Effects and relevance of off-shell transport

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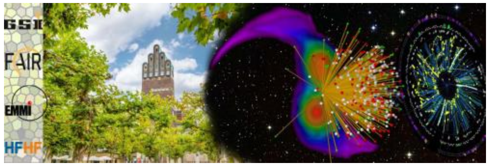
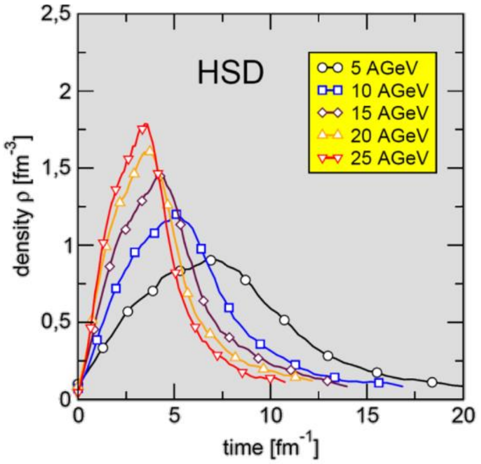
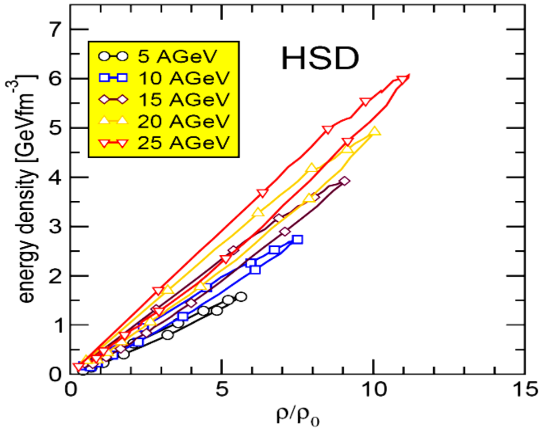


Dense and hot matter created in HICs

Time evolution of baryon density ρ



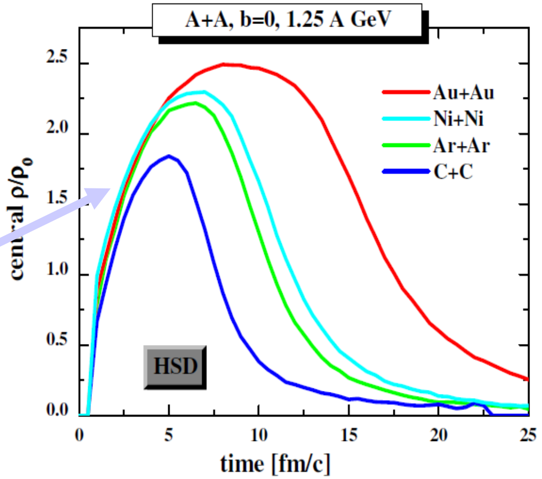
Energy density vs. ρ/ρ_0



Large energy and baryon densities (even above critical $\varepsilon > \varepsilon_{crit} \sim 0.5$ GeV/fm^3) are reached in the central reaction volume at CBM and BM@N/NICA energies (> 5 A GeV)
➔ a phase transition to the QGP

- **At SIS energies: baryon density in central A+A collisions at 1.25 A GeV:**
  - **increases with nuclear size up to 2.5 ρ_0**
  - **the reaction time is larger for heavy systems**

➔ Highly dense matter is created already at SIS energies!



History: Semi-classical BUU equation

Ludwig Boltzmann

Boltzmann-Uehling-Uhlenbeck equation (non-relativistic formulation)
- propagation of particles in the self-generated Hartree-Fock mean-field potential $U(r,t)$ with an on-shell collision term:

$$\frac{\partial}{\partial t} f(\vec{r},\vec{p},t) + \frac{\vec{p}}{m}\vec{\nabla}_{\vec{r}}\, f(\vec{r},\vec{p},t) - \vec{\nabla}_{\vec{r}} U(\vec{r},t)\, \vec{\nabla}_{\vec{p}} f(\vec{r},\vec{p},t) = \left(\frac{\partial f}{\partial t}\right)_{coll}$$

collision term: elastic and inelastic reactions

$f(\vec{r},\vec{p},t)$ **is the single particle phase-space distribution function**
- probability to find the particle at position *r* with momentum *p* at time *t*

- **self-generated Hartree-Fock mean-field potential:**

$$U(\vec{r},t) = \frac{1}{(2\pi\hbar)^3}\sum_{\beta_{occ}}\int d^3r'\, d^3p\;\; V(\vec{r}-\vec{r}',t)\;\, f(\vec{r}',\vec{p},t) \;+\; (Fock\;\; term)$$

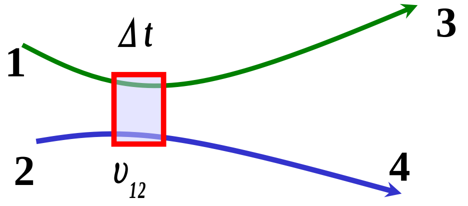
- **Collision term for 1+2→3+4 (let's consider fermions) :**

$$I_{coll} = \frac{4}{(2\pi)^3}\int d^3p_2\, d^3p_3 \int d\Omega\;\; |\upsilon_{12}|\,\delta^3(\vec{p}_1+\vec{p}_2-\vec{p}_3-\vec{p}_4)\cdot\frac{d\sigma}{d\Omega}(1+2\rightarrow 3+4)\cdot P$$

Probability including Pauli blocking of fermions:

$$P = f_3 f_4 (1-f_1)(1-f_2) \;-\; f_1 f_2 (1-f_3)(1-f_4)$$

Gain term: 3+4→1+2 **Loss term: 1+2→3+4**



From weakly to strongly interacting systems

In-medium effects (on hadronic or partonic levels!) = changes of particle properties in the hot and dense medium

