Impact of the momentum dependence of the neutron and proton potentials on pion production in heavy-ion collisions

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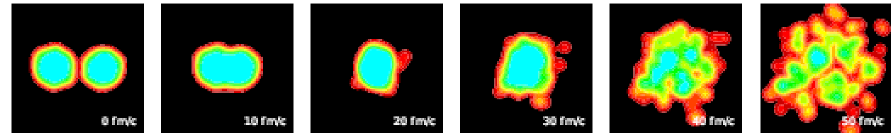
鳥取大学 Tottori University

Transport equation for heavy-ion collisions

^{132}Sn + ^{124}Sn, E/A = 300 MeV, $b \sim 0$



- Transport equation for one-body distribution function $f_a(\mathbf{r}, \mathbf{p}, t)$

BUU eq.

(a = n, p, Δ^-, Δ^0, Δ^+, Δ^{++}, π^-, π^0, π^+)

$$\frac{\partial f_a}{\partial t} + \frac{\partial \varepsilon_a}{\partial \boldsymbol{p}} \cdot \frac{\partial f_a}{\partial \boldsymbol{r}} - \frac{\partial \varepsilon_a}{\partial \boldsymbol{r}} \cdot \frac{\partial f_a}{\partial \boldsymbol{p}} = I_a^{\mathrm{coll}}$$

Mean-field propagation term
ε_a includes potentials U_a

Collision term I^{coll} includes potential U_a
(NN $\leftrightarrow$ NN , NN $\leftrightarrow$ NΔ, $\Delta \leftrightarrow$ Nπ)

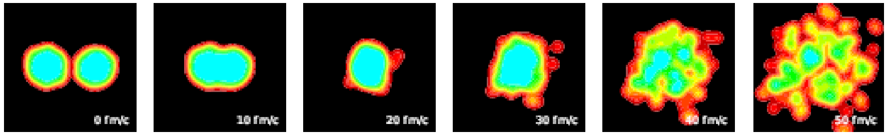
Fully incorporation is still a challenging problem

- Threshold effect (conservation of energy and momentum)
  A few codes: Ferini et al., NPA 762 (2005): M. Cozma, PLB 753, 166 (2016): T. Song and C. M. Ko, PRC 91, 014901 (2015): Z. Zhang and C. M. Ko., PRC 97, 014610 (2018)
- Cross sections
  A.B. Larionov and U. Mosel, NPA728, 135 (2003)
  Y. Cui, Y. X. Zhang and Z. X. Li, PRC98, 054605 (2018)

we rigorously calculate the collision terms of NN $\leftrightarrow$ NΔ and $\Delta \leftrightarrow$ Nπ processes with the precise conservation of energy and momentum under the potentials
(Both threshold effect and cross section in collision term)

Transport equation for heavy-ion collisions

$^{132}\text{Sn} + {}^{124}\text{Sn},\ E/A = 300\ \text{MeV},\ b \sim 0$



- Transport equation for one-body distribution function $f_a(\mathbf{r}, \mathbf{p}, t)$

BUU eq.

$(a = n, p, \Delta^-, \Delta^0, \Delta^+, \Delta^{++}, \pi^-, \pi^0, \pi^+)$

$$\frac{\partial f_a}{\partial t} + \frac{\partial \varepsilon_a}{\partial \boldsymbol{p}} \cdot \frac{\partial f_a}{\partial \boldsymbol{r}} - \frac{\partial \varepsilon_a}{\partial \boldsymbol{r}} \cdot \frac{\partial f_a}{\partial \boldsymbol{p}} = I_a^{\text{coll}}$$

Mean-field propagation term
ε_a includes potentials U_a

Collision term I^{coll} includes potential U_a
($NN \leftrightarrow NN$, $NN \leftrightarrow N\Delta$, $\Delta \leftrightarrow N\pi$)

- Threshold effect (conservation of energy and momentum)
- Cross sections

- Pion production in HIC

<u>Δ^-, π^- production</u>

$$nn \to p\Delta^-$$
$$\Delta^- \to n\pi^-$$

<u>Δ^{++}, π^+ production</u>

$$pp \to n\Delta^{++}$$
$$\Delta^{++} \to p\pi^+$$

Effect of different momentum of U_p and U_n in the high-momentum region seems to be important on the Δ and pion production at E/A=300 MeV

Pion production at SπRIT experiment

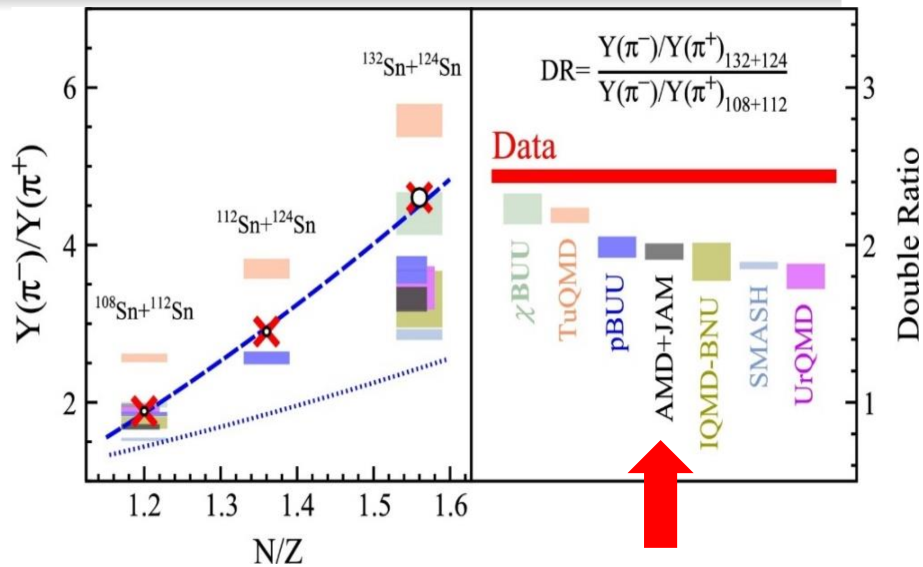
- Charged pion ratio π^-/π^+:

Proposed to be sensitive to symmetry energy at high density B. A. Li, PRL 88 (2002) 192701

- SπRIT experiment @RIBF

J. Estee et al. [SπRIT], PRL26 (2021) 162701

Slope of the symmetry energy is reported to be 42 <L < 117 MeV with dcQMD(TuQMD)



- Transport model evaluation project (TMEP): G.Jhang et al. [SπRIT, TMEP], PLB 813(2021)136016

✓ Most results do not agree with the data of π^-/π^+

✓ The band for each model: Different L effect

✓ Our previous model (AMD+JAM) had similar results to others

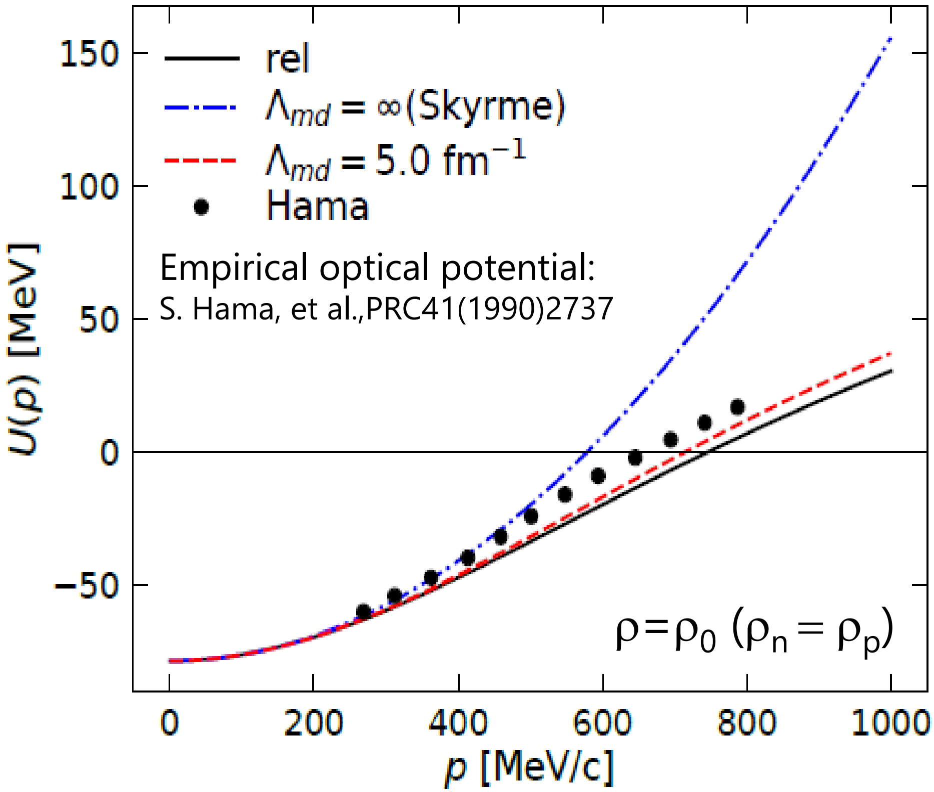
<= Potentials were not taken into account in the collision term (NN ↔ NΔ, Δ ↔ Nπ)

In our study:

✓ Improve the AMD+sJAM model to properly take into account such potentials consistently

✓ See the effect of momentum dependence of the neutron and proton potentials on the pion production

Momentum dependence of the nucleon potentials



- Skyrme interaction (SLy4, $m^*/m=0.70$) (not used)

$$U(\boldsymbol{r},\boldsymbol{p}) = A(\boldsymbol{r})\boldsymbol{p}^2 - 2\boldsymbol{B}(\boldsymbol{r})\cdot\boldsymbol{p} + C(\boldsymbol{r}),$$

U(p) at p>500 MeV/c is important for the Δ, π productions

=> p^2 dependence needs modification in the high-momentum region

- Λ_{md}=5.0 fm^{-1}: Used in AMD

$$U(\boldsymbol{r},\boldsymbol{p}) = A(\boldsymbol{r})\frac{\boldsymbol{p}^2}{1+\boldsymbol{p}^2/\Lambda_{\rm md}^2} + C(\boldsymbol{r})$$

with $B(\boldsymbol{r}) = 0$

- **rel** (relativistic form): Used in sJAM with $\Sigma = (\Sigma^s, \Sigma^0, \boldsymbol{\Sigma})$

