



# Direct comparisons of isospin diffusion measurements with transport models at Fermi energies

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- **INDRA-VAMOS coupling**
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# Nuclear equation of state

$\varphi$

## The Equation of State of a nuclear system

$$\delta = \frac{\rho_n - \rho_p}{\rho_n + \rho_p}$$

- The EOS of a nuclear system is defined by its energy per nucleon :  $\mathcal{E}(\rho, T, \delta)$

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## The Equation of State of a nuclear system

$$\delta = \frac{\rho_n - \rho_p}{\rho_n + \rho_p}$$

- The EOS of a nuclear system is defined by its energy per nucleon :  $\epsilon(\rho, T, \delta)$

Taylor-Young dev. around  $\delta=0$

$$\epsilon(\rho, \delta) = \epsilon_0(\rho, \delta = 0) + \epsilon_{sym}(\rho, \delta) \cdot \delta^2 + \mathcal{O}\{\delta^4\}$$
$$\epsilon_{sym} = \frac{1}{2} \left. \frac{\partial^2 \epsilon(\rho, \delta)}{\partial \delta^2} \right|_{\delta=0}$$

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## The Equation of State of a nuclear system

$$\delta = \frac{\rho_n - \rho_p}{\rho_n + \rho_p}$$

- The EOS of a nuclear system is defined by its energy per nucleon :  $\epsilon(\rho, T, \delta)$
- The density dependence of the symmetry energy term  $\epsilon_{sym}(\rho, T)$  remains a major issue in modern nuclear physics :
  - describes the energetic cost of converting isospin symmetric matter into neutron matter ;
  - constraints well established for  $T=0K$  and  $\rho=\rho_0$  by fitting with nuclear masses ;
  - largely unknown as soon as we move away from saturation density.

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Dev. around  $\rho_0$

(see W. G. Lynch talk)

$$\epsilon_{sym}(\rho) = S_0 + \frac{L}{3} \left( \frac{\rho - \rho_0}{\rho_0} \right) + \frac{K_{sym}}{18} \left( \frac{\rho - \rho_0}{\rho_0} \right)^2 + \mathcal{O} \left\{ \left( \frac{\rho - \rho_0}{\rho_0} \right)^3 \right\}$$

$$L = 3\rho_0 \left. \frac{\partial \epsilon_{sym}(\rho)}{\partial \rho} \right|_{\rho=\rho_0} \quad \leftarrow \text{« Slope » param.}$$

$$K_{sym} = 9\rho_0^2 \left. \frac{\partial^2 \epsilon_{sym}(\rho)}{\partial \rho^2} \right|_{\rho=\rho_0} \quad \leftarrow \text{« Incompressibility » param.}$$

# Isospin transport



## The Equation of State of a nuclear system

$$\delta = (N - Z)/A$$

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Taylor



- $\epsilon_{sym}(\rho)$  largely unknown as soon as we move from  $\rho_0$
- Essential ingredient to describe :
  - Structure of exotic nuclei and neutron skin [1] ;
  - Giant Dipole Resonances and Pygmy Resonances [2] ;
  - **The dynamic of Heavy Ion Collisions**

De

- ... but also stellar matter :
  - Modelling of neutron stars, CCSN and compact binary stars mergers [3] ;

[1] M. Thiel et al., Journal of Physics G., 46, 093003 (2019)

[3] J. M. Lattimer and M. Prakash, Phys. Rep. 621, 127 (2016)

[2] G. Colò et al., EPJA 50, 26 (2014)

# Context and motivations

## Heavy Ion Collisions

- Unique tool to probe the nuclear EOS at finite temperature under laboratory-controlled conditions
- Study the formation of exotic nuclei over a wide range of n/p asymmetry where relatively high  $E^*/A$  can be reached
- Probe thermodynamical properties of the expanding nuclear system (cluster formation, see T. Genard talk)

## Intermediate energies

- $15 \text{ A MeV} \leq E_{inc} \leq 100 \text{ A MeV}$
- Binary-like dissipative collisions
- Isospin transport phenomena



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## Transport model (ImQMD05)

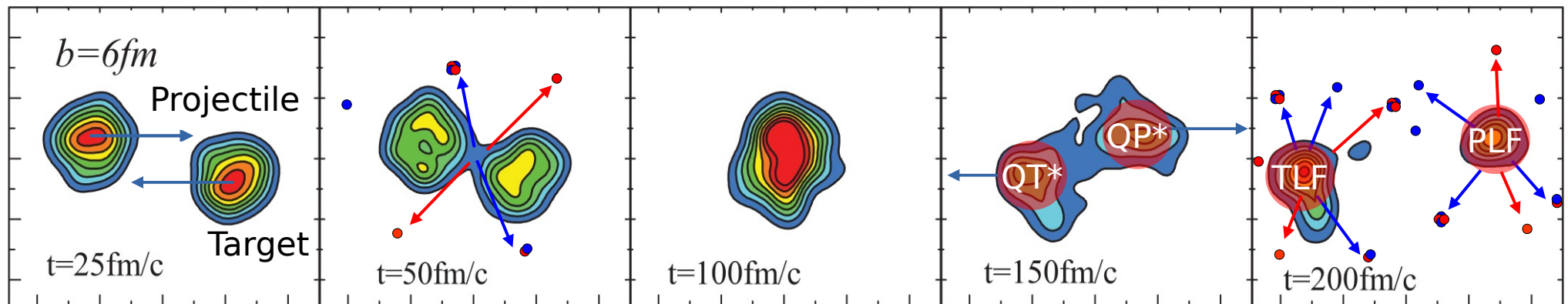
- $^{124}\text{Sn} + ^{124}\text{Sn}$  @ 50 A MeV

Early out-of-eq. emissions

Mixing

Fragments formation

Sequential decays



Zhang et al., PRC 85:024602

Isospin transport

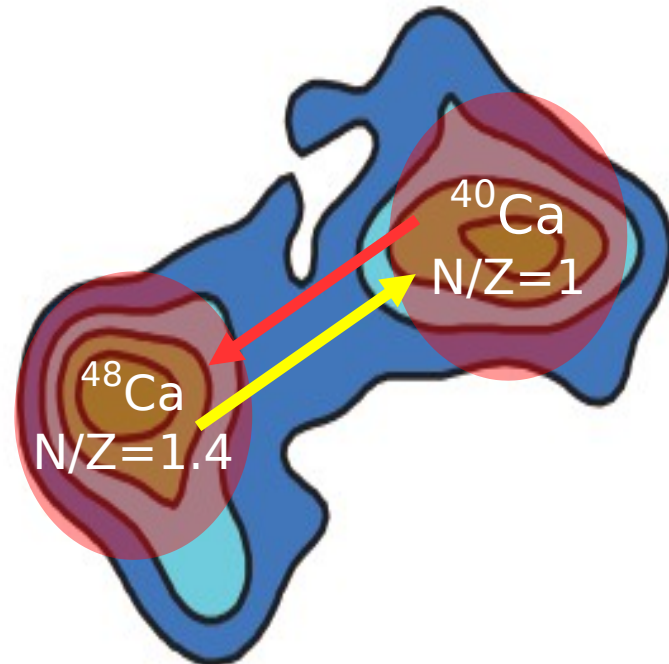
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## Transport mode

- $^{124}\text{Sn} + ^{124}\text{Sn}$  @



V. Baran et al., Nuc. Phys. A 730 (2004)

## Intermediate energies

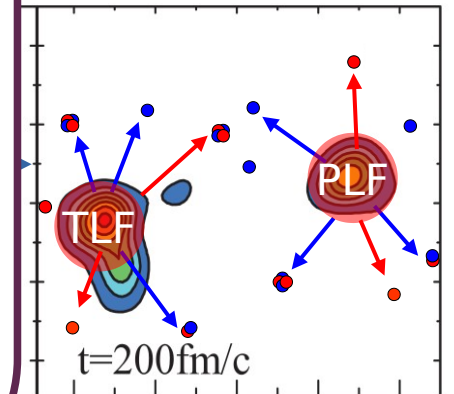
- $15 \text{ A MeV} \leq E_{inc} \leq 100 \text{ A MeV}$
- Dissipative collisions

## Isospin diffusion

- Minimisation of the  $N/Z$  concentration gradient  
→ neutron/proton currents between projectile/target  
→ Linked to  $\epsilon_{sym}$

(see also C. Ciampi talk)

## Transport phenomena



# Context and motivations

## Heavy Ion Collisions

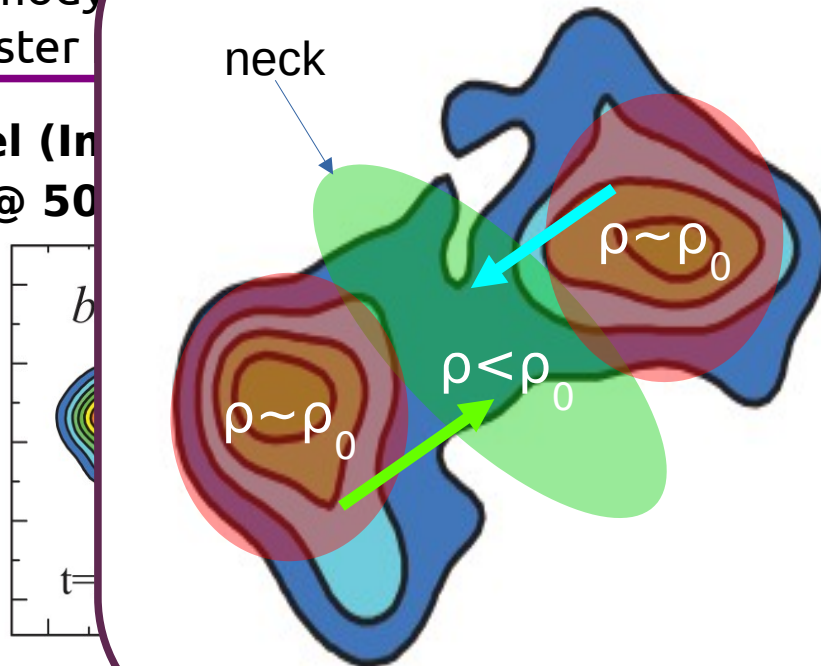
- Unique tool to probe the nuclear EOS at finite temperature under laboratory-controlled conditions
- Study the formation of exotic nuclei over a wide range of n/p asymmetry where relatively high  $E^*/A$  can be reached
- Probe thermodynamic system (cluster)

## Intermediate energies

- $15 \text{ A MeV} \leq E_{inc} \leq 100 \text{ A MeV}$
- Dissipative collisions

### Transport model (In

- $^{124}\text{Sn} + ^{124}\text{Sn}$  @ 50

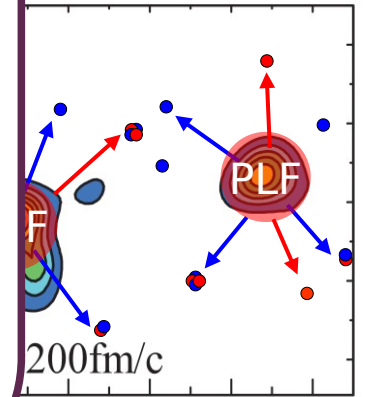


## Isospin migration

- Density gradient
- Neutron-enrichment of the neck

→ Related to  $\frac{\partial \epsilon_{sym}(\rho)}{\partial \rho}$

## Transport phenomena



V. Baran et al., Nuc. Phys. A 730 (2004)

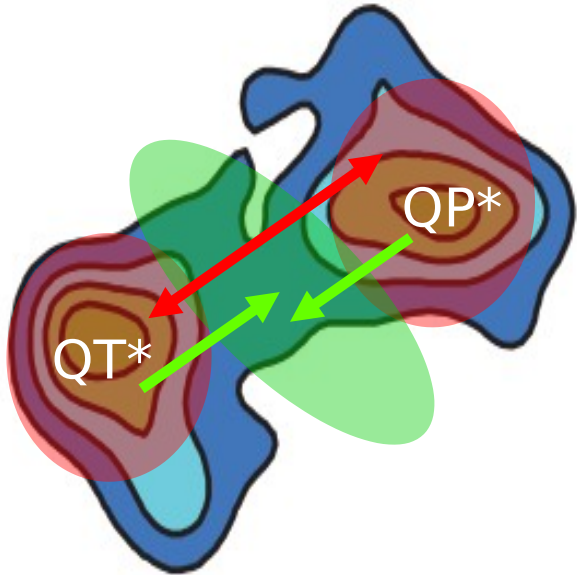
# Context and motivations

## Heavy Ion Collisions

- Unique tool to probe the nuclear EOS at finite temperature under laboratory-controlled conditions
- Study the formation of exotic nuclei over a wide range of n/p asymmetry
- Probe the system (cl)

### Transport models

- $^{124}\text{Sn} + ^{124}\text{Sn}$



## Isospin transport

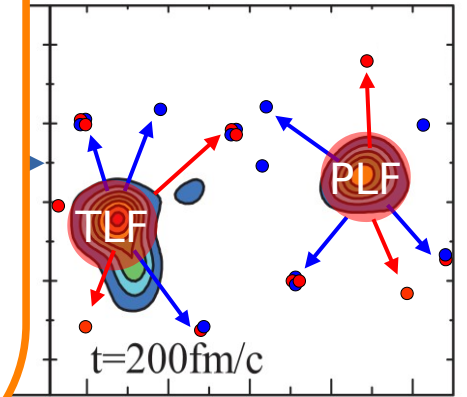
- Competition between the isospin **migration** and **diffusion**
- Transport phenomena directly **linked to  $\mathcal{E}_{sym}$**
- Depends on the **interaction time between projectile and target**  
→ beam energy, impact parameter
- Experimental study requires :  
→ **high isotopic resolution ;**  
→ **special attention to evaporation process ;**  
→ **evaluation of the centrality of the collision.**

## Intermediate energies

- $15 \text{ A MeV} \leq E_{inc} \leq 100 \text{ A MeV}$

### Disruptive collisions

transport phenomena



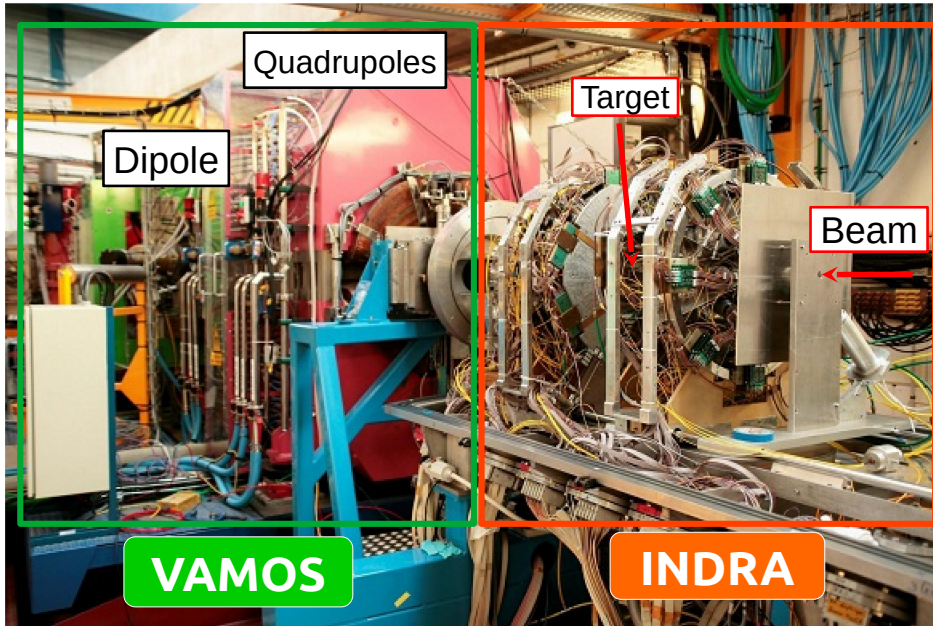
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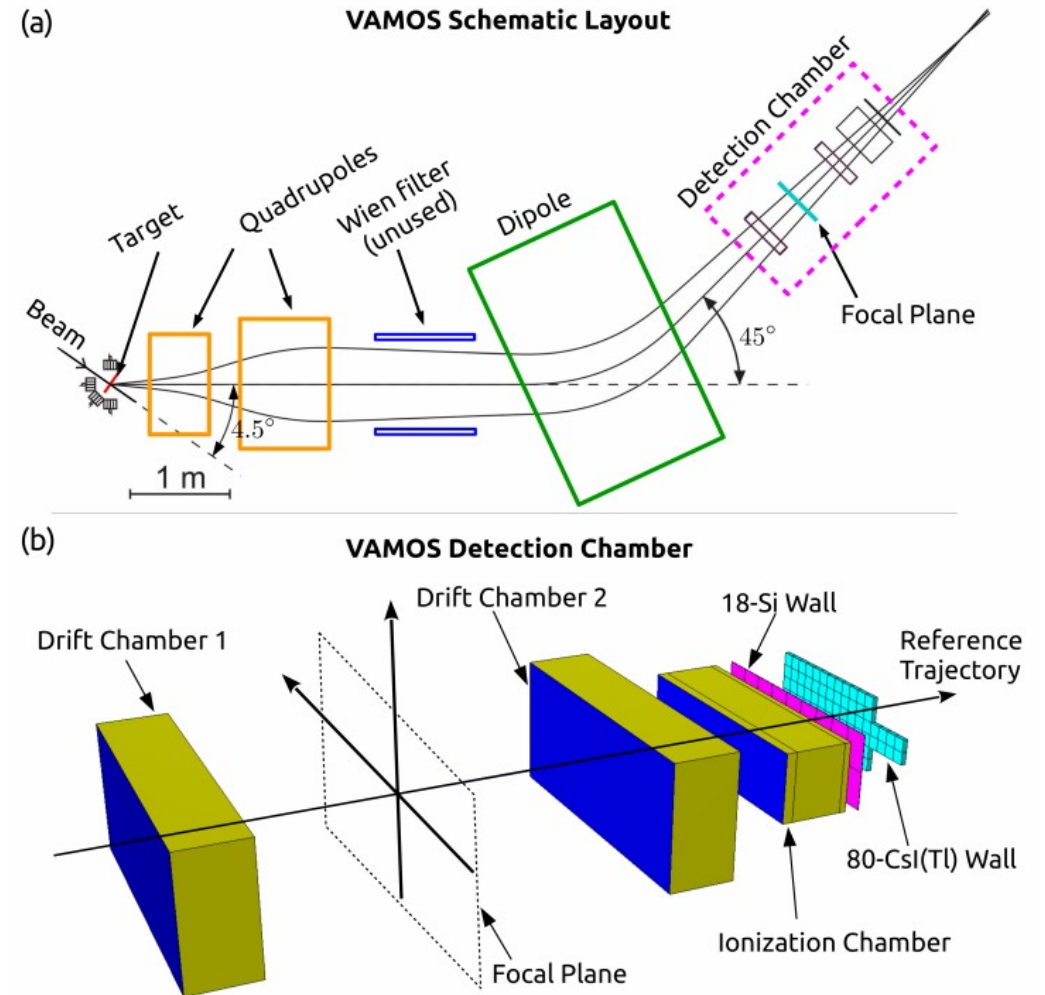
# INDRA-VAMOS coupling @ GANIL

E503 experiment

$^{40,48}\text{Ca} + ^{40,48}\text{Ca}$  @ 35 MeV/nuc



Picture of the experimental setup of the INDRA-VAMOS coupling.