Examples: **hadronic medium** - vector mesons, strange mesons, baryons
QGP – dressing of partons

Many-body theory:
Strong interaction ➔ large widths ➔ broad spectral functions ➔ quantum objects

Semi-classical on-shell BUU: applies for small collisional width, i.e. for a weakly interacting systems of particles

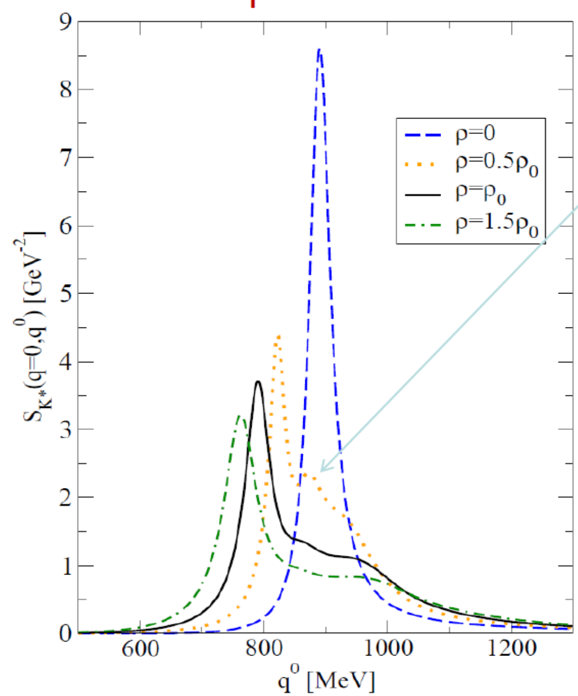
- **How to describe the dynamics of broad strongly interacting quantum states in transport theory?**

❑ **semi-classical BUU**

first order gradient expansion of quantum Kadanoff-Baym equations

❑ **generalized transport equations based on Kadanoff-Baym dynamics**

Kbar* spectral function



$\Lambda(1783)N^{-1}$ and $\Sigma(1830)N^{-1}$ exitations

Barcelona / Valencia group

Dynamical description of strongly interacting systems

Quantum field theory ➔

Kadanoff-Baym dynamics for resummed single-particle Green functions $S^<$

(1962)

$$\hat{S}^{-1}_{0x}\; S^{<}_{xy} \;=\; \Sigma^{ret}_{xz} \odot S^{<}_{zy} \;+\; \Sigma^{<}_{xz} \odot S^{adv}_{zy}$$

Integration over the intermediate spacetime

Green functions $S^<$ / self-energies Σ :

$$iS^{<}_{xy} = \eta\langle\{\Phi^{+}(y)\Phi(x)\}\rangle$$

$$iS^{>}_{xy} = \langle\{\Phi(y)\Phi^{+}(x)\}\rangle$$

$$iS^{c}_{xy} = \langle T^{c}\{\Phi(x)\Phi^{+}(y)\}\rangle \quad - causal$$

$$iS^{a}_{xy} = \langle T^{a}\{\Phi(x)\Phi^{+}(y)\}\rangle \quad - anticausal$$

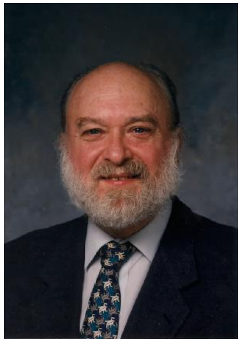
$$S^{ret}_{xy} = S^{c}_{xy} - S^{<}_{xy} = S^{>}_{xy} - S^{a}_{xy} \quad - retarded$$

$$S^{adv}_{xy} = S^{c}_{xy} - S^{>}_{xy} = S^{<}_{xy} - S^{a}_{xy} \quad - advanced$$

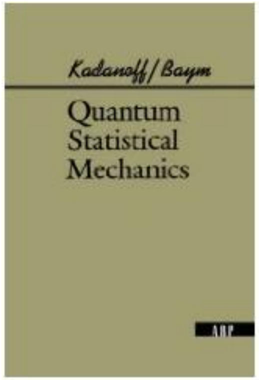
$$\hat{S}^{-1}_{0x} \equiv -(\partial^{\mu}_{x}\partial^{x}_{\mu} + M^{2}_{0})$$

$$\eta = \pm 1(bosons\,/\,fermions)$$

$$T^{a}(T^{c}) - (anti-)time - ordering\ operator$$



Leo Kadanoff



Gordon Baym

1st application for spacially homodeneous system with deformed Fermi sphere:
P. Danielewicz, Ann. Phys. 152, 305 (1984); … H.S. Köhler, Phys. Rev. 51, 3232 (1995); …

From Kadanoff-Baym equations to generalized transport equations

After the first order gradient expansion of the Wigner transformed Kadanoff-Baym equations and separation into the real and imaginary parts one gets:

Generalized transport equations (GTE):

drift term | Vlasov term | backflow term | collision term = ‚gain' - ‚loss' term

$$\diamond\{P^2 - M_0^2 - Re\Sigma^{ret}_{XP}\}\{S^<_{XP}\} - \diamond\{\Sigma^<_{XP}\}\{ReS^{ret}_{XP}\} = \frac{i}{2}[\Sigma^>_{XP}S^<_{XP} - \Sigma^<_{XP}S^>_{XP}]$$

Backflow term incorporates the off-shell behavior in the particle propagation
! vanishes in the quasiparticle limit $A_{XP} \to \delta(p^2-M^2)$

- GTE: Propagation of the Green's function $iS^<_{XP}=A_{XP}N_{XP}$, which carries information not only on the number of particles (N_{XP}), but also on their properties, interactions and correlations (via A_{XP})

- Spectral function:

$$A_{XP} = \frac{\Gamma_{XP}}{(P^2 - M_0^2 - Re\Sigma^{ret}_{XP})^2 + \Gamma^2_{XP}/4}$$

