When and where are clusters formed in expanding systems?

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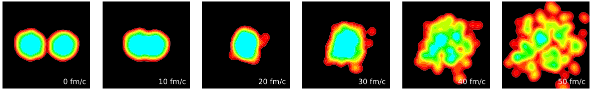
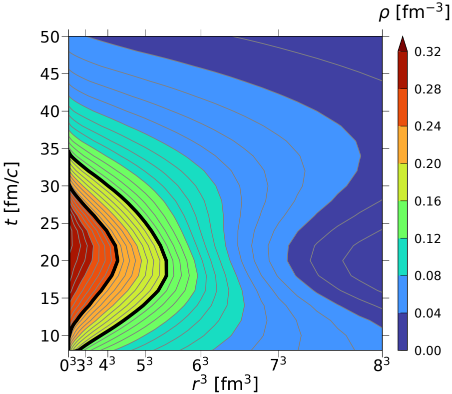
- How clusters appear in HIC, in AMD calculations.
- Symmetry energy information in compressed and expanding matter, and cluster observables.

Space-time evolution of density



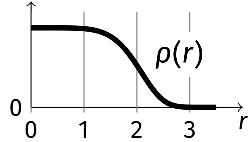
Sn+Sn @300 MeV/u

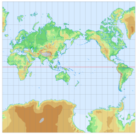


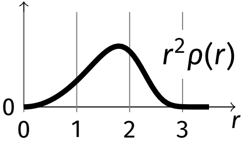
Density $\rho(r,t) = \int \frac{d\Omega}{4\pi}\, \rho(\boldsymbol{r},t)$ in c.m.

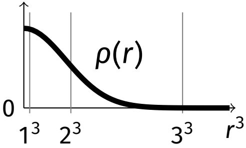
AMD calculation

^{132}Sn + ^{124}Sn, E/A = 270 MeV, b < 1 fm









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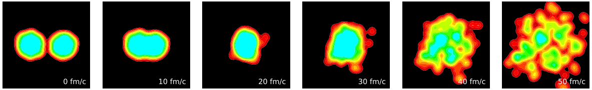

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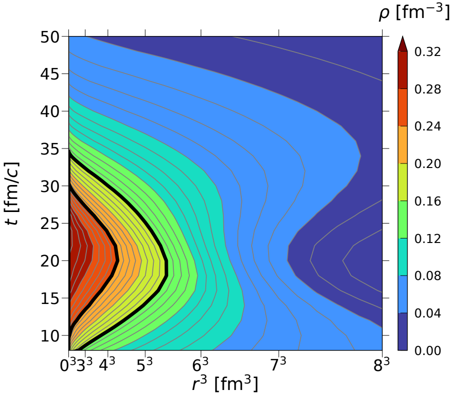

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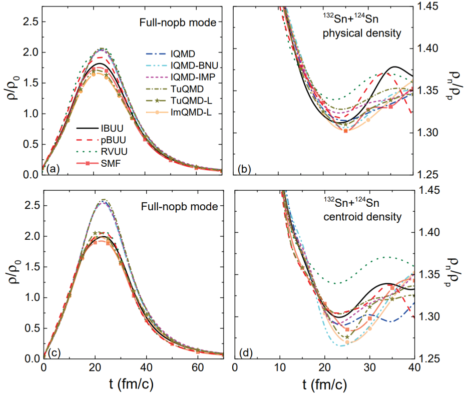
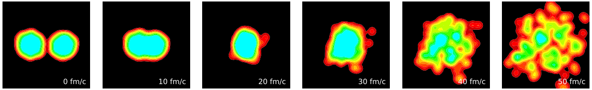


^{132}Sn + ^{124}Sn, E/A = 270 MeV, b = 4 fm

Comparison of transport models

Jun Xu et al. (TMEP), arXiv:2308.05347

Space-time evolution of density



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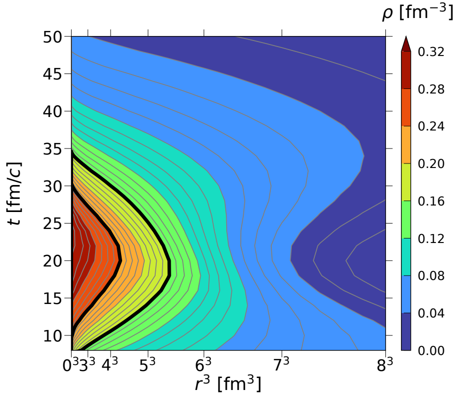

