NUSYM23
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on Nuclear Symmetry Energy

Toward a quantitative evaluation of the nuclear EOS

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Content

1. Phenomenological part :

- *Seyler-Blanchard* effective interaction as a specific *2N* interaction (and beyond)
- Implementation in a semi-classical framework : *Thomas-Fermi* approximation
- Stochastic bayesian method for parameter estimation
- Results for *GS* properties : binding energies, charge radii and n-skin thicknesses
- Consequences for nuclear matter : nuclear *EOS* and empirical parameters

2. Experimental part :

- Non-parametric regression from experimental measurements
- Bayesian Gaussian Process Emulator

Discussion and conclusions : what can we learn ...

Seyler-Blanchard effective interaction

The ***SB*** interaction was first developed in the *60's* and is a **momentum-** and ***2-body*** semi-empirical interaction, here supplemented by a **density term** to cope with **higher order correlations (medium).**

$$u(r_{12},p_{12}) = C.Y(r_{12})[-\alpha + \beta(p_{12}/P_{sat})^2 - \gamma(P_{sat}/p_{12}) + \sigma(2\bar{\rho}/\rho_0)^{2/3}]$$

Yukawa : Finite r-dependence

Non-satur. Attractive

Momentum-dependence Repulsive + attractive

Density-dependent Repulsive

C = 2T$_{sat}$***/ρ***$_{sat}$ is the coupling constant of the interaction, with $\rho_{sat} = 3/(4\pi\ r_0^3)$ and $r_0=1.14$ fm

P$_{sat}$, ***T***$_{sat}$ are the Fermi momentum and energy at saturation density

Normalized ***Yukawa* function** with a finite range ***a=0.59294 fm*** :

$$Y(r_{12}) = \frac{1}{4\pi a^3} \frac{e^{-r_{12}/a}}{r_{12}/a}.$$

This is a **phenomenological interaction** requiring **6 parameters** :

$$\alpha_{\ell,u} = \tfrac{1}{2}(1 \mp \xi)\alpha. \quad \beta_{\ell,u} = \tfrac{1}{2}(1 \mp \zeta)\beta, \quad \gamma_{\ell,u} = \tfrac{1}{2}(1 \mp \zeta)\gamma, \quad \sigma_{\ell,u} = \tfrac{1}{2}(1 \mp \zeta)\sigma,$$

where the indices *l,u* stand for *like (-, pp,nn)* and *unlike (+,np)* particles

4 are for the isocalar part : α, β, γ, σ
2 for the isovector part : ξ, ζ

*W.D. Myers, and W.J. Swiatecki, Nucl. Phys. A **601** (1996) 141*

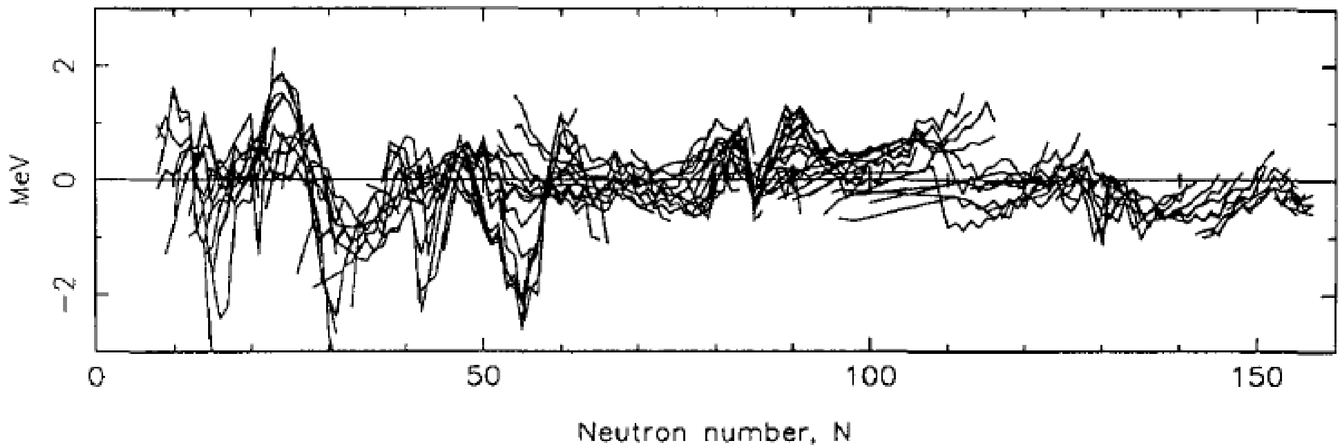
Extended Thomas-Fermi model with SB interaction

Using a **semi-classical approach** : extended **Thomas-Fermi** model

- **2 protons/neutrons** per phase space cell, within a lattice with mesh $\boldsymbol{r_0}$***=1.14 fm***
- **SB interaction** between all nucleons (no gradient corrections),
- **Coulomb repulsion** (no **exchange term** but **Slater approx.**) between protons,
- **Minimization of the total energy** $E = E_{kin} + E_{nuc} + E_{coul}$ by **Metropolis sampling** of the proton, neutron **positions and momenta** on the *TF* lattice

We get **ground-state masses** (binding energies) corrected from **pairing and Wigner terms** to compare with **experimental masses :** Audi & Wapstra's *Atomic Mass Evaluation (AME*).

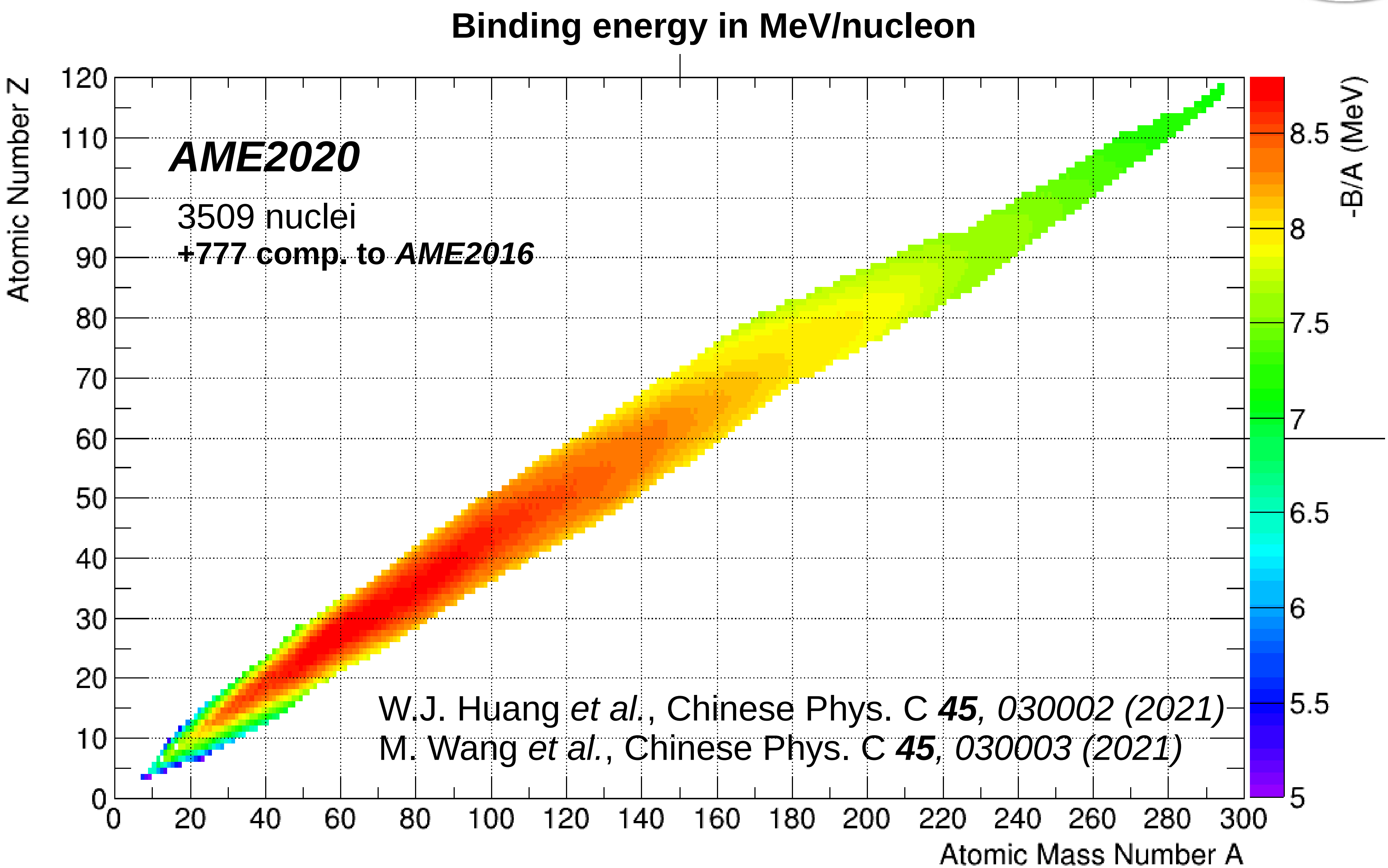
$$\alpha = 1.94684, \quad \beta = 0.15311, \quad \gamma = 1.13672, \quad \sigma = 1.05$$
$$(\therefore\ B = 1.02811), \quad \xi = 0.27976, \quad \zeta = 0.55665, \quad a = 0.59294\ \text{fm}.$$



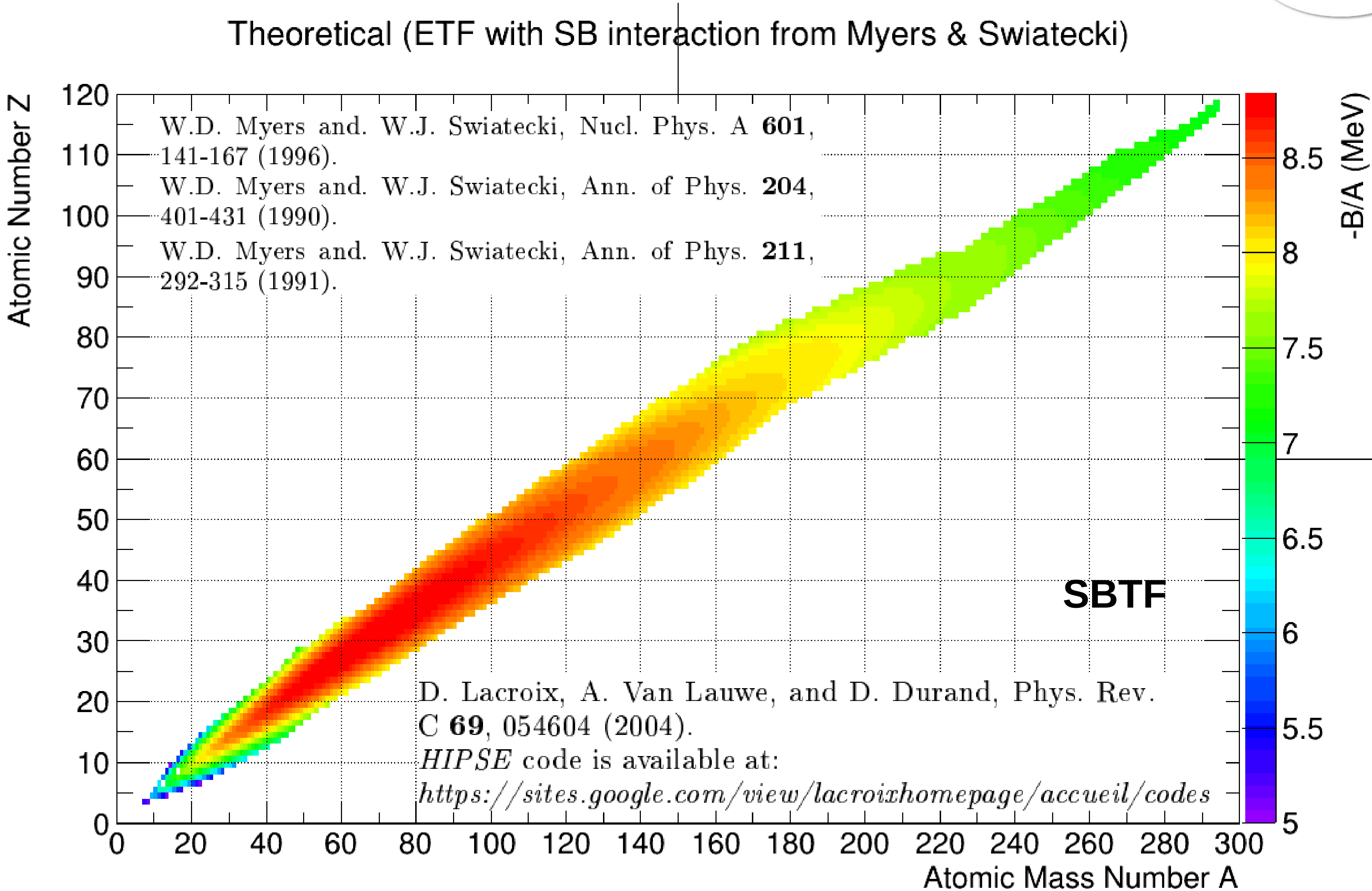
W.D. Myers, and W.J. Swiatecki, Nucl. Phys. A ***601*** *(1996) 141*

rms* deviations** are obtained from the comparison with **1654 nuclei**. The average *rms* deviation over all considered nuclei is ***σ = 0.655 MeV

Ground-State Masses : experimental evaluation *AME*



Ground-State Masses from Extended Thomas-Fermi

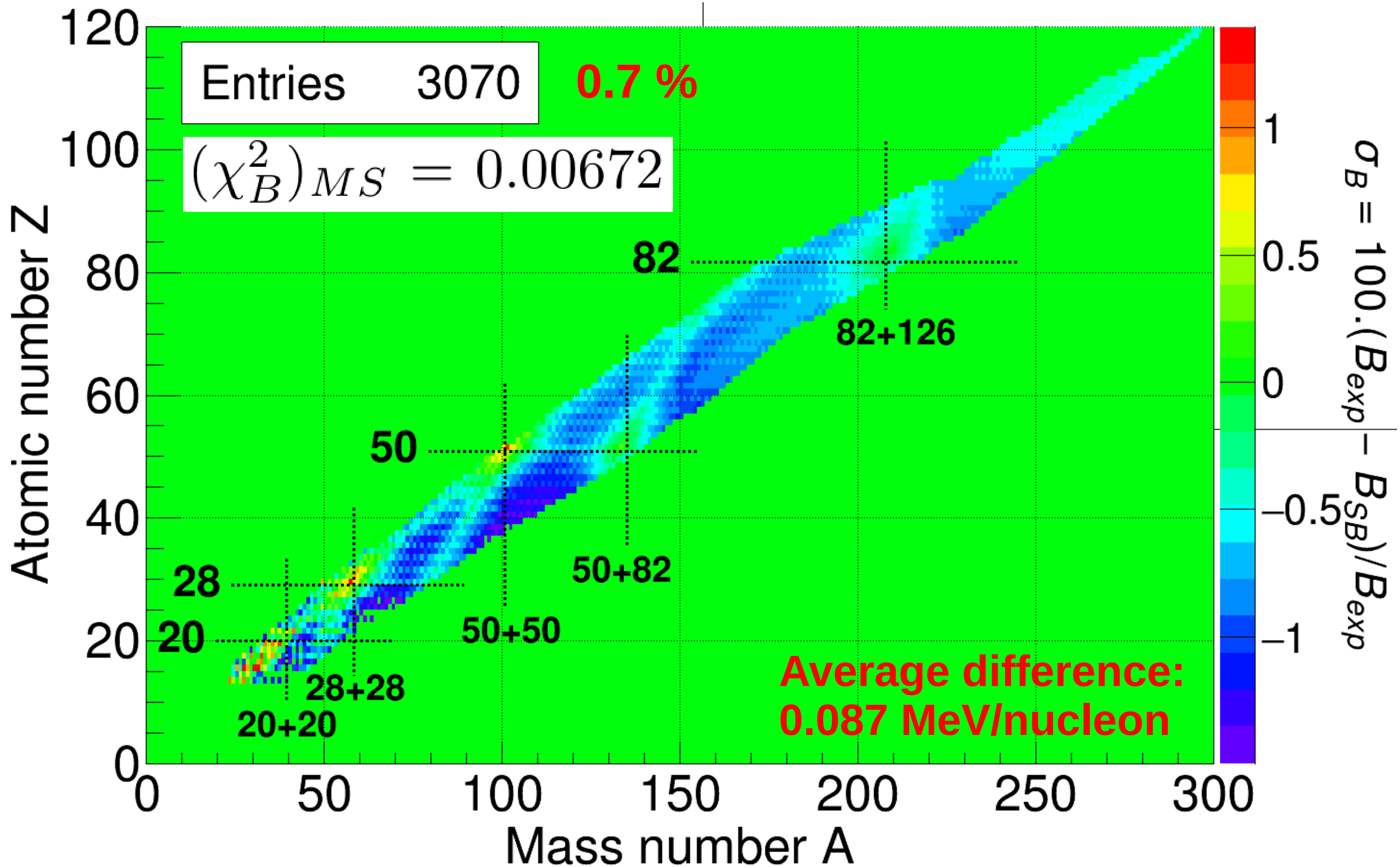


Part of HIPSE package for initialization: Seyler-Blanchard Interaction in a Thomas-Fermi Model with Markov Chains Monte Carlo (MCMC) in a similar formulation as Myers & Swiatecki