- [1] J. Pouthas et al., NIM A 357, 418 (1995)
- [2] S. Pullanhiotan et al., NIM A 593, 343 (2008)
- [3] Q. Fable et al., PRC 106, 024605 (2022)

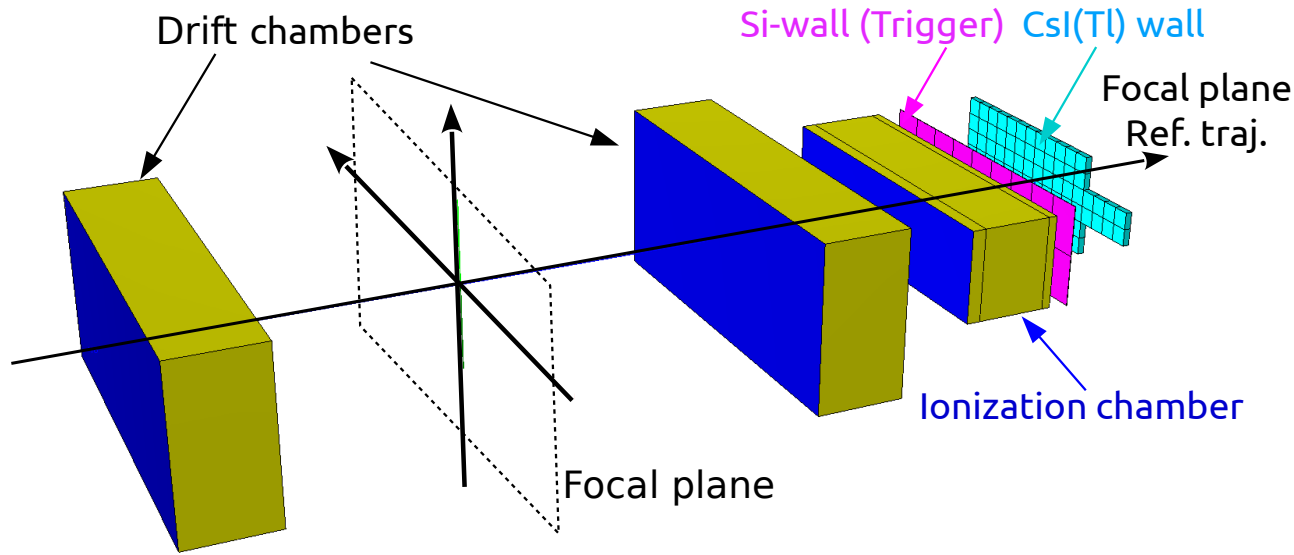
# INDRA-VAMOS coupling @ GANIL

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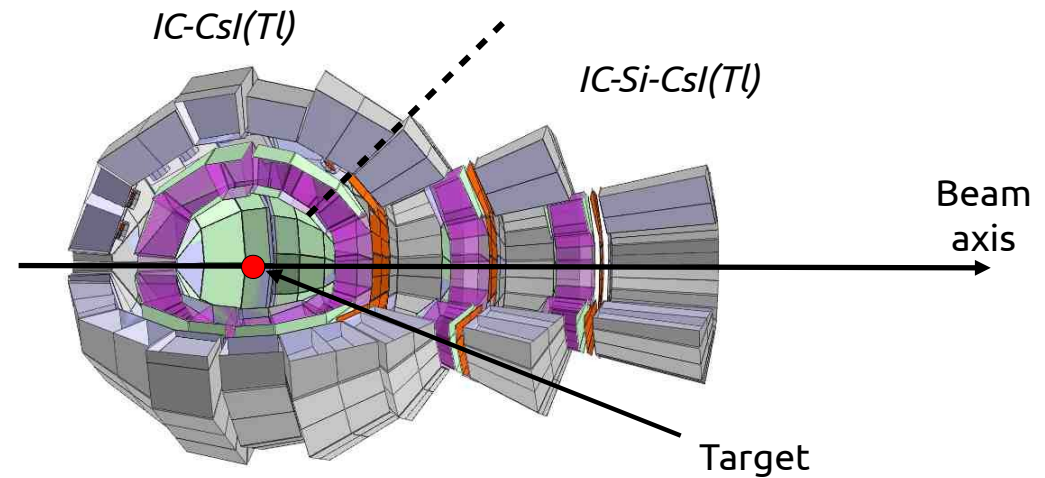
VAMOS

$^{40,48}\text{Ca} + ^{40,48}\text{Ca}$  @ 35 MeV/nuc

INDRA

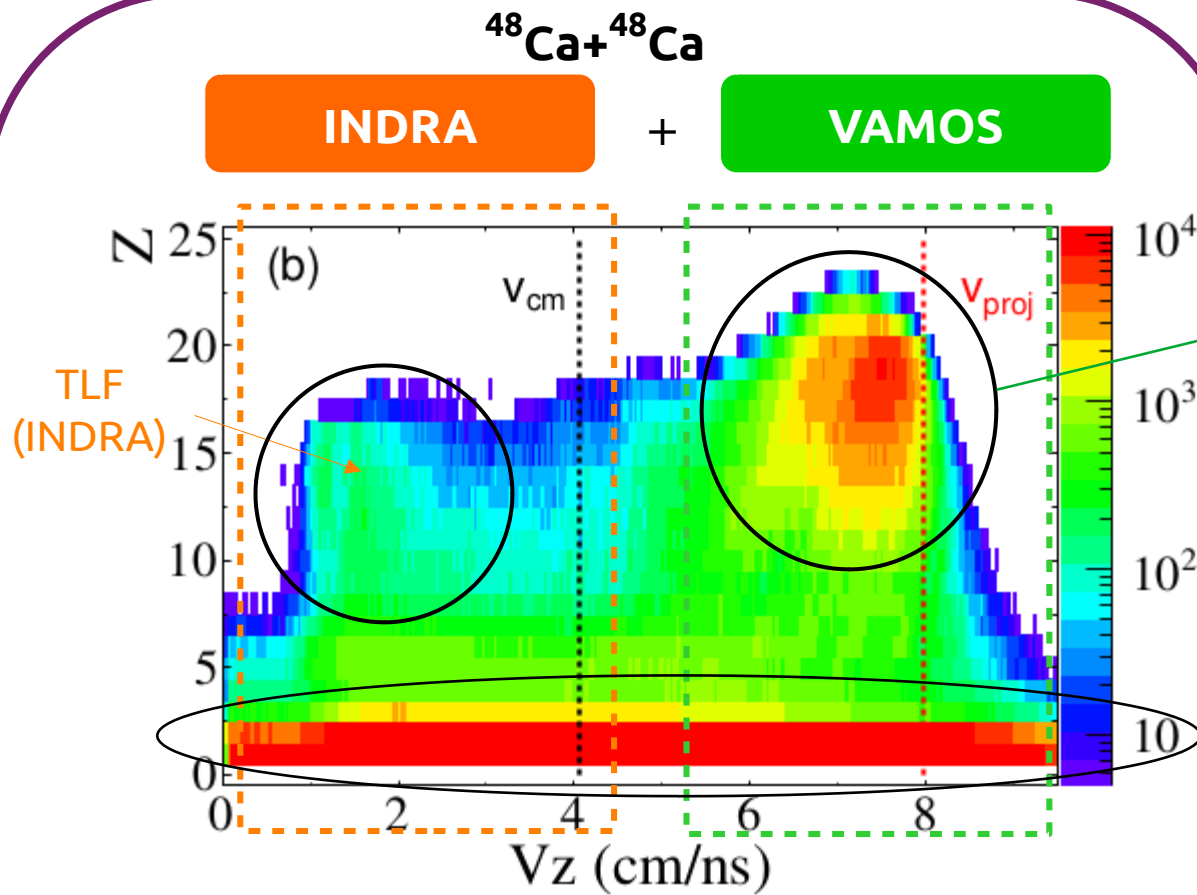


- Si-wall → Acq. Trigger (peripheral collisions)
- Quasiprojectile remnant identification (Z,A)
- $\theta_{LAB} \approx 2.5^\circ - 6.5^\circ$
- $\varphi_{LAB} \approx 220^\circ - 320^\circ$
- 12 Bp settings -  $Bp_0 \approx 0.661 - 2.220$  T.m

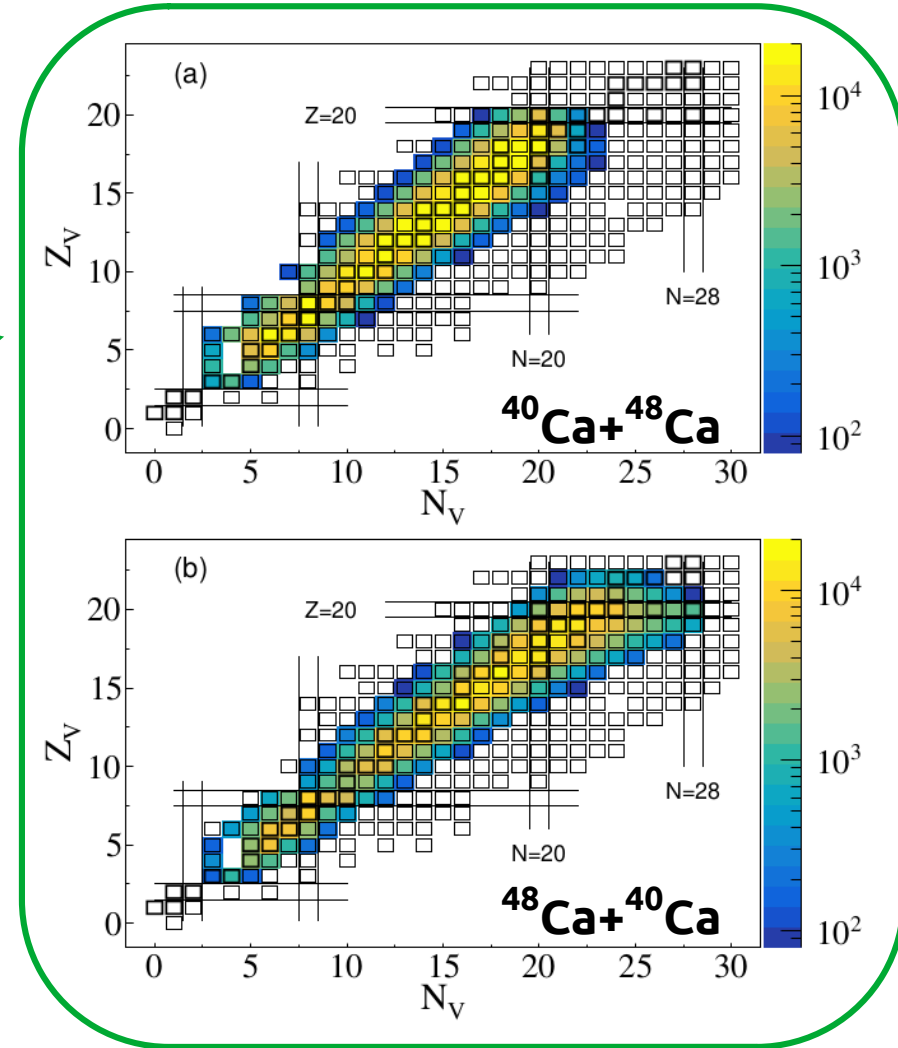


- 14 rings (~300 identification modules)
- Identification
- → (Z,A) for Light Charge Particles (Z≤5)
- → Z up to Z~25
- $\theta_{LAB} \approx 7^\circ - 176^\circ$
- Event characterization ( $b, E^*, \dots$ )

# INDRA-VAMOS : topology



- Dissipative collisions topology
- **3 regions :**
  - LCP emissions around  $v_{\text{CM}}$
  - PLF and TLF from either side of  $v_{\text{CM}}$



- PLF (Vamos)**
- $V_z \geq 6 \text{ cm/ns}$
  - $\rightarrow V_z \sim v_{\text{PROJ}}$
  - $\rightarrow Z \sim Z_{\text{PROJ}}$

→ The fragment identified in VAMOS is assumed to be the PLF

[6] Q. Fable et al., PRC 106, 024605 (2022)



# AMD calculations

- QMD-type model widely used to describe various features of GS nuclei and out-of-equilibrium many-body dynamics
- 2 versions employed (with or without cluster formation) ;
- Triangular input impact parameter distribution (from 0 to  $b_{\max}$ ) ;
- Collisions followed up to a limit time  $t_{\lim}$

## AMD-CC (Cluster Correlations)

- Trees from S. Piantelli and A. Camaiani [1] :
  - $t_{\lim}=500$  fm/c ;
  - $b_{\max} \sim b_{gr} \sim 10$  fm ;
- Mean-Field description based on Skyrme :
  - Sly4,  $K_{\text{sat}}=230$  MeV,  $E_{\text{sym}}=32$  MeV,  $\rho_0=0.16$  fm<sup>-3</sup>
    - Soft :  $L = 46$  MeV ;
    - Stiff :  $L = 108$  MeV.
- GEMINI ++ as afterburner :
  - 50 sec. event / primary event.

## AMD-NC (No Cluster Correlations)

- Former version of AMD (2013) [2-3]:
  - $t_{\lim}=300$  fm/c ;
  - $b_{\max} \sim 8$  fm ;
- Mean-Field description :
  - $\rho_0=0.16$  fm<sup>-3</sup>
    - Soft : Gogny ;
    - Stiff : Gogny-AS.
- GEMINI ++ as afterburner :
  - 100 sec. event / primary event.

[1] A. Camaiani *et al.* PRC 102, 044607 (2020)

[2] A. Ono *et al.*, PRC 70, 041604 (2004)

[3] Q. Fable *et al.*, PRC 106, 024605 (2022)

# AMD calculations

- QMD-type model widely used to describe various features of GS nuclei and out-of-equilibrium many-body dynamics
- 2 versions employed (with or without cluster formation) ;
- Triangular input impact parameter distribution (from 0 to  $b_{\max}$ ) ;
- Collisions followed up to a limit time  $t_{\text{lim}}$

## AMD-CC (Cluster Correlations)

	Proj	Targ	$E^*/A$	$b_{\max}$ [fm]	$N_{\text{pr}}$
stiff	$^{40}\text{Ca}$	$^{40}\text{Ca}$	35	9.72	54340
	$^{48}\text{Ca}$	$^{40}\text{Ca}$	35	10.07	34642
	$^{48}\text{Ca}$	$^{48}\text{Ca}$	35	10.42	35379
soft	$^{40}\text{Ca}$	$^{40}\text{Ca}$	35	9.72	43435
	$^{48}\text{Ca}$	$^{40}\text{Ca}$	35	10.07	34136
	$^{48}\text{Ca}$	$^{48}\text{Ca}$	35	10.42	27636

~ 40.000 primary events/system

## AMD-NC (No Cluster Correlations)

	Proj	Targ	$E^*/A$	$b_{\max}$ [fm]	$N_{\text{pr}}$
stiff	$^{40}\text{Ca}$	$^{40}\text{Ca}$	35	7.70	102551
	$^{48}\text{Ca}$	$^{40}\text{Ca}$	35	7.96	106878
	$^{48}\text{Ca}$	$^{48}\text{Ca}$	35	8.22	110821
soft	$^{40}\text{Ca}$	$^{40}\text{Ca}$	35	7.70	97947
	$^{48}\text{Ca}$	$^{40}\text{Ca}$	35	7.96	107312
	$^{48}\text{Ca}$	$^{48}\text{Ca}$	35	8.22	181831

~ 100.000 primary events/system

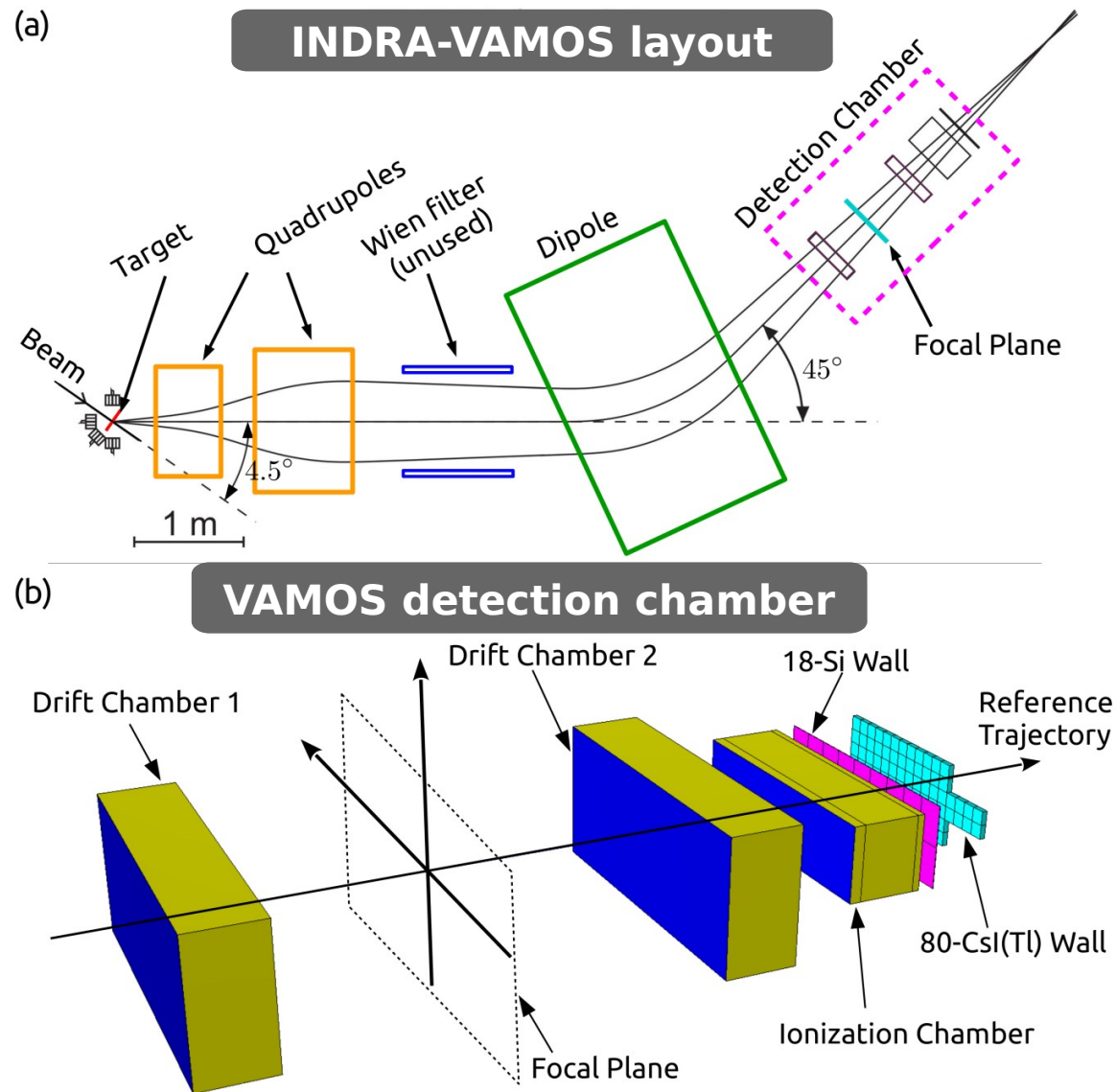
# AMD calculations : filter and data selection

## Experimental setup

- **VAMOS high-acceptance spectrometer :**
  - Exp. trigger (Si-Wall) ;
  - Up to 12  $B\rho_0$  settings (0.661-2.220 Tm)
  - $\Theta_{\text{lab}} = 2.5^\circ - 6.5^\circ$  ;  $\Phi_{\text{lab}} = 220^\circ - 320^\circ$  ;
  - PLF (Z,A) identification ;
- **INDRA multi-det array :**
  - $\Theta_{\text{lab}} = 7^\circ - 176^\circ$  ;
  - (Z,A) ident. up to  $Z = 4-5$  ;

## Numerical filter (KaliVeda)

- **VAMOS filter "by hand" :**
  - Acceptance in lab + energy thresholds ;
- **INDRA standard filter in KaliVeda :**
  - Geometry, identification and energy thresholds ;
- **Same offline selections than the experiment :**
  - Only one hit in VAMOS Si-wall ;
  - Remove elastic-like events
  - Completeness :  $Z_{\text{tot}} > 10$  and  $\Sigma(Z \cdot p_z) > 0.5 \cdot p_{\text{beam}}$

