# Studying the effects of the symmetry energy in hybrid stars

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CREDO THE QUEST FOR THE UNEXPECTED

PHAROS THE MULTI-MESSENGER PHYSICS AND ASTROPHYSICS OF NEUTRON STARS

compstar

# Outline

- A brief introduction to the physics of compact stars.
- The role of the symmetry energy in determination of the properties of compact stars.
- The symmetry energy in the framework of hybrid compact stars.

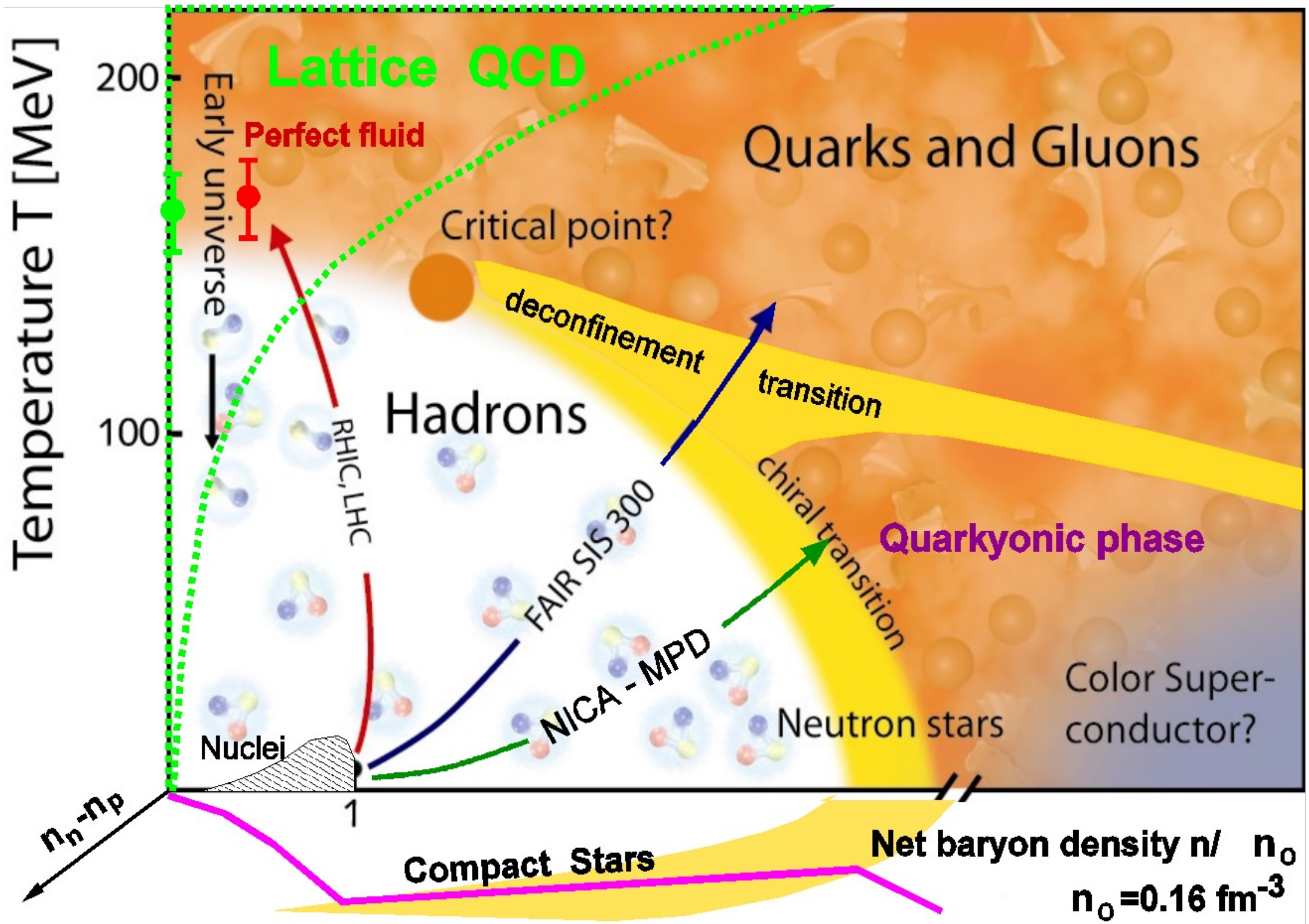
# Motivation

- New channels of multi-messenger observations like gravitational radiation from merger events of binary systems of compact stars or radio and X-ray signals from isolated pulsars allow to study their most basic structural properties like mass, radius, compactness, cooling rates and compressibility of their matter.

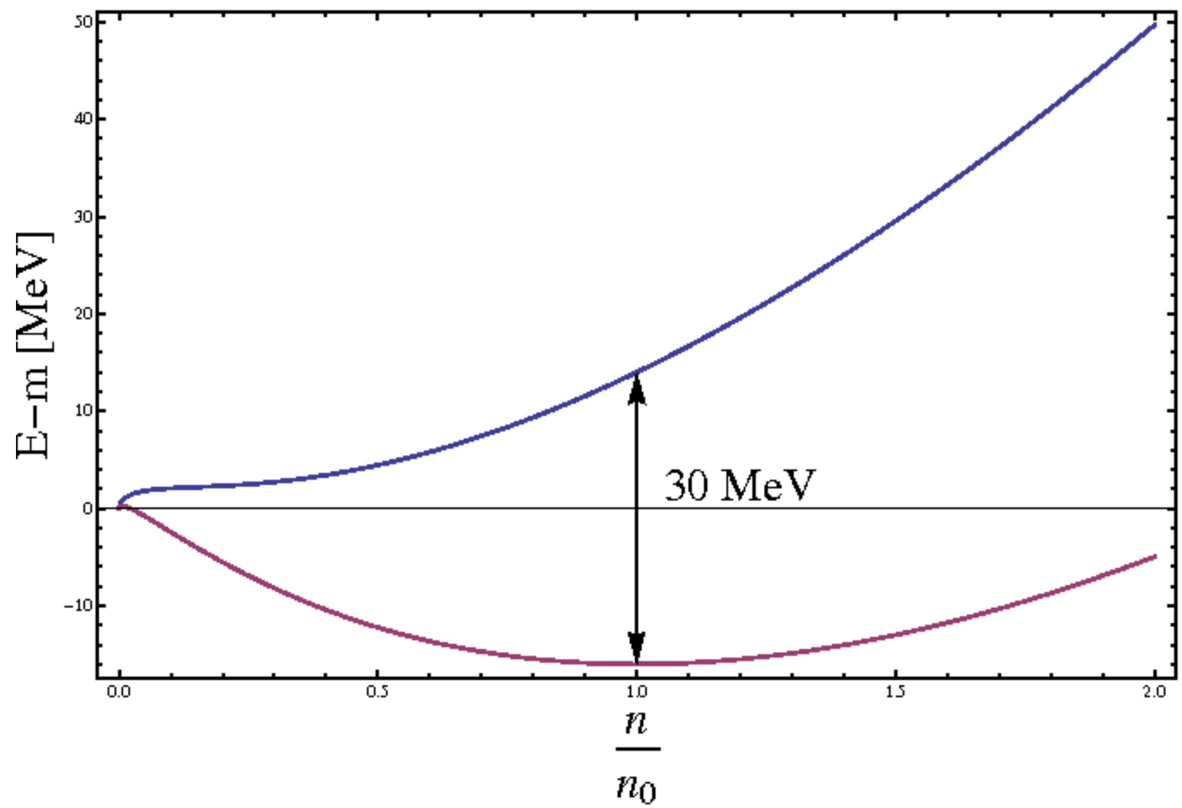
- Nuclear measurement and experiments have narrowed the Equation of State (EoS) uncertainty in the lowest to intermediate density range.

- The nuclear symmetry energy plays an important role in the neutron star structure and cooling rates that can be studied.