Qualification of the agreement

χ^2 residual : $\chi_B^2 = (1/n)\sum_i^n \sigma_B^2$ n=3070 (*Z,N>10*)

$\sigma_B = (B_{exp,i} - B_{SB,i})/B_{exp,i}$

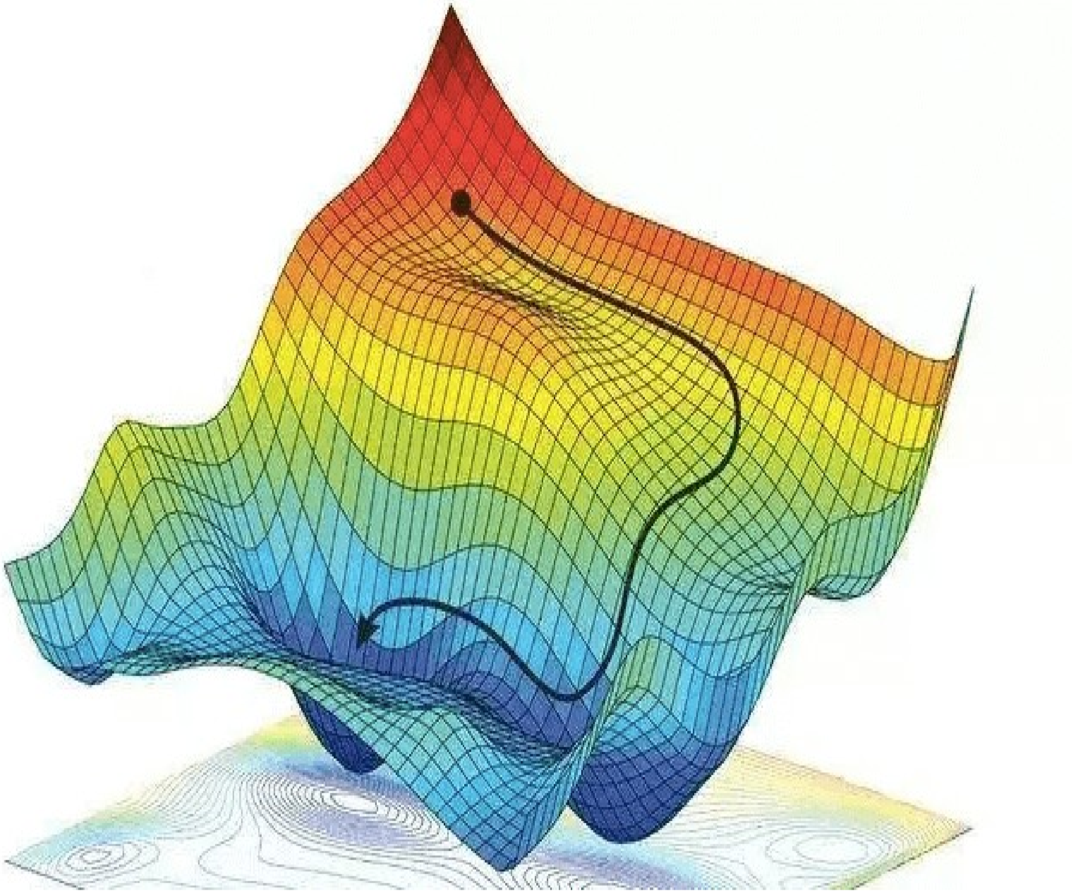


Bayesian Monte Carlo with gradient descent algorithm

Relative distance : $\lambda = (B_{exp}-B_{SBTF,i})/B_{exp}$

For any parameter x_i at iteration i

$$x_{i+1} = x_i + \Delta B_i/\lambda_i$$
$$\Delta B_i = B_i - B_{i-1}$$



Stopping condition when :

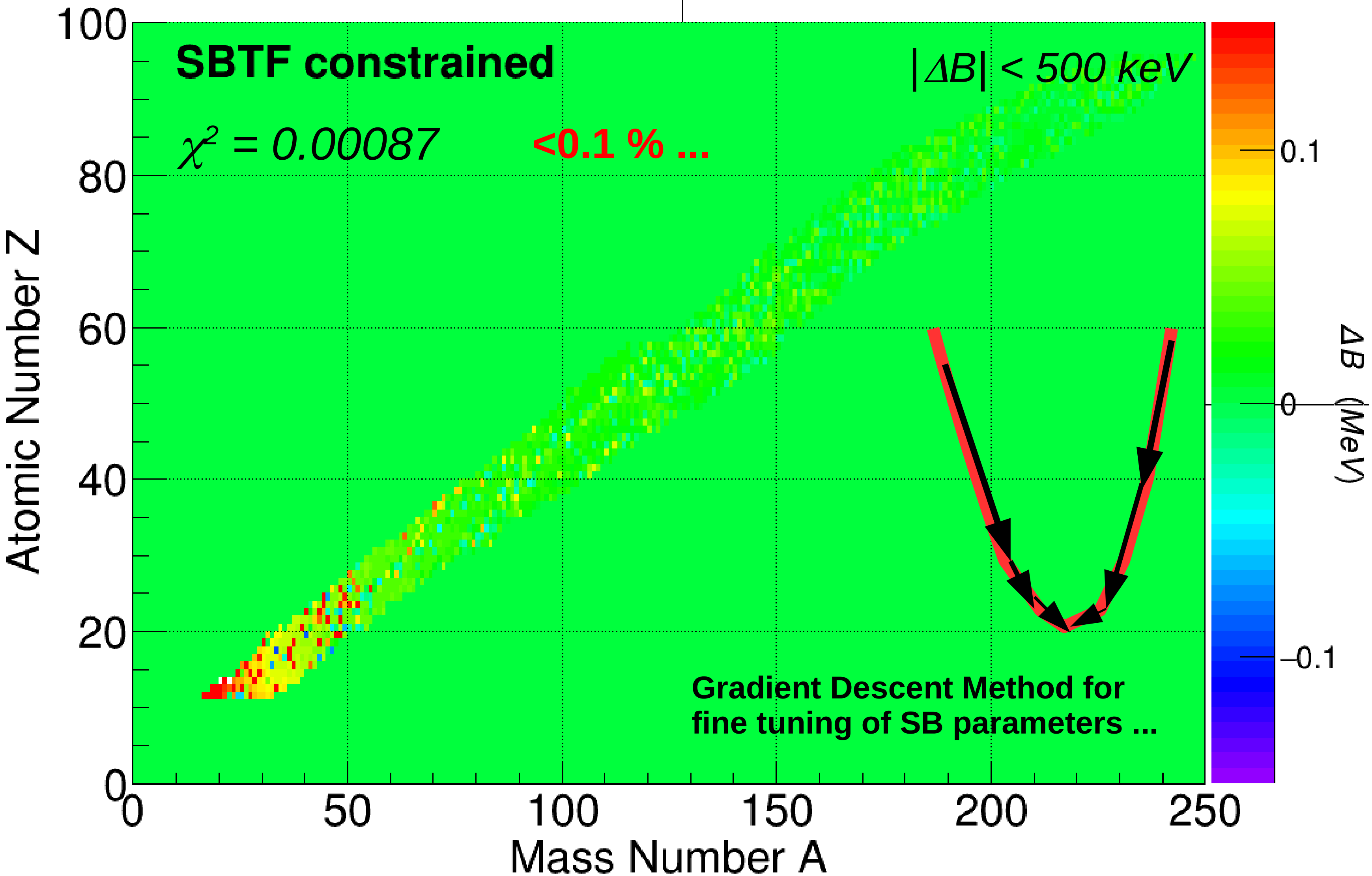
$|B_{exp}-B_{SBTF,i}| < 500$ ***keV***

Initial sampling from uniform prior distributions :

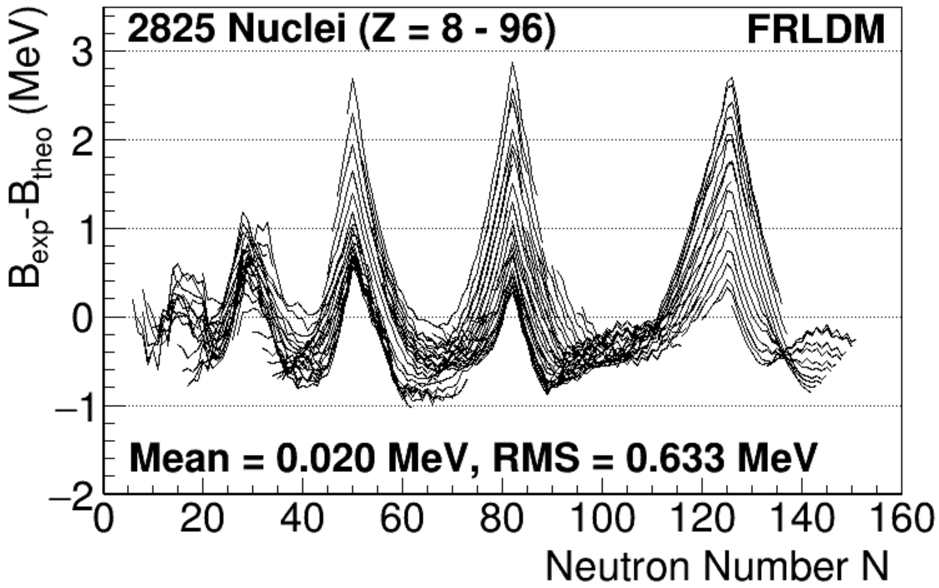
$x_i = x_0 \pm 20\ \%$

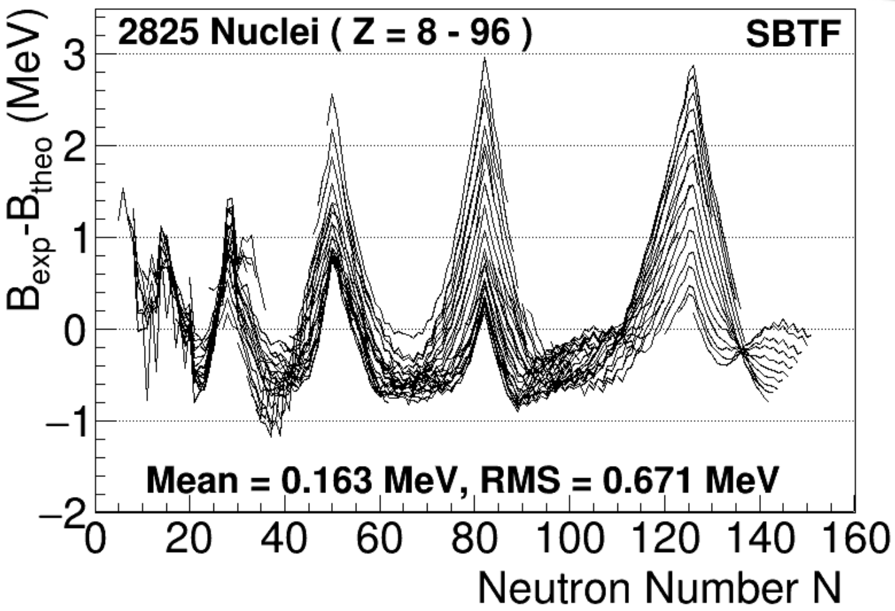
Qualification of the agreement (II)

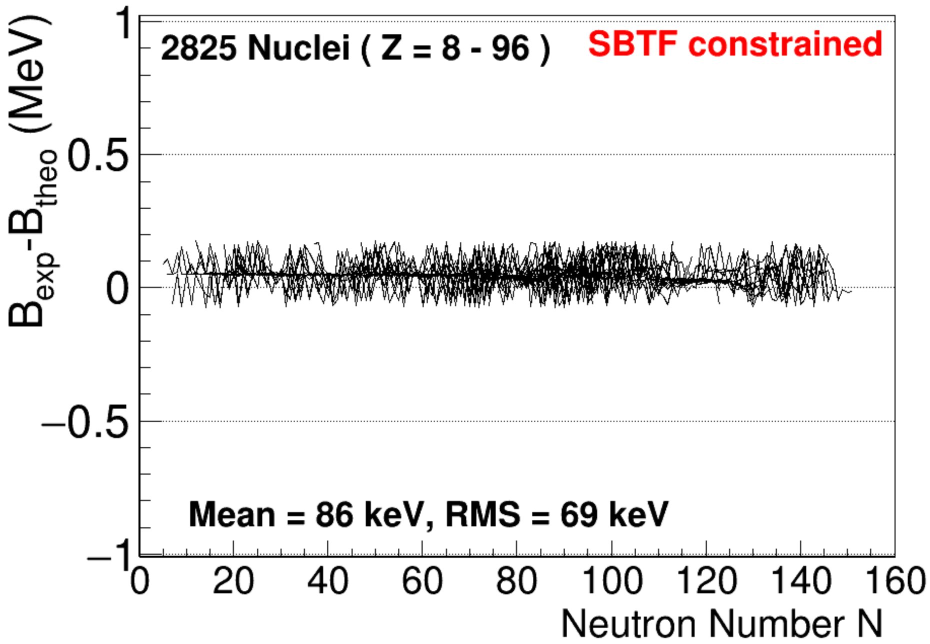
χ^2 residual : $\chi^2_B = (1/n)\sum_i^n \sigma^2_B$ n=3070 (*Z,N>10*)



Qualification of the agreement (III)







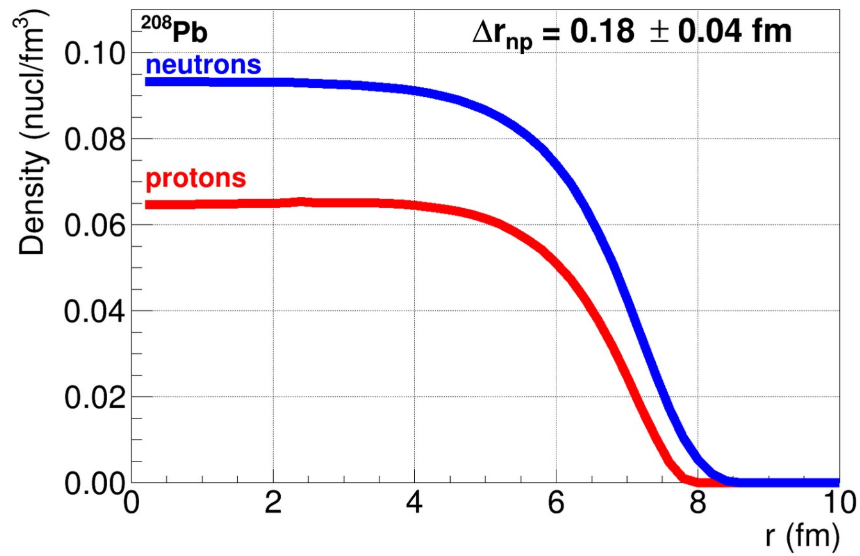
x10 accuracy ...