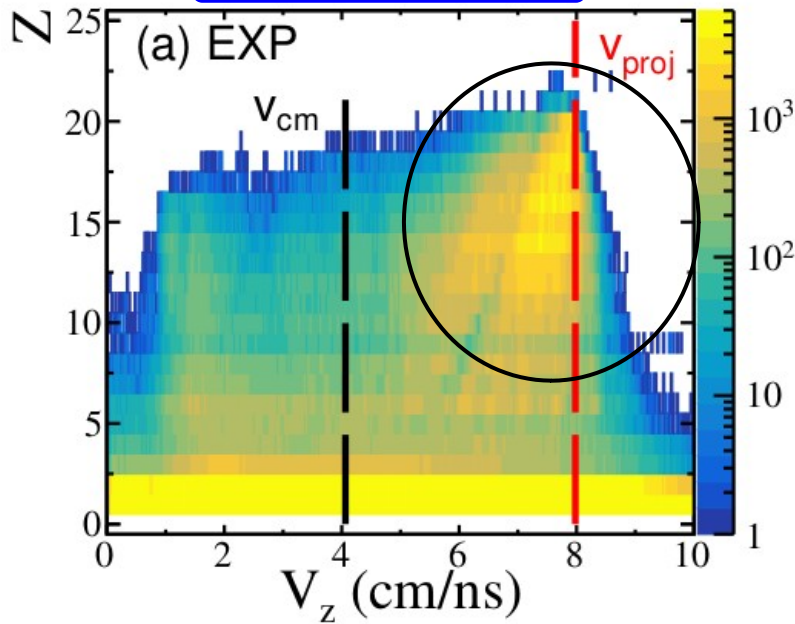


# Model comparisons with the data

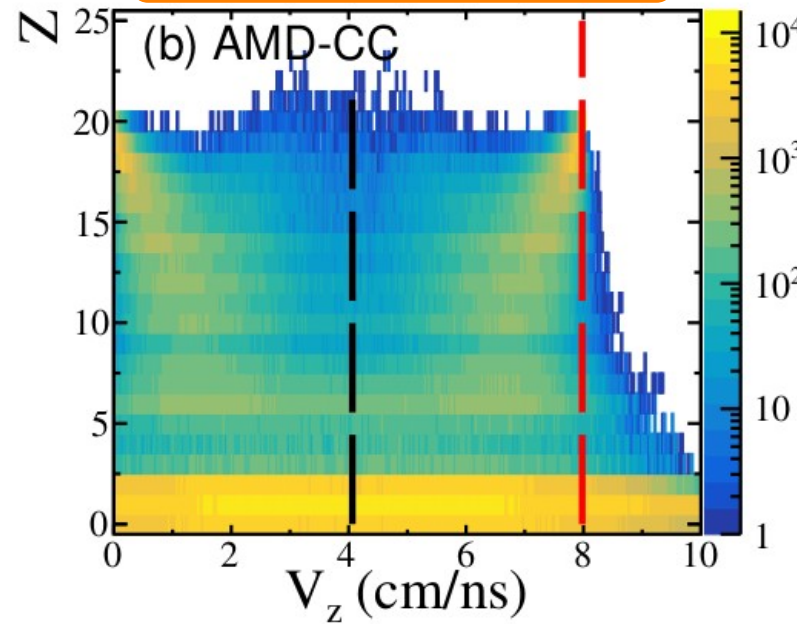
Topology : charge vs velocity

$^{48}\text{Ca}+^{48}\text{Ca}$

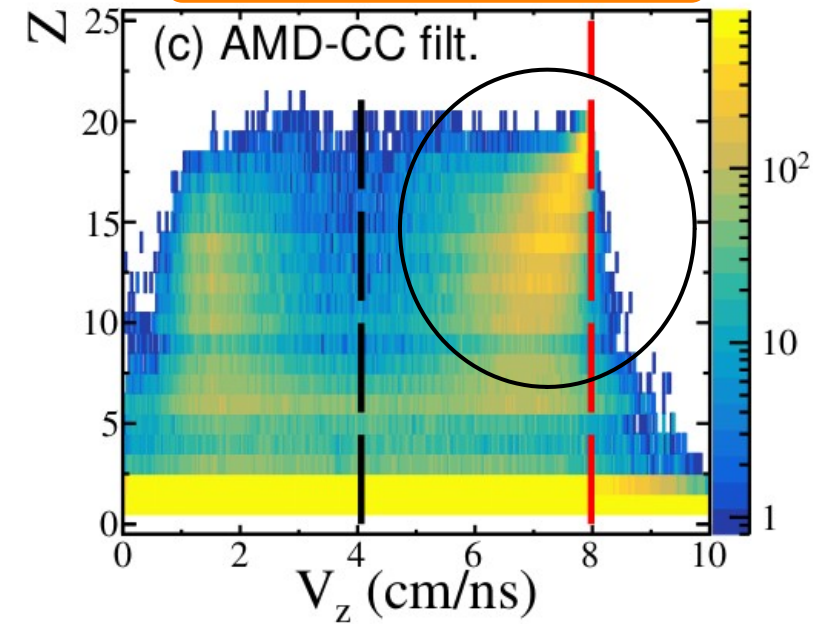
EXPERIMENT



AMD-CC : no filter



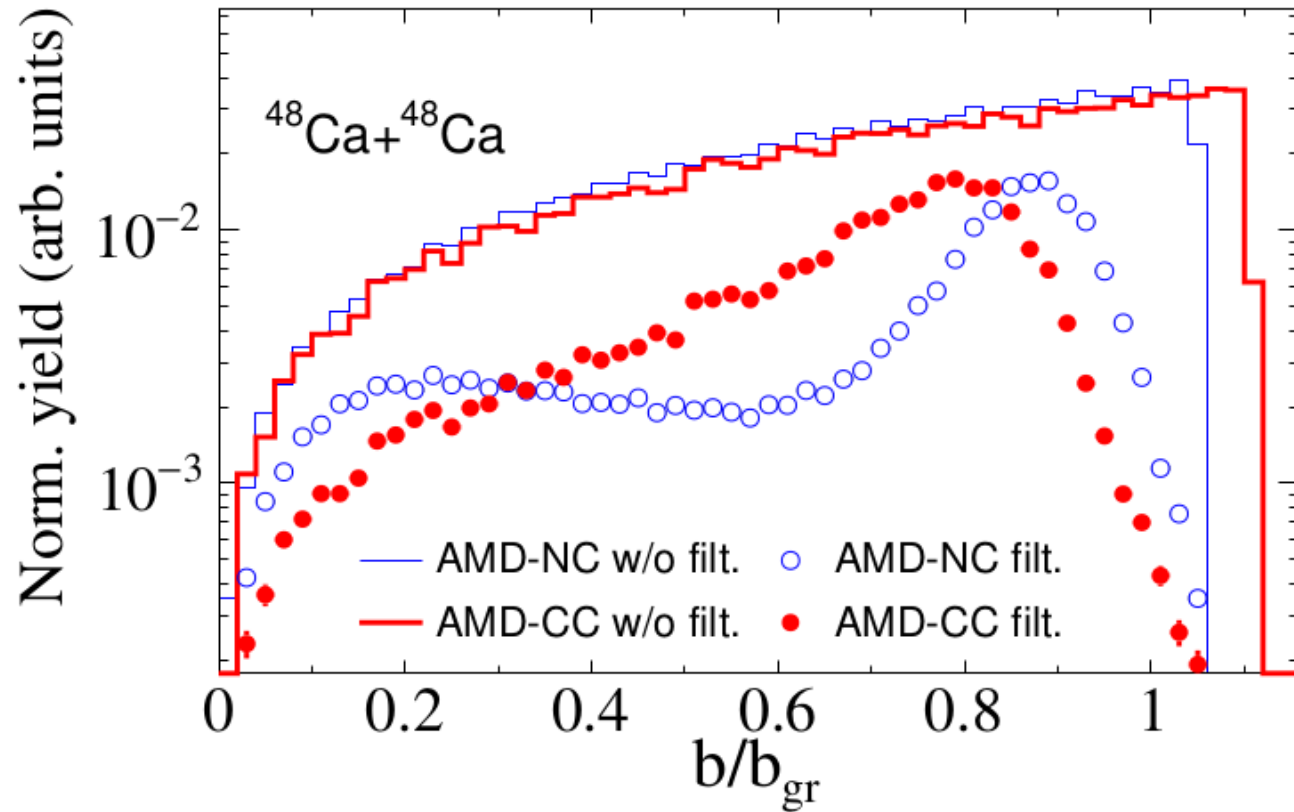
AMD-CC : filtered



- Similarly to the experiment, both AMD models present a topology characteristic of dissipative binary collisions
  - **3 regions :**
    - LCP emissions around  $v_{\text{CM}}$
    - PLF and TLF from either side of  $v_{\text{CM}}$
- The fragments detected in VAMOS are mostly ( $\sim 95\%$ ) the product of the QP decay (PLF) ;

# Model comparisons with the data

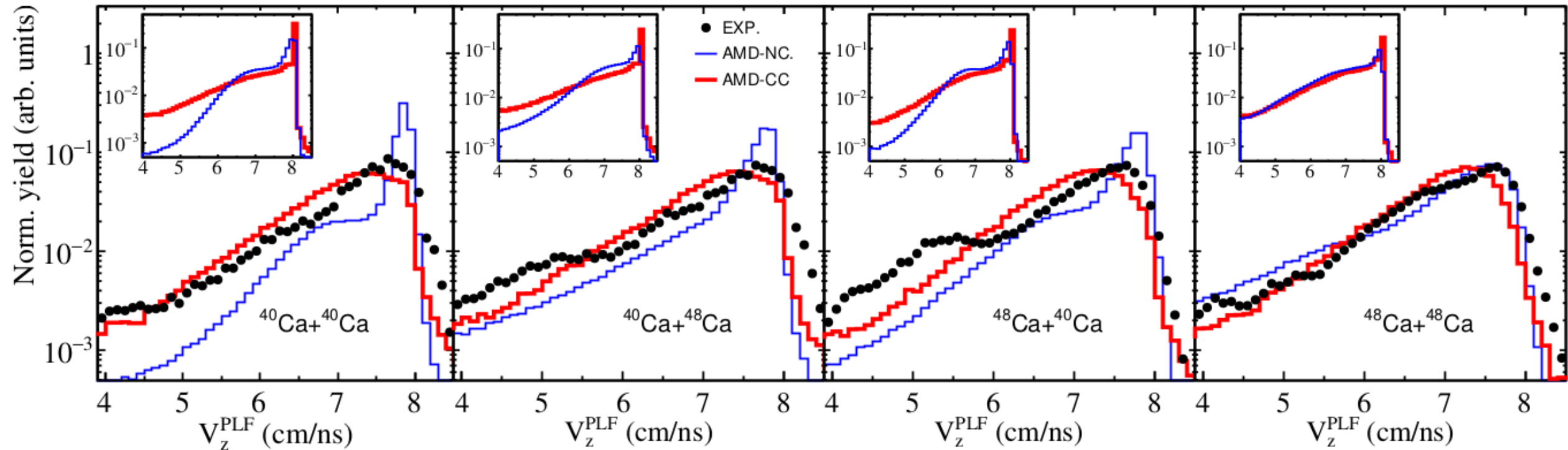
## Impact parameter distribution



- Slight difference between AMD-CC and AMD-NC input distribution ( $b_{max}$ );
  - Strong disagreement between the two versions of the code once the filter is applied:
    - More peripheral events filtered for AMD-NC
  - Not trivial:
    - Model dynamics?
    - Detector acceptance?
    - Offline selections?
- Different subset of events from a model to another
- A detailed study of the primary fragments in AMD-NC shows that this version tends to overproduce inelastic-like events

# Model comparisons with the data

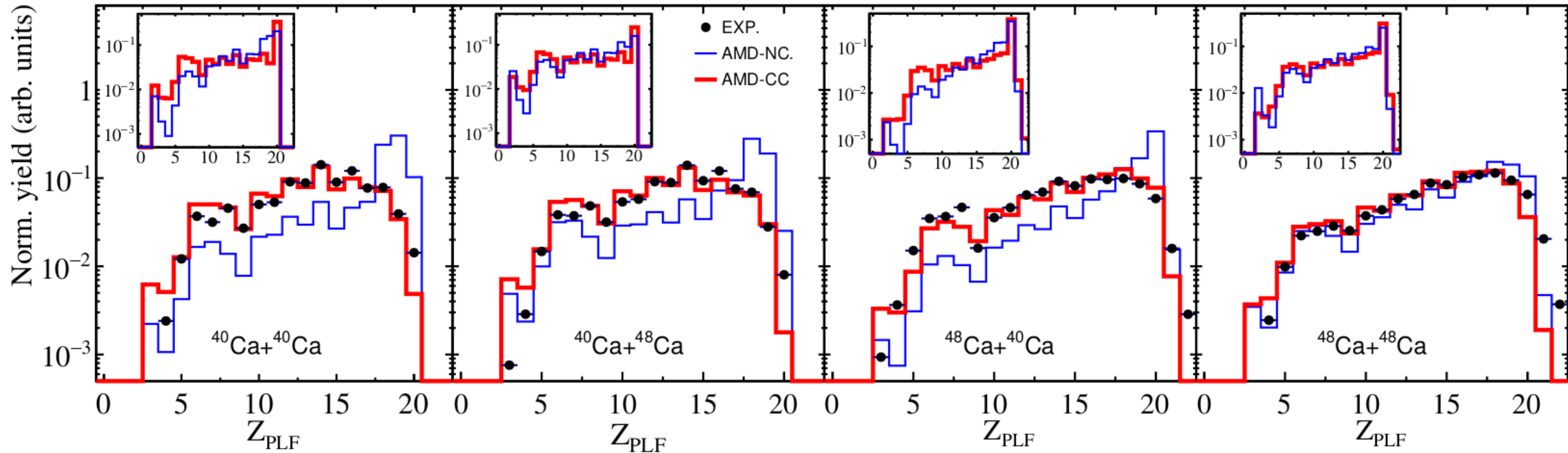
## PLF : velocity distributions



- Inner plots : no filter applied ( $V_z > 4$  cm/ns)
- Increasing values of the yields with the velocity  
→ reflect the dissipation of the collision
- Better agreement with AMD-CC  
→ overproduction of inelastic-like events by AMD-NC

# Model comparisons with the data

## PLF : charge distributions



- Inner plots : no filter applied ( $V_z > 4$  cm/ns)
- Both models reproduce the experimental odd-even staggering ;
- Better agreement with AMD-CC  
→ overproduction of inelastic-like events by AMD-NC
- Similar results are obtained with the mass distributions

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# Impact parameter estimation : method

## Centrality

- By def., the inclusive distribution of an observable  $X$  is :

$$P(X) = \int_0^{\infty} P(b) P(X | b) db$$

- By def., the b-centrality is :

$$c_b = \int_0^b P(b') db'$$

conditional probability

# Impact parameter estimation : method

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Using  
both equations

$$P(X) = \int_0^1 P(c_b)P(X | c_b)dc_b$$
$$= \int_0^1 P(X | c_b)dc_b \text{ (as } P(c_b) = 1 \forall c_b)$$

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**Conclusion : by fitting the inclusive distribution of an observable  $X$ , we can determine  $P(X|c_b)$**

# Impact parameter estimation : method

[1] R. Rogly et al., PRC 98, 024902 (2018)

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**Conclusion : by fitting the inclusive distribution of an observable  $X$ , we can determine  $P(X|c_b)$**

$$P(X | c_b) = f [\bar{X}(c_b), \theta(c_b)]$$

**Proposed form of conditional probability  
(here gamma-distribution [1])**

**→ fit to the inclusive distributions**

# Impact parameter estimation : method

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Using both equations

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Conclusion : by fitting the inclusive distribution of an observable  $X$ , we can determine  $P(X|c_b)$

Once  $P(X|c_b)$  is determined by the fit, it can be used to estimate the centrality distribution for any given sampling of data from Bayes' theorem :

$$P(c_b | \mathcal{S}) = \frac{P(X | c_b) \frac{P(X|\mathcal{S})}{P(X)} dX}{P(X | \mathcal{S}) dX}$$

Experimental sample

Sample distribution of  $X$   
(i.e histogram of  $X$  filled with sample events)

$$P(X | c_b) = f [\bar{X}(c_b), \theta(c_b)]$$

Proposed form of conditional probability  
(here gamma-distribution)

→ fit to the inclusive distributions

# Impact parameter estimation : method

## Centrality

- By def., the inclusive distribution of an observable  $X$  is :

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- By def., the centrality is :

$$c_b = \int_0^b P(b')db'$$

Using  
both equations

$$P(X) = \int_0^1 P(c_b)P(X | c_b)dc_b$$
$$= \int_0^1 P(X | c_b)dc_b \text{ (as } P(c_b) = 1 \forall c_b)$$

Conclusion : by fitting the inclusive distribution of an observable  $X$ , we can determine  $P(X|c_b)$

Once  $P(X|c_b)$  is determined by the fit, it can be used to estimate the centrality distribution for any given sampling of data from Bayes' theorem :

$$P(c_b|\mathcal{S}) = \frac{P(X | c_b) \frac{P(X|\mathcal{S})}{P(X)} dX}{P(X | \mathcal{S}) dX}$$

## Absolute impact param. dist.

$$P(b | \mathcal{S}) = P(b)P(c_b(b)|\mathcal{S})$$

**NOTE** : it is necessary to assume a specific form of the impact parameter  $P(b)$  and calculate the corresponding relationship between  $c_b, b$  and  $c_b(b)$ ...

# Impact parameter estimation : method

- By def., the inclusive distribution of an observable  $X$  is :

$$P(X) = \int_0^\infty P(b)P(X | b)db$$

- By def., the centrality is :

$$c_b = \int_0^b P(b')db'$$

Using  
both equations

$$P(X) = \int_0^1 P(c_b)P(X | c_b)dc_b$$
$$= \int_0^1 P(X | c_b)dc_b \text{ (as } P(c_b) = 1 \forall c_b)$$

Conclusion : by fitting the inclusive distribution of an observable  $X$ , we can determine  $P(X|c_b)$

$$P(X | c_b) = f [\bar{X}(c_b), \theta(c_b)]$$

Proposed form of conditional probability  
(here gamma-distribution)

→ fit to the inclusive distributions

For more details...

Reconstructing the impact parameter of proton-nucleus and nucleus-nucleus collisions

Rudolph Rogly, Giuliano Giacalone, and Jean-Yves Ollitrault  
Phys. Rev. C **98**, 024902 – Published 2 August 2018

PHYSICAL REVIEW C  
covering nuclear physics

Model independent reconstruction of impact parameter distributions for intermediate energy heavy ion collisions

J. D. Frankland, D. Gruyer, E. Bonnet, B. Borderie, R. Bougault, A. Chbihi, J. E. Ducret, D. Durand, Q. Fable, M. Henri, J. Lemarié, N. Le Neindre, I. Lombardo, O. Lopez, L. Manduci, M. Pârlog, J. Quicray, G. Verde, E. Vient, and M. Vigilante (INDRA Collaboration)  
Phys. Rev. C **104**, 034609 – Published 8 September 2021

PHYSICAL REVIEW C  
covering nuclear physics

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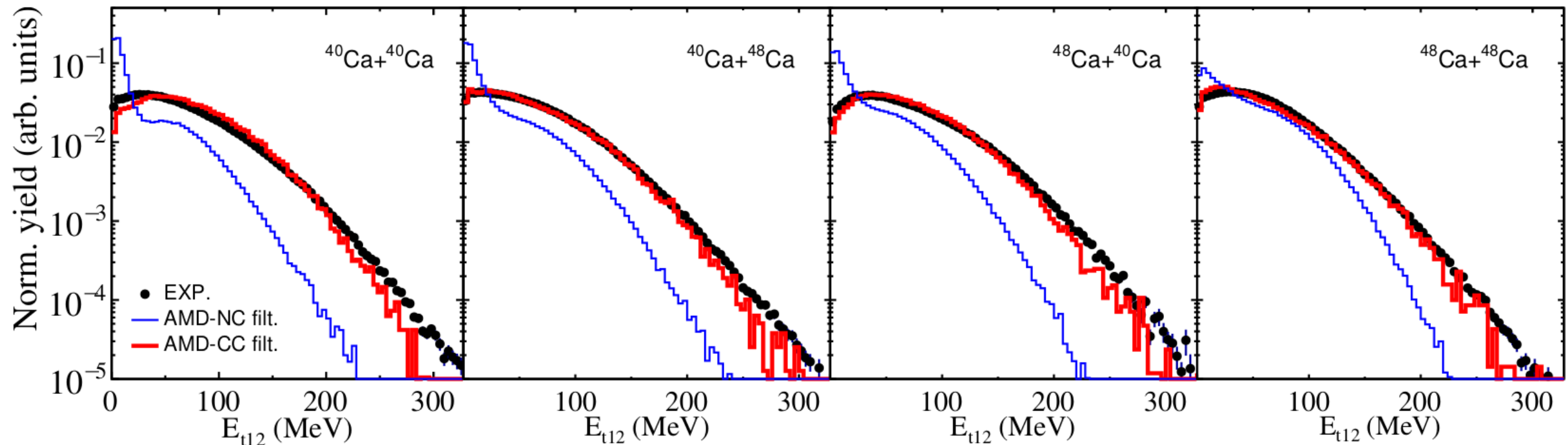


# Impact parameter estimation

## Total transverse kinetic energy

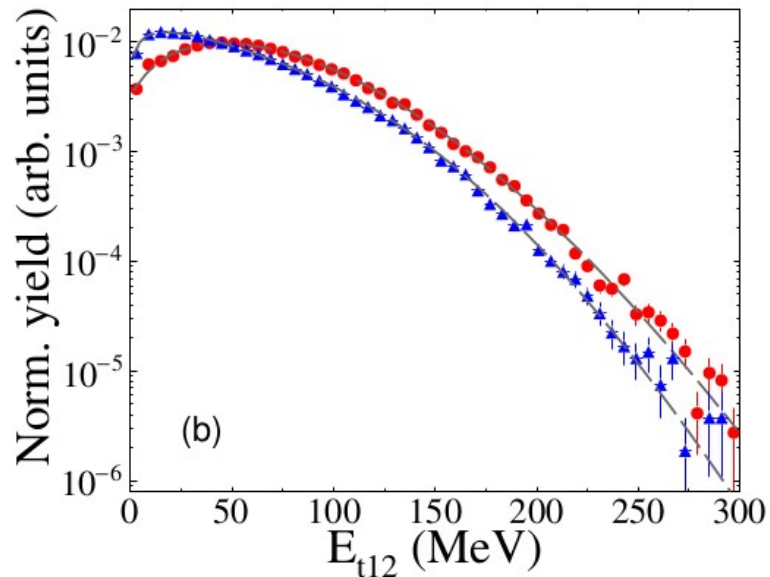
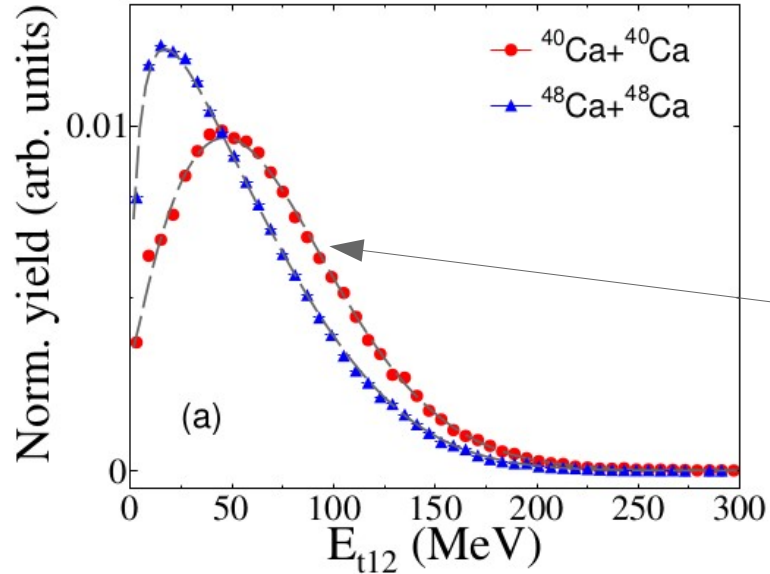
$$E_{t12} = \sum_{Z=1,2} Ek_i \sin^2(\theta_i)$$

- Often used as centrality sorting observable
- 90% efficiency detection of LCP with INDRA
- **Better reproduced by the AMD-CC version**  
→ Relevance of considering clusters to reproduce experimental kinetic energy spectra [1]



[1] C. Frosin et al., PRC 107, 044614 (2023)