# Critical Endpoint in QCD



# Nuclear Symmetry Energy




is the difference between symmetric nuclear matter and pure neutron matter:

$$E(n,x) = E(n,x=1/2) + E_s(n) * \alpha^2(x) + E_q(n) * \alpha^4(x) + O(\alpha^6(x))$$

where $\alpha=1-2x$

# Neutron Star Equation of State

The energy per nucleon in neutron star core matter is given by:

$$\begin{aligned} E_{\rm tot}(n,\{x_i\}) &= E_{\rm b}(n,x_p) + E_{\rm lep}(n,x_e,x_\mu)\ , \\ E_{\rm b}(n,x_p) &= E_0(n) + S(n,x_p) \\ E_{\rm lep}(n,x_e,x_\mu) &= E_e(n,x_e) + E_\mu(n,x_\mu)\ , \end{aligned}$$

where $n = n_p + n_n$ is the total baryon density and $x_i = n_i/n$, $i = p, e, \mu$ are the fractions of protons, electrons and muons, respectively. The baryonic part is very well described by the parabolic approximation w.r.t. the asymmetry

$$\alpha \;=\; \frac{n_n - n_p}{n_n + n_p} = 1 - 2x_p,$$

resulting in $S(n,x_p) = (1-2x_p)^2 E_s(n)$. The leptonic contribution is a sum of the Fermi gas expressions for the contributing leptons $l = e, \mu$

$$E_l(n,x_l) = \frac{1}{n}\frac{p_{F,l}^4}{4\pi^2}\left[\sqrt{1+z_l^2}\left(1+\frac{z_l^2}{2}\right) - \frac{z_l^4}{2}\mathrm{Arsinh}\left(\frac{1}{z_l}\right)\right]\ ,$$

where $z_l = m_l/p_{F,l}$. For massless leptons ($z_l \to 0$), this expression goes over to

$$E_l(n,x_l)\big|_{m_l=0} = \frac{1}{n}\frac{p_{F,l}^4}{4\pi^2} = \frac{3}{4}\left(3\pi^2 n\right)^{1/3} x_l^{4/3}\ .$$

# Charge neutrality and β-equillibrium

Under neutron star conditions charge neutrality holds,

$$x_p = x_e + x_\mu \ .$$

The $\beta-$ equilibrium with respect to the weak interaction processes $n \to p + e^- + \bar{\nu}_e$ and $p + e^- \to n + \nu_e$ (and similar for muons), for cold neutron stars (temperature $T$ below the neutrino opacity criterion $T < T_\nu \sim 1$ MeV) implies

$$\mu_n - \mu_p = \mu_e = \mu_\mu \ .$$

The chemical potentials are defined as

$$\mu_i = \frac{\partial \varepsilon_i}{\partial n_i} = \frac{\partial}{\partial x_i} E_i(n, \{x_j\}) \ , \quad i, j = n, p, e, \mu \ ,$$

where $\varepsilon_i = n \ E_i(n, \{x_j\})$ is the partial energy density of species $i$ in the system. From the above equations:

$$\mu_e = 4(1 - 2x) E_s(n) \ .$$

Since electrons in neutron star interiors are ultrarelativistic,

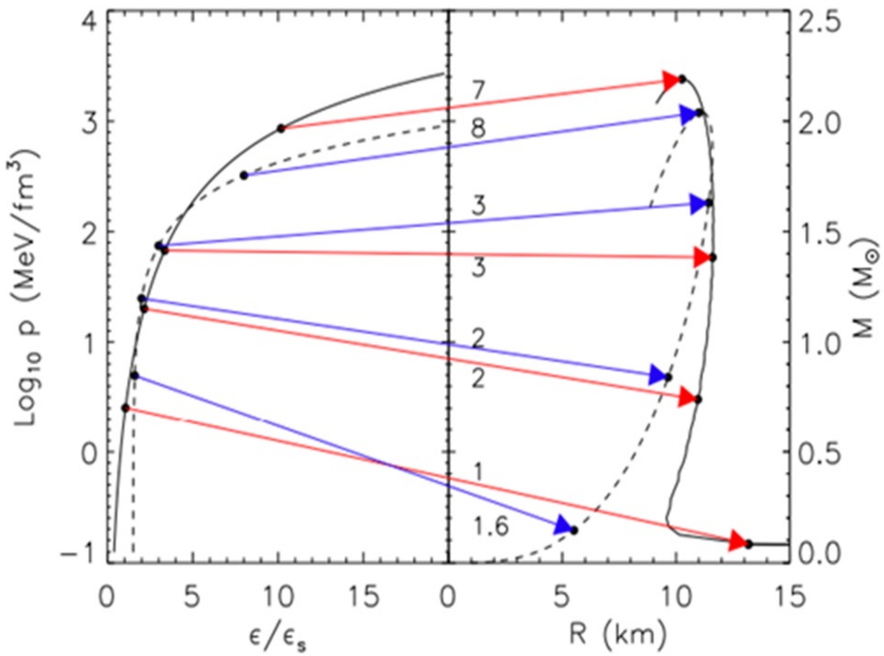
$$\mu_e = \sqrt{p_{F,e}^2 + m_e^2} \approx p_{F,e}, \text{ and } p_{F,e} = (3\pi^2 n_e)^{1/3} = (3\pi^2 n)^{1/3} (x - x_\mu)^{1/3} \ ,$$

$$\frac{x - x_\mu}{(1 - 2x)^3} = \frac{64 E_s^3(n)}{3\pi^2 n} \ , \qquad (x - x_\mu)^{2/3} - x_\mu^{2/3} = \frac{m_\mu^2}{(3\pi^2 n)^{2/3}} \ .$$

The total pressure is then given as $P(n) = n^2 \left( \frac{\partial E_{\rm tot}}{\partial n} \right) \ .$

# Compact Star Sequences

## (M-R ⇔ EoS)




James Lattimer,
Annu. Rev. Nucl. Part. Sci.
62, 485 (2012),
arXiv:1305.3510

- TOV Equations

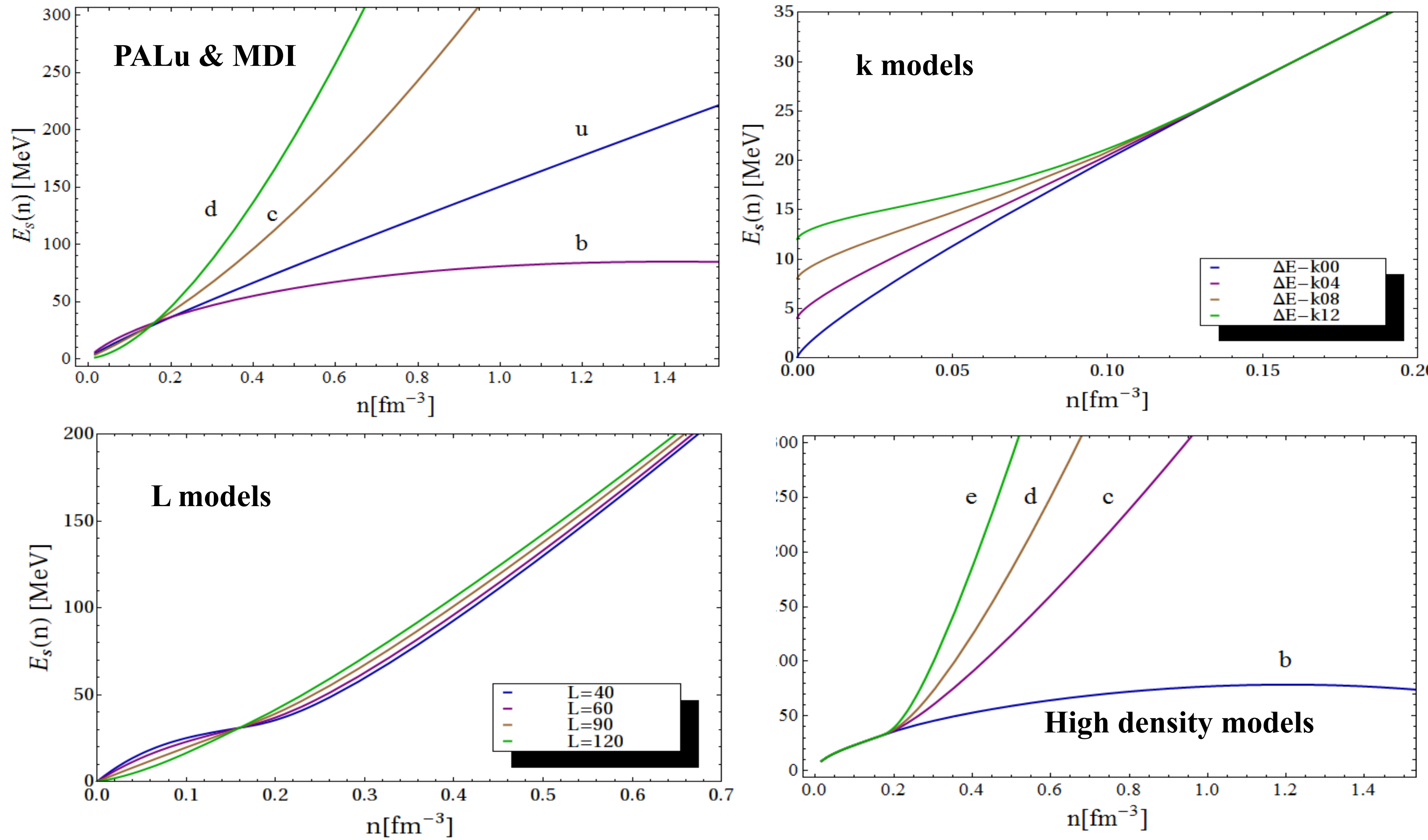
$$\frac{dp}{dr} = -\frac{(\varepsilon + p/c^2)G(m + 4\pi r^3 p/c^2)}{r^2(1 - 2Gm/rc^2)}$$

$$\frac{dm}{dr} = 4\pi r^2 \varepsilon$$

- Equation of State (EoS)

