Bayesian inference of the dense matter equation of state built within mean field models

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Overview

- NS EOS: "Traditional" and statistical approaches
- Bayesian Inferences
- Our aims:
  - the model dependence of NS EOS
  - the role of accounting for **correlations among** (E/A_i) **in PNM**
- Procedure: confront the predictions of two **phenomenological** models subjected to **the same minimal** set of constraint
- Results: posterior PDF and correlations; **role of** m_{eff}
- Conclusions

NS EOS: "traditional" approaches

NS properties depend on 1D EOS, $P(e)$

▷ Schematic parametrizations: piece-wise polytrops; param. of the speed of sound

✓ computationally cheap, flexible
✗ no composition info, no physical pinning, no composition, no use in other circumstances

▷ From an (effective) interaction: phenomenological (relativistic and non-relativistic mean field; various interactions); ab initio (variational, quantum Monte Carlo, coupled cluster expansion, diagrammatic, Brueckner Hartree-Fock, χEFT)

✓ physical underpinning, composition, generalisation to other particle species
? validity away from initial conditions; compliance with causality and astrophys. measurements

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[Dutra+, PhysRevC (2012); PhysRevC (2014)] [Oertel+, RMP (2017)] [Burgio+, ProgrPartNuclPhys (2021)]

NS EOS - Bayesian inferences (I)

- availability of astrophys. constraints ($M_{\max}$, Λ, joint $M-R$ measurements)
- large size of model spaces

require **statistical tools**

Bayes theorem: $P(\Theta|\mathrm{D}) = \mathscr{L}(\mathrm{D}|\Theta) P(\Theta)/\mathcal{Z}$,

Θ: the set of model params, i.e., model (schematic, phenomenological) + priors on parameters

D: the fit data, i.e., constraints (NM param., E/A or P in neutron matter as computed by ab initio, astro. meas.)

$\mathcal{Z}$ = the evidence

Results: Posterior PDF for Θ, $\xi_i(\Theta)$

[Antoniadis+, Science (2013)] [Arzoumanian+, ApJSS (2018)] [Cromartie+, Nature (2020)] [Fonseca+, ApJL (2021)] [Abbott+, PRL (2017)] [Abbott+, PRX (2019)] [Miller+, ApJL (2019)] [Riley+, ApJL (2019)] [Miller+, ApJL (2021)] [Riley+, ApJL (2021)] [Vinciguerra+, arXiv:2308.09469] [Doroshenko+, Nature (2022)]

NS EOS - Bayesian inferences (II)

EOS models: schematic, phenomenological

Perspective: from NS to NM; from NM to NS

Constraints: empirical NM params., microscopic calculations of PNM, heavy ion data, astrophy. observations

Conclusions:

- dependence on the EOS model;
- sensitivity of posterior distrib. to prior distrib.;
- narrowing down of the parameter space upon progressive incorporation of constr.;
- tension between constraints

[Lim & Holt, EPJA (2019)] [Raaijmakers+ (2019)] [Guven+, PhysRevC (2020)] [Raaijmakers+, ApJ (2021)]

[Miller+, ApJL (2021)] [Malik+, ApJ (2022)] [Malik+, PhysRevC (2023)] [Beznogov & Raduta, PhysRevC (2023)]

Our aims

(1) build EOS models based on phenomenological mean field models

(2) study the model dependence of EOS models in Bayesian approaches

(3) study the role of correlations among E/A in PNM at different densities

(4) study the link between density dependence of PNM and NS matter

Setup

Models:

M1. non-relativistic mean-field model of nuclear matter with Skyrme effective interactions,

M2. covariant density functional model with (simplified) density dependent couplings,

Minimal set of constraints:

C1. four best known nuclear empirical parameters (NEP): $n_{\rm sat}$, $E_{\rm sat}$, $K_{\rm sat}$, $J_{\rm sym}$,

C2. $(E/A)(n)$ in pure neutron matter (PNM), as computed by χEFT,

C3. $M_{\rm max}/M_{\odot} \geq 2$,

C4. for non-relativistic mean-field model: causality for $n \leq n_{\rm c}^{*}$

Results:

R1. posterior PDF of eff. int. parameters, NEP, global NS properties ($M_{\rm max}$, $R_{1.4}$, $R_{2.0}$, $\Lambda_{1.4}$, $\Lambda_{2.0}$, M_{DU}, c_s^{2*}, n_c^*, P_c^*),

R2. correlations

[Beznogov & Raduta, PhysRevC (2023)] [Beznogov & Raduta, arXiv:2308.1535]

Mean field model (I): non-relativistic with Skyrme eff. int.

