Nuclear EOS for arbitrary proton fraction and temperature based on chiral EFT and a Gaussian Process Emulator

Kai Hebeler

Darmstadt, September 21, 2023

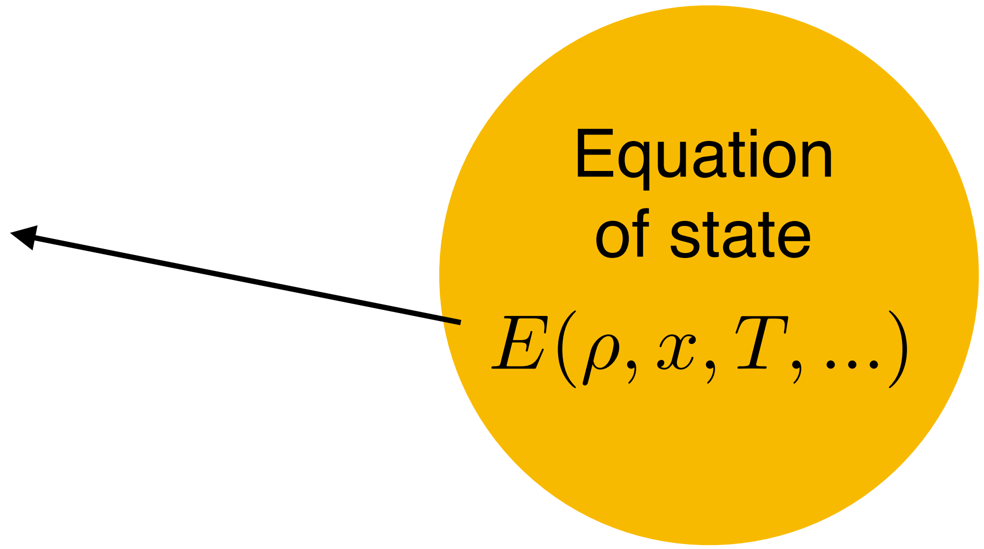
NuSym23:
XIth International Symposium on Nuclear Symmetry Energy

SFB 1245

TECHNISCHE UNIVERSITÄT DARMSTADT

The goal

Related thermodynamic observables: pressure, free energy, speed of sound, chemical potentials, …



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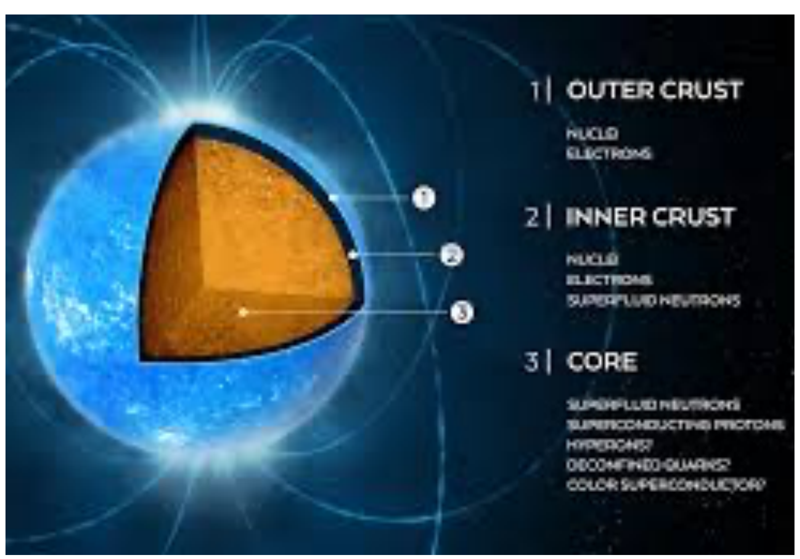
Equation of state

$$E(\rho, x, T, ...)$$

density



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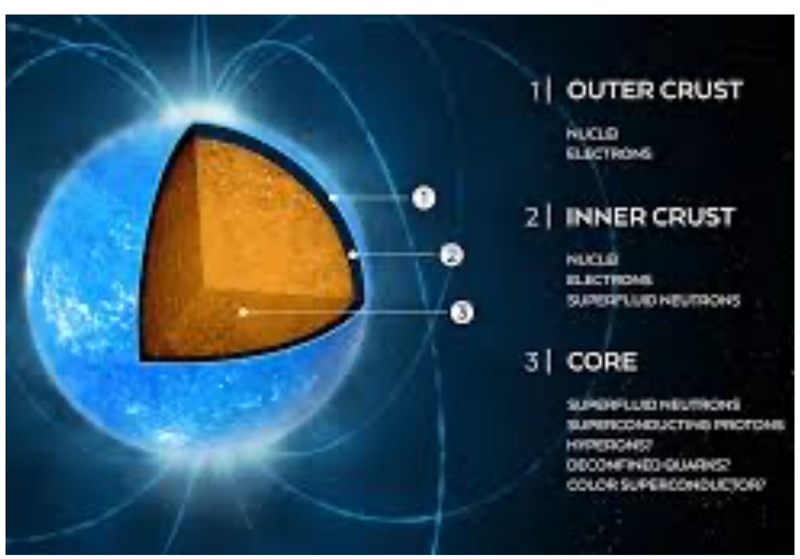

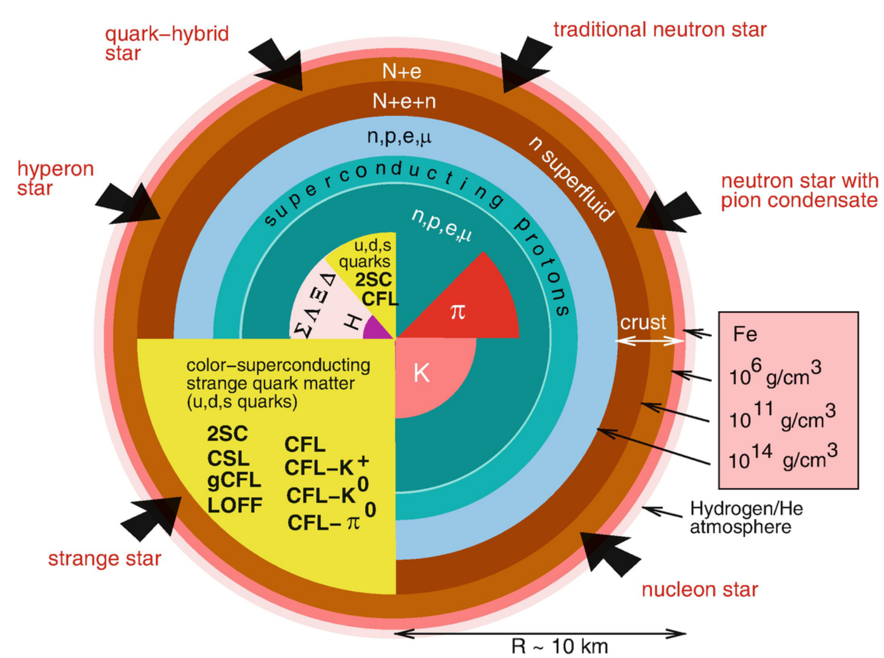

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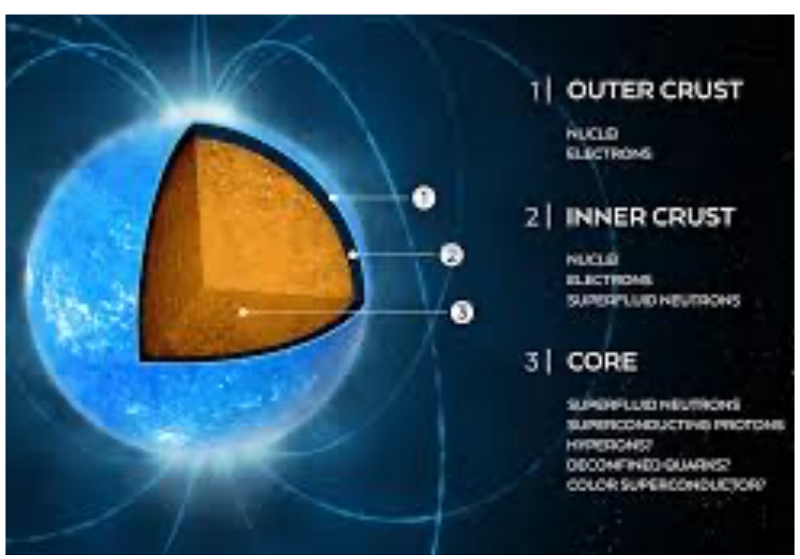

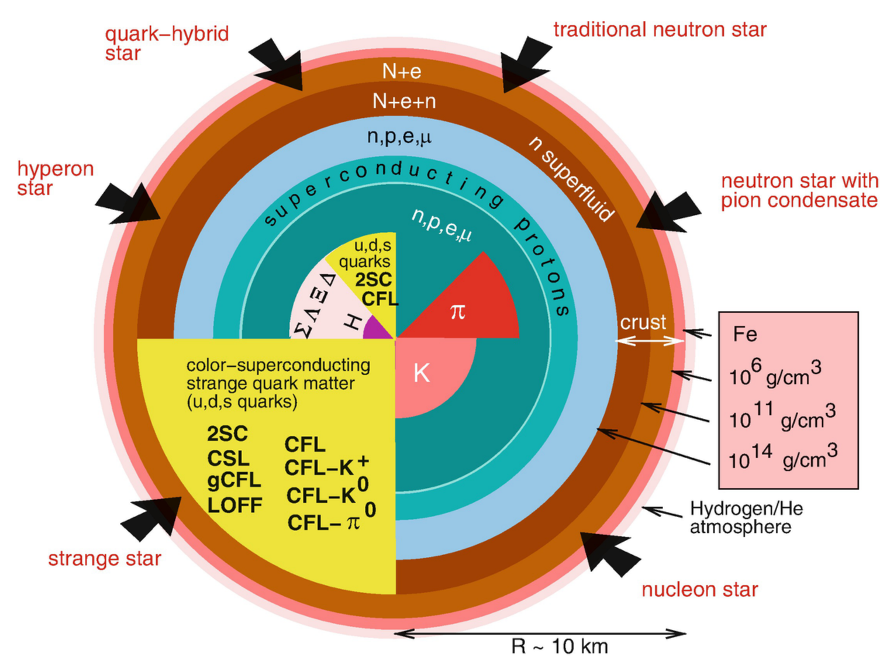

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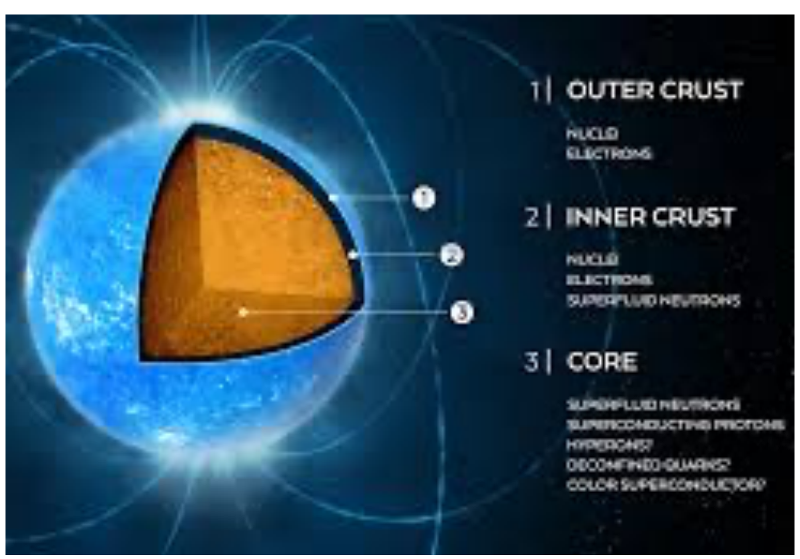
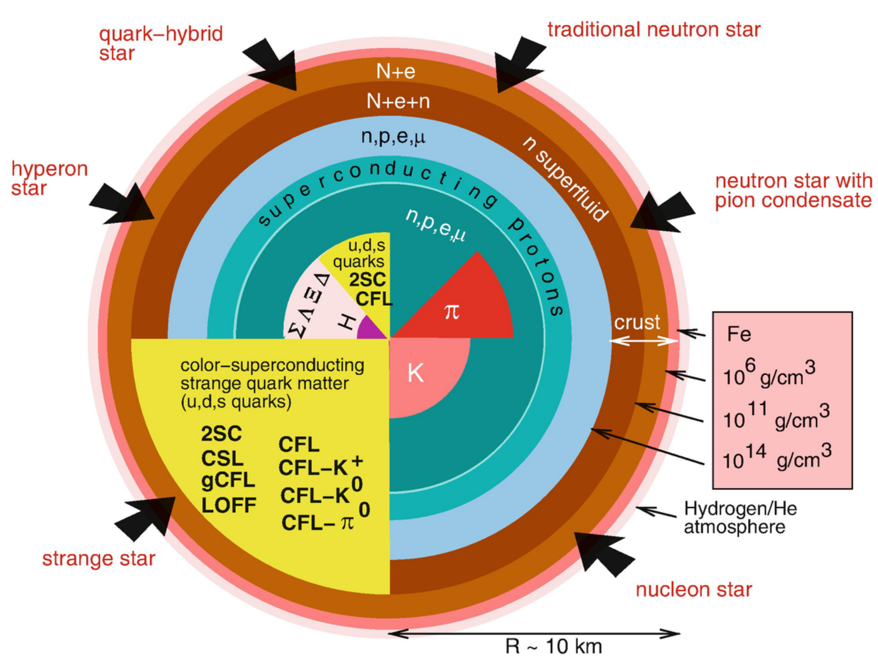
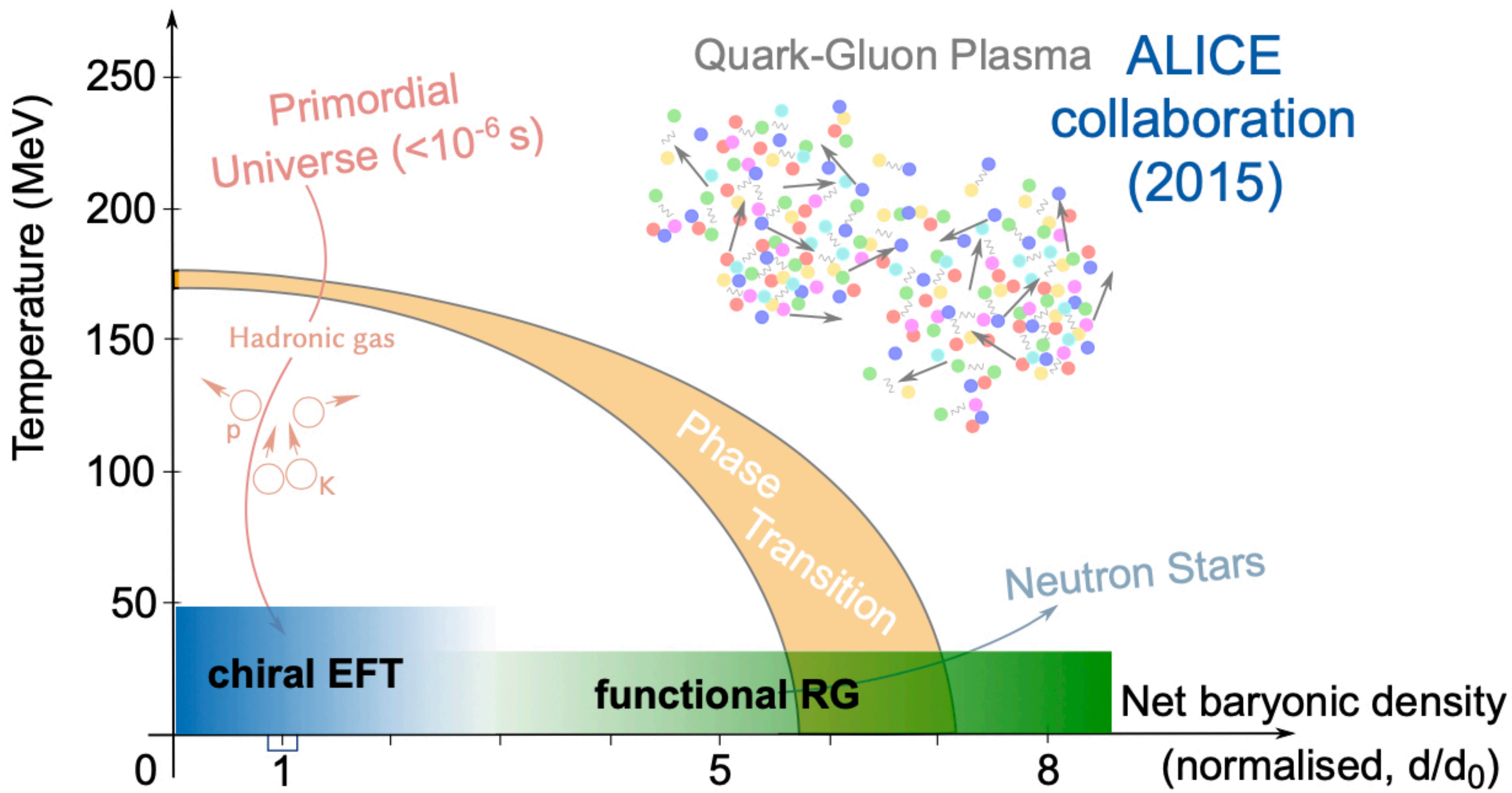
Parameter space should ideally cover full range of values relevant for astrophysical applications





The equation of state of high-density matter: Microscopic approaches



perturbative QCD

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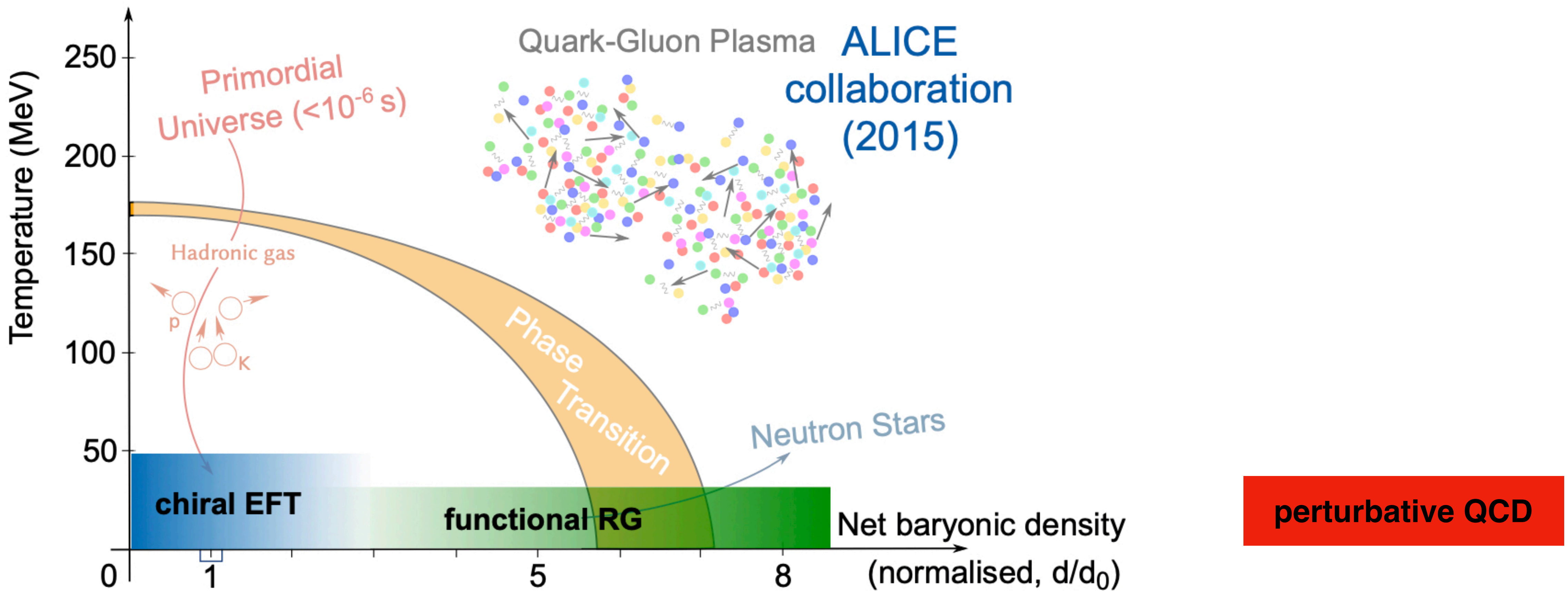

