The Equation of State of Nuclear Matter from Collective Flows in Intermediate Energy Heavy-Ion Collisions

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Overview

Motivation

Model Details

- dcQMD – interaction parametrization
- Threshold effects
- Medium modification of cross-section
- Initial/final state treatment

Study of EoS, Effective Masses, σ*

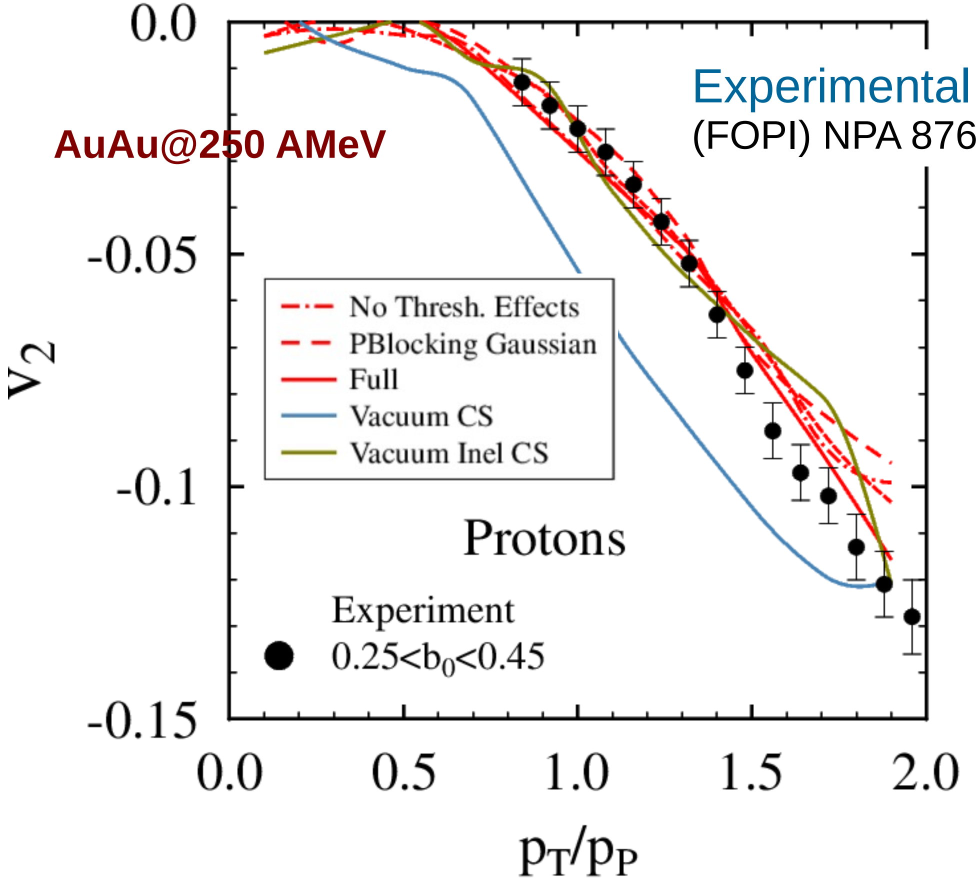
- Framework
- Impact of Different Observables
- Model Dependence
- Probed Density
- Constraints

Summary & Conclusions

Motivation

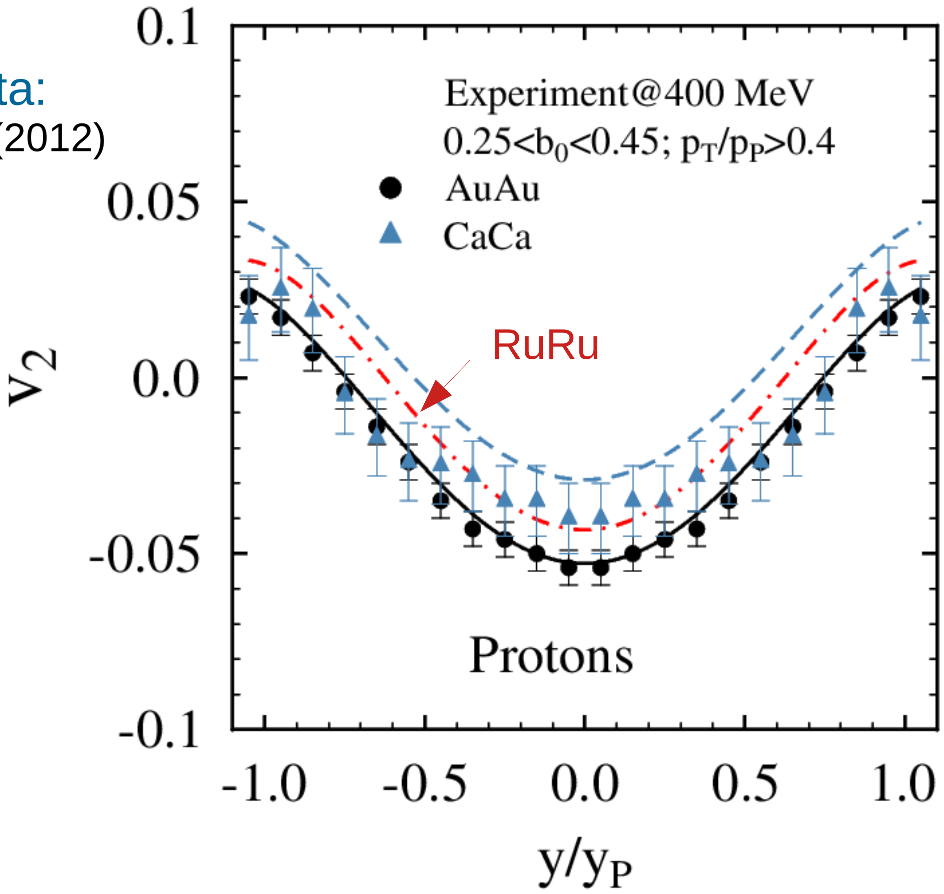
- latest version of the dcQMD model used to study symmetry energy with pion production
- empirical effective isoscalar mass m*=0.70 J. Estee et al., PRL 126, 162701 (2021)
- compressibility modulus close to world average K_0=245 MeV
- in-medium modification factor adjusted to qualitatively describe nucleonic observables

D. Cozma et al.,EPJA 57, 309 (2021)



AuAu@250 AMeV

Experimental data:
(FOPI) NPA 876, 1 (2012)



RuRu

similar quality of description of exp. data for p,d,t,α in the energy range 150-800 AMeV AuAu

Model Details

dcQMD transport model: newest version EPJA 57, 309 (2021)

an upgraded version of TuQMD, see H. Wolter et al.

Prog.Part.Nucl.Phys. 125, 103962 (2022)

Interaction (nucleonic component)

momentum dependent potential **MDI2** -generalization of MDI of

Das, Das Gupta, Gale, Li PRC67, 034611 (2003)

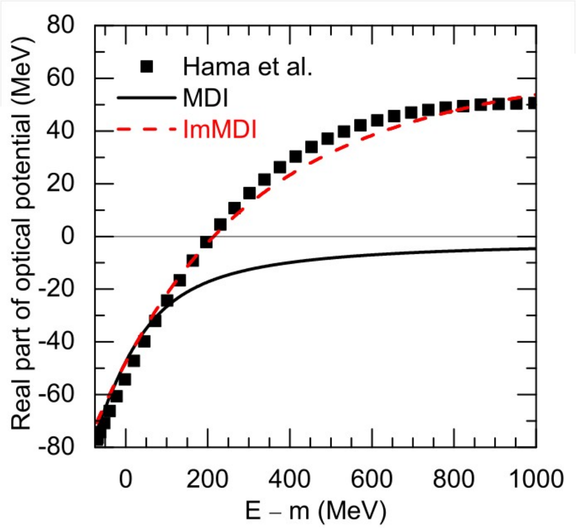
$$\frac{E}{N}(\rho,\beta,x,y)=\frac{1}{2}A_1 u+\frac{1}{2}A_2(x,y)u\beta^2+\frac{Bu^{\sigma}}{\sigma+1}(1-x\beta^2)+\frac{Du^2}{3}(1-y\beta^2)$$

$$+\frac{1}{u\rho_0^2}\sum_{\tau,\tau'}C_{\tau\tau'}\int\int d^3p\,d^3p'\frac{f_{\tau}(p,p')f_{\tau'}(p,p')}{1+(\vec{p}-\vec{p}')^2/\Lambda^2}$$

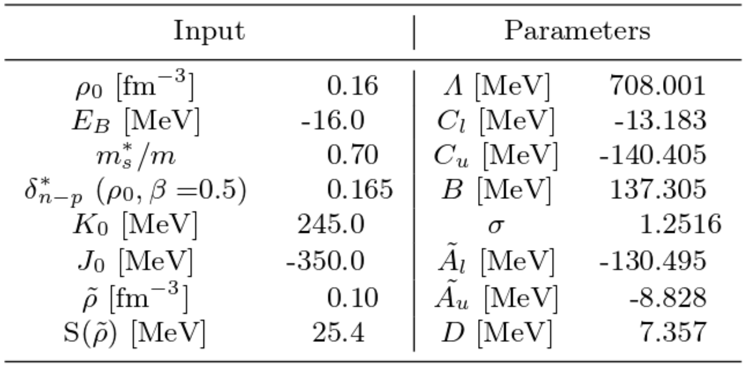
$$A_2(x,y)=A_2^0+\frac{2xB}{\sigma+1}\bar{u}^{\sigma-1}+\frac{2yD}{3}\bar{u} \qquad u=\frac{\rho}{\rho_0}$$