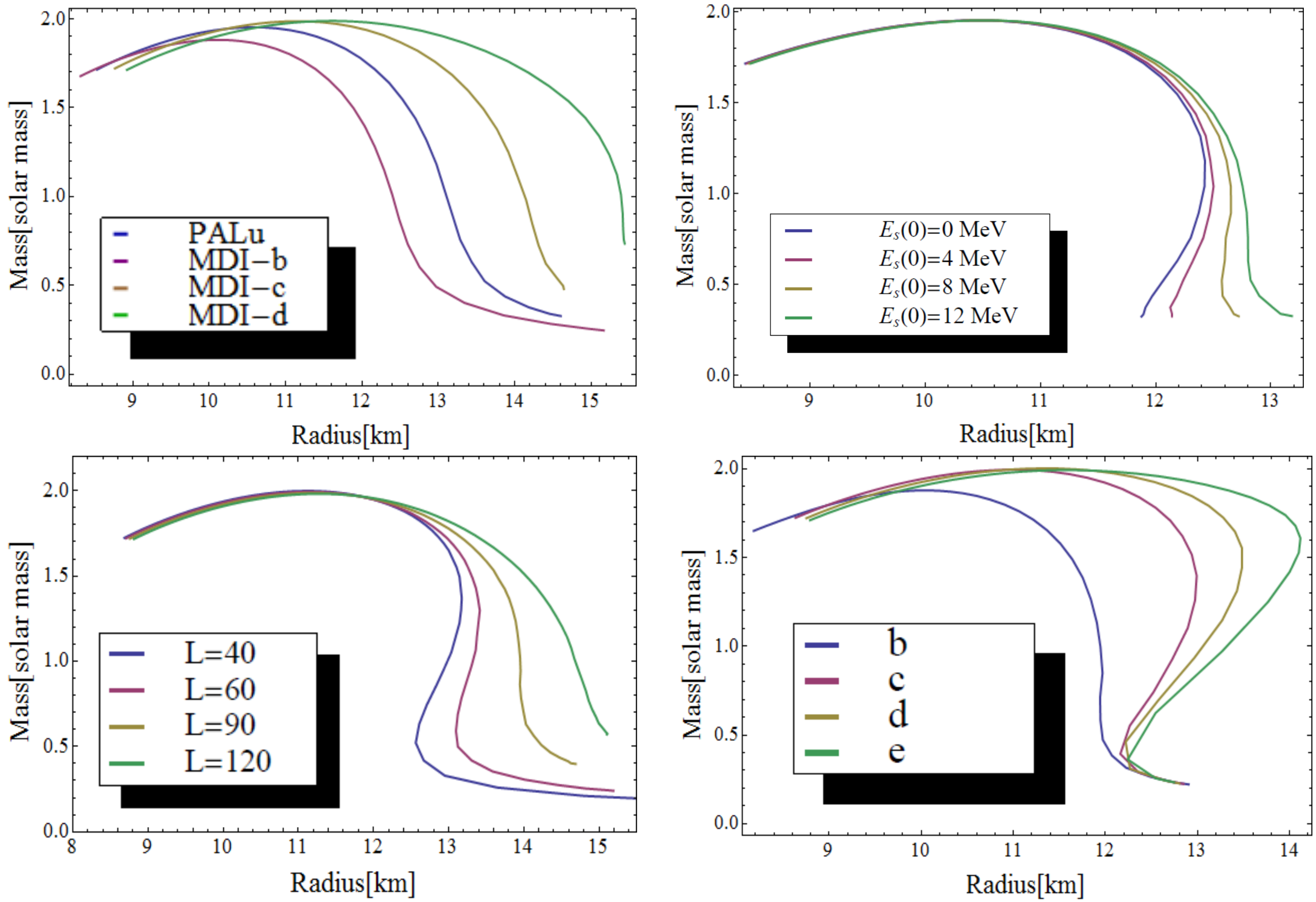
$$p(\varepsilon)$$

# Symmetry energy effects



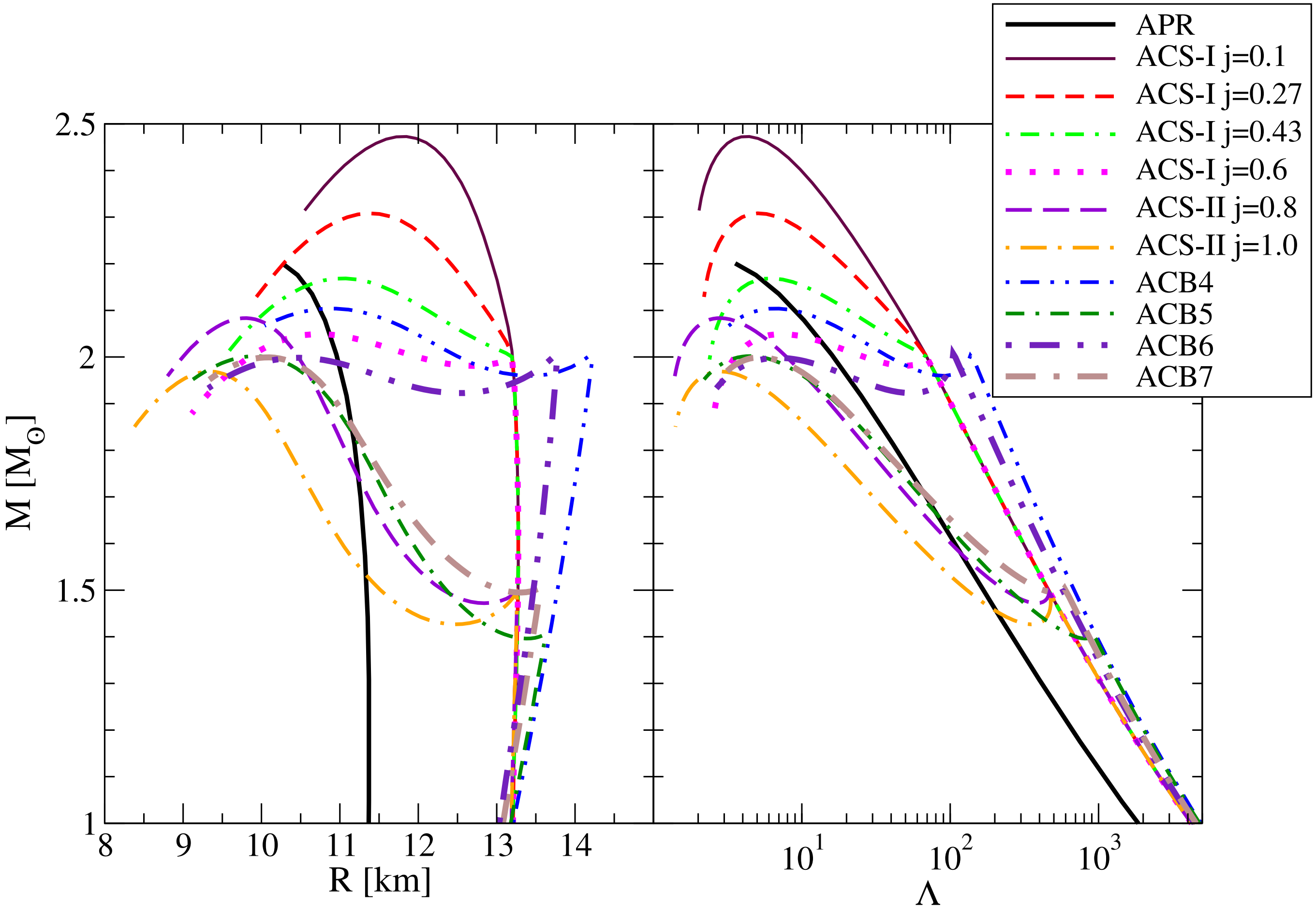

S. Kubis and D. E. Alvarez-Castillo - arXiv:1205.6368

# Symmetry energy effects



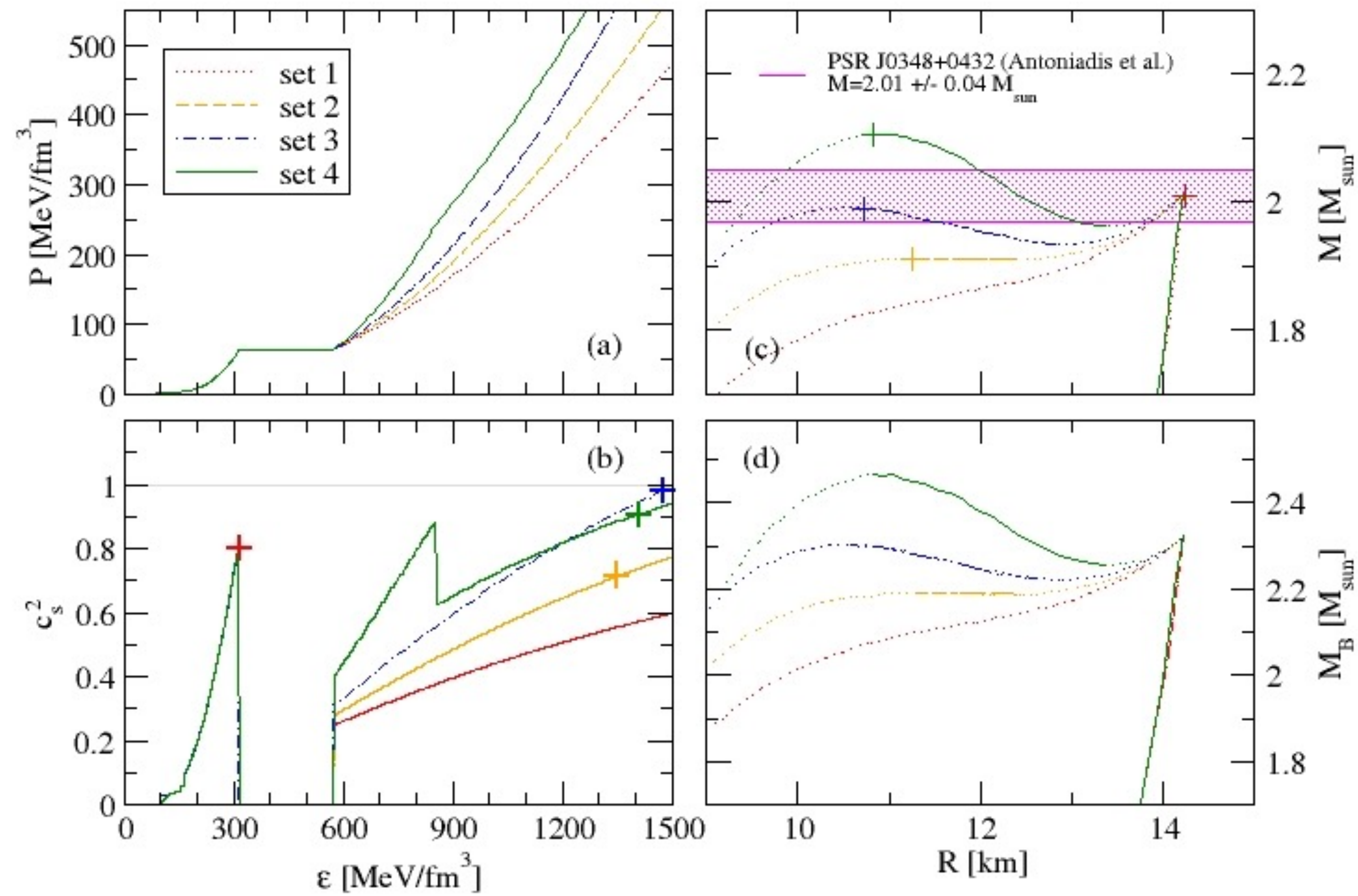

S. Kubis and D. E. Alvarez-Castillo - arXiv:1205.6368