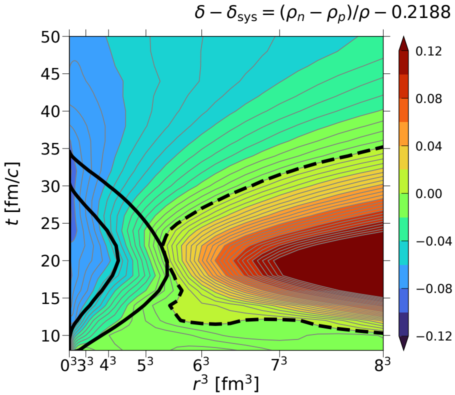

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AMD calculation

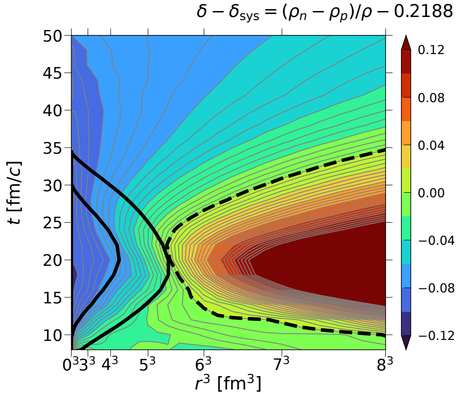
^{132}Sn + ^{124}Sn, E/A = 270 MeV, $b < 1$ fm

Isospin asymmetry: $\delta = (\rho_n - \rho_p)/(\rho_n + \rho_p)$

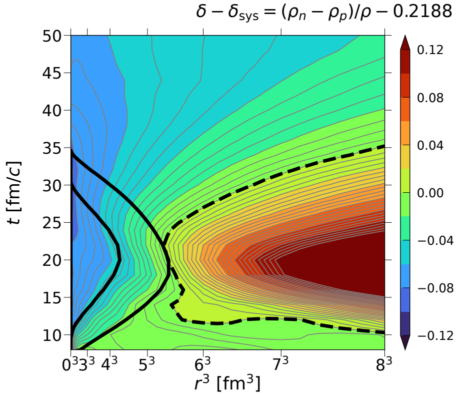
Dashed line: Asymmetry of the system δ_{sys}

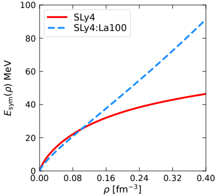
Symmetry energy effect on isospin asymmetry

SLy4:La100



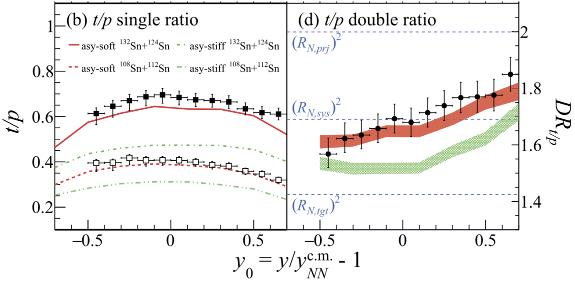
SLy4





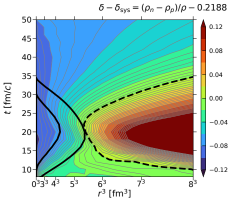
- $E_{sym}^{SLy4:La100}(\rho_1) = E_{sym}^{SLy4}(\rho_1)$ at $\rho_1 = 0.10$ fm^{-3}
- $L(\rho_0) = \begin{cases} 100 \text{ MeV} & \text{(SLy4:La100)} \\ 46 \text{ MeV} & \text{(SLy4)} \end{cases}$

Symmetry energy effects in cluster observables

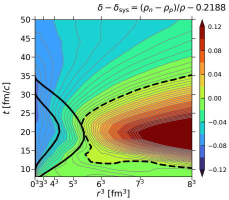


$$\frac{t/p \text{ in } {}^{132}\text{Sn} + {}^{124}\text{Sn}}{t/p \text{ in } {}^{108}\text{Sn} + {}^{112}\text{Sn}} = (t/p \text{ double ratio})$$