Fit:
U_{∞},K,J_0,m* -isoscalar
S(ũ),L,K_{sym},δm_{isv} -isovector

momentum dependent part: similar with that of J. Xu et al. PRC 91, 014611 (2015)
(see also C. Hartnack, J. Aichelin PRC 49, 2801 (1994))
used previously to test model dependence: flow ratio PRC 88, 44912 (2013)
pion multiplicity ratio PLB 753, 166 (2016)
independent part: extra term (vary L vs. K_{sym} and also J_0 vs. K independently)



from J. Xu et al. PRC 91, 014611 (2015)



Input		Parameters	
ρ_0 [fm^{-3}]	0.16	Λ [MeV]	708.001
E_B [MeV]	-16.0	C_l [MeV]	-13.183
m_s^*/m	0.70	C_u [MeV]	-140.405
δ_{n-p}^* $(\rho_0, \beta = 0.5)$	0.165	B [MeV]	137.305
K_0 [MeV]	245.0	σ	1.2516
J_0 [MeV]	-350.0	$\tilde{A}_l$ [MeV]	-130.495
$\tilde{\rho}$ [fm^{-3}]	0.10	$\tilde{A}_u$ [MeV]	-8.828
S$(\tilde{\rho})$ [MeV]	25.4	D [MeV]	7.357

Threshold Effects (dcQMD)

- direct consequence of imposing (total) energy conservation in the medium

$$\sqrt{p_1^2+m_1^2}+U(p_1)+\sqrt{p_2^2+m_2^2}+U(p_2)=\sqrt{p'^2_1+m'^2_1}+U(p'_1)+\sqrt{p'^2_2+m'^2_2}+U(p'_2)$$

- rarely considered in transport models below 1 AGeV, with a few exceptions:

RBUU: G. Ferini et al. PRL 97, 202301 (2006), RVUU: T. Song, C.M. Ko PRC 91, 014901 (2015);
χBUU: Z. Zhang et al, PRC 98, 054614 (2018)

- required for thermodynamical consistency of the model

Z.Zhang et al, PRC 97, 014610 (2018)

- reactions: NN ↔ NR, R ↔ Nπ (R ↔ Nππ not corrected)

- assumptions (dcQMD): - two-body collisions are part of N-body one
  - in-medium two-body collisions modeled as a succession of bare (vacuum-like) collisions followed/preceded by energy exchanges with the fireball, while momentum is conserved
  - reaction with highest probability: corresponds to the one which included the bare collision of highest probability

Example: NN->NΔ

$$\sigma^{(med)}_{NN\to N\Delta}(s^*) = \frac{\mu^{(ini)*}}{\mu^{(ini)}}\,\frac{\mu^{(fin)*}}{\mu^{(fin)}}\,\sigma^{(vac)}_{NN\to N\Delta}(s^*)$$

$s^*=Max\{s^{ini},s^{fin}\}$

Introduced in TuQMD/dcQMD in DC, PLB 753, 166 (2016)

Collision Term

Elastic baryon-baryon collisions

-modified Cugnon parametrization to accurately describe elastic cross-sections at low impact energy (<100 MeV) but also total cross-sections above pion production threshold

J. Cugnon et al., NIMB 111, 215 (1996)

In-medium modification factor

- collision criterion based on effective masses determined using EoM (consistency with the dt→ 0 fm/c limit)
- in-medium modification of elastic cross-sections

$$\sigma^{med} = f(\rho, \delta,)\, \sigma^{vac}_{mod}$$

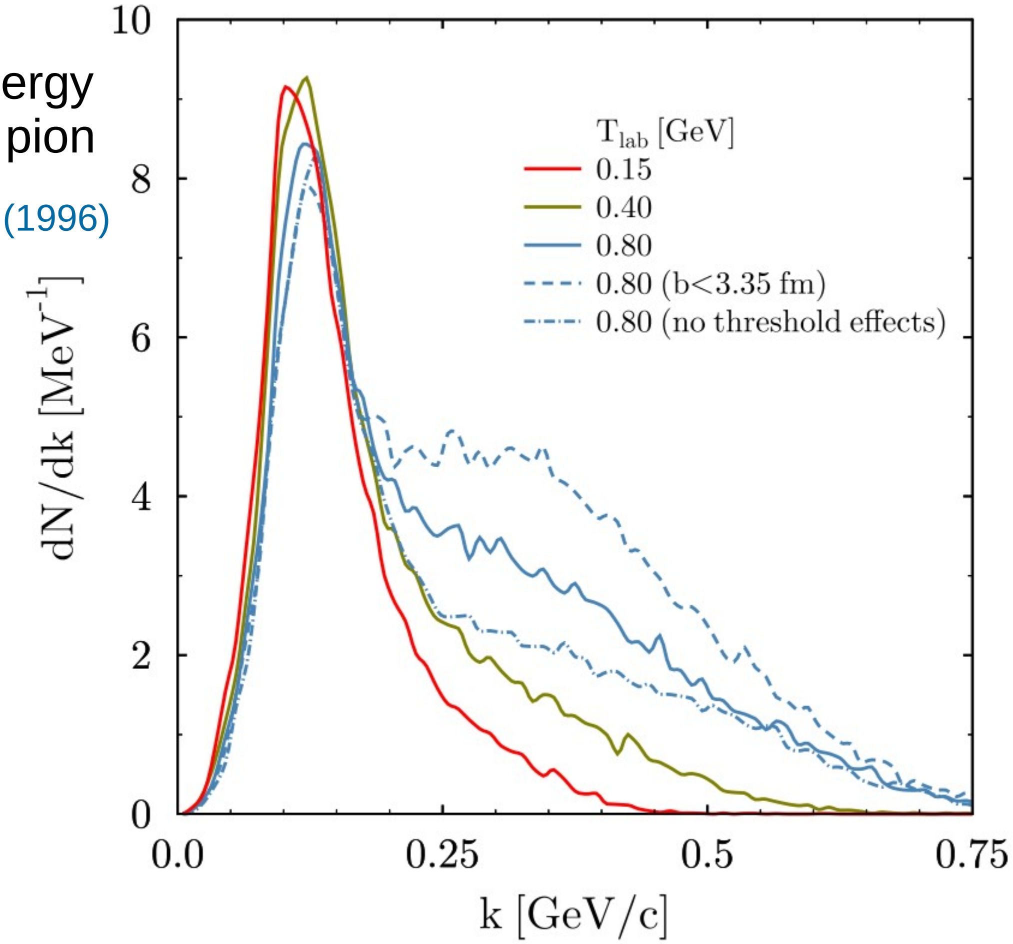
$$f(\rho, \delta) = \exp[\alpha \rho/\rho_0 + \beta_1 \delta\rho/\rho_0 + \beta_2(\tau_1 + \tau_2)\delta\rho/\rho_0]$$

σ^{vac}_{mod} – flux and phase-space factors computed using effective masses

B.A. Li et al. PRC72, 064611 (2005)

f(ρ,δ) – accounts for medium modifications of transition matrix due to departure from the quasi-particle picture

C. Fuchs et al. PRC 64, 024003 (2001)



Resonance production: OBE model

S.Huber et al., NPA 573, 587 (1994)

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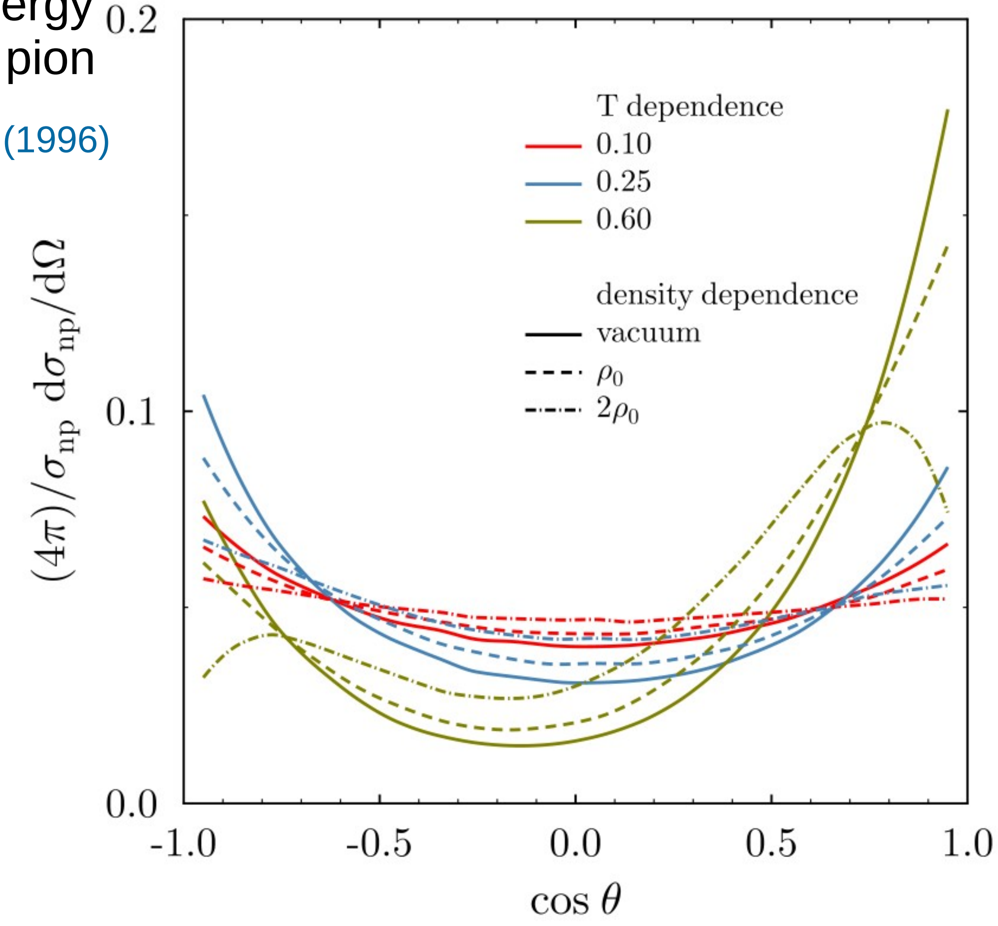
distribution of scattering angle for elastic np scattering