# Hybrid compact stars



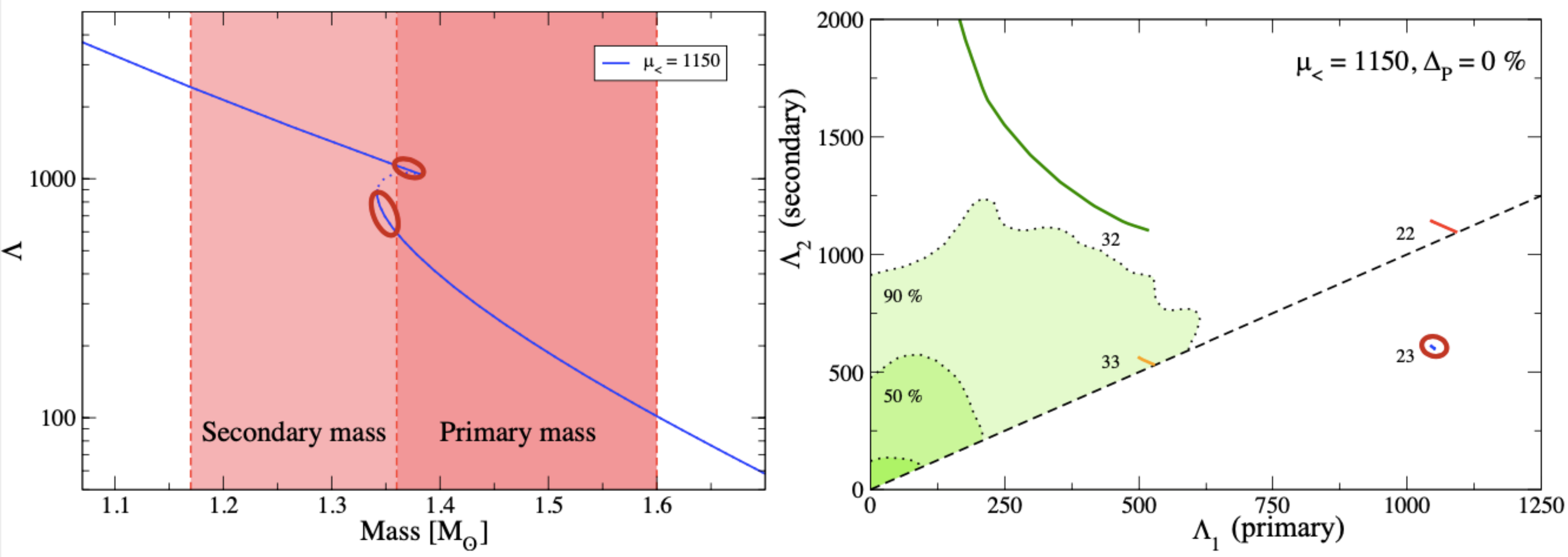

Vasileios Paschalidis, Kent Yagi, David Alvarez-Castillo,
David B. Blaschke, Armen Sedrakian
Phys. Rev. D 97, 084038 (2018), arXiv:1712.00451

# Compact Star Twins




Alvarez-Castillo, Blaschke (2017)
High mass twins from multi-polytrope equations of state
arXiv: 1703.02681v2, Phys. Rev. C 96, 045809 (2017)