GS properties : Neutron skins

$$\Delta r_{np} = <r_n^2>^{1/2} - <r_p^2>^{1/2}$$

SBTF

Nucleus	$<r_n^2>^{1/2}$ (fm)	$\Delta r_{np} \pm \sigma$ (fm)	$\Delta r_{np} \pm \sigma$ [ref] (fm)
^{40}Ca	3.35	-0.03±0.02	-0.04±0.02 [88, 90]
^{48}Ca	3.60	0.16±0.03	0.16±0.02 [89, 90]
^{64}Ni	3.79	0.01±0.02	-0.01±0.02 [87]
^{68}Ni	4.01	0.19±0.03	0.19±0.01 [91]
^{100}Sn	4.35	-0.06±0.02	-0.08±0.01 [92]
^{116}Sn	4.66	0.11±0.02	0.11±0.02 [93]
^{120}Sn	4.73	0.14±0.03	0.15±0.03 [93]
^{132}Sn	4.87	0.20±0.04	0.18±0.02 [94]
^{208}Pb	5.63	0.18±0.04	0.19±0.02 [87, 105]



$\delta(\Delta r_{np}) = 0.025$ fm

From a large-scale analysis incl. 25 NM :
X. Roca-Maza *et al.*, PRL **106**, 252501 (2011)

$$\Delta r_{np} = 0.00147 L_{sym} + 0.101$$

$\Rightarrow$ ***L_{sym} = 57 ± 10 MeV***

Charge radii

$$<r_{ch}^2> \approx <r_p^2> + <R_p^2> + N/Z <R_n^2>$$

Spin-orbit : *0 – 0.05 fm* → **+ *SO* +** $+3/4M^2$

$$\Delta r_{ch} = 0.029 \text{ fm}$$

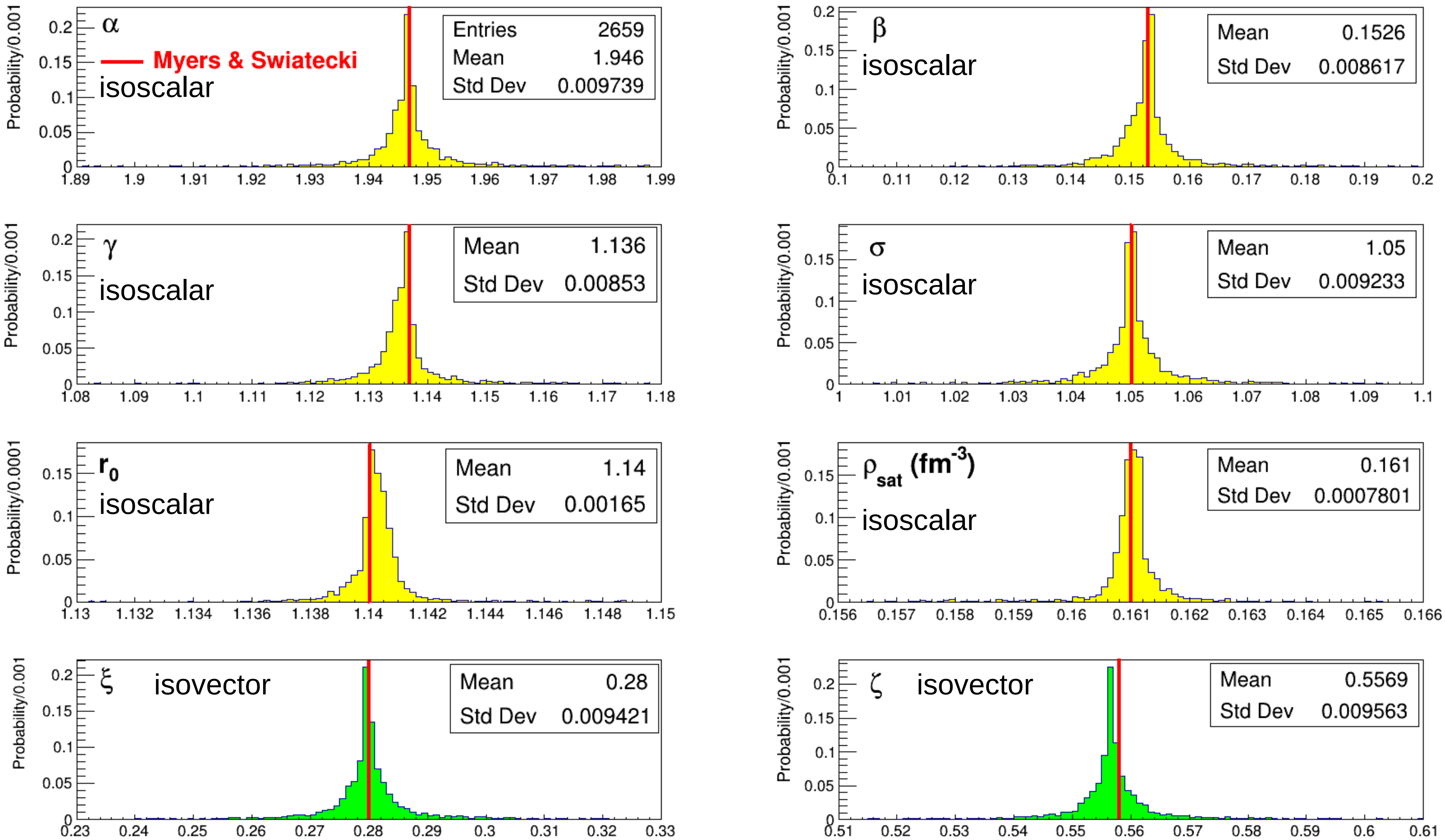
Tension with *PREX-II* ?

$$\Delta r_{np} = 0.283 \pm 0.071 \text{ fm}$$

$$L_{sym} = 106 \pm 37 \text{ MeV}$$

Probability Density Functions (PDF) for SB parameters

25,000 convergences performed



Distributions with **mean values** *and* **consistent standard variations**...

Results for

the nuclear EOS

Derivation for Cold Nuclear Matter : Equation of State

Total energy of a nucleus with neutron, proton densities at *T=0*

$$E = \int d^3r_1\, \mathscr{E}(\mathbf{r}_1),$$

where

$$\rho_n(\mathbf{r}) = (\rho_0/2)\Phi^3(\mathbf{r}), \qquad \rho_p(\mathbf{r}) = (\rho_0/2)\Psi^3(\mathbf{r}),$$

$$\Phi = P_n/P_0, \qquad \Psi = P_p/P_0,$$

$$\mathscr{E}(\mathbf{r}_1) = T_0(\rho_0/2)\frac{3}{5}(\Phi_1^5 + \Psi_1^5) \qquad \text{(Kinetic)}$$

$$-\frac{1}{2}T_0(\rho_0/2)\int d^3r_2\, f\Phi_1^3\Phi_2^3\left\{\alpha_l - \frac{6}{5}B_l\Phi_1^2 + \frac{3}{2}\gamma_l\Phi_>^{-1}\left[1-\frac{1}{5}\frac{\Phi_<^2}{\Phi_>^2}\right]\right\}$$

$$-\frac{1}{2}T_0(\rho_0/2)\int d^3r_2\, f\Psi_1^3\Psi_2^3\left\{\alpha_l - \frac{6}{5}B_l\Psi_1^2 + \frac{3}{2}\gamma_l\Psi_>^{-1}\left[1-\frac{1}{5}\frac{\Psi_<^2}{\Psi_>^2}\right]\right\} \qquad \text{Interaction (SB)}$$

$$-T_0(\rho_0/2)\int d^3r_2\, f\Phi_1^3\Psi_2^3\left\{\alpha_u - \frac{3}{5}B_u(\Phi_1^2+\Psi_2^2) + \frac{3}{2}\gamma_u X_>^{-1}\left[1-\frac{1}{5}\frac{X_<^2}{X_>^2}\right]\right\}$$

$$+\frac{1}{2}e^2(\rho_0/2)^2\int d^3r_2\,\frac{\Psi_1^3\Psi_2^3}{r_{12}}. \qquad \text{(Coulomb)} \qquad (17)$$

→ Integrals are trivial for uniform matter at *T=0* (const. densities and step func.)

W.D. Myers and W.J. Swiatecki,
*Annals of Physics **204**, 401-431 (1990)*

Derivation for Cold Nuclear Matter : Equation of State

Neutron (δ = *1*) and symmetric nuclear matter (δ = *0*) EoS: ε(ρ,δ)=E(ρ,δ)/E$_{sat}$ [29]

$$\begin{cases} \epsilon(\rho,0) = \frac{3}{5}(1-\gamma)\Omega^2 - \frac{\alpha}{2}\Omega^3 + \frac{3}{5}B\Omega^5 \\ \epsilon(\rho,1) = \frac{3}{5}\kappa^2(1-\gamma_l)\Omega^2 - \alpha_l\Omega^3 + \frac{6}{5}\kappa^2 B_l\Omega^5 \end{cases}$$

$$E_{sym}(\rho) = \epsilon(\rho,1) - \epsilon(\rho,0)$$

$$\Omega = (\rho/\rho_{sat})^{1/3} \quad \kappa = 2^{1/3}$$

$$B = \beta + \tfrac{5}{6}\sigma$$

$$B_l = \beta_l + \tfrac{5}{6}\sigma_l$$

≠ parabolic approx. in δ^2
see J. Margueron's talk

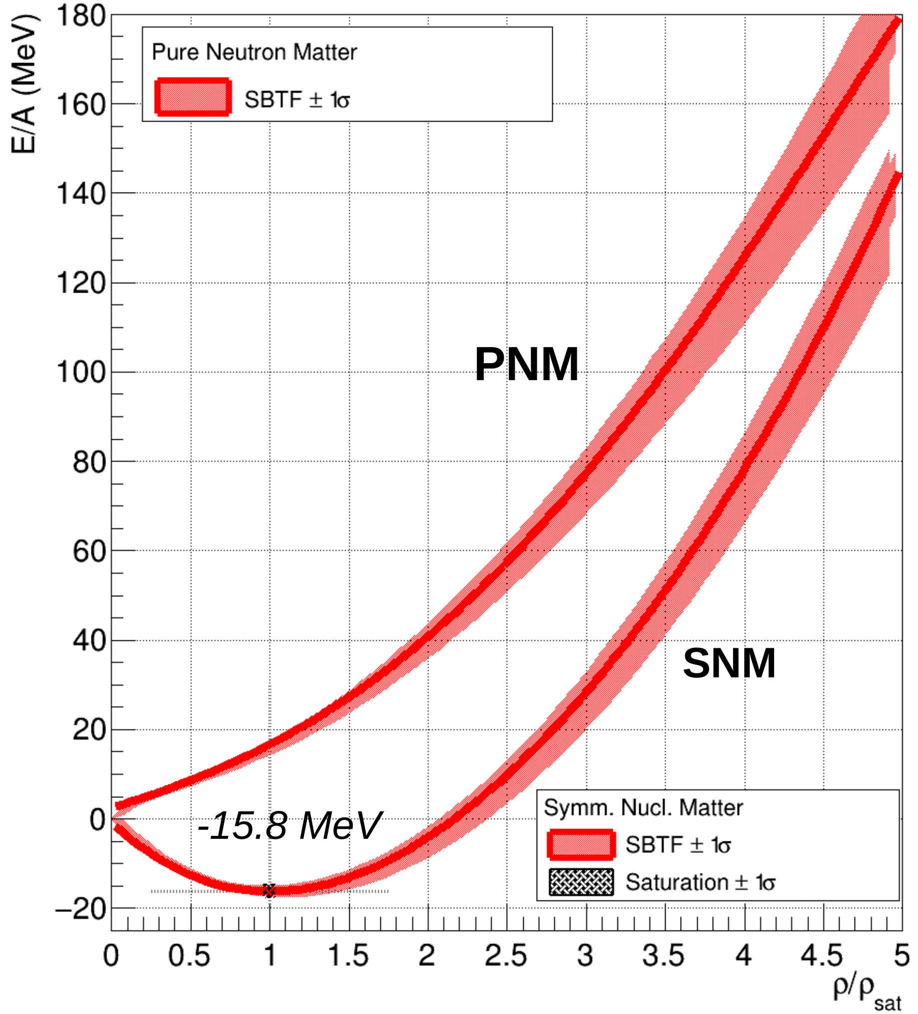
$$E_{sym}/T_{sat} = \frac{3}{5}\{\kappa^2 - 1 + \gamma(1+\zeta/\kappa - 1/\kappa)\}\Omega^2 + \frac{\alpha\xi}{2}\Omega^3 + \frac{3}{5}\{\kappa^2(1-\zeta)-1\}B\Omega^5$$

Kind of EoS	a (Ω^2)	b (Ω^3)	c (Ω^5)
Neutron ($\delta = 1$)	0.44882	-0.35055	0.13674
Symmetric ($\delta = 0$)	-0.08203	-0.97342	0.61687
Asymm. ($\delta = 0.282$)	-0.0171	-0.95176	0.59882
Isovector E_{sym}	0.79443	0.27232	-0.18275

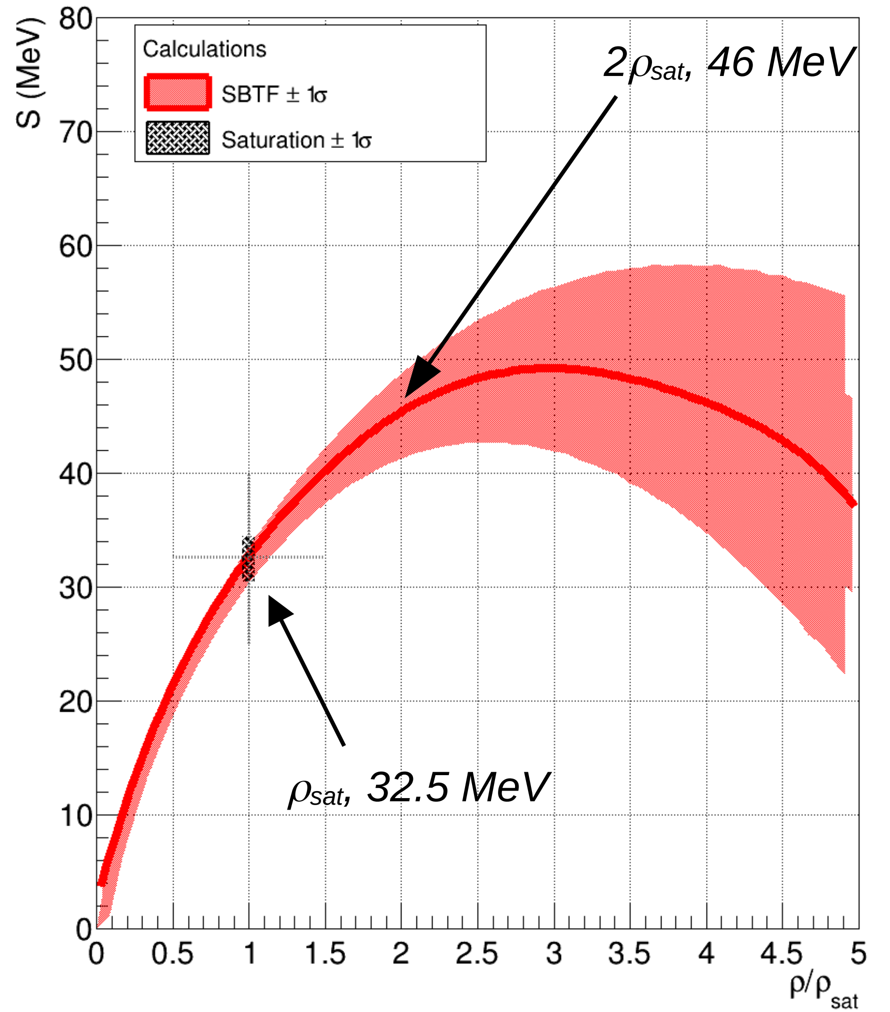
EoS for Cold Nuclear Matter : isoscalar and isovector terms