Stiff $E_{\text{sym}}(\rho)$



Soft $E_{\text{sym}}(\rho)$



M. Kaneko, Murakami, Isobe, Kurata-Nishimura, Ono, Ikeno et al. (SπRIT), PLB 822 (2021) 136681.

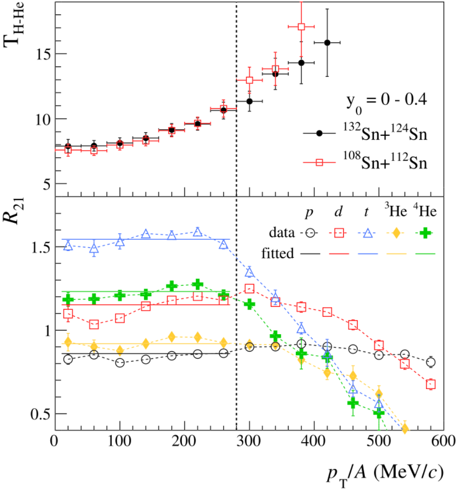
Our argument was:

> *"the strong dependence on the stiffness of the symmetry energy seen in the theoretical $DR_{t/p}$ indicates that the main contribution of the triton production likely stems from the central region of the expanding system rather than the outer region."*

Main aim of today

To confirm this argument by directly investigating the cluster production and evolution during heavy-ion collisions.

Isoscaling in the SπRIT data



Isoscaling ratio:

$$R_{21}(N,Z) = \frac{Y(N,Z) \text{ from } {}^{132}\text{Sn} + {}^{124}\text{Sn}}{Y(N,Z) \text{ from } {}^{108}\text{Sn} + {}^{112}\text{Sn}}$$

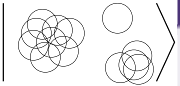
J.W. Lee et al. (SπRIT), EPJA (2022) 201.

Sn + Sn at 270 MeV/u, central events (b < 1.5 fm)

- *Isoscaling phenomenon up to p_T/A < 280 MeV/c is found*
- *but breaks down for cluster particles with p_T/A > 280 MeV/c.*

Today's talk will focus on particles in the region of low p_T, for which interpretation is easier.

Antisymmetrized Molecular Dynamics



AMD wave function

$$|\Phi(Z)\rangle = \det_{ij}\Big[\exp\Big\{-\nu\Big(\mathbf{r}_j - \frac{\mathbf{Z}_i}{\sqrt{\nu}}\Big)^2\Big\}\chi_{\alpha_i}(j)\Big]$$

$$\mathbf{Z}_i = \sqrt{\nu}\mathbf{D}_i + \frac{i}{2\hbar\sqrt{\nu}}\mathbf{K}_i$$

ν : Width parameter = $(2.5\ \mathrm{fm})^{-2}$

χ_{α_i} : Spin-isospin states = $p\uparrow, p\downarrow, n\uparrow, n\downarrow$

Equation of motion for the wave packet centroids Z

$$\frac{d}{dt}\mathbf{Z}_i \quad = \quad \{\mathbf{Z}_i, \mathcal{H}\}_{\mathrm{PB}} \quad + \quad \text{(NN collisions)} \quad + \quad \textit{(some model extensions)}$$

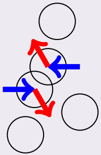
$\{\mathbf{Z}_i, \mathcal{H}\}_{\mathrm{PB}}$: Motion in the mean field

$$\mathcal{H} = \frac{\langle\Phi(Z)|H|\Phi(Z)\rangle}{\langle\Phi(Z)|\Phi(Z)\rangle} + \text{(c.m. correction)}$$

- H: Effective interaction (e.g. Skyrme force)

NN collisions

$$W_{i\to f} = \frac{2\pi}{\hbar}|\langle\Psi_f|V|\Psi_i\rangle|^2\delta(E_f - E_i)$$



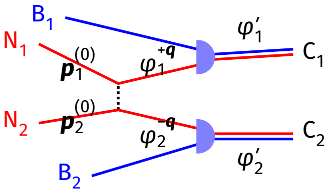
- $|V|^2$ or σ_{NN} (in medium)
- Pauli blocking

Ono, Horiuchi, Maruyama, Ohnishi, Prog. Theor. Phys. 87 (1992) 1185.

