

# Implications of CREX and PREX for energy density functionals

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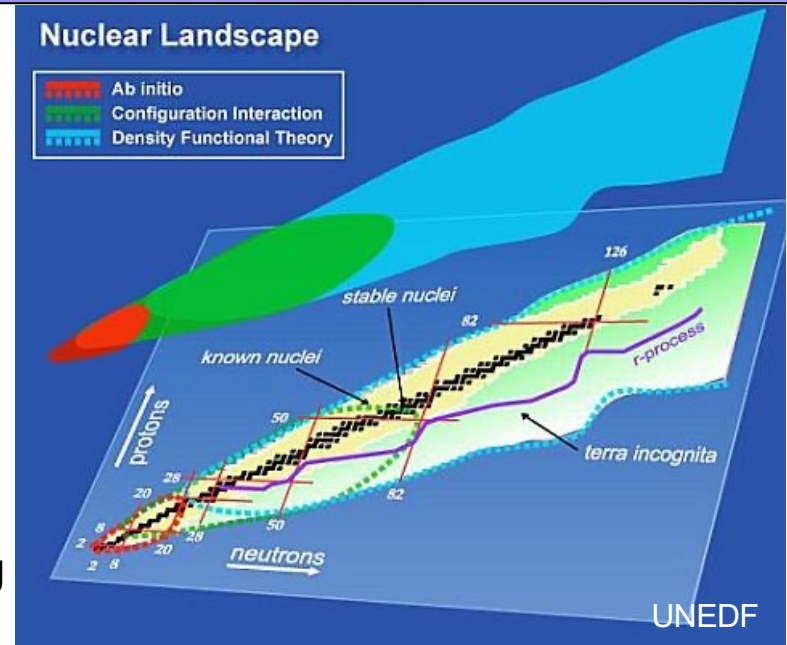


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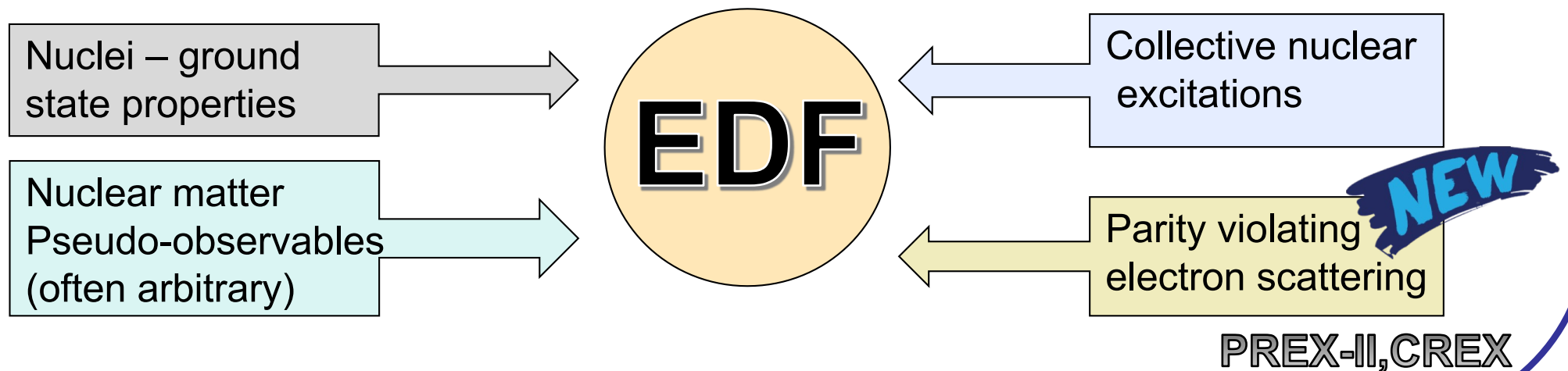
# INTRODUCTION

- **Nuclear energy density functionals (EDF)**

- ❑ In the mean-field approximation the nuclear many-body system is represented by independent nucleons in self-consistent potentials
- ❑ Consistent approach to nuclear properties and EOS
- ❑ Non-relativistic interactions: Skyrme, Gogny
- ❑ Relativistic interactions: meson-exchange, point coupling



- **How are the EDFs constrained by experimental data?**



# RELATIVISTIC NUCLEAR ENERGY DENSITY FUNCTIONAL

- Relativistic density-dependent point coupling interaction → **Effective Lagrangian**

$$\begin{aligned}\mathcal{L} = & \bar{\psi}(i\gamma \cdot \partial - m)\psi \\ & - \frac{1}{2}\alpha_S(\hat{\rho})(\bar{\psi}\psi)(\bar{\psi}\psi) - \frac{1}{2}\alpha_V(\hat{\rho})(\bar{\psi}\gamma^\mu\psi)(\bar{\psi}\gamma_\mu\psi) \\ & - \frac{1}{2}\alpha_{TV}(\hat{\rho})(\bar{\psi}\vec{\tau}\gamma^\mu\psi)(\bar{\psi}\vec{\tau}\gamma_\mu\psi) \\ & - \frac{1}{2}\delta_S(\partial_\nu\bar{\psi}\psi)(\partial^\nu\bar{\psi}\psi) - e\bar{\psi}\gamma \cdot A\frac{(1-\tau_3)}{2}\psi\end{aligned}$$

- Free nucleon terms
- Interaction terms: isoscalar-scalar, isoscalar-vector, isovector-vector
- Derivative term (effects of finite range interaction)
- Electromagnetic interaction

- many-body correlations encoded in density-dependent coupling functions motivated by microscopic calculations but parameterized in a phenomenological way – 10 model parameters

$$\alpha_i(\rho) = a_i + (b_i + c_i x)e^{-d_i x} \quad (i \equiv S, V, TV)$$

- **Nuclear ground state:** Relativistic Hartree-Bogoliubov model (RHB) supplemented with pairing correlations (separable pairing force)
- **Collective excitations in nuclei:** Relativistic quasiparticle random phase approximation (RQRPA)

## CONSTRAINING THE EDFs

- The EDFs have been parametrized by minimizing the  $\chi^2$  function mainly with the experimental data on the ground state properties of finite nuclei

$$\chi^2(\mathbf{p}) = \sum_{i=1}^m \left( \frac{\mathcal{O}_i^{\text{theo.}}(\mathbf{p}) - \mathcal{O}_i^{\text{ref.}}}{\Delta \mathcal{O}_i^{\text{ref.}}} \right)^2$$

- Nuclear ground state properties are often not enough to constrain the effective interaction completely, especially its **isovector channel** (that is especially relevant for the neutron-rich nuclei, neutron skins, symmetry energy, neutron stars, etc.).
- The protocols to determine the EDF's often included additional constraints on the *pseudo-observables* on the nuclear matter properties (often they are arbitrary).
- The neutron skin thickness  $r_{np}$  may be useful probe for the isovector channel of the EDFs. However, the data on  $r_{np}$  are limited and often model dependent.
- New observables are required to constrain the isovector channel of the EDFs.

# NUCLEAR MATTER EQUATION OF STATE

- Nuclear matter equation of state (for the uniform and infinite system)

$$E(\rho, \delta) = E_{SNM}(\rho) + E_{sym}(\rho)\delta^2 + \dots$$

$$\rho = \rho_n + \rho_p \quad \delta = \frac{\rho_n - \rho_p}{\rho}$$

- Symmetry energy  $S_2(\rho)$  describes the increase in the energy of the  $N \neq Z$  system as protons are turned into neutrons

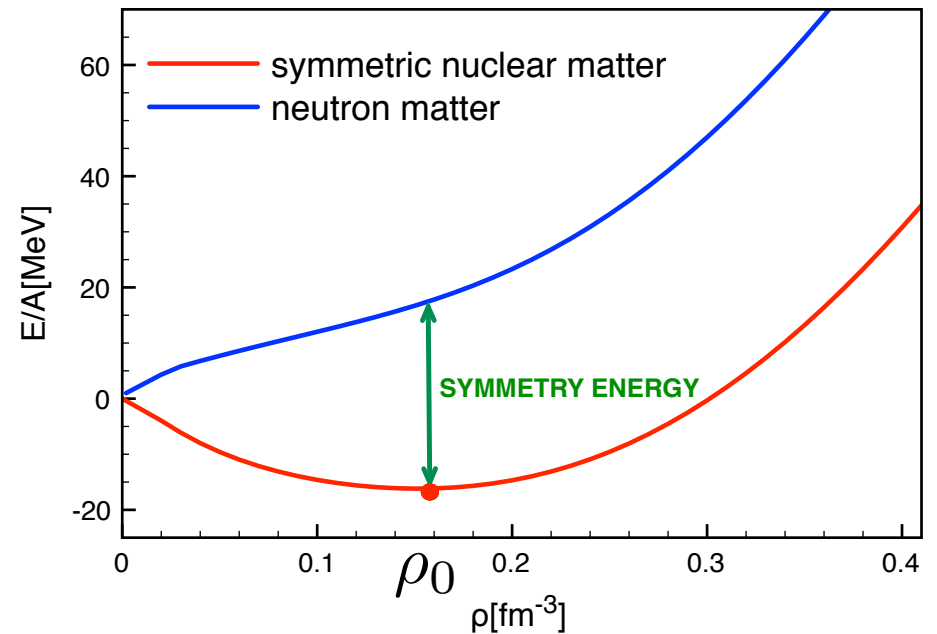
$$E_{sym}(\rho) \equiv S_2(\rho) = J - L\epsilon + \dots$$

$$\epsilon = (\rho_0 - \rho)/(3\rho_0)$$

symmetry energy at saturation density

$$L = 3\rho_0 \left. \frac{dS_2(\rho)}{d\rho} \right|_{\rho_0}$$

slope of the symmetry energy (related to the pressure of neutron matter)



$$K_0 \equiv 9\rho_0^2 \left. \frac{\partial^2 E(\rho, 0)}{\partial \rho^2} \right|_{\rho=\rho_0}$$