Hamiltonian: $\mathscr{H} = k + h_{int}$; $\quad k = \frac{\hbar^2}{2m}\tau$

Effective interaction: $V(\mathrm{r}_1, \mathrm{r}_2) = t_0(1 + x_0 P_\sigma)\delta(\mathrm{r}) + \frac{t_1}{2}(1 + x_1 P_\sigma)\left[\mathrm{k}'^2\delta(\mathrm{r}) + \delta(\mathrm{r})\mathrm{k}^2\right]$
$+ t_2(1 + x_2 P_\sigma)\mathrm{k}' \cdot \delta(\mathrm{r})\mathrm{k} + \frac{t_3}{6}(1 + x_3 P_\sigma)[n(\mathrm{R})]^\sigma \delta(\mathrm{r}),$

Int. energ. dens.: $h_{int} = h_0 + h_3 + h_{\mathrm{eff}}$, with $h_0 = C_0 n^2 + D_0 n_3^2$, $h_3 = C_3 n^{\sigma+2} + D_3 n^\sigma n_3^2$,
$h_{\mathrm{eff}} = C_{\mathrm{eff}} n\tau + D_{\mathrm{eff}} n_3 \tau_3$,

Depends on 7 parameters: σ, C_0, D_0, C_3, D_3, C_{eff}, D_{eff}, or, alternatively,
n_{sat}, E_{sat}, J_{sym}, D_3, C_{eff}, D_{eff}, σ.

Analytic expressions for all thermo quantities, including NEPs:

$$X_{\mathrm{sat}}^{(i)} = \left(\partial^i E_0(n_B, 0)/\partial \chi^i\right)\Big|_{n=n_{\mathrm{sat}}}, \quad X_{\mathrm{sym};\,k}^{(j)} = \left(\partial^j E_{\mathrm{sym};\,k}(n_B, 0)/\partial \chi^j\right)\Big|_{n=n_{\mathrm{sat}}},$$

with $\chi = (n_B - n_{\mathrm{sat}})/3n_{\mathrm{sat}}$, $\delta = n_3/n$

Mean field model (II): relativistic with density dep. couplings

Lagrangian: $\mathscr{L}$

Interactions via exchange of: σ, ω, ρ mesons.

Density-dependent couplings: $\Gamma_M(n) = \Gamma_{M,0} h_M(x)$, $\quad x = n/n_{\rm sat}$, $\quad$ with

$$h_M(x) = \exp[-(x^{a_M} - 1)],\ M = \sigma,\ \omega; \quad h_\varrho(x) = \exp[-a_\varrho(x-1)]$$

Energy density: $e = \frac{1}{\pi^2} \sum_{B=n,p} e_{kin;B} + \frac{1}{2} m_\sigma^2 \sigma^2 + \frac{1}{2} m_\omega^2 \omega^2 + \frac{1}{2} m_\varrho^2 (\varrho_3)^2$,

<u>6 parameters:</u> Γ_σ, Γ_ω, Γ_ρ, a_σ, a_ω, a_ρ

[Typel & Wolter, NuclPhysA (1999)] [Malik+, ApJ (2022)]

Constraints

		Skyrme				RMF	
Quantity	Units	Value	Std. deviation	Ref.	Value	Std. deviation	Ref.
n_{sat}	fm^{-3}	0.16	0.004	[1]	0.153	0.005	[4]
E_{sat}	MeV	−15.9	0.2	[1]	−16.1	0.2	[5]
K_{sat}	MeV	240	30	[1]	230	40	[6,7]
J_{sym}	MeV	30.8	1.6	[1]	32.5	1.8	[8]
$(E/A)_1$	MeV	9.212	0.226	[2]	9.50	0.52	[9]
$(E/A)_2$	MeV	12.356	0.512	[2]	12.68	1.20	[9]
$(E/A)_3$	MeV	15.877	0.872	[2]	16.31	2.13	[9]
M_G^*	$M_\odot$	> 2.0	—	[3]	> 2.0	—	[3]
c_s^{*2}	c^2	< 1	—	—	—	—	—