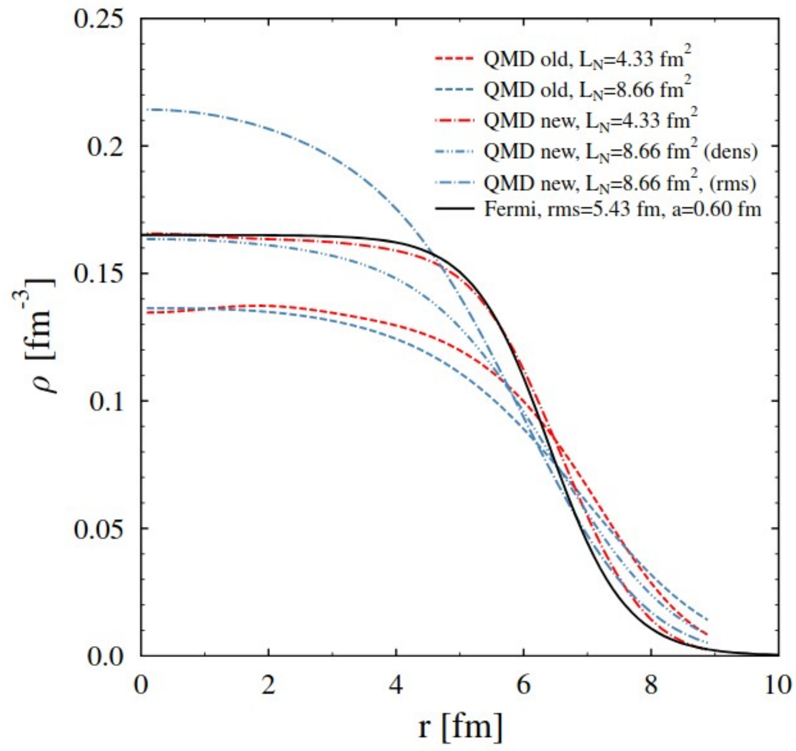


guided by microscopic calculations

Li, Machleidt PRC 48, 1702 (1993),
Li, Machleidt PRC 49, 566 (1994)

Initial/Final State

Initial state density profile of nuclei



- nuclei initialized with realistic charge radii and neutron skins
- larger L^2_N leads to stronger tails and consequently lower reduced impact parameter (flow at projectile/target rapidities affected most visibly)

this study: L^2_N=5.0 fm^2

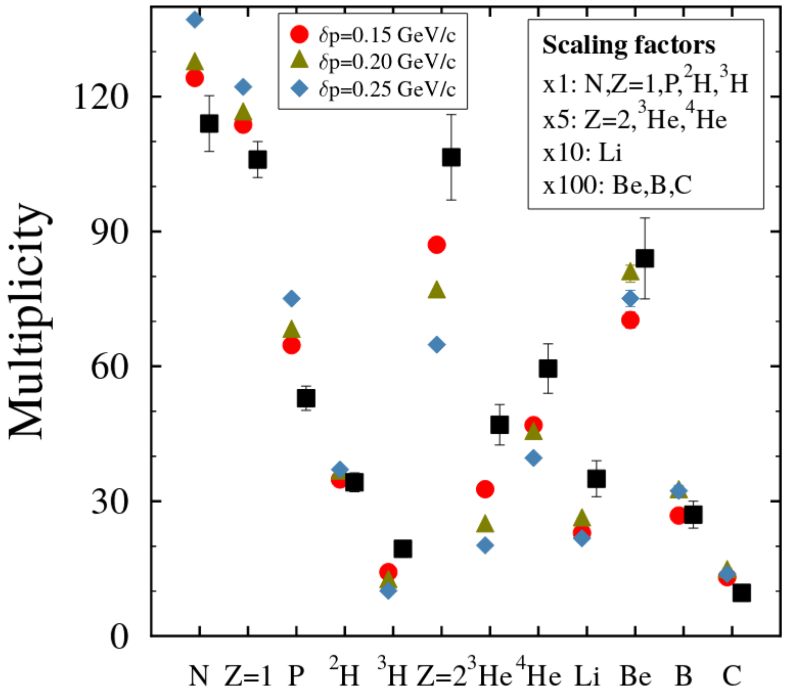
Minimum spanning tree (MST) algorithm

all clusters with A≤15, 23 additional A>15 (B,C,N,O)

Stable : lifetime > 1ms

Unstable : decay into stable using known decay channels

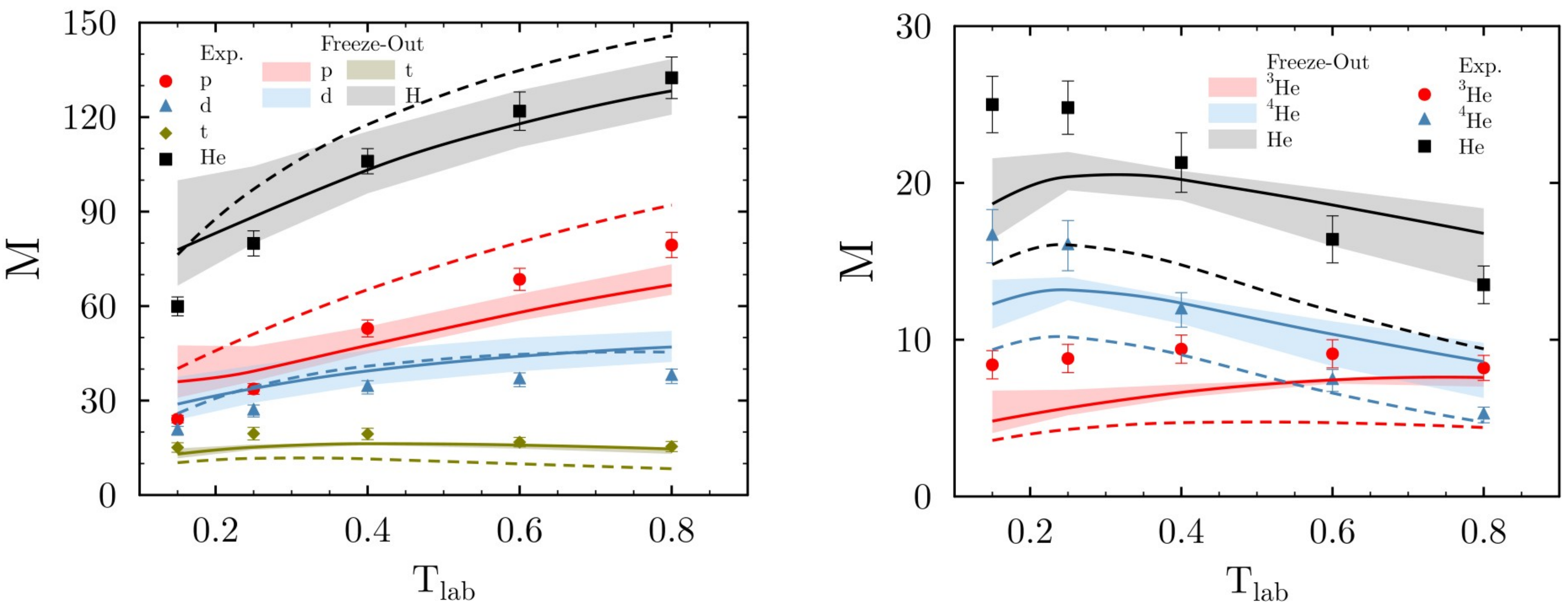
Au+Au @ 400 AmeV b< 2.0 fm



this study: δr=2.5-4.5 fm, δp=0.15-0.35 GeV/c

Cluster multiplicities

- coalescence algorithm applied at local freeze-out time
- model parameter determined from a fit of v_1 and v_2 experimental data
- δr=3.0-4.0 fm, δp=0.2-0.3 GeV/c



Hydrogen

Helium

dashed curves: results with the coalescence model applied at final time (150 fm/c)

Study of the EoS

- flow observables for protons and light clusters

Experimental data set: W. Reisdorf et al.(FOPI), NPA 876, 1 (2012)