- Calculations based on chiral effective field theory interactions

  main focus of this talk

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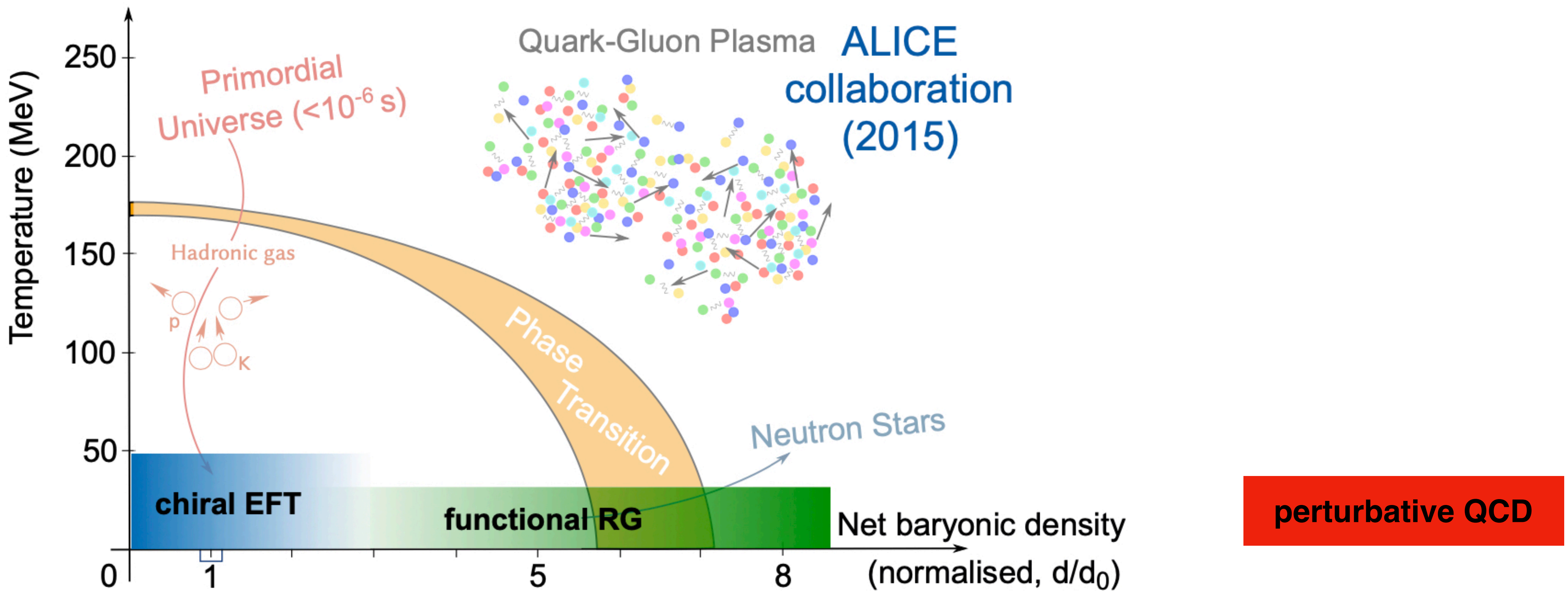

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  Leonhardt et al., PRL 125, 142502 (2020)

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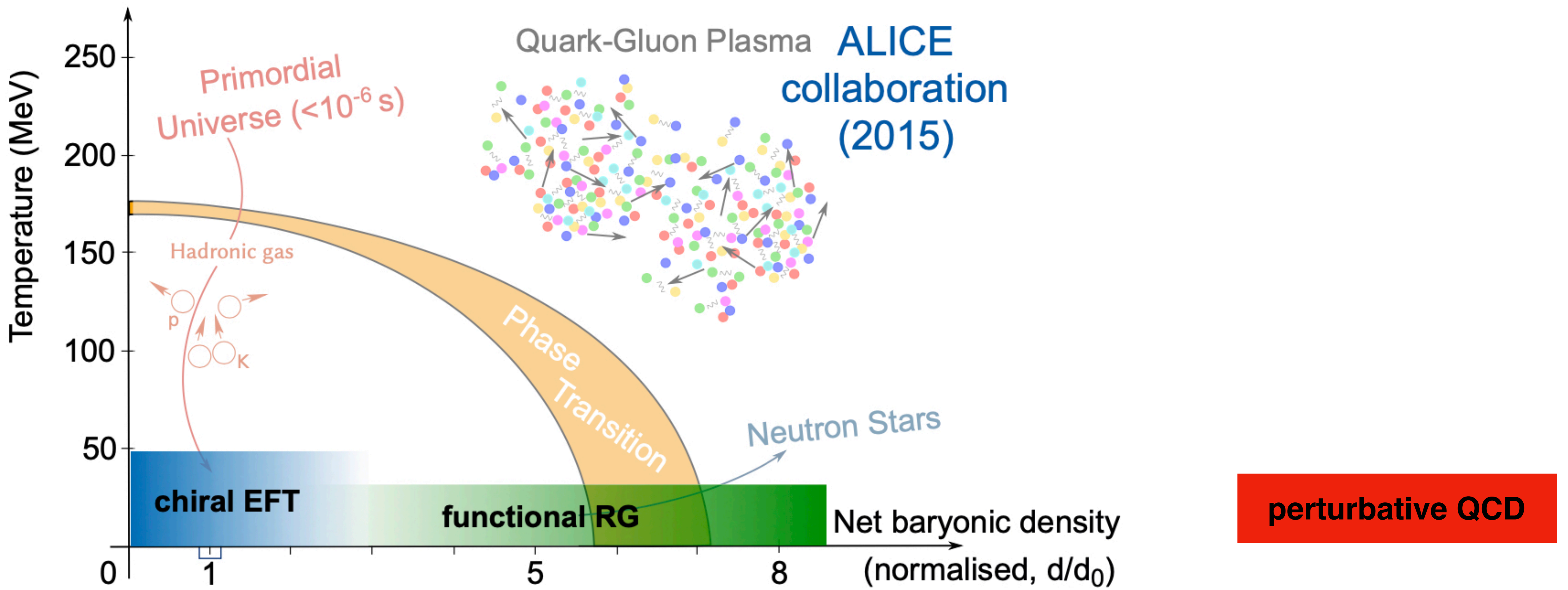

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- Perturbative QCD

  Ghiglieri et al., Phys. Rept. 880, 1 (2020)

The equation of state of high-density matter: constraints from neutron star observations

- observation of heavy neutron stars

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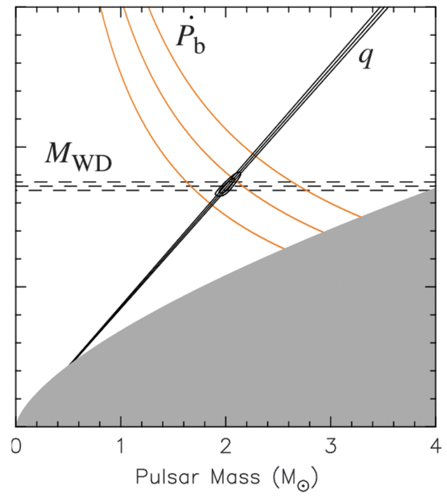

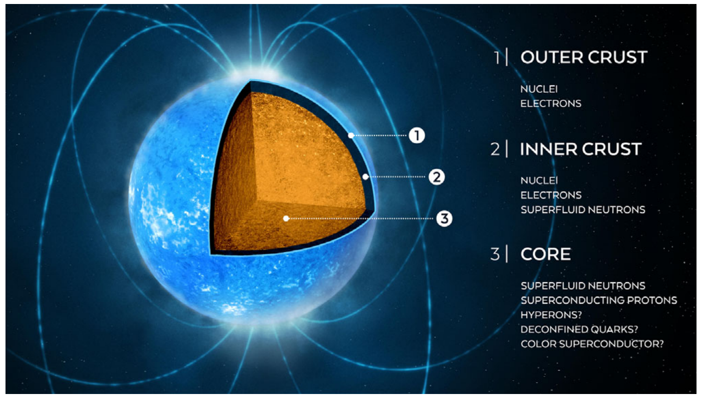

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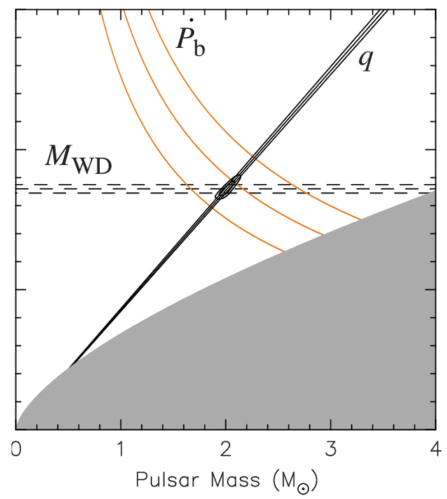
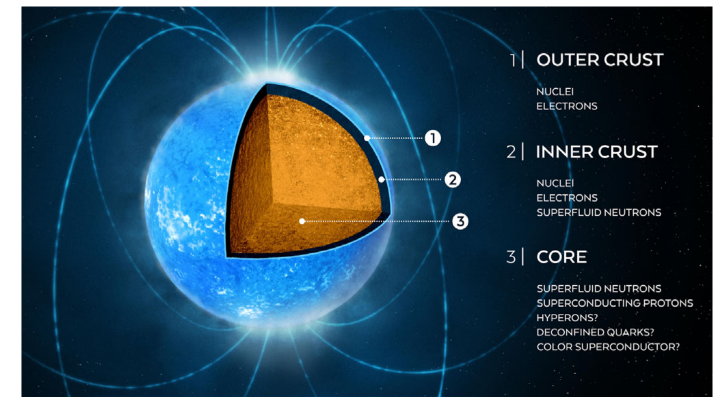
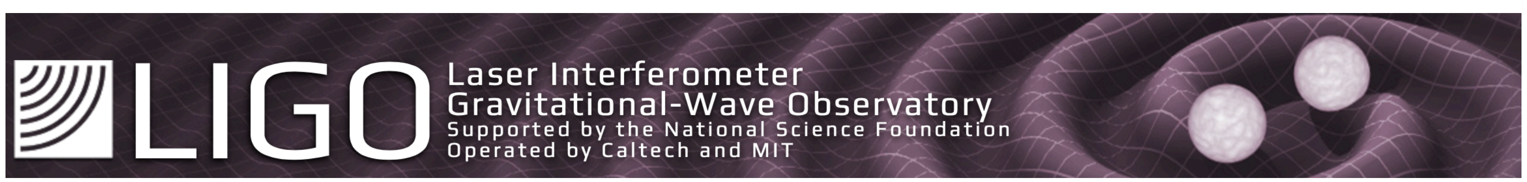
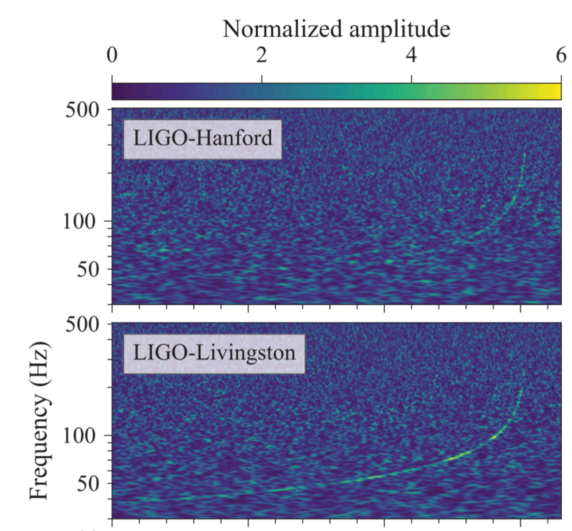




- detection of gravitational waves from neutron star merger event

  Abbott et al., PRL 119, 161101 (2017)





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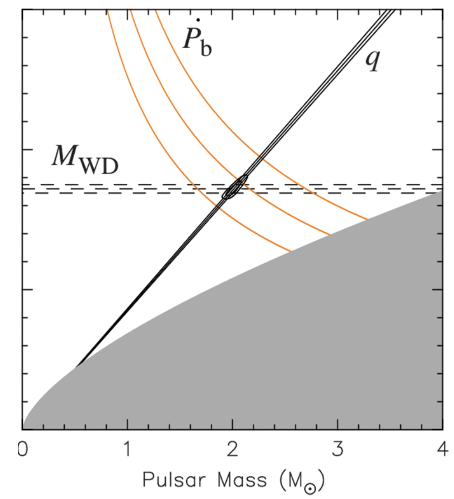

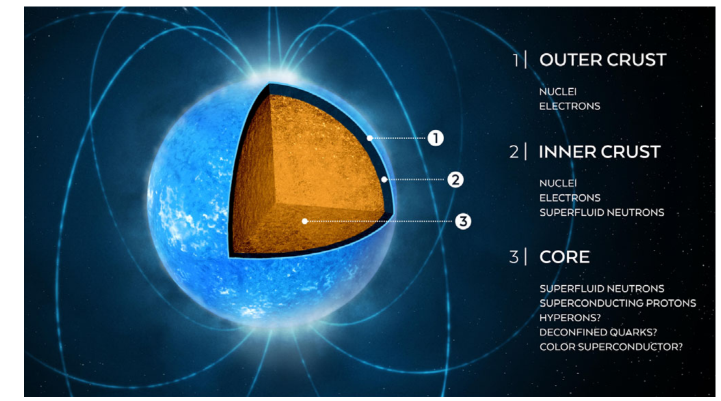

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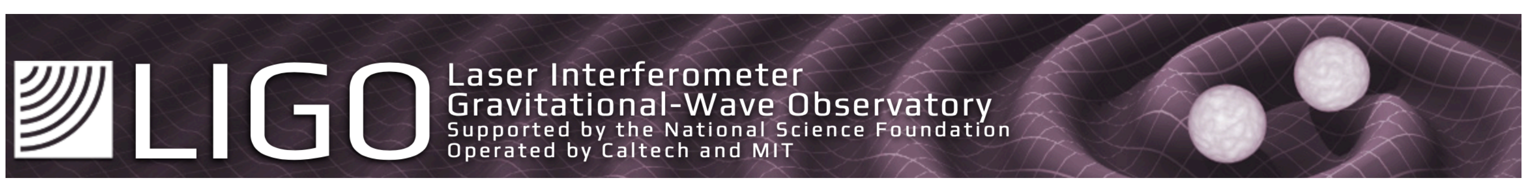
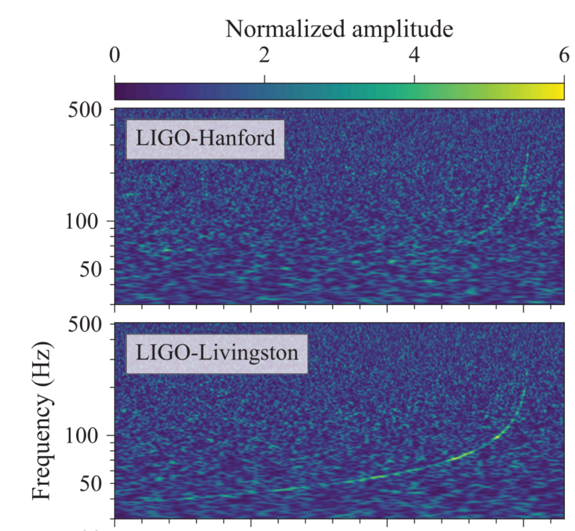




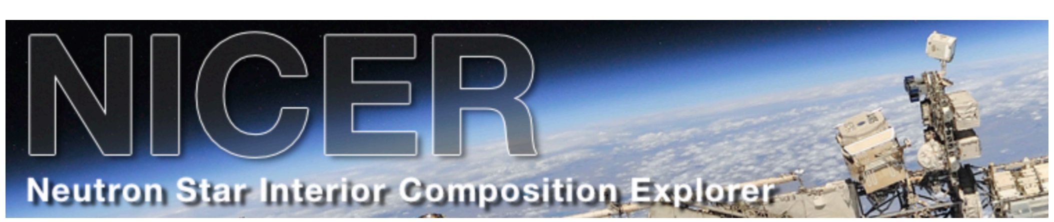
- radius measurements from pulsar x-ray timing

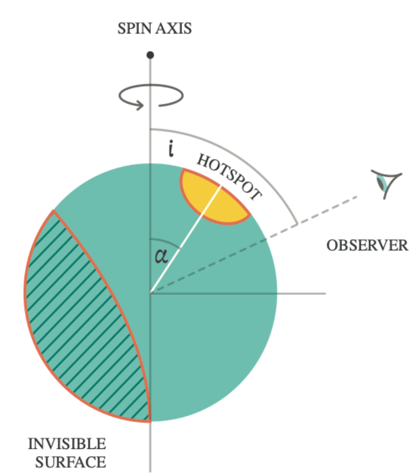
  Watts et al., RMP 88, 021001 (2016)

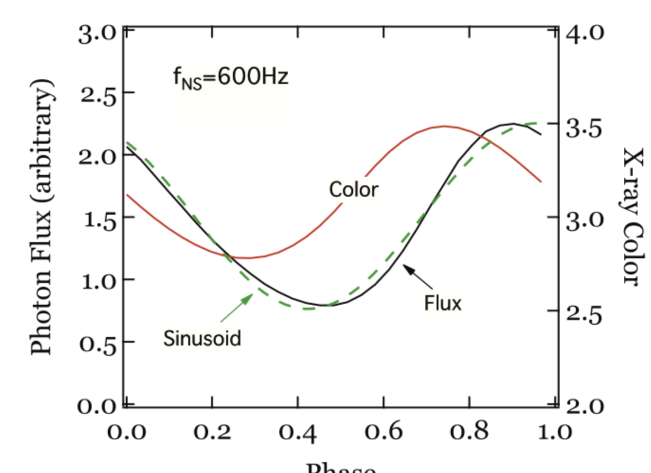
  Riley at al., APJL 887, 21 (2019)