NN collisions with cluster correlations

$$N_1 + B_1 \; + \; N_2 + B_2 \;\to C_1 + C_2$$

- N_1, N_2 : Colliding nucleons
- B_1, B_2 : Spectator nucleons/clusters (maybe empty)
- C_1, C_2 : N, $(2N)$, $(3N)$, $(4N)$ (up to α cluster)



Transition probability

$$W(\mathrm{NBNB} \to \mathrm{CC}) \propto |\langle\varphi_1'|\varphi_1^{+\boldsymbol{q}}\rangle|^2 |\langle\varphi_2'|\varphi_2^{-\boldsymbol{q}}\rangle|^2 |M|^2 \delta(E_\mathrm{f} - E_\mathrm{i}) p_\mathrm{f,rel}^2 dp_\mathrm{f,rel} d\Omega$$

$$\boldsymbol{p}_\mathrm{rel} = \tfrac{1}{2}(\boldsymbol{p}_1 - \boldsymbol{p}_2) = p_\mathrm{rel}\hat{\boldsymbol{\Omega}}$$

$$\boldsymbol{p}_1 = \boldsymbol{p}_1^{(0)} + \boldsymbol{q}$$

$$\boldsymbol{p}_2 = \boldsymbol{p}_2^{(0)} - \boldsymbol{q}$$

$$\varphi_1^{+\boldsymbol{q}} = \exp(+i\boldsymbol{q}\cdot\boldsymbol{r}_{\mathrm{N}_1})\varphi_1^{(0)}$$

$$\varphi_2^{-\boldsymbol{q}} = \exp(-i\boldsymbol{q}\cdot\boldsymbol{r}_{\mathrm{N}_2})\varphi_2^{(0)}$$

$$\frac{d\sigma_{\mathrm{C}_1\mathrm{C}_2}}{d\Omega} = P(\mathrm{C}_1\mathrm{C}_2, p_\mathrm{f,rel}, \Omega) \times \left(\frac{p_\mathrm{i,rel}}{v_\mathrm{i}}\frac{p_\mathrm{f,rel}}{v_\mathrm{f}}\right) \times \left|M(p_\mathrm{i,rel}, p_\mathrm{f,rel}, \Omega)\right|^2 \times \frac{p_\mathrm{f,rel}}{p_\mathrm{i,rel}}$$

$E_\mathrm{f}(p_\mathrm{f}) = E_\mathrm{i}$ and $v_\mathrm{f} = \dfrac{dE_\mathrm{f}}{dp_\mathrm{f}}$ are solved with E_i, $E_\mathrm{f}(p_\mathrm{f}) = \dfrac{\langle\Phi|H|\Phi\rangle}{\langle\Phi|\Phi\rangle}$ + c.m. crct.

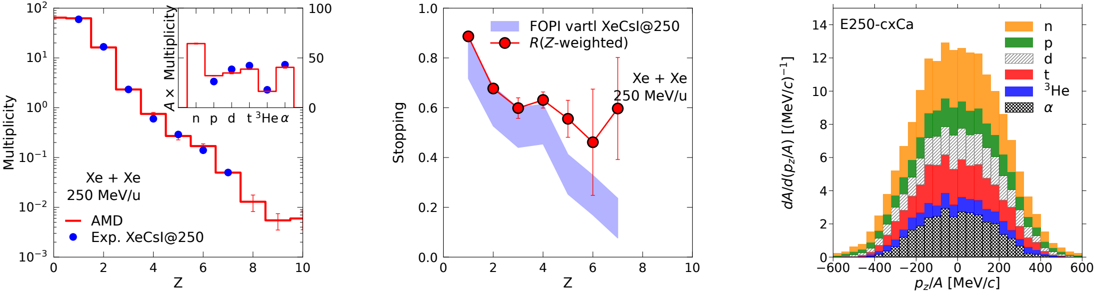
- Energy is conserved precisely.
- Cross section naturally depends on potentials.