***100,000* EOS** computed from **mean values** and **std deviations** of SB parameter **PDFs**

Isoscalar & Neutron



Isovector

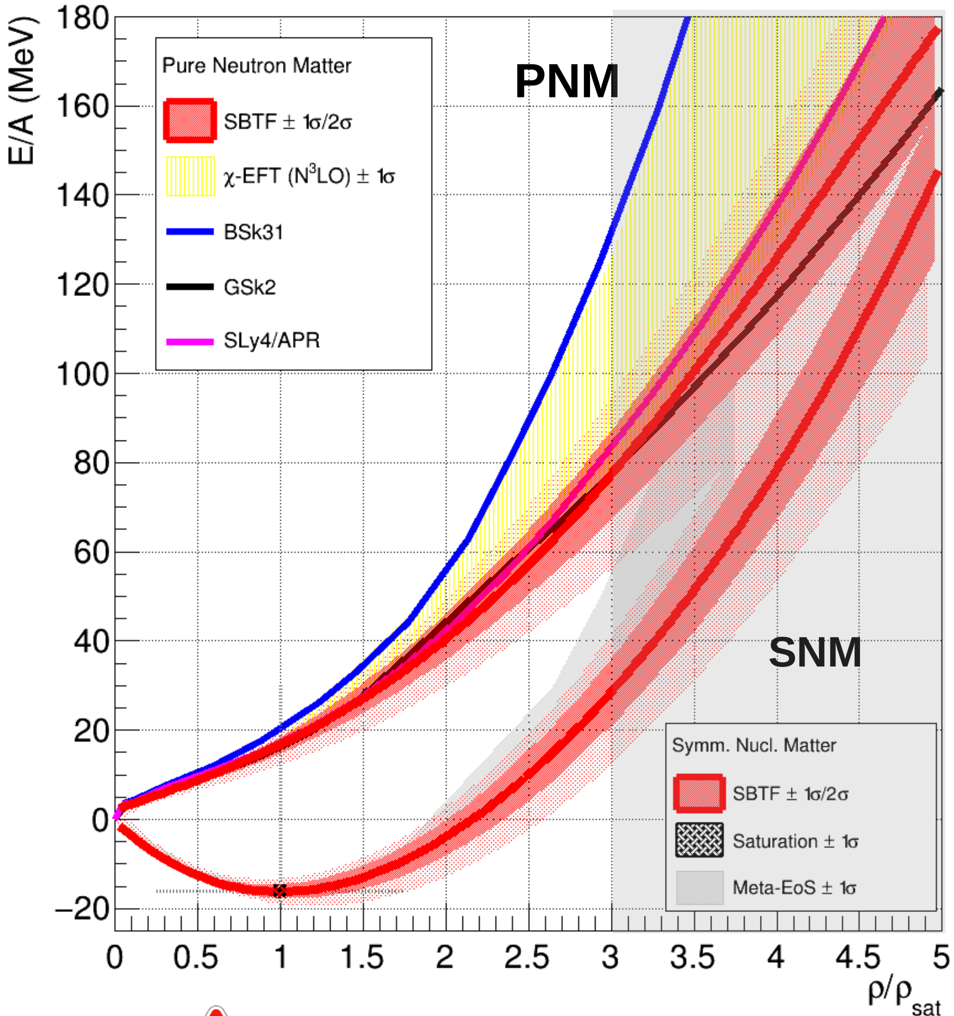


→ **Maximum E_{sym} = 50 ± 8 MeV for ~$3\rho_{sat}$ (1σ)**

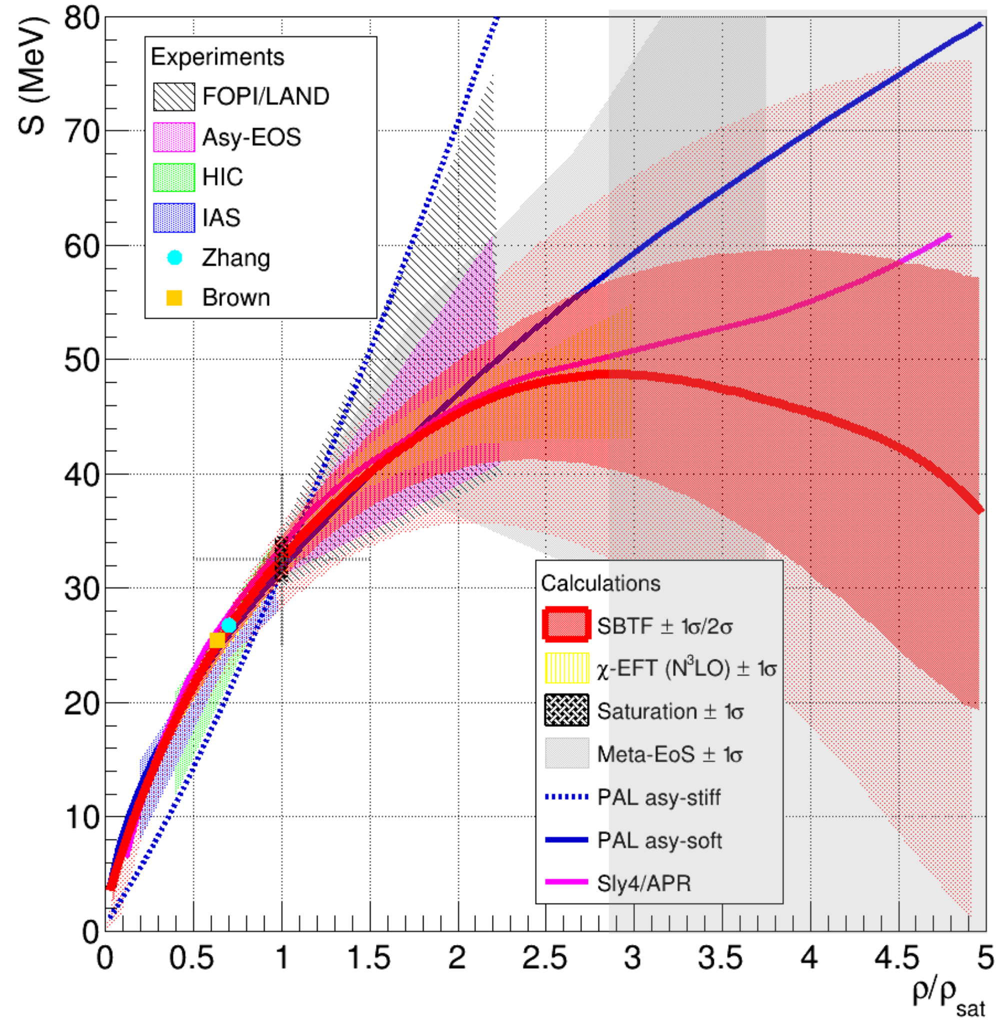
EoS for Cold Nuclear Matter : isoscalar and isovector terms

***100,000* EOS** computed from **mean values** and **std deviations** of SB parameter **PDFs**

Isoscalar & Neutron



Isovector

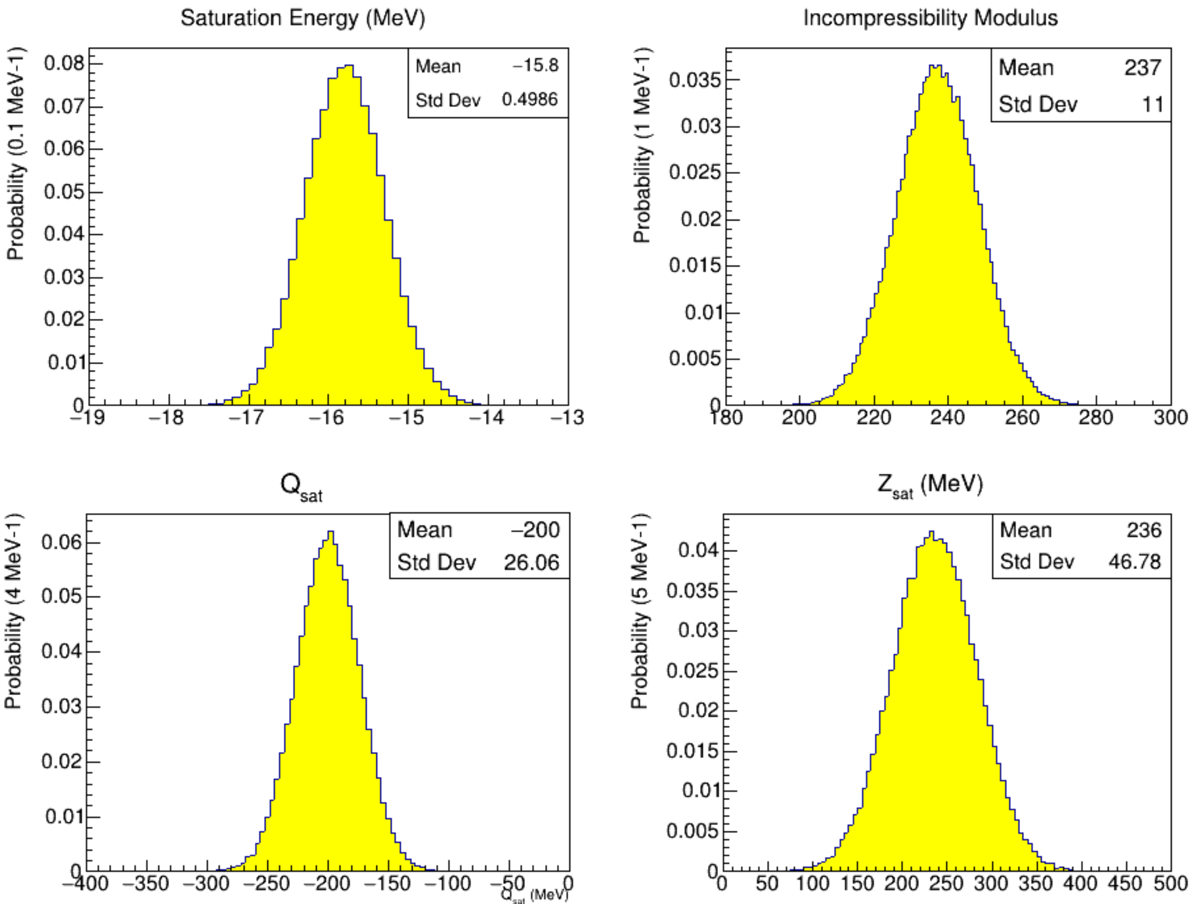


Extrapolation at high density $\rho > 2\rho_{sat}$ …
Phase transition at high ρ ?

EoS empirical parameters from *SBTF : isoscalar*

***100,000* EOS** computed from **mean values** and **std deviations** of SB parameter **PDFs**

Taylor expansion of energy around ρ_{sat}: $e(n_0,n_1) = e_{is}(n_0) + \delta^2 e_{iv}(n_0).$



Higher orders in ρ ($Q_{sat/sym}$,$Z_{sat/sym}$) rule the high density region ρ > 3 ρ_{sat}

$$e_{is} = E_{sat} + \frac{1}{2}K_{sat}x^2 + \frac{1}{3!}Q_{sat}x^3 + \frac{1}{4!}Z_{sat}x^4 + \ldots, \qquad (2)$$

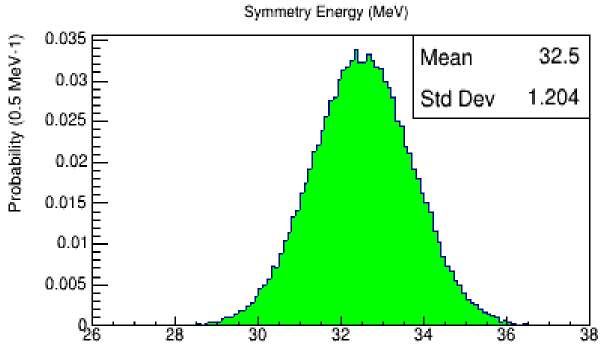
$$e_{iv} = E_{sym} + L_{sym}x + \frac{1}{2}K_{sym}x^2 + \frac{1}{3!}Q_{sym}x^3 + \frac{1}{4!}Z_{sym}x^4 + \ldots,$$

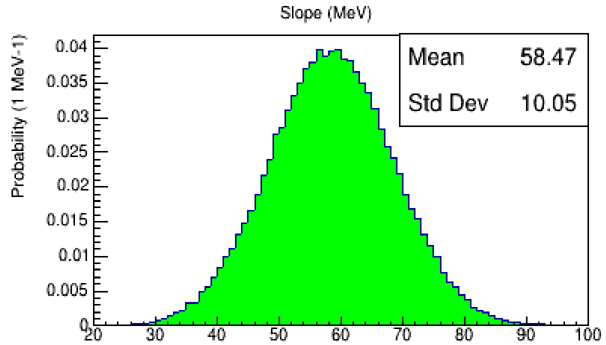
$x = (\rho-\rho_{sat})/(3\rho_{sat})$

EoS empirical parameters from *SBTF : isovector*

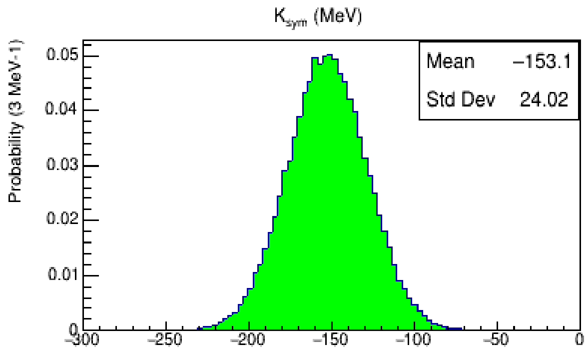
100,000 **EOS** computed from **mean values** and **std deviations** of SB parameter **PDFs**

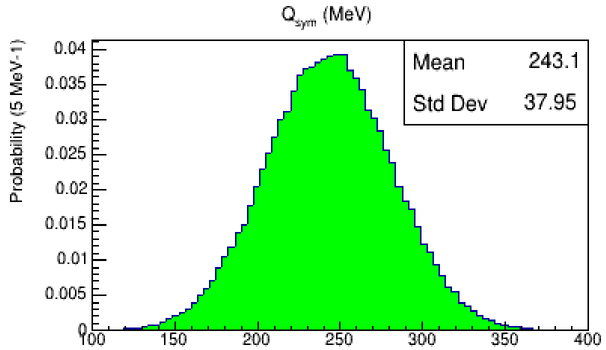
Taylor expansion of energy around ρ_{sat}: $e(n_0, n_1) = e_{is}(n_0) + \delta^2 e_{iv}(n_0).$