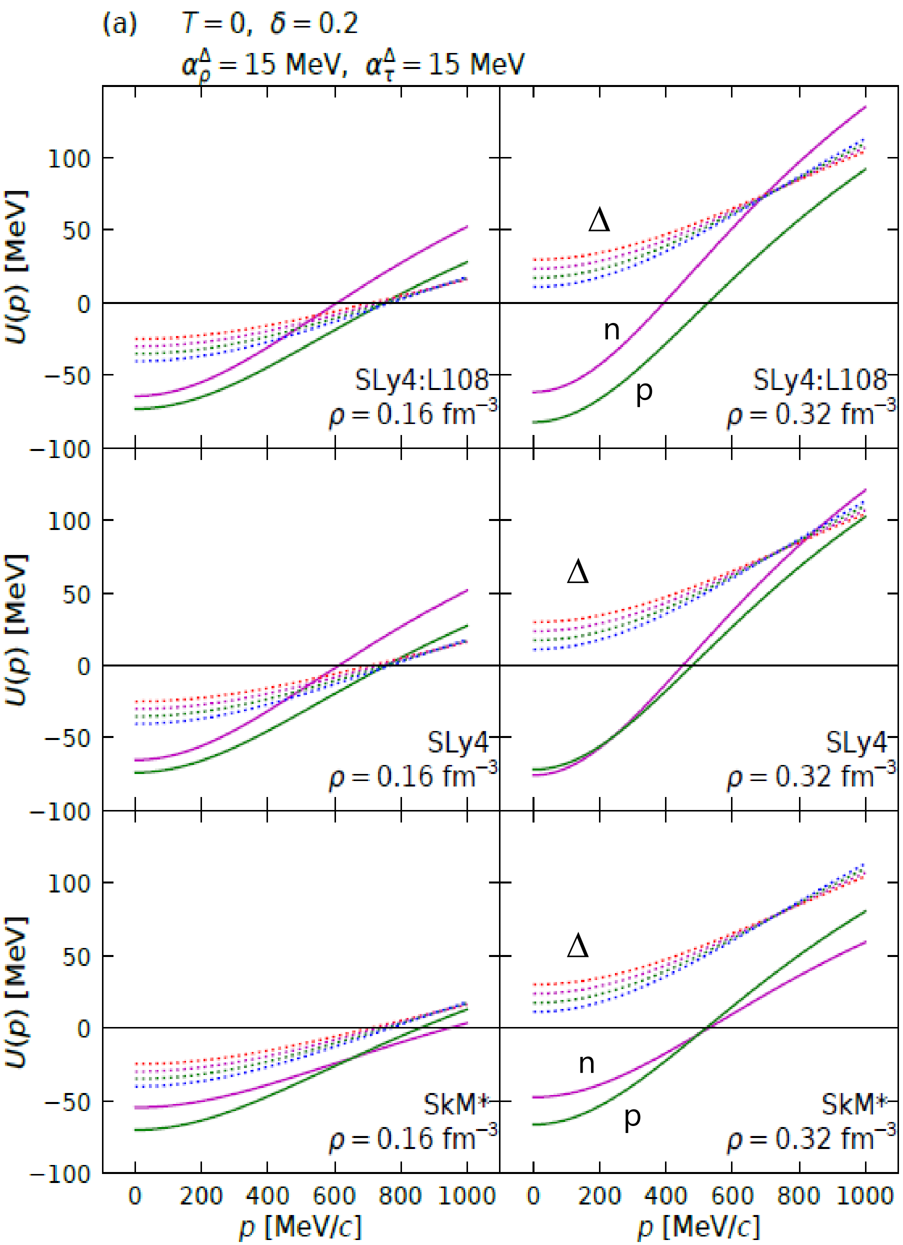
$$U(\boldsymbol{r},\boldsymbol{p}) = \sqrt{(m_N+\Sigma^s)^2 + (\boldsymbol{p}-\boldsymbol{\Sigma})^2} + \Sigma^0 - \sqrt{m_N^2 + \boldsymbol{p}^2}$$

$$m^* = m_N + \Sigma^s, \quad E^* = \sqrt{m_N^{*2} + \boldsymbol{p}^{*2}}, \quad \boldsymbol{p}^* = \boldsymbol{p} - \boldsymbol{\Sigma}$$

(Parametrization from Skyrme interaction: equivalent up to $O(p^2)$

$$m^* = (m_N^{-1} + 2A)^{-1}, \quad \Sigma^s = m^* - m_N, \quad \boldsymbol{\Sigma} = 2m^*\boldsymbol{B}, \quad \Sigma^0 = C - \Sigma^s - \frac{\boldsymbol{\Sigma}^2}{2m^*},$$
)

Nucleon and Δ potentials



- Nucleon potential

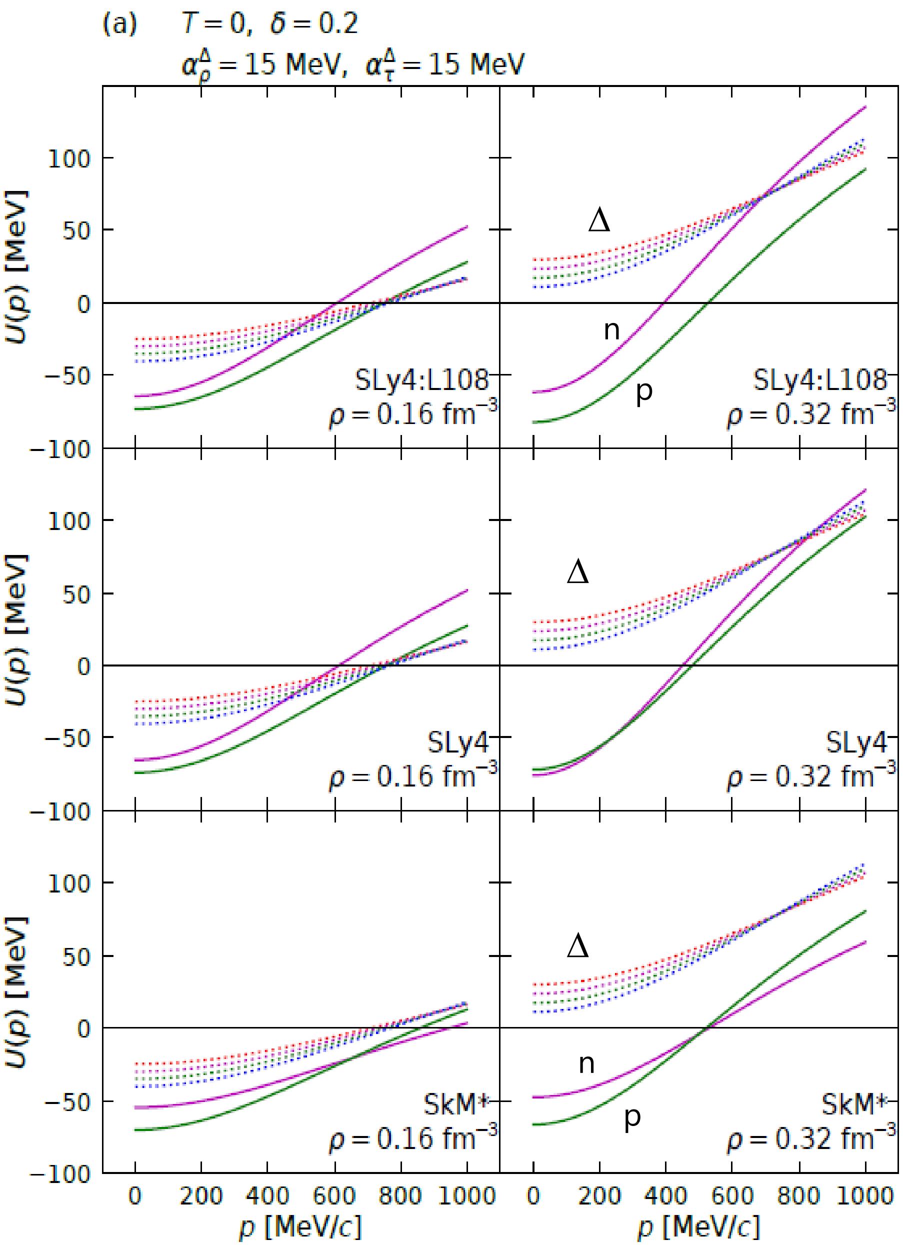
  SLy4:L108 (Stiff), SLy4 (Soft), SkM*

  in the relativistic form

Nuclear matter properties for the effective interactions of Skyrme SLy4, SLy4:L108, and SkM*

	SLy4	SLy4:L108	SkM*
ρ_0 [fm^{-3}]	0.160	0.160	0.160
E/A [MeV]	−15.97	−15.97	−15.77
K [MeV]	230	230	217
m^*/m_N	0.70	0.70	0.79
S_0 [MeV]	32.0	32.0	30.0
L [MeV]	46	108	46
$\Delta m^*_{np}/(m_N\delta)$	−0.18	−0.18	+0.33
in n-rich	$m^*_n < m^*_p$	$m^*_n < m^*_p$	$m^*_n > m^*_p$

Nucleon and Δ potentials



- Nucleon potential

  SLy4:L108 (Stiff), SLy4 (Soft), SkM*

  in the relativistic form

- Δ potentials: $\Sigma_\Delta = (\Sigma_\Delta^s, \Sigma_\Delta^0, \boldsymbol{\Sigma}_\Delta)$

  Consist of isoscalar and isovector part

$$
\begin{aligned}
\Sigma_{\Delta^-} &= \Sigma_{\rm is} + \frac{3}{2}\Sigma_{\rm iv} \\
\Sigma_{\Delta^0} &= \Sigma_{\rm is} + \frac{1}{2}\Sigma_{\rm iv} \\
\Sigma_{\Delta^+} &= \Sigma_{\rm is} - \frac{1}{2}\Sigma_{\rm iv} \\
\Sigma_{\Delta^{++}} &= \Sigma_{\rm is} - \frac{3}{2}\Sigma_{\rm iv}
\end{aligned}
$$

isoscalar part:

$$
\begin{aligned}
\Sigma_{\rm is}^s &= \frac{1}{2}(\Sigma_n^s + \Sigma_p^s)_{\rm SkM*}, \\
\Sigma_{\rm is}^0 &= \frac{1}{2}(\Sigma_n^0 + \Sigma_p^0)_{\rm SkM*} + \alpha_\rho^\Delta \frac{\rho}{\rho_0} + \alpha_\tau^\Delta \frac{\tau}{\tau_0}, \\
\boldsymbol{\Sigma}_{\rm is} &= \alpha_\rho^\Delta \frac{\boldsymbol{J}}{\rho_0},
\end{aligned}
$$

isovector part:

$$
\begin{aligned}
\Sigma_{\rm iv}^s &= \frac{\gamma^\Delta}{3}(\Sigma_n^s - \Sigma_n^s)_{\rm SkM*}, \\
\Sigma_{\rm iv}^0 &= \frac{\gamma^\Delta}{3}(\Sigma_n^0 - \Sigma_p^0)_{\rm SkM*}, \\
\boldsymbol{\Sigma}_{\rm iv} &= \boldsymbol{0}.
\end{aligned}
$$

based on the nucleon potential in the SkM* parametrization

Free parameters: $\alpha_\rho^\Delta, \alpha_\tau^\Delta, \gamma^\Delta$

- No Pion potential

Collision term $\frac{\partial f_a}{\partial t} + \frac{\partial \varepsilon_a}{\partial \boldsymbol{p}} \cdot \frac{\partial f_a}{\partial \boldsymbol{r}} - \frac{\partial \varepsilon_a}{\partial \boldsymbol{r}} \cdot \frac{\partial f_a}{\partial \boldsymbol{p}} = I_a^{\mathrm{coll}}$

- Formulation of NN→NΔ under potentials

N(1) + N(2) → N(3) + Δ(4) reaction:

$$d\sigma = f_{\mathrm{in}} f_{\mathrm{out}} \frac{|\mathcal{M}|^2_{\Sigma=0}}{16\pi\tilde{s}} \frac{[p_{\mathrm{f}}^*]_{\mathrm{out}}}{[p_{\mathrm{i}}^*]_{\mathrm{in}}} \frac{A(m_4)dm_4}{2\pi} \frac{d\Omega_{\mathrm{f}}^*}{4\pi}$$

- Phase space factor $f_{\mathrm{in}} f_{\mathrm{out}} [p^*{}_{\mathrm{f}}]_{\mathrm{out}} / [p^*{}_{\mathrm{i}}]_{\mathrm{in}}$:

Depends on the potential (Σ_i^s, Σ_i^0, $\boldsymbol{\Sigma}_i$) of the initial and final state particles

$$[p_f^*]_{\mathrm{out}} = \sqrt{\frac{[s_{\mathrm{out}}^* - (m_3^* + m_4^*)^2][s_{\mathrm{out}}^* - (m_3^* - m_4^*)^2]}{4 s_{\mathrm{out}}^*}}$$

$$\begin{aligned} s_{\mathrm{out}}^* &= (E_3^* + E_4^*)^2 - (\boldsymbol{p}_3^* + \boldsymbol{p}_4^*)^2 \\ &= (E_1^* + E_2^* + \Sigma_1^0 + \Sigma_2^0 - \Sigma_3^0 - \Sigma_4^0)^2 \\ &\quad -(\boldsymbol{p}_1^* + \boldsymbol{p}_2^* + \boldsymbol{\Sigma}_1 + \boldsymbol{\Sigma}_2 - \boldsymbol{\Sigma}_3 - \boldsymbol{\Sigma}_4)^2 \end{aligned}$$

Energy and momentum conservation

- Δ spectral function A(m):