Ono, J. Phys. Conf. Ser. 420 (2013) 012103.
Ikeno, Ono et al., PRC 93 (2016) 044612.
Ono, JPS Conf. Proc. 32 (2020) 010076.

Choice of NN matrix element and basic cluster observables

$$\frac{d\sigma}{d\Omega}(NNNB \to C_1C_2) = P(C_1C_2, p_{\mathrm{f,rel}}, \Omega) \times \left(\frac{p_{\mathrm{i,rel}}}{v_{\mathrm{i}}}\frac{p_{\mathrm{f,rel}}}{v_{\mathrm{f}}}\right) \times \left|M(p_{\mathrm{i,rel}}, p_{\mathrm{f,rel}}, \Omega)\right|^2 \times \frac{p_{\mathrm{f,rel}}}{p_{\mathrm{i,rel}}}$$

$$|M|^2 = \left(\frac{2}{m_N}\right)^2 \frac{d\sigma_{NN}}{d\Omega} \quad \text{with} \quad \sigma_{NN}(\rho',\epsilon) = \sigma_0 \tanh\left(\frac{\sigma_{\mathrm{free}}(\epsilon)}{\sigma_0}\right), \quad \sigma_0 = 0.8\,(\rho')^{-2/3}$$



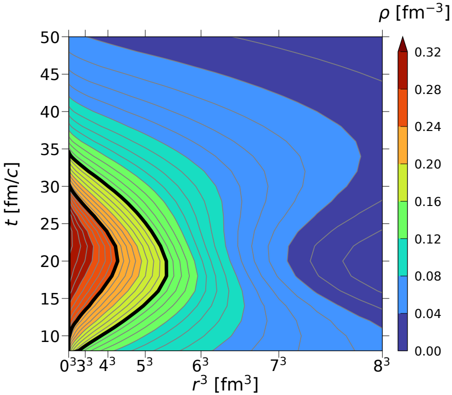
Central Xe + CsI (Xe + Xe) collisions at 250 MeV/nucleon

FOPI Data: Reisdorf et al., NPA 848 (2010) 366.

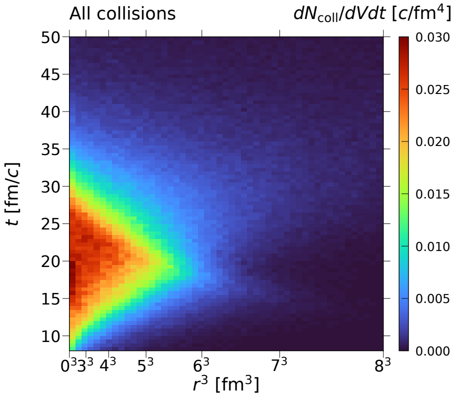
"Density" $\rho_i'^{(\mathrm{ini/fin})} = \left(\frac{2\nu}{\pi}\right)^{\frac{3}{2}} \sum_{k(\neq i)} \theta\left(p_{\mathrm{cut}} > |\boldsymbol{P}_i^{(\mathrm{ini/fin})} - \boldsymbol{P}_k|\right) e^{-2\nu(\boldsymbol{R}_i-\boldsymbol{R}_k)^2}$ with a momentum cut $p_{\mathrm{cut}} = (375\ \mathrm{MeV}/c)\, e^{-\epsilon_{\mathrm{cm}}/(225\ \mathrm{MeV})}$.

Space-time distribution of *NN* collisions

Density $\rho(r,t)$



All $NNBB \to CC$ collisions



$\dfrac{dN_{\mathrm{coll}}}{dVdt}$:

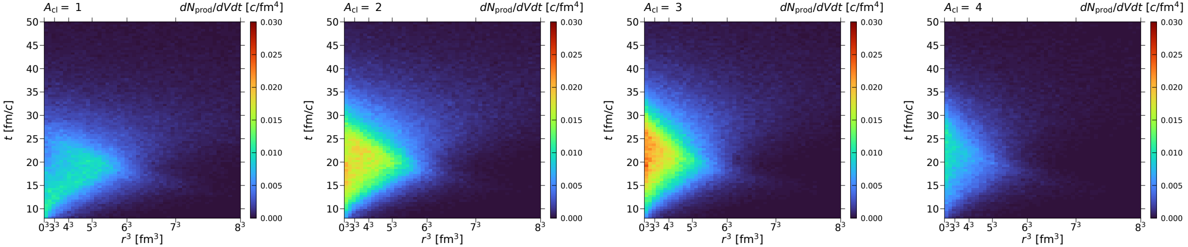
Distribution of $(\boldsymbol{r}_{N_{1,2}}, t)$ of $N_1 + N_2 + B_1 + B_2 \to C_1 + C_2$ collisions.