  Raaijmakers et al., APJL 887, 22 (2019)

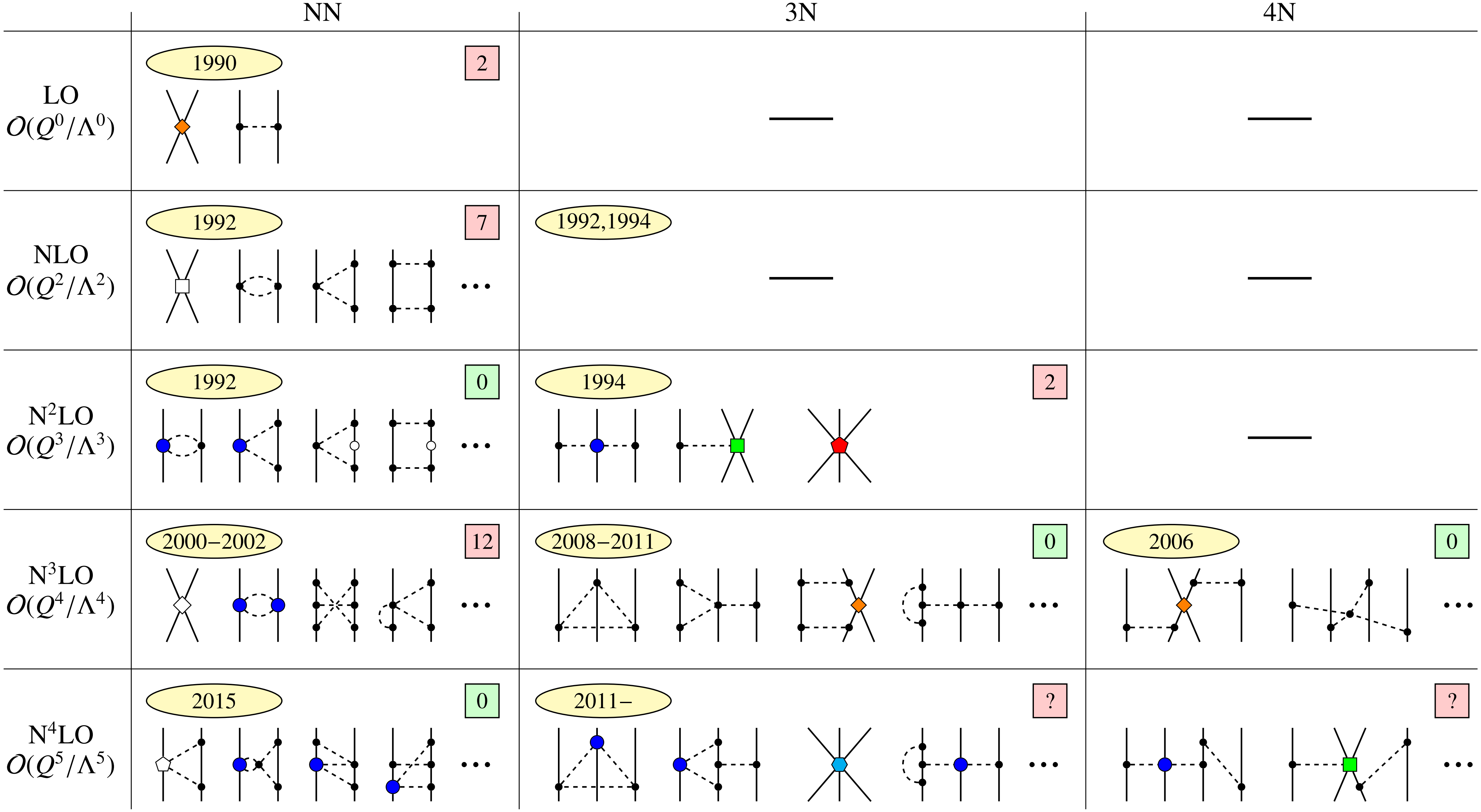
  Raaijmakers et al., APJL 918, 2 (2021)







Chiral effective field theory for nuclear forces



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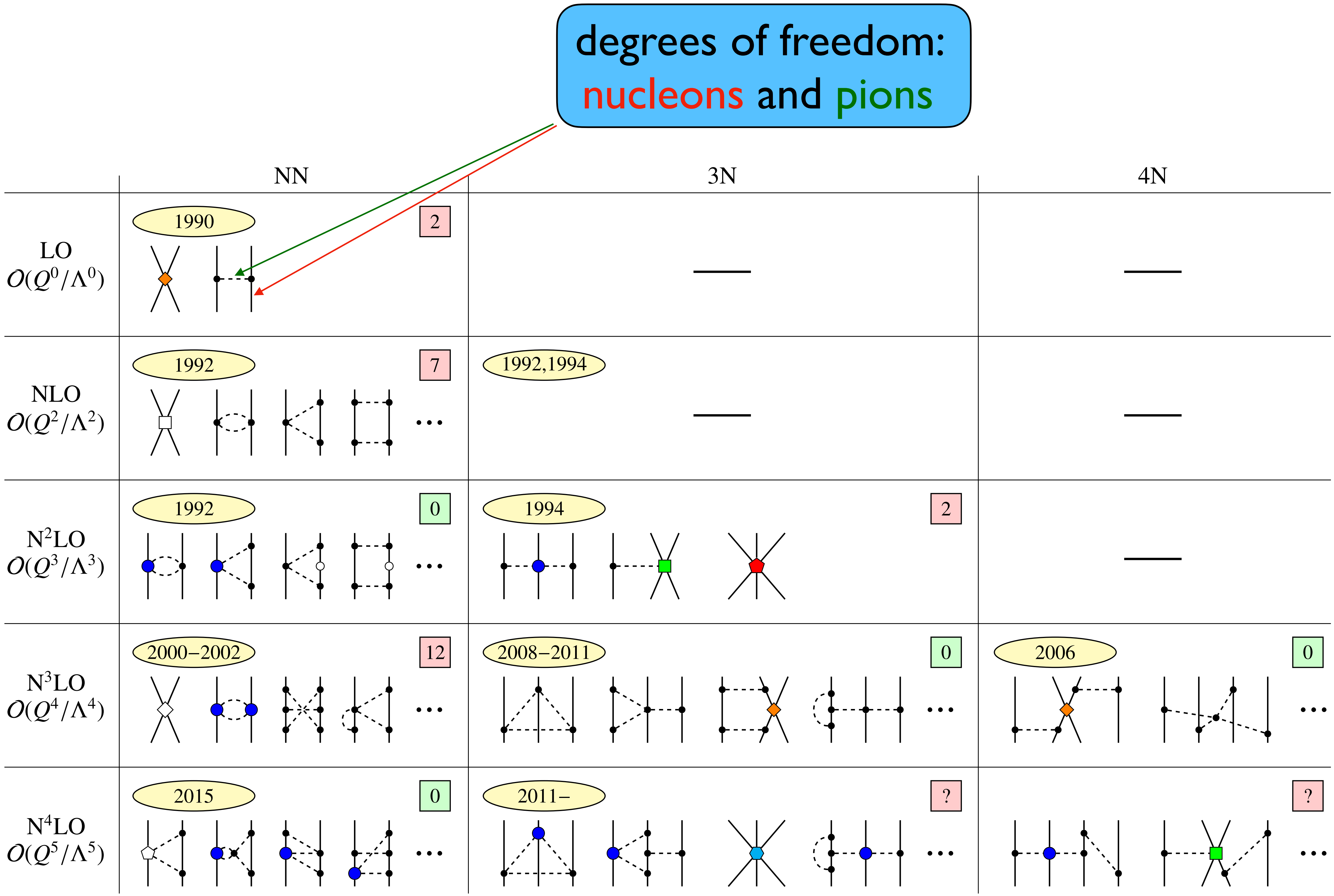

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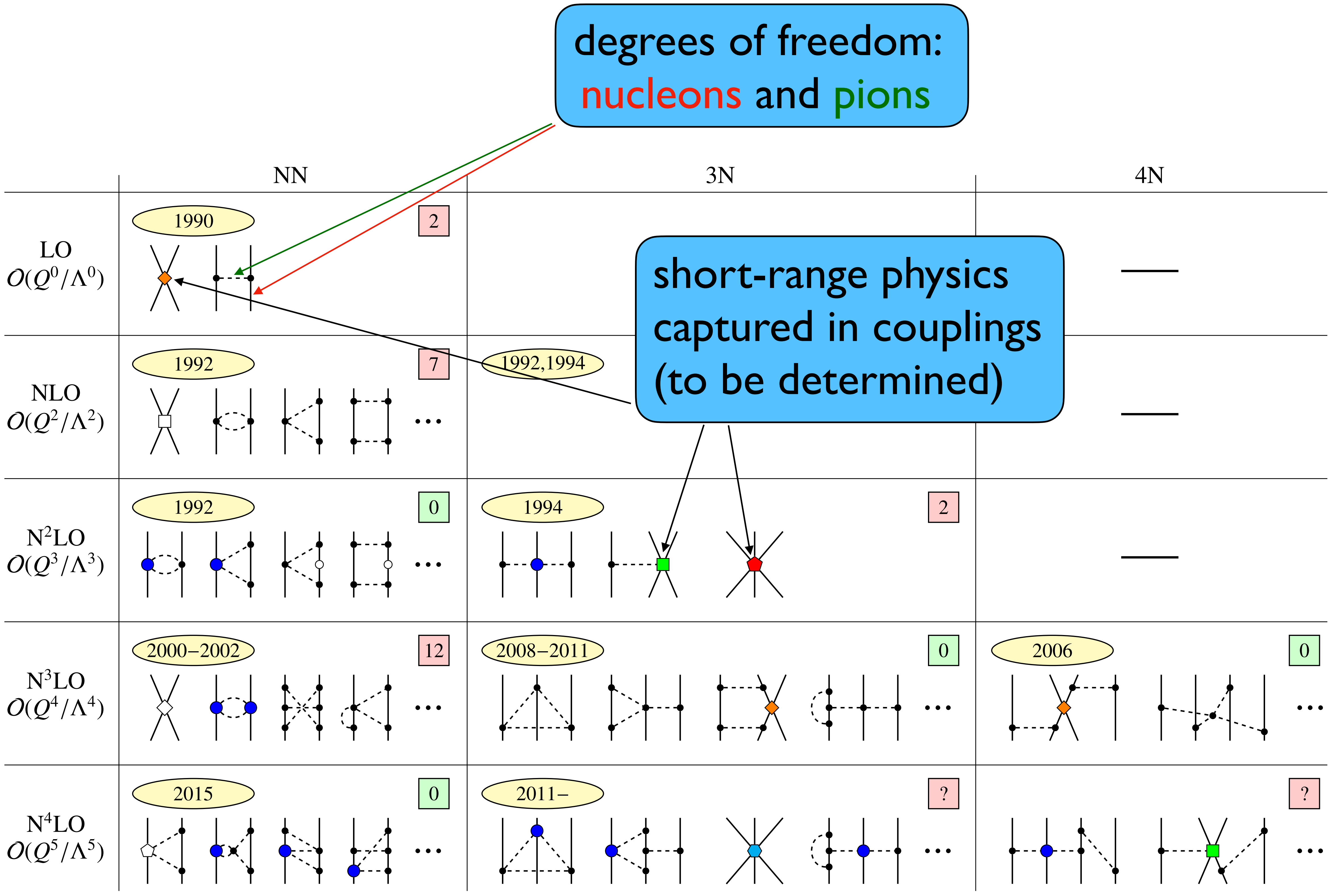
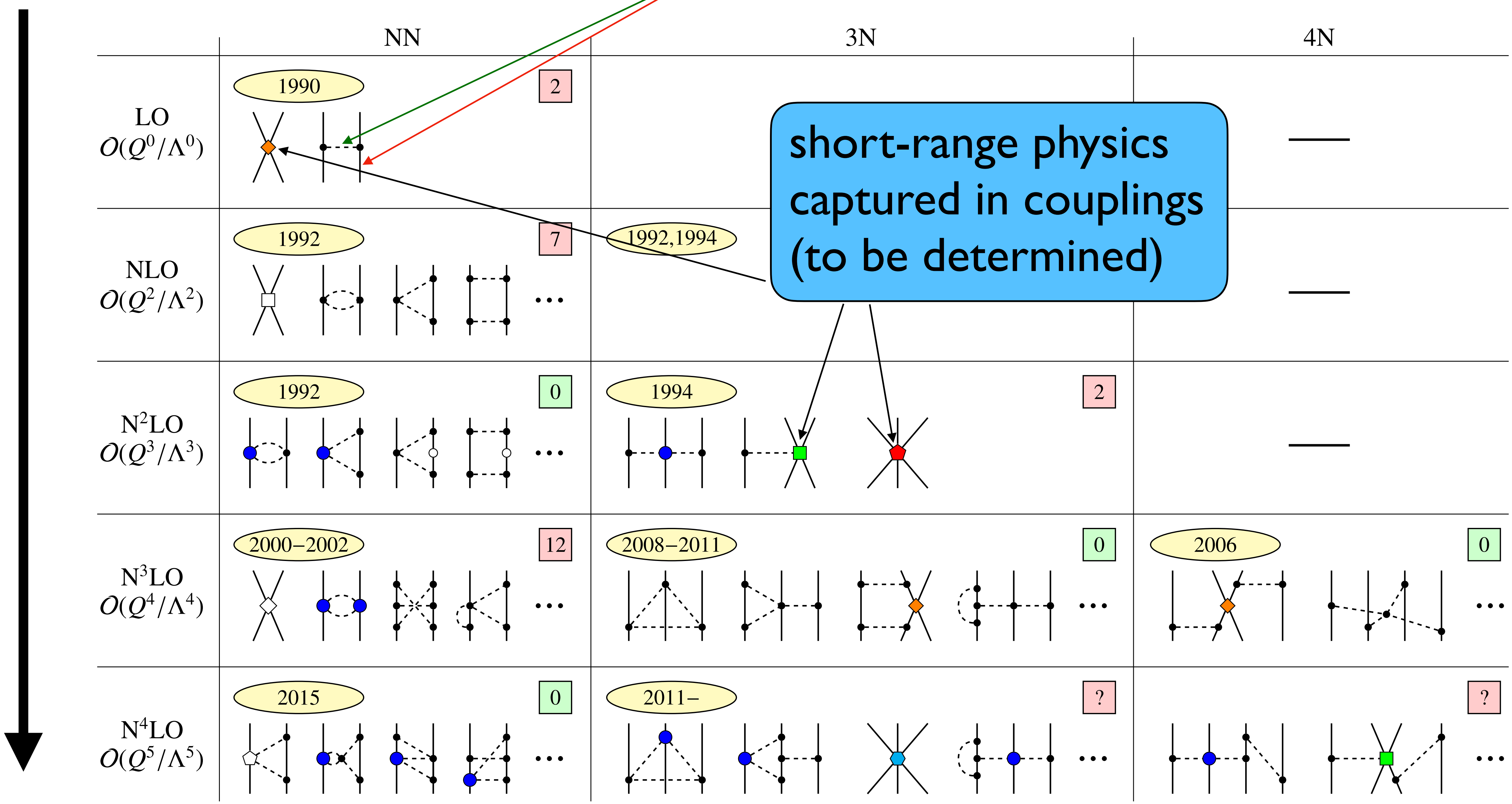


Chiral effective field theory for nuclear forces

power-counting:
expand in Q/Λ, error estimates!