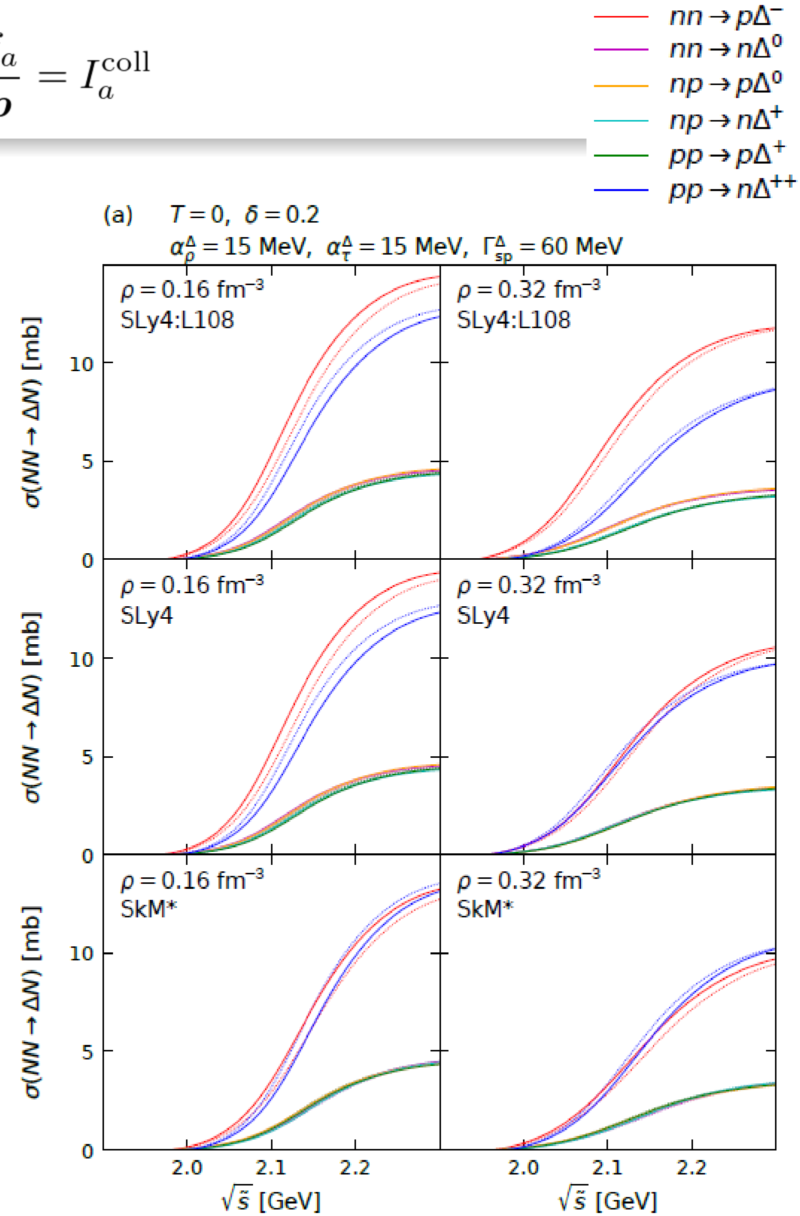
$$A(m) = \frac{4m^2\Gamma_\Delta(m)}{(m^2 - M_\Delta^2)^2 + m^2\Gamma_\Delta(m)^2}$$

$$\Gamma_\Delta(m) = \Gamma_{\mathrm{sp}}\frac{\rho}{\rho_0} + \sum \Gamma_{\Delta\to N\pi}(m)$$

e.g. A.B. Larionov and U. Mosel, NPA728, 135 (2003).

Decay width evaluated with $[p^*{}_{\mathrm{f}}]_{\mathrm{out}}$ in the Δ→ Nπ

$$\Gamma_{\Delta\to N\pi}(m_\Delta) = C_{\Delta N\pi} f_{\mathrm{out}} \frac{M_0\Gamma_0}{m_\Delta}\left(\frac{[p_{\mathrm{f}}^*]_{\mathrm{out}}}{p_0}\right)^3 \frac{p_0^2 + \Lambda^2}{[p_{\mathrm{f}}^*]_{\mathrm{out}}^2 + \Lambda^2},$$



- Potential effect on the cross section
- Different channels for Δ production

Collision term $\frac{\partial f_a}{\partial t} + \frac{\partial \varepsilon_a}{\partial \boldsymbol{p}} \cdot \frac{\partial f_a}{\partial \boldsymbol{r}} - \frac{\partial \varepsilon_a}{\partial \boldsymbol{r}} \cdot \frac{\partial f_a}{\partial \boldsymbol{p}} = I_a^{\mathrm{coll}}$

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- Phase space factor $f_{\mathrm{in}} f_{\mathrm{out}} [p^*_{\mathrm{f}}]_{\mathrm{out}} / [p^*_{\mathrm{i}}]_{\mathrm{in}}$:

Depends on the potential (Σ_i^s, Σ_i^0, $\boldsymbol{\Sigma}_i$) of the initial and final state particles

$$[p^*_f]_{\mathrm{out}} = \sqrt{\frac{[s^*_{\mathrm{out}} - (m^*_3 + m^*_4)^2][s^*_{\mathrm{out}} - (m^*_3 - m^*_4)^2]}{4 s^*_{\mathrm{out}}}}$$

$$\begin{aligned} s^*_{\mathrm{out}} &= (E^*_3 + E^*_4)^2 - (\boldsymbol{p}^*_3 + \boldsymbol{p}^*_4)^2 \\ &= (E^*_1 + E^*_2 + \Sigma^0_1 + \Sigma^0_2 - \Sigma^0_3 - \Sigma^0_4)^2 \\ &\quad - (\boldsymbol{p}^*_1 + \boldsymbol{p}^*_2 + \boldsymbol{\Sigma}_1 + \boldsymbol{\Sigma}_2 - \boldsymbol{\Sigma}_3 - \boldsymbol{\Sigma}_4)^2 \end{aligned}$$

Energy and momentum conservation

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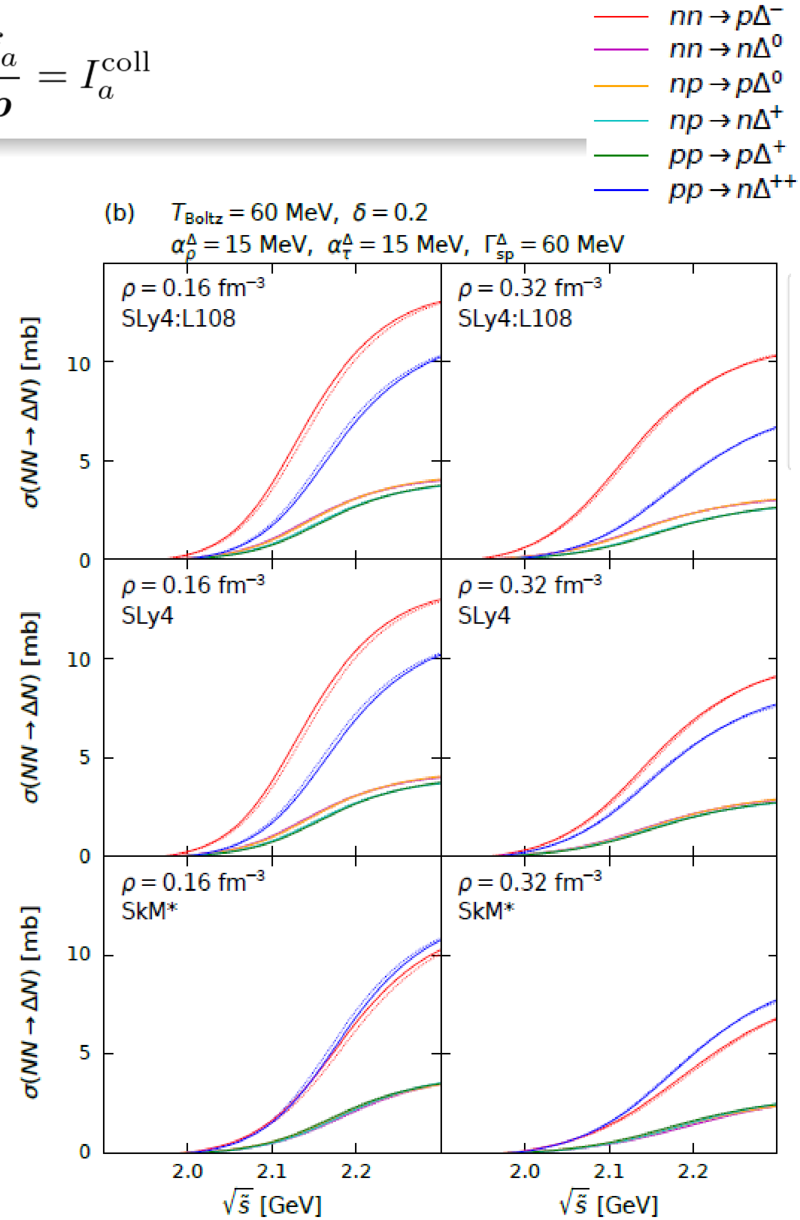
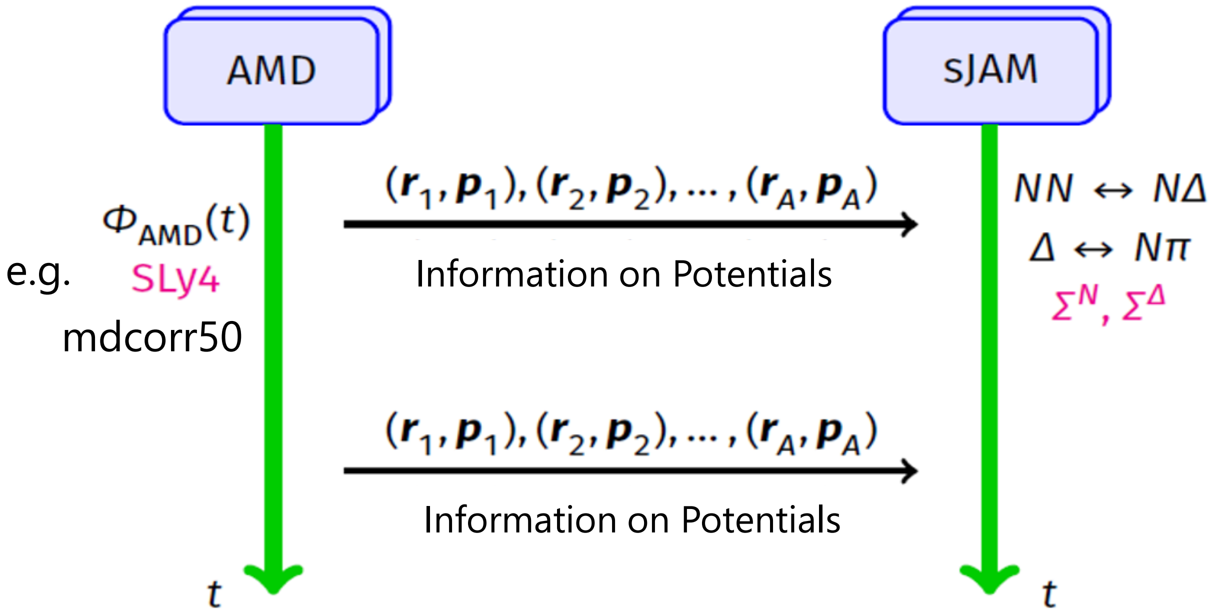
$$\Gamma_{\Delta \to N\pi}(m_\Delta) = C_{\Delta N\pi} f_{\mathrm{out}} \frac{M_0 \Gamma_0}{m_\Delta} \left(\frac{[p^*_{\mathrm{f}}]_{\mathrm{out}}}{p_0}\right)^3 \frac{p_0^2 + \Lambda^2}{[p^*_{\mathrm{f}}]^2_{\mathrm{out}} + \Lambda^2},$$