W. Botermans, R. Malfliet, Phys. Rep. 198 (1990) 115

Reaction rate of particle (at space-time position X):

$$\Gamma_{XP} = -Im\,\Sigma^{ret}_{XP} = 2\,p_0\Gamma$$ where Γ is a ‚width' of spectral function

4-dimentional generalizaton of the Poisson-bracket:

$$\diamond\{F_1\}\{F_2\} := \frac{1}{2}\left(\frac{\partial F_1}{\partial X_\mu}\frac{\partial F_2}{\partial P^\mu} - \frac{\partial F_1}{\partial P_\mu}\frac{\partial F_2}{\partial X^\mu}\right)$$

- Life time $\tau = \frac{\hbar c}{\Gamma}$

W. Cassing , S. Juchem, NPA 665 (2000) 377; 672 (2000) 417; 677 (2000) 445

General testparticle off-shell equations of motion

W. Cassing , S. Juchem, NPA 665 (2000) 377; 672 (2000) 417; 677 (2000) 445

- Employ testparticle Ansatz for the real valued quantity $i\, S^{<}_{XP}$

$$F_{XP} = A_{XP} N_{XP} = i\, S^{<}_{XP} \sim \sum_{i=1}^{N} \delta^{(3)}(\vec{X} - \vec{X}_i(t))\ \delta^{(3)}(\vec{P} - \vec{P}_i(t))\ \delta(P_0 - \epsilon_i(t))$$

insert in generalized transport equations and determine equations of motion !

➔ **Generalized testparticle Cassing-Juchem off-shell equations of motion for the time-like particles:**

$$\frac{d\vec{X}_i}{dt} = \frac{1}{1 - C_{(i)}} \frac{1}{2\epsilon_i} \left[2\,\vec{P}_i + \vec{\nabla}_{P_i} Re\Sigma^{ret}_{(i)} + \frac{\epsilon_i^2 - \vec{P}_i^2 - M_0^2 - Re\Sigma^{ret}_{(i)}}{\Gamma_{(i)}} \vec{\nabla}_{P_i} \Gamma_{(i)} \right],$$

$$\frac{d\vec{P}_i}{dt} = -\frac{1}{1 - C_{(i)}} \frac{1}{2\epsilon_i} \left[\vec{\nabla}_{X_i} Re\Sigma^{ret}_{i} + \frac{\epsilon_i^2 - \vec{P}_i^2 - M_0^2 - Re\Sigma^{ret}_{(i)}}{\Gamma_{(i)}} \vec{\nabla}_{X_i} \Gamma_{(i)} \right],$$

$$\frac{d\epsilon_i}{dt} = \frac{1}{1 - C_{(i)}} \frac{1}{2\epsilon_i} \left[\frac{\partial Re\Sigma^{ret}_{(i)}}{\partial t} + \frac{\epsilon_i^2 - \vec{P}_i^2 - M_0^2 - Re\Sigma^{ret}_{(i)}}{\Gamma_{(i)}} \frac{\partial \Gamma_{(i)}}{\partial t} \right],$$

with $F_{(i)} \equiv F(t, \vec{X}_i(t), \vec{P}_i(t), \epsilon_i(t))$

$$C_{(i)} = \frac{1}{2\epsilon_i} \left[\frac{\partial}{\partial \epsilon_i} Re\Sigma^{ret}_{(i)} + \frac{\epsilon_i^2 - \vec{P}_i^2 - M_0^2 - Re\Sigma^{ret}_{(i)}}{\Gamma_{(i)}} \frac{\partial}{\partial \epsilon_i} \Gamma_{(i)} \right]$$

Realized in PHSD

Note: the common factor $1/(1-C_{(i)})$ can be absorbed in an ‚eigentime' of particle (i) !

On-shell limits

- **$\Gamma(X,P) \rightarrow 0$**

$$A_{XP} = \frac{\Gamma_{XP}}{(P^2 - M_0^2 - Re\Sigma^{ret}_{XP})^2 + \Gamma^2_{XP}/4}$$

quasiparticle approximation :

$$A_{XP} = 2\,p\,\delta(P^2 - M_0^2)$$

- **$\Gamma(X,P)$ such that**

$$\nabla_X \Gamma = 0 \quad \text{and} \quad \nabla_P \Gamma = 0$$

E.g.: Γ = const

$\Gamma = \Gamma_{vacuum}$ (M)

‚Vacuum' spectral function with constant or mass dependent width Γ:
i.e. spectral function A_{XP} does NOT change the shape (and pole position) during propagation through the medium

In on-shell limits the ‚backflow term' - which incorporates the off-shell behavior in the particle propagation - vanishes !

$$\frac{d\vec{X}_i}{dt} = \frac{1}{1 - C_{(i)}} \frac{1}{2\epsilon_i} \left[2\,\vec{P}_i + \vec{\nabla}_{P_i} Re\Sigma^{ret}_{(i)} + \frac{\epsilon_i^2 - \vec{P}_i^2 - M_0^2 - Re\Sigma^{ret}_{(i)}}{\Gamma_{(i)}} \vec{\nabla}_{P_i} \Gamma_{(i)} \right],$$

$$\frac{d\vec{P}_i}{dt} = -\frac{1}{1 - C_{(i)}} \frac{1}{2\epsilon_i} \left[\vec{\nabla}_{X_i} Re\Sigma^{ret}_i + \frac{\epsilon_i^2 - \vec{P}_i^2 - M_0^2 - Re\Sigma^{ret}_{(i)}}{\Gamma_{(i)}} \vec{\nabla}_{X_i} \Gamma_{(i)} \right],$$

$$\frac{d\epsilon_i}{dt} = \frac{1}{1 - C_{(i)}} \frac{1}{2\epsilon_i} \left[\frac{\partial Re\Sigma^{ret}_{(i)}}{\partial t} + \frac{\epsilon_i^2 - \vec{P}_i^2 - M_0^2 - Re\Sigma^{ret}_{(i)}}{\Gamma_{(i)}} \frac{\partial \Gamma_{(i)}}{\partial t} \right],$$