degrees of freedom:
nucleons and pions



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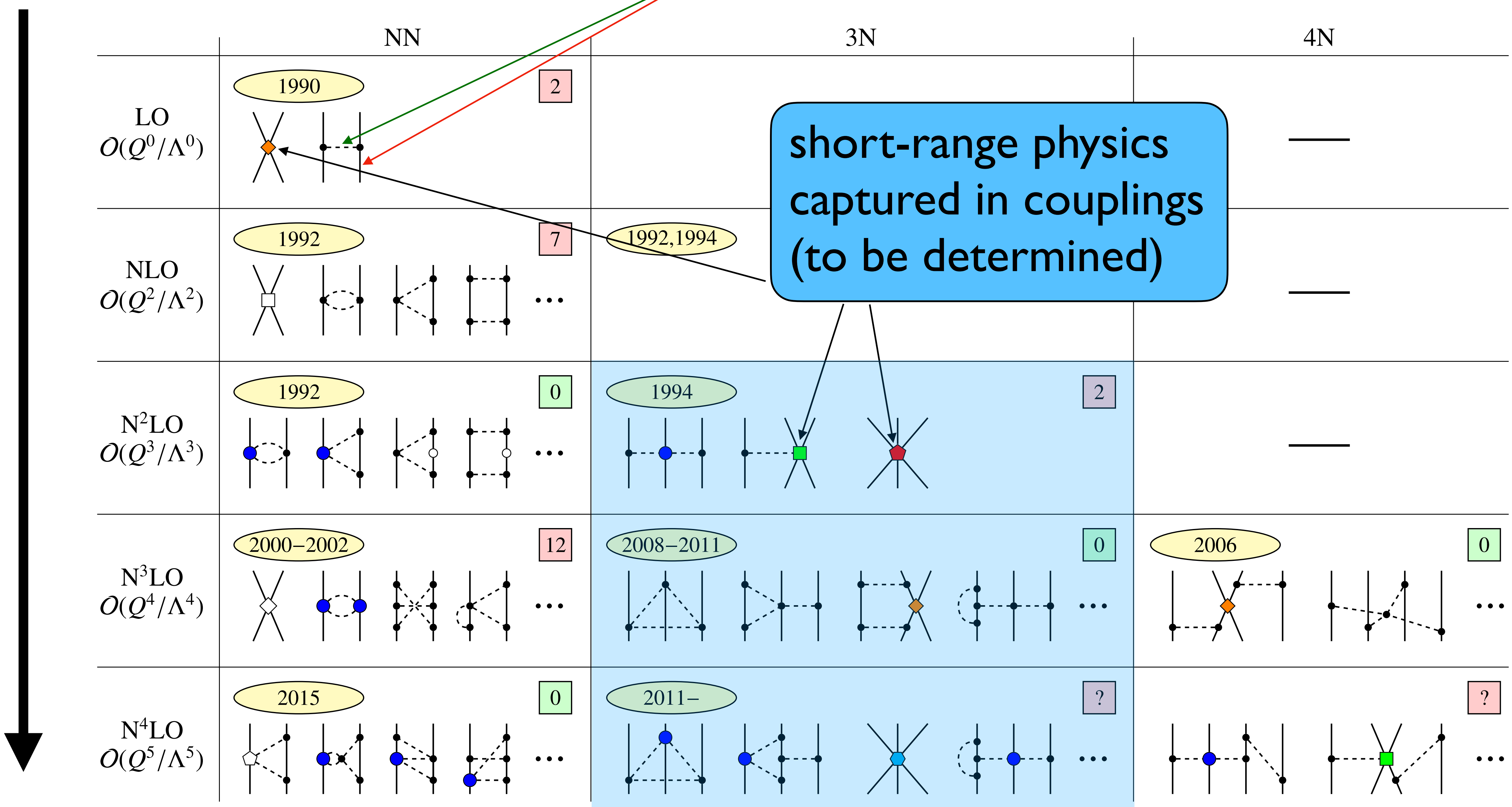
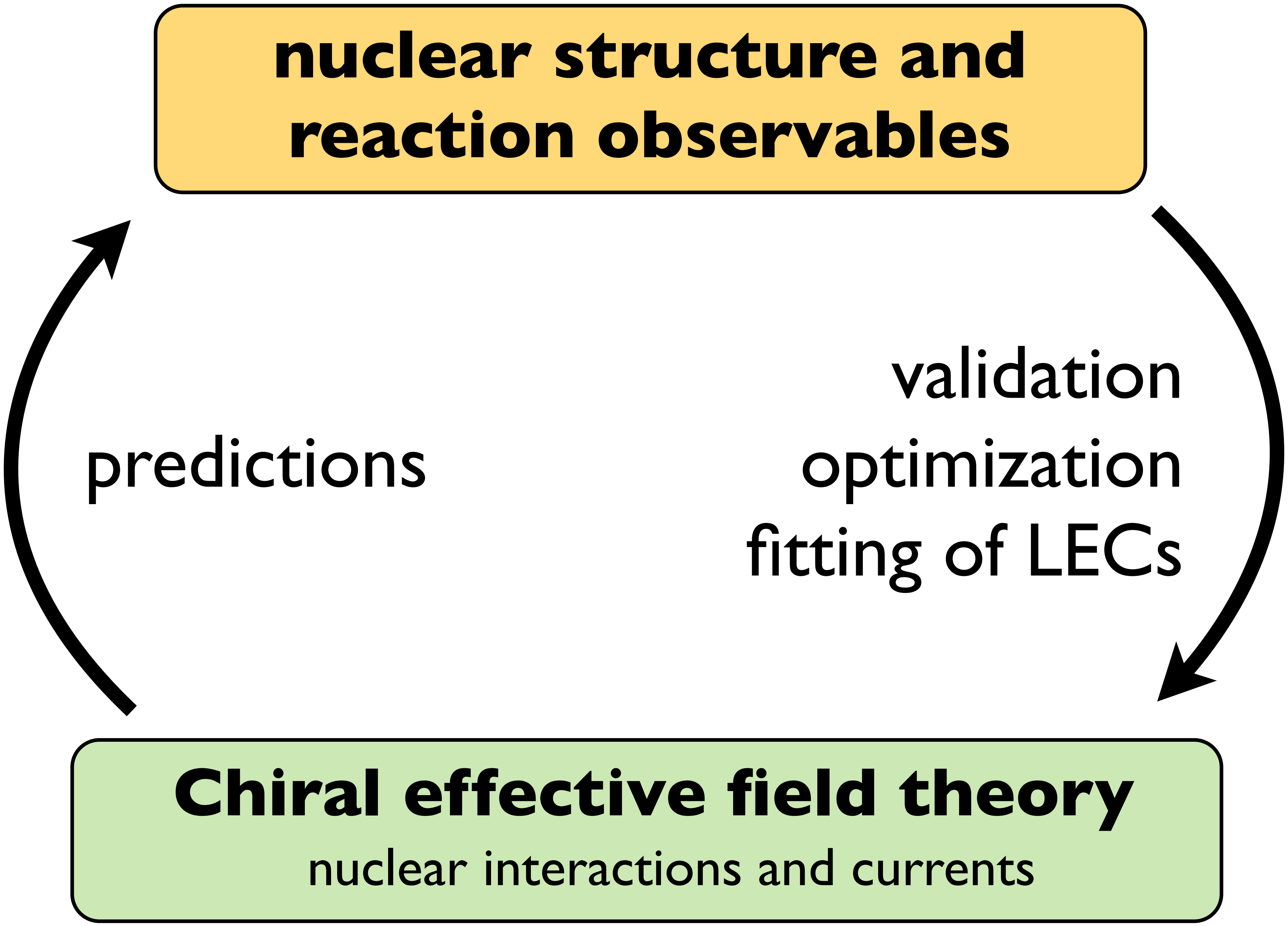


short-range physics
captured in couplings
(to be determined)

3N forces appear naturally

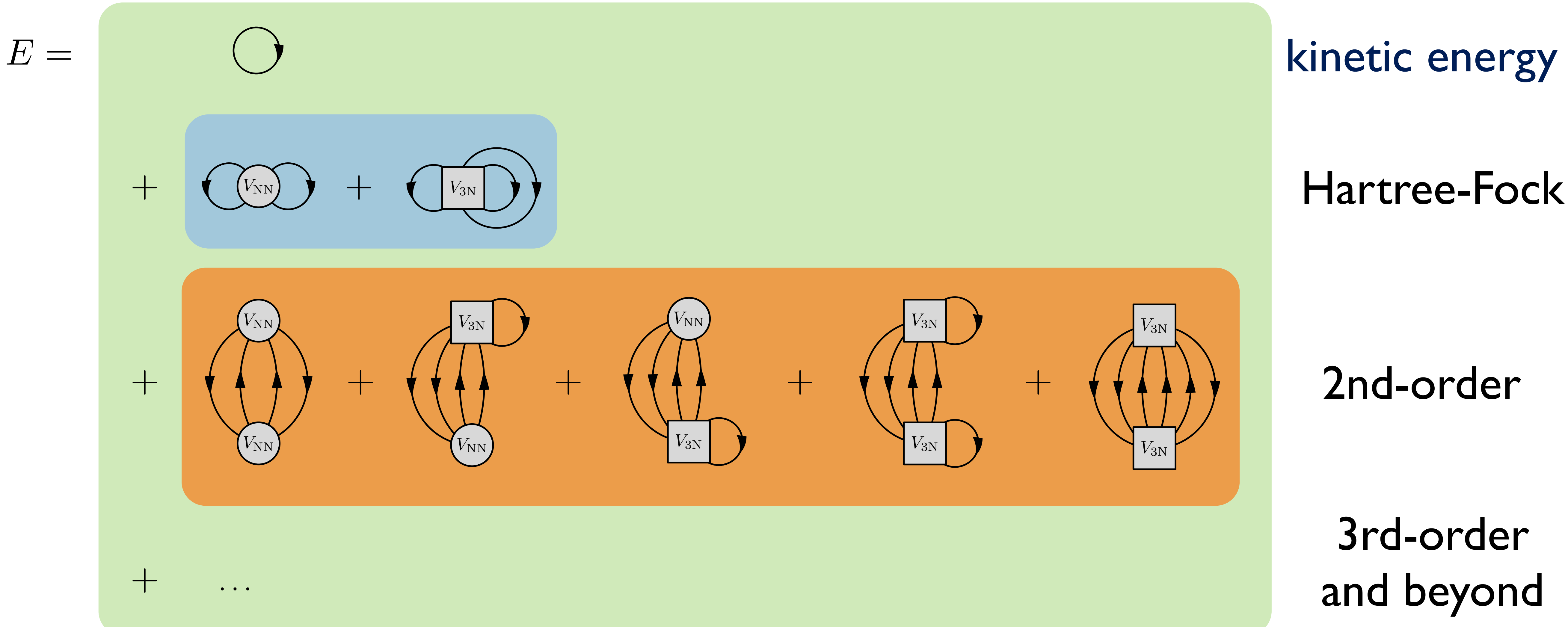
Development of nuclear interactions



Equation of state: Many-body perturbation theory

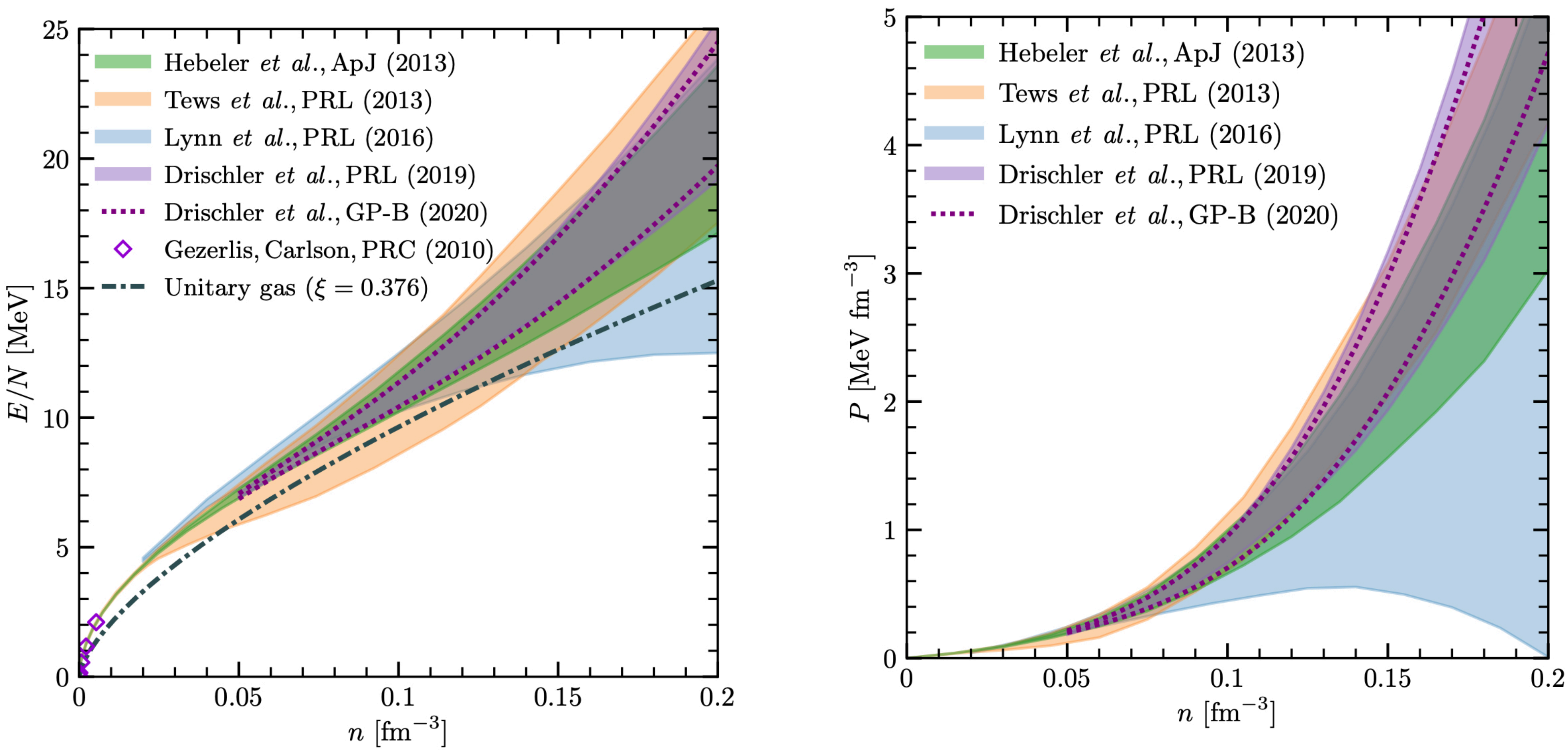
central quantity of interest: energy per particle E/N

$$H(\lambda) = T + V_{\rm NN}(\lambda) + V_{\rm 3N}(\lambda) + \ldots$$



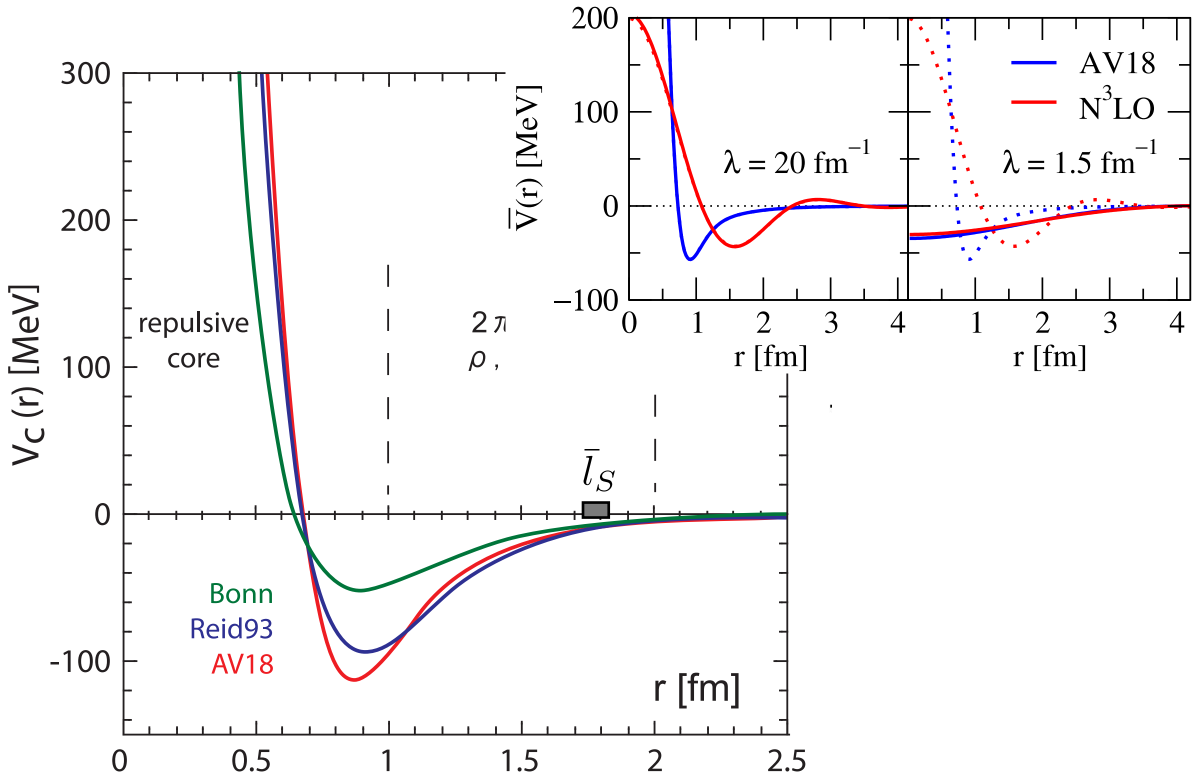
- "hard" interactions require non-perturbative summation of diagrams
- with low-momentum interactions much more perturbative
- inclusion of contributions from 3N interaction crucial and challenging!

Equation of state of neutron matter up to nuclear densities

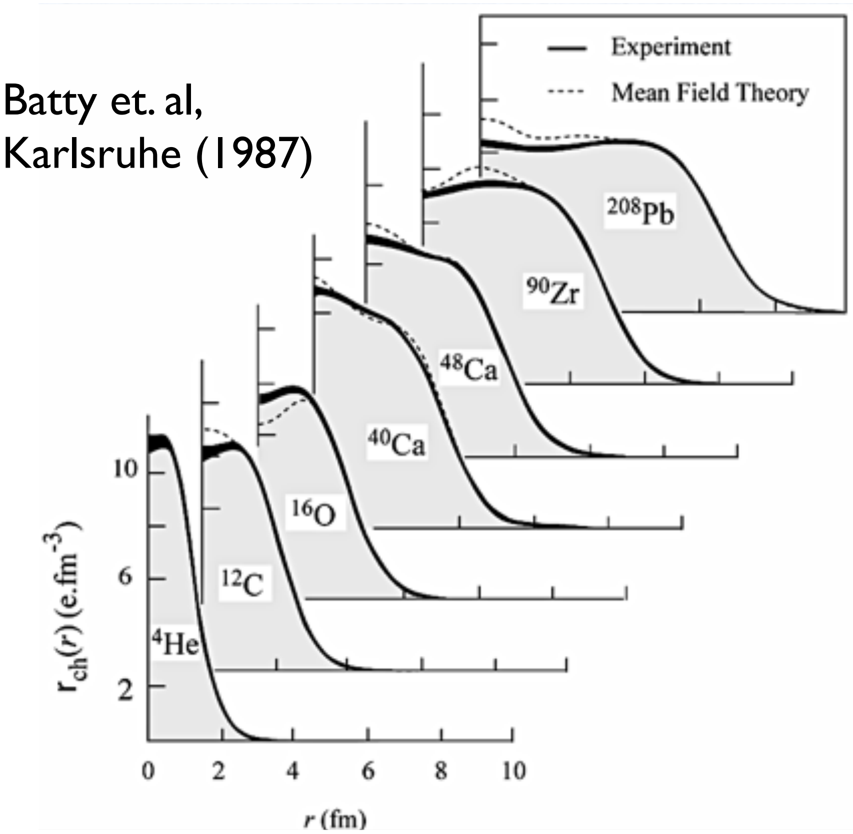


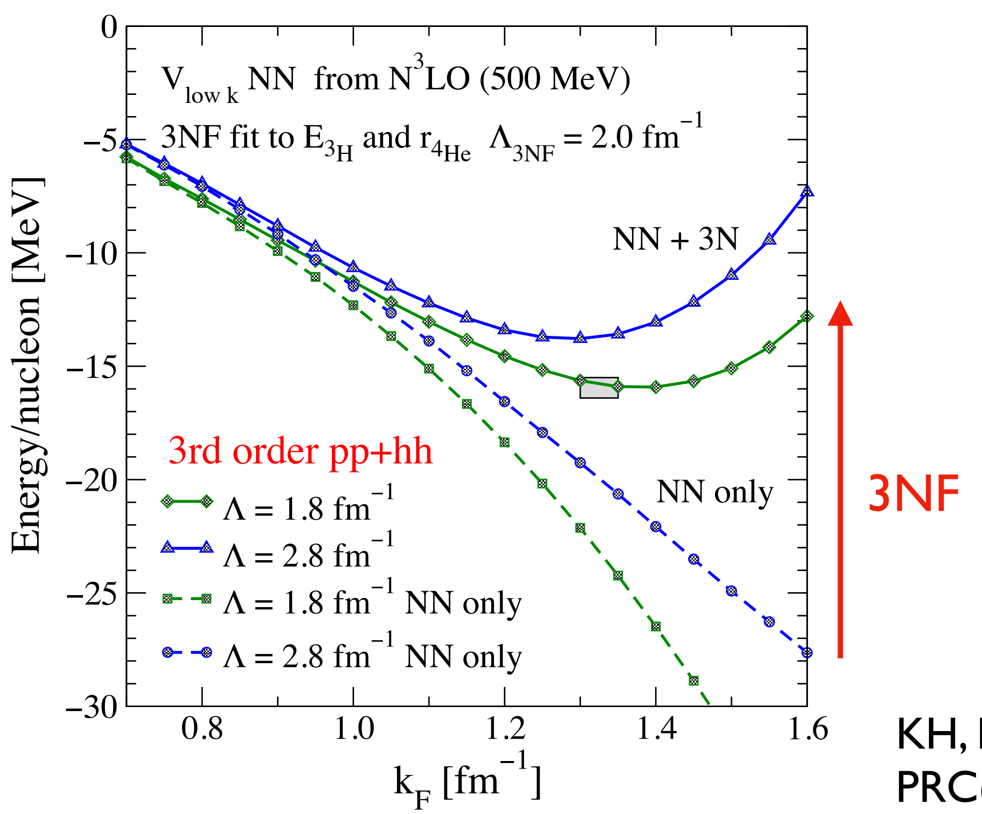
- EOS of neutron matter well constrained by chiral EFT up to nuclear densities
- results insensitive to choices of nuclear forces and many-body methods