- AuAu collisions of impact energy 0.15-0.80 GeV/nucleon
- availability for midcentral collisions (3.35fm < b < 6.0 fm):

v1(y): p, d , A=3, α

v2(y): p, d, α ($T_{lab} \geq 0.4$ GeV/A)

v1(p_T): p, d, A=3, α

v2(p_T): p, d, t

Model:

m*	[0.6, 0.9]	isoscalar effective mass
V_∞	[25, 125] MeV	isoscalar potential p → ∞
K_0	[165, 355] MeV	compressibility modulus
α	[-0.4, 0.8]	in-medium σ_{NN}, δ=0.0
Δm^*_{np}	[-0.25,0.25] at ($\rho=\rho_0$, δ=0.5)	n-p effective mass diff.
L	[15, 145] MeV	slope symmetry energy
β_1	[-0.5,3.5]	in-medium σ_{NN}, δ<>0.0
β_2	[-1.5, 2.5]	σ_{nn}<>σ_{pp}, δ<>0.0

Enforced correlations:

$J_0=-600+(K_0-165)*3.125$

$K_{sym}=-488+L*6.728$

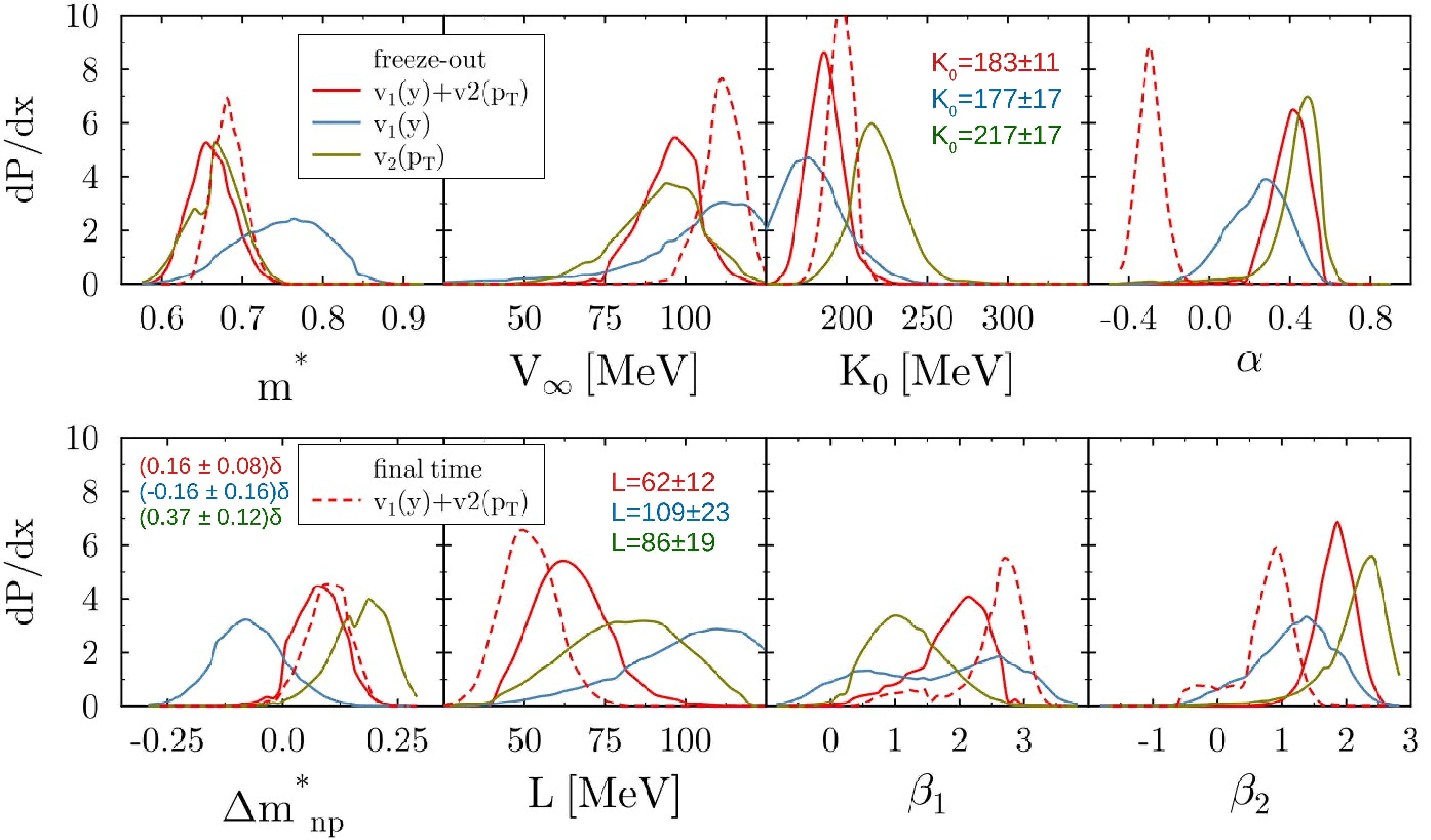
in units of [MeV]

Input:
$E/N(\rho_0)=-16.0$ MeV
$S(0.62\rho_0)=25.5$ MeV

Model uncertainty: statistical+ systematical (δr=2.5-4.5 fm, δp=0.15-0.35 GeV/c) added in quadrature

Model emulator: sum of monomials of degree ≤ 2; checked robustness LOO-CV method

Constraints



$K_0=183\pm11$
$K_0=177\pm17$
$K_0=217\pm17$

$(0.16 \pm 0.08)\delta$
$(-0.16 \pm 0.16)\delta$
$(0.37 \pm 0.12)\delta$

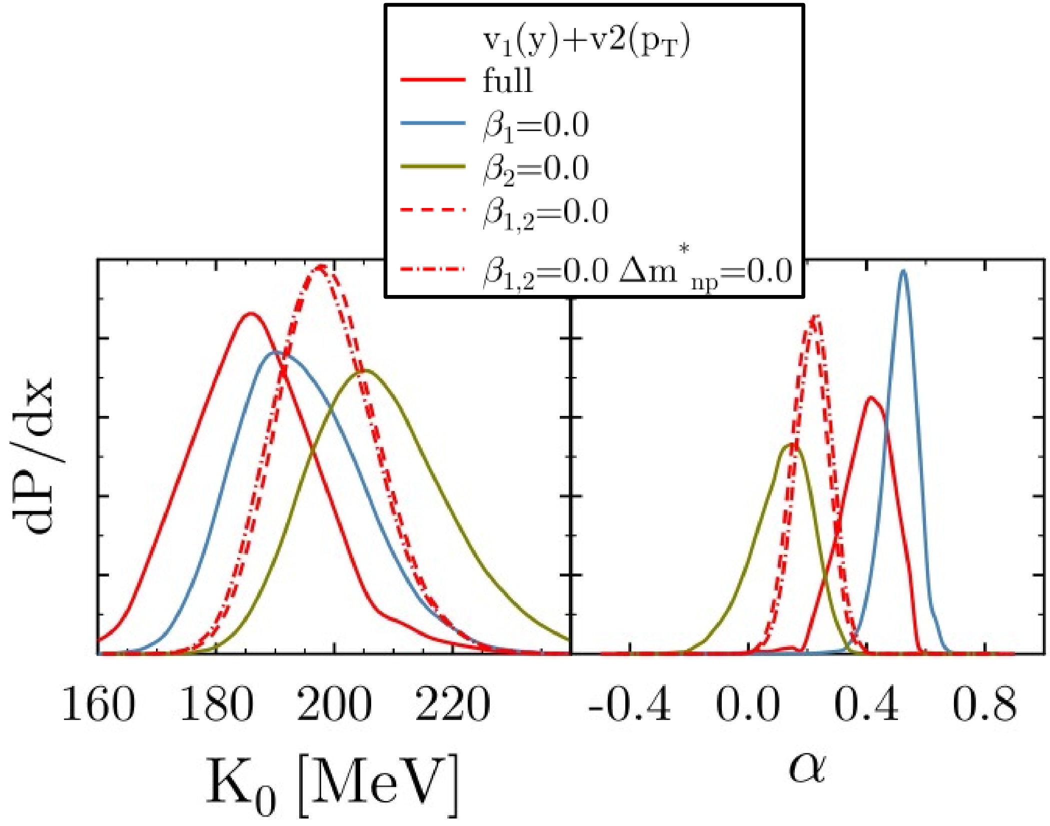
L=62±12
L=109±23
L=86±19

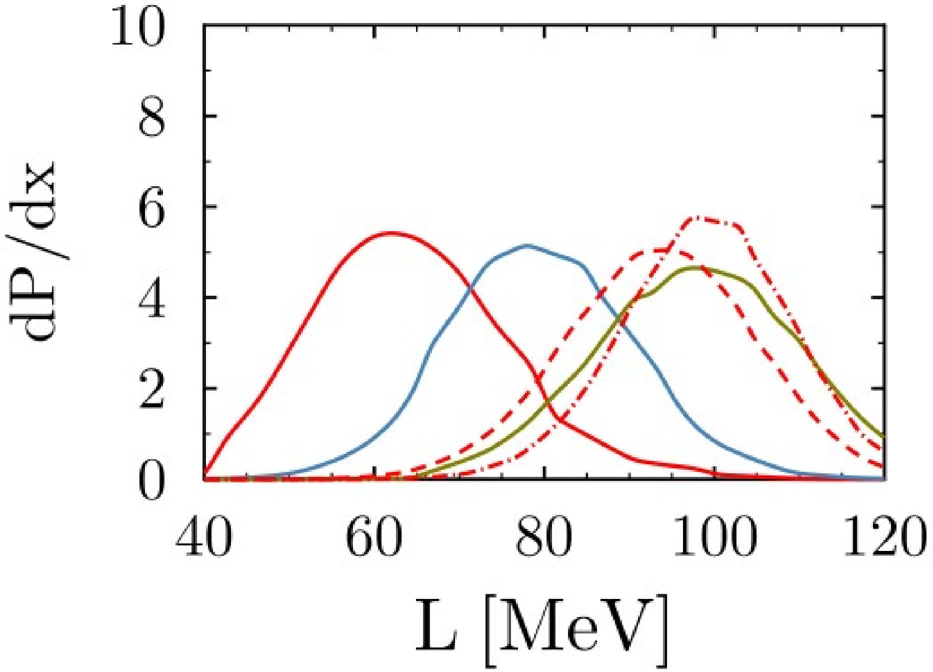
- dominant source of uncertainties: systematical model uncertainty (coalescence afterburner)
- softer SNM EoS deduced from v_1 induced by low p_T experimental data