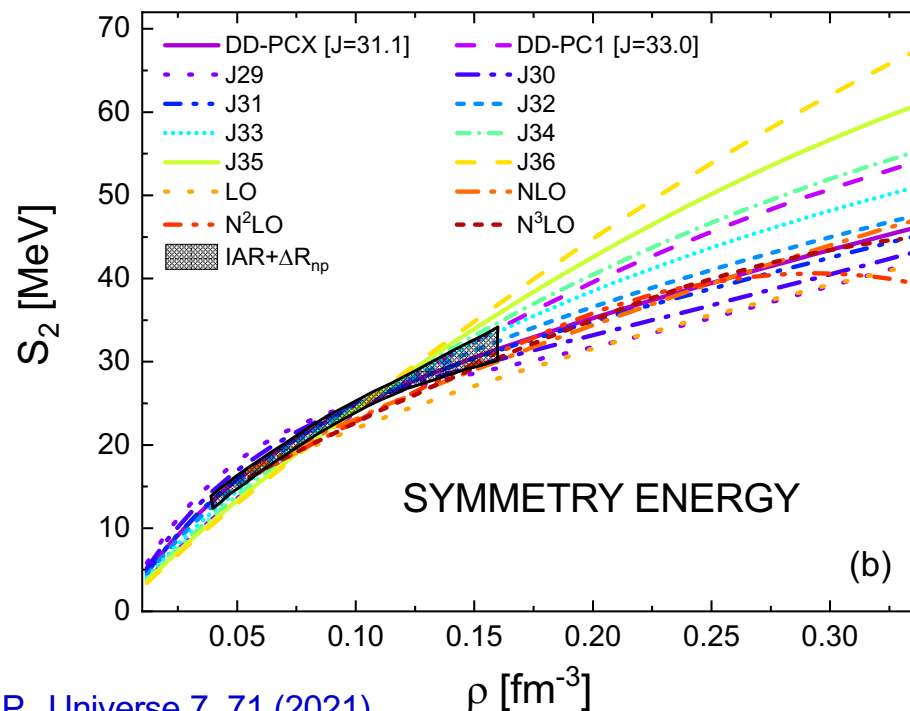
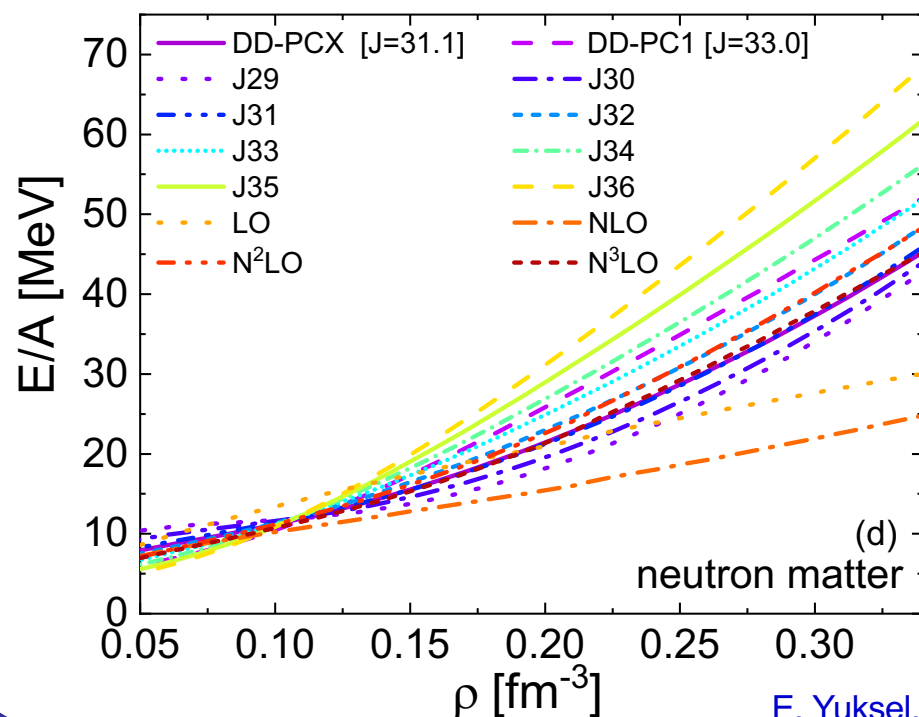
incompressibility of symmetric nuclear matter

The symmetry energy parameters are not well constrained  
 → additional isovector constraints from the experiment are required to optimize the EDFs.

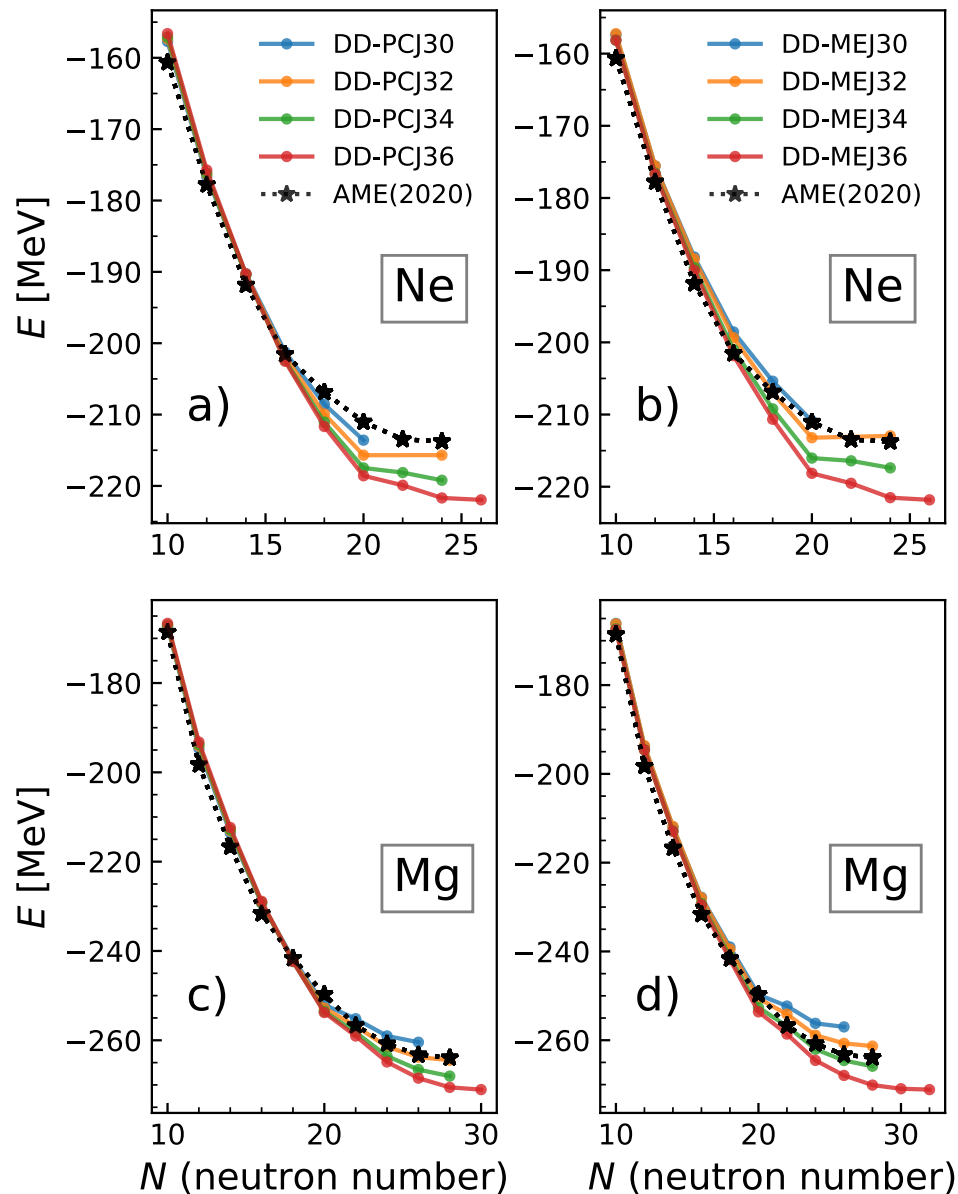
# CONSTRAINING THE SYMMETRY ENERGY IN EDFs

## Family of interactions for studies of symmetry energy:

- Systematic variation of the symmetry energy in constraining the EDF parameters  
→ Established set of 8 relativistic point coupling EDFs for the range of values  $J=29,30,\dots,36$  MeV
- Adjusted properties of 72 spherical nuclei to exp. data (binding energies, charge radii, diffraction radii, surface thickness, pairing gaps).
- Each interaction is determined independently using the same dataset supplemented with an additional constraint on  $J$