NM parameters; astro. constr. and causality.
density behavior of PNM as predicted by χEFT;
1, 2, 3: $n_B = 0.08,\ 0.12\ , 0.16\ \mathrm{fm}^{-3}$

[1] Margueron+, PhysRevC (2018); [2] Somasundaram+, PhysRevC (2021); [3] Fonseca+, ApJL (2021); [4] Typel & Wolter, NuclPhysA (1999); [5] Dutra+, PhysRevC (2014); [6] Todd-Rutel & Piekarezicz, PhysRevLett (2005); [7] Shlomo+, EPJA (2006); [8] Essik+, PhysRevC (2021); [9] Hebeler+, ApJ (2013);

Priors

Priors: uniform (uninformative) distributions

Skyrme: $n_{\rm sat}$, $E_{\rm sat}$, $J_{\rm sym}$, D_3, $C_{\rm eff}$, $D_{\rm eff}$, σ $\leftarrow$ mixed eff. int. and NM parameters
RMF: Γ_σ, Γ_ω, Γ_ρ, a_σ, a_ω, a_ρ $\leftarrow$ eff. int. parameters

For domains, see [Beznogov & Raduta, PhysRevC (2023); arXiv:2308.1535],
M. Beznogov's poster & talk

Likelihood

$$\log \mathscr{L}_q \propto -\chi_q^2 = -\sum_{n=1}^{N} \chi_n^2 - \sum_{m=1}^{M} \chi_m^{'2} - \sum_{p=1}^{P} \chi_p^{''2},$$

1) Uncorrelated obs., e.g., $n_{\rm sat}$, $E_{\rm sat}$, $K_{\rm sat}$, $L_{\rm sym}$:

$$-\chi_n^2 = -\frac{1}{2}\left(\frac{d_i - \xi_i(\Theta)}{\mathcal{I}_i}\right)^2$$

2) Correlated obs., e.g., the values that E/A in PNM takes at various n_i,

$$-\chi_m^{'2} = -\chi_m^2 - \frac{1}{2}\sum_{i=1}^{3}\sum_{j=1}^{3}\left(\mathrm{cov}^{-1}\right)_{ij}\delta\mathscr{E}_i\delta\mathscr{E}_j,$$

[Somasundaram+, PhysRevC (2021)]

3) "Hard-wall", e.g., $M_{\rm G}^*$, c_s^{*2},

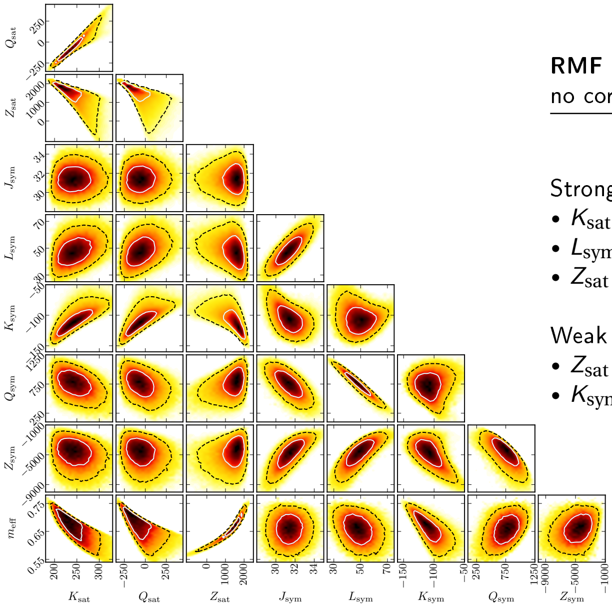
$$-\chi_p^{''2} = -10^{10}, \text{ if } M_{\rm G}^*/M_\odot < 2 \text{ or } c_s^{*2}/c^2 \geq 1$$

