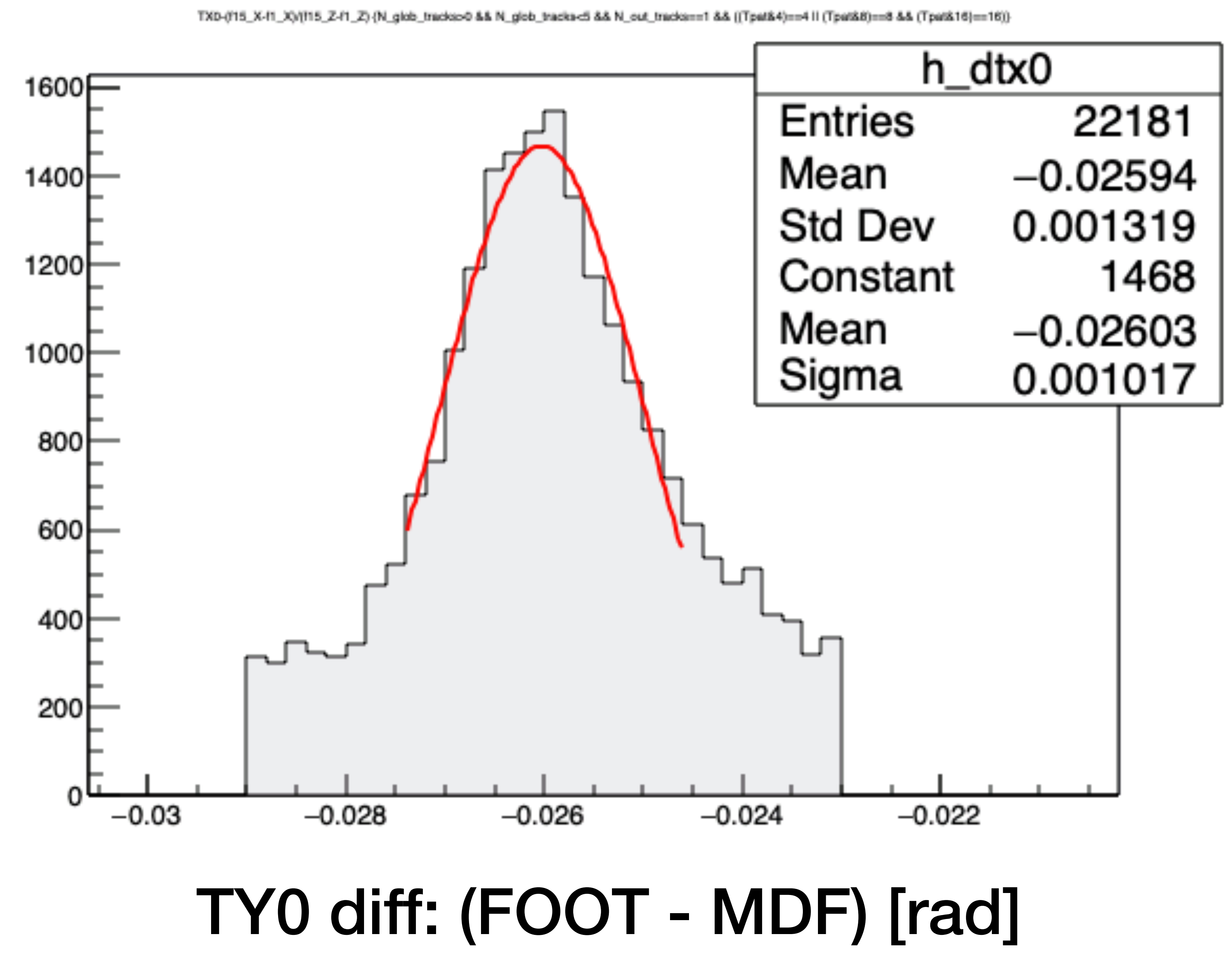


Scattering angles from the target (no alignment) s522 run 131





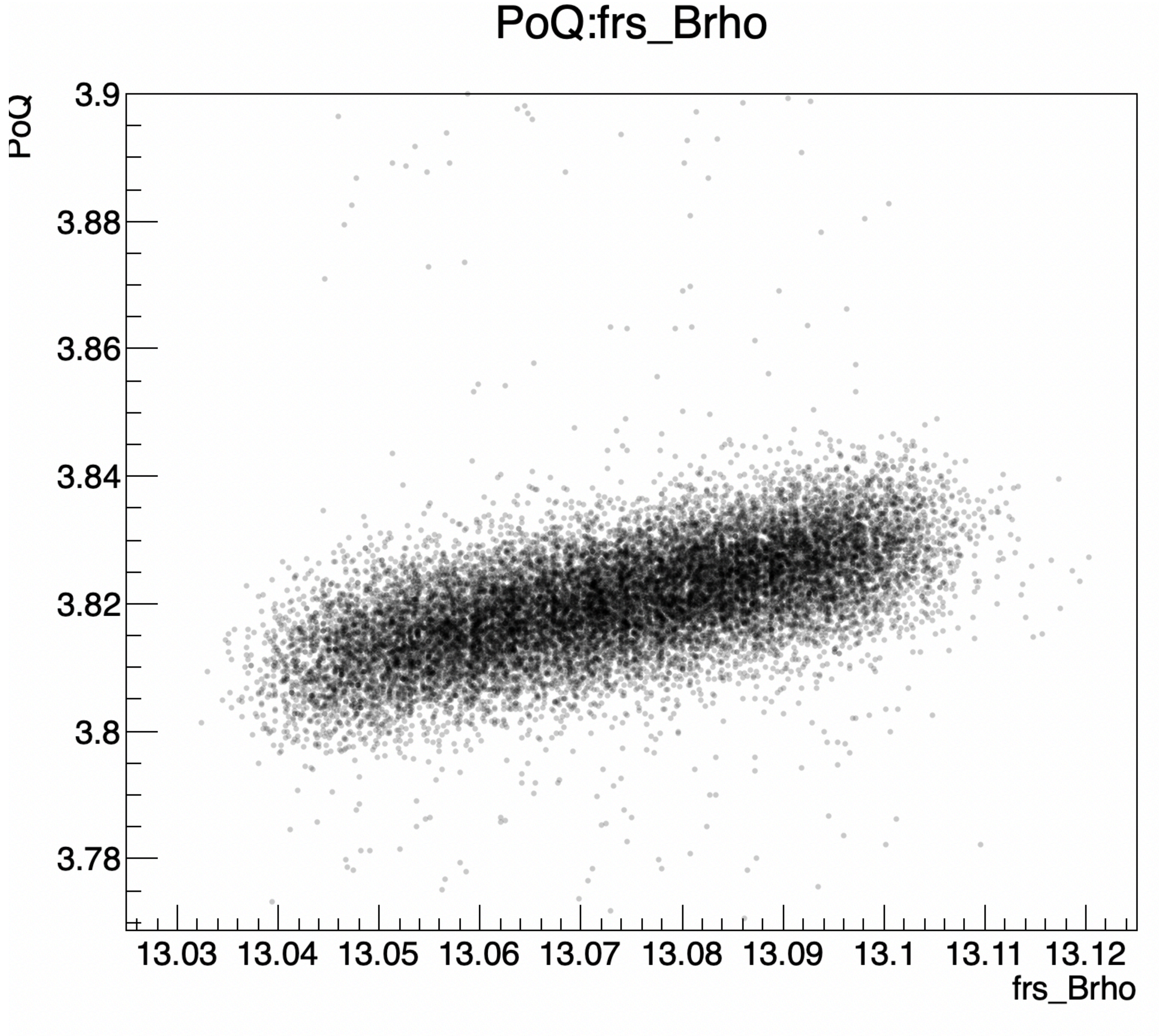


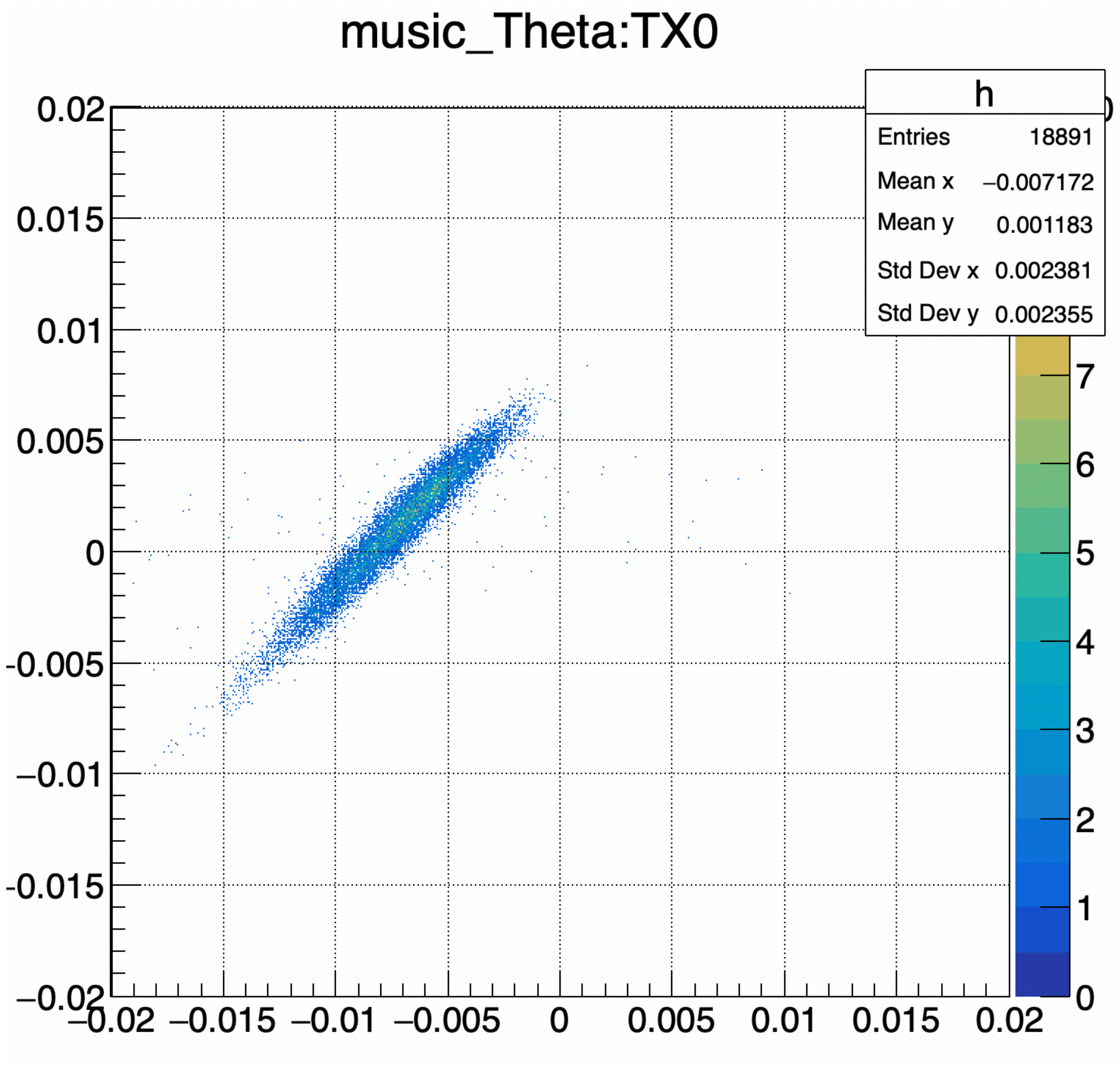


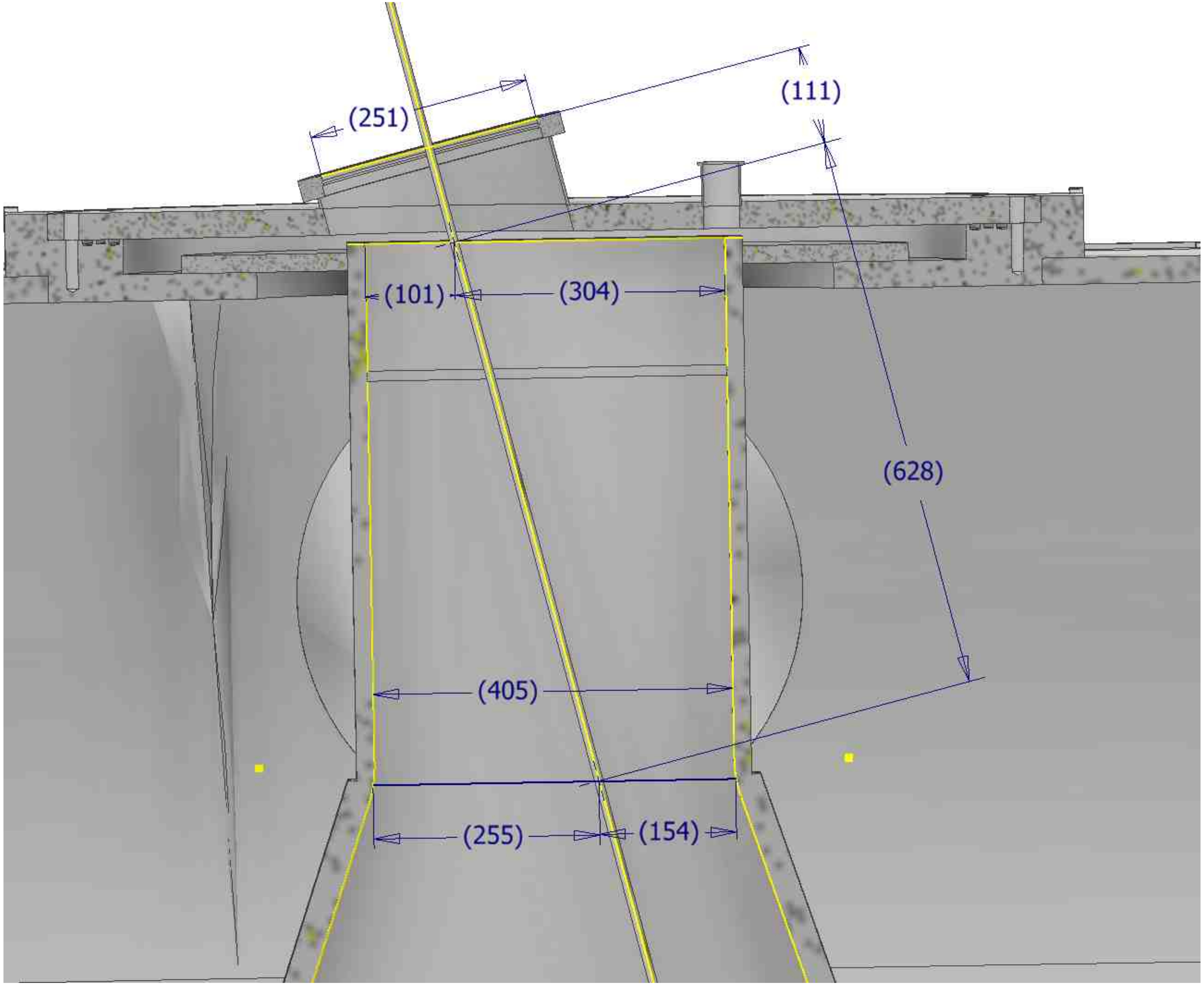
Summary

- MDF is a simple and efficient tracking method in a complex magnetic field
- Easy to configure for different setup configurations, tracking variables, detectors etc.
- Proved to work for various experiments (s515, s522, s509, s473, s454 + SAMURAI, Dubna, etc.)
- MDF functions can be used to perform self-consistent alignment of the all inseam detectors
- Recent updates for the R3BTPropagator to improve particle tracking in air (courtesy of M. Xarepe)
    - Proton tracking with RPC: limited position info + ToF
- Experiment specific MDF functions - need to be prepared individually
- Is there a universal MDF function for any R3B experiment?

Example of MDF reconstruction in S515







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