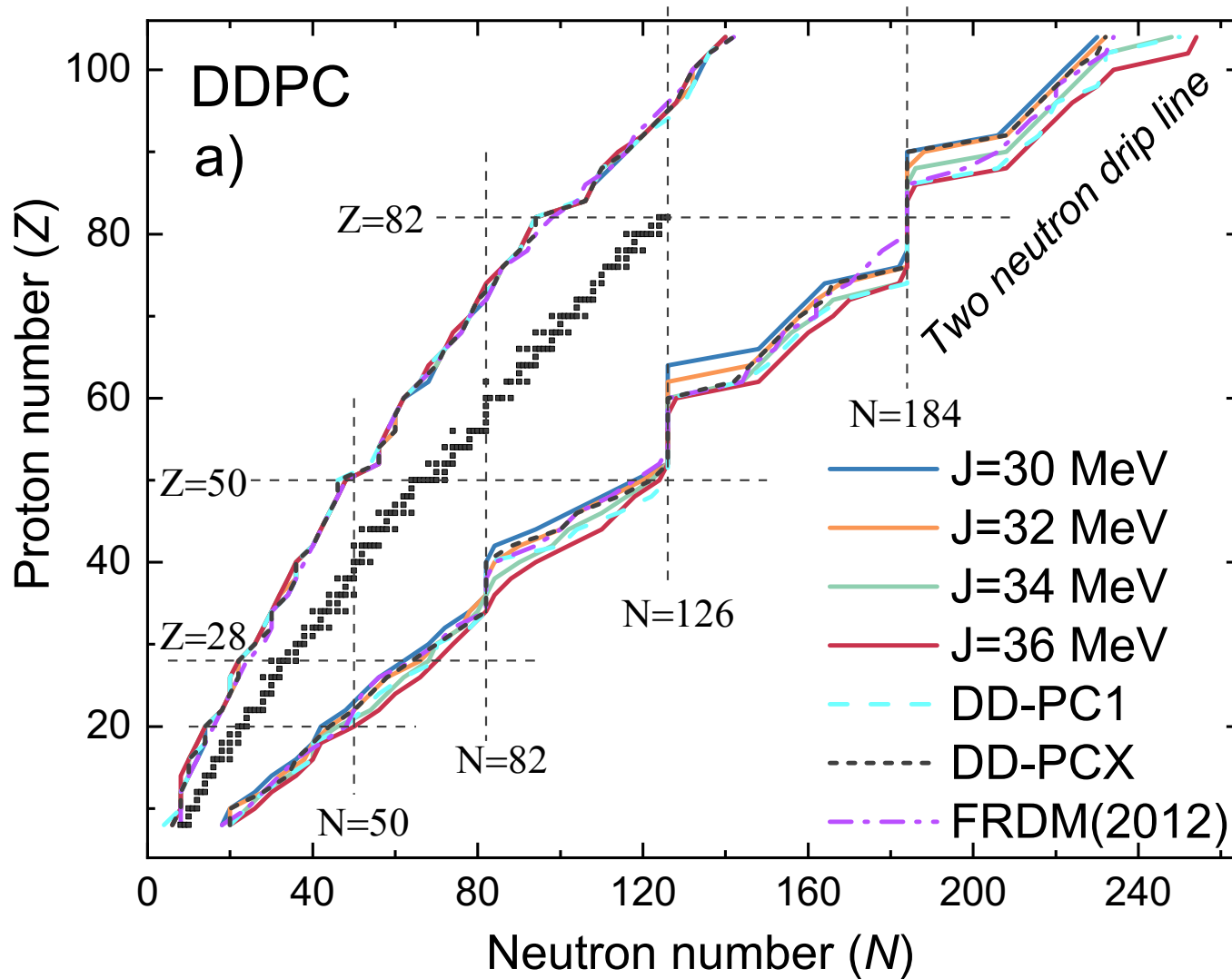


# BINDING ENERGIES AND SYMMETRY ENERGY



- Calculations are performed with DD-PCJ (a,c) and DD-MEJ (b,d) effective interactions and compared with exp. data.
- As the two-neutron drip line is approached, a separation of binding energy curves for different  $J$  values is obtained.
- By increasing the value of  $J$ , the binding energy increases, but so does its slope. Thus the position of the two-neutron drip lines is also affected.
- Drip line nuclei (exp.):
  - $^{34}\text{Ne}$  – favors  $J=32,34$  MeV
  - $^{40}\text{Mg}$  (?) – favors  $J=32,34$  MeV

# DRIP LINES AND SYMMETRY ENERGY



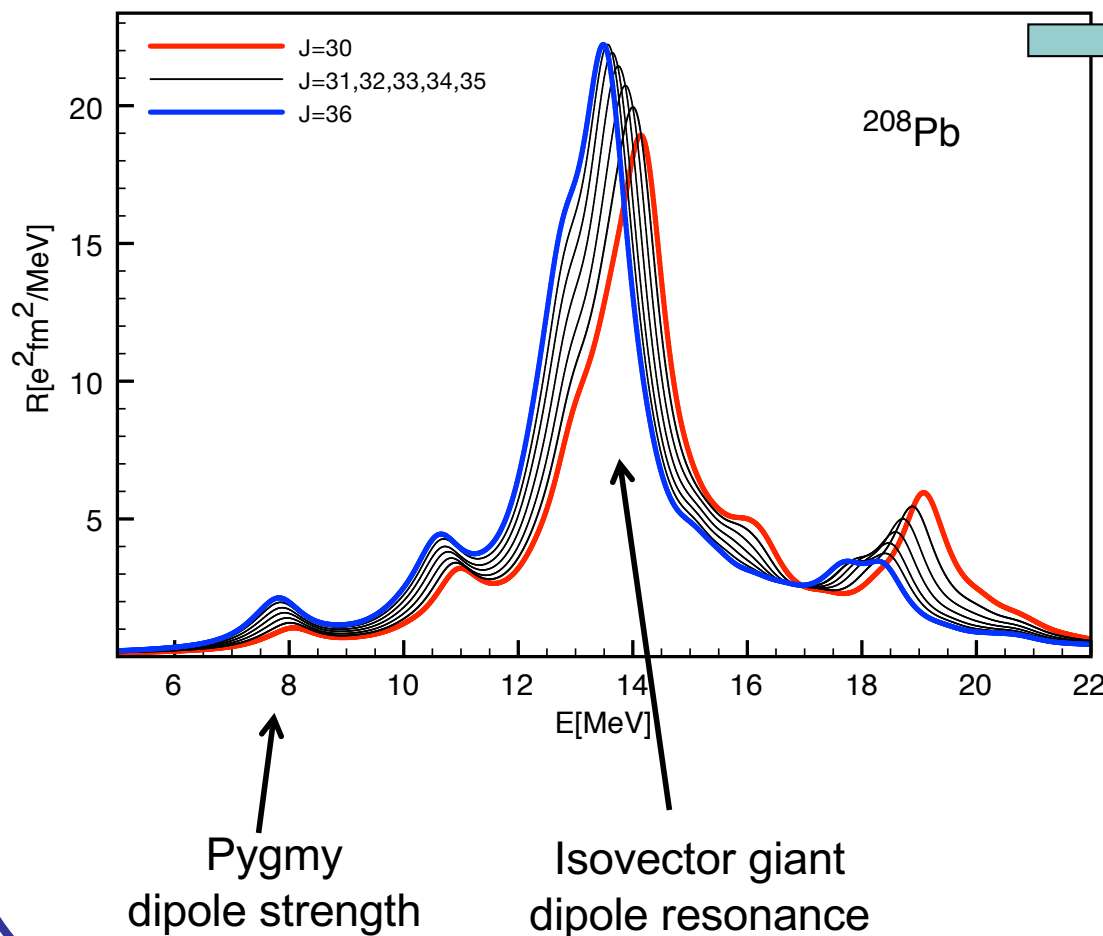
- Two-proton and two-neutron drip lines for DDPCJ interactions with different values of symmetry energy.
- The two-neutron drip line shows a systematic shift towards a higher neutron number with increasing  $J$



# CONSTRAINING THE SYMMETRY ENERGY WITH NUCLEAR EXCITATIONS

## How can we relate the symmetry energy with nuclear dipole transitions?

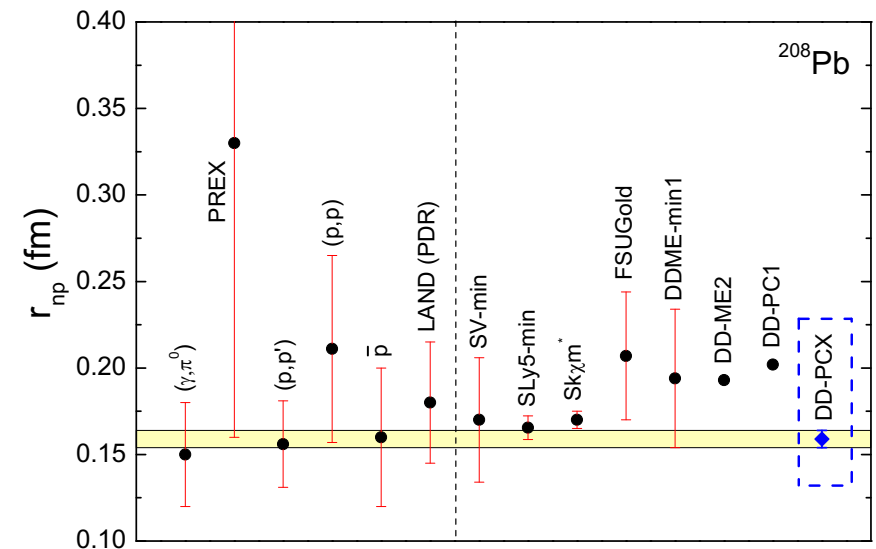
- Isovector dipole transition strength for a set of relativistic point coupling interactions which vary the symmetry energy properties  $\rightarrow J=30,31,\dots,36$  MeV