Equation of state of symmetric nuclear matter: nuclear saturation



Batty et. al,
Karlsruhe (1987)



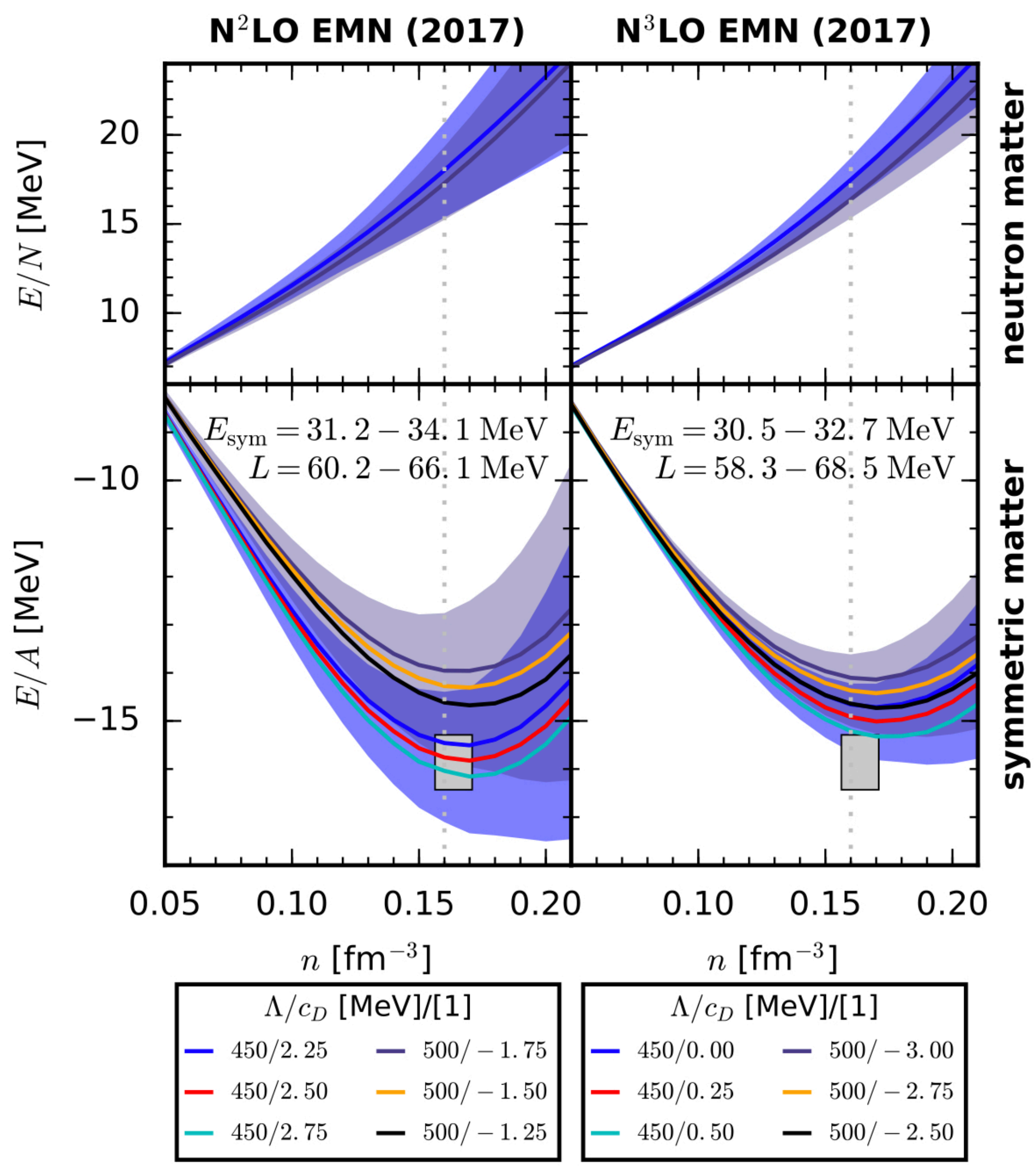


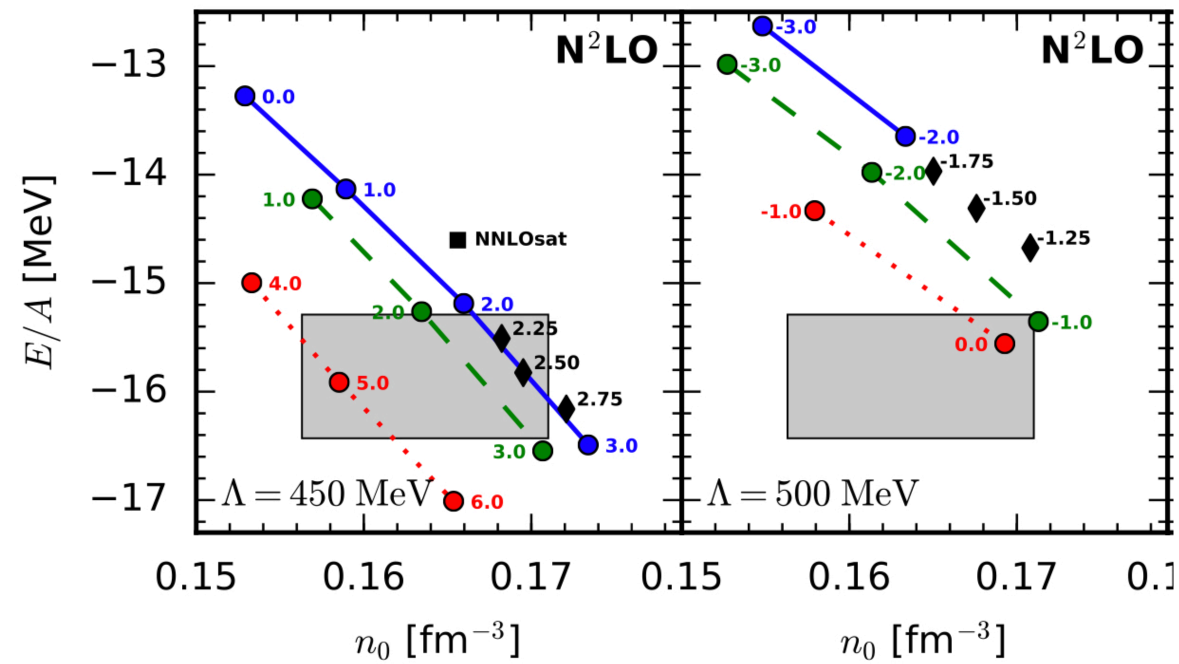
contributions from 3NF crucial for realistic description of nuclear matter

KH, Bogner, Furnstahl, Nogga,
PRC(R) 83, 031301 (2011)

Results for symmetric and neutron matter at T=0

- performed MBPT calculations up to 4th order (complete for NN interactions)
- fits to the empirical saturation point possible
- natural convergence pattern in MBPT and chiral expansion





Drischler, KH, Schwenk, PRL 122 (2019)

Results for symmetric matter at T=0

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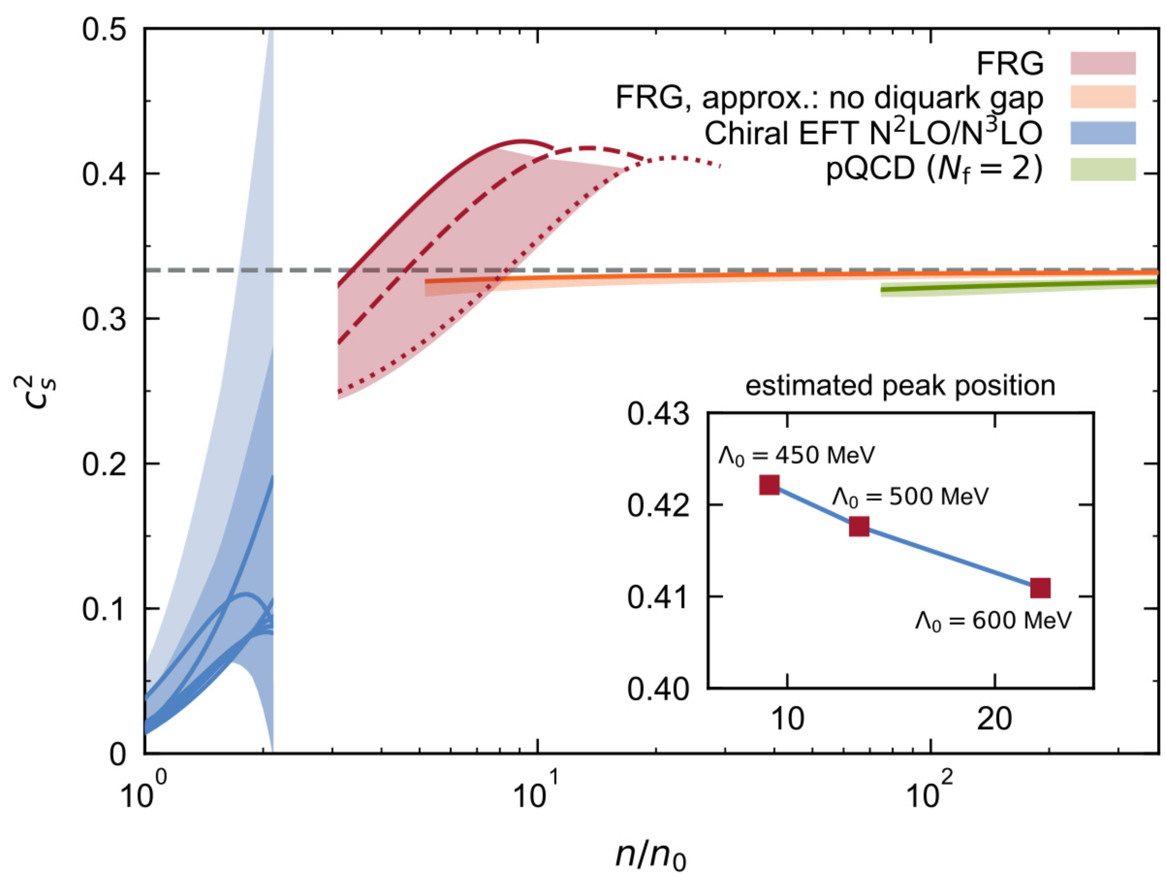
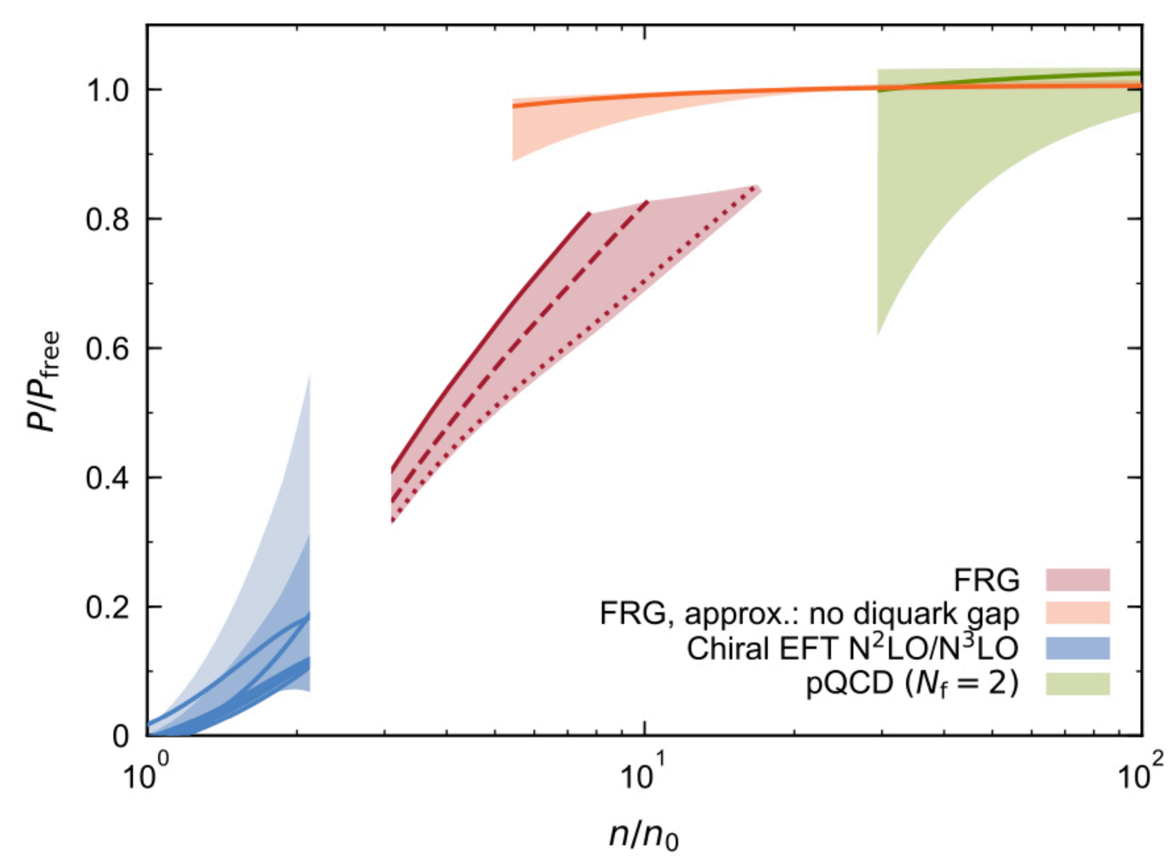
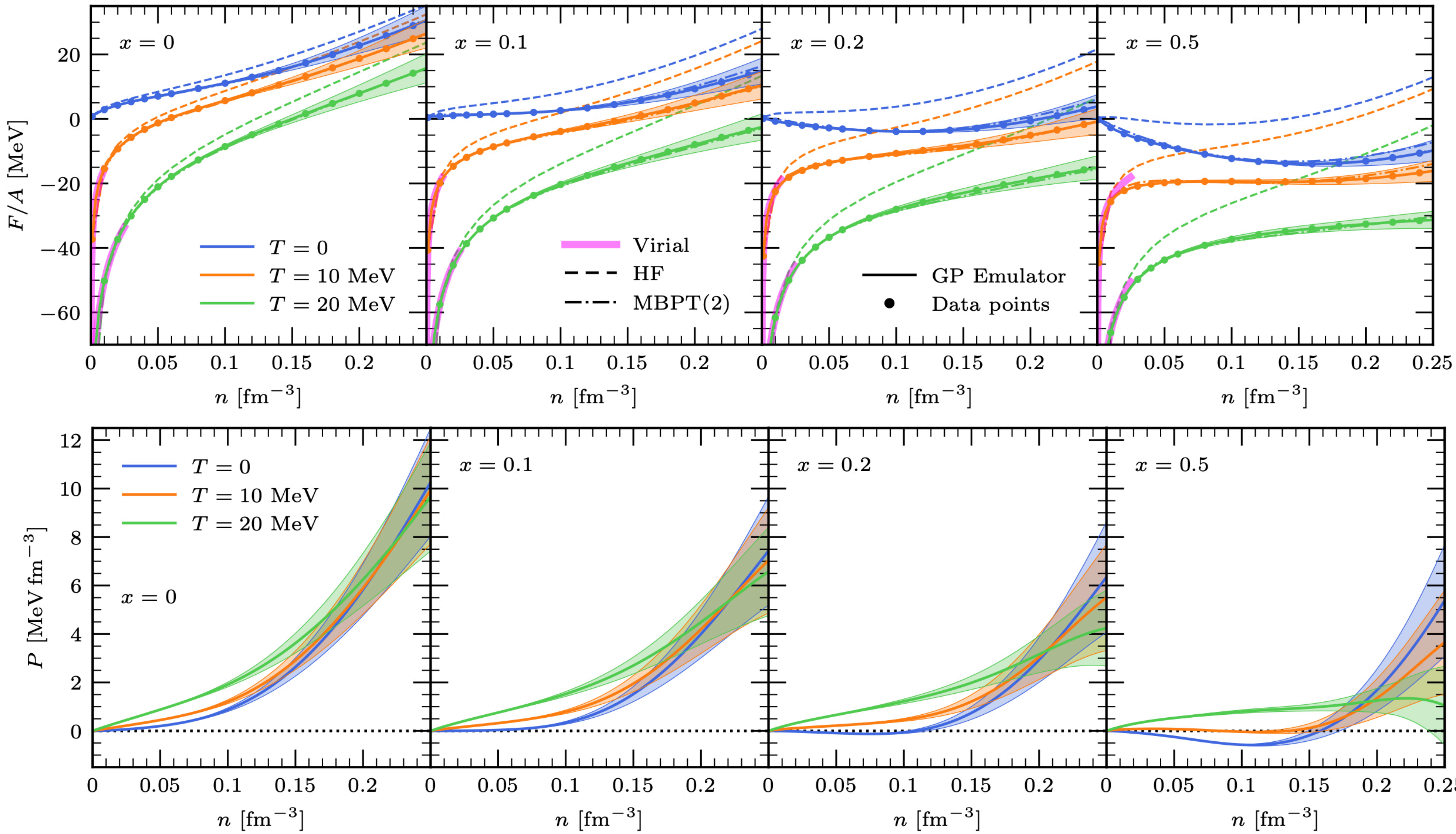
- comparison with Functional Renormalization Group (fRG) calculations based on QCD and perturbative QCD





Leonhardt et al., PRL 125,142502 (2020)

Matter at finite temperature and general proton fractions



- evaluation of the grand canonical potential in MBPT:

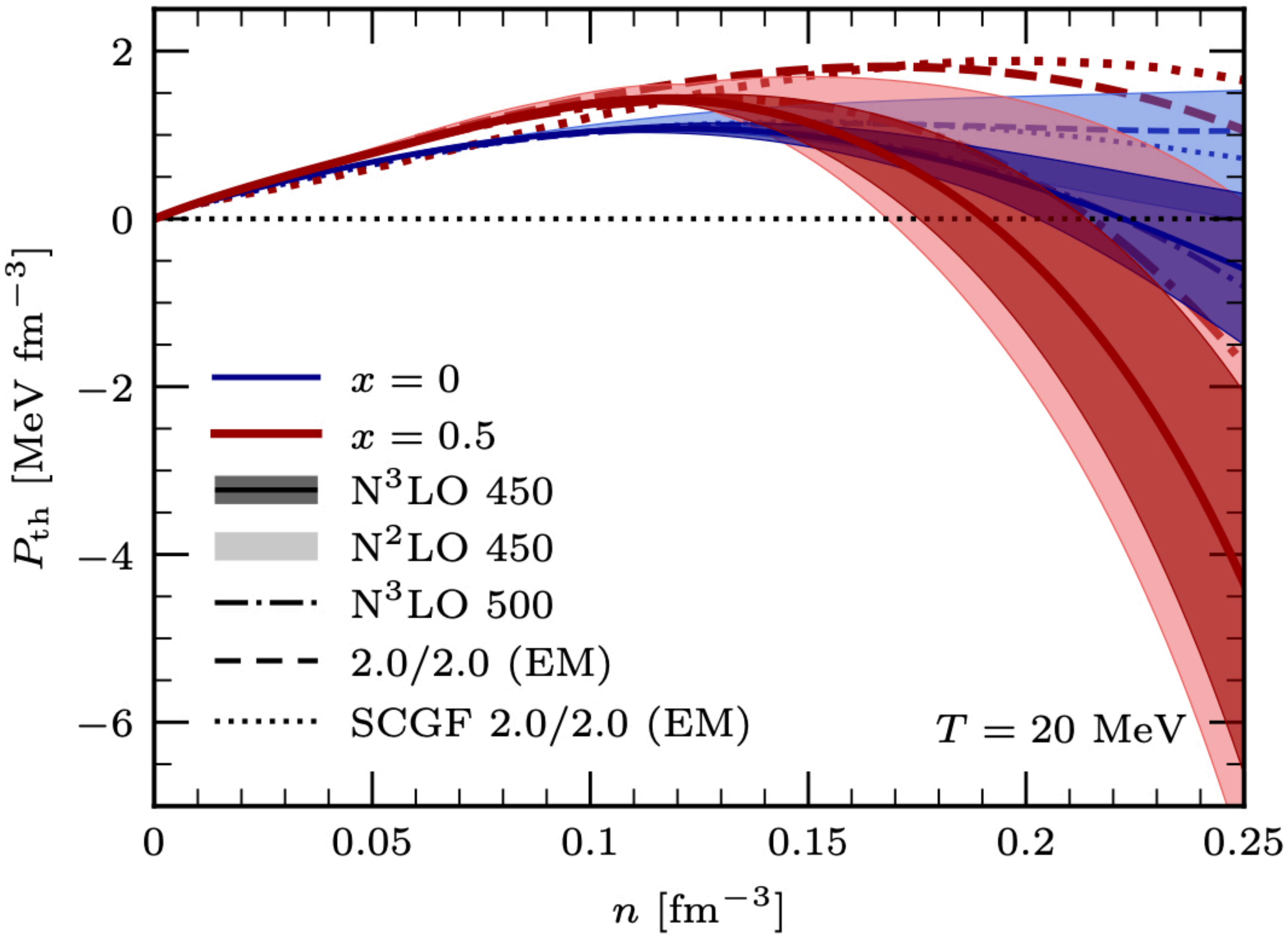
$$\Omega\left(T, \mu_n, \mu_p\right) = -\frac{1}{\beta} \ln \mathrm{Tr}\left(e^{-\beta(H-\mu_n N_n - \mu_p N_p)}\right)$$

- implementation of Gaussian process emulator for efficient interpolation and evaluation of thermodynamic quantities

Keller, KH, Schwenk, PRL 130, 072701

Jonas Keller

Negative thermal pressure due to 3N interaction effects



Keller, KH, Schwenk, PRL 130, 072701

- thermal pressure: $P_{\rm th}(T) = P(T) - P(T=0)$
- $P_{\rm th}(T)$ becomes negative at higher densities due to contributions from 3N interactions
- robust for different chiral interactions, chiral orders and cutoff values

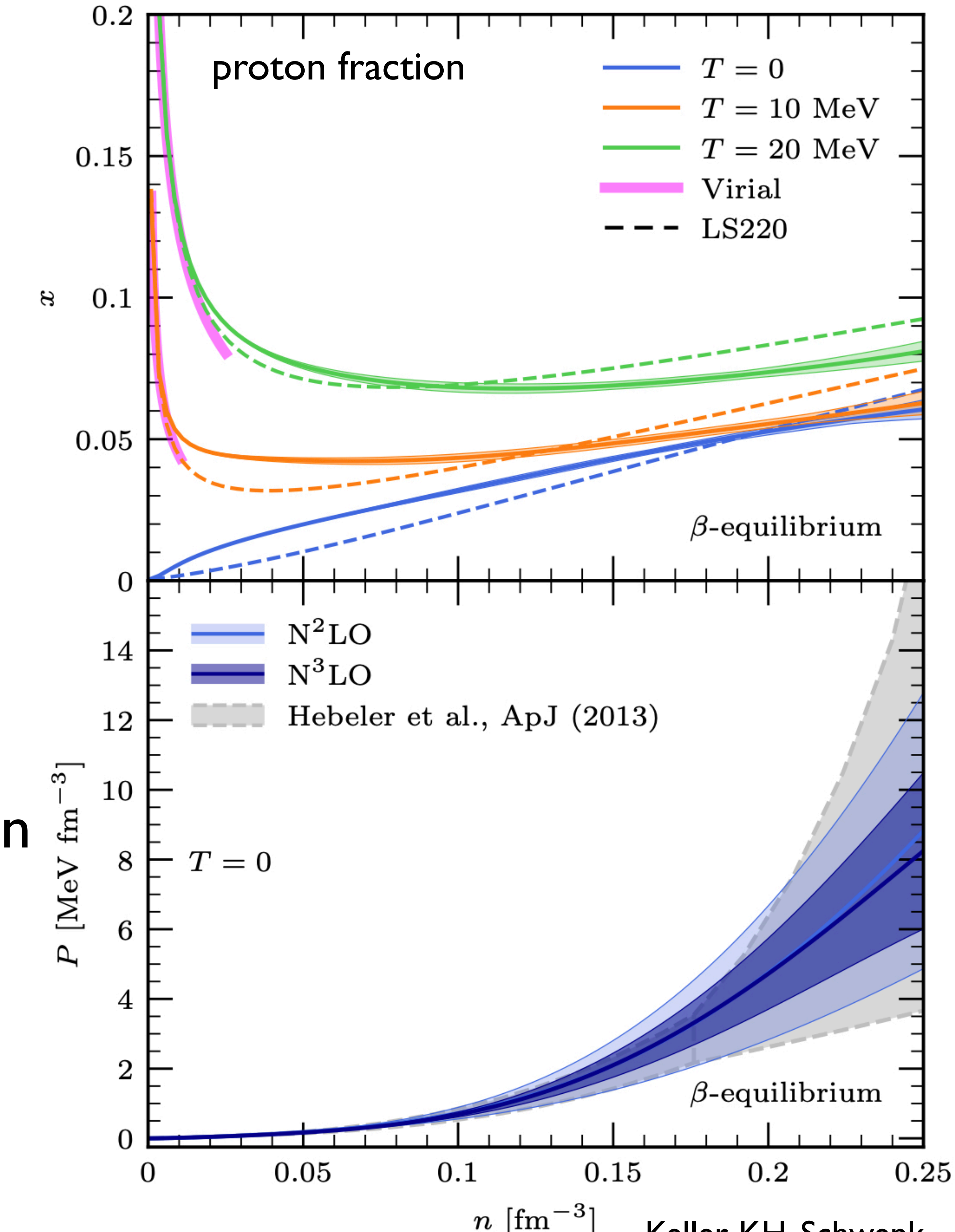
Neutron star matter

- incorporation of beta equilibrium

$$m_n + \mu_n = (m_p + \mu_p) + (m_e + \mu_e)$$

$$\mu_n - \mu_p = -\frac{\partial}{\partial x}\frac{F}{N}$$

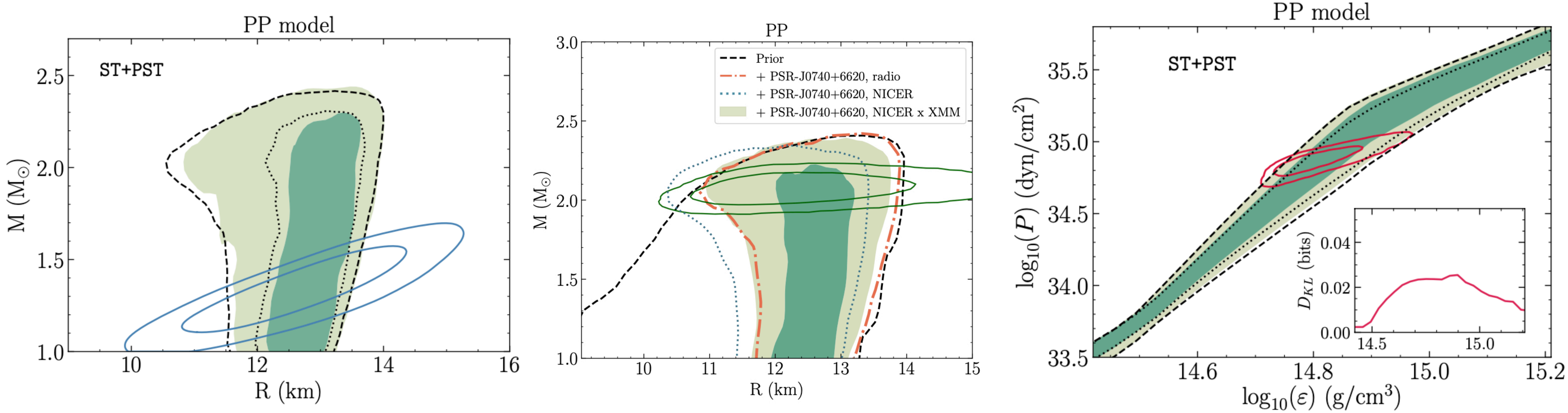
- comparison to uncertainty band (2013):
  » inclusion of interactions up to N3LO
  » no RG transformations
  » systematic EFT convergence
  » no parametrisation in proton fraction
  » no approximations in 3NF treatment in MBPT diagrams
  » calculations to higher densities



Keller, KH, Schwenk, PRL 130, 072701

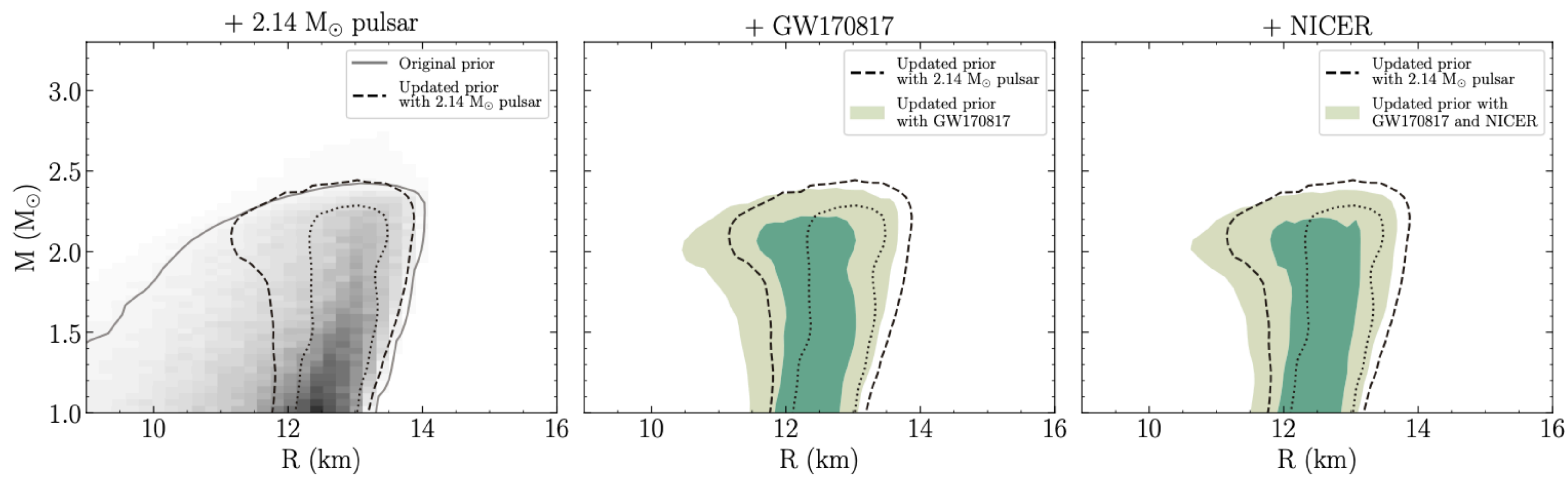
Constraints on neutron star radii

constraints on EOS and NS radii from first NICER observations:



Raaijmakers et al., APJL 887, 22 (2019); Raaijmakers et al., APJL 918, 2 (2021)

additionally incorporating constraints from LIGO and mass measurements:



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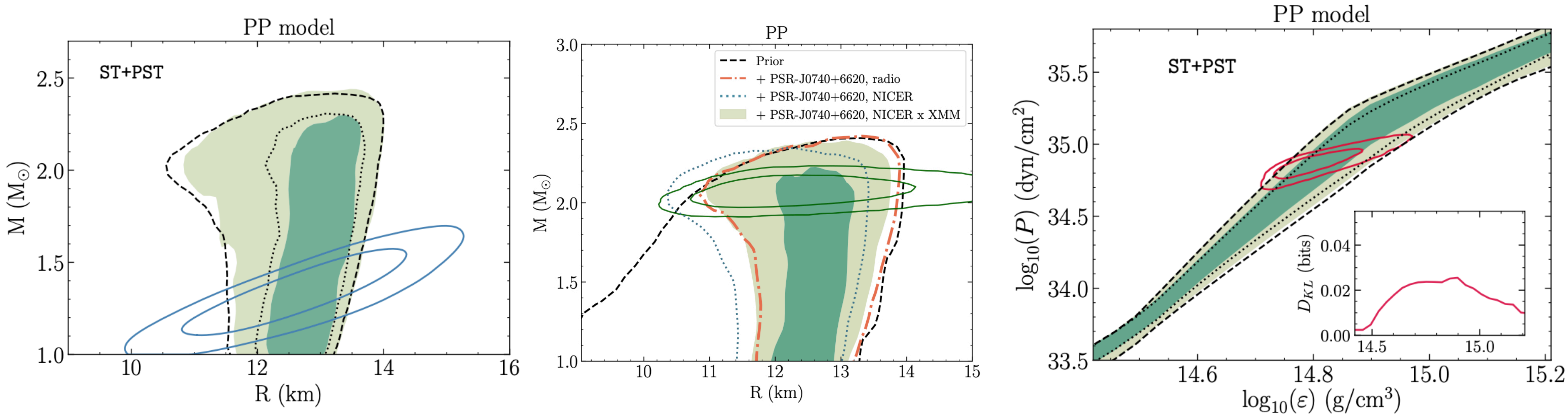

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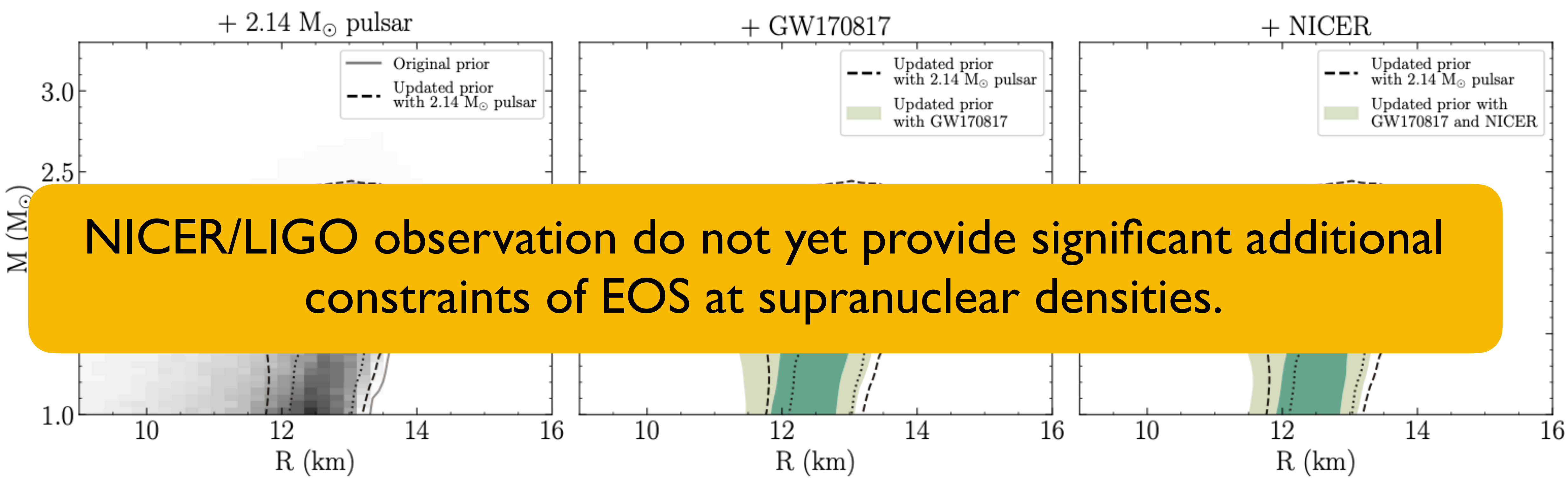
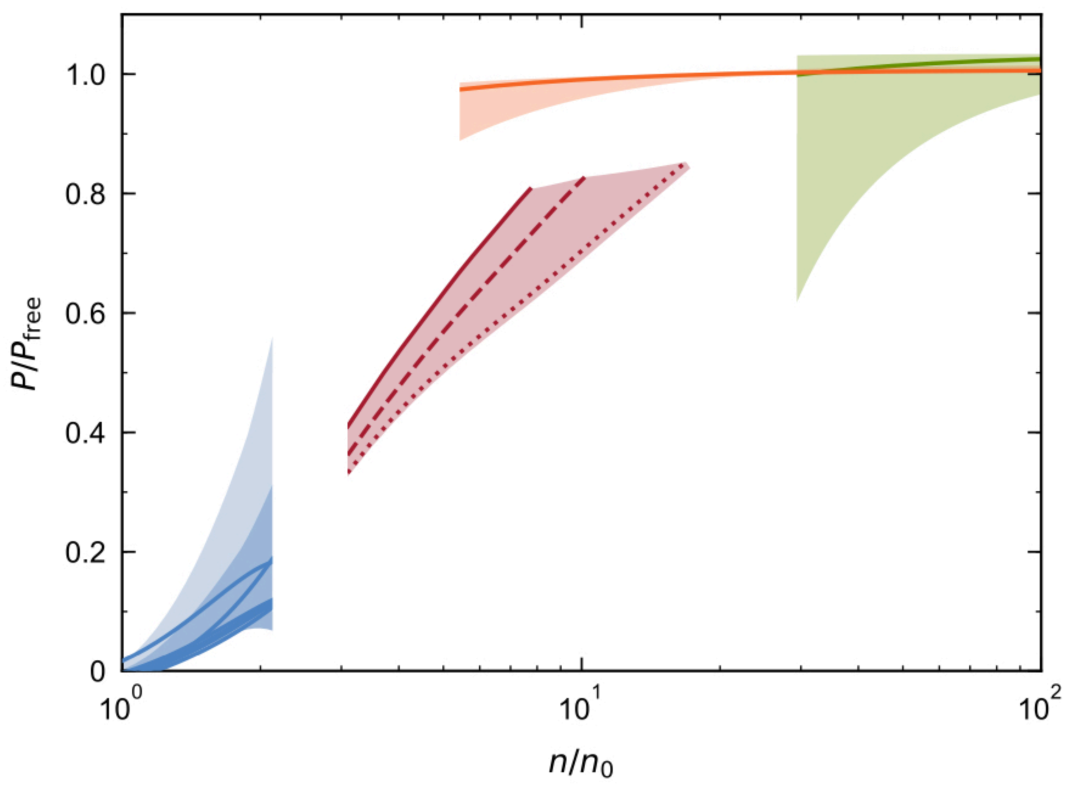


NICER/LIGO observation do not yet provide significant additional constraints of EOS at supranuclear densities.