(Summed over all $C_1, C_2 = n, p, d, t, \ldots$)

Space-time distribution of cluster formation

$\dfrac{dN_{\mathrm{prod}}}{dVdt}$: Distribution of $(\boldsymbol{r}_N, t)$ of $N + N + B + B \to C + X$.

$A_C = 1$ $A_C = 2$ $A_C = 3$ $A_C = 4$

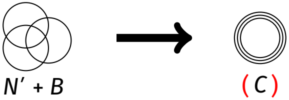


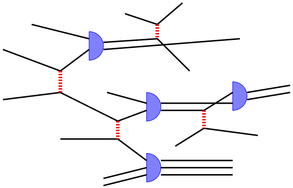
Clusters C ($A_C \geq 2$) are created mainly in the central part of the system.

Cluster in AMD calculation: “nm-cluster”

$$N_1 + N_2 + B_1 + B_2 \to C_1 + C_2$$

When a cluster C (= C_1 or C_2) is created, the nucleon wave packets are placed at the phase space point.



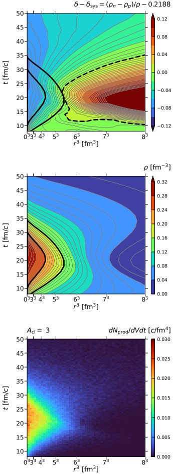


AMD wave function: $\Phi = \mathcal{A}[\varphi_1\varphi_2\varphi_3\varphi_4\varphi_5\varphi_6\varphi_7\cdots] = \mathcal{A}[\underset{^3\mathrm{He}}{(\varphi_1\varphi_2\varphi_3)}\cdot\underset{n}{(\varphi_4)}\cdot(\varphi_5\varphi_6)\cdot(\varphi_7\cdots]$

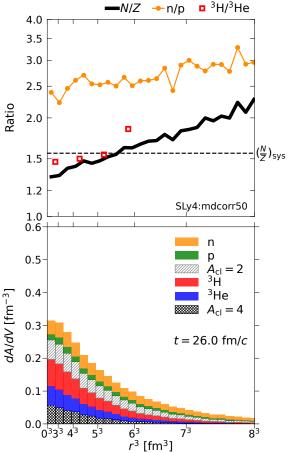
Such a **“nominal cluster (nm-cluster)”** should be distinguished from an emitted cluster in vacuum.

- The time evolution of the state Φ doesn’t depend on whether we recognize a nucleon as a part of a nm-cluster or not, except for some extensions of the model.
- A produced nm-cluster may be broken later during the evolution of the system.
- A nm-cluster may be a part of a larger nucleus. For example, nm-^{3}He + nm-neutron = α.
- A nm-cluster is different from a cluster in vacuum in the internal state and energy, due to antisymmetrization and density-dependent interaction.

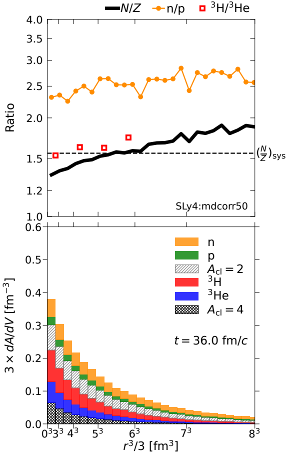
Distribution and evolution of existing nm-clusters



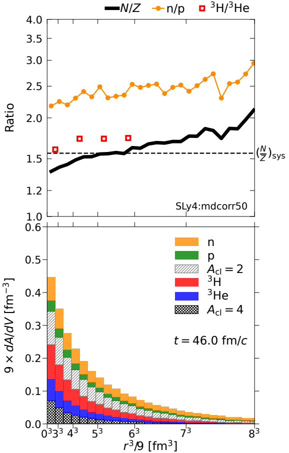
t = 26 fm/c



t = 36 fm/c



t = 46 fm/c



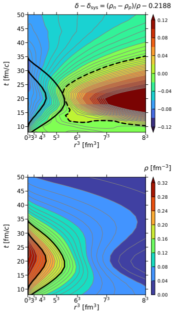
The r^3 axis is scaled as $r^3\to r^3/3\to r^3/9$.