E. Yuksel, N.P., Universe 7, 71 (2021).

Dipole polarizability ( $\alpha_D \sim m_{-1}$ )

- The relativistic point coupling EDF constrained by the nuclear ground state properties and collective excitation properties - dipole polarizability, ISGMR ( $^{208}\text{Pb}$ ):  $\rightarrow$  **DD-PCX**



E. Yuksel, T. Marketin, N.P., PRC 99, 034318 (2019)

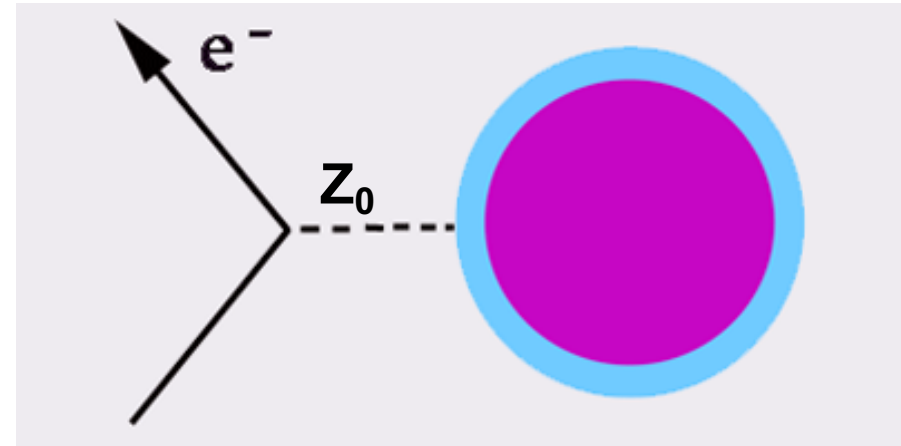
# PREX AND CREX EXPERIMENTS

## Measurement of neutral weak form factor $F_W$ in elastic scattering of electrons on nuclei

- ❖ significantly larger coupling of the  $Z_0$  boson to neutrons compared to protons
- ▶ Parity violating (PV) asymmetry in longitudinally polarized elastic electron scattering

$$A_{PV} = \frac{\sigma_R - \sigma_L}{\sigma_R + \sigma_L} \approx \frac{G_F Q^2 |Q_W| F_W(Q^2)}{4\sqrt{2}\pi\alpha Z F_{ch}(Q^2)}$$

→ nuclear weak form factor  $F_W \rightarrow r_n \rightarrow r_{np}$



### $^{48}\text{Ca}$ (CREX)

$$F_W(q=0.8733 \text{ fm}^{-1}) = 0.1304 \pm 0.0052(\text{stat}) \pm 0.0020(\text{syst.})$$

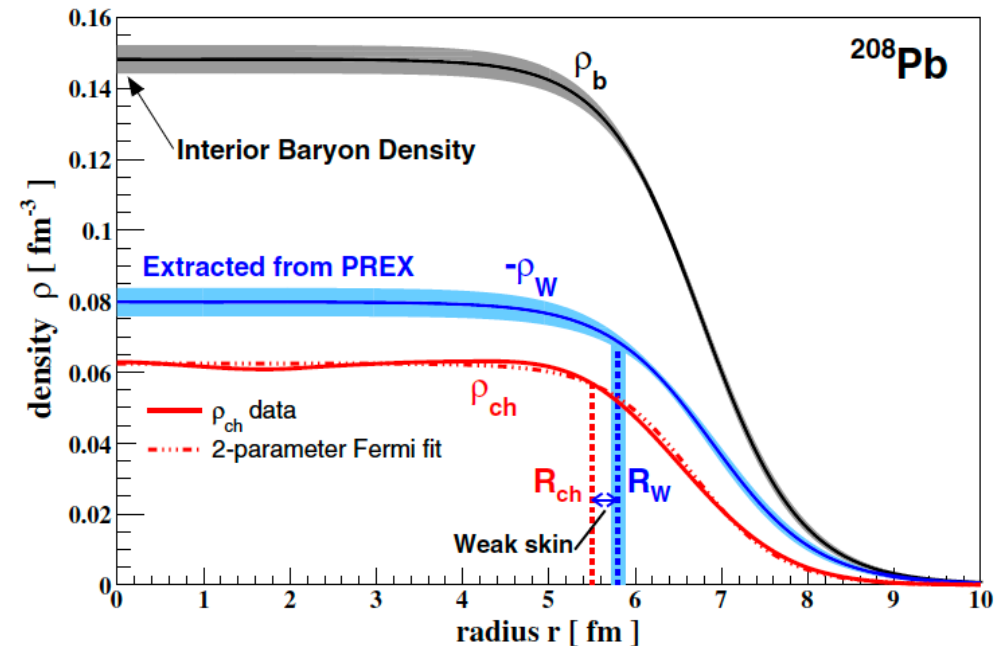
D. Adhikari et al., PRL 129, 042501 (2022).

### $^{208}\text{Pb}$ (PREX II)

$$F_W(q=0.3978 \text{ fm}^{-1}) = 0.368 \pm 0.013$$

D. Adhikari et al., PRL 126, 172502 (2021).

S. Abrahamyan et al., PRL 108, 112502 (2012).



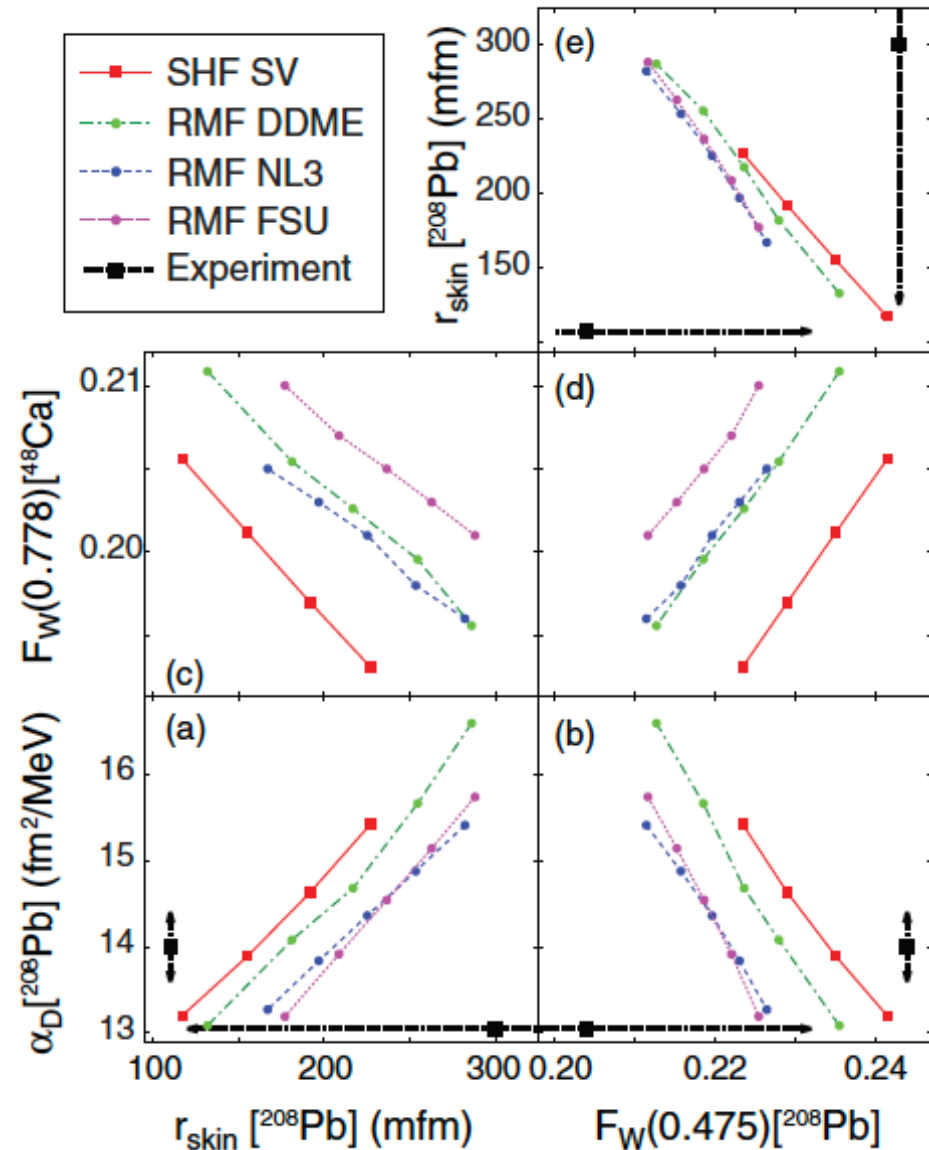
D. Adhikari et al., PRL 126, 172502 (2021).