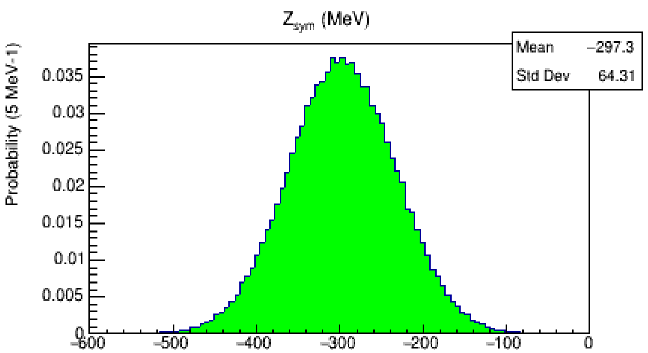




Higher orders in ρ ($Q_{sat/sym}$,$Z_{sat/sym}$) rule the high density region ρ > 3 ρ_{sat}







$$e_{is} = E_{sat} + \frac{1}{2}K_{sat}x^2 + \frac{1}{3!}Q_{sat}x^3 + \frac{1}{4!}Z_{sat}x^4 + \ldots, \quad (2)$$

$$e_{iv} = E_{sym} + L_{sym}x + \frac{1}{2}K_{sym}x^2 + \frac{1}{3!}Q_{sym}x^3 + \frac{1}{4!}Z_{sym}x^4 + \ldots,$$

$x = (\rho-\rho_{sat})/3\rho_{sat}$

EoS empirical parameters : isoscalar + isovector terms

Taylor expansion of energy around n_0 (ρ_{sat}):

$$e(n_0,n_1) = e_{is}(n_0) + \delta^2 e_{iv}(n_0).$$

$$e_{is} = E_{sat} + \frac{1}{2}K_{sat}x^2 + \frac{1}{3!}Q_{sat}x^3 + \frac{1}{4!}Z_{sat}x^4 + \ldots, \quad (2)$$

$$x = (\rho-\rho_{sat})/3\rho_{sat}$$

$$e_{iv} = E_{sym} + L_{sym}x + \frac{1}{2}K_{sym}x^2 + \frac{1}{3!}Q_{sym}x^3 + \frac{1}{4!}Z_{sym}x^4 + \ldots,$$

Parameter (order)	Mean ± σ (MeV)
E_{sat} (0)	-15.8 ± 0.5
K_{sat} (2)	237 ± 11
Q_{sat} (3)	-200 ± 26
Z_{sat} (4)	236 ± 47
J_{sym} (0)	32.5 ± 1.2
L_{sym} (1)	58.5 ± 10
K_{sym} (2)	-153 ± 24
Q_{sym} (3)	243 ± 38
Z_{sym} (4)	-297 ± 64

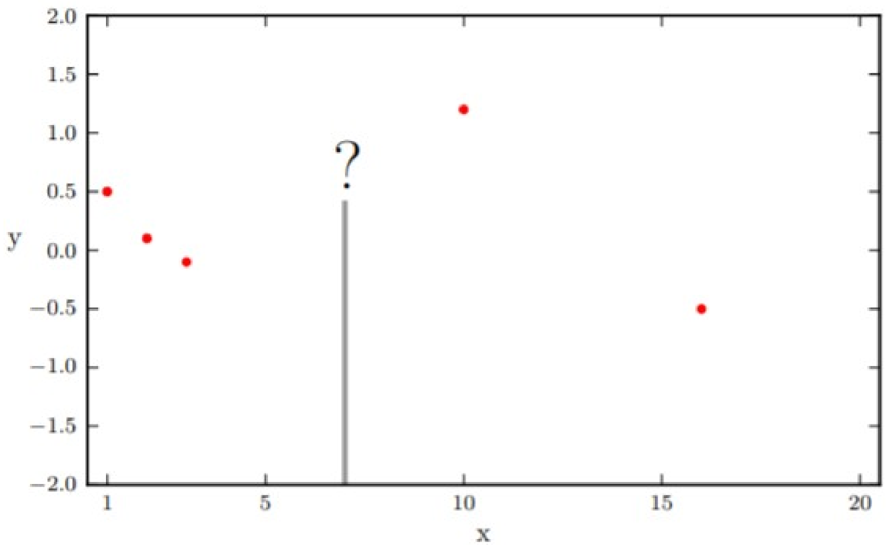
Third and fourth order param. *Q,Z* could be likely uncertain due to the extrapolation to high densities $\rho >> 2\rho_{sat}$ ...

It is here TRUE values for isovector EP ! (cf. J. Margueron's talk)

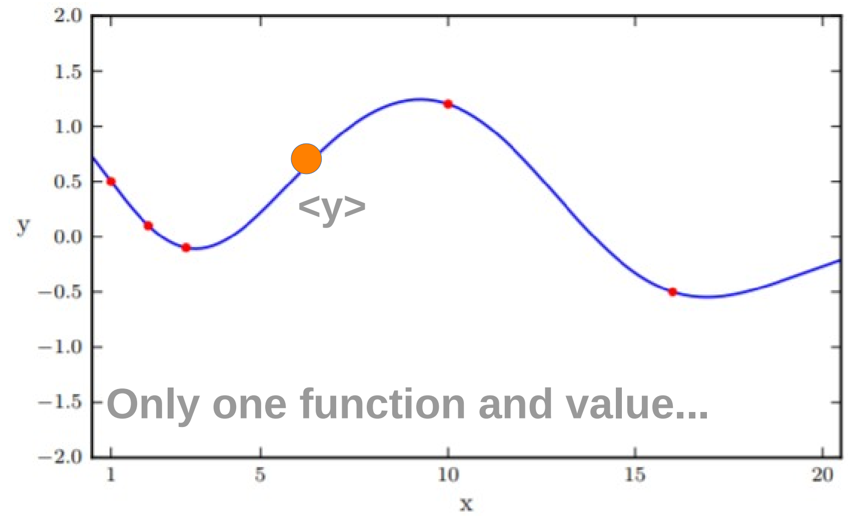
Gaussian Process Regression : non-linear & bayesian

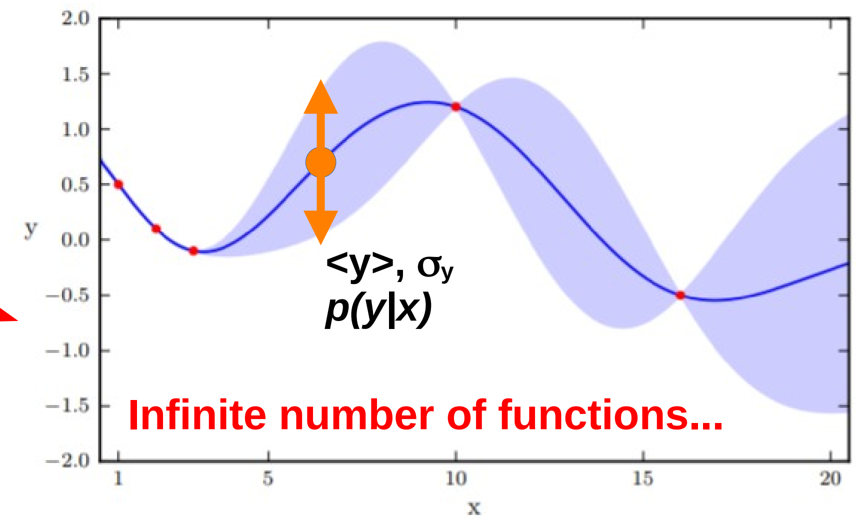
Polynomial regression

Finding a function *f* to solve *y=f(x)* ...



… given the red points





Gaussian Process Regression

Probabilistic (bayesian) inference

Find the best **non-parametric** solutions and quantify their **dispersion**

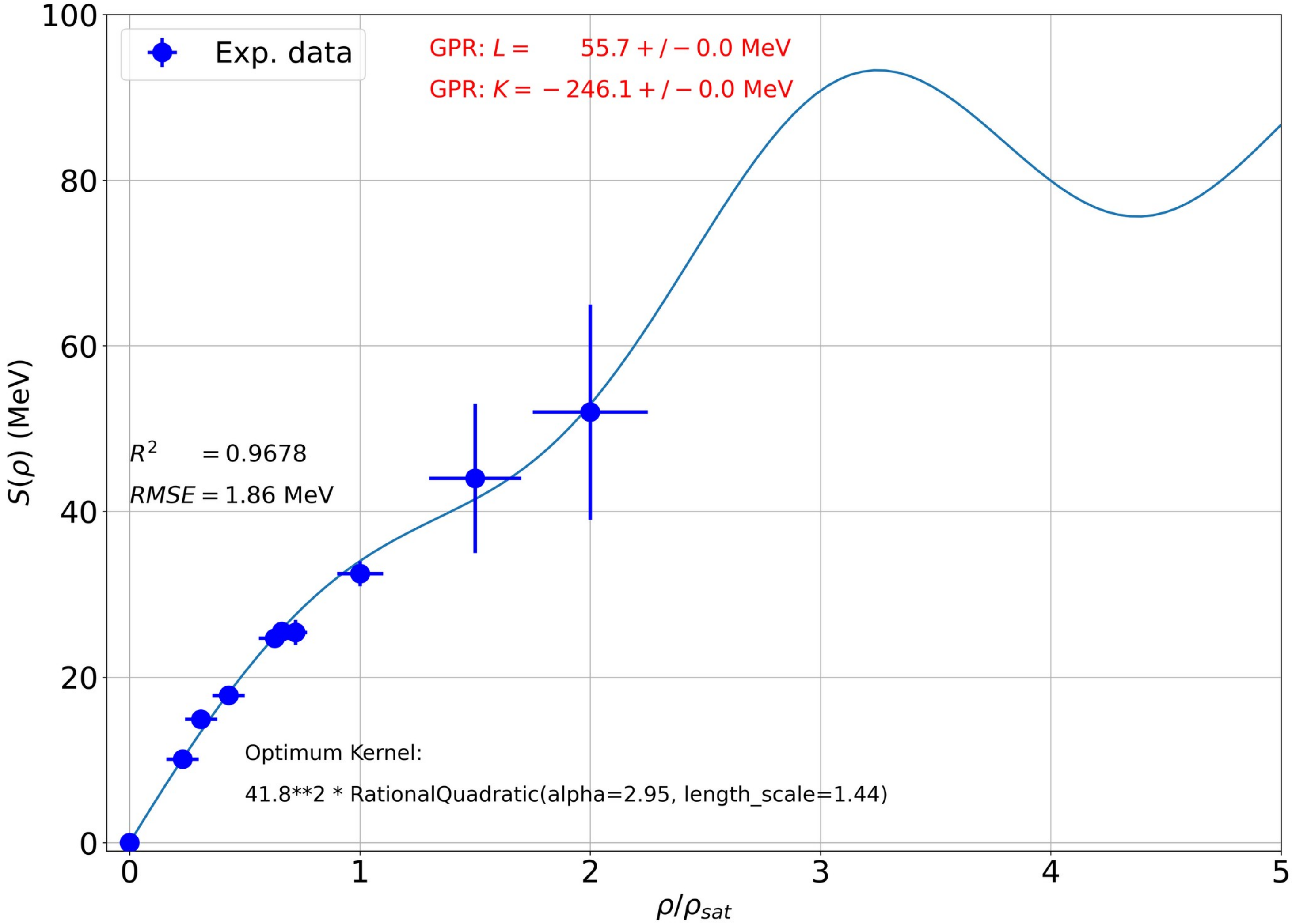
Gaussian Processes for Machine Learning
C.E. Rasmussen & C.K.I. Williams, MIT Press (2006)
`www.GaussianProcess.org/gpml`

GPR : step by step

Matérn kernel

1 posterior

$$C_\nu(d) = \sigma^2 \frac{2^{1-\nu}}{\Gamma(\nu)} \left(\sqrt{2\nu}\frac{d}{\rho}\right)^\nu K_\nu\left(\sqrt{2\nu}\frac{d}{\rho}\right),$$

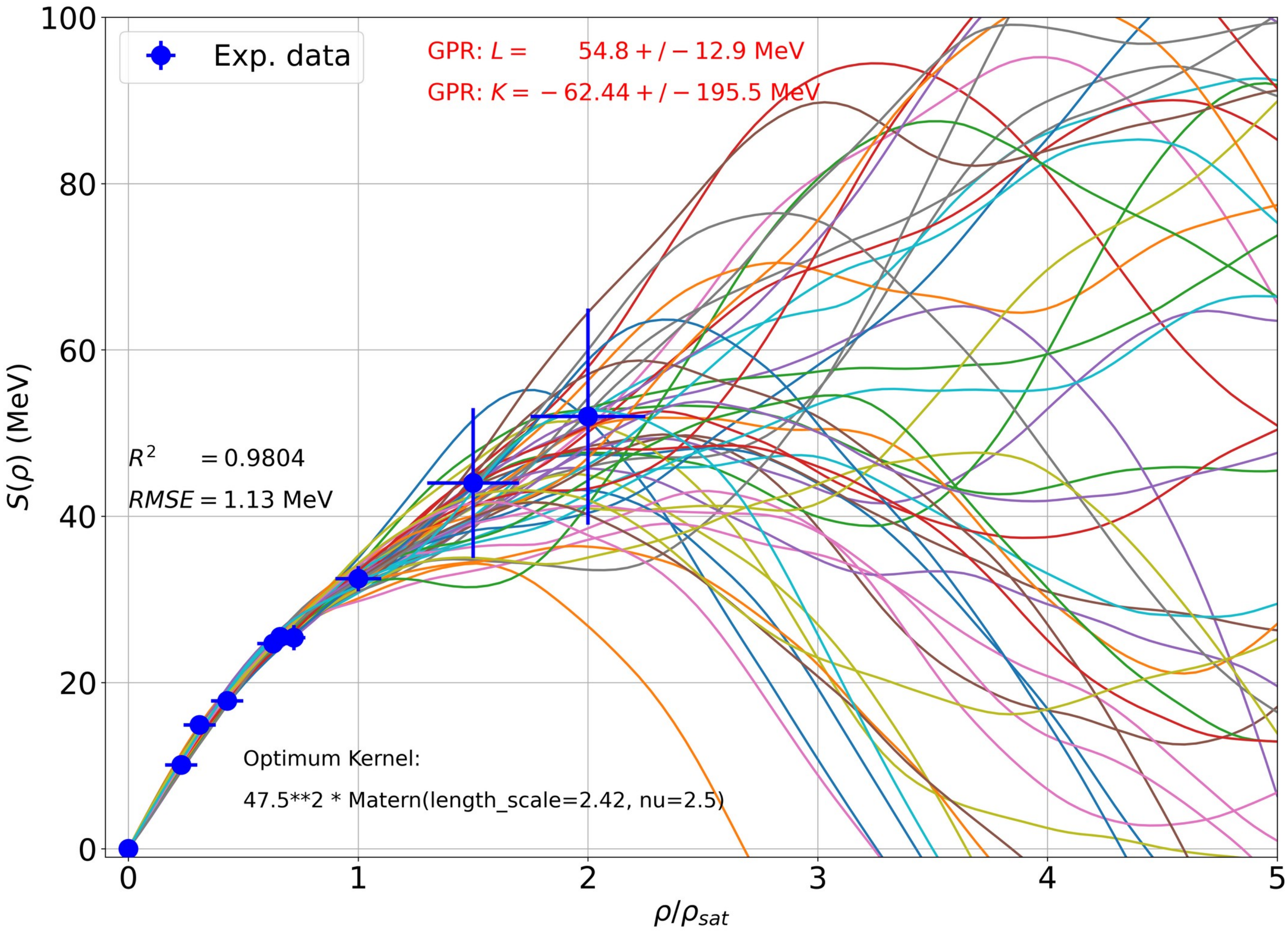


GPR step by step

Matérn kernel

50 posteriors

$$C_\nu(d) = \sigma^2 \frac{2^{1-\nu}}{\Gamma(\nu)} \left(\sqrt{2\nu}\frac{d}{\rho}\right)^\nu K_\nu\left(\sqrt{2\nu}\frac{d}{\rho}\right),$$