# Impact parameter estimation : AMD-CC



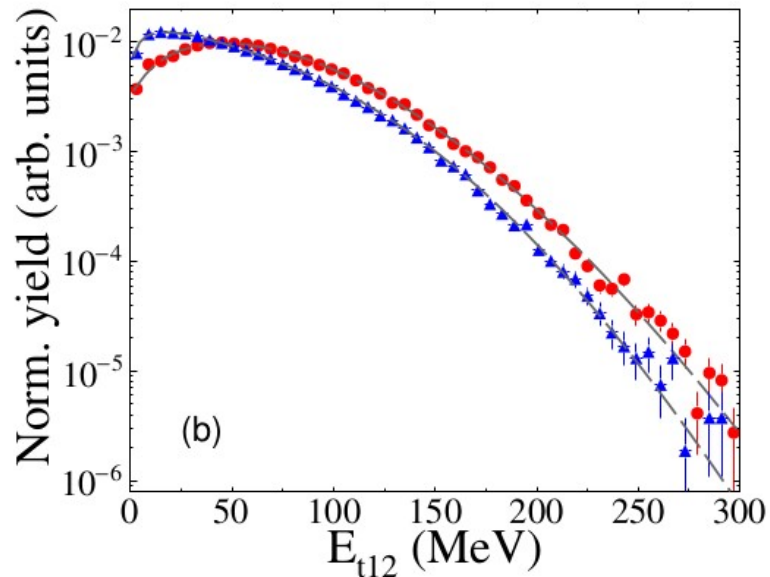
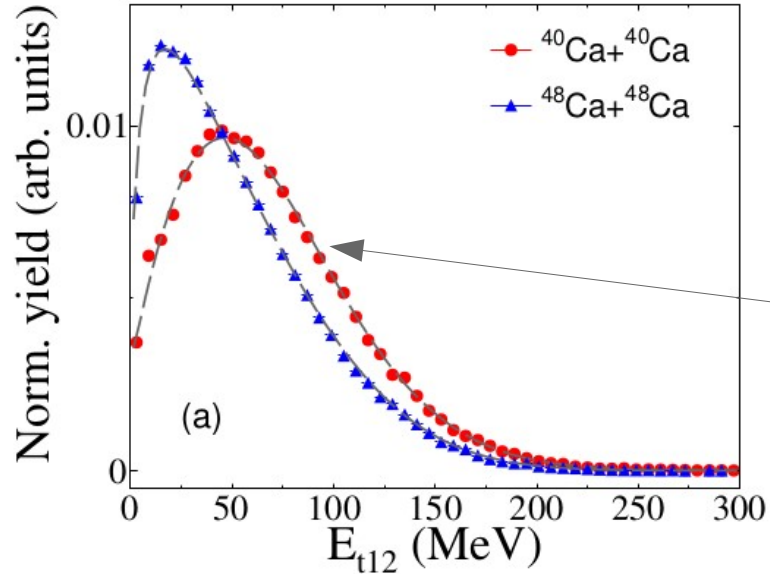
$$P(X) = \int_0^1 P(X | c_b) dc_b \quad \text{with } X = Et_{12} = \sum_{Z=1,2} Ek_i \sin^2(\theta_i)$$

**Proposed form of conditional probability : gamma distribution (5 fit parameters)  
→ fit to the inclusive distributions**

$$P(X | c_b) = f [\bar{X}(c_b), \theta(c_b)]$$

Model	System	$\alpha$	$\gamma$	$\theta$ [MeV]	$X_{min}$ [MeV]	$X_{max}$ [MeV]	$\chi^2$
AMD-CC stiff	$^{40}\text{Ca}+^{40}\text{Ca}$	0.10	0.52	8.95	1	265	1.2
	$^{40}\text{Ca}+^{48}\text{Ca}$	0.26	0.81	9.42	4	197	1.7
	$^{48}\text{Ca}+^{40}\text{Ca}$	0.12	0.59	8.68	5	251	1.4
	$^{48}\text{Ca}+^{48}\text{Ca}$	0.33	0.93	8.48	8	179	1.1
AMD-CC soft	$^{40}\text{Ca}+^{40}\text{Ca}$	0.15	0.58	7.12	1	246	1.5
	$^{40}\text{Ca}+^{48}\text{Ca}$	0.31	0.84	9.65	5	187	1.2
	$^{48}\text{Ca}+^{40}\text{Ca}$	0.15	0.60	8.71	5	241	1.3
	$^{48}\text{Ca}+^{48}\text{Ca}$	0.35	0.96	9.68	10	175	1.1

# Impact parameter estimation : AMD-CC



$$P(X) = \int_0^1 P(X | c_b) dc_b \quad \text{with } X = Et_{12} = \sum_{Z=1,2} Ek_i \sin^2(\theta_i)$$

**Proposed form of conditional probability : gamma distribution (5 fit parameters)**  
**→ fit to the inclusive distributions**

$$P(X | c_b) = f [\bar{X}(c_b), \theta(c_b)]$$

**For a generic experimental sample, the b-centrality distribution is :**

$$P(c_b | \mathcal{S}) = \frac{P(X | c_b) \frac{P(X | \mathcal{S})}{P(X)} dX}{P(X | \mathcal{S}) dX}$$

**Where  $\mathcal{S}$  is the sample distribution of  $X$**   
**→ histogram of  $X$  filled from the events in the sample**

**In our case :**

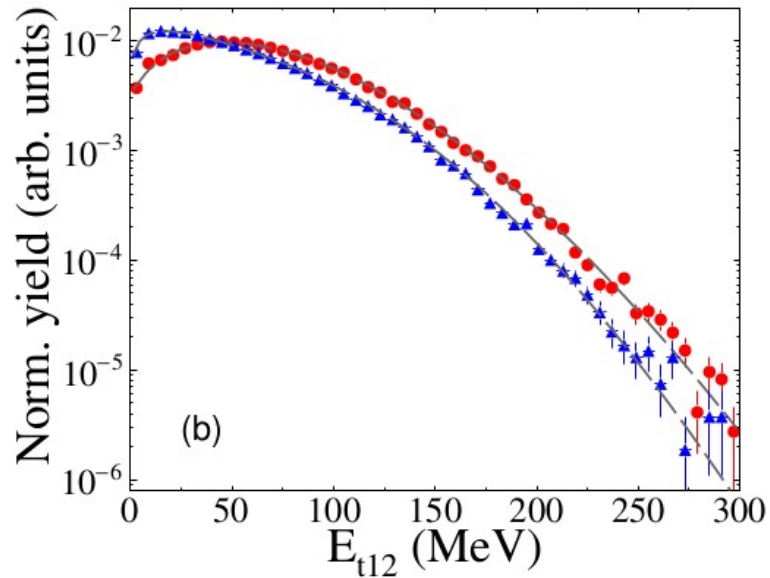
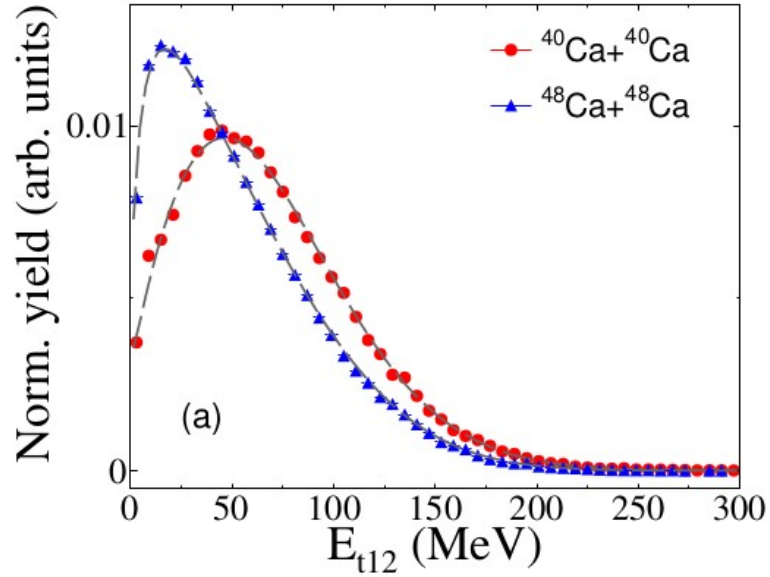
**$X = E_{t12}$**

**Samples of 5 % in  $C_{E_{t12}}$**

**Sampling in centrality**

$$c_{E_{t12}} \equiv \int_{E_{t12}}^{+\infty} P(\tilde{E}_{t12}) d\tilde{E}_{t12}$$

# Impact parameter estimation : AMD-CC



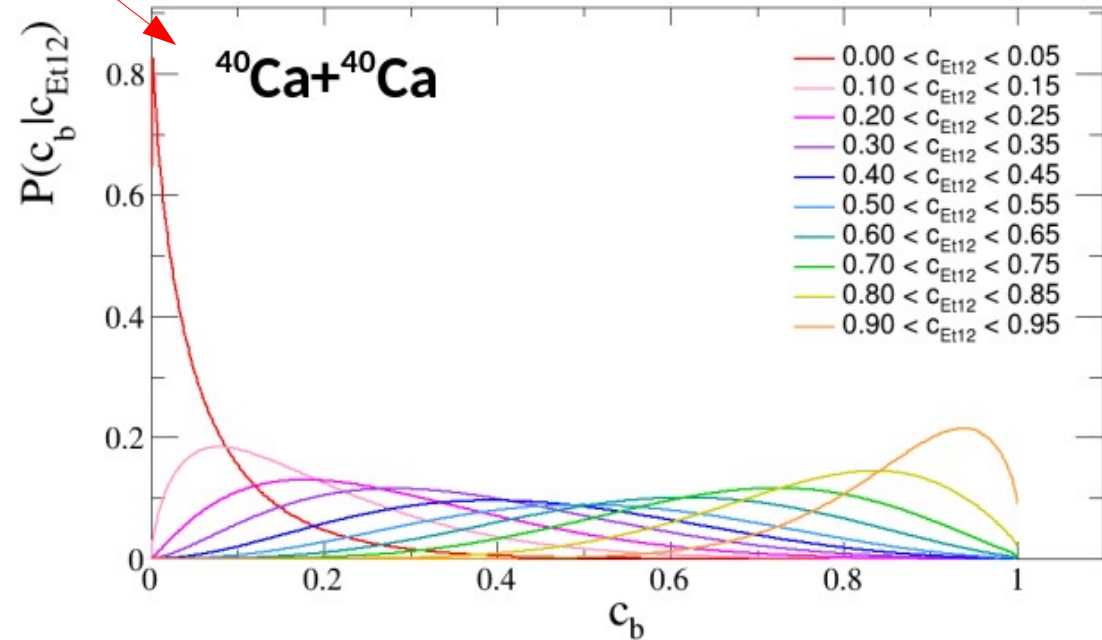
For a generic experimental sample, the b-centrality distribution is :

$$P(c_b | \mathcal{S}) = \frac{P(X | c_b) \frac{P(X | \mathcal{S})}{P(X)} dX}{P(X | \mathcal{S}) dX}$$

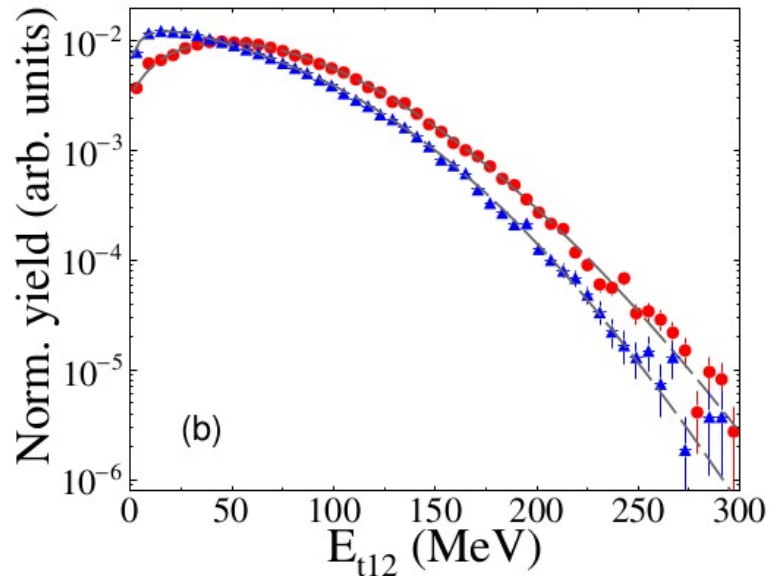
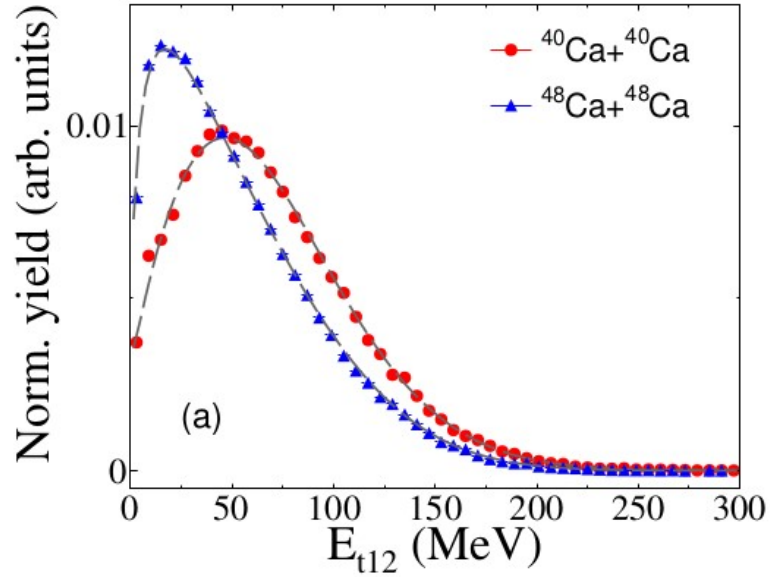
In our case :

$X = E_{t12}$

Samples of 5 % in  $C_{E_{t12}}$



# Impact parameter estimation : AMD-CC



For a generic experimental sample, the b-centrality distribution is :

$$P(c_b | \mathcal{S}) = \frac{P(X | c_b) \frac{P(X | \mathcal{S})}{P(X)} dX}{P(X | \mathcal{S}) dX}$$

In our case :

$X = E_{t12}$

Samples of 5 % in  $C_{E_{t12}}$

Absolute impact parameter distribution

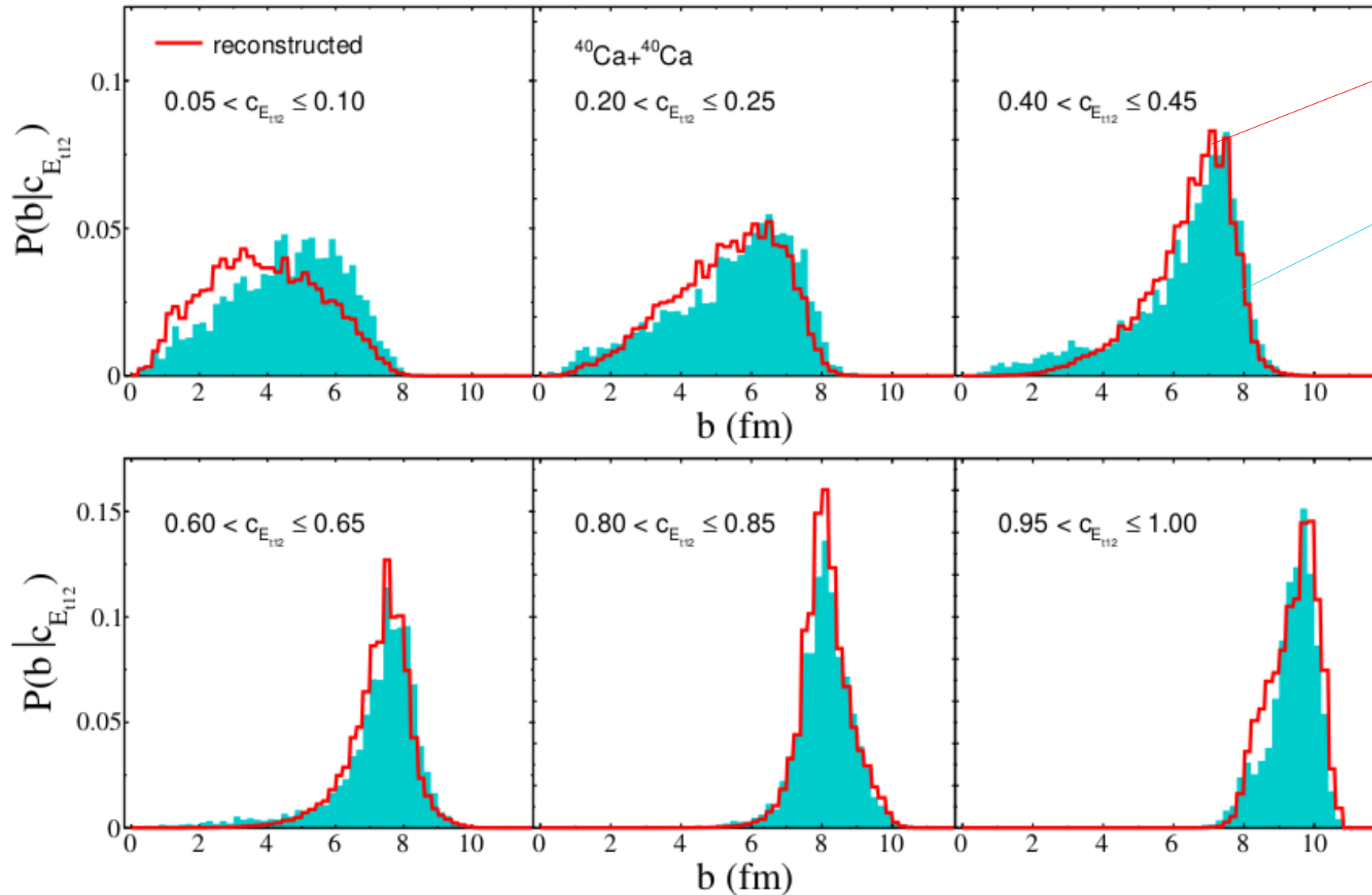
$$P(b | \mathcal{S}) = P(b) P(c_b(b) | \mathcal{S})$$

NOTE : it is necessary to assume a specific form of the impact parameter  $P(b)$  and calculate the corresponding relationship between  $c_b, b$  and  $c_b(b)$ ...

# Impact parameter estimation : AMD-CC

## Conditional probability results

AMD-CC stiff  
 $^{40}\text{Ca}+^{40}\text{Ca}$



- Reasonable agreement for most of the samplings
- Small shift for the most central sampling  
→ slight underestimation of the impact. param.

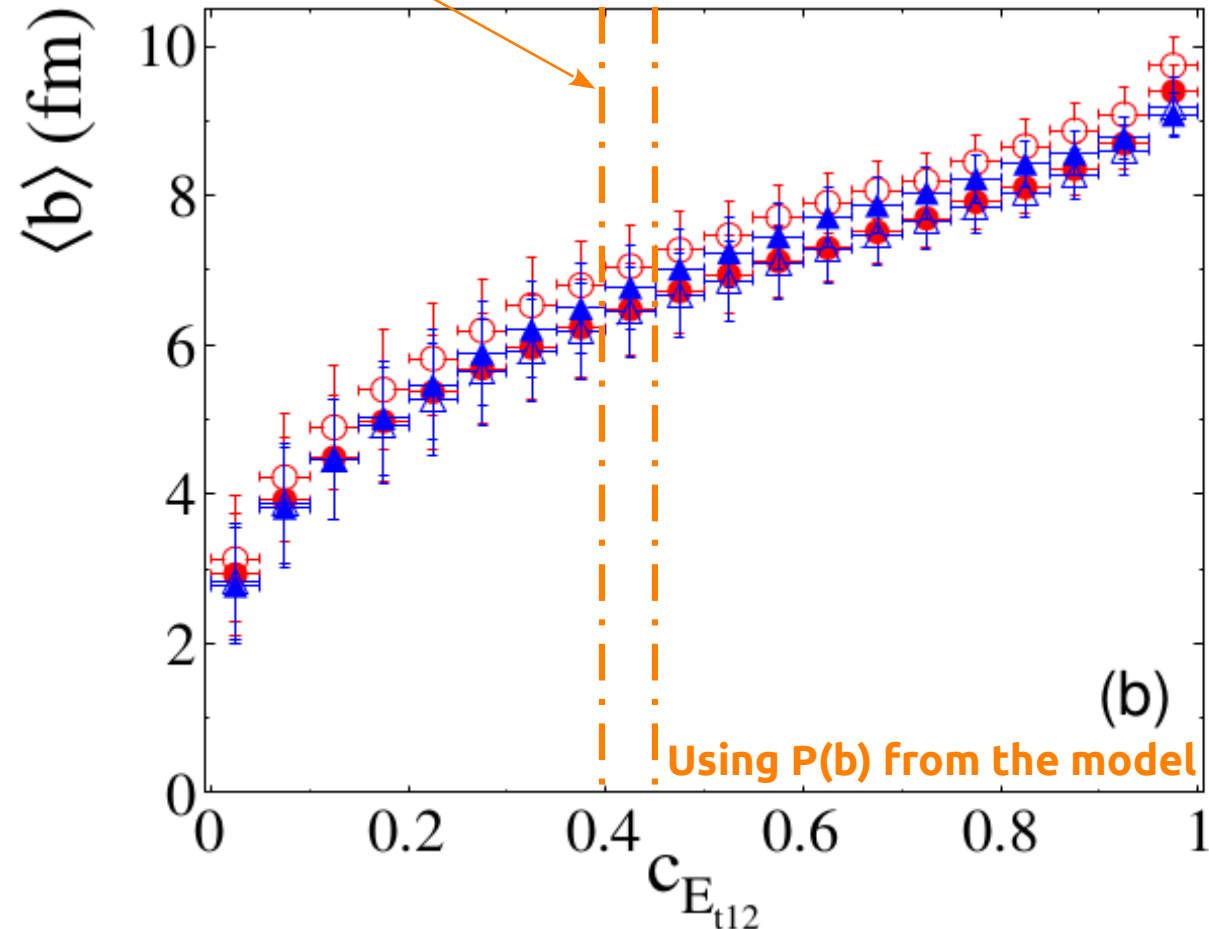
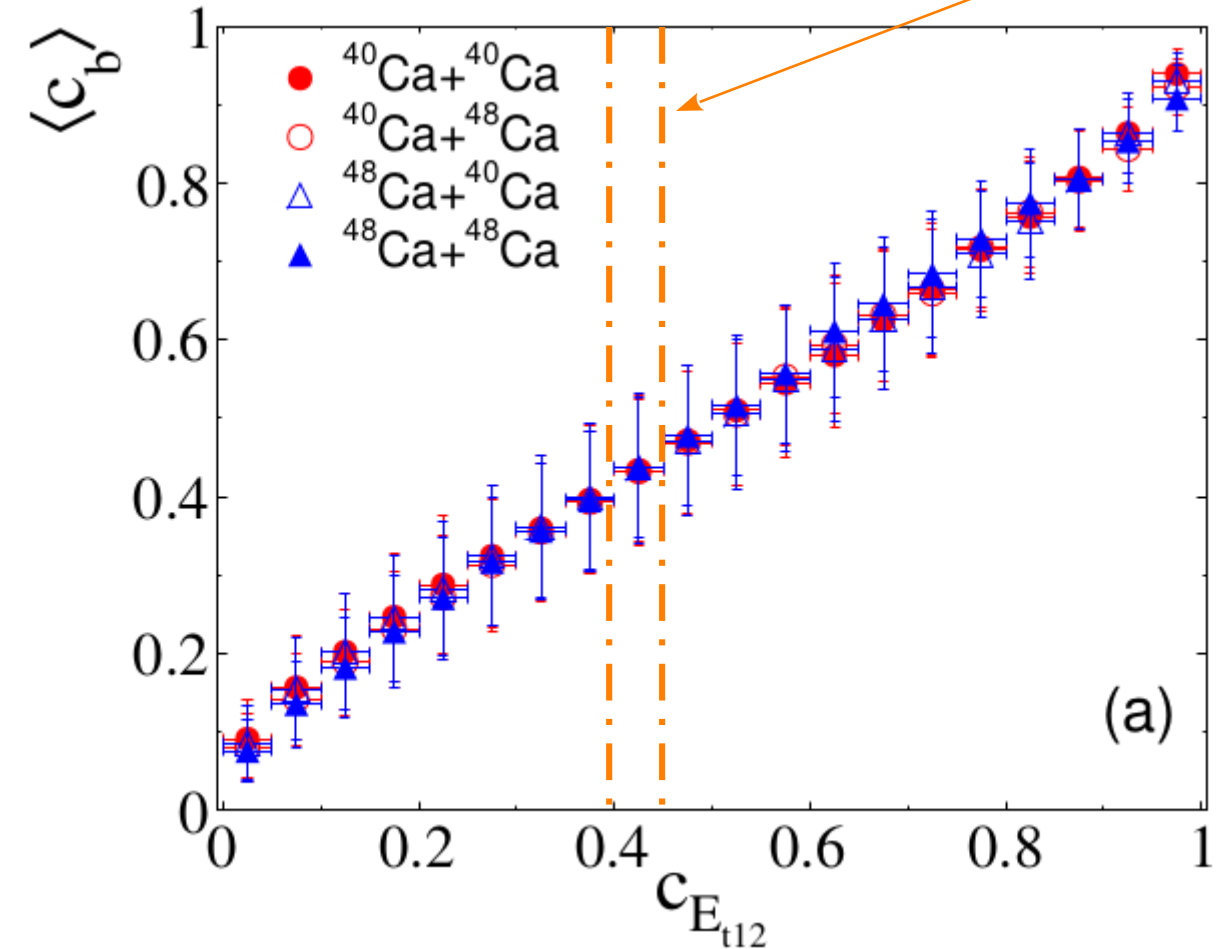
# Impact parameter estimation : AMD-CC

$$P(c_b | \mathcal{S}) = \frac{P(X | c_b) \frac{P(X|\mathcal{S})}{P(X)} dX}{P(X | \mathcal{S}) dX}$$