# INFORMATION CONTENT OF THE WEAK CHARGE FORM FACTOR

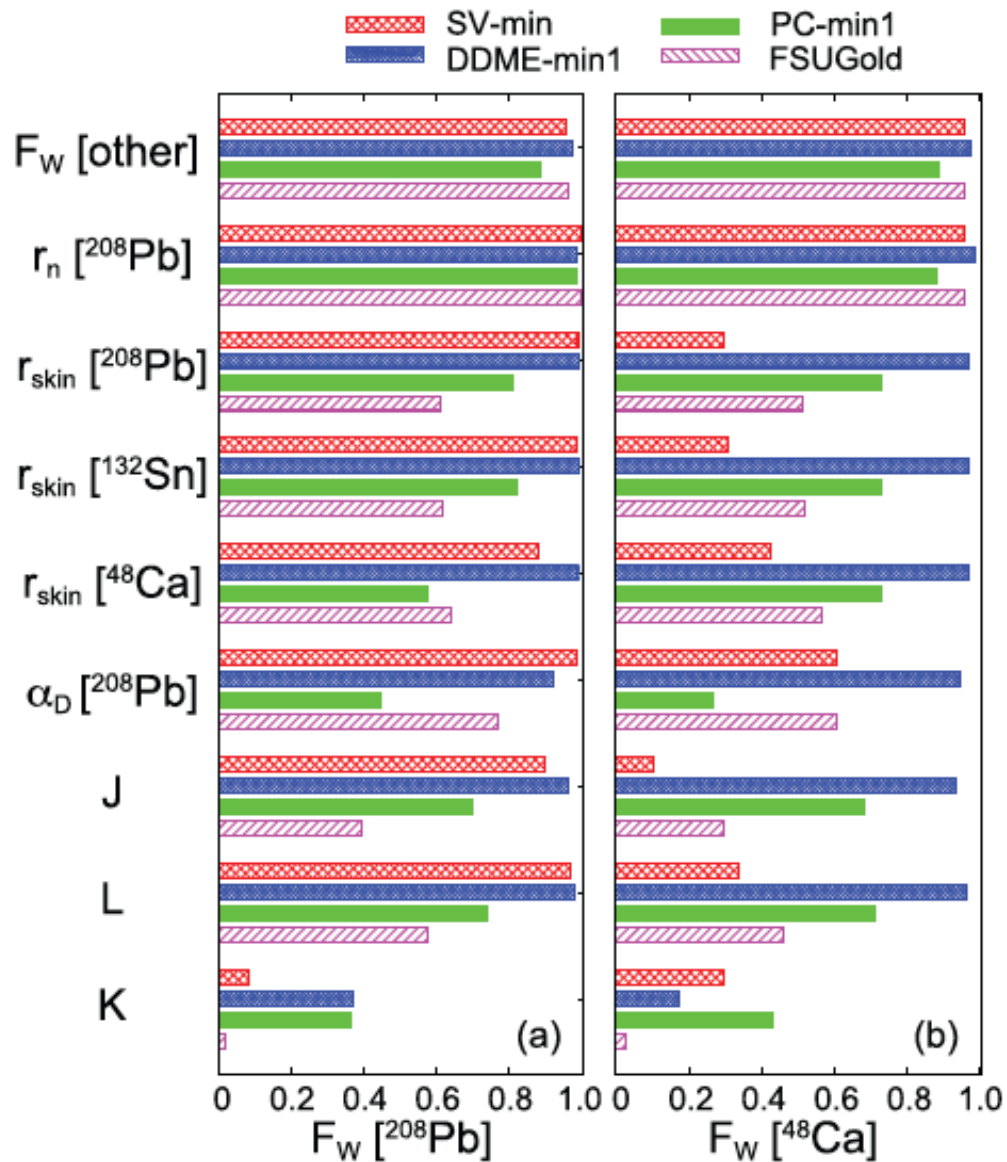
P.-G. Reinhard, et.al., Phys. Rev. C 88, 034325, (2013)

What is the impact of precise measurements of the weak-charge form factors of  $^{48}\text{Ca}$  and  $^{208}\text{Pb}$  on a variety of nuclear observables, such as the neutron skin and the electric-dipole polarizability?

- several accurately calibrated nonrelativistic and relativistic EDFs are employed to assess the degree of correlation between nuclear observables and weak charge form factors for  $^{48}\text{Ca}$  and  $^{208}\text{Pb}$
- strong correlation between the weak-charge form factor and the neutron radius, that should allow for an accurate determination of the neutron skin of neutron-rich nuclei.



# INFORMATION CONTENT OF THE WEAK CHARGE FORM FACTOR

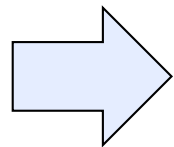


Correlation coefficients derived from a covariance analysis between the weak-charge form factors of  $^{208}\text{Pb}$  and  $^{48}\text{Ca}$ , and a variety of nuclear observables, including:

- the strong isovector indicators such as neutron skin, electric dipole polarizability, symmetry energy  $J$ , slope of the symmetry energy  $L$ ,
- the strong isoscalar indicator: incompressibility  $K$  at saturation density
- **Strong correlation between  $F_w$  for  $^{48}\text{Ca}$  and  $^{208}\text{Pb}$  - PREX II and CREX should provide complementary information**

# CONSTRAINING EDFs WITH PREX AND CREX EXPERIMENTS

How can we use the CREX and PREX II data to constrain the symmetry energy and isovector properties of nuclei?



We introduce EDFs directly constrained by  $F_W$ :

Nuclear ground state properties  
- the same like for DD-PCX interaction using constraint on dipole polarizability  
( $E_B$ ,  $r_c$ , pairing gaps)



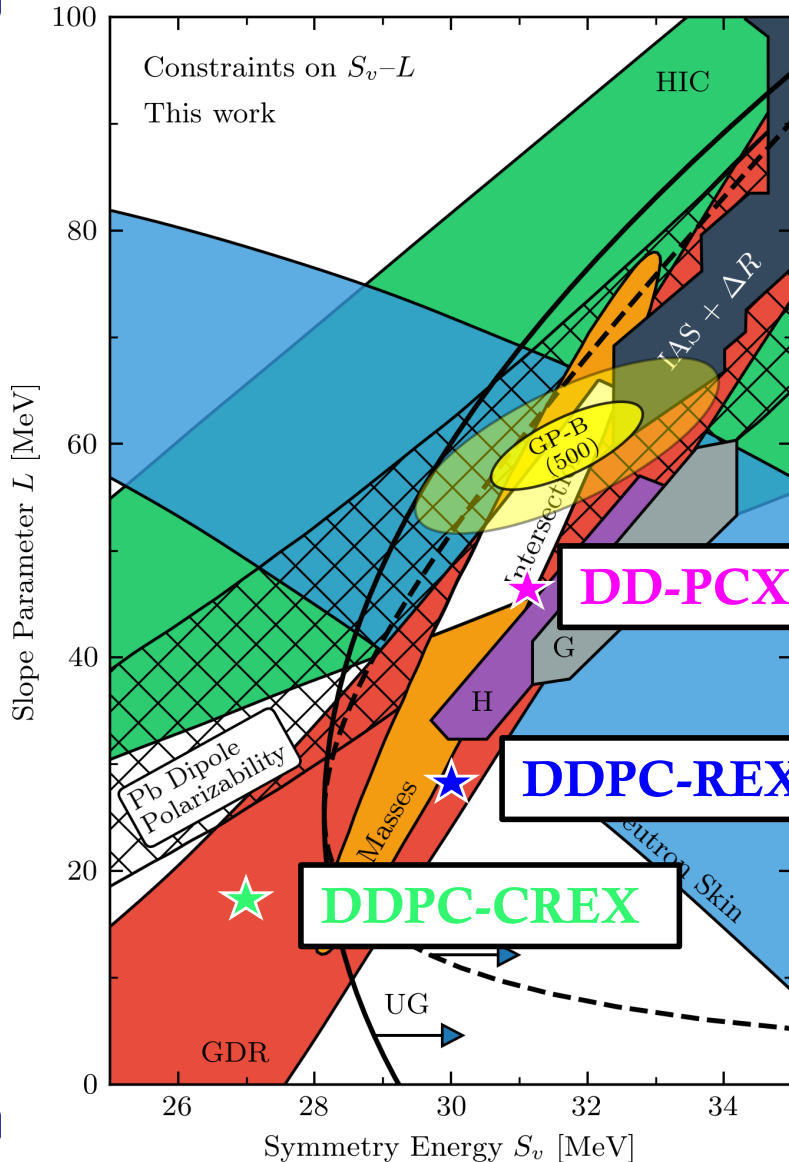
The weak-form factors  $F_W$  from the CREX and PREX-II experiments

→ 3 new relativistic point coupling interactions:

- **DDPC-CREX** – constrained by  $F_W(^{48}\text{Ca})$
- **DDPC-PREX** – constrained by  $F_W(^{208}\text{Pb})$
- **DDPC-REX** – constrained by  $F_W(^{48}\text{Ca})$  &  $F_W(^{208}\text{Pb})$