Upper panel: $N/Z=\dfrac{\sum_{i\in\{\text{nm-clusters}\}}N_i}{\sum_{i\in\{\text{nm-clusters}\}}Z_i}$

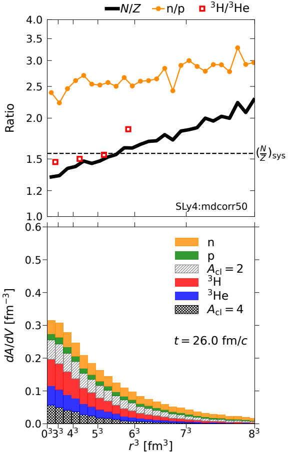
Lower panel: **Mass-weighted distribution**

- Nm-clusters are located in the inner part.
- Expansion keeps nm-cluster fractions.
- $E_{\rm sym}(\rho)$ effect remains in N/Z.

From dense phase (t = 26 fm/c) to experimental observable (t = ∞)



t = 26 fm/c



For simple expansion,
we can expect r/t → velocity.

Observable (after statistical decay)

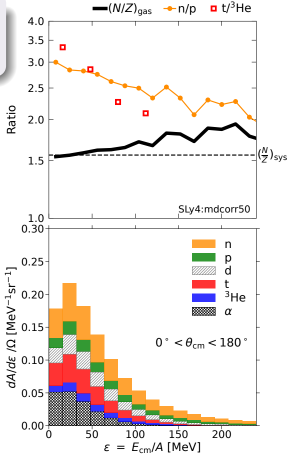
Mass-weighted spectrum of $\varepsilon = E_{\rm cm}/A$ for nucleons and light clusters ('gas' particles).

$$(N/Z)_{\rm gas} = \frac{Y_n + Y_d + 2Y_t + Y_h + 2Y_\alpha}{Y_p + Y_d + Y_t + 2Y_h + 2Y_\alpha}$$

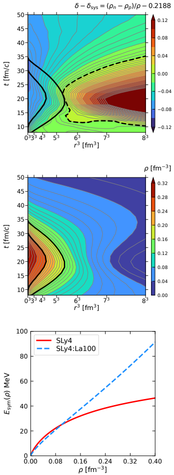
Often called "C.I. spectrum".

Showing the spectrum as a function of $\varepsilon^{3/2}$ would be more suitable, but ...

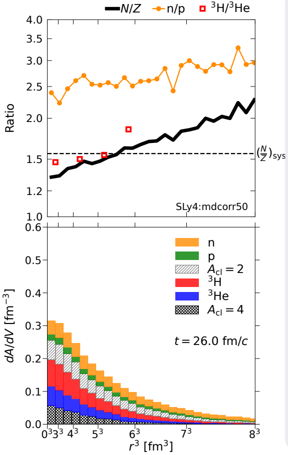
t = ∞



From dense phase (t = 26 fm/c) to experimental observable (t = ∞)