Hamilton equations of motion (independent on Γ) → BUU limit

W. Cassing, Eur. Phys. J. ST 168 (2009) 3

Collision term in off-shell transport models

Collision term for reaction 1+2->3+4:

$$I_{coll}(X,\vec{P},M^2) = Tr_2 Tr_3 Tr_4 A(X,\vec{P},M^2) A(X,\vec{P}_2,M_2^2) A(X,\vec{P}_3,M_3^2) A(X,\vec{P}_4,M_4^2)$$

$$|G((\vec{P},M^2)+(\vec{P}_2,M_2^2) \to (\vec{P}_3,M_3^2)+(\vec{P}_4,M_4^2))|^2_{\mathcal{A},\mathcal{S}} \; \delta^{(4)}(P+P_2-P_3-P_4)$$

$$[N_{X\vec{P}_3M_3^2} \, N_{X\vec{P}_4M_4^2} \, \bar{f}_{X\vec{P}M^2} \, \bar{f}_{X\vec{P}_2M_2^2} \; - \; N_{X\vec{P}M^2} \, N_{X\vec{P}_2M_2^2} \, \bar{f}_{X\vec{P}_3M_3^2} \, \bar{f}_{X\vec{P}_4M_4^2}]$$

'gain' term　　　　'loss' term

with $\bar{f}_{X\vec{P}M^2} = 1 + \eta \, N_{X\vec{P}M^2}$ and $\eta = \pm 1$ for bosons/fermions, respectively.

The trace over particles 2,3,4 reads explicitly

for fermions

$$Tr_2 = \sum_{\sigma_2,\tau_2} \frac{1}{(2\pi)^4} \int d^3P_2 \frac{dM_2^2}{2\sqrt{\vec{P}_2^2 + M_2^2}}$$

for bosons

$$Tr_2 = \sum_{\sigma_2,\tau_2} \frac{1}{(2\pi)^4} \int d^3P_2 \frac{dP_{0,2}^2}{2}$$

additional integration

The transport approach and the particle spectral functions are fully determined once the in-medium transition amplitudes G are known in their off-shell dependence!

Mean-field potential in off-shell transport models

- **Many-body theory:** Interacting relativistic particles have a **complex self-energy:**

$$\Sigma^{ret}_{XP} = Re\,\Sigma^{ret}_{XP} + i\,Im\,\Sigma^{ret}_{XP}$$

The neg. imaginary part $\Gamma_{XP} = -Im\,\Sigma^{ret}_{XP} = 2p_0\Gamma$ is related via the width $\Gamma = \Gamma_{coll} + \Gamma_{dec}$ to the inverse livetime of the particle $\tau \sim 1/\Gamma$

- The **collision width** Γ_{coll} is determined from the **loss term** of the collision integral I_{coll}

$$-I_{coll}(loss) = \Gamma_{coll}(X,\vec{P},M^2)N_{X\vec{P}M^2}$$

- By **dispersion relation** we get a contribution to the **real part of self-energy**:

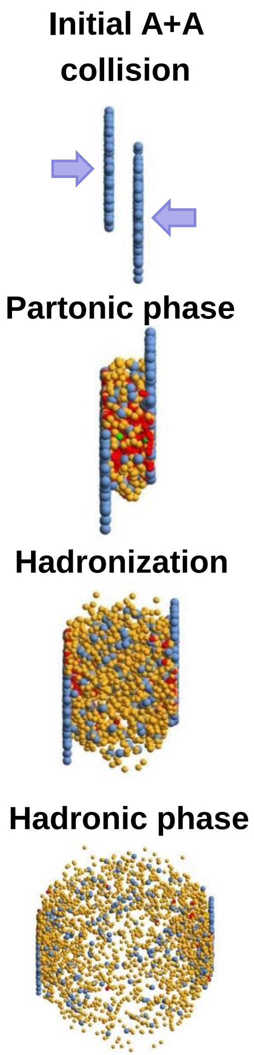
$$Re\,\Sigma^{ret}_{XP}(p_0) = P\int_0^\infty dq\,\frac{Im\,\Sigma^{ret}_{XP}(q)}{(q-p_0)}$$

which gives **a mean-field potential U_{XP}** via: $Re\,\Sigma^{ret}_{XP}(p_0) = 2p_0U_{XP}$

→ The **complex self-energy** relates in a self-consistent way to the **self-generated mean-field potential and collision width**

* Cf. Giessen group: J. Lehr et al., NPA703 (2002) 393 ‚Nuclear matter spectral functions by transport theory'

Parton-Hadron-String-Dynamics (PHSD)



PHSD is a non-equilibrium microscopic transport approach for the description of strongly-interacting hadronic and partonic matter created in heavy-ion collisions

- **Dynamics:**
  based on the solution of generalized off-shell Cassing-Juchem transport equations derived from Kadanoff-Baym many-body theory

- **Generalized off-shell collision integral:**
  for $n \leftrightarrow m$ selected reactions (for strangeness, anti-baryons, deuteron production)

- **QGP:**
  strongly interacting quasiparticles (quarks and gluons) with dynamical temperature and density spectral functions

W. Cassing, E. Bratkovskaya, PRC 78 (2008) 034919; NPA831 (2009) 215; W. Cassing, EPJ ST 168 (2009) 3

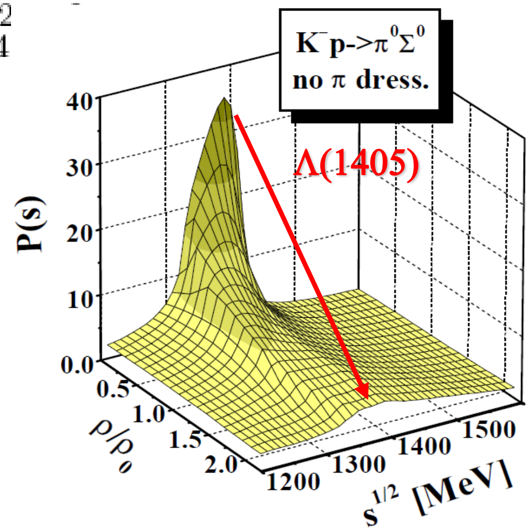
In-medium transition rates: G-matrix approach