# NUCLEAR MATTER PROPERTIES

## ★ DDPC-PREX



	$E/A$ (MeV)	$m_D^*/m$	$K_0$ (MeV)	$J$ (MeV)	$L$ (MeV)
DDPC-CREX	-15.989(15)	0.5672(13)	225.48(4.69)	27.01(16)	19.60(64)
DDPC-PREX	-16.108(17)	0.5680(15)	235.41(5.20)	36.18(47)	101.78(4.87)
DDPC-REX	-16.019(15)	0.5696(7)	242.95(2.04)	28.86(15)	30.03(63)
DD-PC1	-16.061	0.580	230.0	33.0	70.1
DD-PCX	-16.026(18)	0.5598(8)	213.03(3.54)	31.12(32)	46.32(1.68)

DD-PCX

Constrained by collective nuclear excitations

DDPC-REX

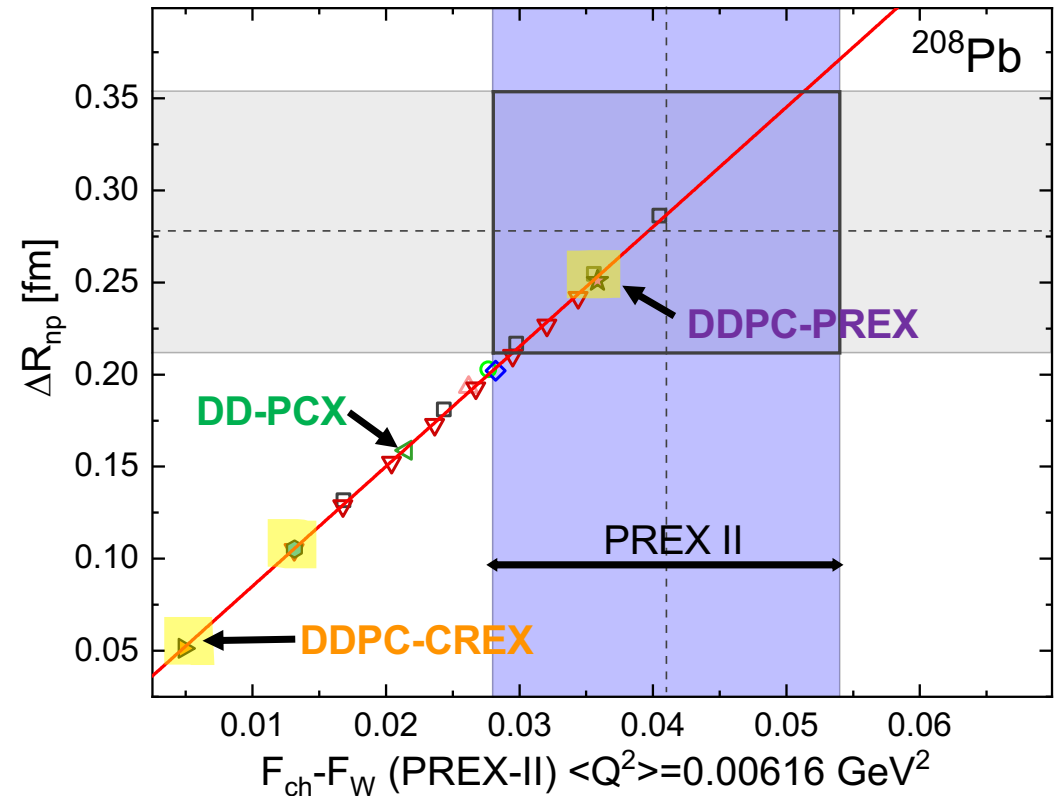
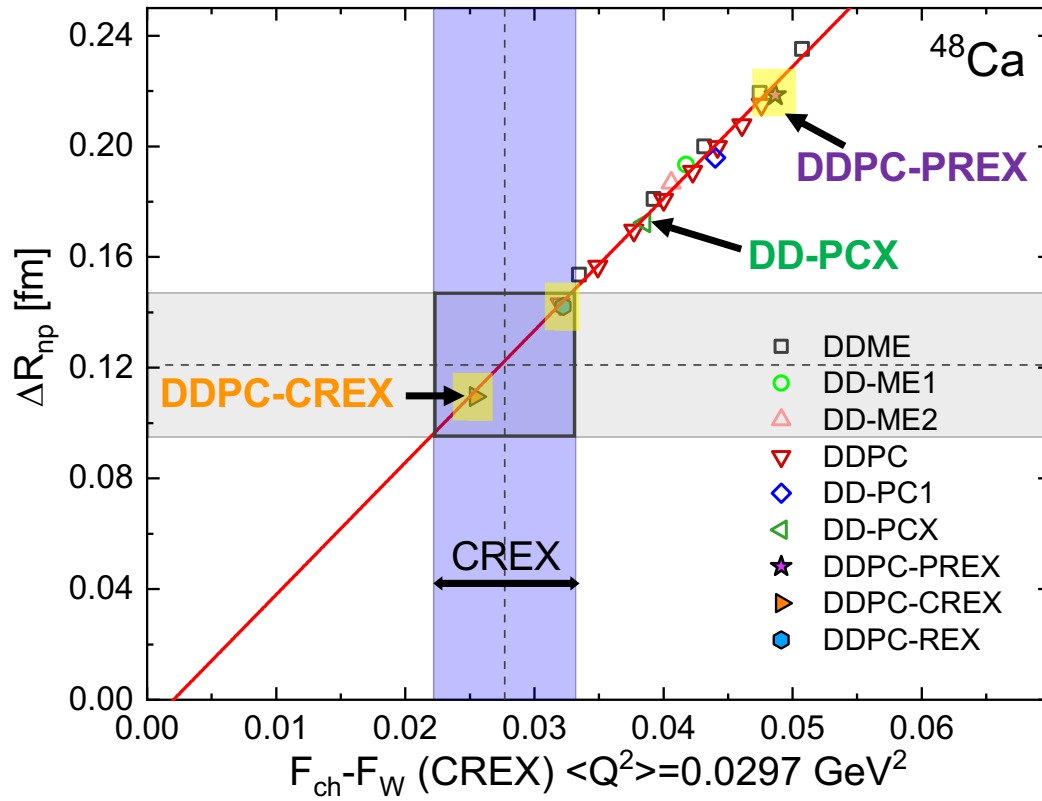
- Point coupling EDFs constrained by CREX and PREX data result in rather different values of the symmetry energy ( $J$ ) and its slope parameter ( $L$ )

DDPC-CREX

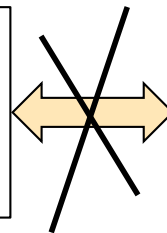
- Discrepancy with the DD-PCX interaction (the same EDF formalism) constrained by nuclear ground state and collective excitation properties

# PROBING THE NEUTRON SKIN THICKNESS

The neutron skin thickness  $\Delta R_{np}$  of  $^{48}\text{Ca}$  and  $^{208}\text{Pb}$  as a function of the form factor difference  $F_{ch} - F_W$  using relativistic EDFs  $\rightarrow$  linear dependence (almost model independent)