It is necessary to assume a specific form for the impact param. distribution or use the one from the model

Sampling:  
steps of 5% in  
 $c_{Et12}$

$$P(b | \mathcal{S}) = P(b) P(c_b(b) | \mathcal{S})$$

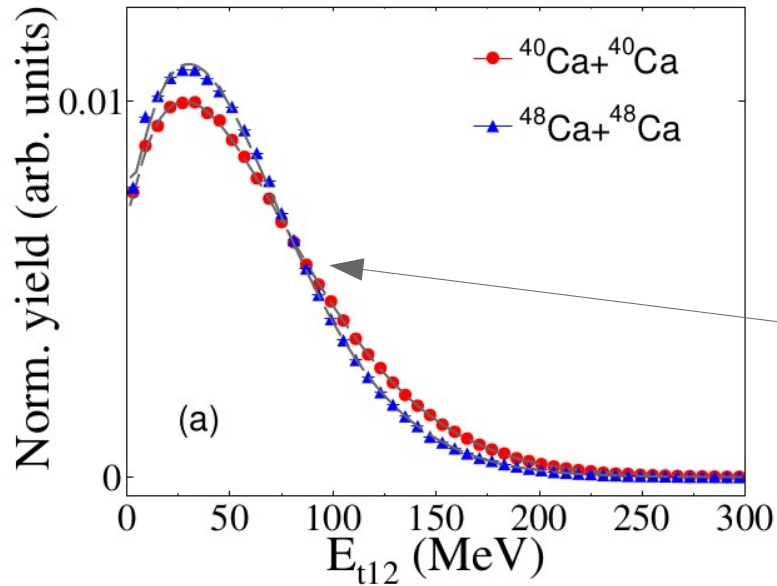


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# Impact parameter estimation : EXP

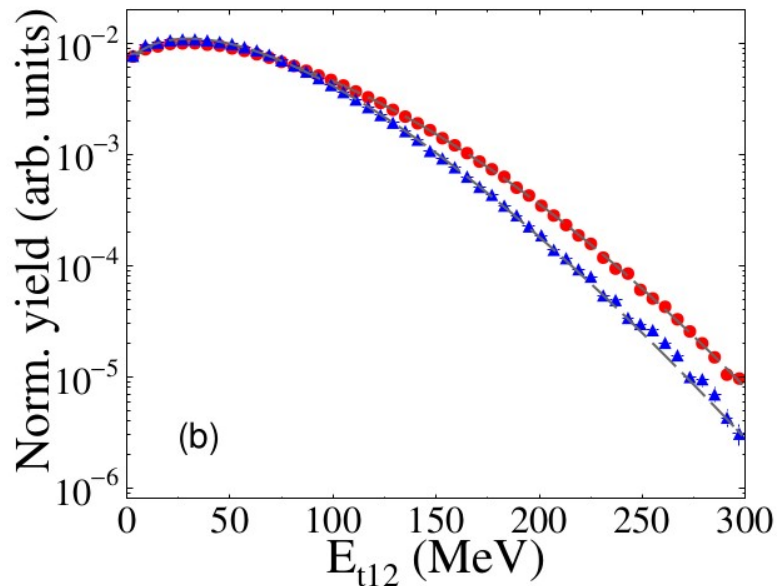


$$P(X) = \int_0^1 P(X | c_b) dc_b$$

$$\text{with } X = Et_{12} = \sum_{Z=1,2} Ek_i \sin^2(\theta_i)$$

**Proposed form of conditional probability : gamma distribution (5 fit parameters)**  
**→ fit to the inclusive distributions**

$$P(X | c_b) = f [\bar{X}(c_b), \theta(c_b)]$$

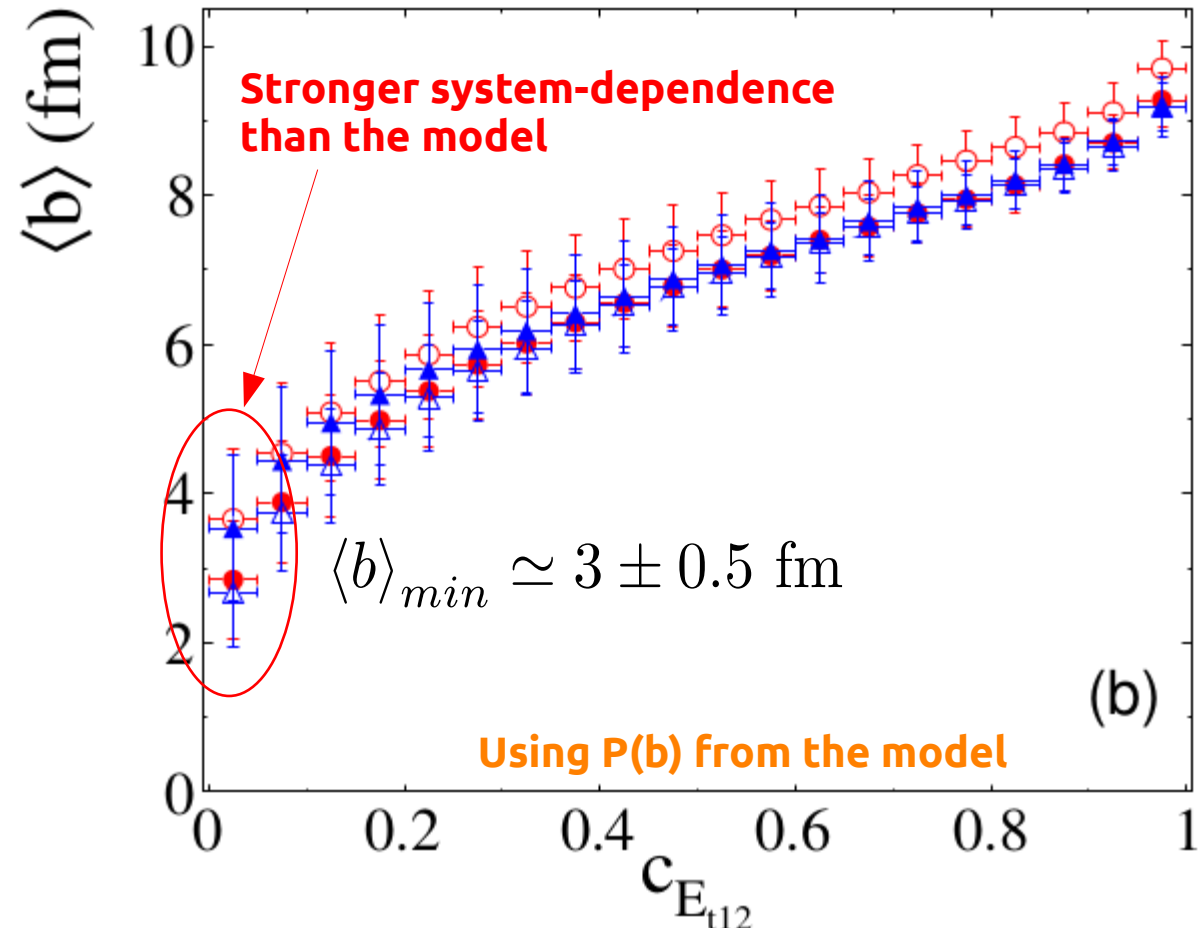
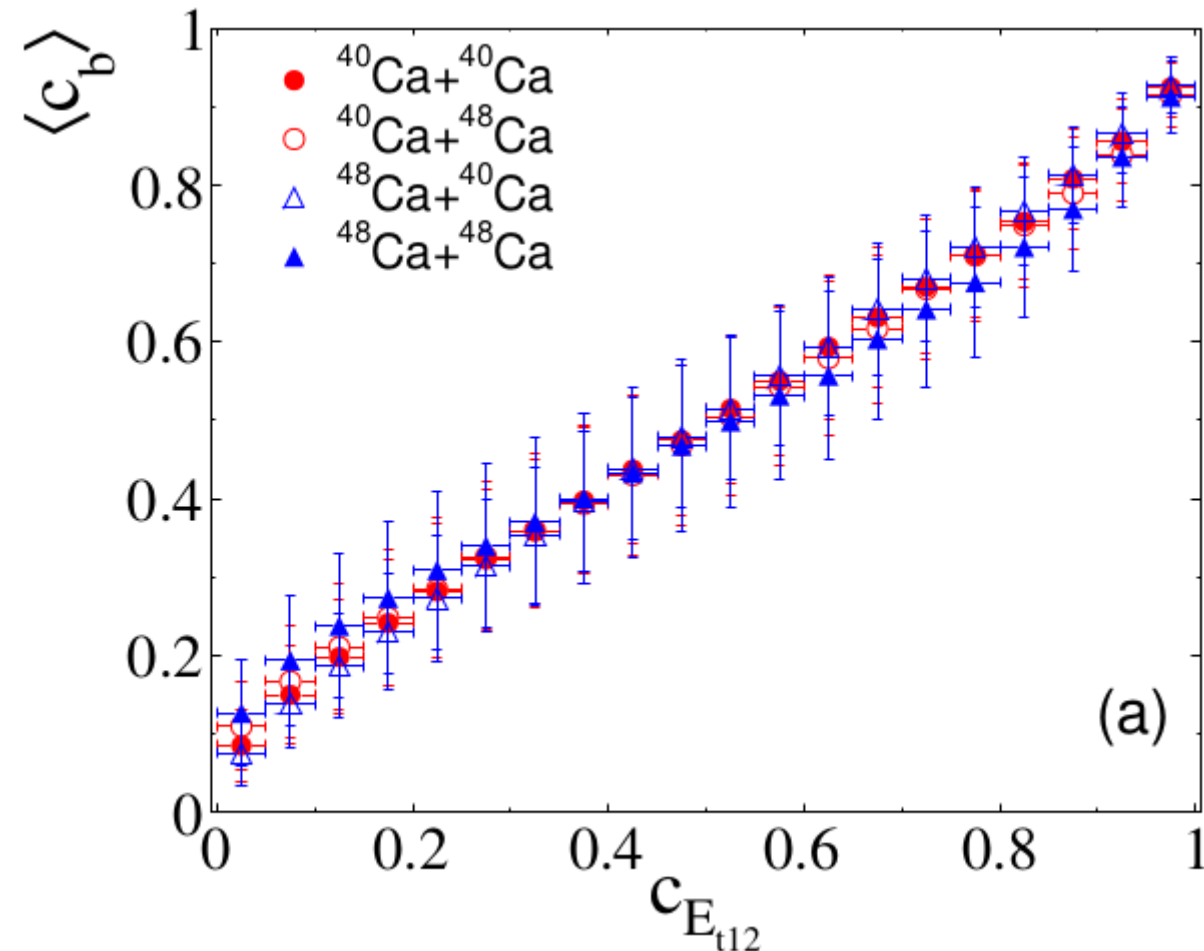


System	$\alpha$	$\gamma$	$\theta$ [MeV]	$X_{min}$ [MeV]	$X_{max}$ [MeV]	$\chi^2$
$^{40}\text{Ca}+^{40}\text{Ca}$	0.14	0.69	10.29	278	4	1.3
$^{40}\text{Ca}+^{48}\text{Ca}$	0.37	0.82	12.90	167	7	1.3
$^{48}\text{Ca}+^{40}\text{Ca}$	0.17	0.71	9.11	257	6	1.1
$^{48}\text{Ca}+^{48}\text{Ca}$	0.10	0.55	13.01	233	1	1.4

# Impact parameter estimation : EXP

$$P(c_b | \mathcal{S}) = \frac{P(X | c_b) \frac{P(X|\mathcal{S})}{P(X)} dX}{P(X | \mathcal{S}) dX}$$

$$P(b | \mathcal{S}) = P(b) P(c_b(b) | \mathcal{S})$$



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# Isospin transport ratio

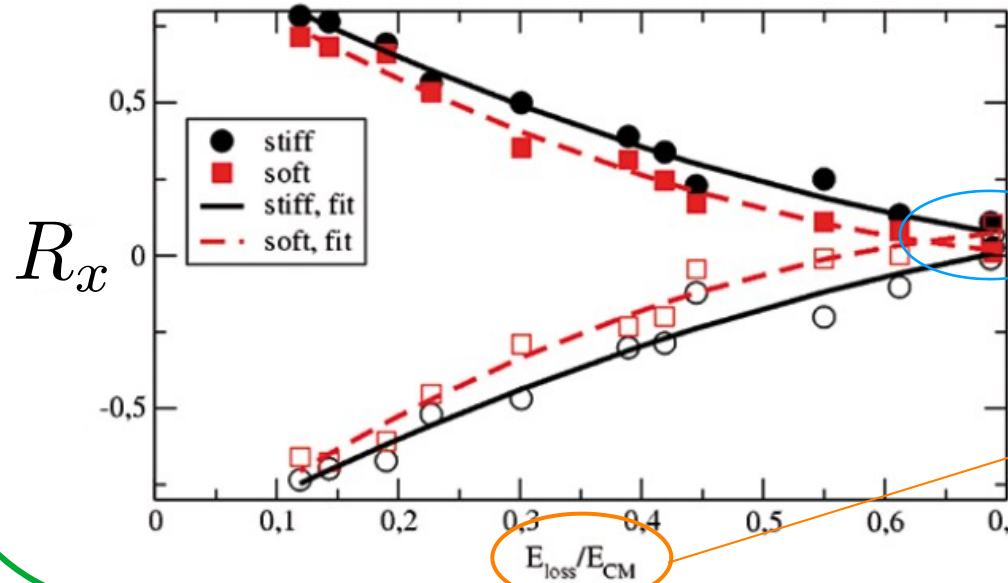
## Isospin transport ratio

$$R_x = \frac{2x^M - x^{NR} - x^{ND}}{x^{NR} - x^{ND}}$$

M : asymmetric system  
 NR : neutron-rich system  
 ND : neutron-deficient system

$$\delta = \frac{N - Z}{A}$$

Complete eq.

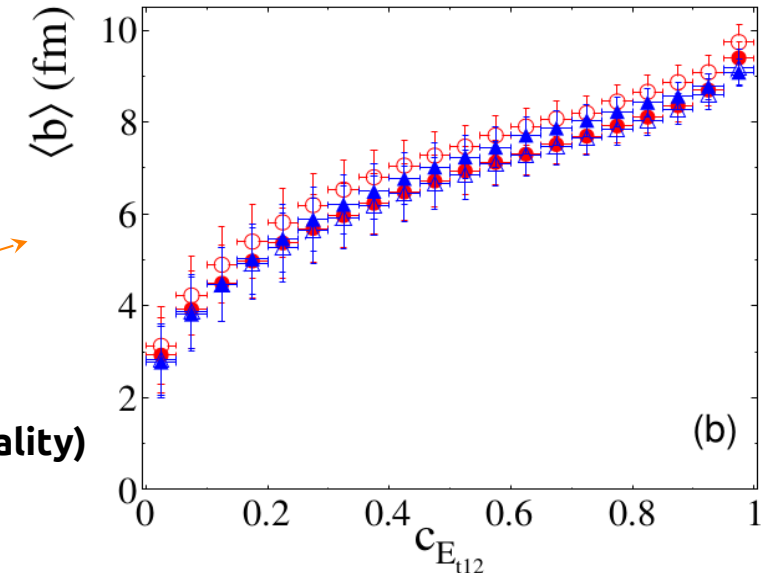


## Centrality - impact. param

“b-centrality”  $\rightarrow c_b \equiv \int_0^b P(b') db'$

“exp-centrality”  $\rightarrow c_x \equiv \int_x^{+\infty} P(X) dX$   
 increasing X with decreasing b

or  $X = Et_{12} = \sum_{Z=1,2} Ek_i \sin^2(\theta_i)$



“order parameter”  
 (correlated to the centrality)

M. Colonna et al., EPJA 50: 30 (2014)

F. Rami et al., PRL 84, 1120 (2000)

# QP reconstruction : selection

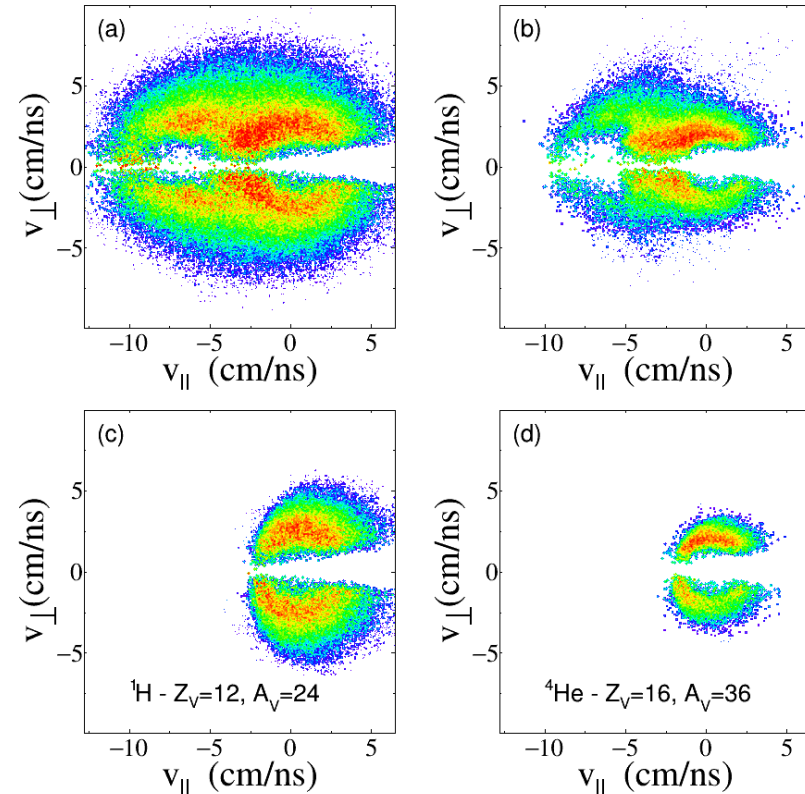
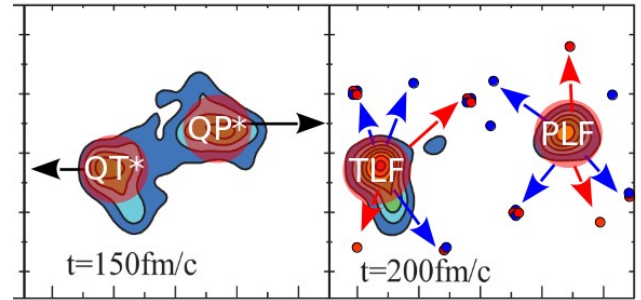
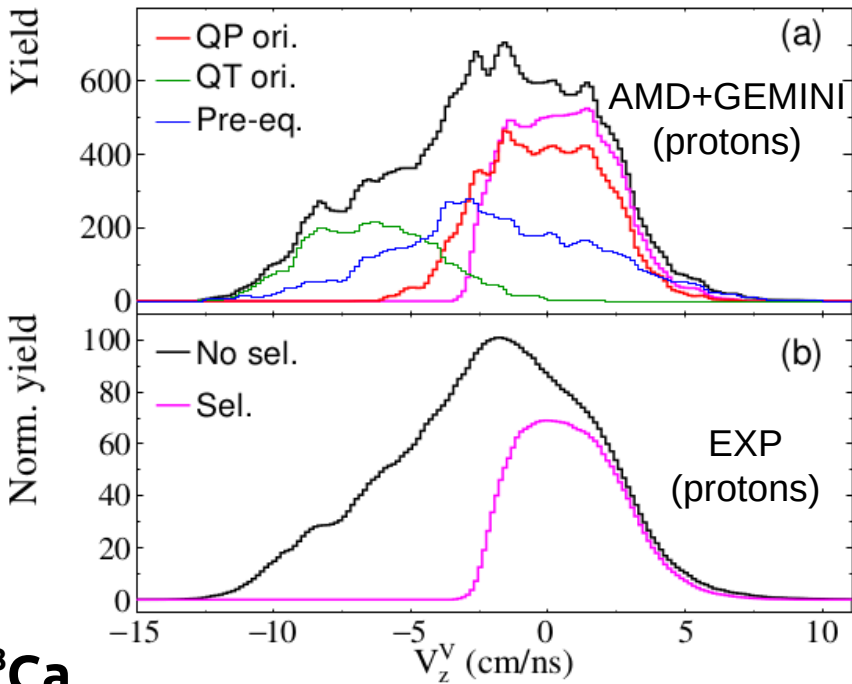
**QP reconstruction** based on the relative velocities between the reaction products detected with INDRA and :

- (i) The PLF identified with VAMOS ;
- (ii) The largest fragment identified in charge with INDRA at backward angles (TLF)

$$V_{rel,TLF}/V_{rel,PLF} > 1.35, \text{ if } Z = 1$$

- Fragment selection :  $V_{rel,TLF}/V_{rel,PLF} > 1.75, \text{ if } Z \geq 2$

- Optimized from filtered AMD+GEMINI calculations



No sel.