# Was GW170817 a canonical neutron star merger?

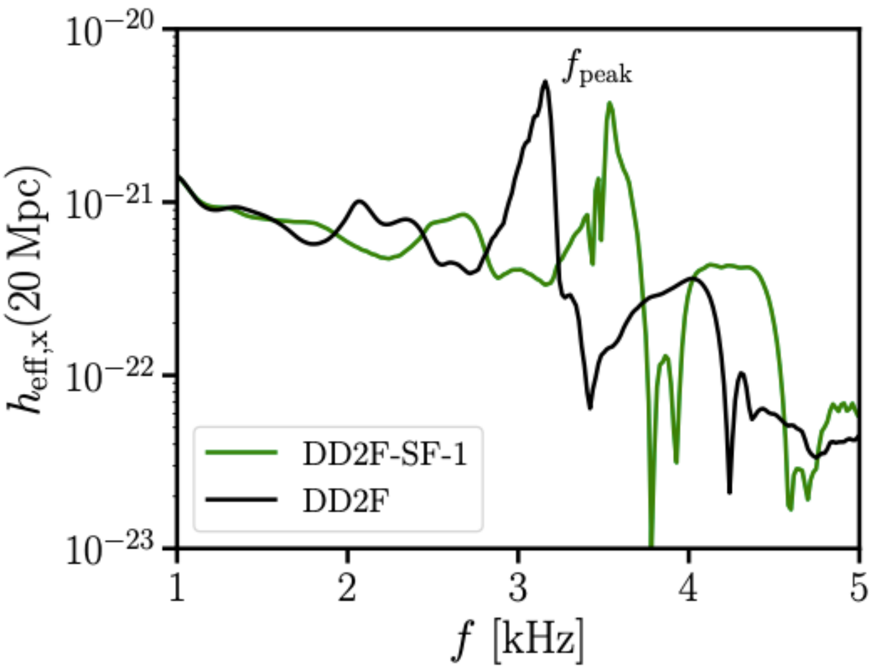


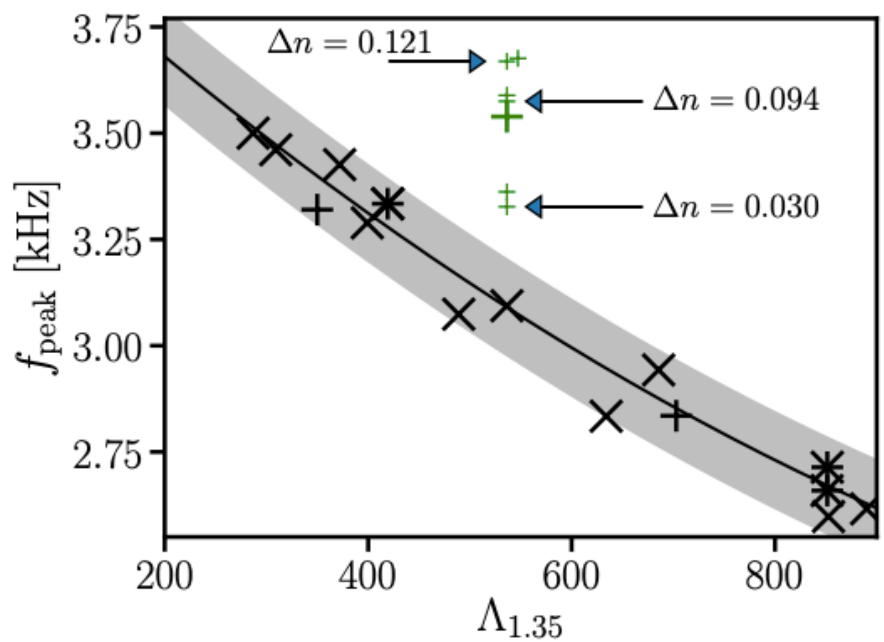

A. Ayriyan, D.Alvarez-Castillo, D. Blaschke and H. Grigorian,
Universe 6, 81 (2020)

D. Alvarez-Castillo, D. Blaschke, G. Grunfeld, V. Pagura
Phys. Rev. D 99, 063010 (2019) - arXiv: 1805.04105

# Gravitational Wave Signals

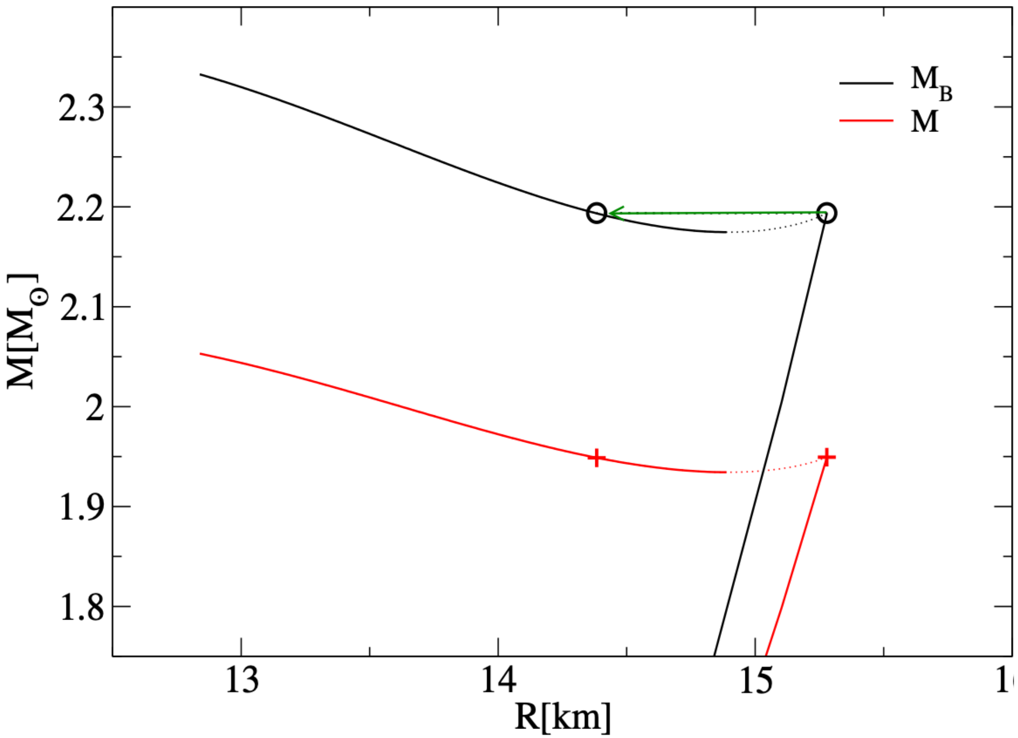
## First Order Phase Transitions

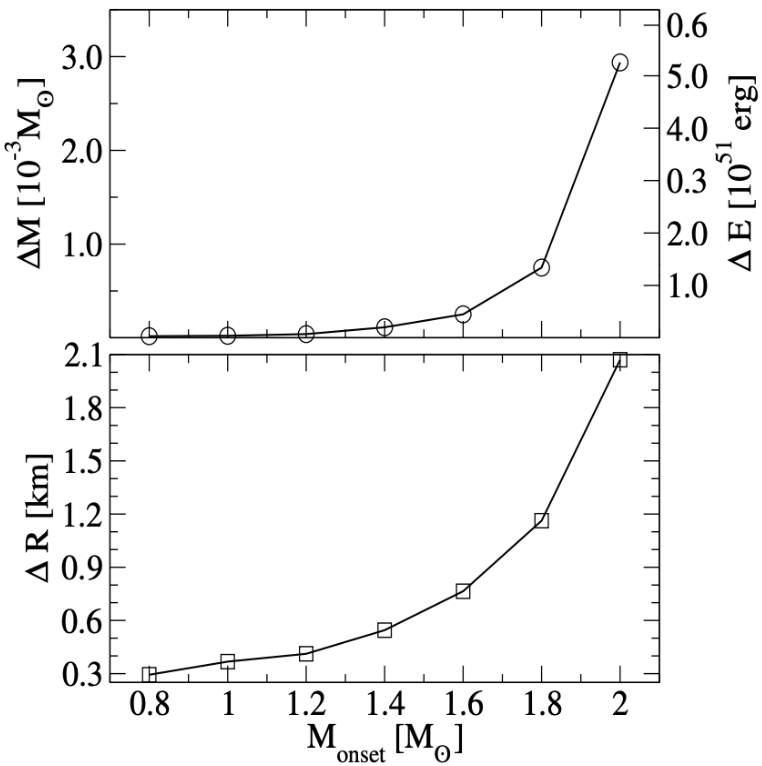



A. Bauswein et al. - arXiv: 1904.01306, PRL 122 (2019) 061102

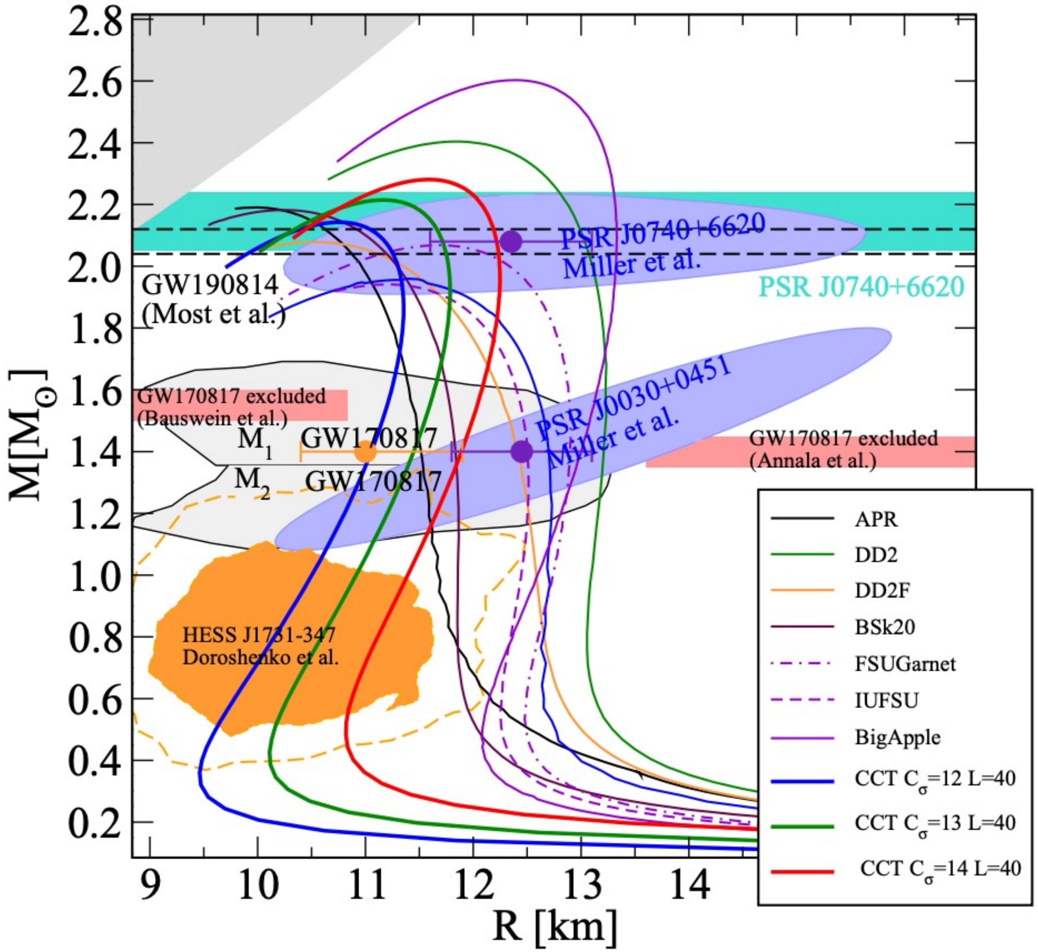
# Mass Twins – Energy Released




DD2MEV-CSS EoS, D. A-C, Astronomischen Nachrichten (2021) 1–6, arXiv: 2011.11145

# HESS J1731-347



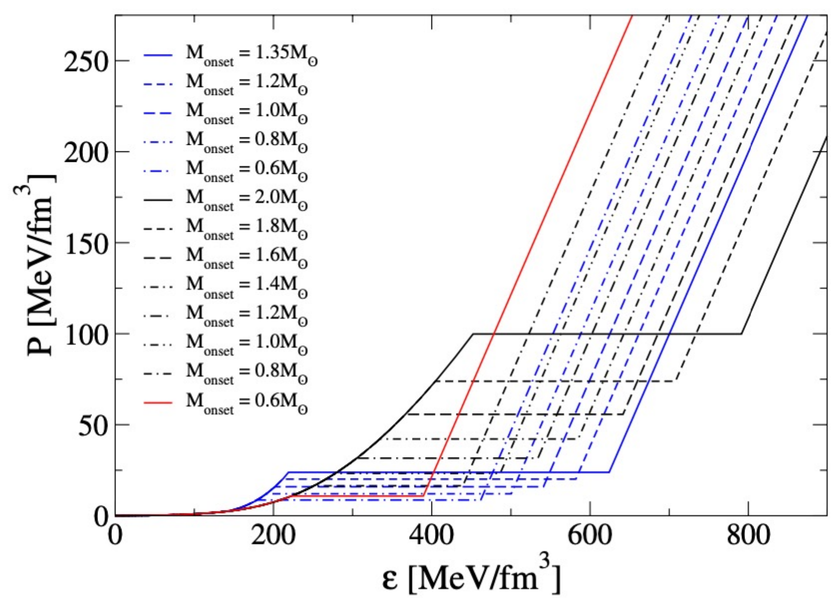