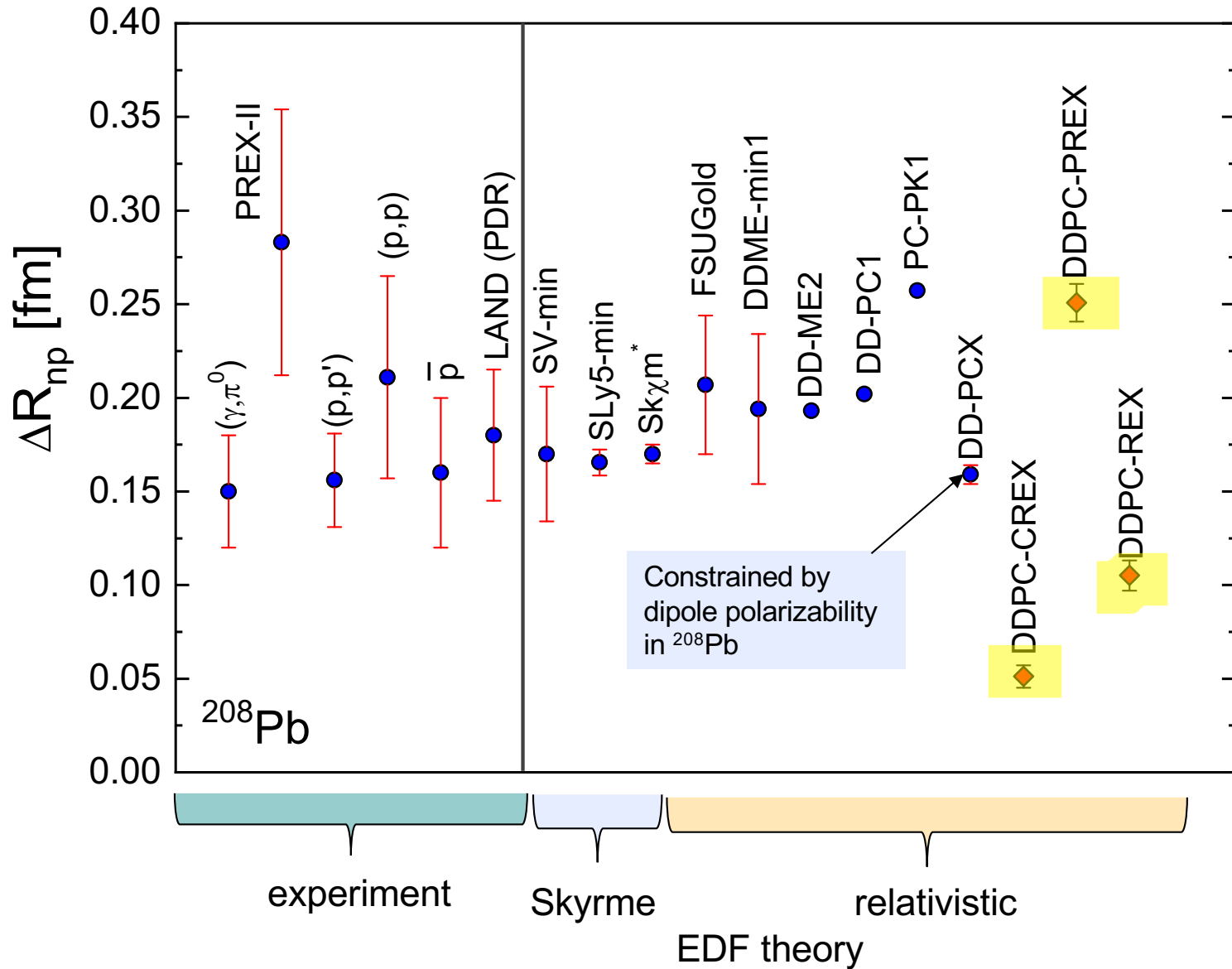


Interactions DDPC-CREX ( $J=27.01 \text{ MeV}$ ), DDPC-REX ( $J=28.86 \text{ MeV}$ ) and DD-PC ( $J=29 \text{ MeV}$ ) fit into exp. range. All other interactions (known as successful in describing nuclear properties) are out of the box.



Larger exp. errors and more interactions fit into exp. range. Only interactions with  $J \geq 34 \text{ MeV}$  (DD-ME, DD-PC) + DDPC-PREX ( $J=36.18 \text{ MeV}$ ) are consistent with PREX II data.

# NEUTRON SKIN THICKNESS IN $^{208}\text{Pb}$



$^{208}\text{Pb}$

Model averaged value (EDFs)

J. Piekarewicz et al.,  
PRC 85, 041302 (2012)

$\Delta R_{np} = 0.168 \pm 0.022$  fm

Ab initio theory

B. Hu et al.,  
Nature Phys. 18, 1196 (2022)

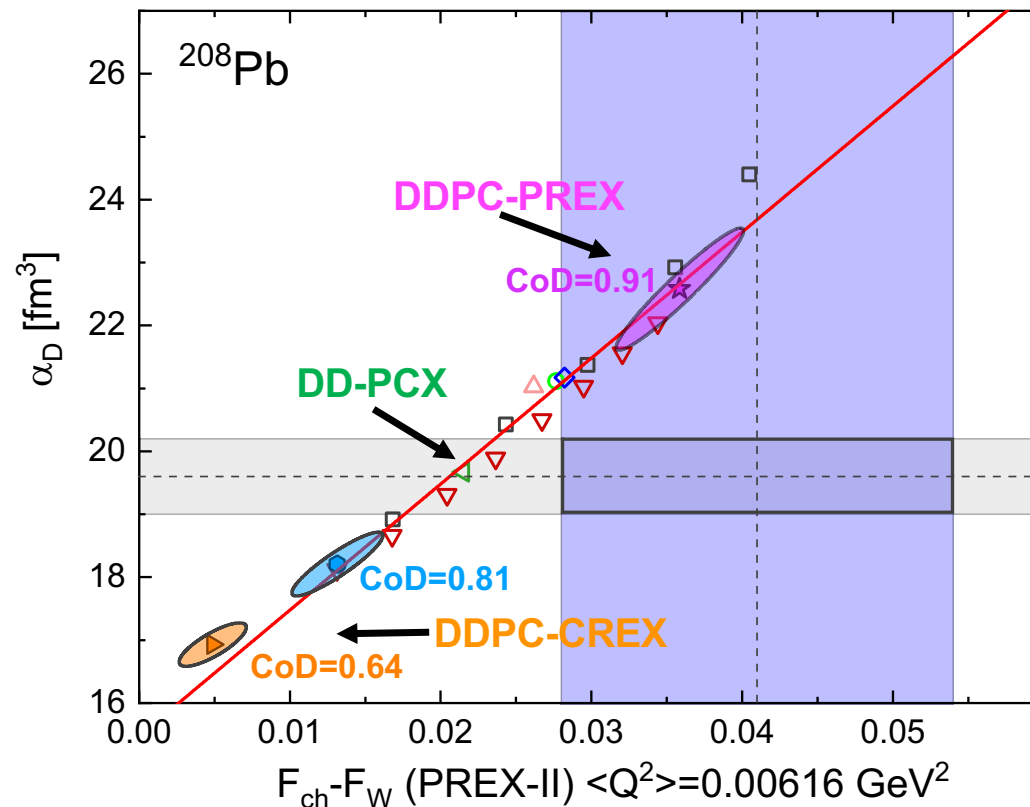
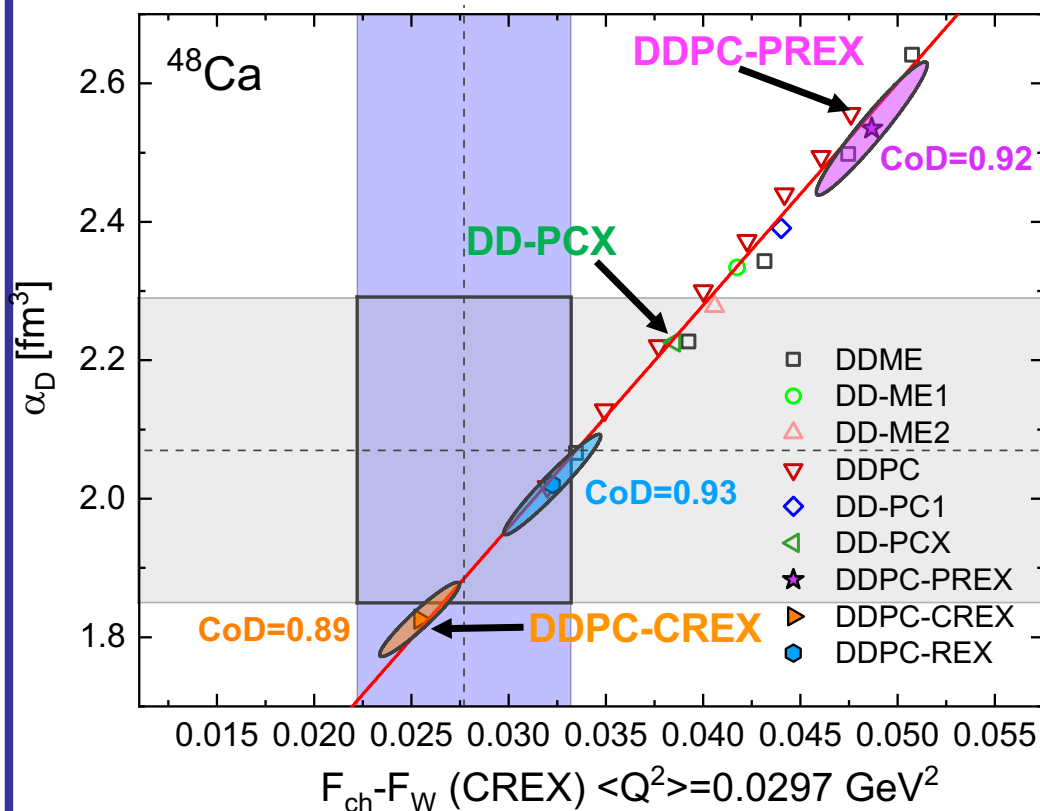
$\Delta R_{np} = 0.14-0.20$  fm

The limits on  $\Delta R_{np}$  given by new interactions DDPC-PREX, DDPC-CREX are not consistent with other EDF and ab initio studies



# PROBING THE DIPOLE POLARIZABILITY

The dipole polarizability  $\alpha_D$  of  $^{48}\text{Ca}$  and  $^{208}\text{Pb}$  as a function of the form factor difference  $F_{\text{ch}} - F_{\text{W}}$  using relativistic EDFs.