Summary and open questions

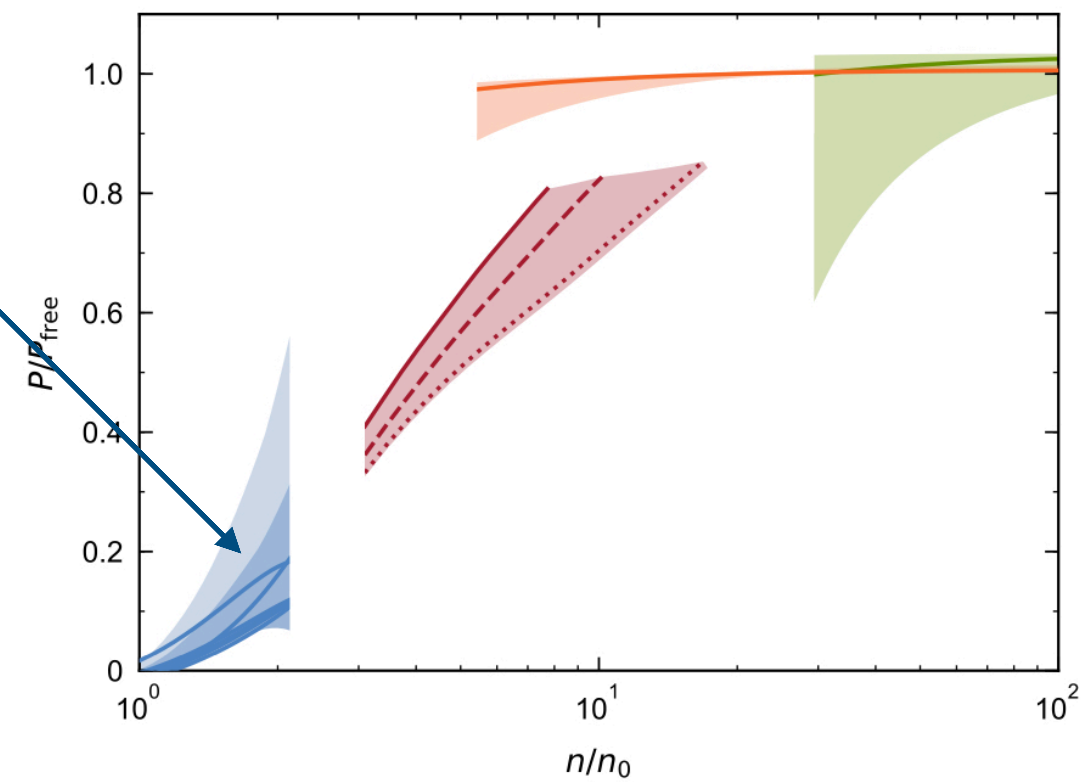


Summary and open questions

chiral EFT interactions

$n \sim [0.5, 1.5-2] n_0$

calculations for wide range of conditions, tightest constraints for neutron-rich matter



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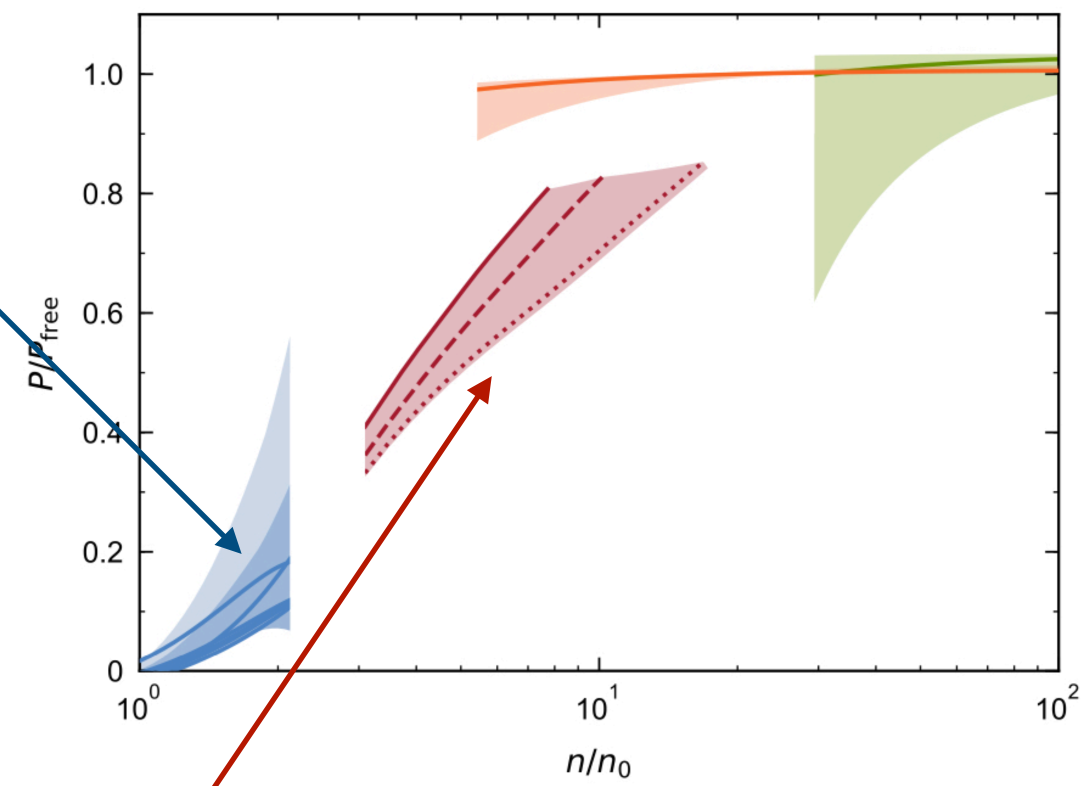

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$n > 3n_0 - 4n_0$

still under active development, can be extended to general conditions

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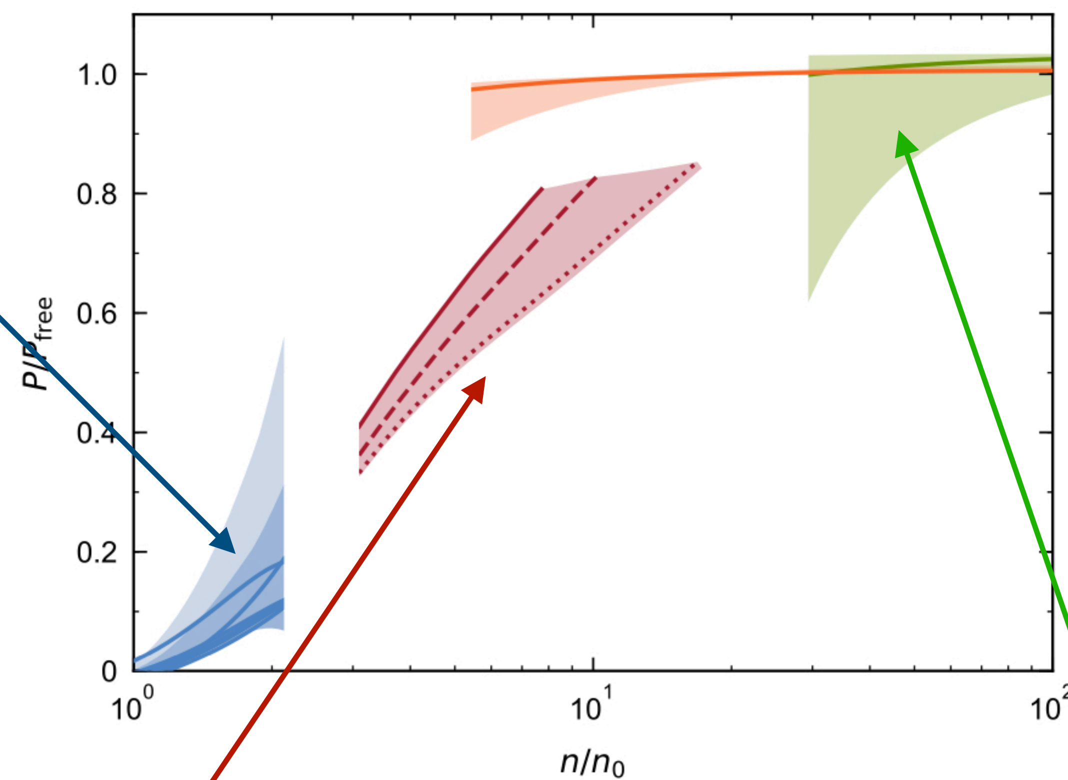

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$n > 50\, n_0$

including thermodynamic relations:

$n > 10\, n_0$

still under discussion to what extend it provides additional constraints for EOS relevant for NSs

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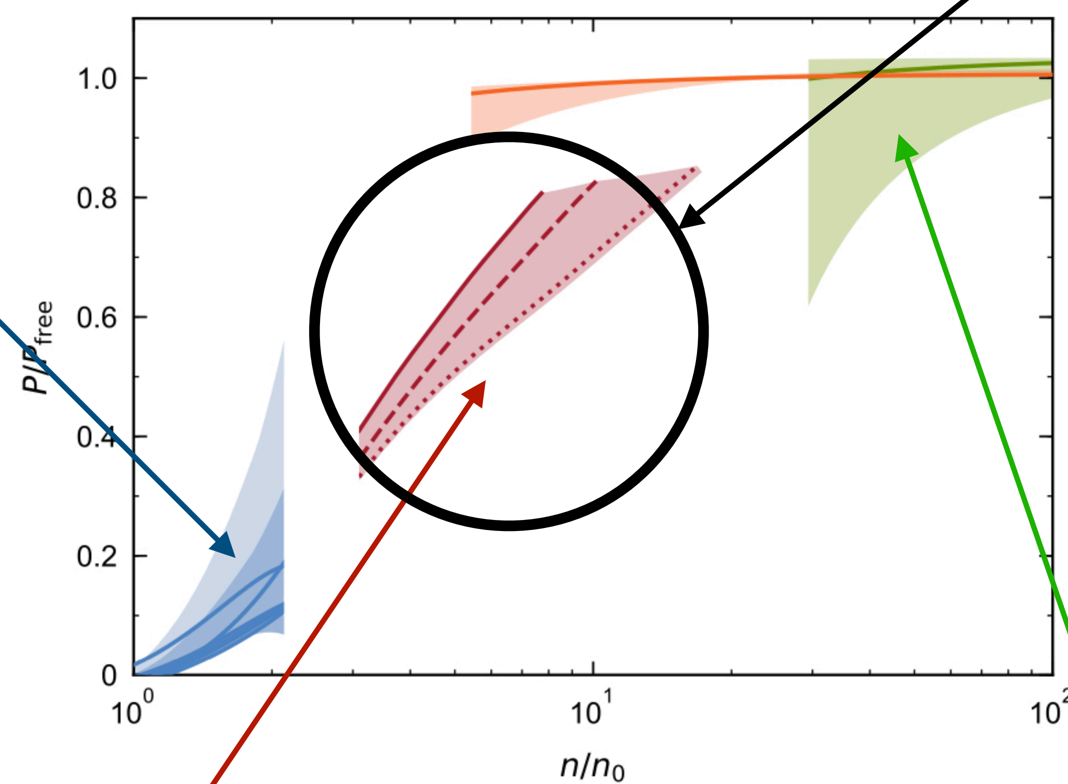
chiralEFT+causality +NS mass constraints

$n \sim [1.5 - 8] n_0$

provides bulk part of model-independent constraints at intermediate densities



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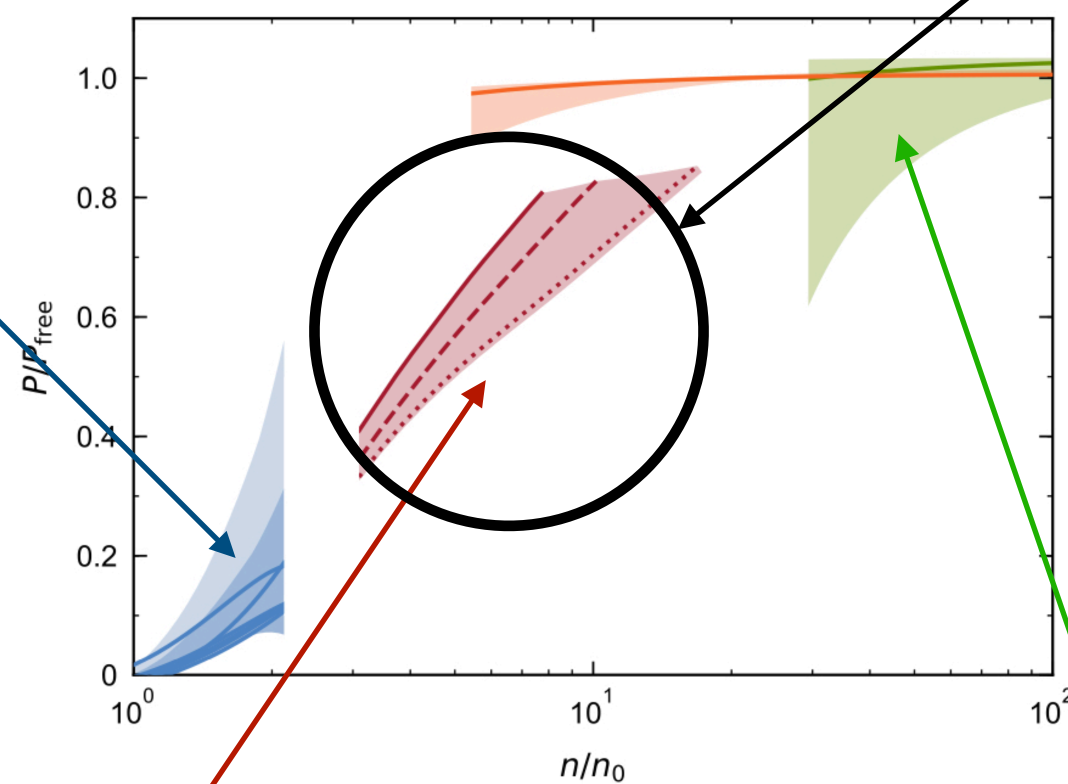

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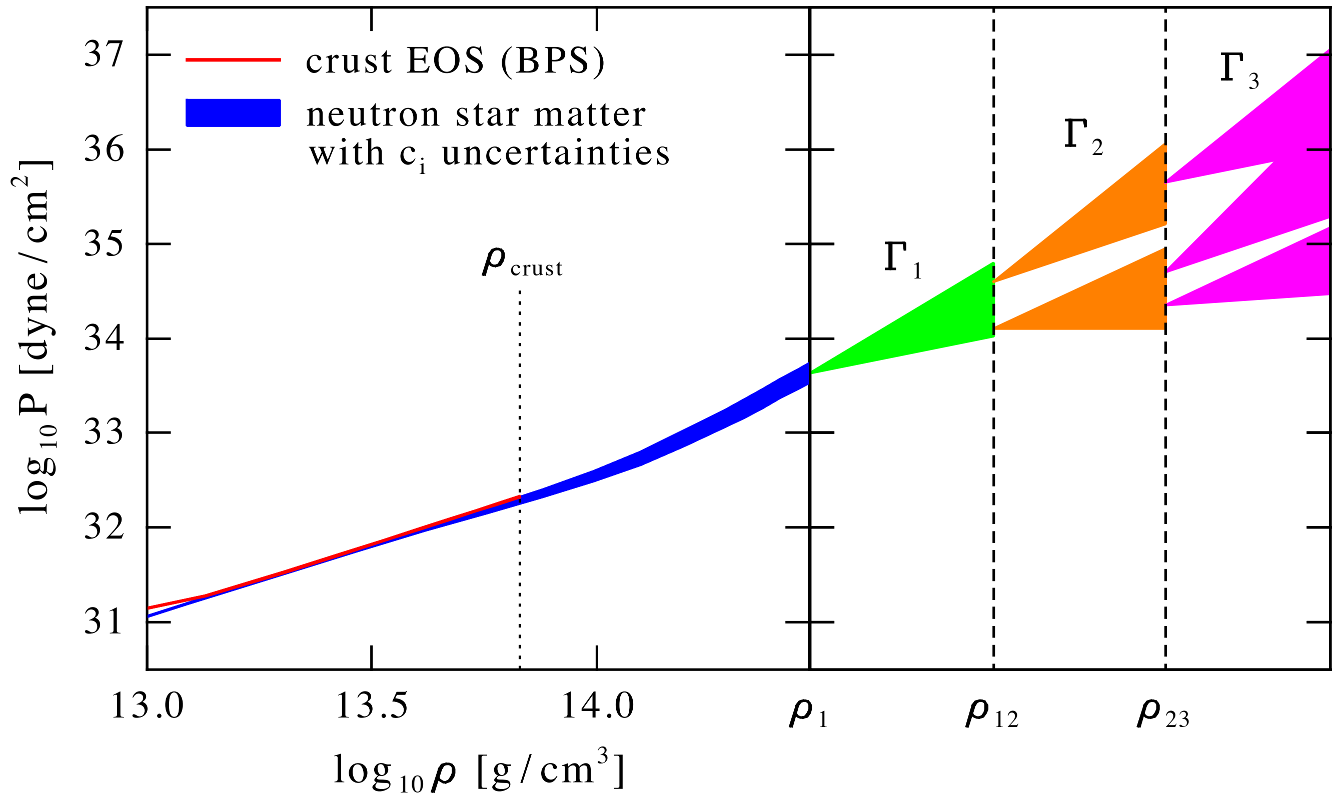
$n > 3n_0 - 4n_0$

still under active development, can be extended to general conditions

current data from GW, NICER and HIC information are consistent with other constraints, but do not lead to significantly improved EOS uncertainties (yet).