Relativistic mean field model for ultra-compact low mass neutron star of HESS J1731-347
Kubis et al. arXiv:2307.02979

# Compact Star Mass Twins

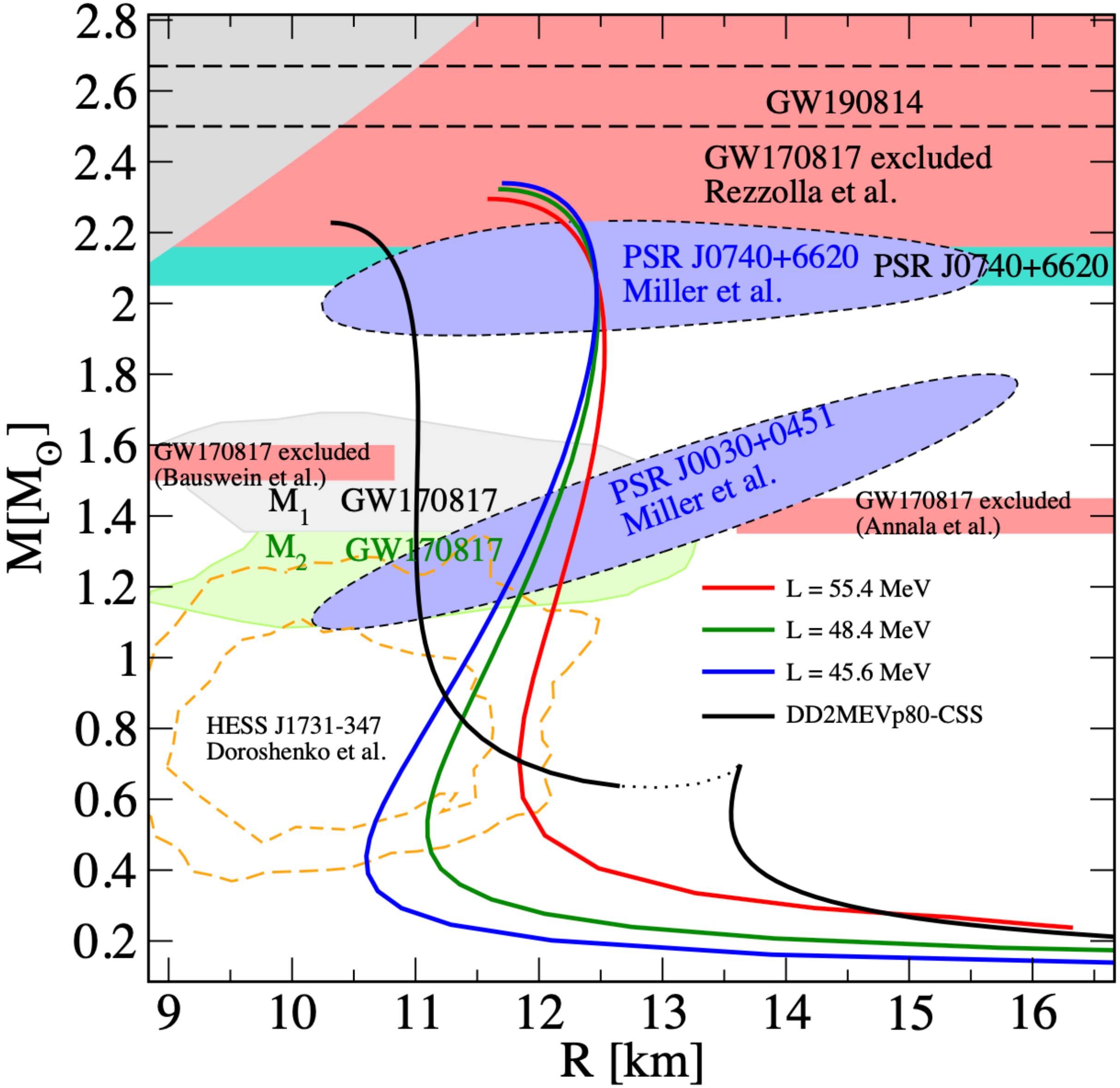
$$\varepsilon(p) = \begin{cases} \varepsilon_{\rm NM}(p) & p < p_{\rm trans} \\ \varepsilon_{\rm NM}(p_{\rm trans}) + \Delta\varepsilon + c_{\rm QM}^{-2}(p - p_{\rm trans}) & p > p_{\rm trans} \end{cases}$$

| Model | $M_{onset}$ $[M_\odot]$ | $n_{trans}$ $[1/\mathrm{fm}^3]$ | $\varepsilon_{trans}$ $[\mathrm{MeV/fm}^3]$ | $p_{trans}$ $[\mathrm{MeV/fm}^3]$ | $\Delta\varepsilon$ $[\mathrm{MeV/fm}^3]$ | $c_{QM}$ $[c]$ |
|---|---|---|---|---|---|---|
| DD2p80 | 0.7 | 0.193 | 185.223 | 10.3131 | 268.573 | 0.9 |

$$\frac{\Delta\varepsilon_{\rm crit}}{\varepsilon_{\rm trans}} = \frac{1}{2} + \frac{3}{2}\frac{p_{\rm trans}}{\varepsilon_{\rm trans}}$$