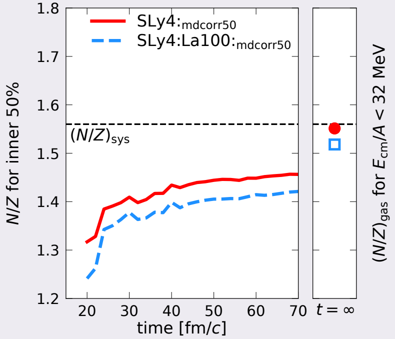


t = 26 fm/c

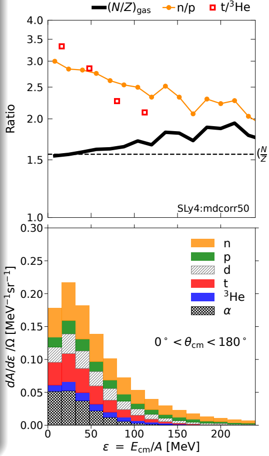


N/Z of the inner part

- Sphere including 50% of the system.
- $E_{cm}/A < 32$ MeV

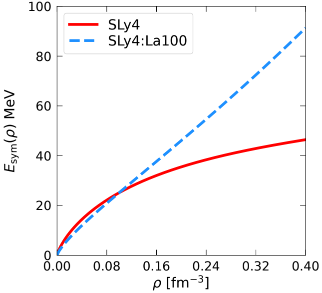


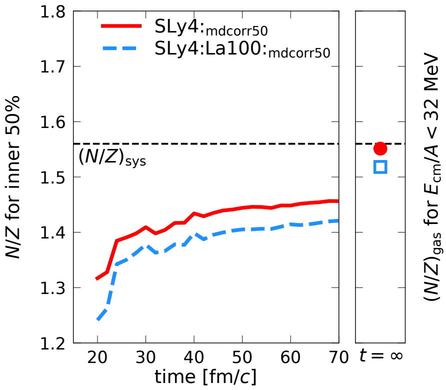
t = ∞



- **The $E_{sym}(\rho)$ effect remains from dense phase to the observable.**
- N/Z doesn't → the observable $(N/Z)_{gas}$. A simple reason is because $(N/Z)_{gas}$ does not include $A > 4$.

More considerations with more cases of EOS



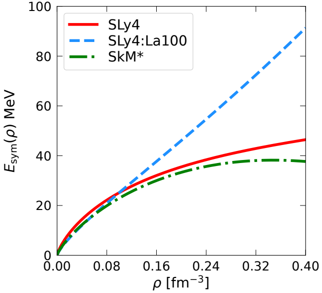


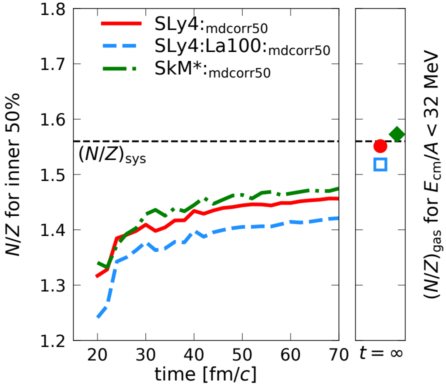
- N/Z of the inner part gradually increases during the expansion phase, due to
  - Coulomb force.

    Note: Both neutrons and protons in a cluster C are accelerated as if they have a common effective charge Z_C/A_C.
  - Possible isospin diffusion between inner and outer regions.

More considerations with more cases of EOS

	SLy4	SkM*	
ρ_0	0.16	0.16	fm^{-3}
E_0	−15.97	−15.77	MeV
K	230	217	MeV
m^*	0.69	0.79	m_N
S_0	32.0	30.0	MeV
L	46	46	MeV
$m_n^* - m_p^*$	−0.18	+0.33	$\delta \cdot m_N$





- N/Z of the inner part gradually increases during the expansion phase, due to
  - Coulomb force.

    Note: Both neutrons and protons in a cluster C are accelerated as if they have a common effective charge Z_C/A_C.
  - Possible isospin diffusion between inner and outer regions.
- The effective masses ($m_n^* < m_p^*$ or $m_n^* > m_p^*$) have some effect on the increase of N/Z before $t \approx 30$ fm/c.

Summary

Central ^{132}Sn + ^{124}Sn collisions at 270 MeV/nucleon simulated by AMD with cluster correlations.

- Cluster correlations may start to appear in the central region of the compressed and expanding system. (But they are nominal clusters at first.)
- The expansion is simple so that the $E_{\text{sym}}(\rho)$ effect in the compressed phase remains in the N/Z ratio of the inner part of the expanding system.
- The N/Z ratio of the inner part of the expanding system is mapped well to the $(N/Z)_{\text{gas}}$ of the observed low-energy light particles.
- Some effect of the isovector effective mass ($m_n^* < m_p^*$ or $m_n^* > m_p^*$) on $(N/Z)_{\text{gas}}$ at low energies.

Key Question

Model dependence of these conclusions.