Isospin asymmetry dependent σ_{NN}

$$\sigma^{med} = f(\rho, \delta,)\, \sigma^{vac}_{mod}$$

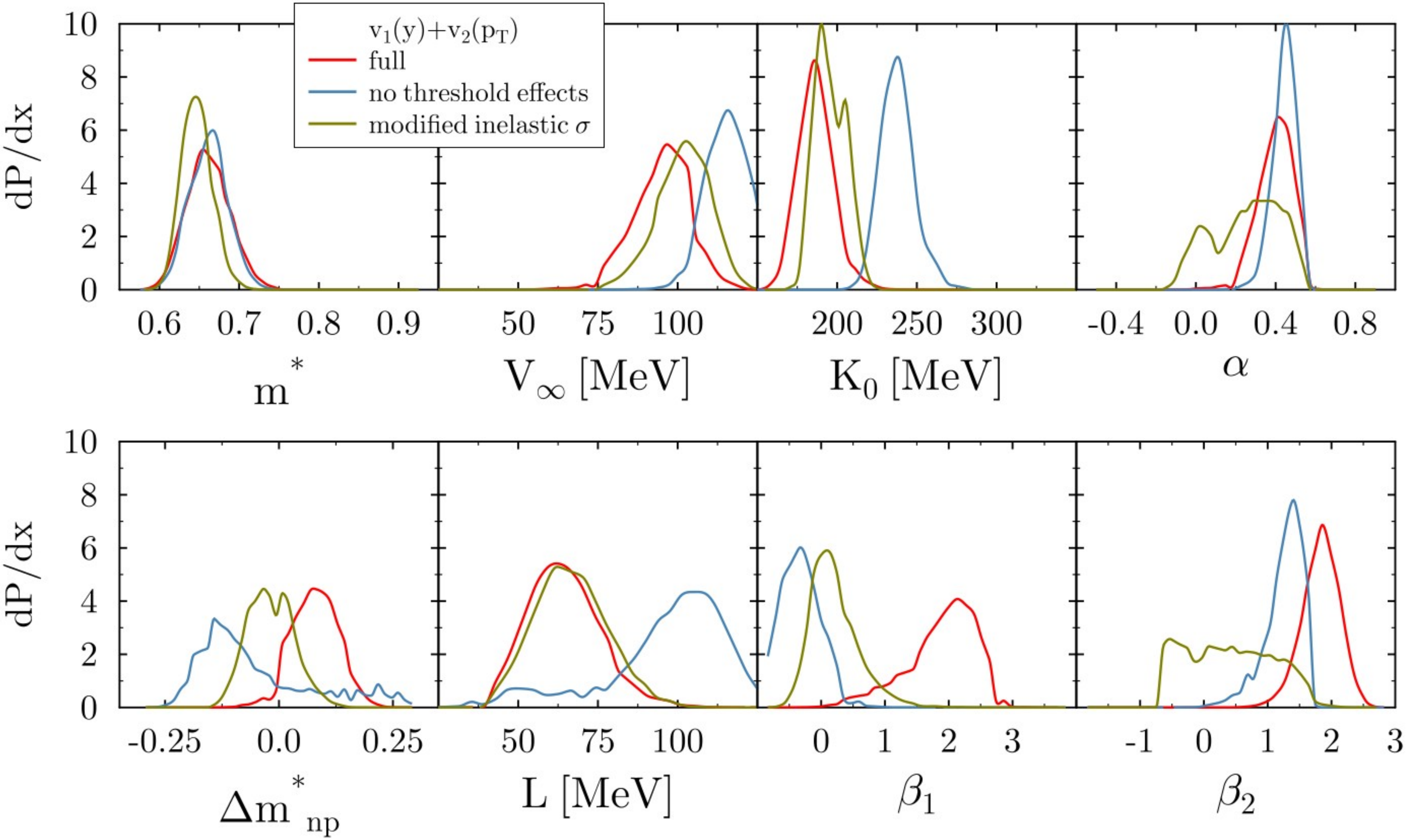
$$f(\rho,\delta) = \exp[\alpha \rho/\rho_0 + \beta_1 \delta\rho/\rho_0 + \beta_2 (\tau_1 + \tau_2)\delta\rho/\rho_0]$$ ← in-medium modification of the transition amplitude





- impact on effective masses is small
- setting, additionally, α=0.0 results in K_0=227±8 MeV, L=81±12 MeV

Threshold Effect and Inelastic Channels



Probed Density (Free Protons)

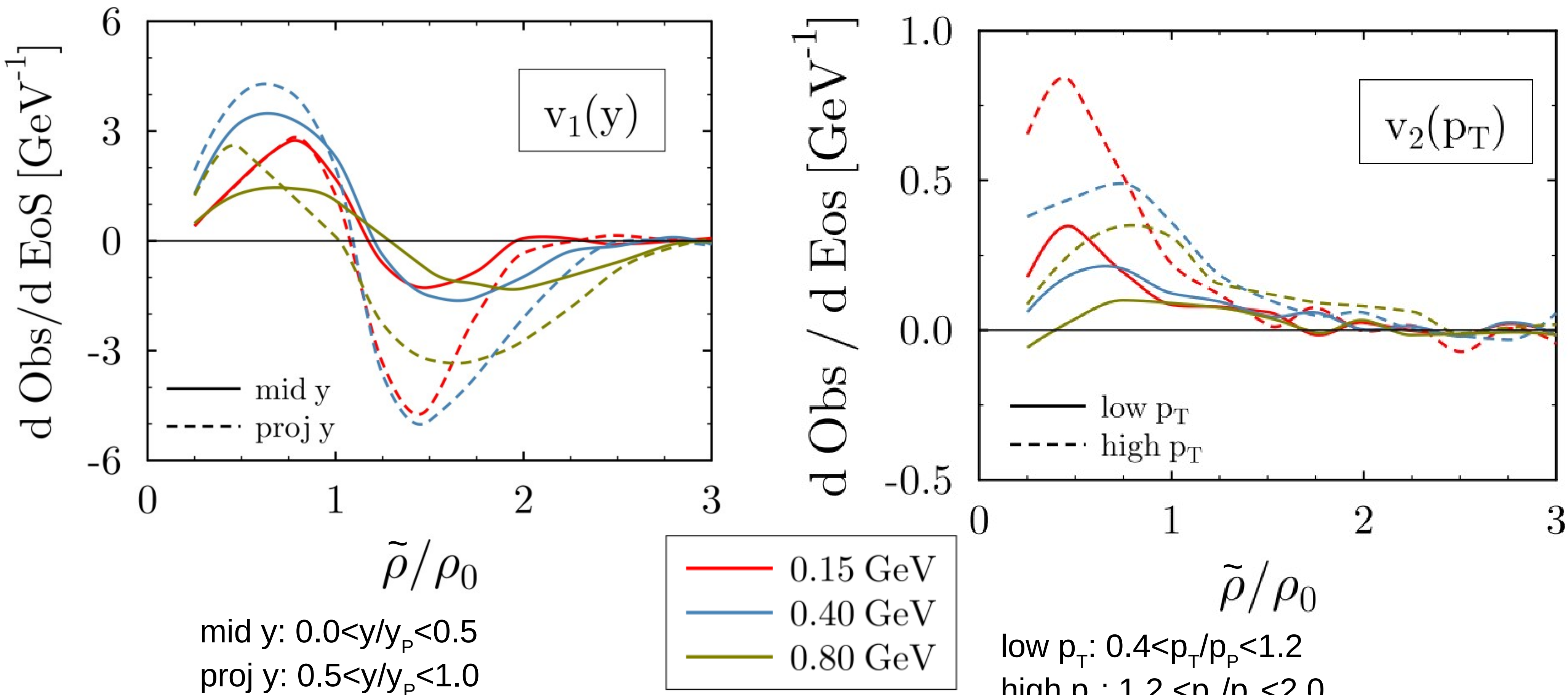
- observables are functionals of the EoS
- sensitivity: functional derivatives w.r.t to EoS or d EoS/ dρ, etc.

$$\frac{d\,Obs}{d\,EoS}=\frac{\lim_{\epsilon\to 0} Obs\left[\frac{dEoS}{d\rho}(\rho)+\epsilon\,\delta(\rho-\tilde{\rho})\right]-Obs\left[\frac{dEoS}{d\rho}(\rho)-\epsilon\,\delta(\rho-\tilde{\rho})\right]}{2\,\epsilon}$$