Need to know in-medium transition amplitudes G and their off-shell dependence

$$|G((\vec{P}, M^2) + (\vec{P}_2, M_2^2) \rightarrow (\vec{P}_3, M_3^2) + (\vec{P}_4, M_4^2$$

↓

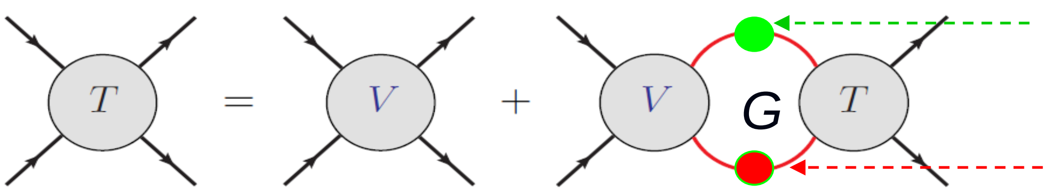
Coupled channel G-matrix approach



Transition probability :

$$P_{1+2\rightarrow 3+4}(s) = \int d\cos(\theta)\ \frac{1}{(2s_1+1)(2s_2+1)} \sum_i \sum_\alpha\ G^\dagger G$$

with G(p,ρ,T) - G-matrix from the solution of coupled-channel equations:



Meson selfenergy and spectral function

Baryons: Pauli blocking and potential dressing

$$\mathcal{T}_{ij}(\rho, T) = V_{ij} + V_{il}\ G_l(\rho, T)\ \mathcal{T}_{lj}(\rho, T)$$

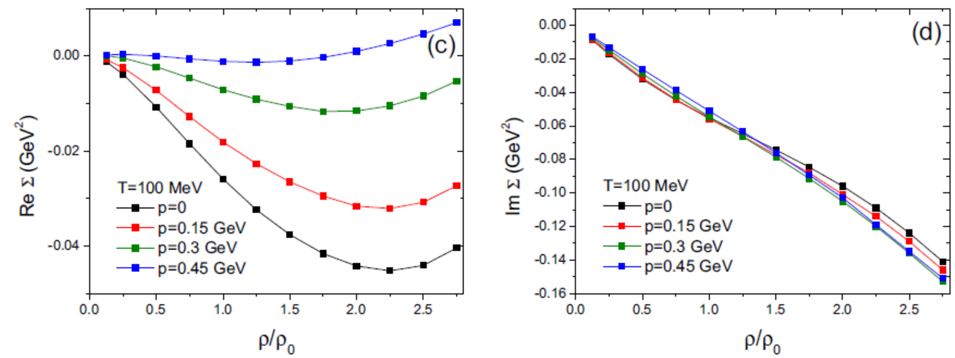
For strangeness:
D. Cabrera, L. Tolos, J. Aichelin, E.B., PRC C90 (2014) 055207; W. Cassing, L. Tolos, E.B., A. Ramos, NPA727 (2003) 59; T. Song et al., PRC 103, 044901 (2021)

Off-shell dynamics for antikaons at SIS energies

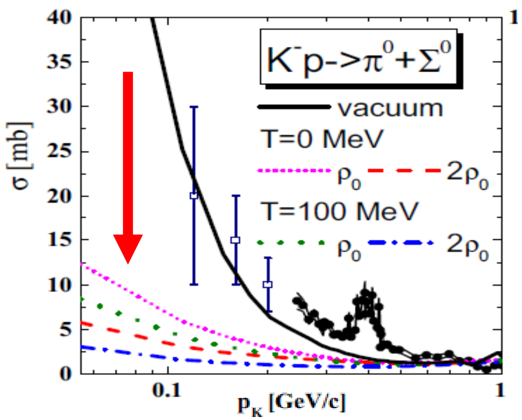
Spectral function of K^- within the G-matrix approach:

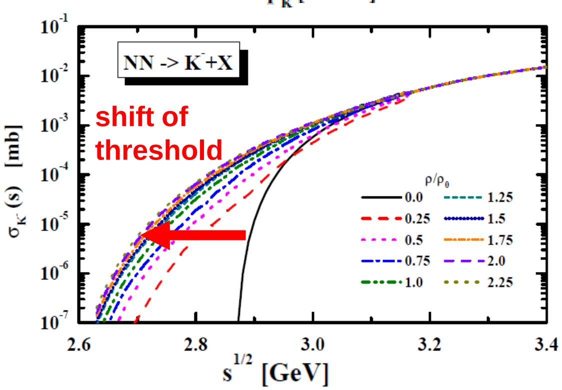
$$S_{\bar{K}}(k_0,\vec{k};T) = -\frac{1}{\pi}\frac{\mathrm{Im}\,\Sigma_{\bar{K}}(k_0,\vec{k};T)}{\left|k_0^2-\vec{k}^{\,2}-m_{\bar{K}}^2-\Sigma_{\bar{K}}(k_0,\vec{k};T)\right|^2}.$$