see M. Beznogov's poster & talk

Results

- EOS of nuclear matter & NS
  - posterior PDF for nuclear matter ($n_{\rm sat}$, $X^{(i)}_{\rm sat}$, $X^{(j)}_{\rm sym}$, $m_{\rm eff}$)
  - posterior PDF for NS properties (M^*_{max}, $R_{1.4}$, $R_{2.0}$, $\Lambda_{1.4}$, $\Lambda_{2.0}$, M_{DU})
  - **correlations among NM params.; among params. of MN and properties of NS**
- model dependence
- correlations between $(E/A)_i$ in PNM; strong constraints on NS EOS

NM parameters: Correlations (I)



RMF

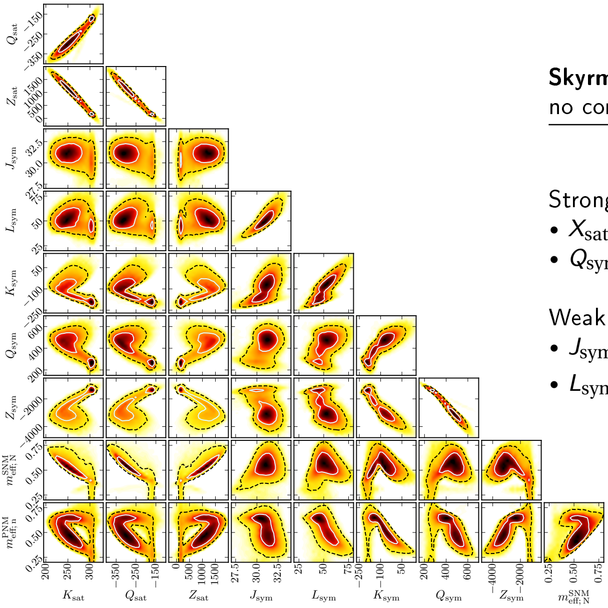
no correl. between $(E/A)_i$ in PNM

Strong correlations:

- K_{sat} and Q_{sat}
- L_{sym} and Q_{sym}
- Z_{sat} and m_{eff}

Weak correlations:

- Z_{sat} with K_{sat}, Q_{sat}
- K_{sym} and X_{sat}, with $X = K, Q, Z$



Skyrme

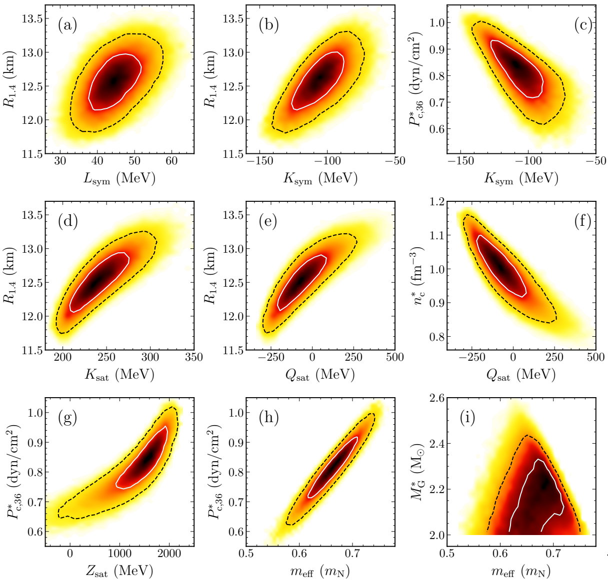
no correl. between $(E/A)_i$ in PNM

Strong correlations:

- X_{sat} and Y_{sat}, $X, Y = K, Q, Z$
- Q_{sym} and Z_{sym}

Weak correlations:

- J_{sym} and L_{sym}
- L_{sym} and K_{sym}; K_{sym} and Q_{sym}

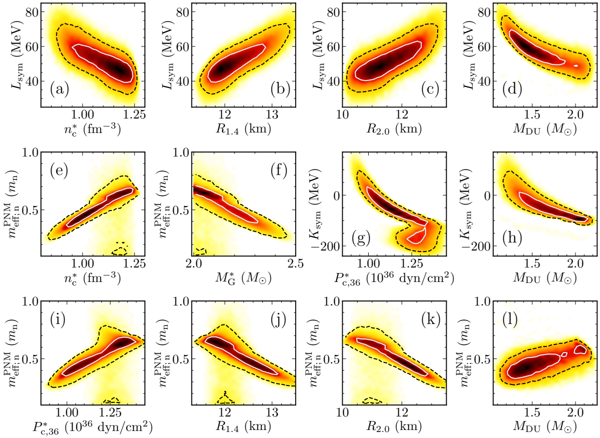


RMF

- L_{sym}, K_{sym} are weakly correl. with $R_{1.4}$
- K_{sat}, Q_{sat} are weakly correl. with $R_{1.4}$
- K_{sym}, Z_{sat} are weakly correl. with P^*_c
- m_{eff} is strongly corr. with P^*_c

no correl. $(E/A)_i$ in PNM

NM parameters vs. NS properties (II)



no correl. $(E/A)_i$ **in PNM**

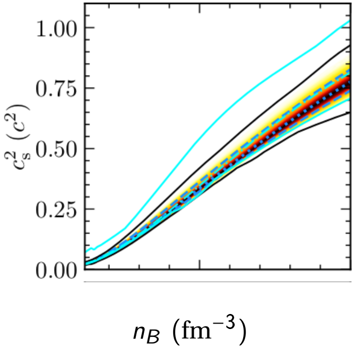
Skyrme

- L_{sym}, K_{sym} are correl. with $R_{1.4}$, $R_{2.0}$, M_{DU}
- L_{sym} is correl. with n_c^*
- m_{eff} is correl. with n_c^*, P_c^*, M_G^*, $R_{1.4}$, $R_{2.0}$, M_{DU}

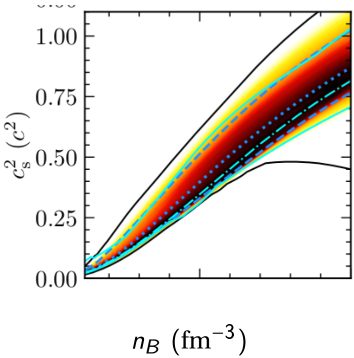
Correlations between $(E/A)_i$: $c_S^2(n_B)$ in NS

Skyrme

with corr.



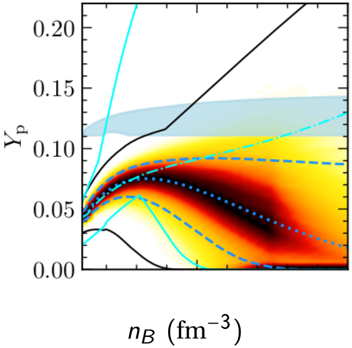
no correl. among $(E/A)_i$ in PNM



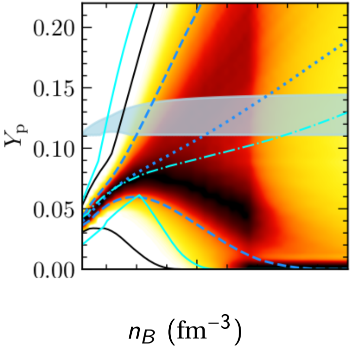
- accounting for correl. further constraints the EOS

Skyrme

with corr.