Isoscalar EoS

AuAu 3.35<b<6.0 fm

Symmetry Energy



mid y: $0.0<y/y_P<0.5$
proj y: $0.5<y/y_P<1.0$

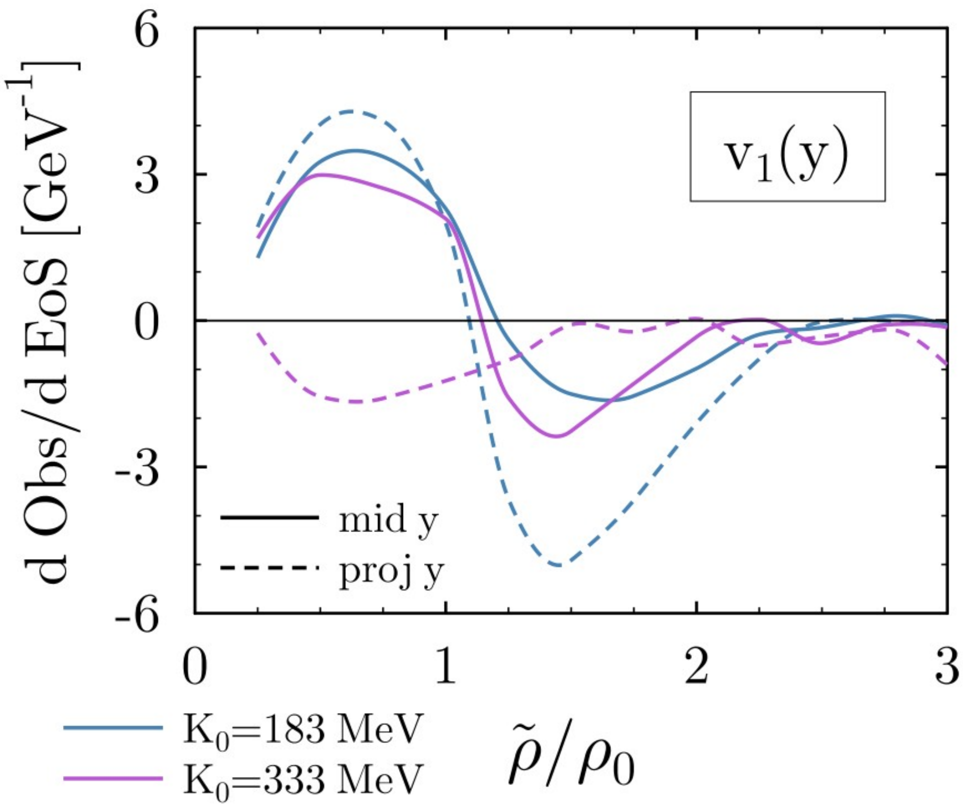
low p_T: $0.4<p_T/p_P<1.2$
high p_T: $1.2<p_T/p_P<2.0$

Impact of EoS on Probed Density

AuAu 3.35<b<6.0 fm
@ 400 MeV/A

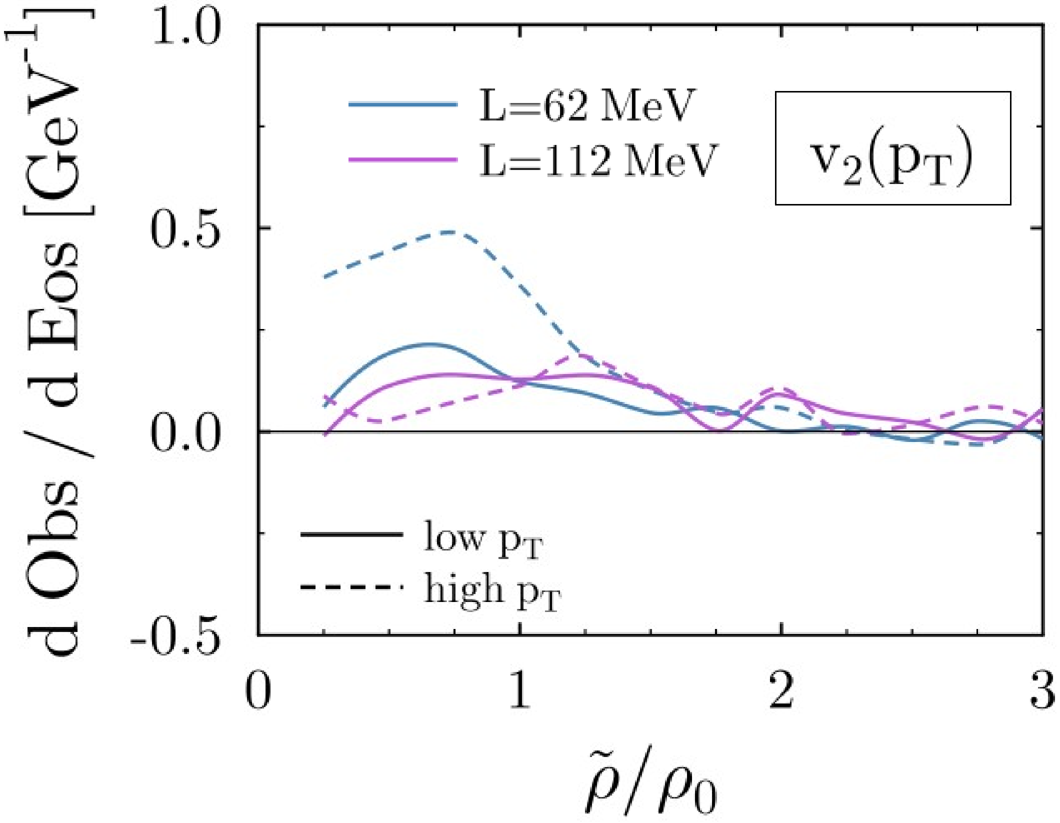
L=62 MeV → K_{sym}=-71 MeV
L=112 MeV → K_{sym}=+266 MeV

Isoscalar EoS



mid y: 0.0<y/y_P<0.5
proj y: 0.5<y/y_P<1.0

Symmetry Energy



low p_T: 0.4<p_T/p_P<1.2
high p_T: 1.2 <p_T/p_P<2.0

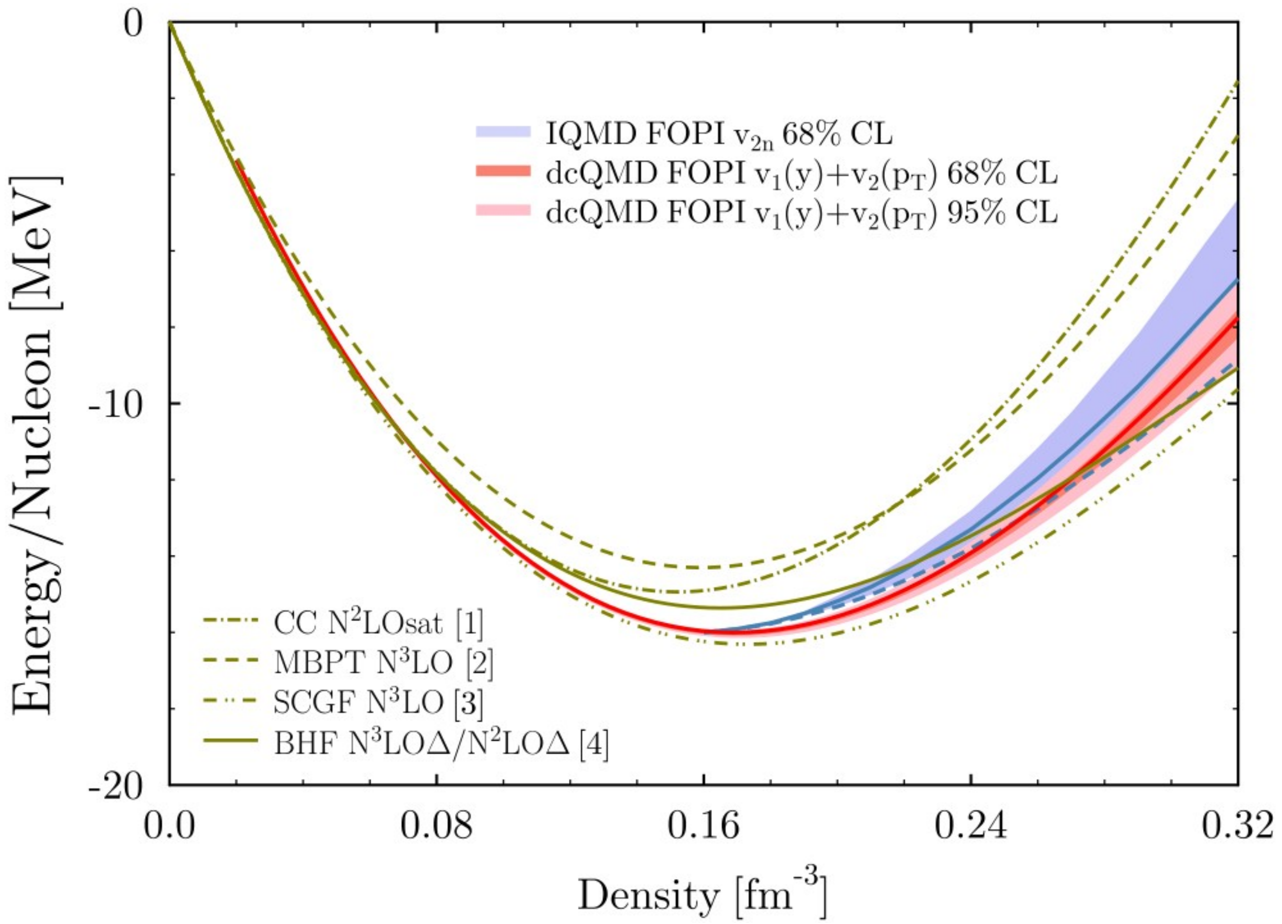
EoS of Symmetric Matter

Experimental data set: p,d,t v_2 (p_T)

p,d,A=3, α v_1(y)