D. Cabrera et al., Phys.Rev.C 90 (2014) 055207

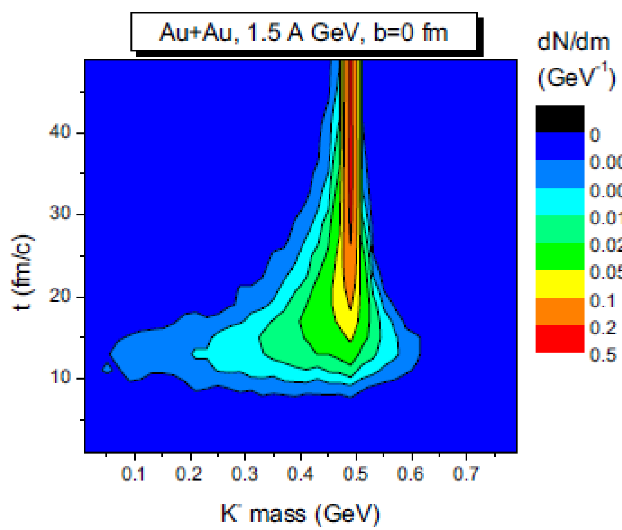


In-medium cross sections for K^- production and absorption are strongly modified in the medium:

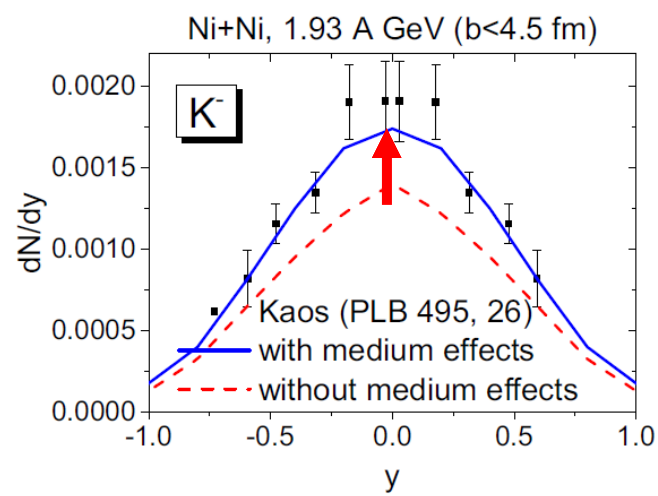




Time evolution of the K^- masses



In-medium effects are mandatory for the description of experimental K^- spectra



T. Song et al., PRC 103, 044901 (2021)

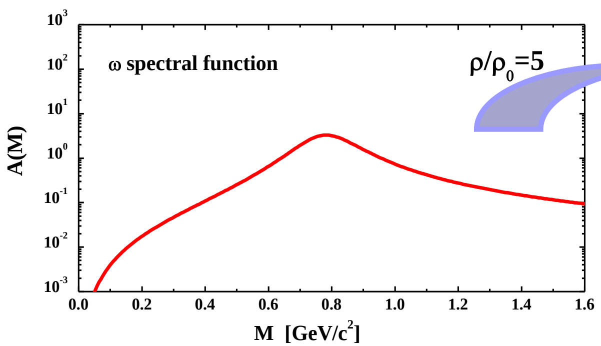
Short-lived resonances in semi-classical transport models

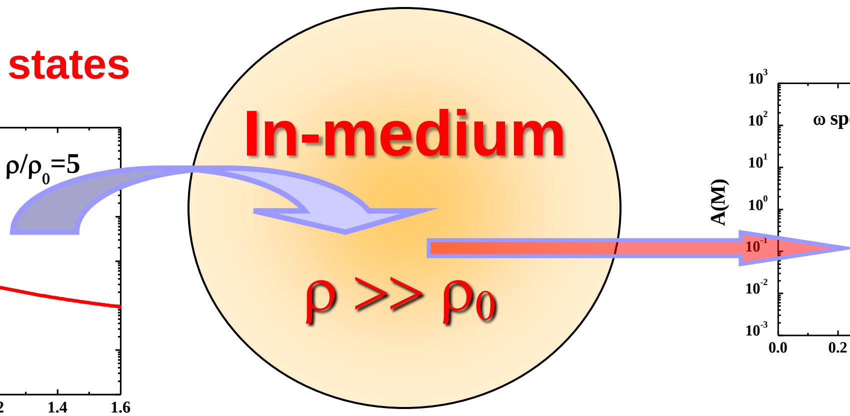
Spectral function of vector mesons (ρ, ω, ϕ):

$$A(M,p,\rho)=\frac{2}{\pi}\frac{M^2\Gamma_{tot}(M,p,\rho)}{(M^2-M_0^2-Re\Sigma^{ret})+\left(M\Gamma_{tot}(M,p,\rho)\right)^2},$$

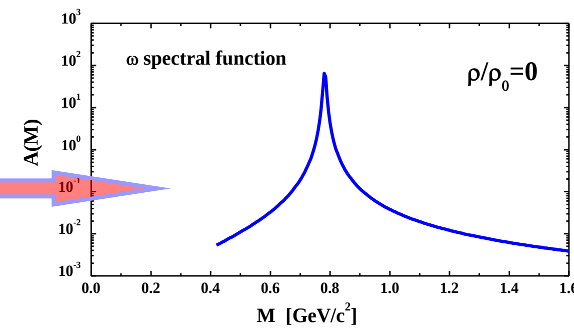
width $\Gamma \sim -\mathrm{Im}\,\Sigma^{ret}/M$

In-medium:
production of broad states



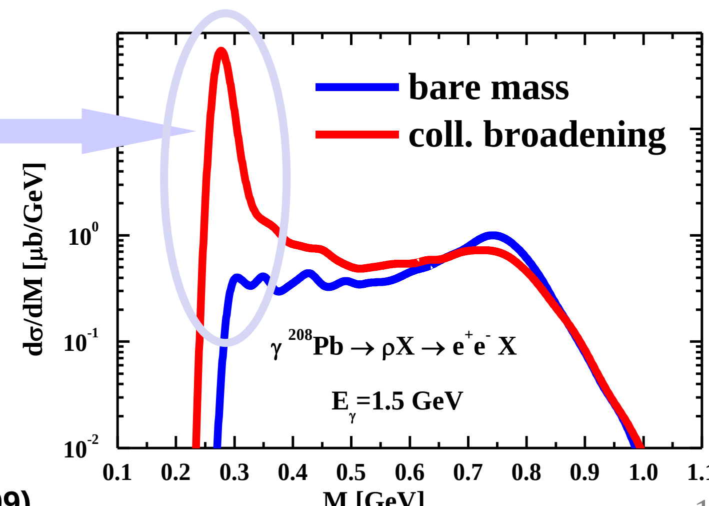


Vacuum (ρ =0)
narrow states



Example :
ρ-meson propagation through the medium within on-shell BUU model

➔ **broad in-medium spectral function does not become on-shell in vacuum by propagation within ‚on-shell' transport models!**