Backup slides

EOS at high density and neutron star radius constraints



KH, Lattimer, Pethick, Schwenk, ApJ 773, 11 (2013)

parametrize our ignorance via piecewise high-density extensions of EOS:

- use polytropic ansatz $p \sim \rho^\Gamma$ (results insensitive to particular form)
- range of parameters $\Gamma_1, \rho_{12}, \Gamma_2, \rho_{23}, \Gamma_3$ limited by physics

Incorporate constraints from chiral EFT, causality and neutron star masses

EOS at high density and neutron star radius constraints

use the constraints:

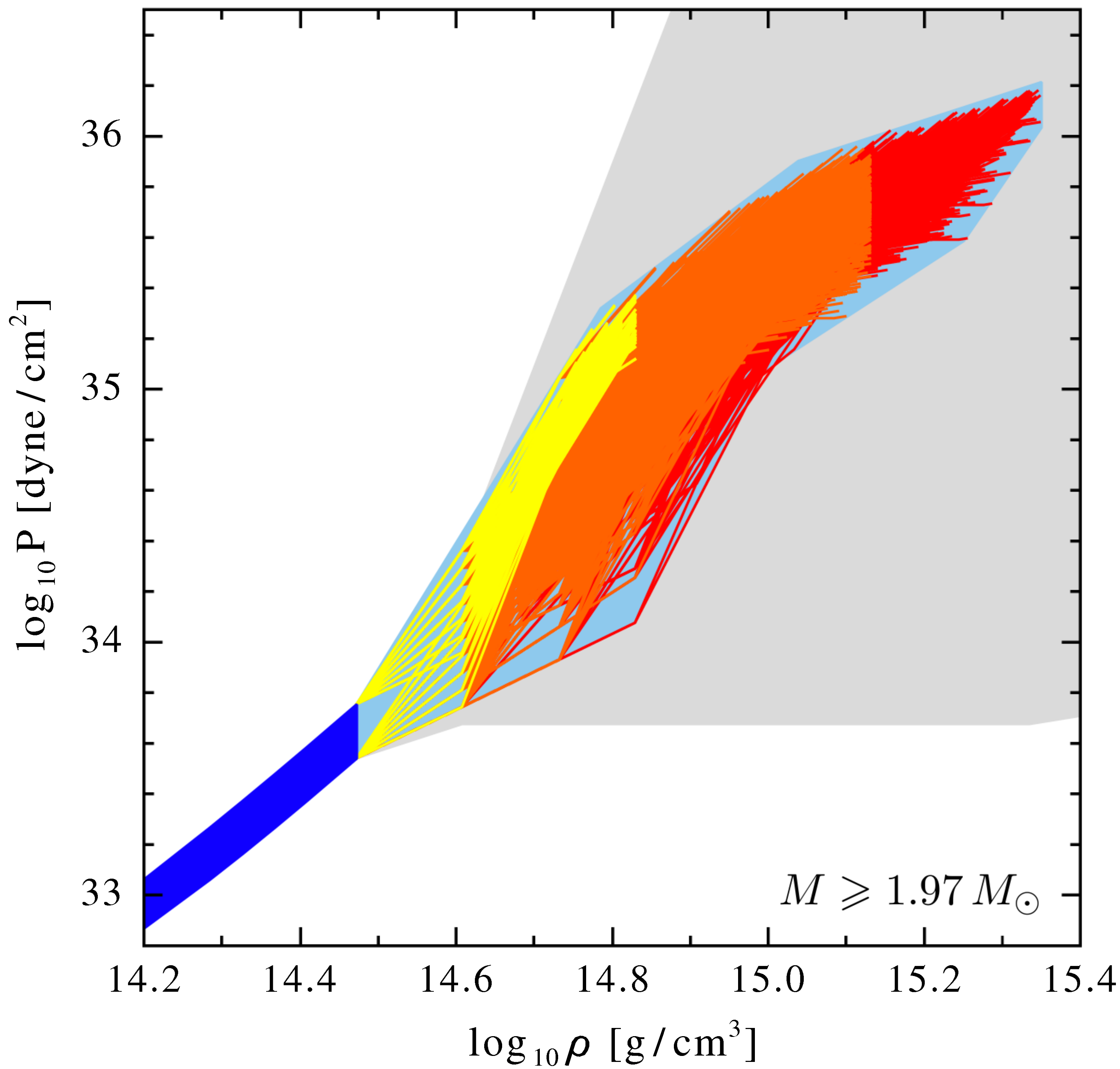
recent NS observations

$$M_{\max} > 1.97\,M_{\odot}$$

causality

$$v_s(\rho) = \sqrt{dP/d\varepsilon} < c$$

KH, Lattimer, Pethick, Schwenk, ApJ 773, 11 (2013)



constraints lead to significant reduction of EOS uncertainty band

EOS at high density and neutron star radius constraints

use the constraints:

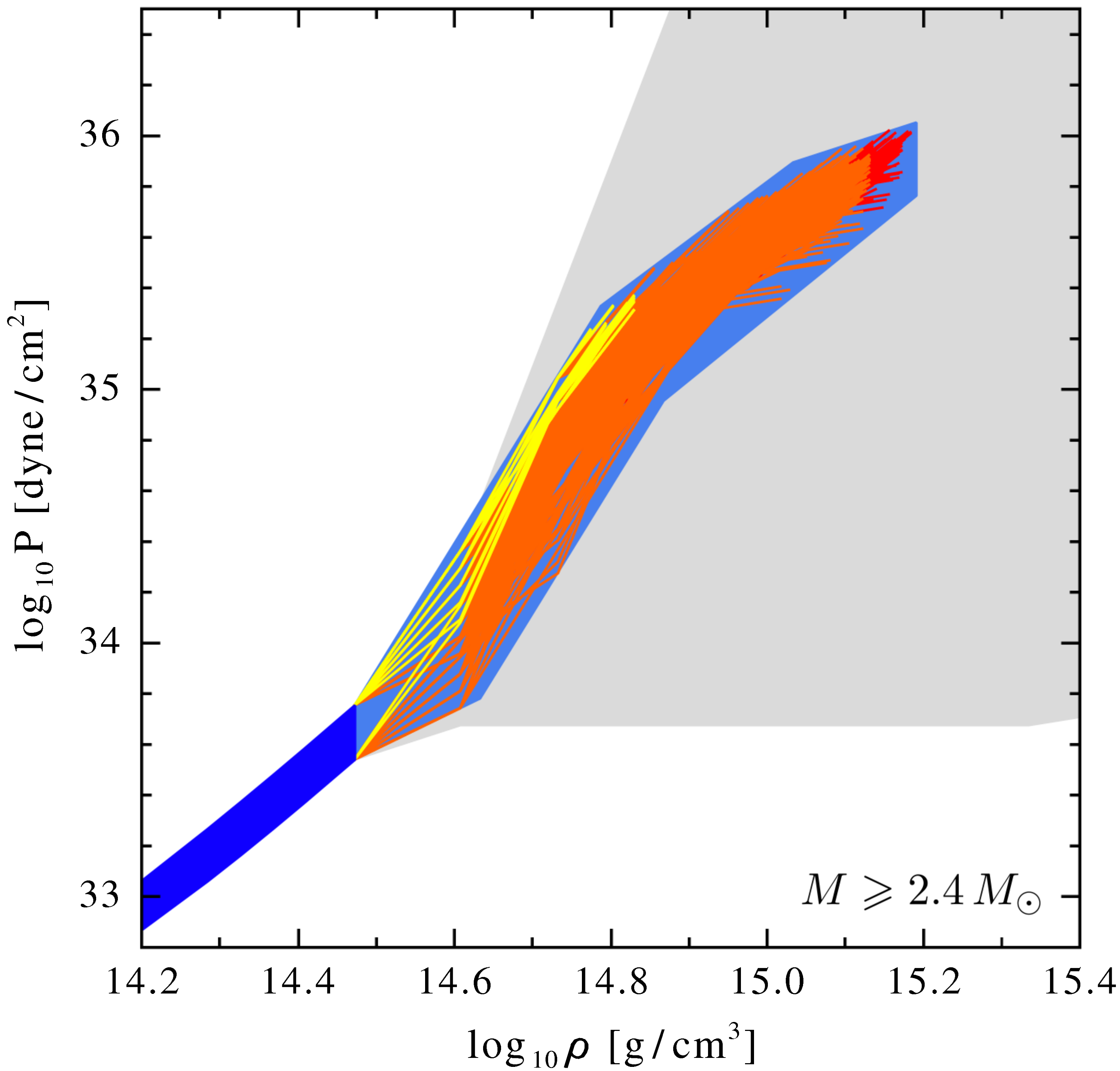
fictitious NS mass

$$M_{\max} > 2.4\, M_\odot$$

causality

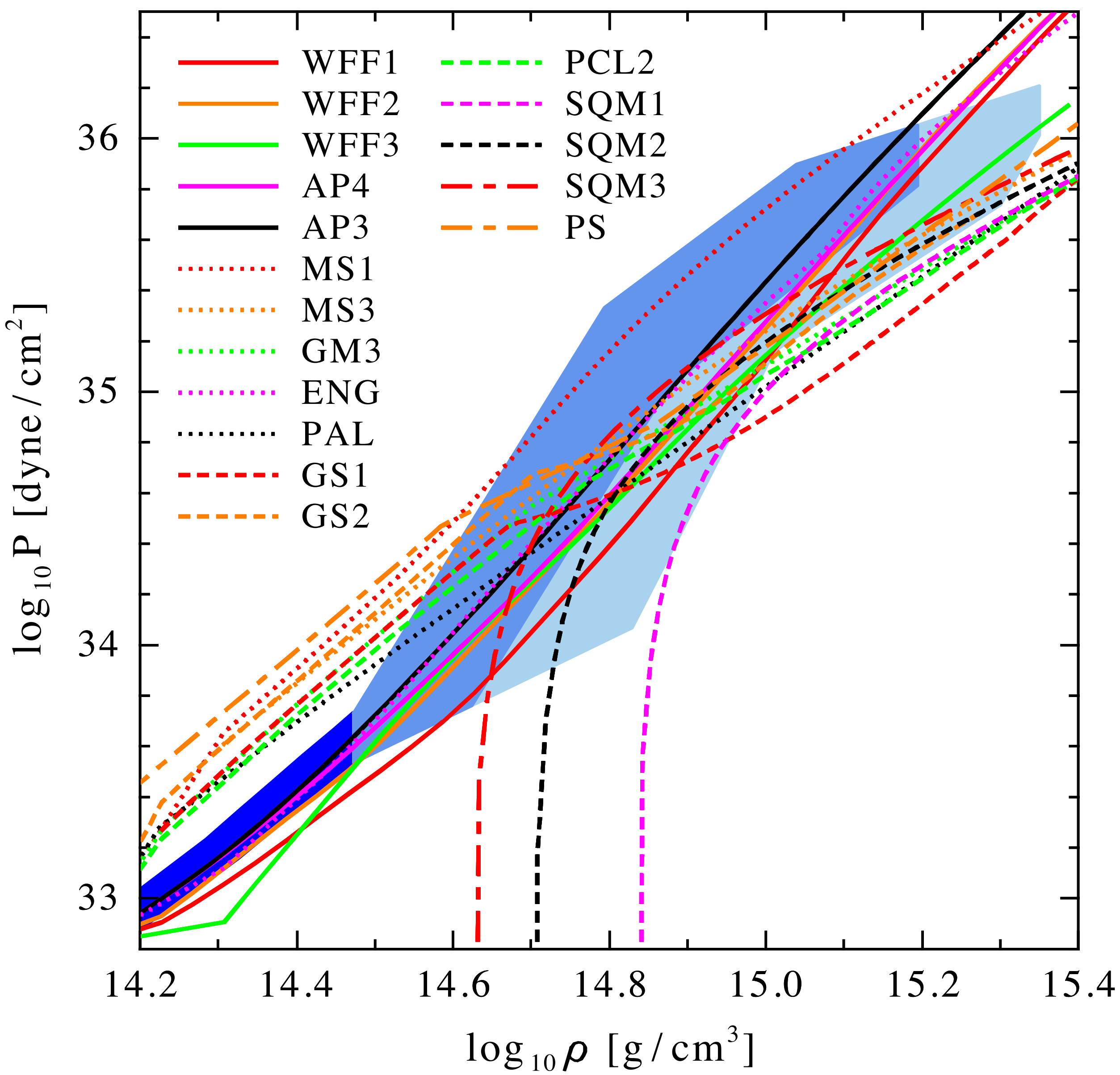
$$v_s(\rho) = \sqrt{dP/d\varepsilon} < c$$

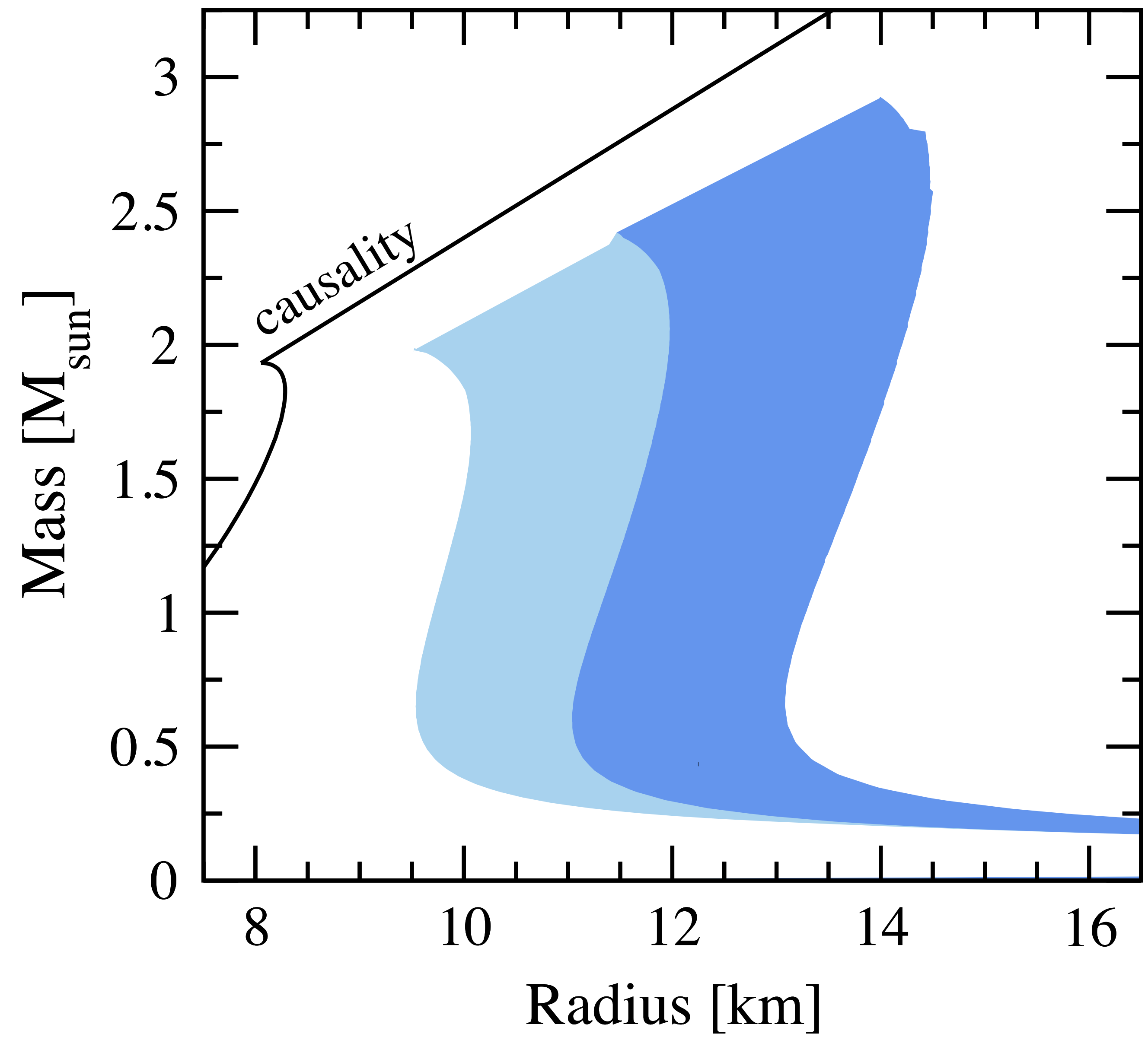
KH, Lattimer, Pethick, Schwenk, ApJ 773, 11 (2013)



increased $M_{\max}$ systematically reduces width of band

EOS at high density and neutron star radius constraints



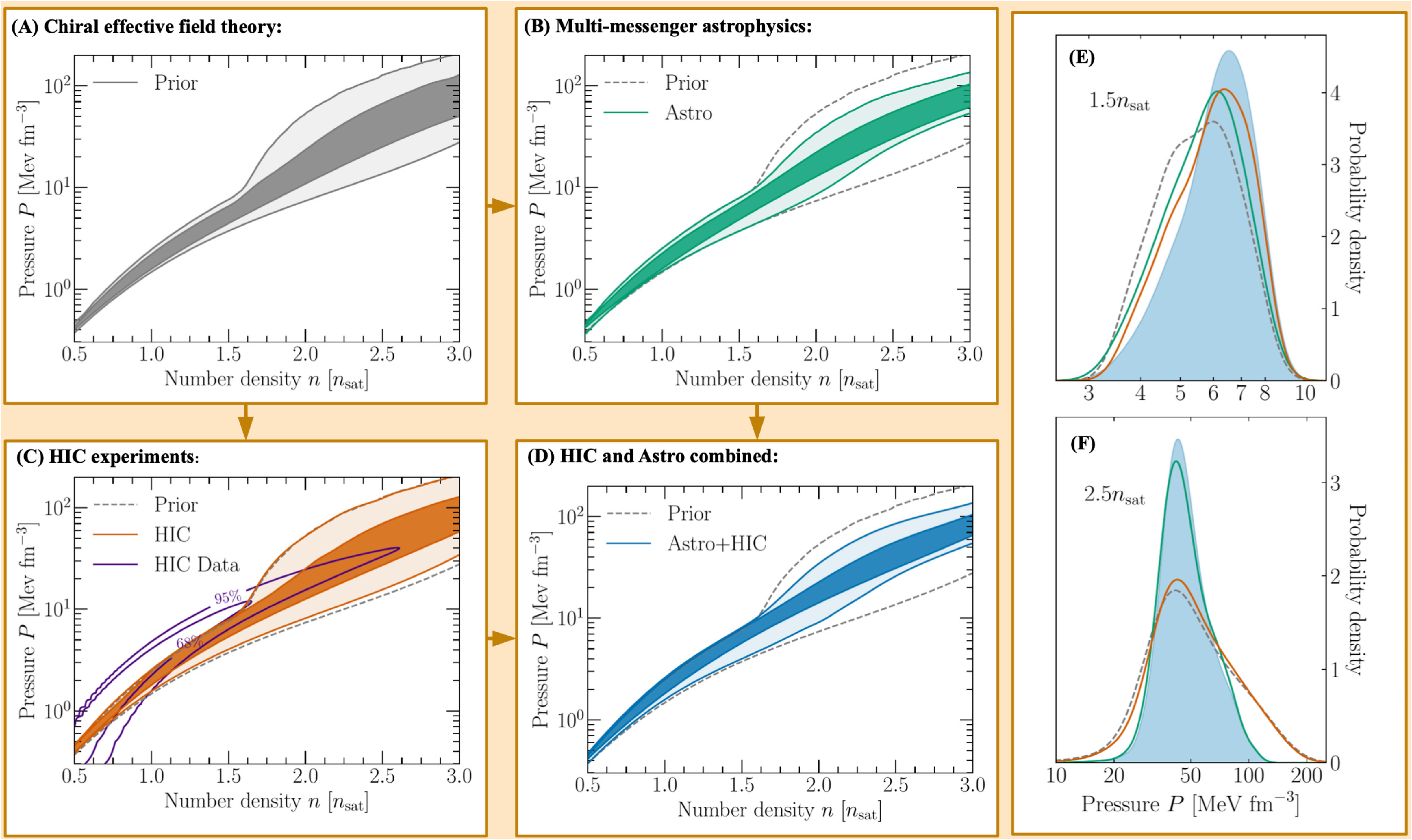


KH, Lattimer, Pethick, Schwenk, ApJ 773, 11 (2013)
see also KH, Lattimer, Pethick, Schwenk, PRL 105, 161102 (2010)

- low-density part of EOS sets scale for allowed high-density extensions
- current radius prediction for typical $1.4\,M_{\odot}$ neutron star: $9.7 - 13.9\,\mathrm{km}$

Constraints from multimessenger astrophysics and heavy ion experiments

Huth at al., Nature 606, 276 (2022)



Current HIC data is consistent with astrophysical constraints, but does not lead to further reduction of EOS uncertainties