D. Alvarez-Castillo - Astron. Nachr. 1–6 (2021)



2.8
2.6
2.4
2.2
2
1.8
1.6
1.4
1.2
1
0.8
0.6
0.4
0.2
M[M⊙]
9
10
11
12
13
14
15
16
R [km]
GW190814
GW170817 excluded
Rezzolla et al.
PSR J0740+6620
Miller et al.
PSR J0740+6620
PSR J0030+0451
Miller et al.
GW170817 excluded
(Bauswein et al.)
M1 GW170817
M2 GW170817
GW170817 excluded
(Annala et al.)
HESS J1731-347
Doroshenko et al.
L = 55.4 MeV
L = 48.4 MeV
L = 45.6 MeV
DD2MEVp80-CSS

# CRUST-CORE Transition

The shear modulus in the crust is given by

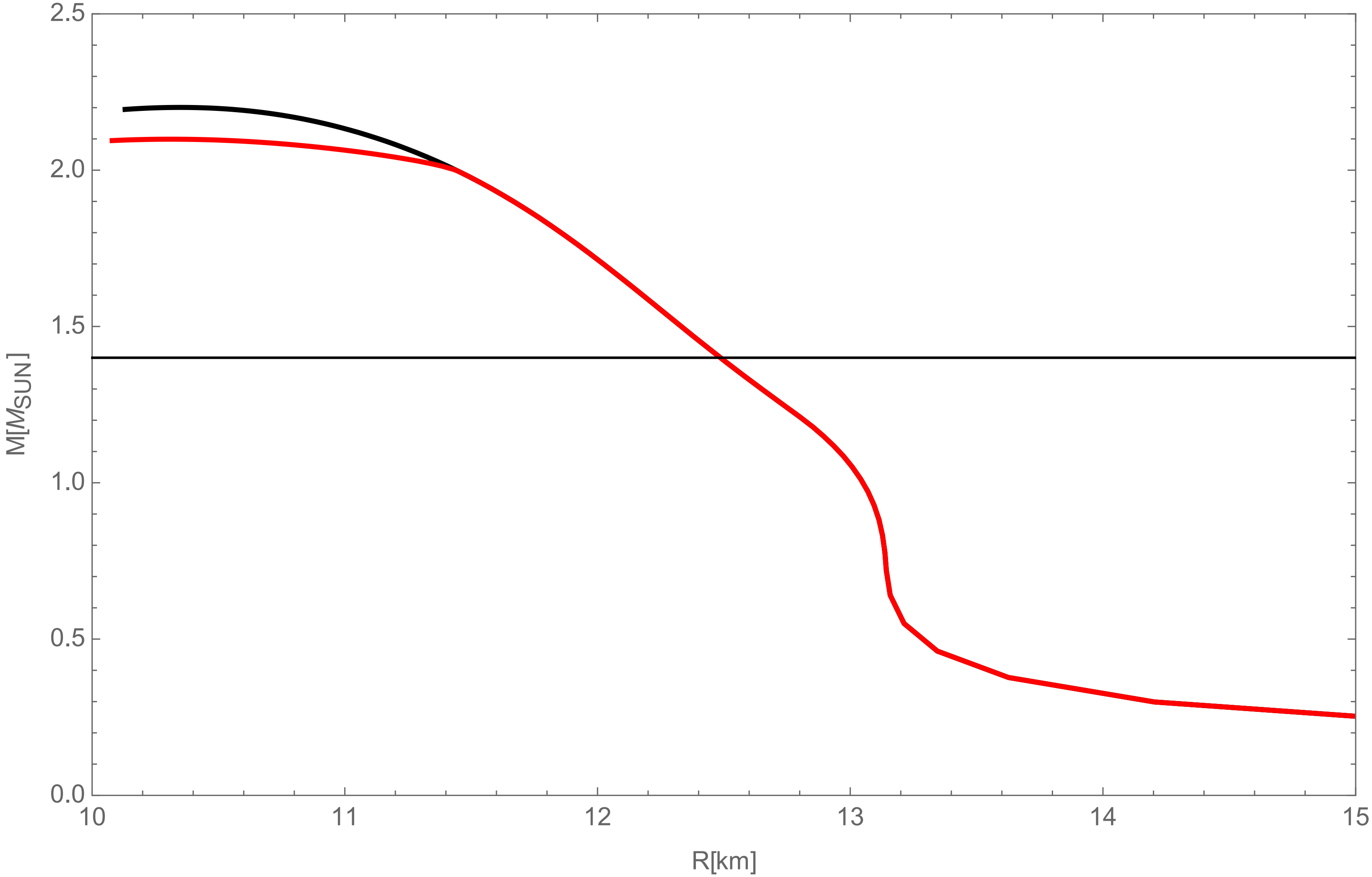
$$\mu = \frac{0.1106}{1 + 17810\left(\frac{ak_bT}{(Ze)^2}\right)^2} \frac{n_i\,(Ze)^2}{a}, \tag{5}$$

where $T$ is the temperature, $n_i$ is the ion density, $Z$ is the proton number of the nuclei, and

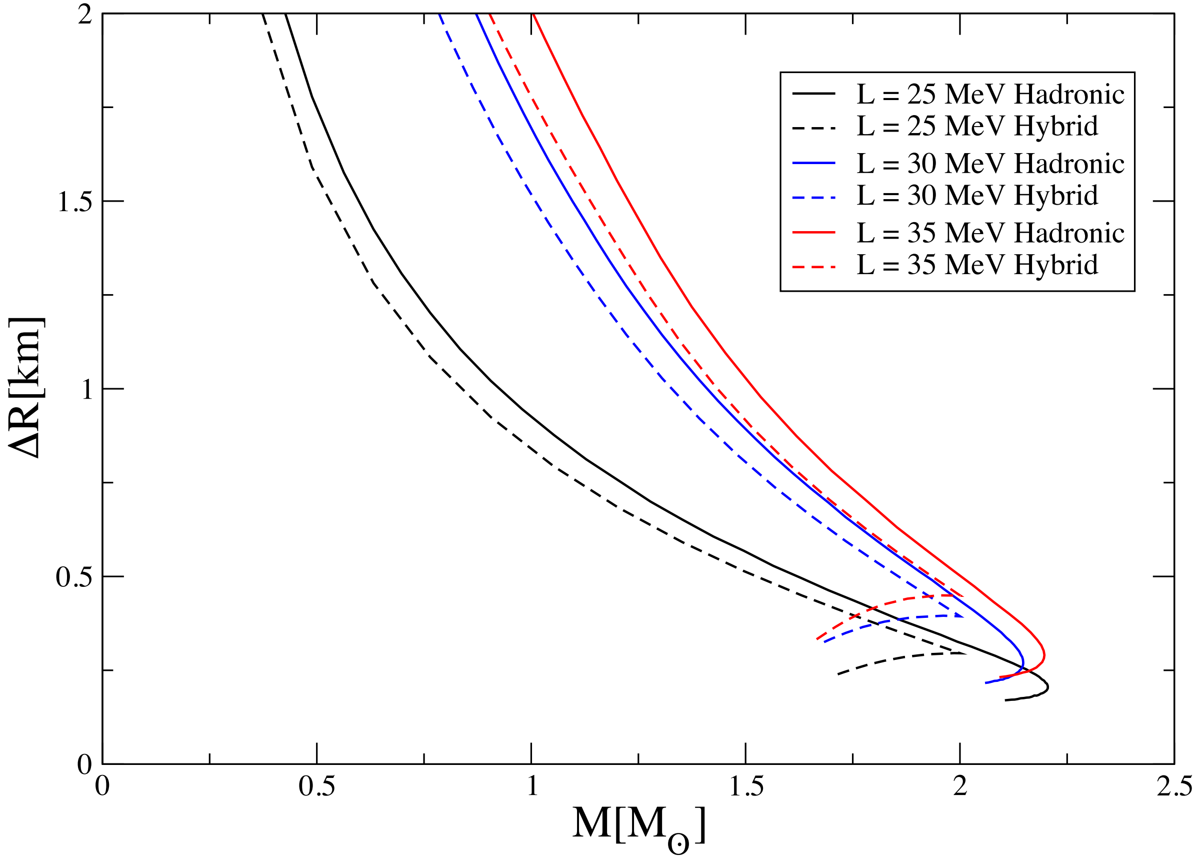
$$a = \left(\frac{3}{4\pi n_i}\right)^{\frac{1}{3}}. \tag{6}$$