With sel.

# QP reconstruction : neutron estimation

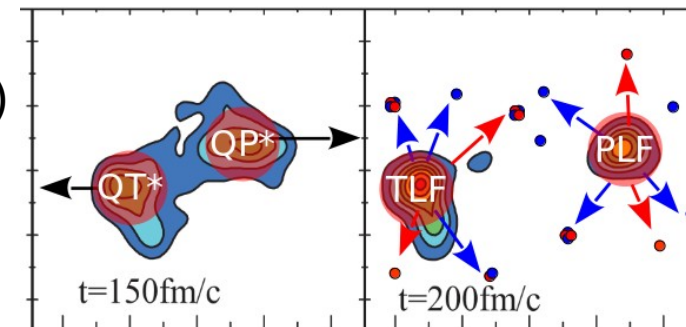
**QP reconstruction** based on the relative velocities between the reaction products detected with INDRA and :

- (i) The PLF identified with VAMOS ;
- (ii) The largest fragment identified in charge with INDRA at backward angles (TLF)

- Fragment selection :  $V_{rel,TLF}/V_{rel,PLF} > 1.35$ , if  $Z = 1$
- Fragment selection :  $V_{rel,TLF}/V_{rel,PLF} > 1.75$ , if  $Z \geq 2$

- Optimized from filtered AMD+GEMINI calculations

- **Estimation of the evaporated neutrons from the simulations**



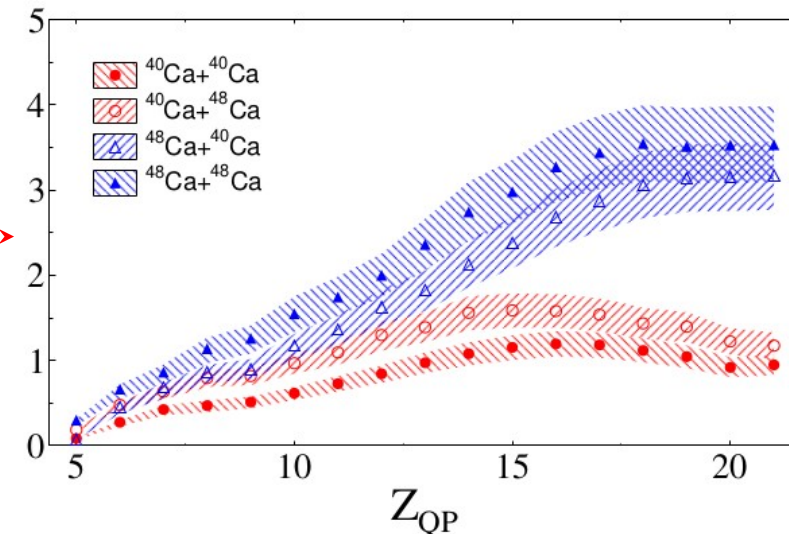
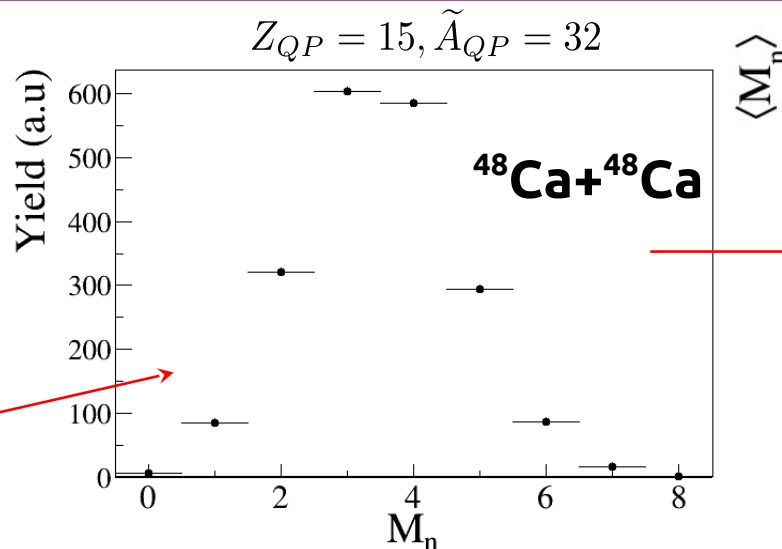
(Z,A) estimation

$$Z_{QP} = Z_V + \sum_i^{M_I} Z_i$$

$$\tilde{A}_{QP} = A_V + \sum_i^{M_I} A_i$$

Random selector

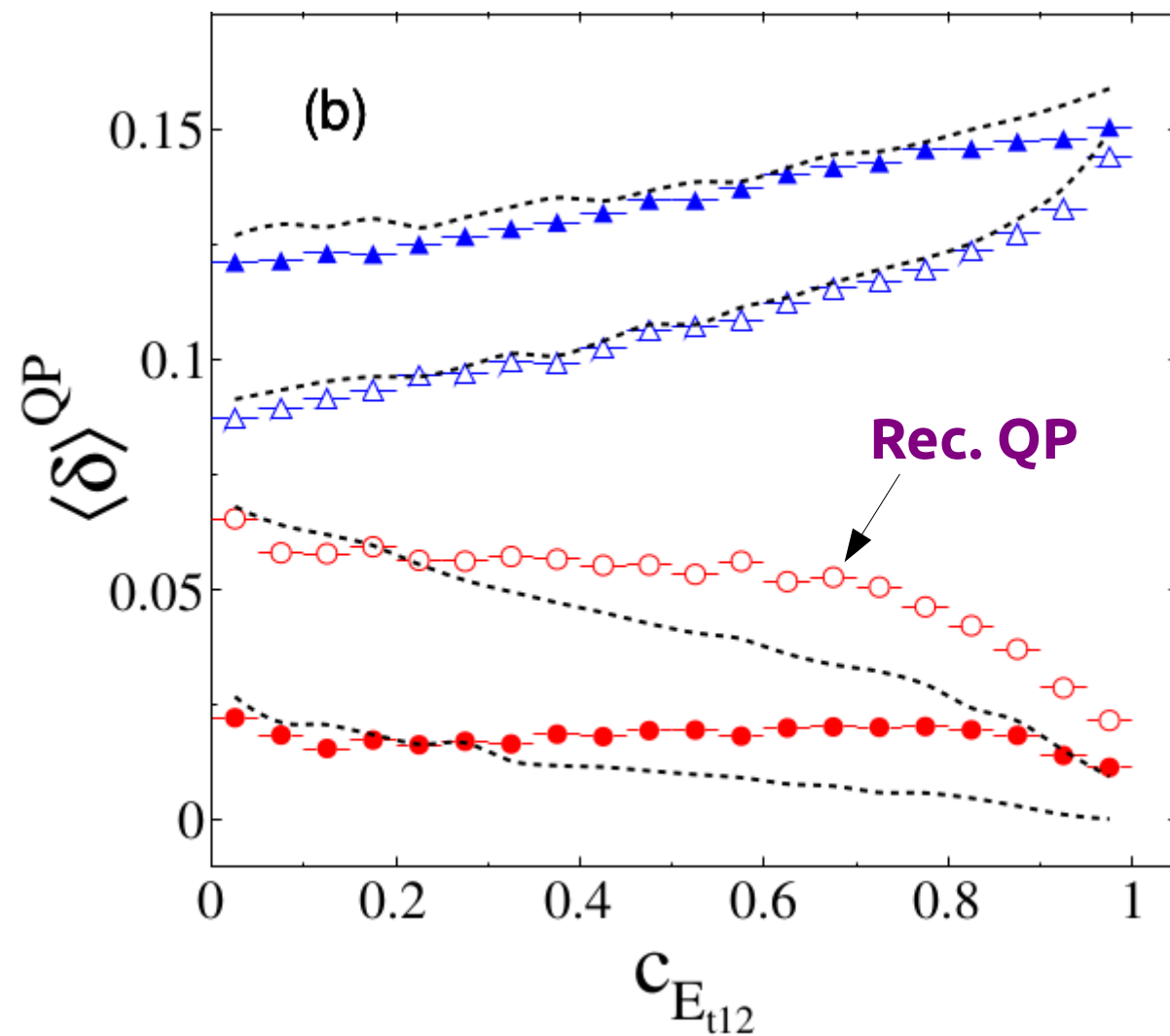
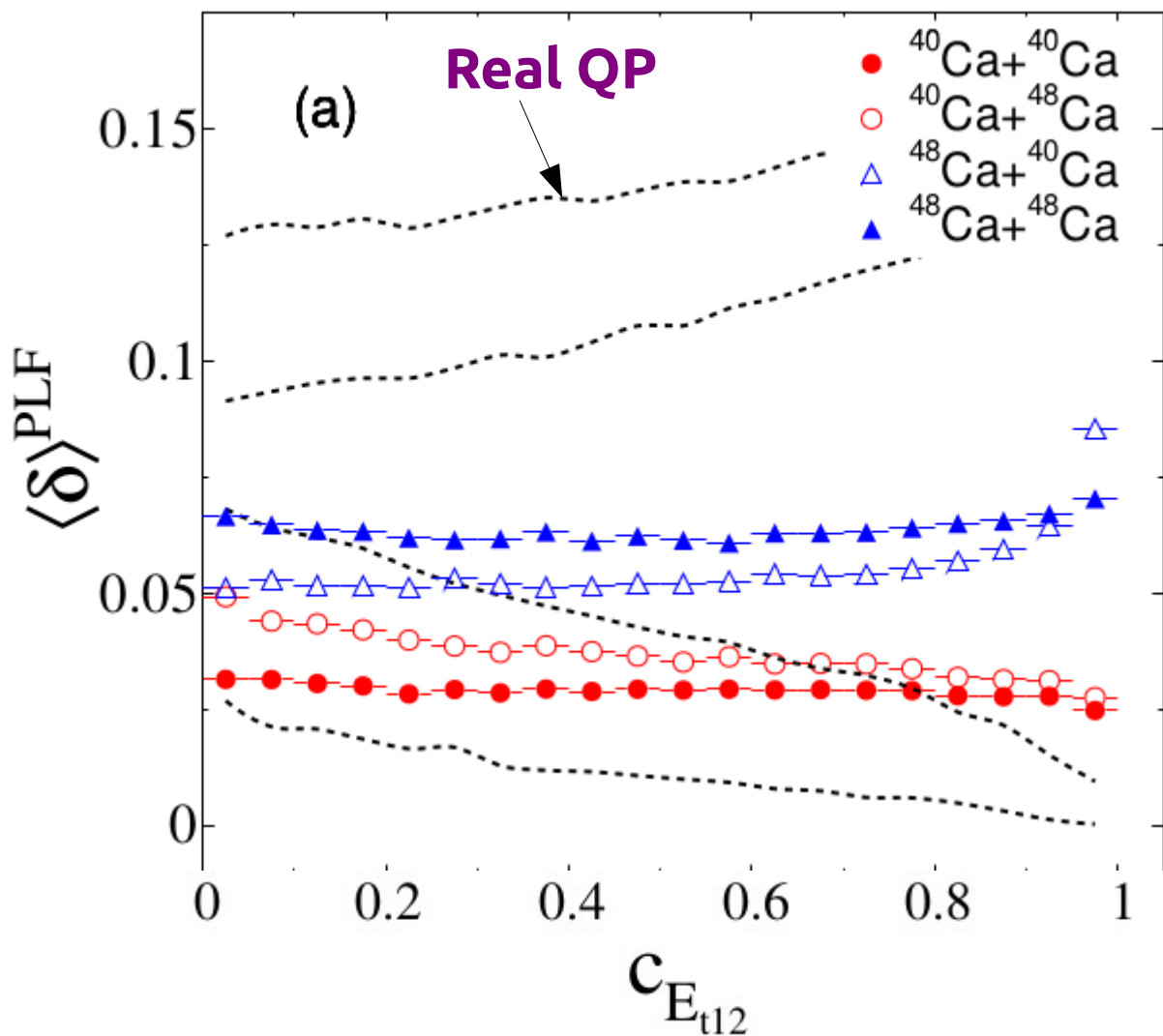
$$A_{QP} = \tilde{A}_{QP} + M_n^{rdm}(\tilde{A}_{QP}, Z_{QP})$$



# Isospin diffusion in AMD-CC

AMD-CC stiff

$$\delta = (N - Z)/A$$

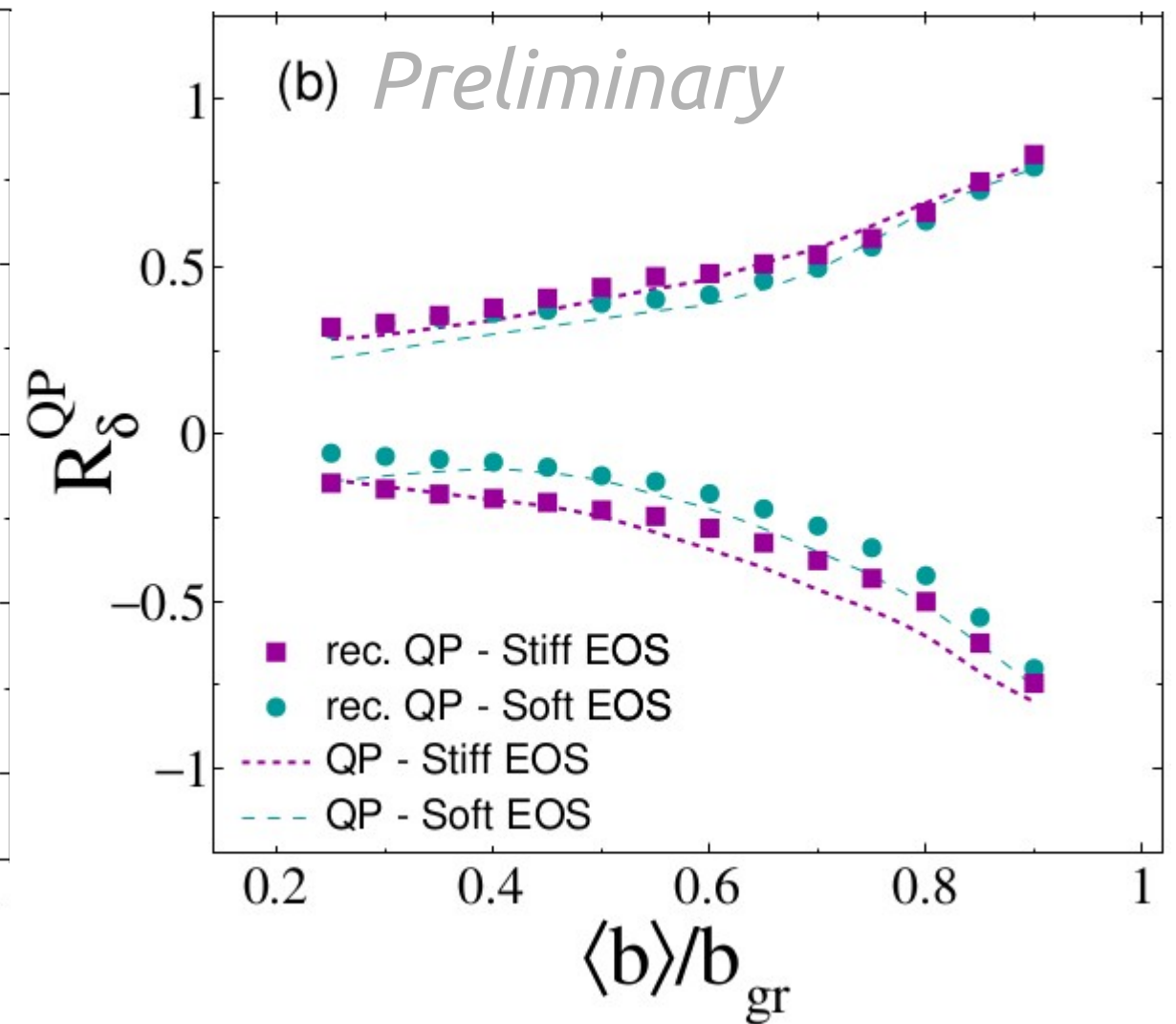
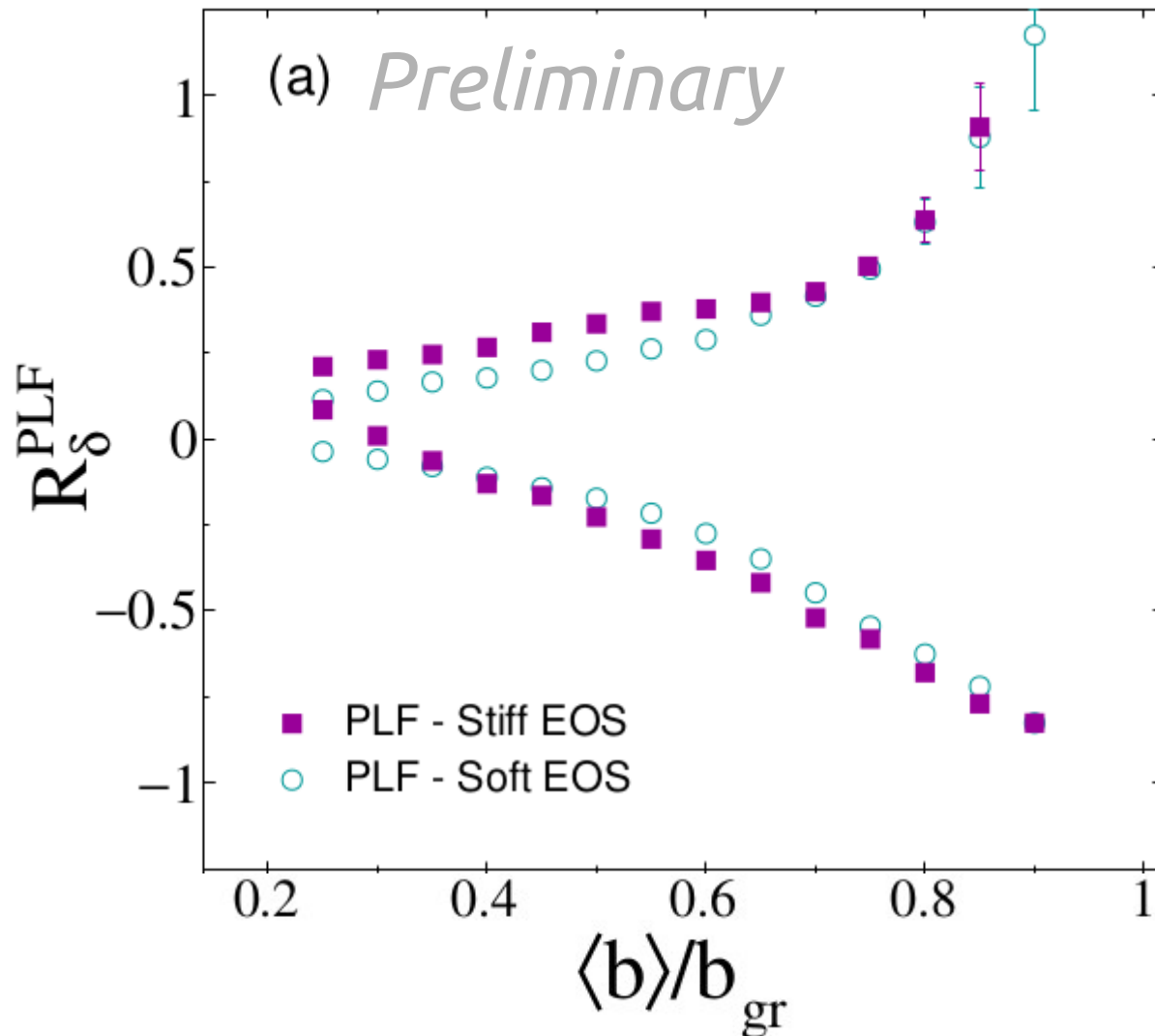