- Potential effect on the cross section
- Different channels for Δ production

AMD+sJAM transport model

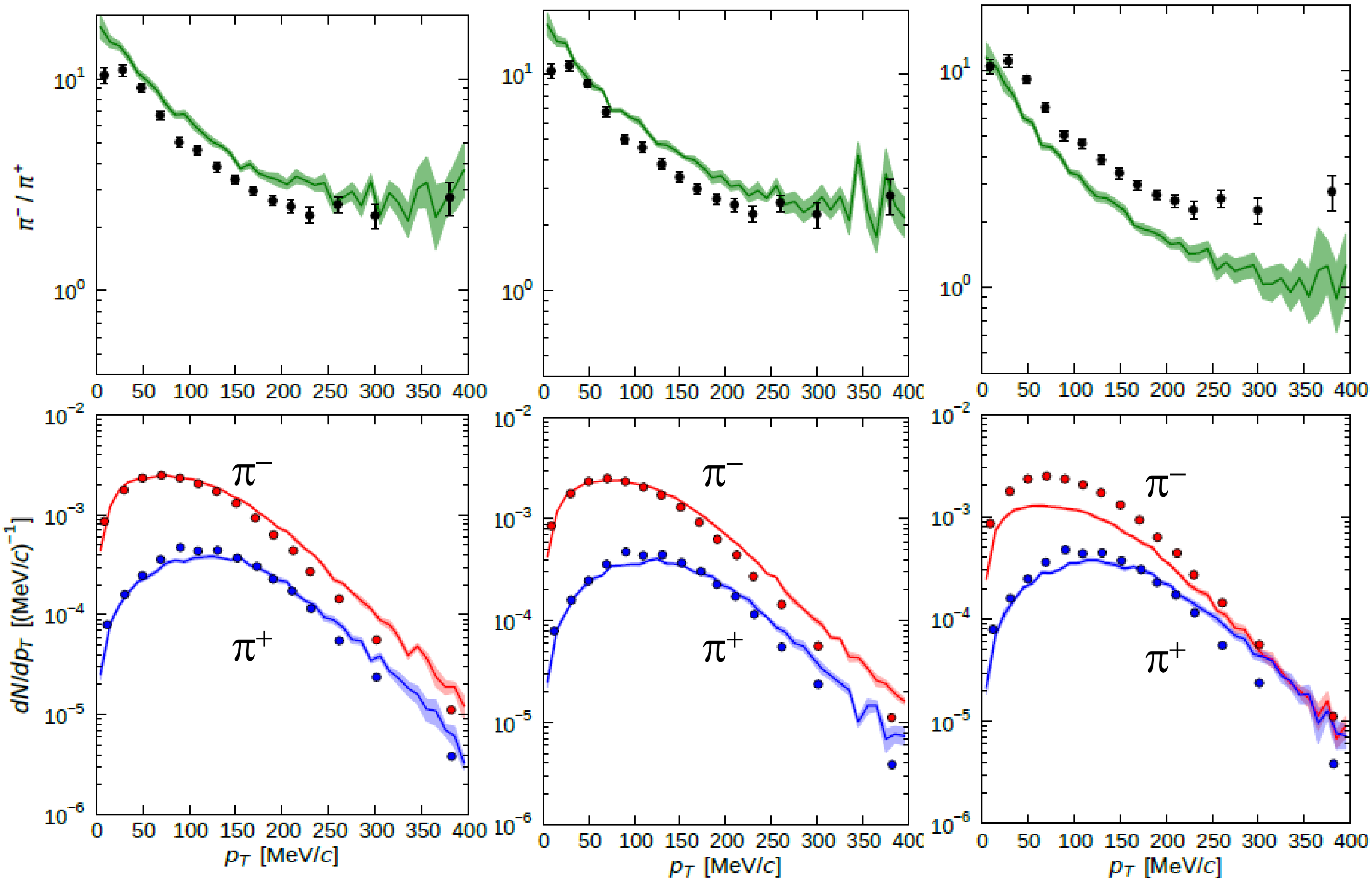


- In the (N, Δ, π) system, sJAM works identically to JAM if $\Sigma = 0$
- The $NN \leftrightarrow N\Delta$, $\Delta \leftrightarrow N\pi$ processes are calculated in sJAM under the potentials (Σ_i^s, Σ_i^0, $\boldsymbol{\Sigma}_i$) with a precise treatment of energy conservation
- Potential dependence on cross section is also considered in a natural way

Potential information is sent from AMD to sJAM together with the nucleon information (test particle) at every time step of 1 fm/c

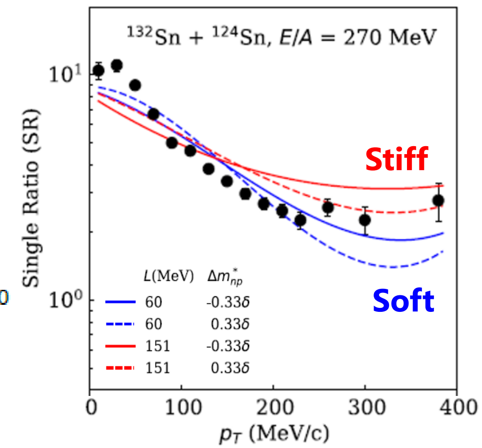
Effect of nucleon potential on pion production

SLy4:L108 (Stiff) $(m^*_n < m^*_p)$ SLy4 (Soft) SkM* $(m^*_n > m^*_p)$



J. Estee et al. [SπRIT], PRL26,162701(2021).

Data: SπRIT, Cal: dcQMD

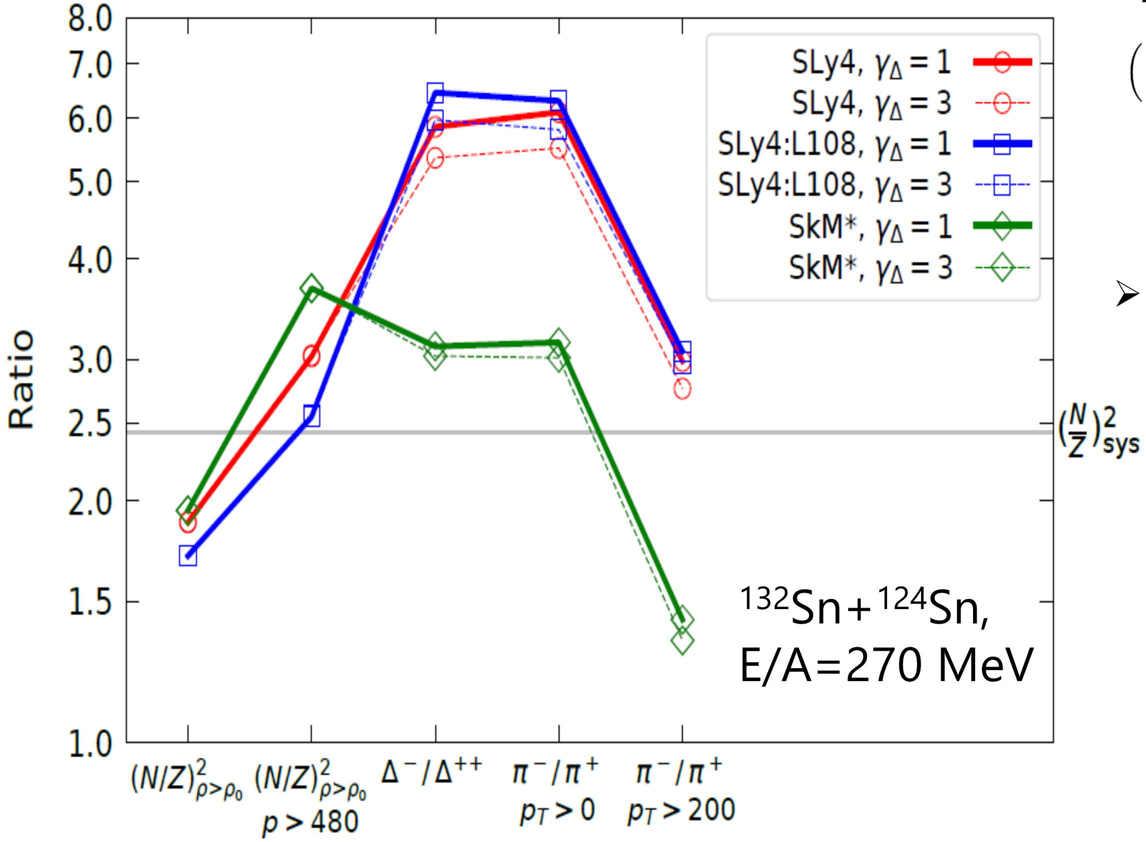


$^{132}Sn + ^{124}Sn$, E/A=270 MeV

Impact parameter: 0< b <3 fm

- ✓ SLy4 vs. SLy4:L108: Relatively small dependence of symmetry energy (L) on pion production
- ✓ SLy4 vs. SkM*: Momentum dependence of U_n and U_p has a strong effect on pion production

From nucleons to pion ratios



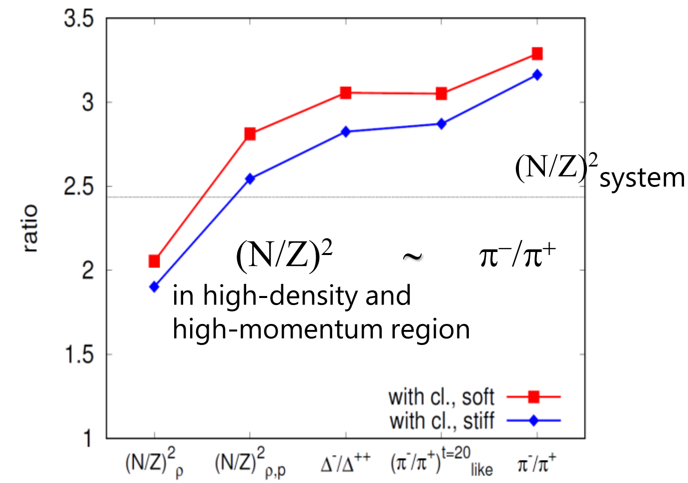
Representative ratios:

$$\left(\frac{N}{Z}\right)^2 = \frac{\int_0^\infty N(t)^2 dt}{\int_0^\infty Z(t)^2 dt} \qquad \frac{\Delta^-}{\Delta^{++}} = \frac{\int_0^\infty (nn \to p\Delta^-) dt}{\int_0^\infty (pp \to n\Delta^{++}) dt}$$

N(t), Z(t) : Numbers of nucleon which satisfy the conditions

- Without consideration of potential effects

N. Ikeno, A. Ono, Y. Nara, A. Ohnishi,
PRC93 (2016) 044612; PRC97(2018) 069902(E)

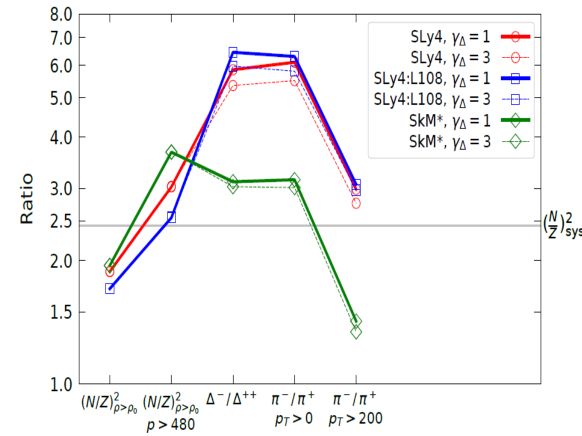


- ✓ L dependence (SLy4 vs SLy4:L108) in N/Z is inverted in the Δ production.
- ✓ Effect of the symmetry energy L (SLy4 vs SLy4:L108) : Relatively small on pion production

- ✓ Effect of the momentum dependence of U_n and U_p (SLy4 vs SkM*): Strong
- ✓ π^-/π^+ carries strong information on the momentum-dependence of U_n and U_p

Summary

- We use the AMD+sJAM transport model, modified to correctly incorporate the nucleon and Δ resonance potentials in the collision processes of $NN \leftrightarrow N\Delta$, $\Delta \leftrightarrow N\pi$
- The momentum dependence of the nucleon potential has a very strong influence on the $NN \leftrightarrow N\Delta$ process (SLy4 vs. SkM*)
- Charged pion ratios also strongly reflect information on the momentum dependence of nucleon potentials
- As the high-density symmetry energy effect, L-dependence in the N/Z ratio is reversed for Δ production (SLy4 vs. SLy4:L108)



<u>Conclusion and Question</u>:

- Pion ratios are more sensitive to the momentum dependence of Un and Up than to the effect of the high-density symmetry energy.

--> Seems hard to determine the high-density symmetry energy from only pion observable.

- Need to check other ingredients like pion potentials (on going)

- Better observables and ways to determine the symmetry energy?

Pion + nucleon fragments + ...