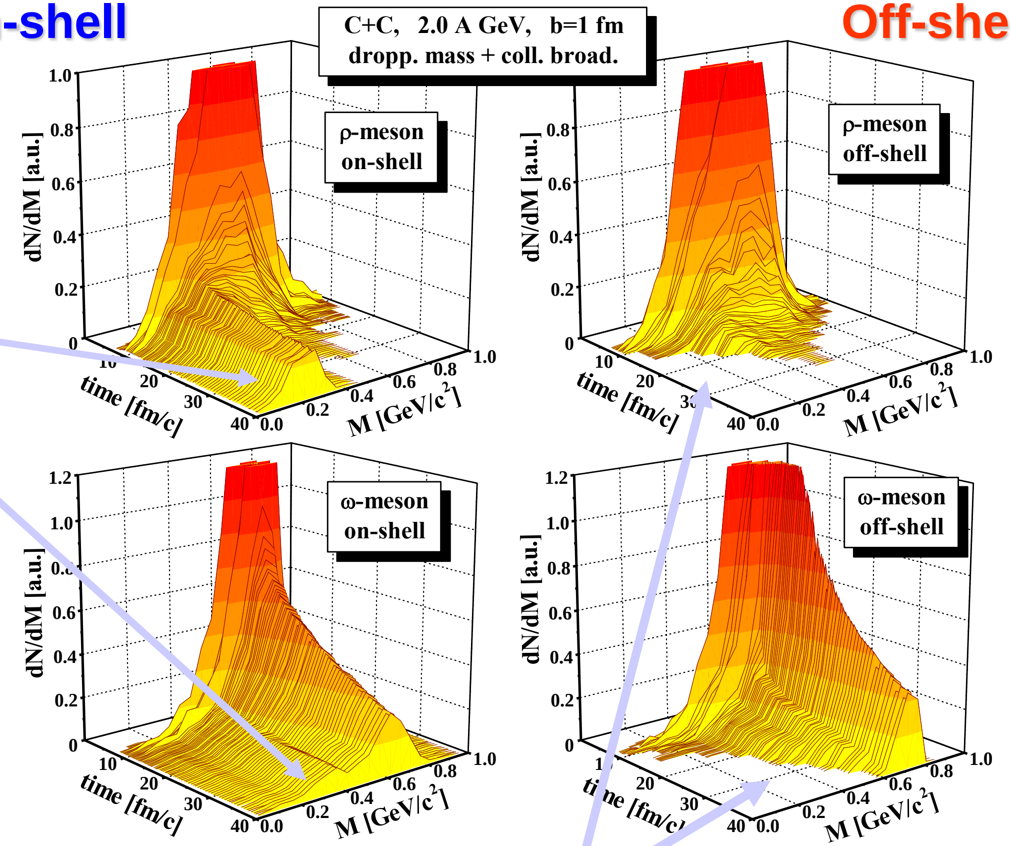
eBUU: M. Effenberger et al, PRC60 (1999)

Off-shell vs. on-shell transport dynamics

Time evolution of the mass distribution of ρ and ω mesons for central C+C collisions (b=1 fm) at 2 A GeV for dropping mass + collisional broadening scenario

On-shell **Off-shell**

On-shell model:
low mass ρ and ω
mesons live forever
and shine dileptons!