no correl. among $(E/A)_i$ in PNM

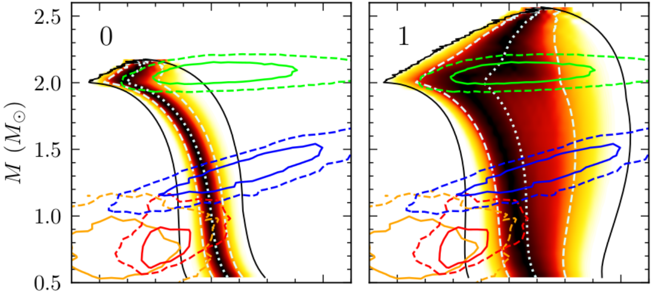


- accounting for correl. further constraints the EOS

Correlations between $(E/A)_i$: M-R

Skyrme

with corr. | no correl. among $(E/A)_i$ in PNM

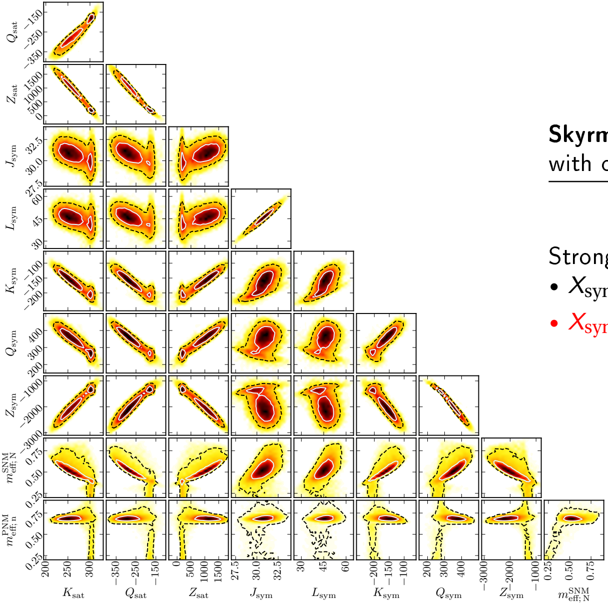


R (km) | R (km)

- accounting for correl. further constraints the EOS

green: PSR J0740+6620 [Miller+, ApJL (2021)]; blue: PSR J0030+0451 [Miller+, ApJL (2019)];

red, orange: HESS J1731-34 [Doroshenko+, Nature (2022)]



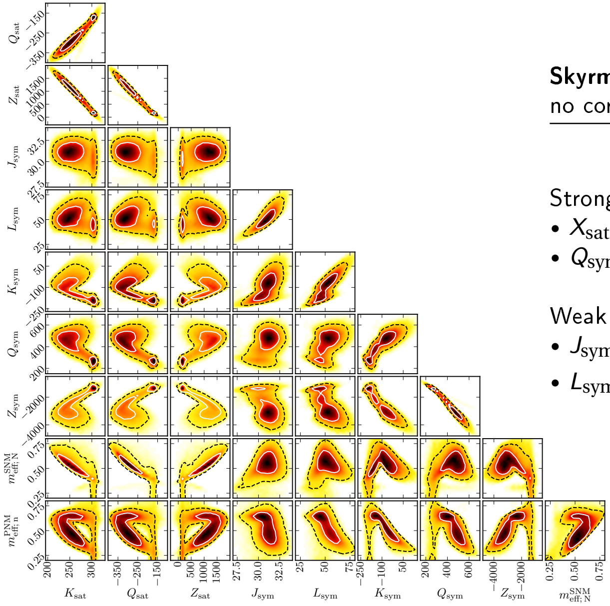
Skyrme

with correl. between $(E/A)_i$ in PNM

Strong correlations:

- X_{sym} with Y_{sym}, $X, Y = J, L, K, Q, Z$
- X_{sym} with Y_{sat}, $X, Y = K, Q, Z$

NM parameters: Correlations (II)



Skyrme

no correl. between $(E/A)_i$ in PNM

Strong correlations:

- X_{sat} and Y_{sat}, $X, Y = K, Q, Z$
- Q_{sym} and Z_{sym}

Weak correlations:

- J_{sym} and L_{sym}
- L_{sym} and K_{sym}; K_{sym} and Q_{sym}

Conclusions

- Bayesian inference of dense matter EOS derived within
  - Non-rel. mean field approach with Skyrme eff. int.
  - RMF model with density dep. couplings

- minimally constrained
  - four NEP: $n_{\rm sat}$, $E_{\rm sat}$, $K_{\rm sat}$, $J_{\rm sym}$
  - $E/A(n)$ in PNM as predicted by χEFT
  - $M_{\rm max}/M_{\odot} \geq 2$, $c_s^2/c^2 \leq 1$

- posterior PDF of NEP and their correlations depend on the model; on the constraints

- NS prop. show correlations with Landau/Dirac effective masses

- accounting for correl. among E/A in PNM adds constraints

[Beznogov & Raduta, PhysRevC (2023)] [Beznogov & Raduta, arXiv:2308.1535]