Delta potential (isoscalar and isovector)

- Effects of the **isovector part** of U_Δ
- Δ potentials: $\Sigma_\Delta = (\Sigma^s_\Delta, \Sigma^0_\Delta, \boldsymbol{\Sigma}_\Delta)$

  Consist of isoscalar and isovector part

$$\Sigma_{\Delta^-} = \Sigma_{\rm is} + \frac{3}{2}\Sigma_{\rm iv}$$
$$\Sigma_{\Delta^0} = \Sigma_{\rm is} + \frac{1}{2}\Sigma_{\rm iv}$$
$$\Sigma_{\Delta^+} = \Sigma_{\rm is} - \frac{1}{2}\Sigma_{\rm iv}$$
$$\Sigma_{\Delta^{++}} = \Sigma_{\rm is} - \frac{3}{2}\Sigma_{\rm iv}$$

isoscalar part:

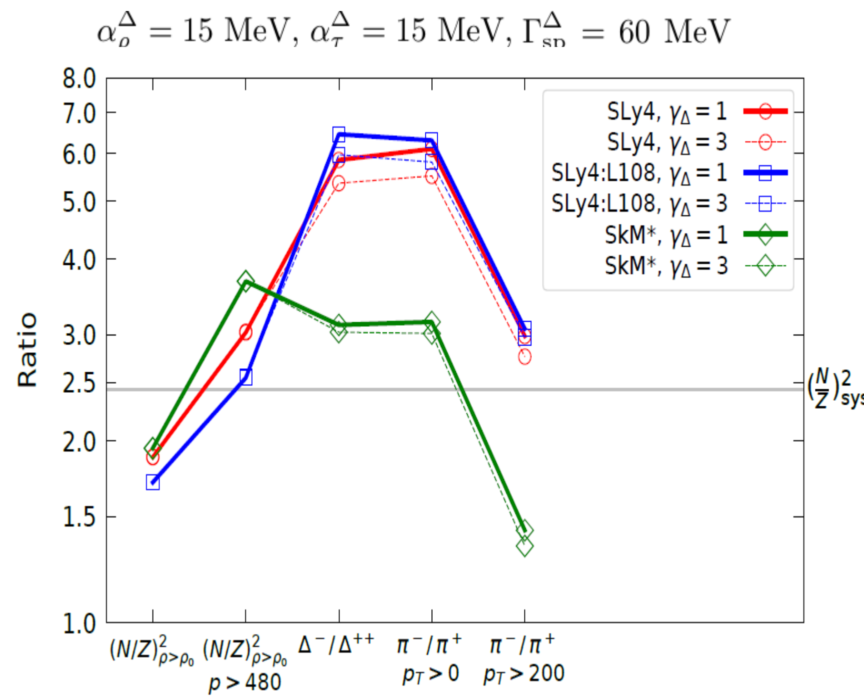
$$\Sigma^s_{\rm is} = \frac{1}{2}(\Sigma^s_n + \Sigma^s_p)_{\rm SkM^*},$$
$$\Sigma^0_{\rm is} = \frac{1}{2}(\Sigma^0_n + \Sigma^0_p)_{\rm SkM^*} + \alpha^\Delta_\rho \frac{\rho}{\rho_0} + \alpha^\Delta_\tau \frac{\tau}{\tau_0},$$
$$\boldsymbol{\Sigma}_{\rm is} = \alpha^\Delta_\rho \frac{\boldsymbol{J}}{\rho_0},$$

isovector part:

$$\Sigma^s_{\rm iv} = \frac{\gamma^\Delta}{3}(\Sigma^s_n - \Sigma^s_n)_{\rm SkM^*},$$
$$\Sigma^0_{\rm iv} = \frac{\gamma^\Delta}{3}(\Sigma^0_n - \Sigma^0_p)_{\rm SkM^*},$$
$$\boldsymbol{\Sigma}_{\rm iv} = \boldsymbol{0}.$$

based on the nucleon potential in the SkM* parametrization

Free parameters: $\alpha^\Delta_\rho, \alpha^\Delta_\tau, \gamma^\Delta$



Solid line $\gamma_\Delta = 1$
Dashed line $\gamma_\Delta = 3$

$$\gamma_\Delta = 1 \Rightarrow \Sigma_{\Delta^-} - \Sigma_{\Delta^{++}} = \Sigma_n - \Sigma_p$$
$$\gamma_\Delta = 3 \Rightarrow \Sigma_{\Delta^0} - \Sigma_{\Delta^+} = \Sigma_n - \Sigma_p$$

✓ Effect of the isospin splitting of the Δ potential ($\gamma_\Delta = 1$ vs. $\gamma_\Delta = 3$) is of the same order as that of the nuclear symmetry energy (SLy4 vs SLy4:L108).

Delta potential (isoscalar and isovector)

- Effects of the **isoscalar part** of U_Δ and spreading width Γ^Δ
- Δ potentials: $\Sigma_\Delta = (\Sigma^s_\Delta, \Sigma^0_\Delta, \boldsymbol{\Sigma}_\Delta)$

$\alpha^\Delta_\rho = 0,\ \alpha^\Delta_\tau = 0 \quad \Gamma^\Delta_{sp} = 0.$ (No repulsive terms)

Consist of isoscalar and isovector part

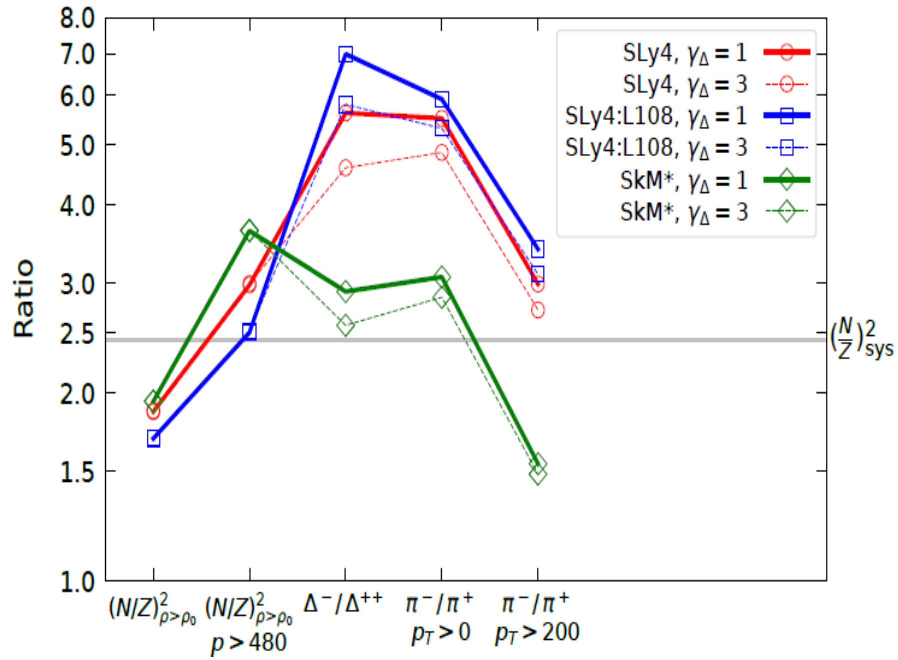
$$\Sigma_{\Delta^-} = \Sigma_{is} + \frac{3}{2}\Sigma_{iv}$$
$$\Sigma_{\Delta^0} = \Sigma_{is} + \frac{1}{2}\Sigma_{iv}$$
$$\Sigma_{\Delta^+} = \Sigma_{is} - \frac{1}{2}\Sigma_{iv}$$
$$\Sigma_{\Delta^{++}} = \Sigma_{is} - \frac{3}{2}\Sigma_{iv}$$

isoscalar part:

$$\Sigma^s_{is} = \frac{1}{2}(\Sigma^s_n + \Sigma^s_p)_{SkM^*},$$
$$\Sigma^0_{is} = \frac{1}{2}(\Sigma^0_n + \Sigma^0_p)_{SkM^*} + \alpha^\Delta_\rho \frac{\rho}{\rho_0} + \alpha^\Delta_\tau \frac{\tau}{\tau_0},$$
$$\boldsymbol{\Sigma}_{is} = \alpha^\Delta_\rho \frac{\boldsymbol{J}}{\rho_0},$$

isovector part:

$$\Sigma^s_{iv} = \frac{\gamma^\Delta}{3}(\Sigma^s_n - \Sigma^s_n)_{SkM^*},$$
$$\Sigma^0_{iv} = \frac{\gamma^\Delta}{3}(\Sigma^0_n - \Sigma^0_p)_{SkM^*},$$
$$\boldsymbol{\Sigma}_{iv} = \boldsymbol{0}.$$



Solid line $\gamma_\Delta = 1$
Dashed line $\gamma_\Delta = 3$

- spreading width Γ^Δ $\quad \Gamma_\Delta(m) = \Gamma_{sp}\frac{\rho}{\rho_0} + \sum \Gamma_{\Delta\to N\pi}(m)$

Δ spectral function A(m): $\quad A(m) = \dfrac{4m^2\Gamma_\Delta(m)}{(m^2 - M^2_\Delta)^2 + m^2\Gamma_\Delta(m)^2}$

- ✓ Results are similar qualitatively
- ✓ Effect of the symmetry energy (SLy4 vs SLy4:L108) is now stronger
- ✓ Effect of the difference in the momentum dependence of U_n and U_p (SLy4 vs SkM*) is always the most significant

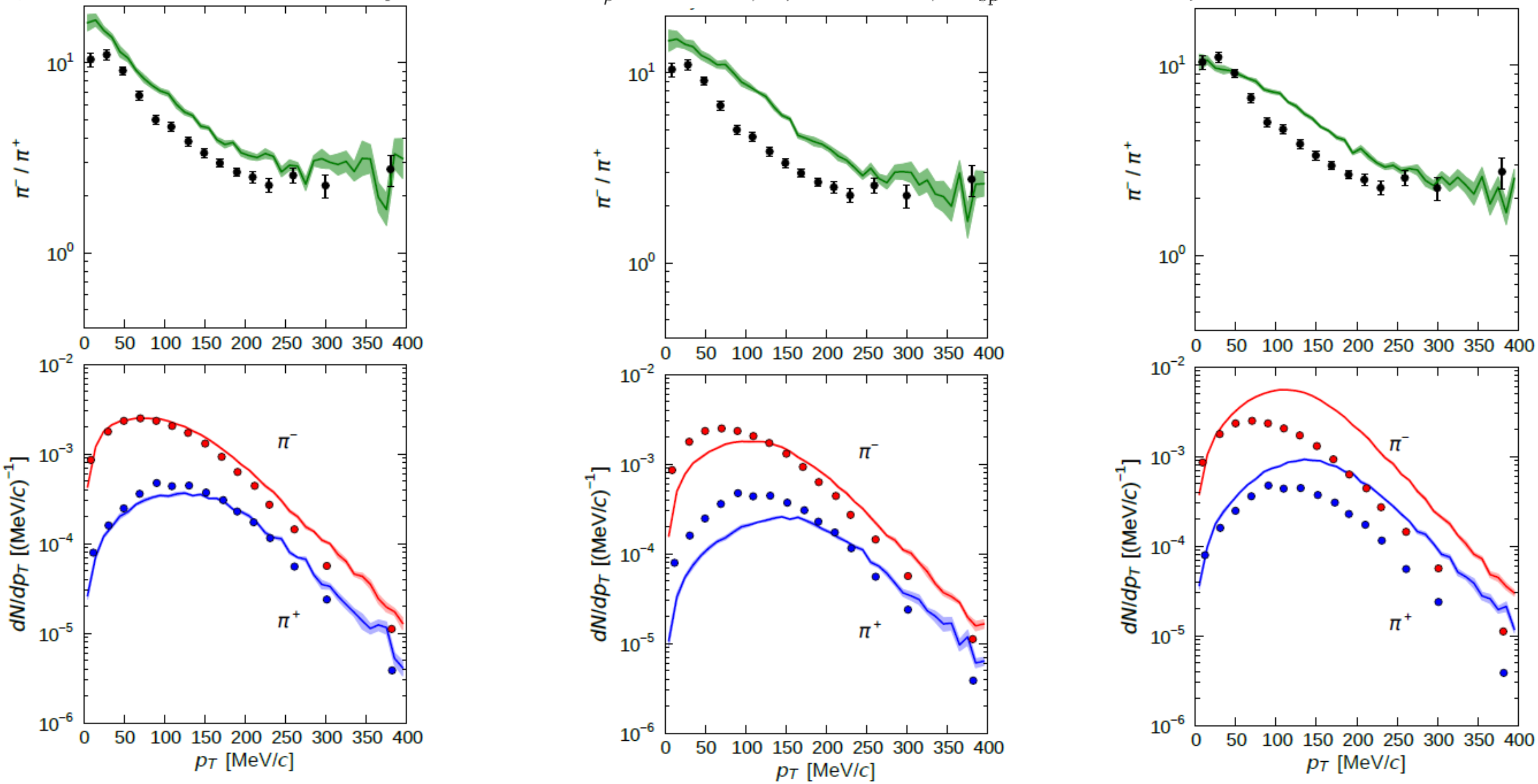
Delta potential (isoscalar and isovector)