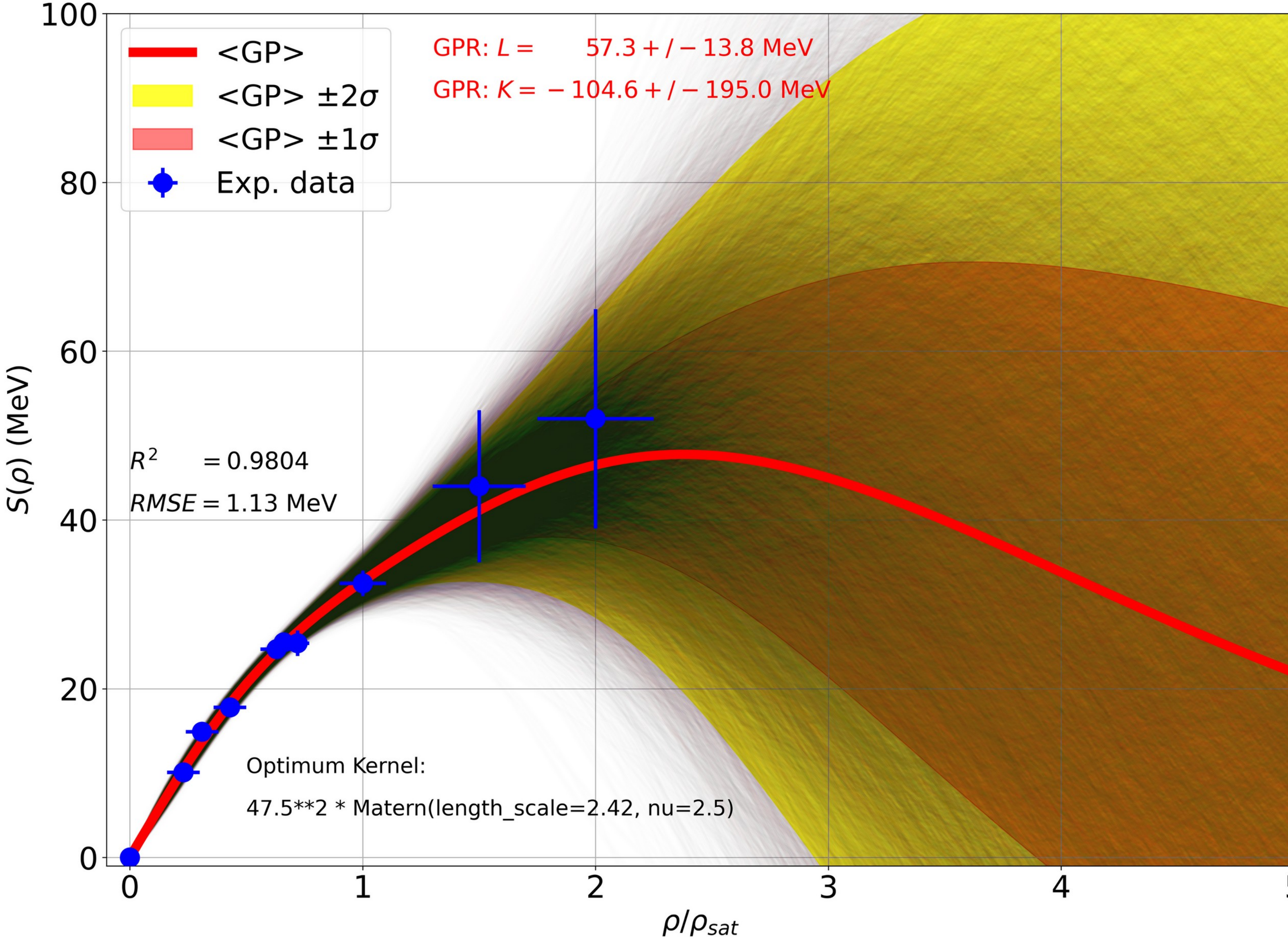


Gaussian Process Regression

Matérn kernel

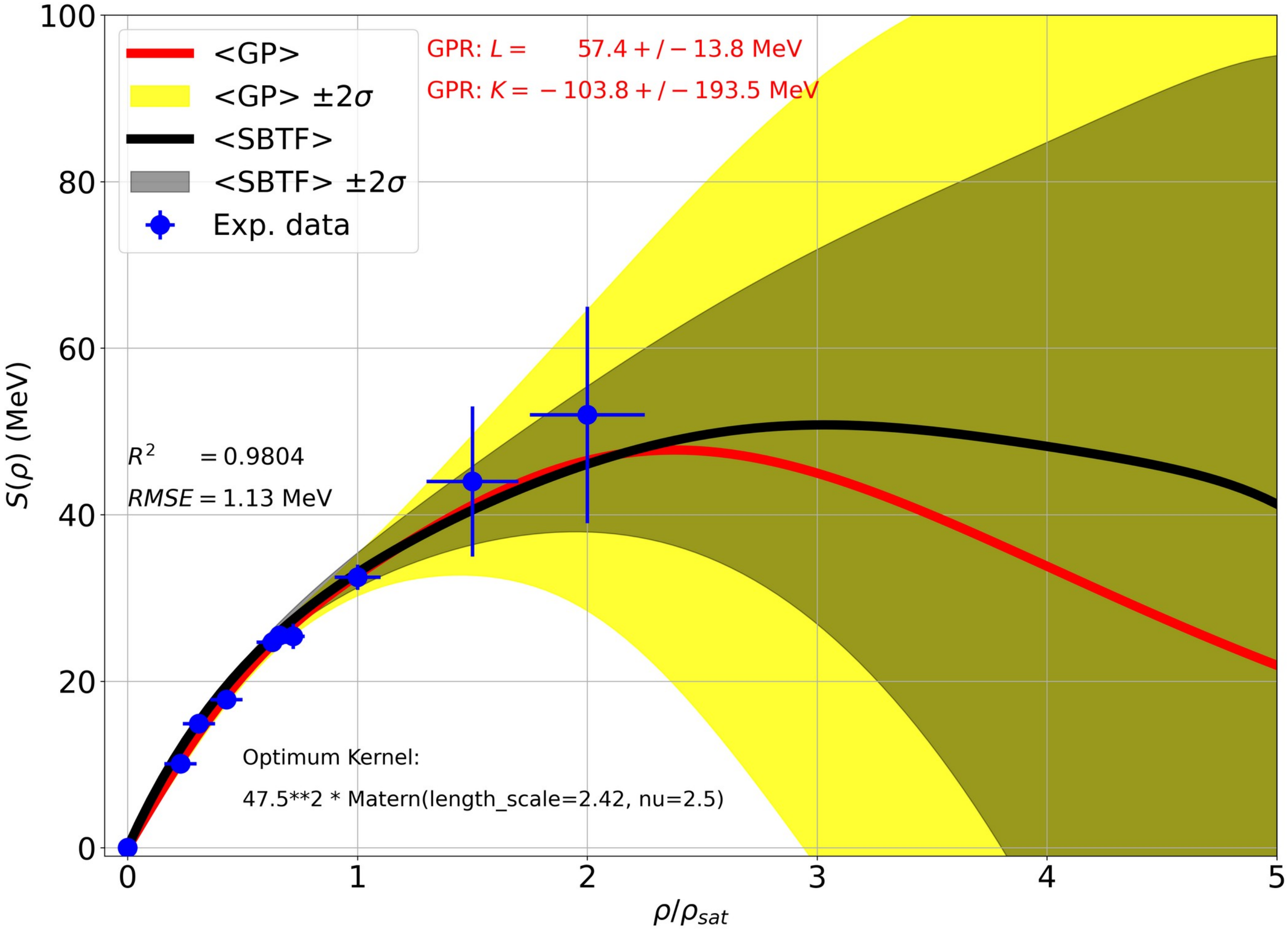
$$C_\nu(d) = \sigma^2 \frac{2^{1-\nu}}{\Gamma(\nu)} \left(\sqrt{2\nu}\frac{d}{\rho}\right)^\nu K_\nu\left(\sqrt{2\nu}\frac{d}{\rho}\right),$$

50,000 posteriors trained on 10 exp. points



EoS for Cold Nuclear Matter : isovector part

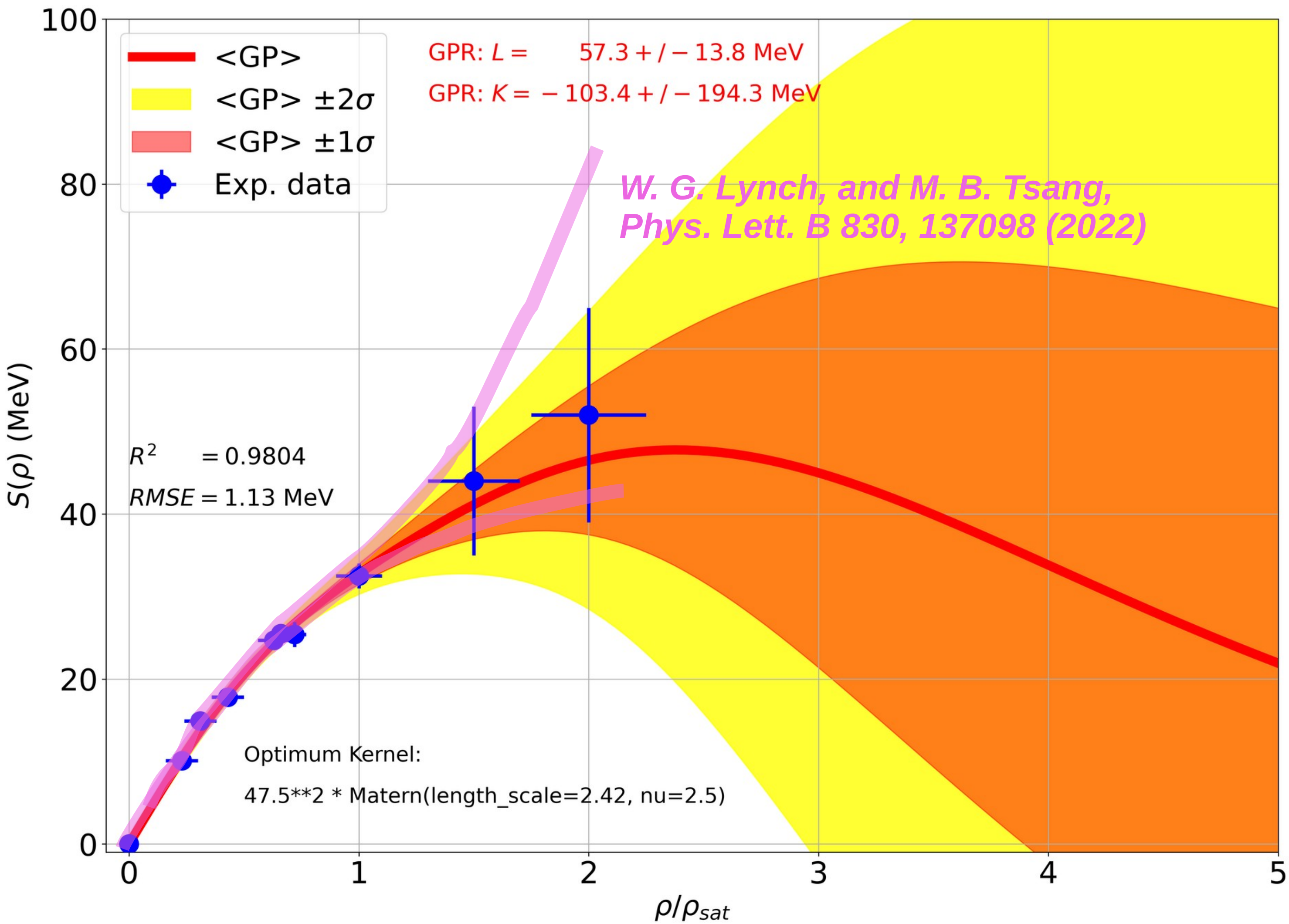
50,000 posteriors trained on 10 exp. points