# Isospin diffusion in AMD-CC

$$\delta = (N - Z)/A$$

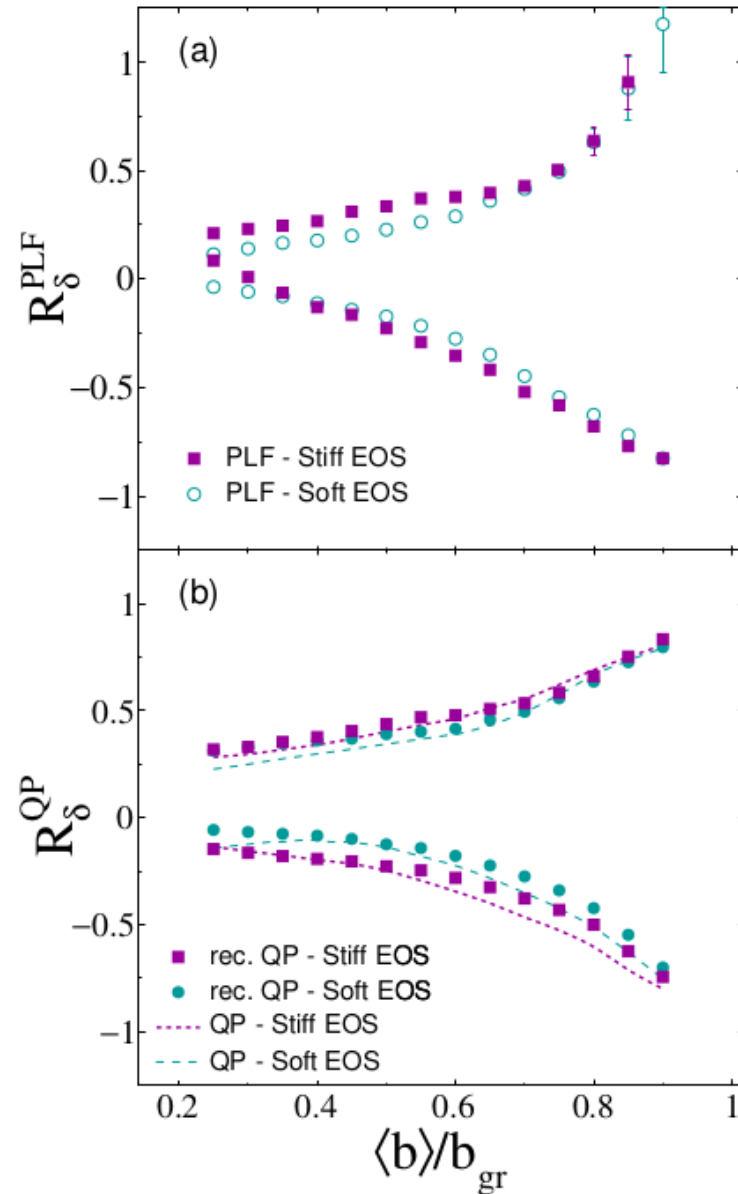
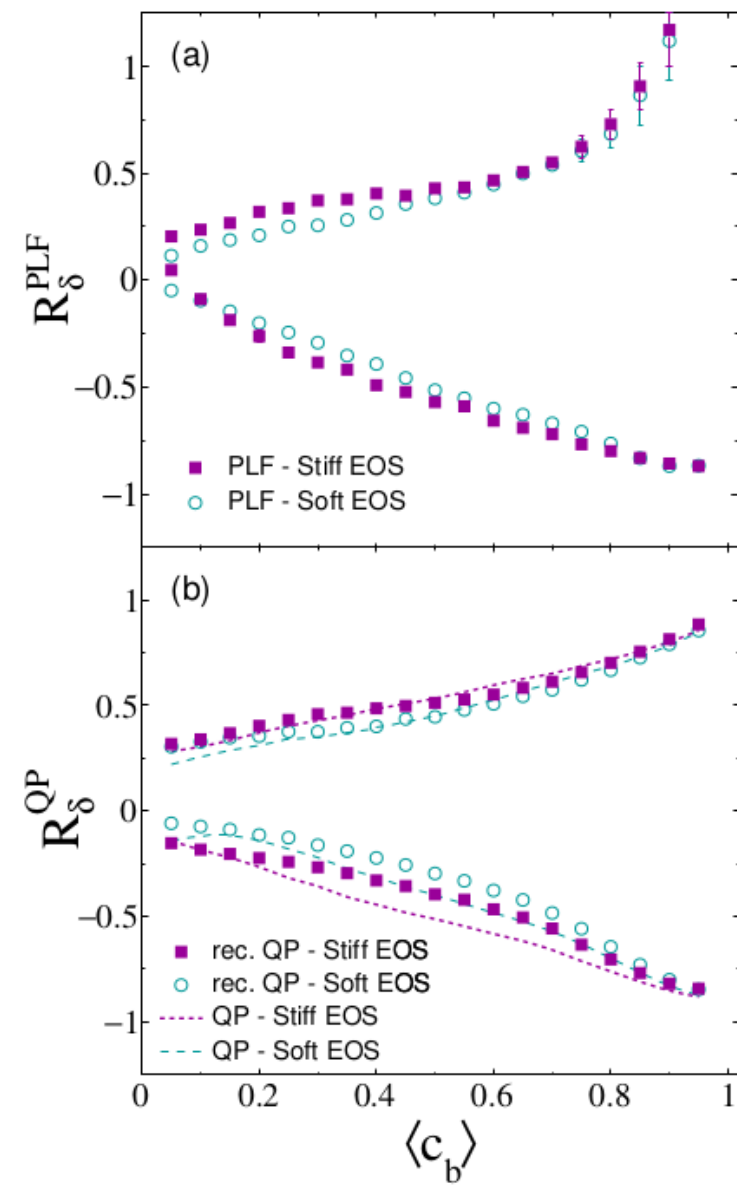
$$R_x = \frac{2x^M - x^{NR} - x^{ND}}{x^{NR} - x^{ND}}$$

M:  $^{40}\text{Ca}+^{48}\text{Ca}$  or  $^{48}\text{Ca}+^{40}\text{Ca}$   
 NR:  $^{48}\text{Ca}+^{48}\text{Ca}$   
 ND:  $^{40}\text{Ca}+^{40}\text{Ca}$





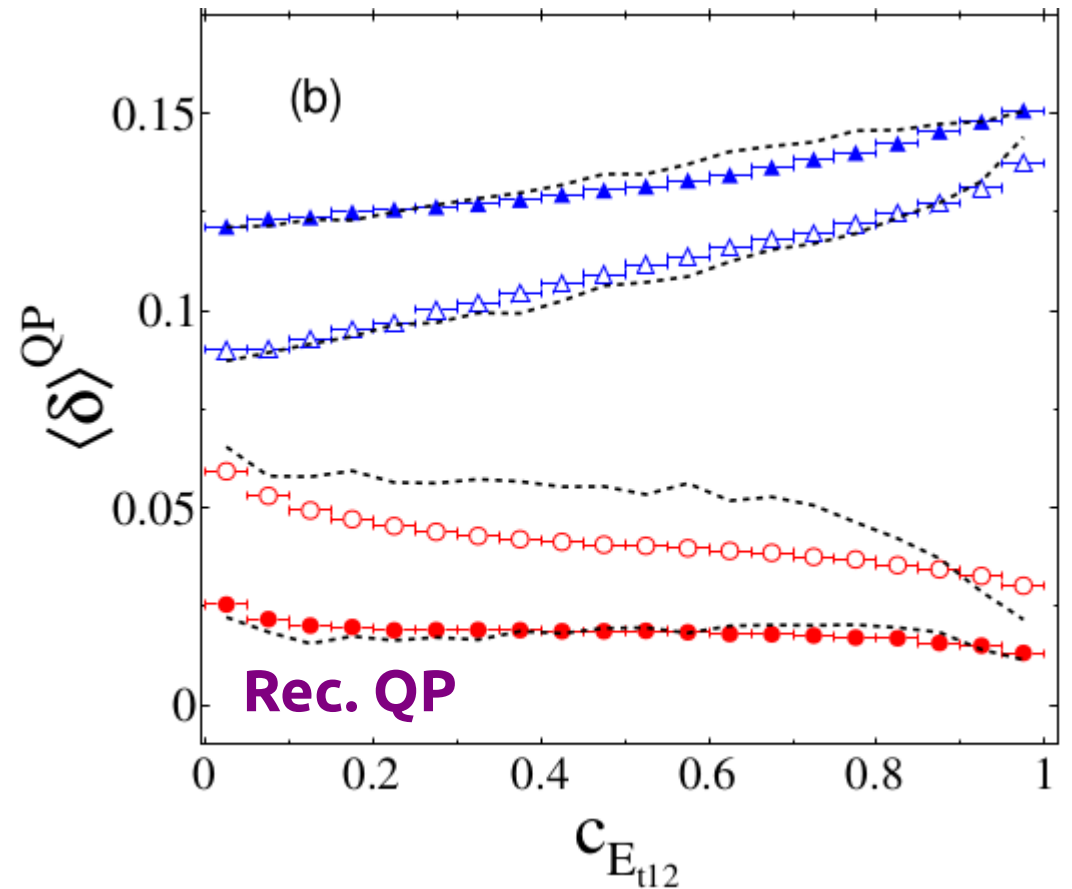
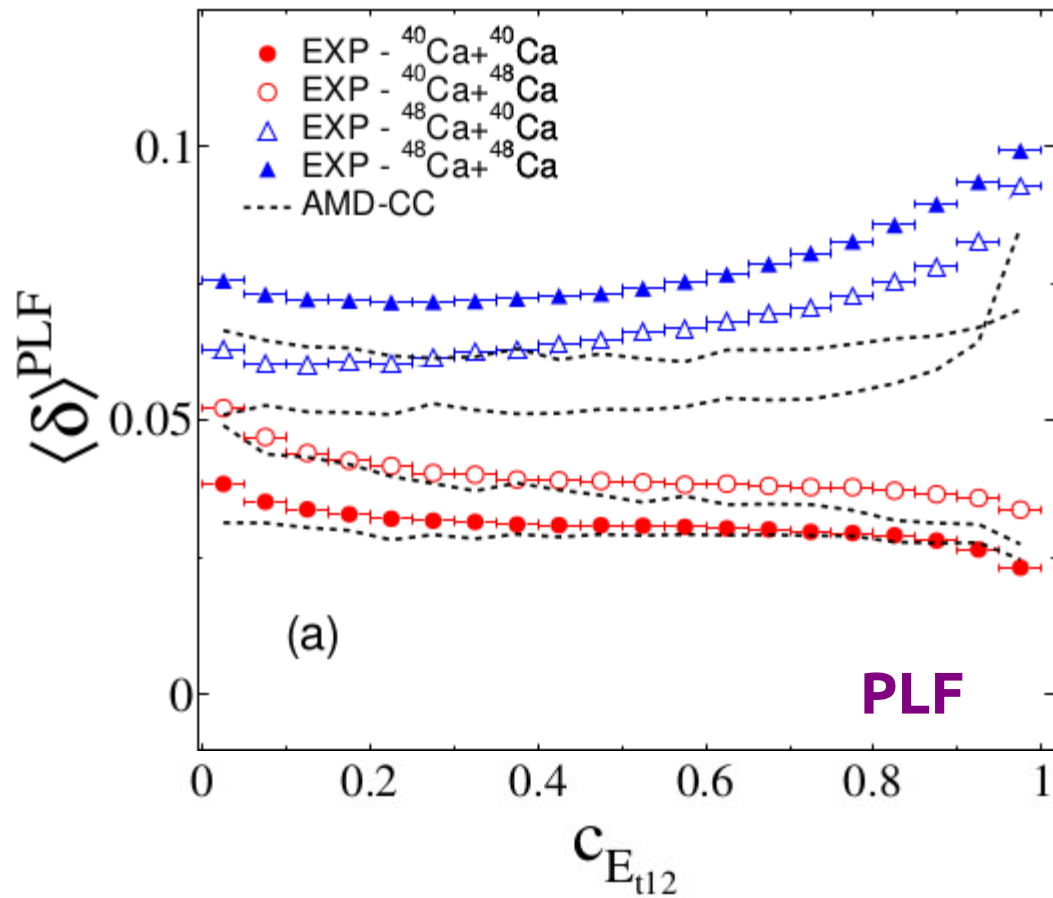
# Isospin transport ratio in AMD-CC



- Decrease of  $R_\delta$  with the dissipation of the collision  
→ from  $R_\delta = \pm 0.75$  to  $R_\delta = \pm 0.25$
- AMD indicates a regular evolution towards isospin equilibration  
→ Smoother for the QP than the PLF
- For both PLF and QP, we observe a sensitivity to the stiffness of the EOS
- Also observed for the reconstructed QP

# Isospin transport ratio in exp. data

$$\delta = (N - Z)/A$$



# Isospin transport ratio in exp. data

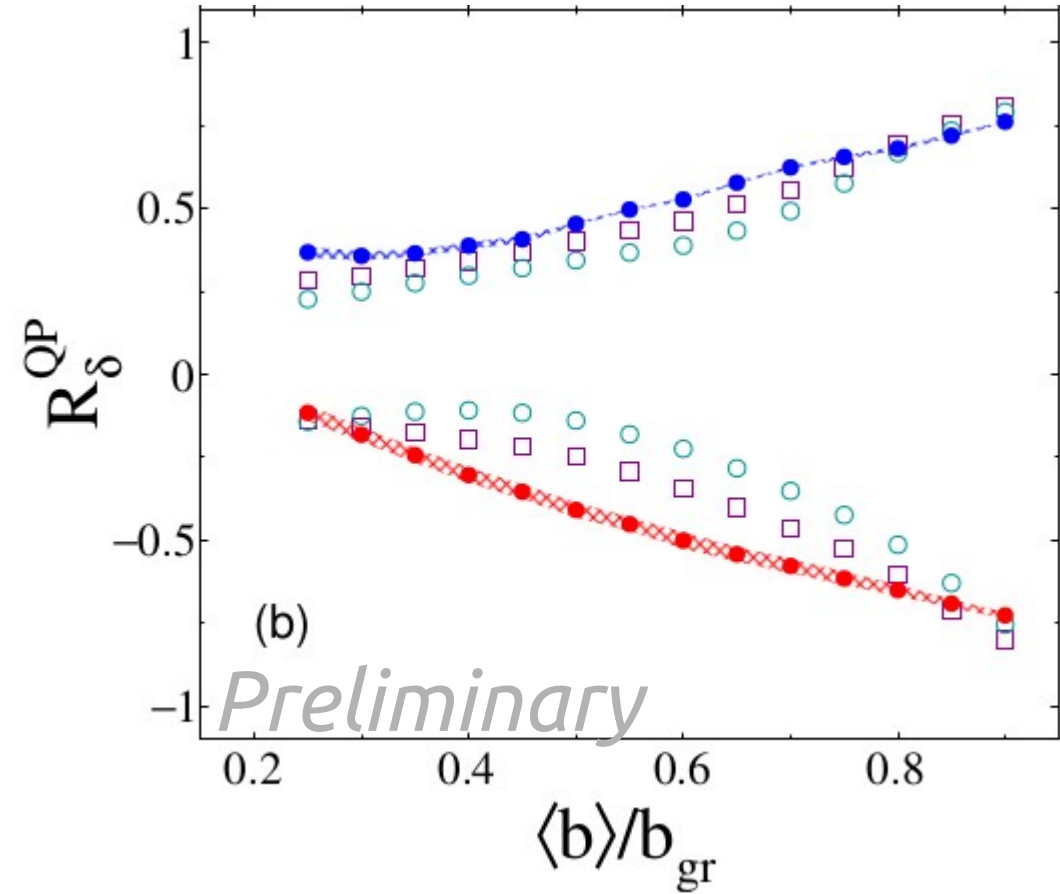
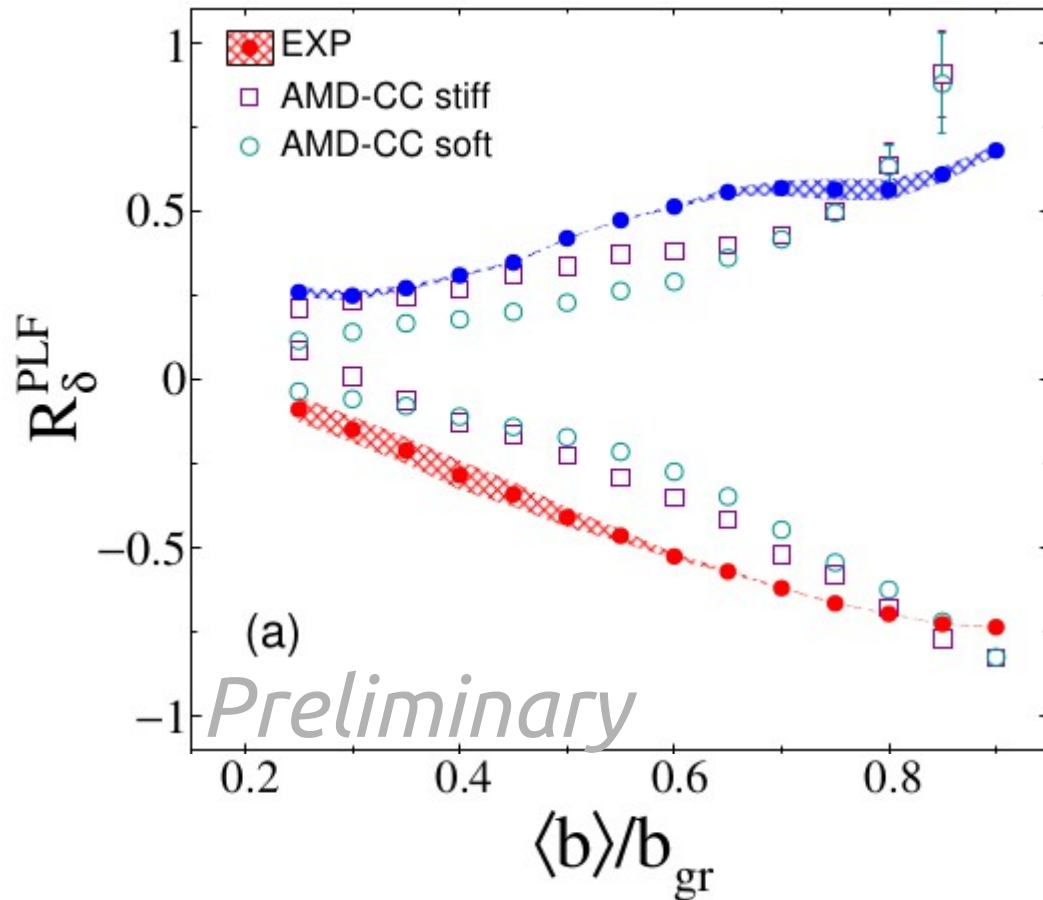
$$\delta = (N - Z)/A$$

$$R_x = \frac{2x^M - x^{NR} - x^{ND}}{x^{NR} - x^{ND}}$$

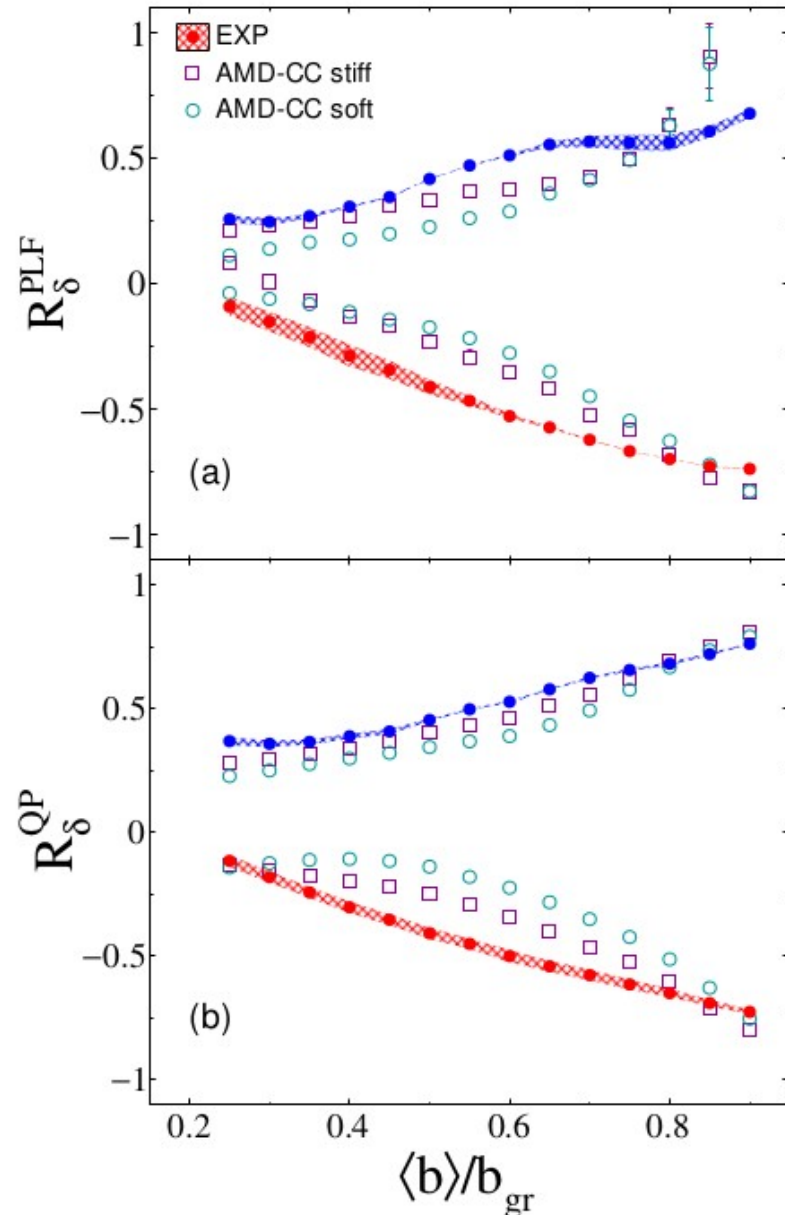
M:  $^{40}\text{Ca} + ^{48}\text{Ca}$  or  $^{48}\text{Ca} + ^{40}\text{Ca}$

NR:  $^{48}\text{Ca} + ^{48}\text{Ca}$

ND:  $^{40}\text{Ca} + ^{40}\text{Ca}$



# Isospin diffusion in exp. data

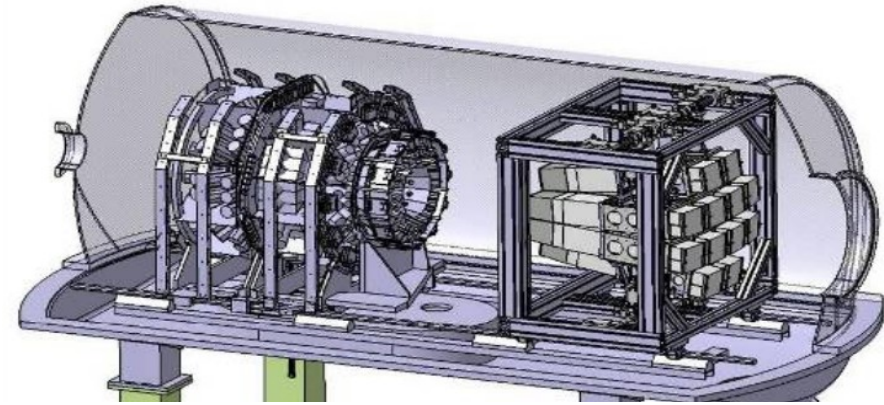


- Decrease of  $R_\delta$  with the dissipation of the collision
- The experimental data also indicates a regular evolution towards isospin equilibration
  - For both PLF and rec. QP
  - also from  $R_\delta = \pm 0.75$  to  $R_\delta = \pm 0.25$
- Strong difference in the slopes between the two mixed reactions
  - Better agreement with  $^{48}\text{Ca} + ^{40}\text{Ca}$  as mixed reaction
  - Weak indication in favor of a stiff symmetry energy with AMD-CC [1] ?