- Effects of the isoscalar part of U_Δ and spreading width Γ^Δ [No repulsive terms]

$\alpha_\rho^\Delta = 15$ MeV, $\alpha_\tau^\Delta = 15$ MeV, $\Gamma_{\rm sp}^\Delta = 60$ MeV

$\alpha_\rho^\Delta = 15$ MeV, $\alpha_\tau^\Delta = 15$ MeV, $\Gamma_{\rm sp}^\Delta = 0.$

$\alpha_\rho^\Delta = 0,\ \alpha_\tau^\Delta = 0\ \ \Gamma_{\rm sp}^\Delta = 0.$

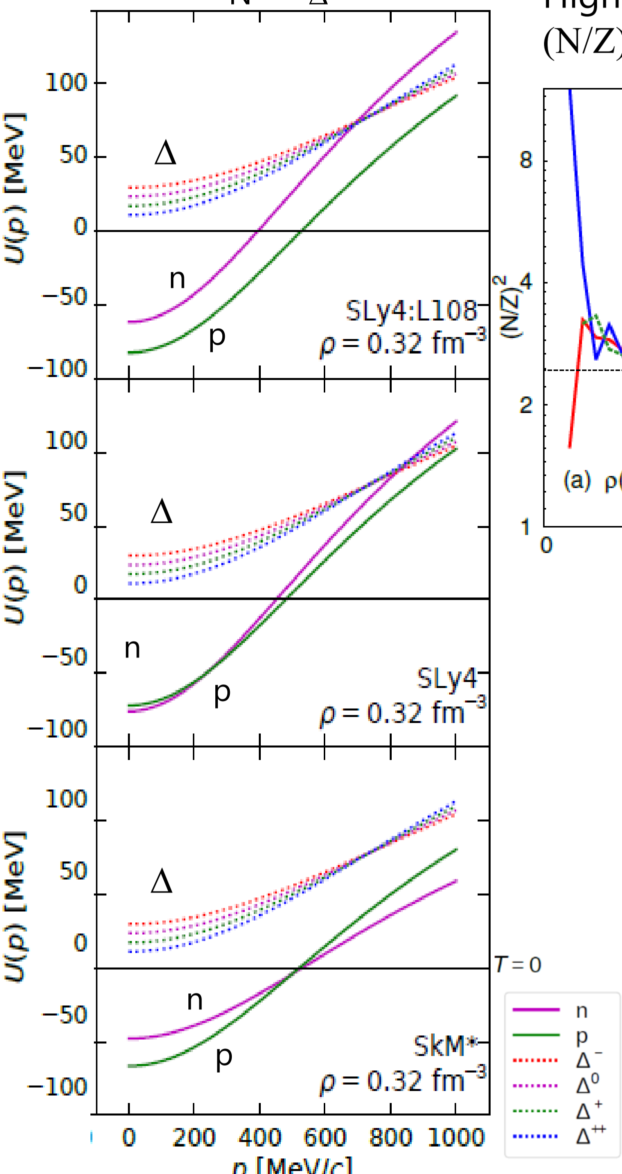


π^-/π^+ ratio of the spectra is not affected much

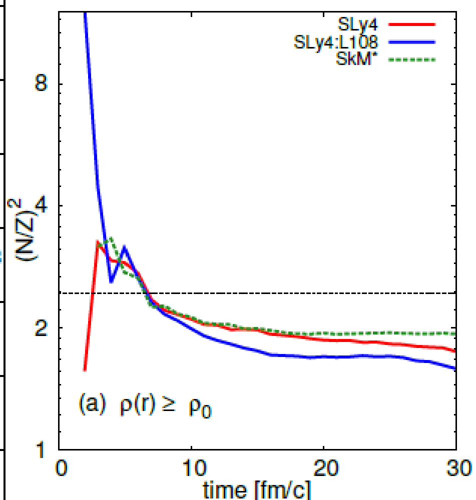
- Low momentum region of the spectra is significantly affected by Γ^Δ
- Pion yield is overestimated due to the lack of the repulsive terms in U_Δ

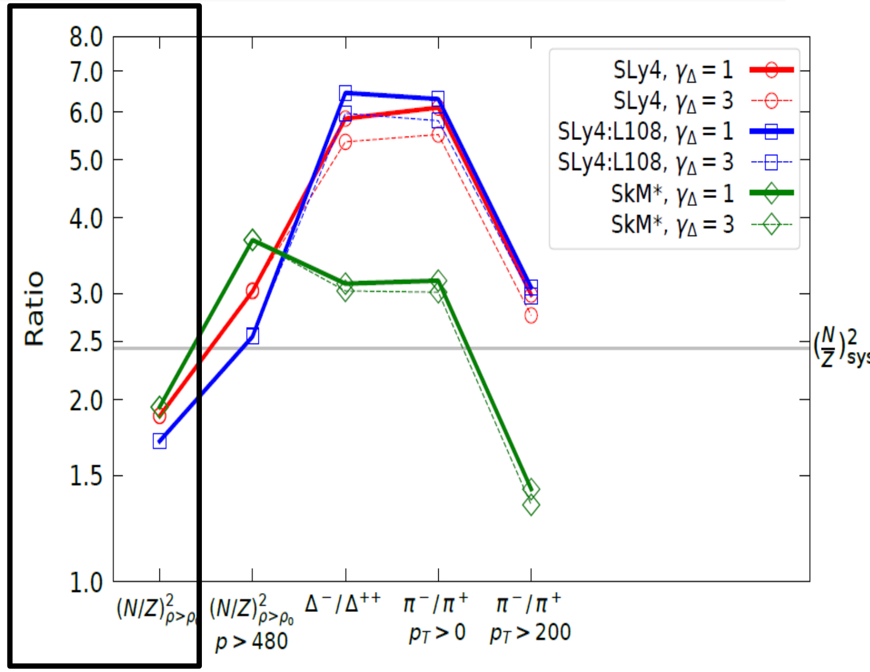
How to understand the effects in Nucleon dynamics

U_N, U_Δ



High density region
$(N/Z)^2$





Representative ratios: $\left(\frac{N}{Z}\right)^2 = \frac{\int_0^\infty N(t)^2 dt}{\int_0^\infty Z(t)^2 dt}$

N(t), Z(t) : Numbers of nucleon which satisfy the conditions

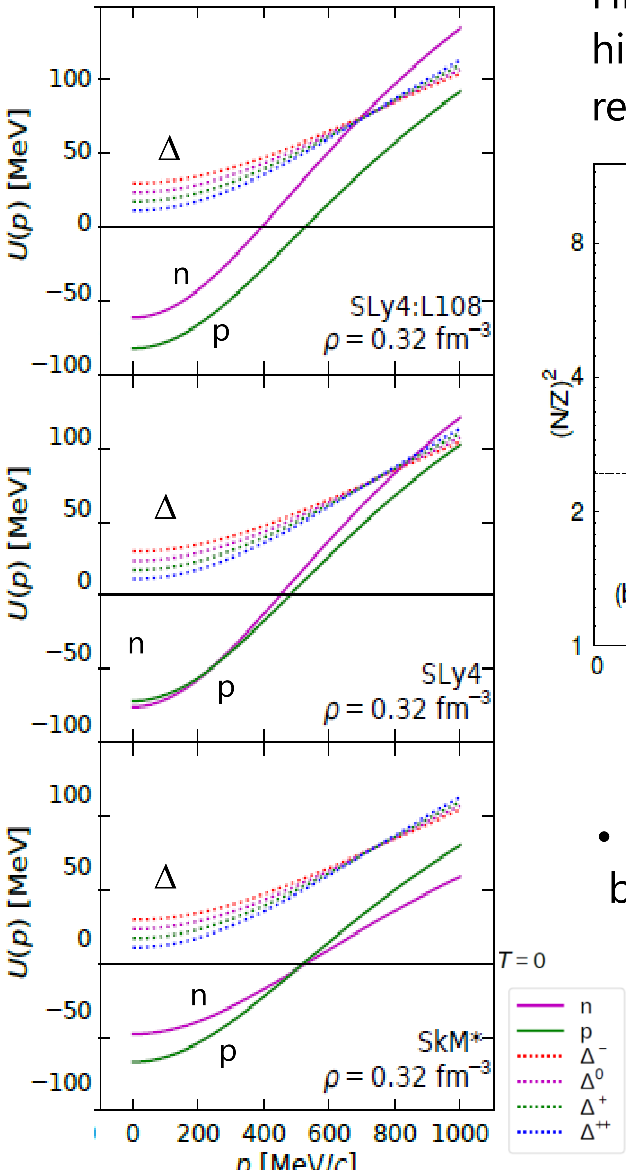
From Ratio's Fig, we can see

- Effect of the symmetry energy L (SLy4 vs SLy4:L108)
- Effect of the momentum dependence of U_n and U_p (SLy4 vs SkM*)

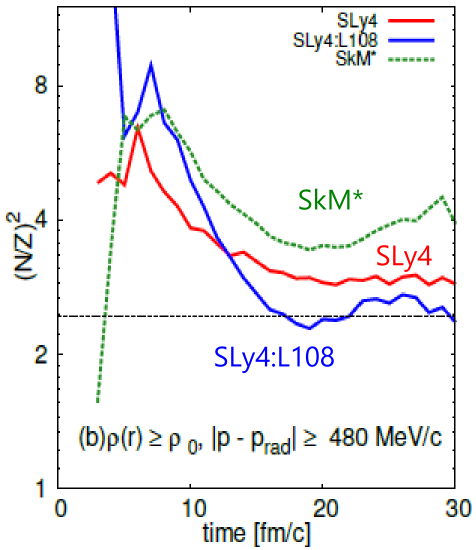
=> There is clearly the effect of the symmetry energy L between SLy4 and SLy4:L108

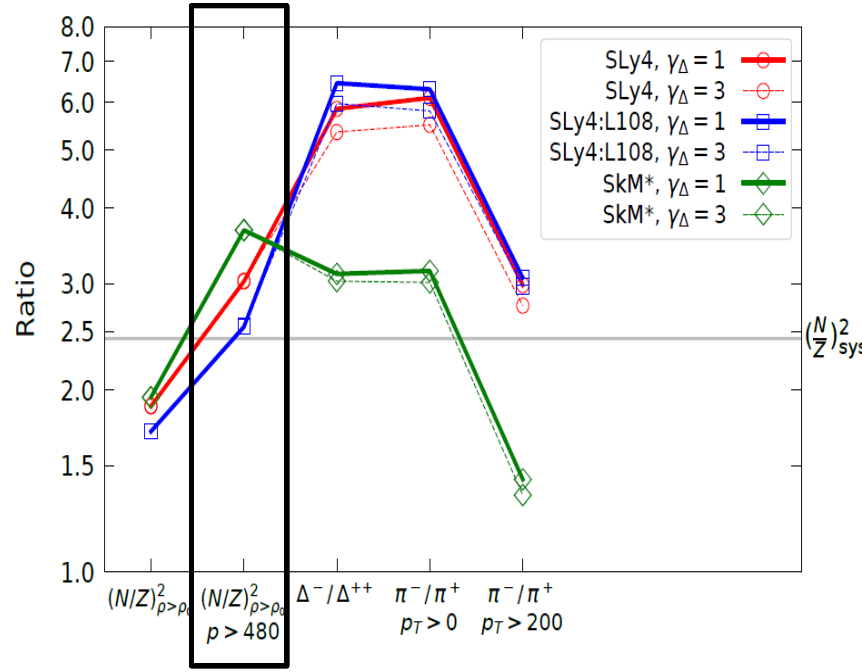
How to understand the effects in Nucleon dynamics