→ ***GP* and *SBTF* are in agreement up to $\rho = 3\rho_{sat}$**

Benchmark

50000 posteriors trained on 10 exp. points

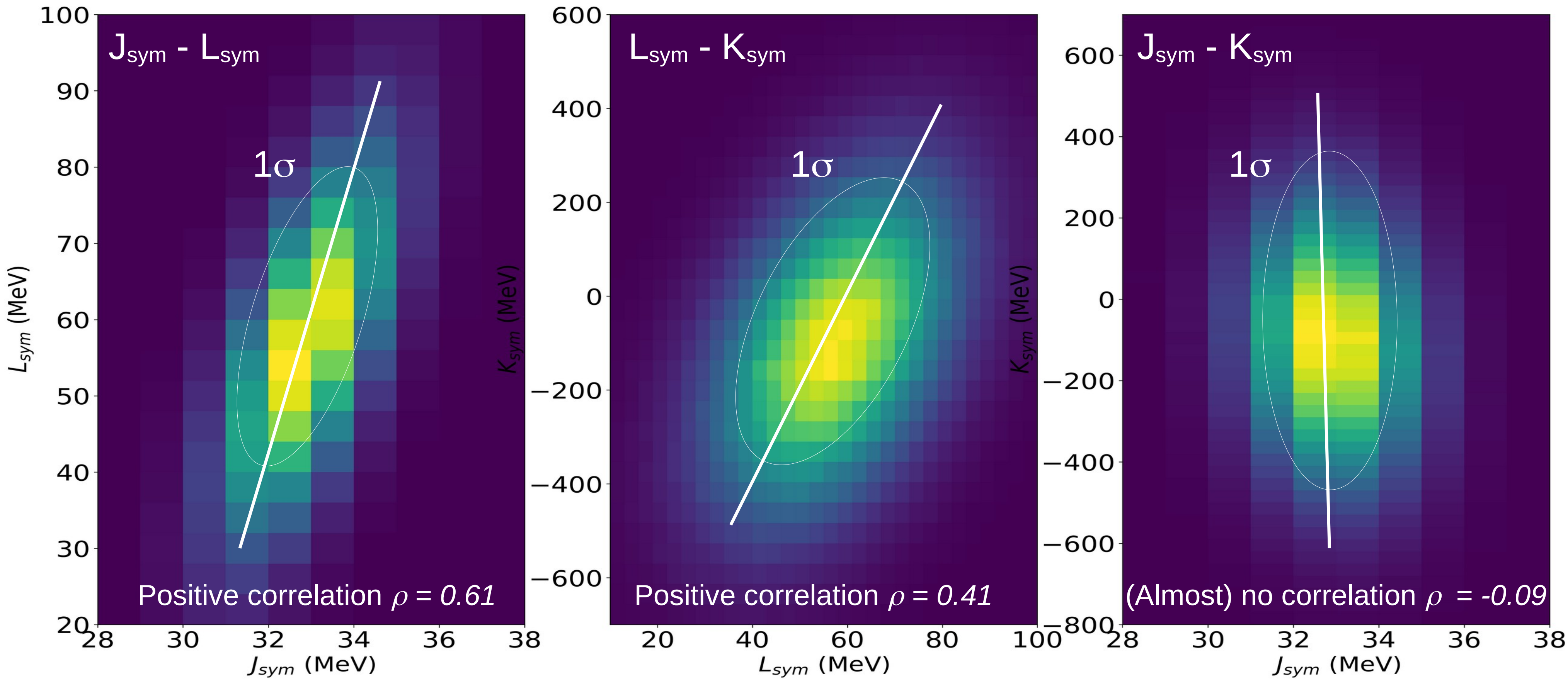


$$RMSE = \sqrt{\sum_{i=1}^{n} \frac{(\hat{y}_i - y_i)^2}{n}}$$

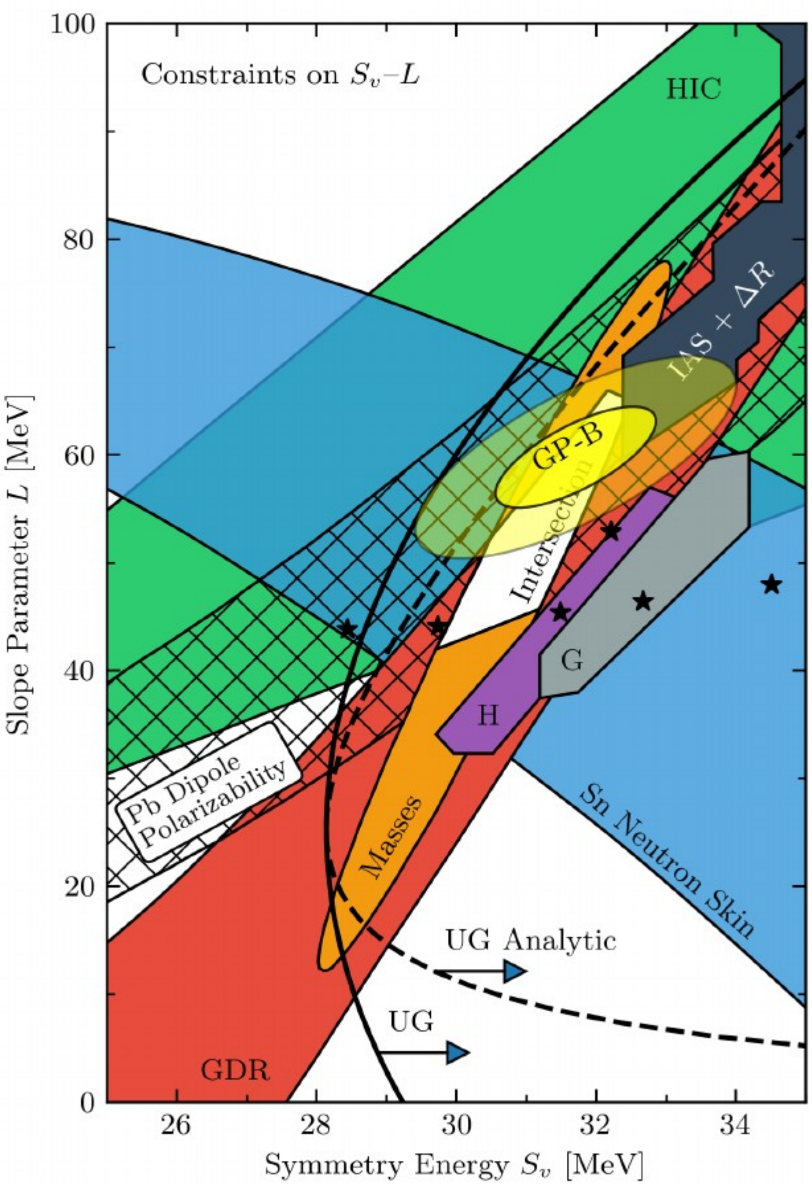
$$R^2 = \frac{\text{Variance explained by the model}}{\text{Total variance}}$$

GPR : J_{sym} – L_{sym} - K_{sym} 2D correlations → Ellipse plots

	J_{sym} - L_{sym}	L_{sym} - K_{sym}	J_{sym} - K_{sym}
Mean μ =	$\begin{pmatrix} 32.4 \\ 57 \end{pmatrix}$	$\begin{pmatrix} 57 \\ -104 \end{pmatrix}$	$\begin{pmatrix} 32.4 \\ -104 \end{pmatrix}$
Covariance Σ =	$\begin{pmatrix} 1.8 & 13 \\ 13 & 255 \end{pmatrix}$	$\begin{pmatrix} 255 & 1472 \\ 1472 & 51633 \end{pmatrix}$	$\begin{pmatrix} 1.8 & -33 \\ -33 & 51633 \end{pmatrix}$



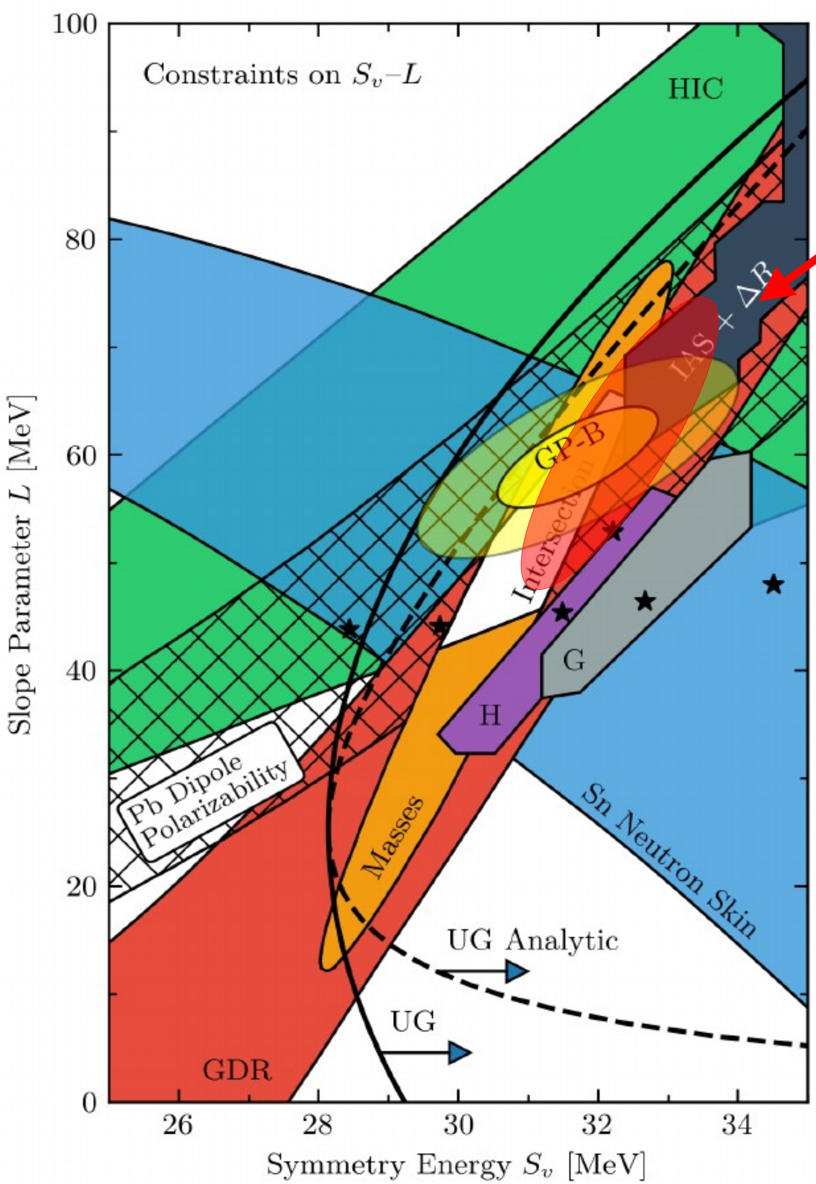
Comparison with experimental and theoretical constraints



GP-B : Gaussian process *BUQ-EYE*

*C. Drischler, R. J. Furnstahl, J. A. Melendez and D. R. Phillips, PRL **125**, 202702 (2020)*

Comparison with experimental and theoretical constraints



Gaussian Process Regression
1σ ellipse

GP-B : Gaussian process *BUQ-EYE*

*C. Drischler, R. J. Furnstahl, J. A. Melendez and D. R. Phillips, PRL **125**, 202702 (2020)*

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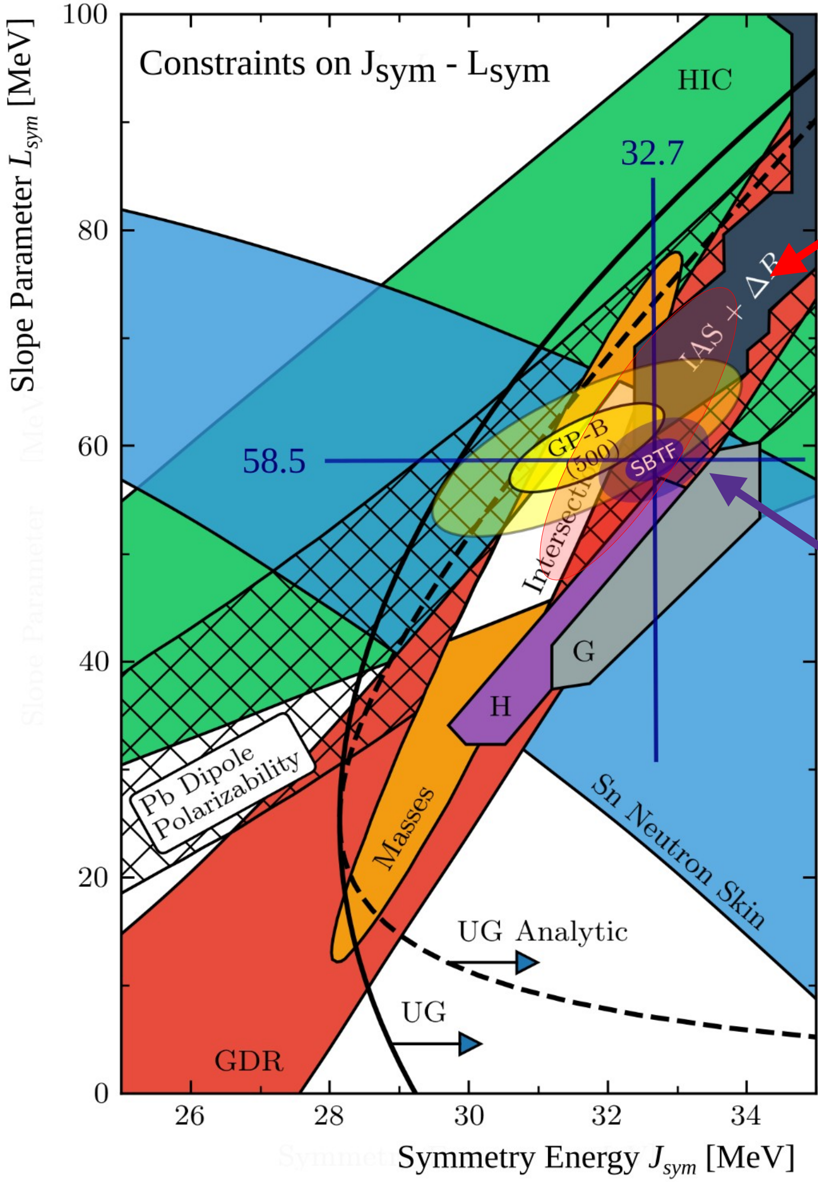


Gaussian Process Regression 1σ ellipse, Episode I

GP-B : Gaussian process *BUQ-EYE*

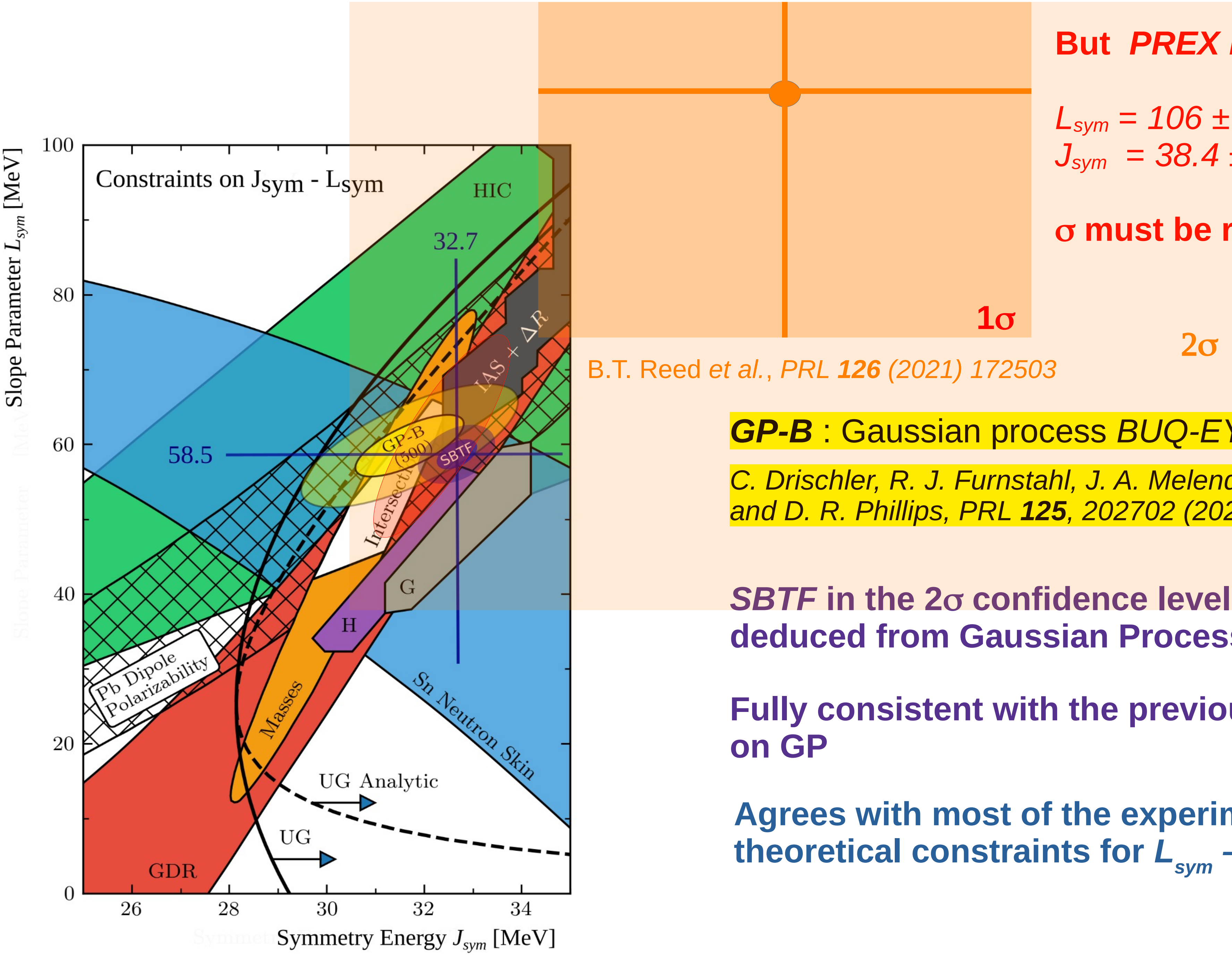
*C. Drischler, R. J. Furnstahl, J. A. Melendez and D. R. Phillips, PRL **125**, 202702 (2020)*

SBTF* in the 2σ confidence level area deduced from Gaussian Process *GP-B

Fully consistent with the previous analysis with *GPR*

Agrees with most of the experimental + theoretical constraints for $L_{sym} - J_{sym}$

Comparison with experimental and theoretical constraints



B.T. Reed *et al.*, *PRL* ***126*** *(2021) 172503*

But *PREX II ?*

L_{sym} = 106 ± 37 MeV
J_{sym} = 38.4 ± 4.2 MeV

σ must be reduced !