[1] S. Piantelli, PRC 103, 014603 (2021)

# Conclusions and outlooks

- **Isospin transport :**
  - INDRA-VAMOS experiment allows to probe the isospin diffusion at Fermi energies ;
  - Direct comparisons are possible with AMD model thanks to the impact parameter estimation method ;
- **INDRA-FAZIA coupling :**
  - Complementary results (see C. Ciampi talk) ;
  - Effect of beam energy (density) ?
  - Application of the impact parameter reconstruction ?
  - Probed impact parameter ?
- **Extensive comparisons** with different models to link the observations to transport properties :
  - BLOB, QMD, AMD...
  - (Work undergoing with BLOB) ;
  - Bayesian analysis ;
  - Need a versioning of the codes.
- What differences can we expect from a QMD-like approach and a BUU-like approach ?



# Thanks for your attention

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(INDRA collaboration)

## Special thanks to :

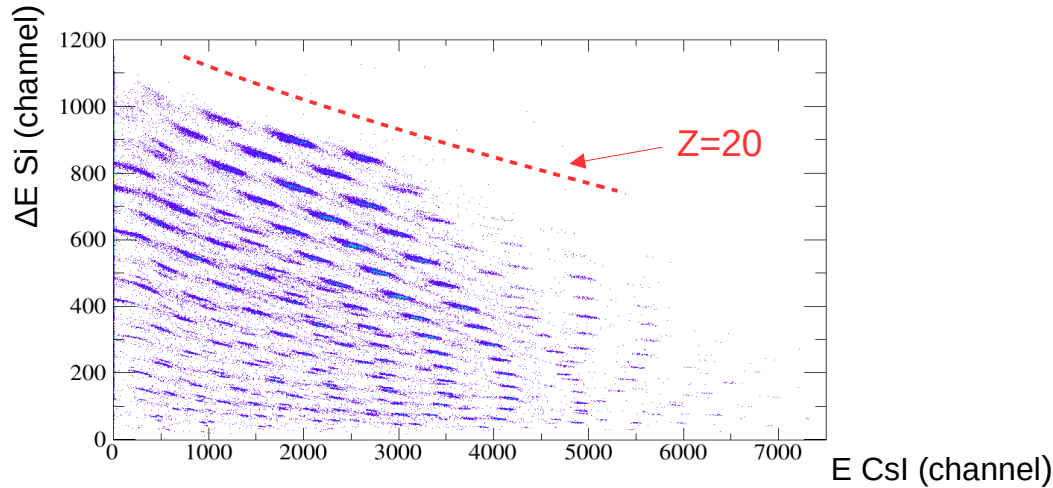
- Alberto Camaiani and Silvia Piantelli for the AMD files
- Maria Colonna and Paolo Napolitani for productive discussions about the models

# Back-up slide : Particle ID

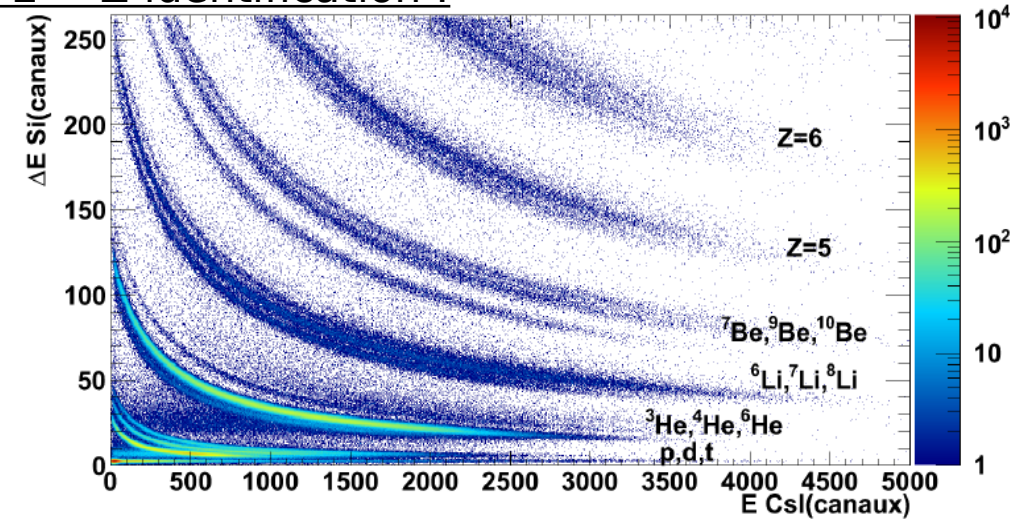
VAMOS

INDRA

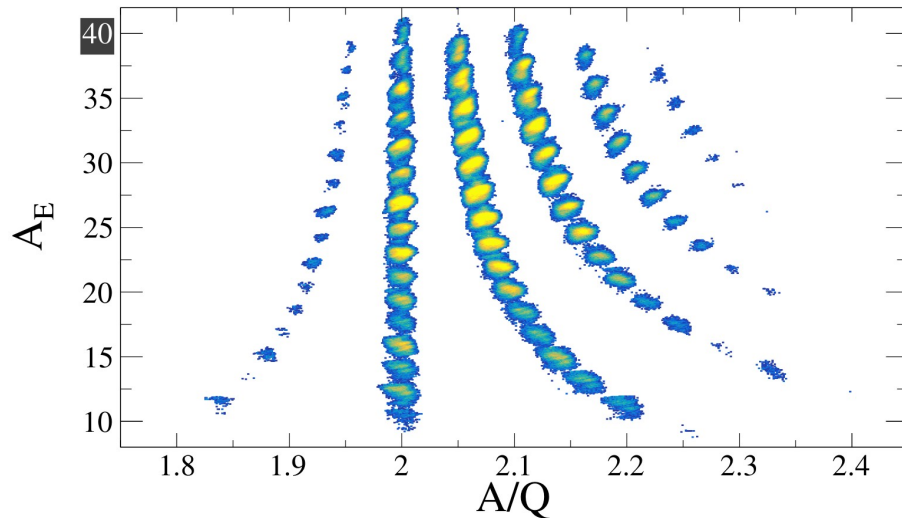
•  $\Delta E$ -E  $\rightarrow$  Z-identification:



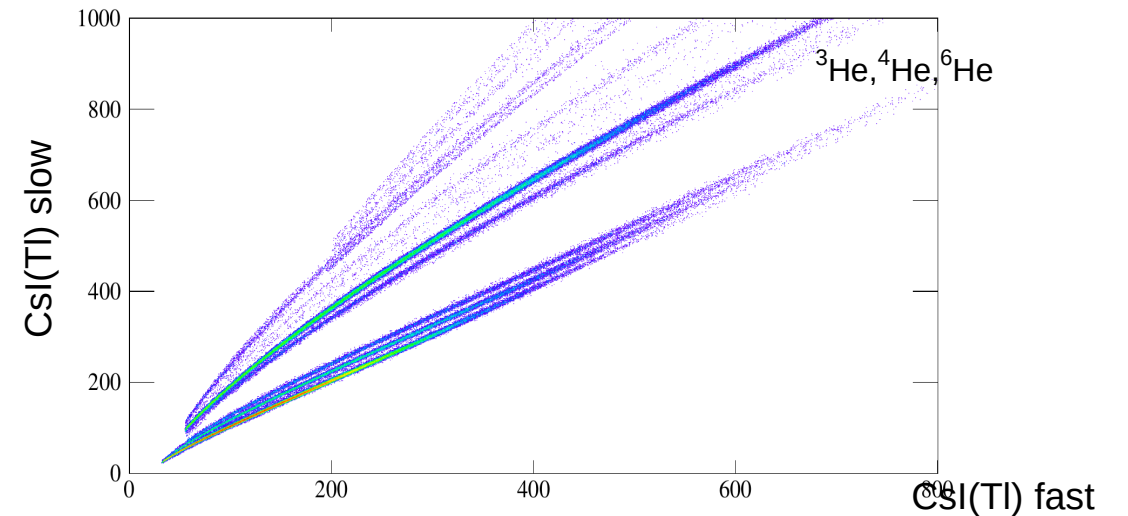
•  $\Delta E$ -E  $\rightarrow$  Z-identification :



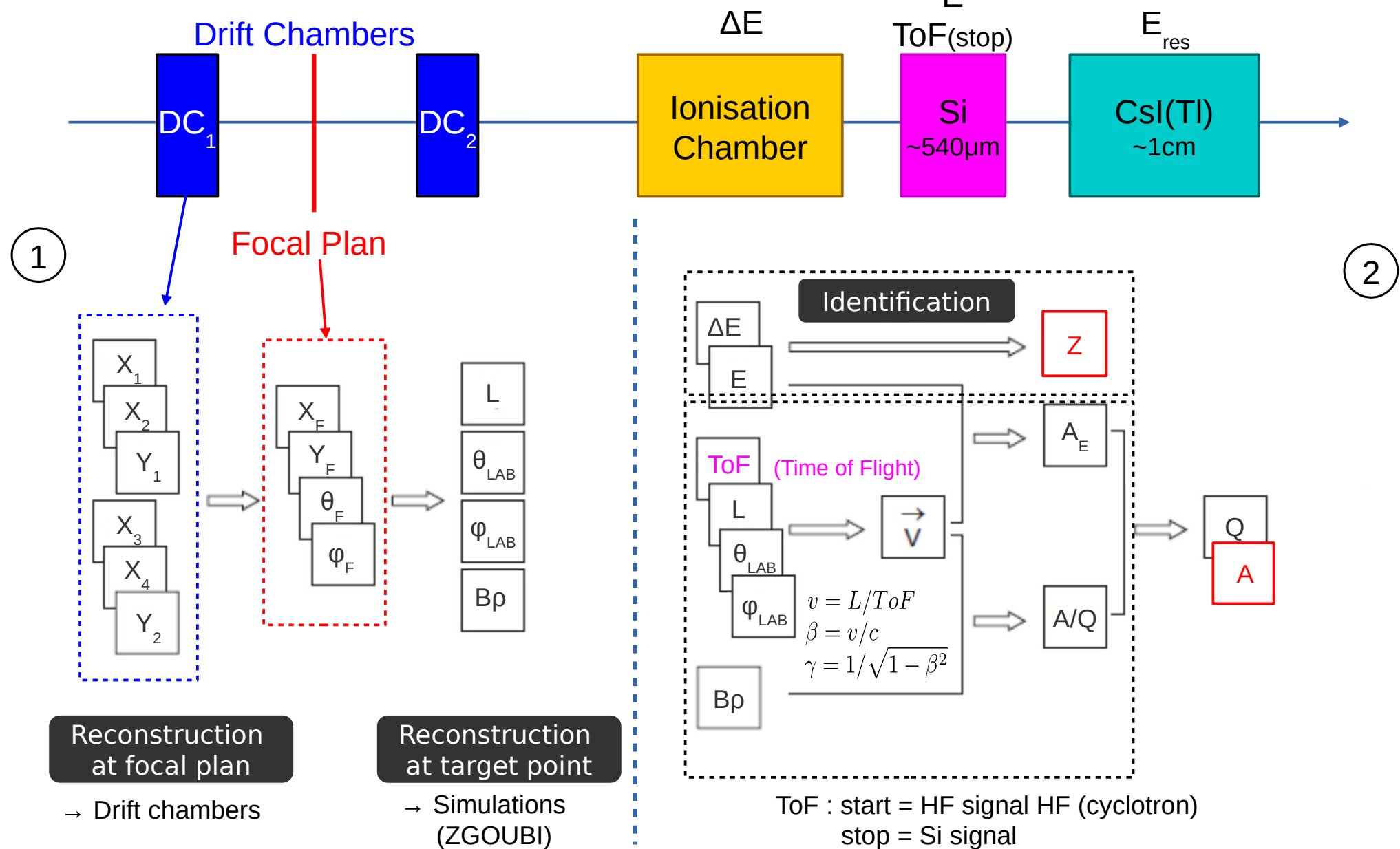
• A-identification :



• Pulse-shape (slow/fast) CsI(Tl) :

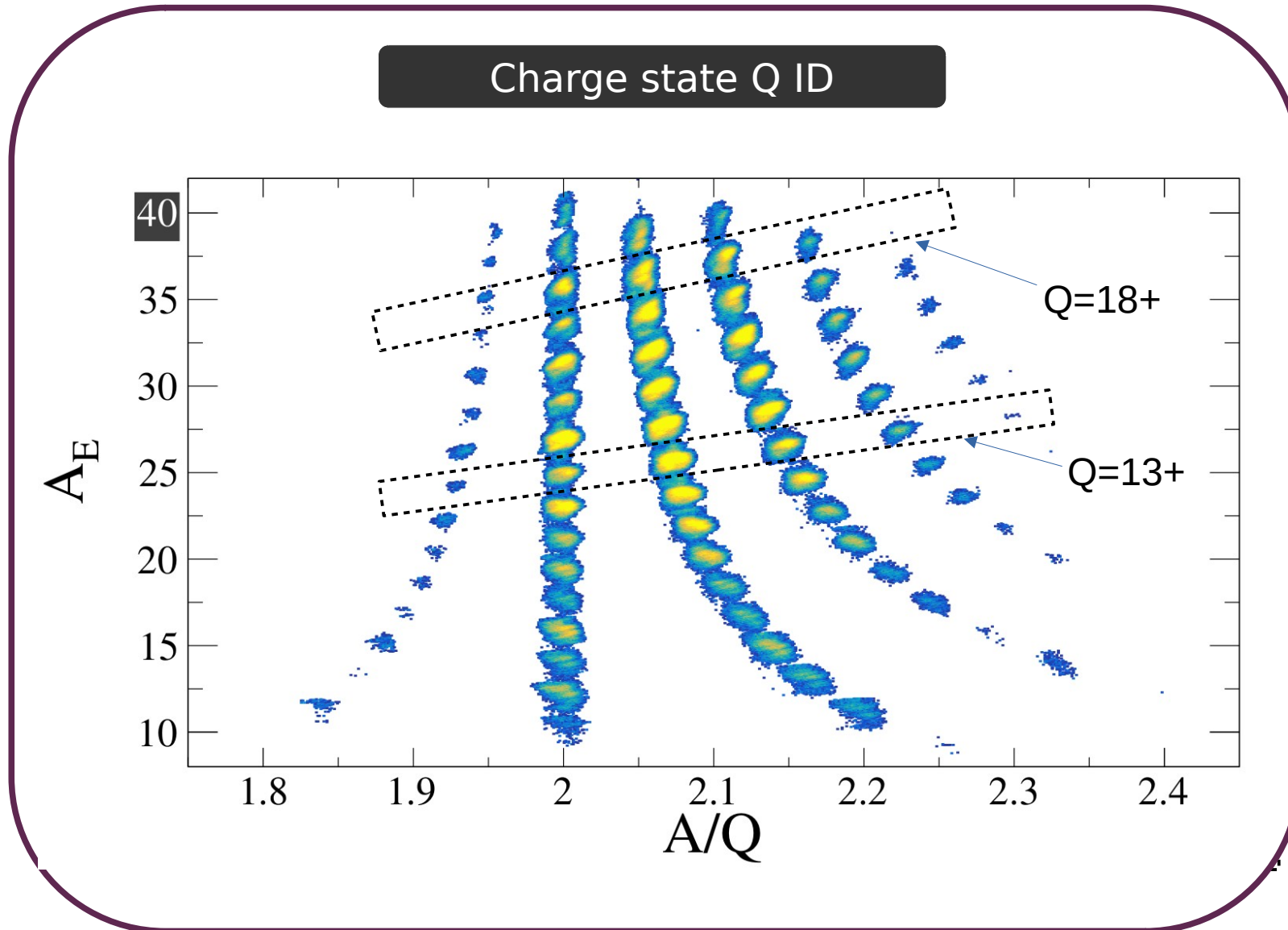


# Back-up slide : Particle ID with VAMOS





# Back-up slide : Particle ID with VAMOS



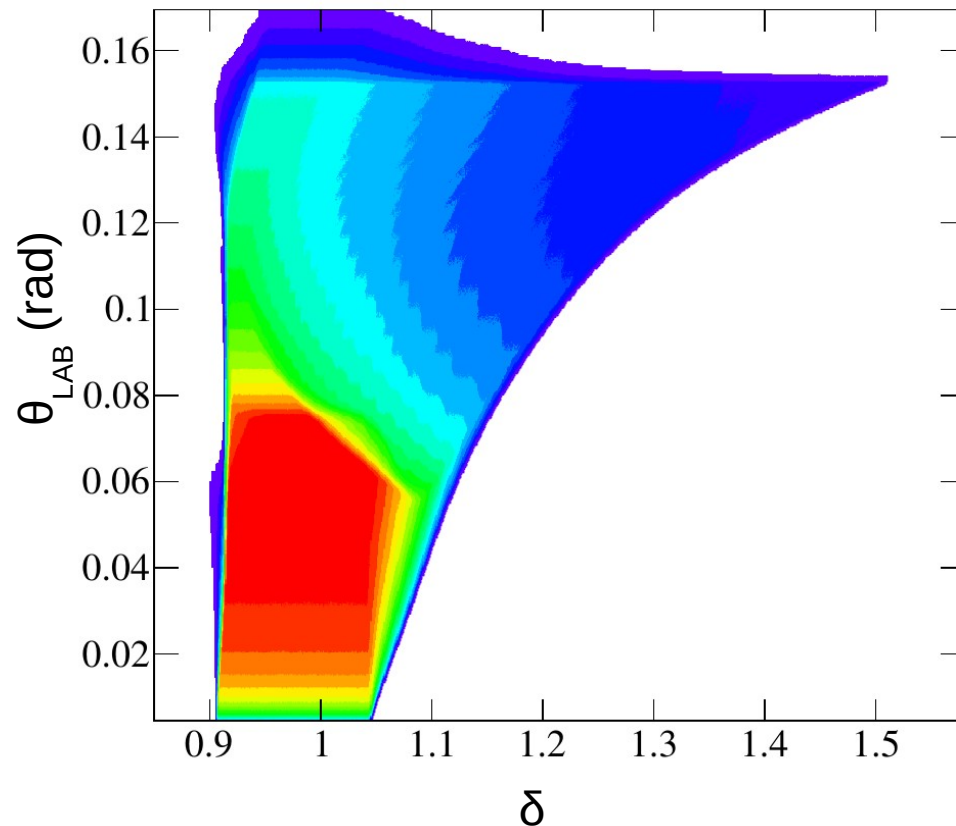
# Back-up slide : Data normalization

- Beam intensity corrections  $\rightarrow I_{beam}$

- Dead Time corrections  $\rightarrow DT$

- 

VAMOS geometrical efficiency



- Magnetic rigidity overlaps  $\rightarrow \delta$

- VAMOS acceptance corrections :

$$\rightarrow \epsilon_{geo}(\delta, \theta_{LAB}) = \frac{\Delta^2 \Omega(\delta, \theta_{LAB})}{4\pi}$$

efficacité  
géométrique

angle solide  
effectif

- $\rightarrow$  Simulation of more than  $10^6$  trajectories
- with Zgoubi to estimate  $\epsilon_{geo}(\delta, \theta_{LAB})$

Weight  $W(I_{beam}, DT, \delta, \theta_{LAB})$  applied  
event-by-event

# Back-up slide : QP estimation

## Isotopic yields (reconstructed) : AMD with clusters vs EXP

$^{40}\text{Ca}+^{40}\text{Ca}$

$^{48}\text{Ca}+^{48}\text{Ca}$