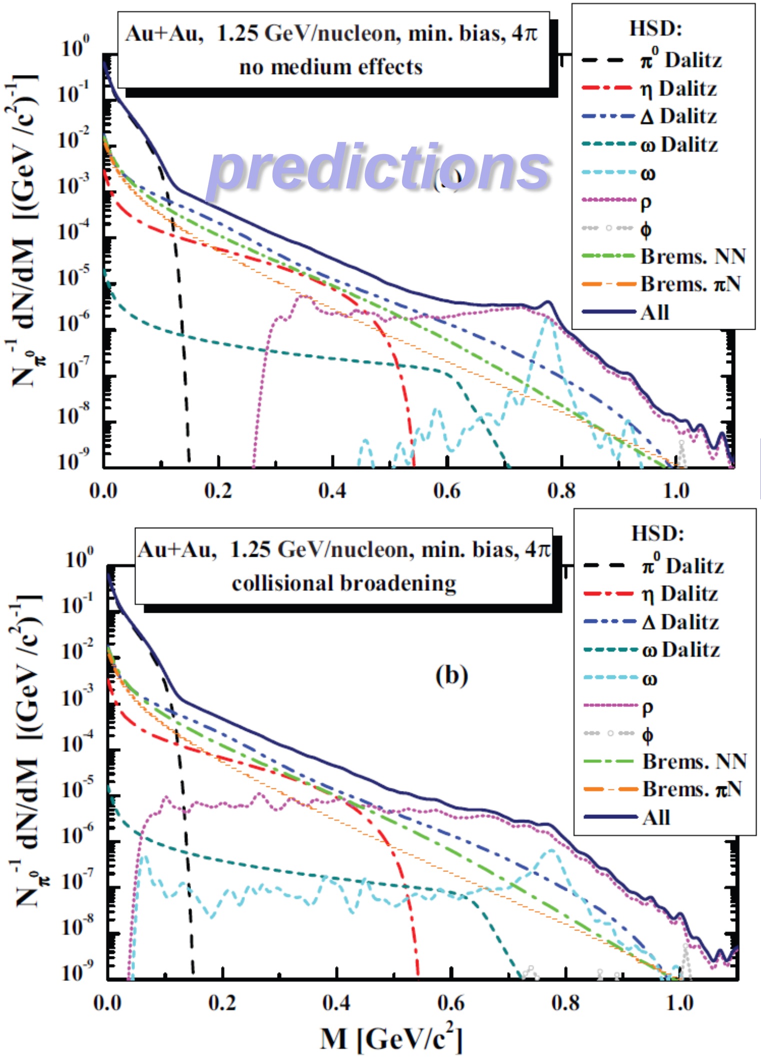


EB, W. Cassing,
NPA 807 (2008) 214

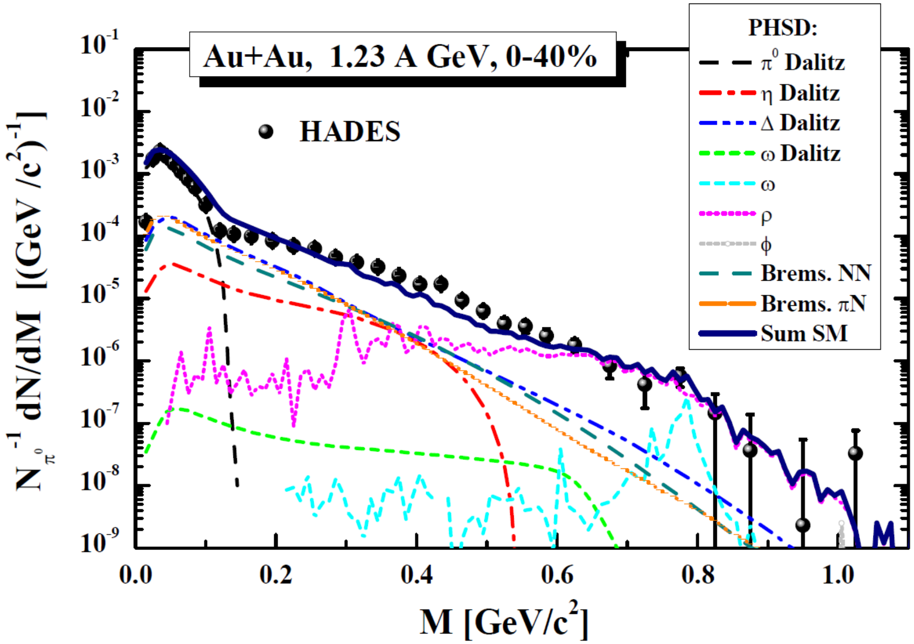
The off-shell spectral function becomes on-shell in the vacuum dynamically by propagation through the medium within off-shell KB

Dileptons at SIS energies - HADES

- **HADES:** Au+Au at 1.23 AGeV - dilepton yield dN/dM scaled with the number of $N_{\pi 0}$



E. B., J. Aichelin, M. Thomere, S. Vogel, and M. Bleicher, PRC 87 (2013) 064907



- In-medium effects are stronger for large systems (Au+Au)

HADES: J. Adamczewski-Musch et al., Nat. Phys. 15 (2019), 1040.
PHSD: I. Schmidt, E.B., M. Gumberidze, R. Holzmann, Phys. Rev. D 104 (2021), 015008

Advantages of Kadanoff-Baym dynamics vs Boltzmann

Kadanoff-Baym equations:

- propagate two-point Green functions
  $G^<(x,p) \rightarrow A(x,p)*N(x,p)$
  in 8 dimensions $x=(t,\vec{r})$ $p=(p_0,\vec{p})$
- $G^<$ carries information not only on the occupation number N_{XP}, but also on the particle properties, interactions and correlations via spectral function A_{XP}

Boltzmann equations

- propagate phase space distribution function $f(\vec{r},\vec{p},t)$ in 6+1 dimensions
- works well for small coupling = weakly interacting system,
  ➔ on-shell approach

- Applicable for strong coupling = strongly interaction system
- Includes memory effects (time integration) and off-shell transitions in collision term
- Dynamically generates a broad spectral function for strong coupling
- KB can be solved exactly for model cases as Φ^4 – theory
- KB can be solved in 1st order gradient expansion in terms of generalized transport equations (in test particle ansatz) for realistic systems of HICs

➔ W. Cassing, *`Transport Theories for Strongly-Interacting Systems'*, Springer Nature: Lecture Notes in Physics 989, 2021
DOI: 10.1007/978-3-030-80295-0

Summary

The developments in microscopic transport theory in the last decades - based on the solution of generalized transport equations as derived from Kadanoff-Baym dynamics - made it applicable for the description of strongly-interacting hadronic and partonic matter as created in heavy-ion collisions from SIS to LHC energies

Note:
for a consistent description of HICs the input from lQCD and many-body theory is mandatory:
properties of partonic and hadronic degrees-of-freedom and their in-medium interactions at finite density and temperature!

Key questions:

For a proper extraction of the EoS from a comparison of transport calculations with exp. data one has to consider that:

1) Hadrons modify their propertis in a hot and dense medium:
- in-medium spectral functions with complex self energies depending on density, temperature and momentum (as shown for antikaons)
- in-medium cross sections also change the dynamics and consequently mean-field potentials

➔ influence on observables - yields, spectra, collective flows v_n

⇨ Off-shell transport is MANDATORY for the proper description of observables as measured by HIC experiments!

2) What matter consists of?
i.e. what are the degrees-of-freedom at the collision energy considered (hadronic, partonic)?
- baryonic resonance matter at SIS energies (U. Mosel at NuSym23)
(resonances change their properties – go to 1)
Nucleon potential ≠ Delta potential?
- Hadron-parton matter at AGS-SPS
- Dominant partonic matter at RHIC and LHC