Resonant Shattering Flares as Multimessenger Probes of the Nuclear Symmetry Energy
Duncan Neill, William G. Newton, David Tsang, MNRAS Volume 504, Issue 1 (2021)
arXiv: 2012.10322

# Hybrid and Hadronic Stars

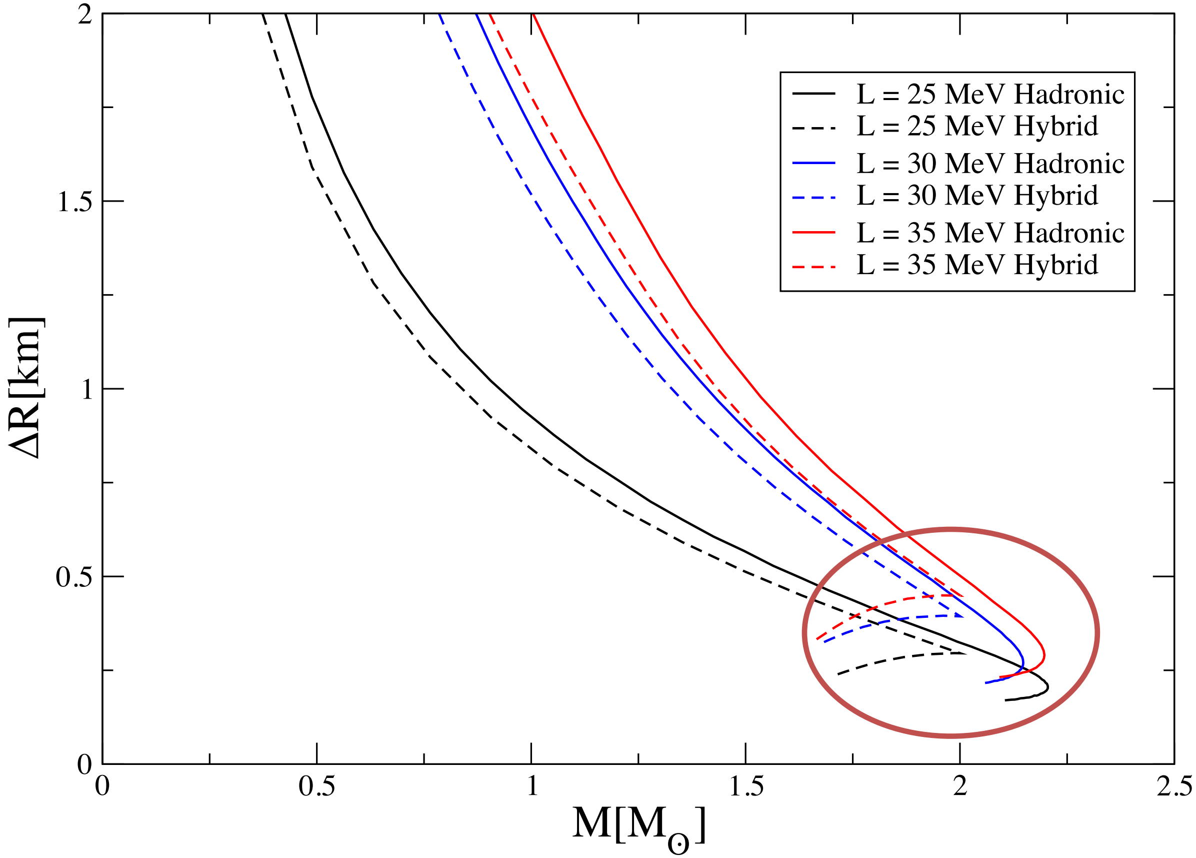


# CS Crust Thickness



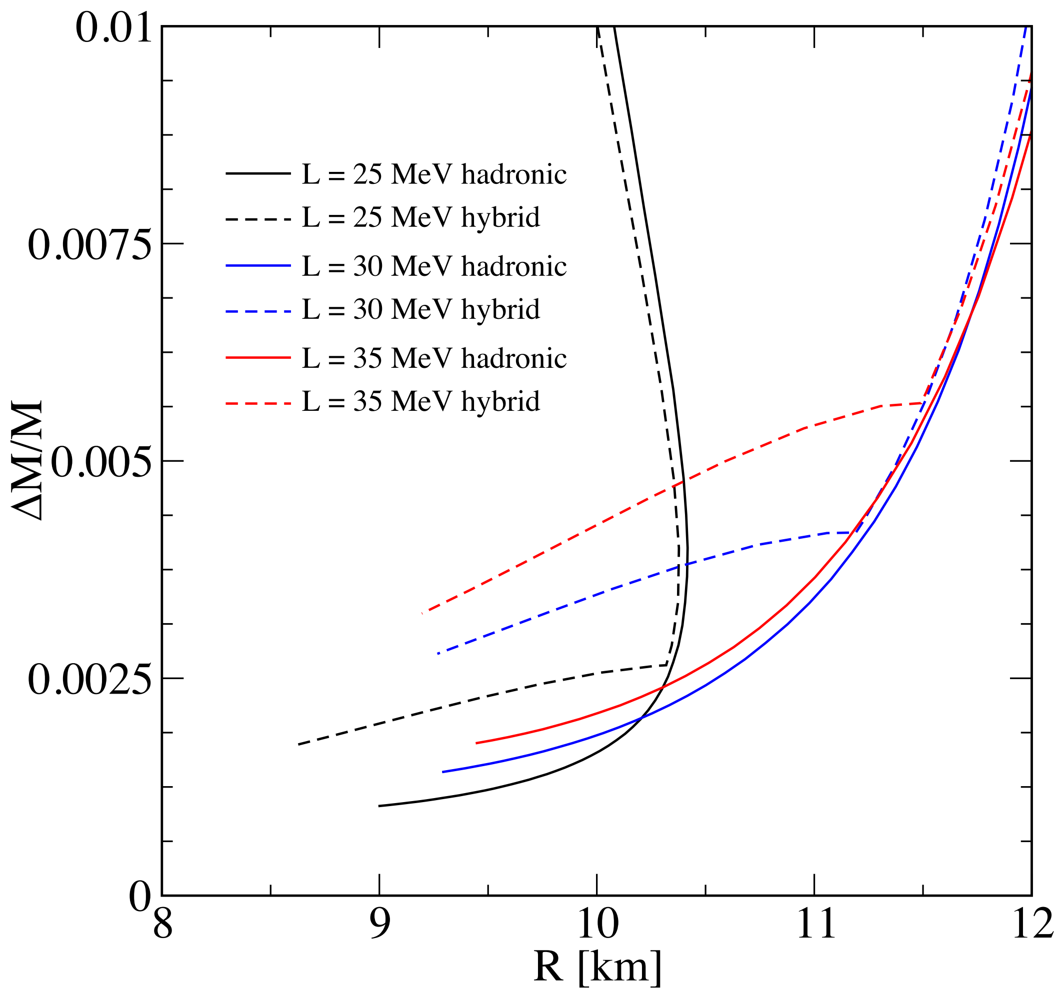

*Hybrid stars within the CSS model with $c_s$=1

# CS Crust Thickness




*Hybrid stars within the CSS model with $c_s=1$

# Fractional Mass of the CS Crust




*Hybrid stars within the CSS model with $c_s$=1

# Outlook

- Collect multi-messenger astronomy and collider experiments results to probe the properties of dense matter.
- Employ Bayesian Analysis and Machine Learning methods in order to estimate unknown physical parameters of the EoS.
- Study and detect CS excitations, see for instance: arXiv: 2309.08775
- Employ the universal I-Love-Q relations to study CS crusts.
- Study finite temperature effects relevant for CS mergers.
- **Key question**: is it possible to study the crust of neutron stars in order to determine whether they are are hybrid stars or not?