68% CL Result

K_0=183 ± 11 MeV



K_0=195 ± 5 MeV (t=150 fm/c)

K_0=179 ± 5 MeV $v_1(y) \rightarrow v_1(p_T)$

$p_T/p_P>1.0$

IQMD result:

A.Le Fevre et al., NPA 945, 112 (2016)

Microscopic calculations:

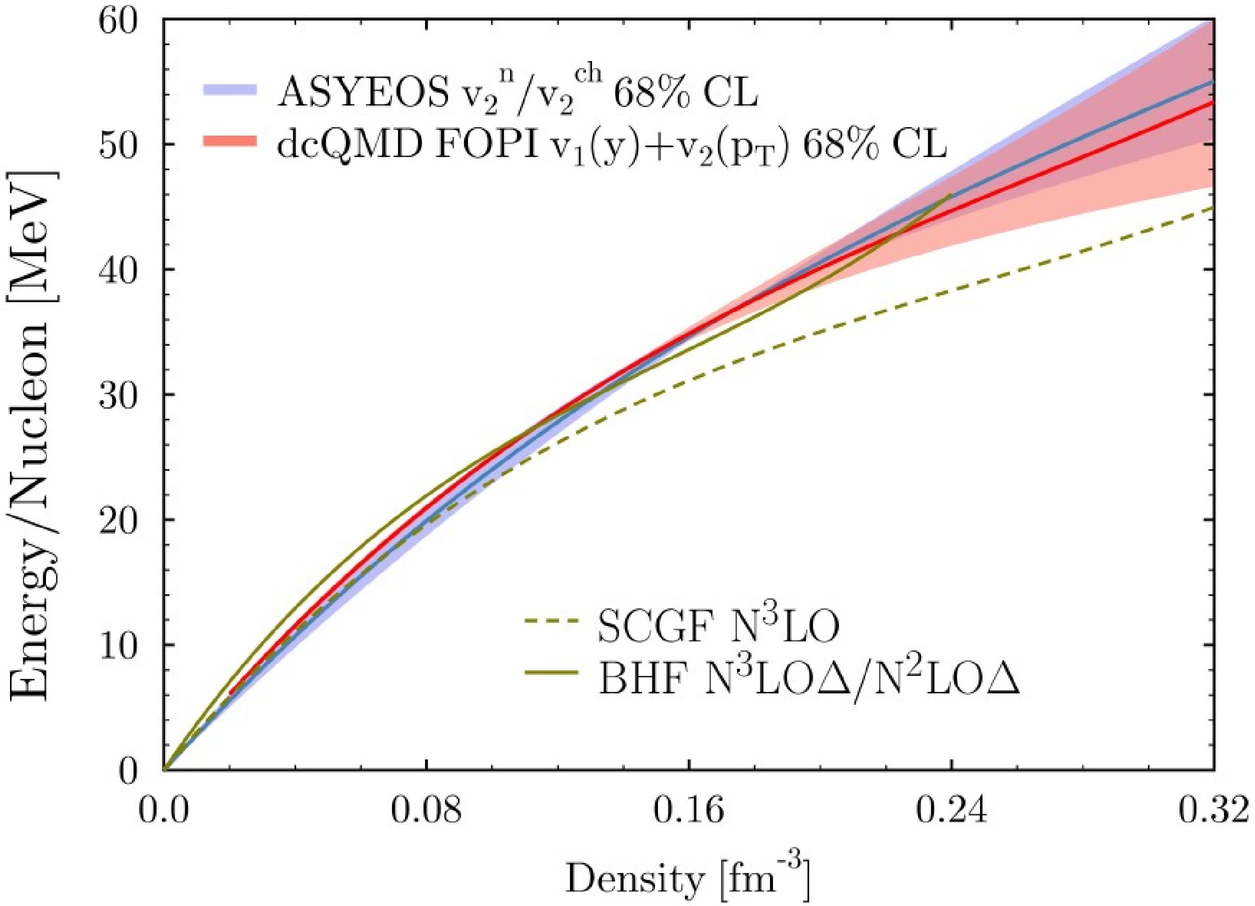
1. A. Ekstrom et al., PRC 91, 051301 (2015)
2. C. Drischler et al., PRC 102, 054315 (2020)
3. A. Carbone, PRR 2, 023227 (2020)
4. D. Logoteta, PRC 94, 064001 (2016)c

Symmetry Energy

Experimental data set: p,d,t v_2 (p_T)
p,d,A=3, α v_1(y)

Input: S($0.62\rho_0$)=25.5 MeV

68% CL Result
L=62 ± 12 MeV
S(ρ_0)=34.1 ± 1.2 MeV
S($2\rho_0$)=53.8 ± 13.2 MeV



L=50 ± 7 MeV (t=150 fm/c)
L=67 ± 7 MeV $v_1(y) \rightarrow v_1(p_T)$
p_T/p_P>1.0

ASYEOS result:

P. Russotto et al., PRC 94, 034608 (2016)

Microscopic calculations:

SCGF: A. Carbone, PRR 2, 023227 (2020)

BHF: D. Logoteta, PRC 94, 064001 (2016)c

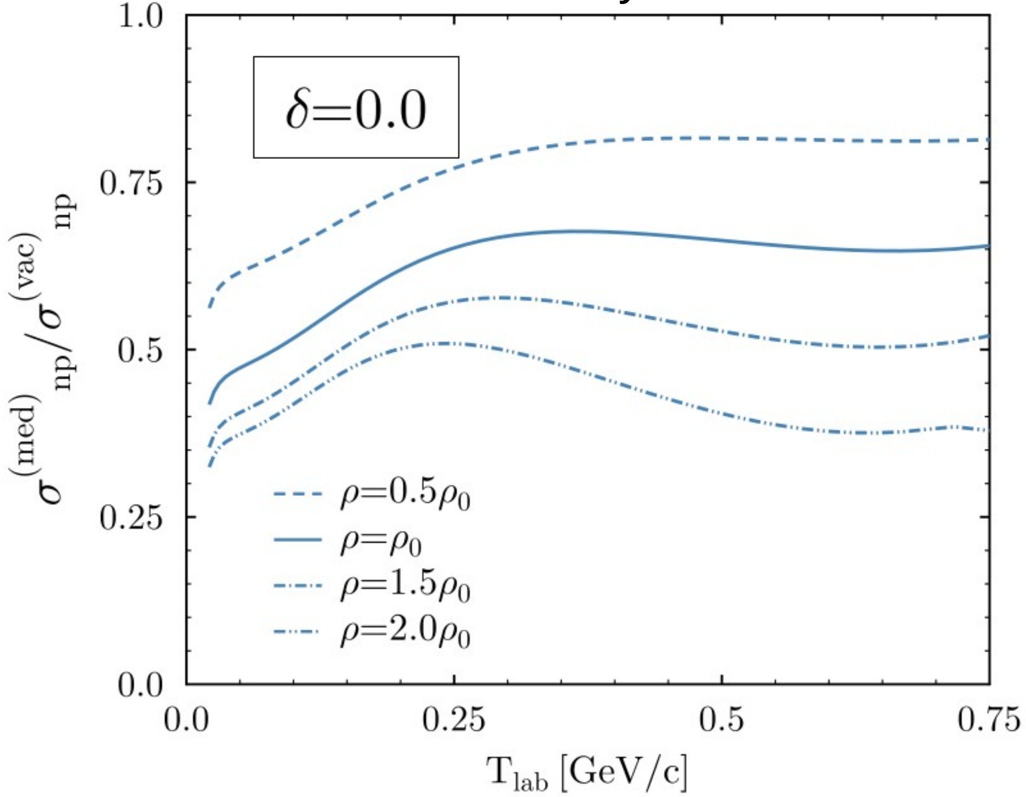
In-medium σ_{NN} (T=0 MeV Fermi)

$$\boxed{\frac{d\sigma^{(med)}}{d\Omega} = (2\pi)^4 \frac{m_1^* \, m_2^*}{k_i^* \sqrt{s_i^*}} |M_{fi}^{(med)}(\rho, \delta, \{\tau\})|^2 \frac{k_f^* \, m_{1'}^*, m_{2'}^*}{\sqrt{s_f^*}}}$$