GP-B : Gaussian process *BUQ-EYE*

*C. Drischler, R. J. Furnstahl, J. A. Melendez and D. R. Phillips, PRL **125**, 202702 (2020)*

***SBTF* in the 2σ confidence level area deduced from Gaussian Process GP-B**

Fully consistent with the previous analysis on GP

Agrees with most of the experimental + theoretical constraints for L_{sym} – J_{sym}

EoS empirical parameters : isoscalar + isovector terms

50,000 EoS

Taylor expansion of energy around n_0 (ρ_{sat}):

$$e(n_0, n_1) \approx e_{is}(n_0) + \delta^2 e_{iv}(n_0).$$

$$e_{is} = E_{sat} + \frac{1}{2}K_{sat}x^2 + \frac{1}{3!}Q_{sat}x^3 + \frac{1}{4!}Z_{sat}x^4 + \ldots, \quad (2)$$

$$e_{iv} = E_{sym} + L_{sym}x + \frac{1}{2}K_{sym}x^2 + \frac{1}{3!}Q_{sym}x^3 + \frac{1}{4!}Z_{sym}x^4 + \ldots,$$

$x = (\rho-\rho_{sat})/3\rho_{sat}$

Parameter (order)	Mean ± σ (MeV)
E_{sat} (0)	*-15.8 ± 0.5*
K_{sat} (2)	*237 ± 11*
Q_{sat} (3)	*-200 ± 26*
Z_{sat} (4)	*236 ± 47*
J_{sym} (0)	*32.5 ± 1.2*
L_{sym} (1)	*58.5 ± 10*
K_{sym} (2)	*-153 ± 24*
Q_{sym} (3)	*243 ± 38*
Z_{sym} (4)	*-297 ± 64*

Third and fourth order param. *Q,Z* are likely uncertain due to the extrapolation to high densities $\rho >> 2\rho_{sat}$ ...

From GPR

L_{sym}	*57 ± 14*
K_{sym}	*-104 ± 194*

EoS for Cold Nuclear Matter, experimental constraints

Parameter (order)	Mean ± σ *SBTF* (MeV)	Mean ± σ *GPR* (MeV)	Mean ± σ *Exp.* * (MeV)
E_{sat} (0)	*-15.8 ± 0.5*		*-15.8 ± 0.3*
K_{sat} (2)	*237 ± 11*		*230 ± 20*
Q_{sat} (3)	*-200 ± 26*		*300 ± 400*
Z_{sat} (4)	*236 ± 47*		*-500 ± 1000*
J_{sym} (0)	*32.5 ± 1.2*	*32.5 ± 2*	*32.5 ± 2*
L_{sym} (1)	*58.5 ± 10*	*57 ± 14*	*60 ± 15*
K_{sym} (2)	*-153 ± 24*	*-104 ± 194*	*-100 ± 100*
Q_{sym} (3)	*243 ± 38*		*100 ± 400*
Z_{sym} (4)	*-297 ± 64*		*-500 ± 1000*

- **Semi-classical** approach : **extended Thomas-Fermi** Model with **Seyler-Blanchard** interaction
- **Plausible set of EOS** obtained from a **bayesian analysis** based on ground-state masses (*AME2020*)
- **EOS empirical parameters** consistent with **all experimental results but PREX I/II**

* Extracted from Table VII *ibid* J. Margueron, R. Hoffmann Casali, and F. Gulminelli Phys. Rev. C **97**, 025805, 025806 (2018).

Conclusions

- ***SBTF*** agrees with ***GPR*** **up to** $2\rho_{sat}$ with **experimental / theoretical** estimates

- Adjusted to the **latest** atomic masses evaluation ***AME2020***

- Dedicated **gradient descent algorithm** has provided a kind of **bayesian analysis** on *SB* interaction parameters and has defined the **theoretical uncertainties**

- Following **Myers & Swiatecki's** work, ***PDF EOS*** **empirical parameters** have been derived accordingly

- For the **isoscalar** empirical parameters (***95% CL,*** 2σ) :

$E_{sat} = -15.8 \pm 1.0$ MeV , $K_{sat} = 237 \pm 22$ MeV,
$Q_{sat} = -200 \pm 52$ MeV , $Z_{sat} = 236 \pm 94$ MeV

- For the **isovector** empirical parameters (***95% CL,*** 2σ) :

$J_{sym} = 32.5 \pm 2.6$ MeV , $L_{sym} = 58.5 \pm 20$ MeV,
$K_{sym} = -153 \pm 48$ MeV , $Q_{sym} = 243 \pm 76$ MeV , $Z_{sym} = -297 \pm 128$ MeV

Results are consistent with (almost) all known experimental and theoretical constraints including non-linear regressions (GPR/GP-B)

But agreement with PREX/CREX results ?

→ uncertainties should be reduced… PREX / CREX/ ASY-EOS , ...

The end

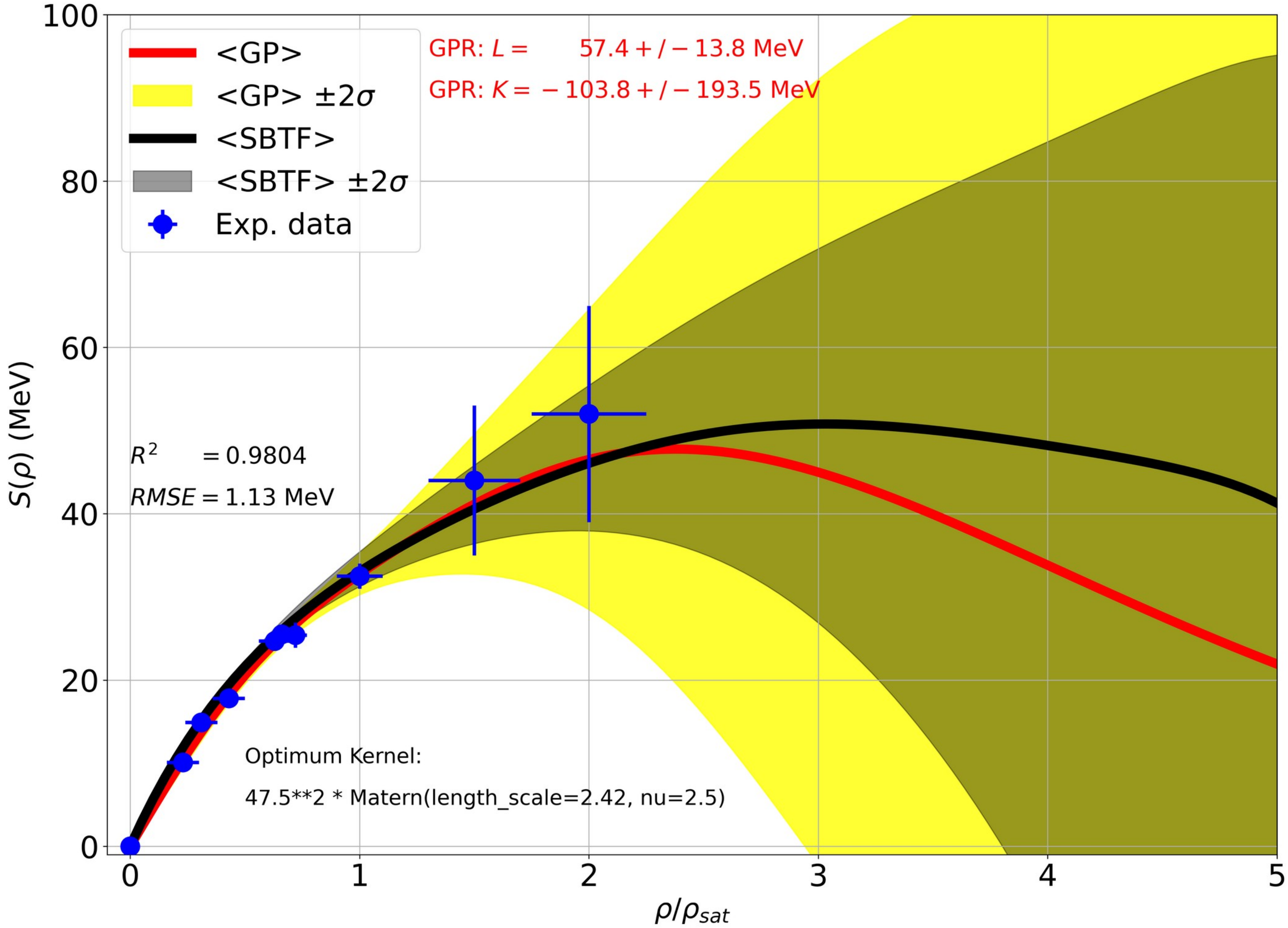
Epilogue :

Taylor expansion for *EOS* still valid at high ρ, especially for $\rho > 2\rho_{sat}$?

Can we assess a possible phase transition at large densities from HIC and/or NS ??

EoS for Cold Nuclear Matter : isovector part

SBTF *: 50,000* EOS computed from **bayesian inference** using ***AME2020 + charge radii***
GP *:* **Gaussian Process** with **Matérn kernel** trained on **10 experimental data**



EoS for Cold Nuclear Matter, experimental constraints

Parameter (order)	Mean ± σ *SBTF* (MeV)	Mean ± σ *GPR* (MeV)	Mean ± σ *Exp.* * (MeV)
E_{sat} (0)	*-15.8 ± 0.5*		*-15.8 ± 0.3*
K_{sat} (2)	*237 ± 11*		*230 ± 20*
Q_{sat} (3)	*-200 ± 26*		*300 ± 400*
Z_{sat} (4)	*236 ± 47*		*-500 ± 1000*
J_{sym} (0)	*32.5 ± 1.2*	*32.5 ± 2*	*32.5 ± 2*
L_{sym} (1)	*58.5 ± 10*	*57 ± 14*	*60 ± 15*
K_{sym} (2)	*-153 ± 24*	*-104 ± 194*	*-100 ± 100*
Q_{sym} (3)	*243 ± 38*		*100 ± 400*
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- **Semi-classical** approach : **extended Thomas-Fermi** Model with **Seyler-Blanchard** interaction
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* Extracted from Table VII *ibid* J. Margueron, R. Hoffmann Casali, and F. Gulminelli Phys. Rev. C **97**, 025805, 025806 (2018).

Results

Parameter (order)	Mean ± σ *GP* (MeV)	Mean ± σ *Exp.* * (MeV)	Mean ± σ SπRIT *2021* ** (MeV)
J_{sym} (0)	*32.5 ± 2*	*32.5 ± 2*	*35.3 ± 2.8*
L_{sym} (1)	*57 ± 14*	*60 ± 15*	*79.5 ± 38*
K_{sym} (2)	*-104 ± 194*	*-100 ± 100*	*47 ± 252*

NS Crust-Core transition	Mean ± σ *GP*	Mean ± σ *Lynch2022****
Density ρ_{cc} *(fm^{-3})*	*0.075 ± 0.007*	*0.069 ± 0.006*
Proton fraction Y_{cc}	*0.027 ± 0.005*	*0.021 ± 0.005*
Pressure P_{cc} *(MeV/fm^3)*	*0.32 ± 0.05*	*0.33 ± 0.05*

→ GPR results compatible with current (experimental) estimates

* Extracted from Table VII *ibid* J. Margueron, R. Hoffmann Casali, and F. Gulminelli Phys. Rev. C **97**, 025805, 025806 (2018).

** J. Estee *et al.*, Phys. Rev. Lett. **126**, 162701 (2021)

*** W. G. Lynch, and M. B. Tsang, Phys. Lett. B **830**, 137098 (2022)

Charge radii

From the *rms* proton density $< r_p^2 >$:

$$< r_{ch}^2 > \approx < r_p^2 > + < R_p^2 > + N/Z < R_n^2 >$$

+ *SO* $+3/4M^2$

with : $< R_p^2 >= 0.7080 \text{ fm}^2$

$< R_n^2 >= -0.117 \text{ fm}^2$

$\Delta r_{ch} = 0.029 \text{ fm}$ over *706* nuclei with *Z* = *8 - 96*

Spin-orbit contribution : *0 – 0.05 fm*

I. Angeli, and K.P. Marinova, Atomic Data and Nucl. Data Tables **99**, 69-95 (2013).

From in-source resonance-ion laser spectroscopy at *ISOLDE*:

SBTF

$< r_{ch}^2(^{202}Hg) >= 29.8870 \text{ fm}^2$ $\quad < r_{ch,exp}^2(^{202}Hg) >= 29.8592 \text{ fm}^2$

$< r_{ch}^2(^{206}Hg) >= 30.1829 \text{ fm}^2$ $\quad < r_{ch,exp}^2(^{206}Hg) >= 30.0712 \text{ fm}^2$

$< r_{ch}^2(^{208}Hg) >= 30.3326 \text{ fm}^2$ $\quad < r_{ch,exp}^2(^{208}Hg) >= 30.2862 \text{ fm}^2$

T. Day Goodacre, *et al.*, Phys. Rev. Lett. **126**, 032502 (2021).

Non-parametric regression: Gaussian Processes (GP)

Taking advantage of the experimental estimates from literature

→ *W. G. Lynch, and M. B. Tsang, Phys. Lett. B* ***830****, 137098 (2022)*

- **High density ρ / ρ_{sat} >1 : 2 mean values + 1σ uncertainties**

FOPI-LAND — P. Russotto *et al.*, *Phys. Lett. B* ***697****, 471 (2011)* : *elliptic flow*
ASY-EOS — P. Russotto *et al.*, *Phys. Rev. C* ***94****, 034608 (2016)* : *elliptic flow*
SπRIT — *J. Estee et al., Phys. Rev. Lett.* ***126****, 162701 (2021)* : *π^+/π^- ratio*

$$E_{sym}([1.45\pm0.2]\rho_{sat}) = 52 \pm 13\ \text{MeV},\ E_{sym}([2\pm0.25]\rho_{sat}) = 56 \pm 15\ \text{MeV}$$

- **Saturation density ρ / ρ_{sat} = 1 : 1 combined value + 1σ uncertainty**

J. Margueron, R. Hoffmann Casali, and F. Gulminelli, *Phys. Rev.* ***97****, 025806 (2018)*:
Large body of theoretical predictions based on nuclear masses

$$E_{sym}([1\pm0.1]\rho_{sat}) = 32.5 \pm 2\ \text{MeV}$$

- **Low density $\rho_{sat}/5 \leq \rho$ / ρ_{sat} <1 : 6 mean values + 1σ uncertainties**

P. Danielewicz and J. Lee, *Nucl. Phys. A* ***922****, 1 (2014)* : *Isobaric Analog States*
M. B. Tsang, *et al.*, *Phys. Rev. Lett.* ***102****, 122701 (2009)* : *Isospin diffusion*
B. A. Brown, *Phys. Rev. Lett.* ***111****, 232502 (2013)* : *GS for doubly magic nuclei*
Z. Zhang, and L. W. Chen, *Phys. Lett. B* ***726****, 234 (2013)* : *neutron skins*

$$E_{sym}([0.21\pm0.11]\rho_{sat}) = 10.1 \pm 1.0\ \text{MeV},\ E_{sym}([0.31\pm0.03]\rho_{sat}) = 15.9 \pm 1.0\ \text{MeV}$$
$$E_{sym}([0.43\pm0.05]\rho_{sat}) = 16.8 \pm 1.2\ \text{MeV},\ E_{sym}([0.63\pm0.03]\rho_{sat}) = 24.7 \pm 0.8\ \text{MeV}$$
$$E_{sym}([0.66\pm0.04]\rho_{sat}) = 25.5 \pm 1.1\ \text{MeV},\ E_{sym}([0.72\pm0.01]\rho_{sat}) = 25.4 \pm 1.1\ \text{MeV}$$

- **Limit boundary (no uncertainty) :** $E_{sym}(0) = 0 \pm 0$ MeV