U_N, U_Δ



High-density and high-momentum region $(N/Z)^2$

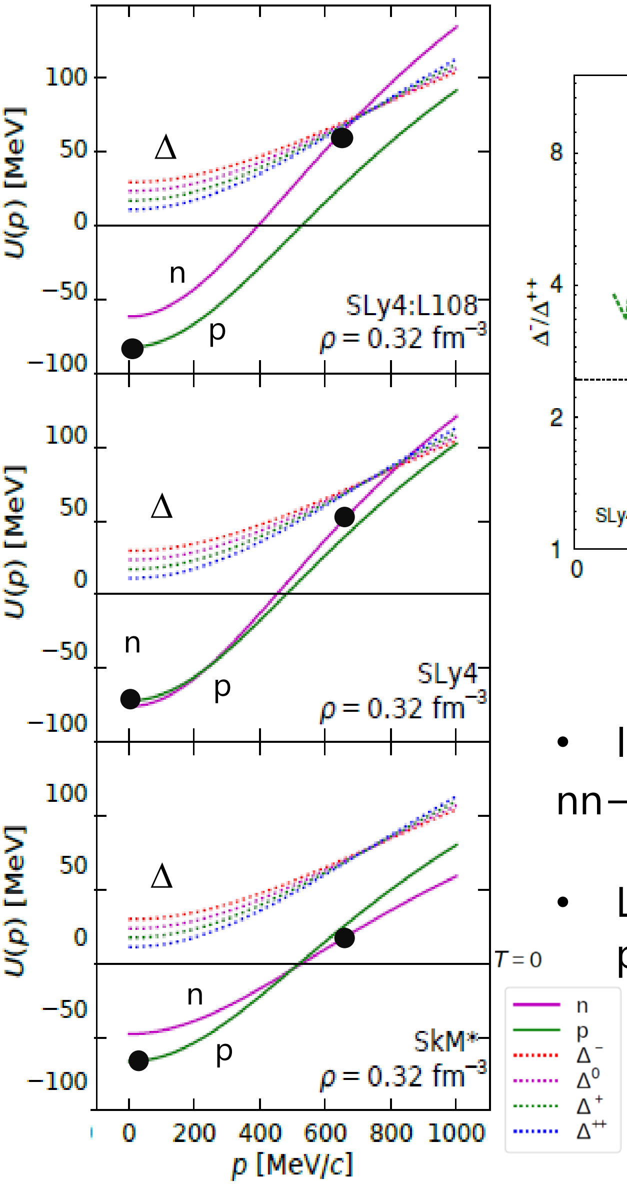




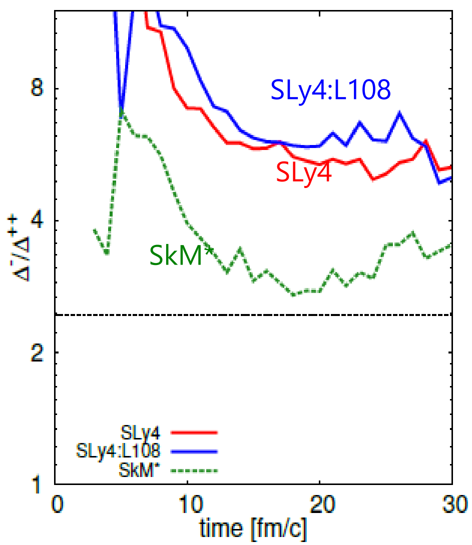
- In the high ρ and p region, $(N/Z)^2$ of SkM* drastically increases because U_n has weaker momentum dependence than that in SLy4 (m^*_n(SkM*) > m^*_n(SLy4))

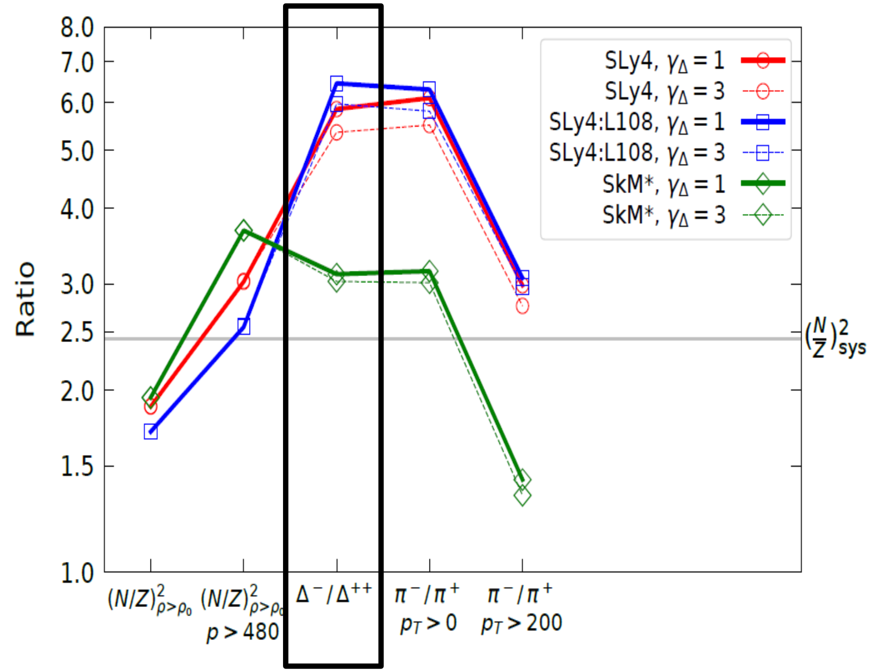
How to understand the effects in Delta and pion

U_N, U_Δ



(nn→pΔ⁻)
/(pp→nΔ⁺⁺)



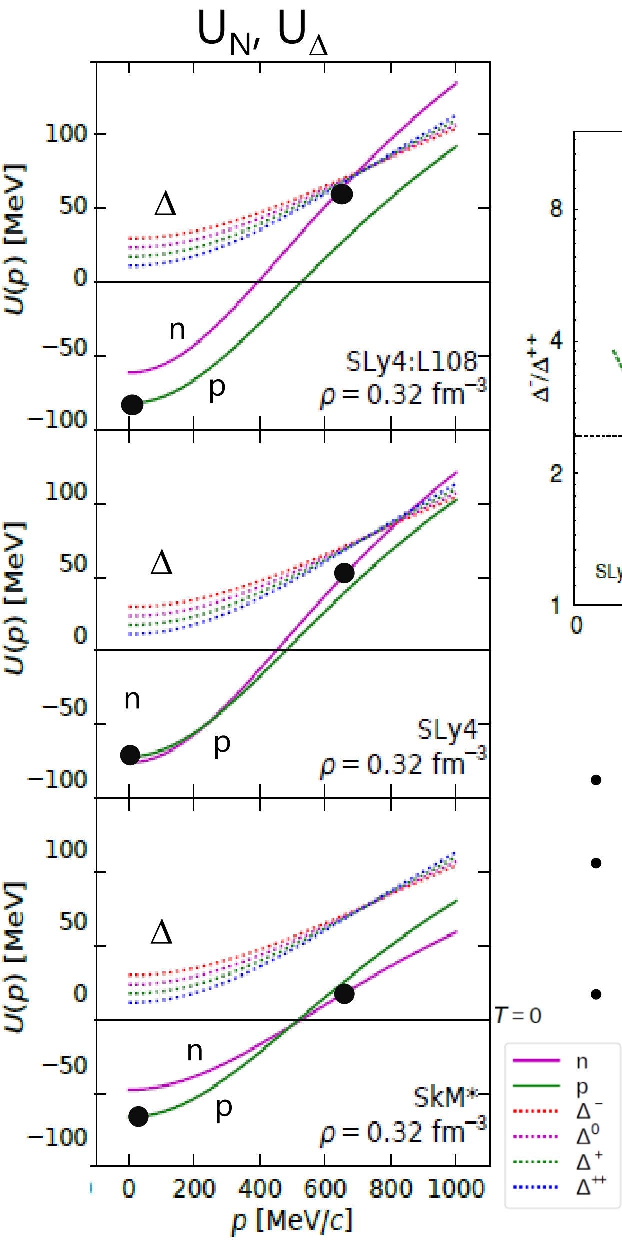


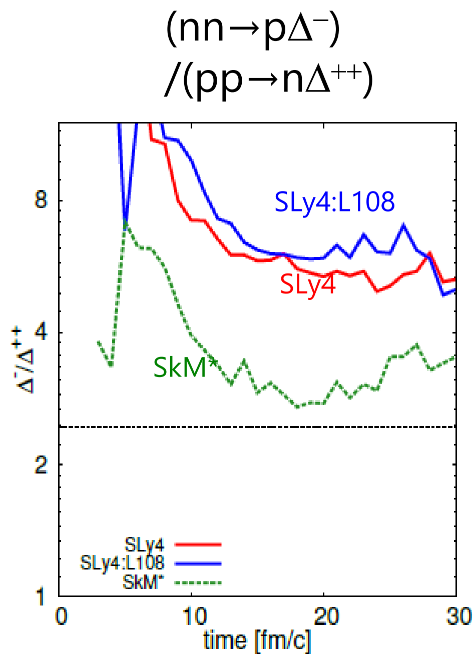
- In the Δ production, Δ^-/Δ^{++} in SLy4:L108 increases drastically.

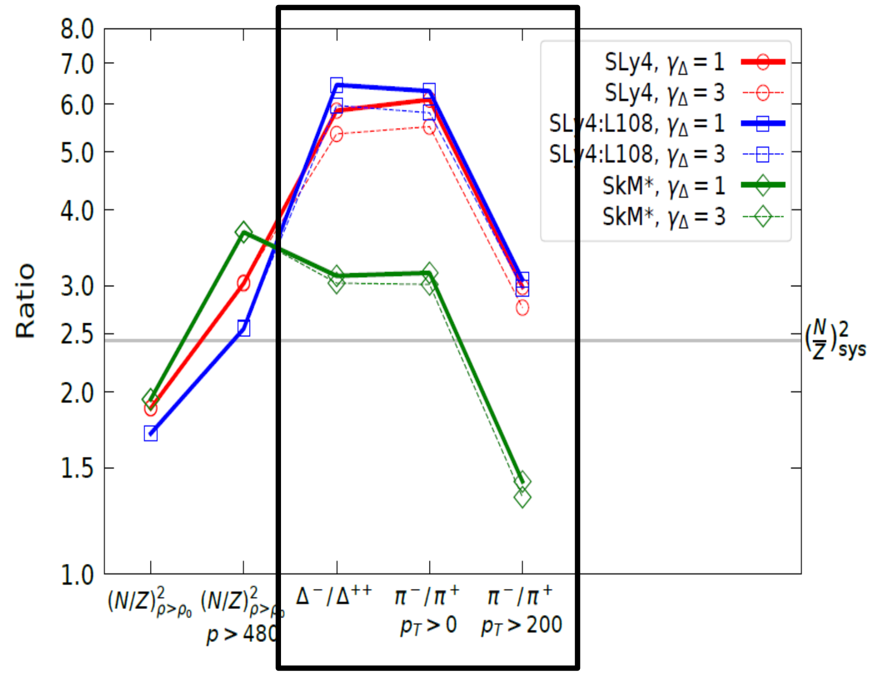
nn→pΔ⁻ is favored, due to the momentum dependence of U_n and U_p.

- L dependence (SLy4 vs SLy4:L108) in N/Z is inverted in the Δ production.