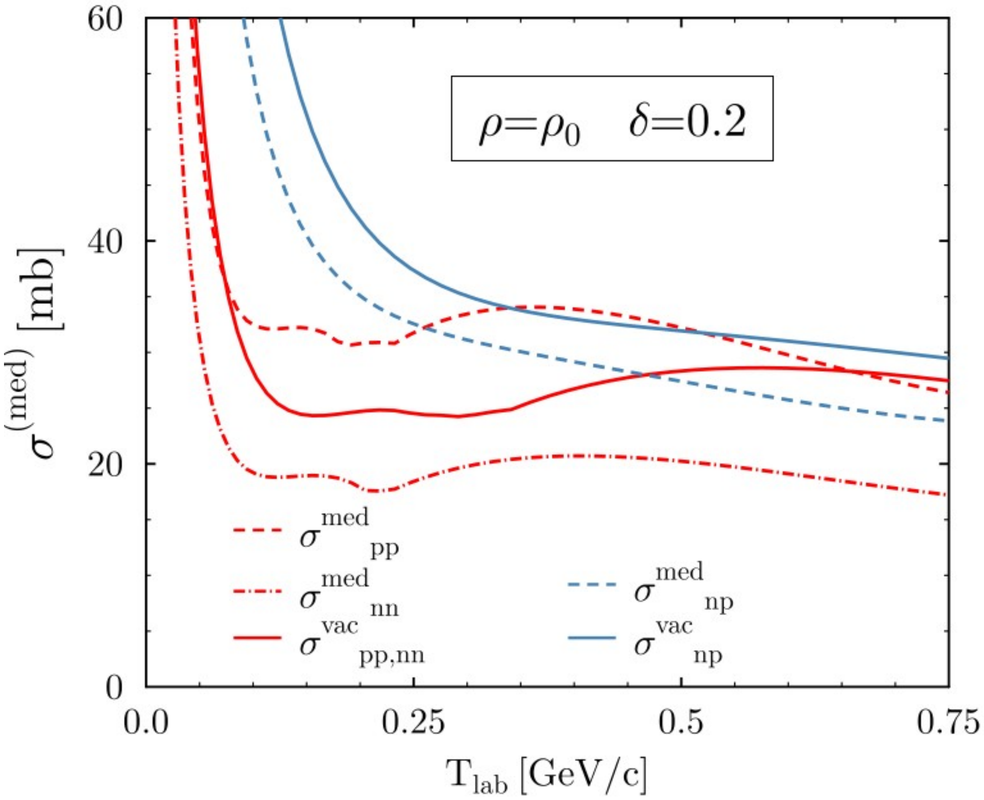
$$|M_{fi}^{(med)}(\rho, \delta, \{\tau\})|^2 = \frac{1}{2}(|M_{fi}^{(vac)}(\tilde{s}_i)|^2 + |M_{fi}^{(vac)}(\tilde{s}_f)|^2) \longleftarrow \sqrt{\tilde{s}_{i,f}} - 2\, m_N = \sqrt{s_{i,f}^*} - \sqrt{s_{th}^*} + U_{i,f} - U_{th}$$

$$\times \exp\left[(\alpha + \beta_1\, \delta + \beta_2\, (\tau_1 + \tau_2)\delta)\frac{\rho}{\rho_0}\right]$$

reduction factor in symmetric matter



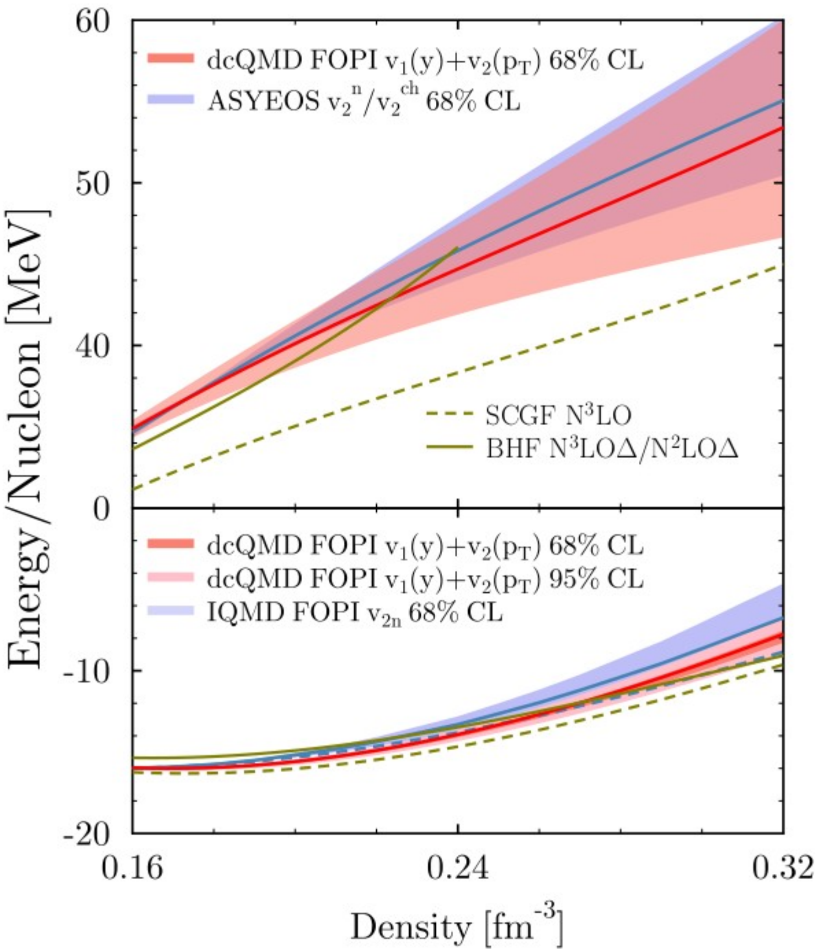
asymmetric nuclear matter



qualitative agreement with microscopical models

H. Zhang et al., IJMPE 19, 1788 (2010)
F. Sammarruca, EPJA 50, 22 (2014)

Summary & Conclusions



Study of EoS, effective masses and σ* using nucleonic observables in AuAu collisions of intermediate impact energy (0.15-0.80 GeV/nucleon)

- in-medium effects on elastic collisions that depend on density, isospin asymmetry and isospin projections

- clusterization algorithm applied at local freeze-out time

- systematic uncertainty due to coalescence parameters; dominating effect on extracted constraints

- constraints extracted from FOPI experimental data for $v_1(y)$ and $v_2(p_T)$

- model dependence: threshold effects and isospin asymmetry dependence of σ* have significant impact

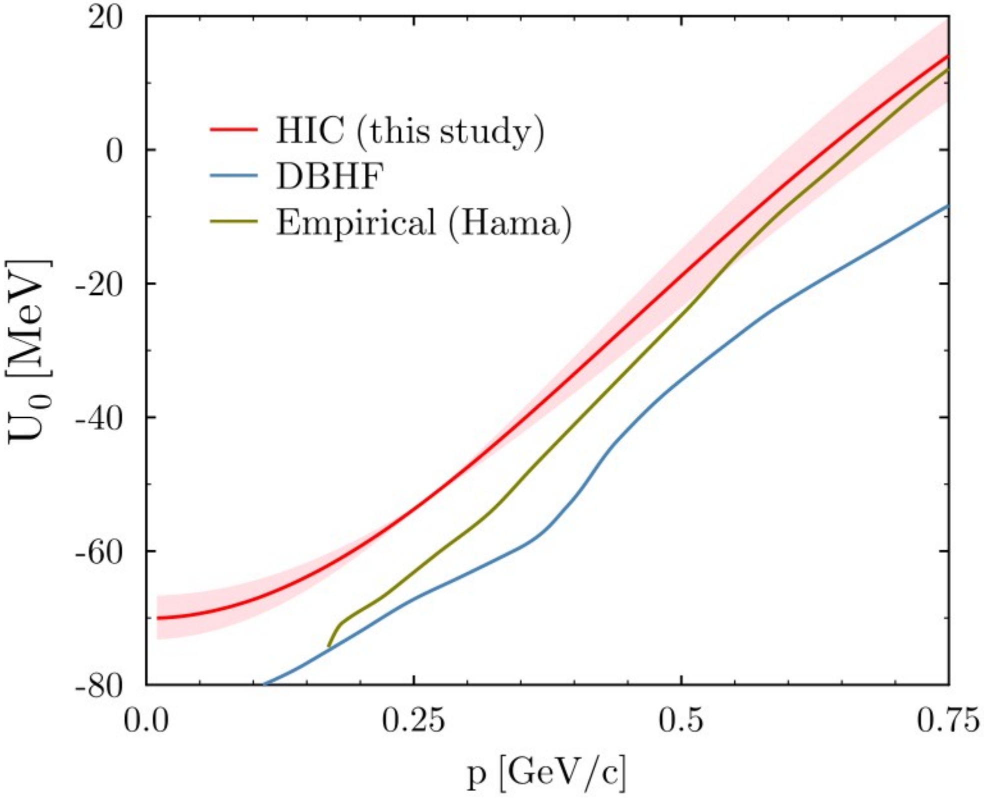
68% CL Result

$m^*=0.65 \pm 0.03$ $V_\infty=97 \pm 8$ MeV
$\Delta m^*_{np}=(0.16 \pm 0.08)\delta$
$K_0=183 \pm 11$ MeV
$L=62 \pm 12$ MeV

Perspectives: - remove imposed correlation between $L_0 \leftrightarrow J_0$ and $L \leftrightarrow K_{sym}$; allow for a variation of $E/N(\rho_0)$ and $S(0.1\ \text{fm}^{-3})$.
- include explicit cluster degrees of freedom to be able to use experimental data sets to their full potential
- extend the model to be able to make robust studies of the EoS around $3\rho_0$ using FOPI and HADES data

Momentum dependent optical potential

Isoscalar Potential



Lane Potential