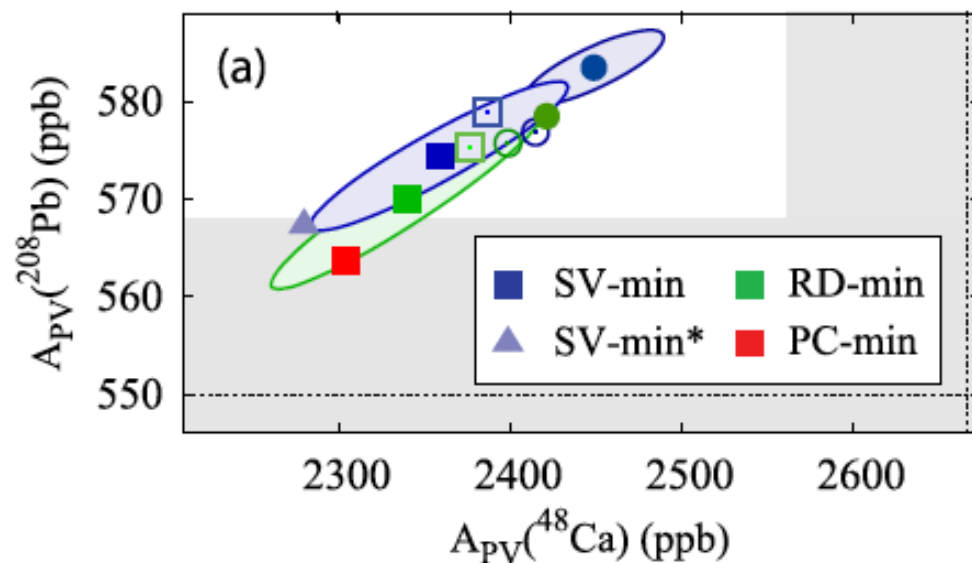
- Vertical bands denote  $F_{\text{ch}} - F_{\text{W}}$  range of values from the CREX and PREX-II  
D. Adhikari et al., PRL 126, 172502 (2021); PRL 129, 042501 (2022).
- Horizontal bands correspond to the experimental data on  $\alpha_D$   
J. Birkhan et al., PRL 118, 252501 (2017); A. Tamii et al., PRL 107, 062502 (2011); X. Roca-Maza et al., PRC 92, 064304 (2015).

## A FEW REMARKS FROM OTHER THEORETICAL STUDIES

### Theoretical analysis of parity violating asymmetry

*P.-G. Reinhard, X. Roca-Maza, W. Nazarewicz, PRL 129, 232501 (2022)*

- The measured parity violating asymmetry  $A_{PV}$  in  $^{48}\text{Ca}$  and  $^{208}\text{Pb}$  has been analyzed using non-relativistic and relativistic EDFs



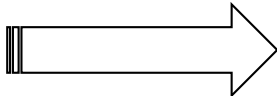
→ there are difficulties to describe  $A_{PV}$  simultaneously in  $^{48}\text{Ca}$  and  $^{208}\text{Pb}$

### CREX- and PREX-II-motivated relativistic interactions and their implications

*M. Kumar, S. Kumar, V. Thakur, R. Kumar, B. K. Agrawal, S. K. Dhiman, PRC 107, 055801 (2023)*

→ The RMF model parametrization obtained with the CREX data acquires much smaller value of symmetry energy ( $J = 28.97 \pm 0.99$  MeV) and its slope parameter ( $L = 30.61 \pm 6.74$  MeV) in comparison to those obtained with PREX-II data ( $J = 34.41 \pm 2.71$  MeV,  $L = 77.08 \pm 28.87$  MeV)

## CONCLUDING REMARKS

- The implementations of EDF methods have evolved to allow including new observables on nuclei going beyond ground state properties, directly in constraining the EDF parameters
  - weak charge form factor, dipole polarizability,...
- In this way, the symmetry energy of the EDFs is constrained directly by relevant experimental data
- **DDPC-CREX, DDPC-PREX, DDPC-REX** functionals established using the weak form factors from parity violating electron scattering experiments CREX and PREX II
  - The symmetry energy and isovector properties for these 3 functionals are in strong tension
  - Dipole polarizability requires largely different isovector properties than CREX and PREX II
- **KEY QUESTION: How can we resolve strong inconsistency between the symmetry energy constraints from parity violating electron scattering (CREX, PREX) and constraints from nuclear ground state and excitations?**
- **New experiments?** 



P2 – Parity violation at MESA (Mainz)

## THANKS TO COLLABORATORS

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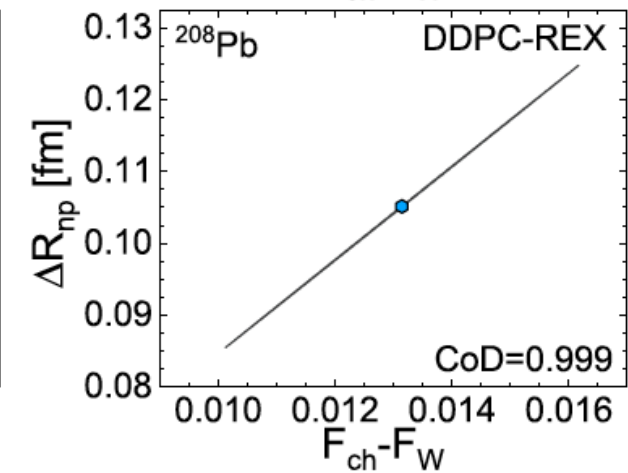
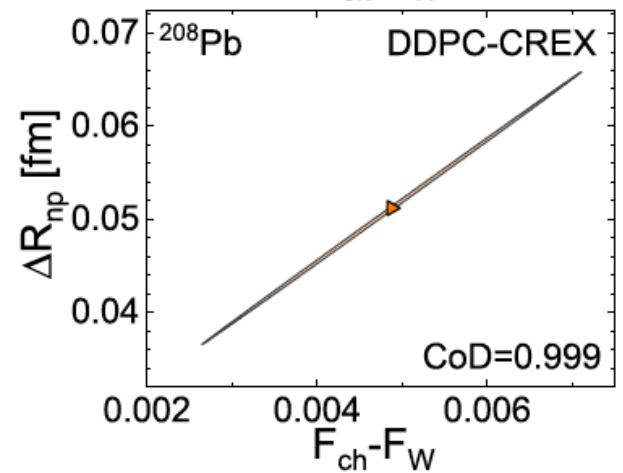
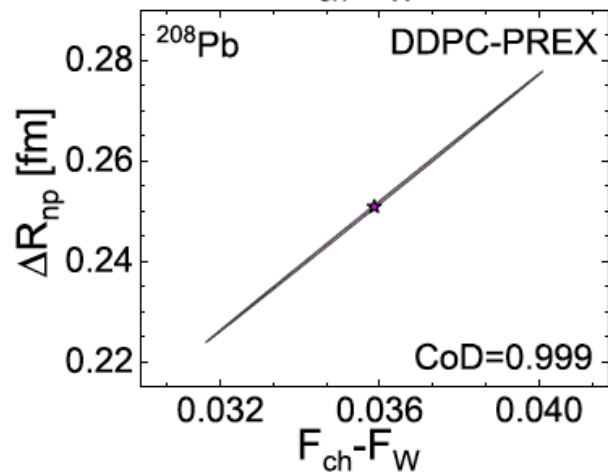
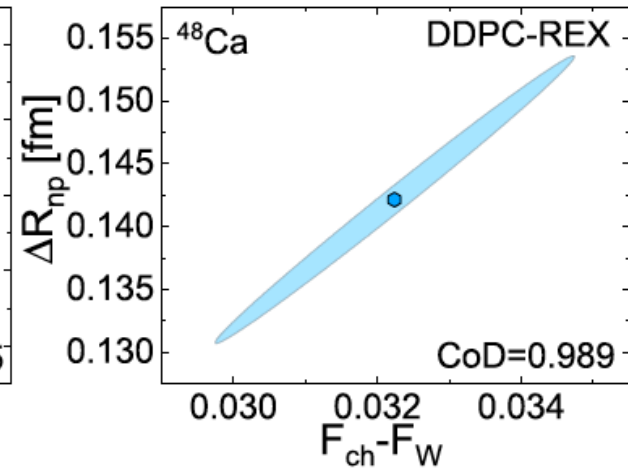
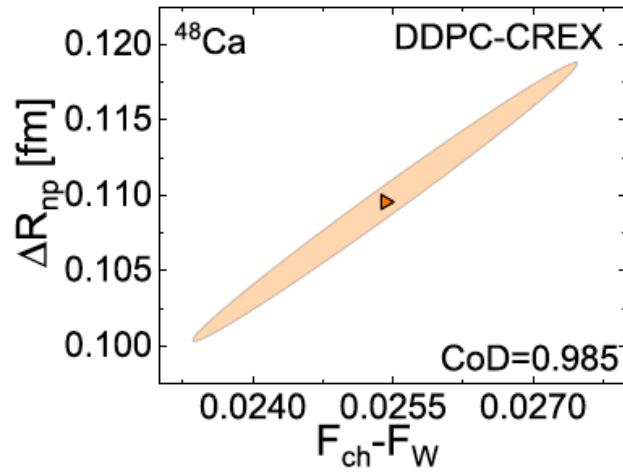
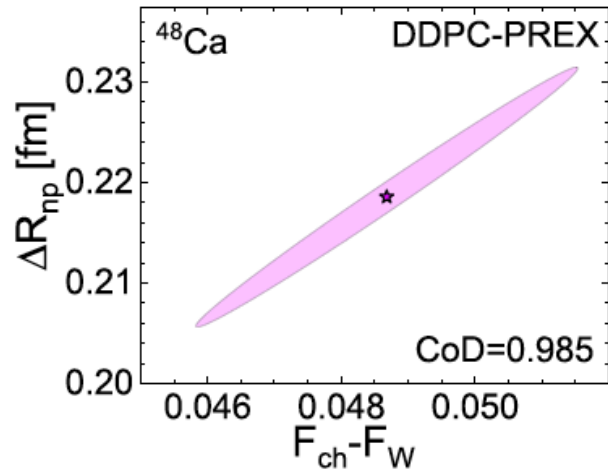


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# ERROR ELLIPSOID FOR NEUTRON SKIN THICKNESS AND $F_{ch}-F_W$



# ERROR ELLIPSOID FOR DIPOLE POLARIZABILITY AND $F_{ch}-F_W$