How to understand the effects in Delta and pion

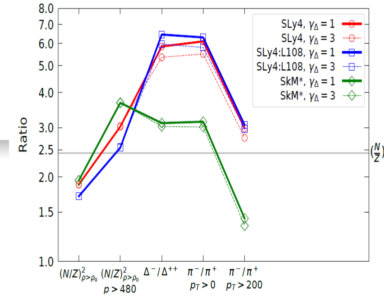






- Then π^-/π^+ seems to reflect to Δ^-/Δ^{++}
- Effect of the symmetry energy L (SLy4 vs SLy4:L108) : Relatively small on pion production
- Effect of the momentum dependence of U_n and U_p (SLy4 vs SkM*): Strong effect on pion production

Interactions: SLy4, SLy4:L108, SkM*



- Energy density:

$$\mathcal{E}_{\rm int}(\boldsymbol{r}) = \sum_{\alpha\beta}\Big\{U^{t_0}_{\alpha\beta}\rho_\alpha(\boldsymbol{r})\rho_\beta(\boldsymbol{r}) + U^{t_3}_{\alpha\beta}\rho_\alpha(\boldsymbol{r})\rho_\beta(\boldsymbol{r})[\rho(\mathbf{r})]^\gamma + U^{\tau}_{\alpha\beta}\tilde{\tau}_\alpha(\boldsymbol{r})\rho_\beta(\boldsymbol{r}) + U^{\nabla}_{\alpha\beta}\nabla\rho_\alpha(\boldsymbol{r})\nabla\rho_\beta(\boldsymbol{r})\Big\},$$

Densities: $\rho_\alpha(\boldsymbol{r}) = \int \frac{d\boldsymbol{p}}{(2\pi\hbar)^3} f_\alpha(\boldsymbol{r},\boldsymbol{p}),\quad \tilde{\tau}_\alpha(\boldsymbol{r}) = \int \frac{d\boldsymbol{p}}{(2\pi\hbar)^3}\frac{[\boldsymbol{p}-\bar{\boldsymbol{p}}(\boldsymbol{r})]^2}{1+[\boldsymbol{p}-\bar{\boldsymbol{p}}(\boldsymbol{r})]^2/\Lambda^2_{\rm md}} f_\alpha(\boldsymbol{r},\boldsymbol{p}),$

with $\bar{\boldsymbol{p}}(\boldsymbol{r}) = \frac{1}{\sum_\alpha \rho_\alpha(\boldsymbol{r})}\sum_\alpha \int \frac{d\boldsymbol{p}}{(2\pi\hbar)^3}\boldsymbol{p} f_\alpha(\boldsymbol{r},\boldsymbol{p}).$

The coefficients are related to the Skyrme parameters

$$U^{t_0}_{\alpha\beta} = \langle\alpha\beta|\frac{1}{2}t_0(1+x_0P_\sigma)|\alpha\beta-\beta\alpha\rangle,$$

$$U^{t_3}_{\alpha\beta} = \langle\alpha\beta|\frac{1}{12}t_3(1+x_3P_\sigma)|\alpha\beta-\beta\alpha\rangle,$$

$$U^{\tau}_{\alpha\beta} = \langle\alpha\beta|\frac{1}{4}t_1(1+x_1P_\sigma)|\alpha\beta-\beta\alpha\rangle + \langle\alpha\beta|\frac{1}{4}t_2(1+x_2P_\sigma)\,|\alpha\beta+\beta\alpha\rangle,$$

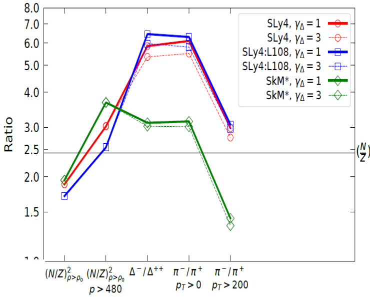
$$U^{\nabla}_{\alpha\beta} = \langle\alpha\beta|\frac{3}{16}t_1(1+x_1P_\sigma)|\alpha\beta-\beta\alpha\rangle - \langle\alpha\beta|\frac{1}{16}t_2(1+x_2P_\sigma)\,|\alpha\beta+\beta\alpha\rangle,$$

In the case of cut-off parameter $\Lambda_{\rm md} = \infty$, interaction is equivalent to the Skyrme type interaction

$$v_{ij} = t_0(1+x_0P_\sigma)\delta(\mathbf{r}) + \tfrac{1}{2}t_1(1+x_1P_\sigma)[\delta(\mathbf{r})\mathbf{k}^2+\mathbf{k}^2\delta(\mathbf{r})] + t_2(1+x_2P_\sigma)\mathbf{k}\cdot\delta(\mathbf{r})\mathbf{k} + \tfrac{1}{6}t_3(1+x_3P_\sigma)[\rho(\mathbf{r}_i)]^\gamma\delta(\mathbf{r}),$$

the spin–isospin label α (or β) $= p\uparrow, p\downarrow, n\uparrow$ and $n\downarrow$

Interactions: SLy4, SLy4:L108, SkM*



- Momentum-dependent potential **(in AMD):**

$$U_\alpha(\boldsymbol{r},\boldsymbol{p}) = (2\pi\hbar)^3\frac{\delta}{\delta f_\alpha(\boldsymbol{r},\boldsymbol{p})}\int \mathcal{E}_{\rm int}(\boldsymbol{r})d\boldsymbol{r} = A_\alpha(\boldsymbol{r})\frac{[\boldsymbol{p}-\bar{\boldsymbol{p}}(\boldsymbol{r})]^2}{1+[\boldsymbol{p}-\bar{\boldsymbol{p}}(\boldsymbol{r})]^2/\Lambda_{\rm md}^2} + \tilde{C}_\alpha(\boldsymbol{r}),$$

with $A_\alpha(\boldsymbol{r}) = \sum_\beta U^\tau_{\alpha\beta}\rho_\beta(\boldsymbol{r})$

$$\tilde{C}_\alpha(\boldsymbol{r}) = \sum_\beta\Big\{2U^{t_0}_{\alpha\beta}\rho_\beta(\boldsymbol{r}) + 2U^{t_3}_{\alpha\beta}\rho_\beta(\boldsymbol{r})[\rho(\boldsymbol{r})]^\gamma + U^\tau_{\alpha\beta}\tilde{\tau}_\beta(\boldsymbol{r}) - 2U^\nabla_{\alpha\beta}\nabla^2\rho_\beta(\boldsymbol{r})\Big\} + \Big(\sum_{\alpha'\beta'}U^{t_3}_{\alpha'\beta'}\rho_{\alpha'}(\boldsymbol{r})\rho_{\beta'}(\boldsymbol{r})\Big)\gamma[\rho(\mathbf{r})]^{\gamma-1}.$$

- Relativistic version **(in sJAM):**

Nucleon single-particle energy $E_a(\boldsymbol{r},\boldsymbol{p}) = \sqrt{(m_N+\Sigma^s_a(\boldsymbol{r}))^2 + (\boldsymbol{p}-\boldsymbol{\Sigma}_a(\boldsymbol{r}))^2} + \Sigma^0_a(\boldsymbol{r}).$

Parametrization from Skyrme interaction: equivalent up to $O(p^2)$:

$$\frac{\boldsymbol{p}^2}{2m_N} + A_a(\boldsymbol{p}-\bar{\boldsymbol{p}})^2 + \tilde{C}_a + m_N \approx \sqrt{(m_N+\Sigma^s_a)^2 + (\boldsymbol{p}-\boldsymbol{\Sigma}_a)^2} + \Sigma^0_a$$

c.f. Zhen Zhang and Che Ming Ko, PRC 98 (2018) 054614.

$\Sigma^s_a = m^*_a - m_N$ with the nucleon effective mass $m^*_a = (m_N^{-1} + 2A_a)^{-1}$

$$\boldsymbol{\Sigma}_a = 4A_a m^*_a\bar{\boldsymbol{p}} = 2m^*_a\sum_b U^\tau_{ab}\boldsymbol{J}_b$$

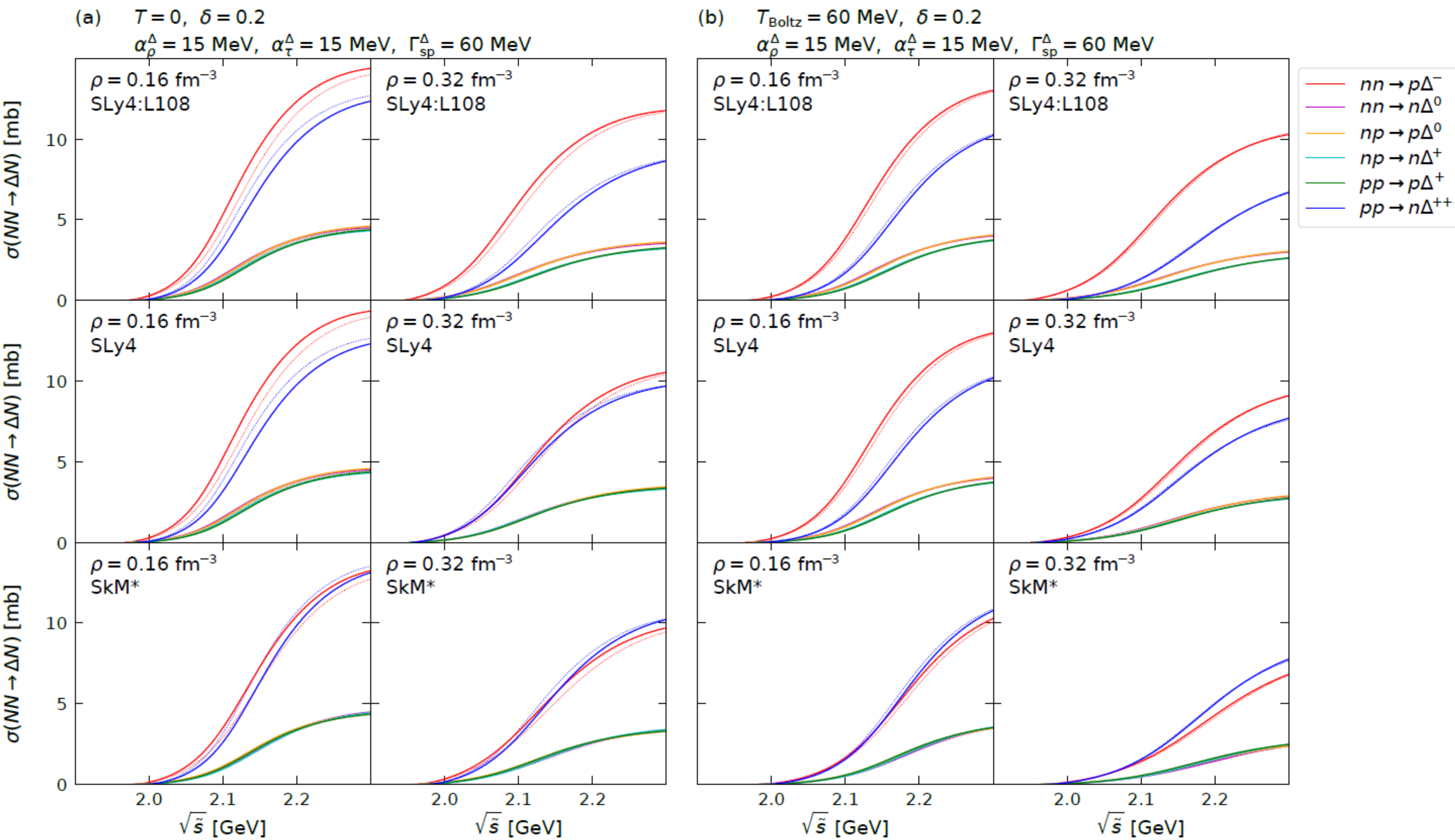
$$\Sigma^0_a = \tilde{C}_a - \Sigma^s_a + A_a\bar{\boldsymbol{p}}^2 - 8m^*_a A^2_a\bar{\boldsymbol{p}}^2 = C_a - \Sigma^s_a - \frac{\boldsymbol{\Sigma}^2_a}{2m^*_a}$$

$$\boldsymbol{J}_b(\boldsymbol{r}) = \int\frac{d\boldsymbol{p}}{(2\pi\hbar)^3}\boldsymbol{p}f_b(\boldsymbol{r},\boldsymbol{p})$$

$$\tau_b(\boldsymbol{r}) = \int\frac{d\boldsymbol{p}}{(2\pi\hbar)^3}\boldsymbol{p}^2 f_b(\boldsymbol{r},\boldsymbol{p}).$$

$$U_{\rm rel}(p) = \sqrt{(m_N+\Sigma^s)^2 + p^2} + \Sigma^0 - \sqrt{m_N^2 + p^2}$$

Cross section NN→NΔ under potentials



Cross section NN→NΔ under potentials