The symmetry energy from ground and excited state properties of atomic nuclei

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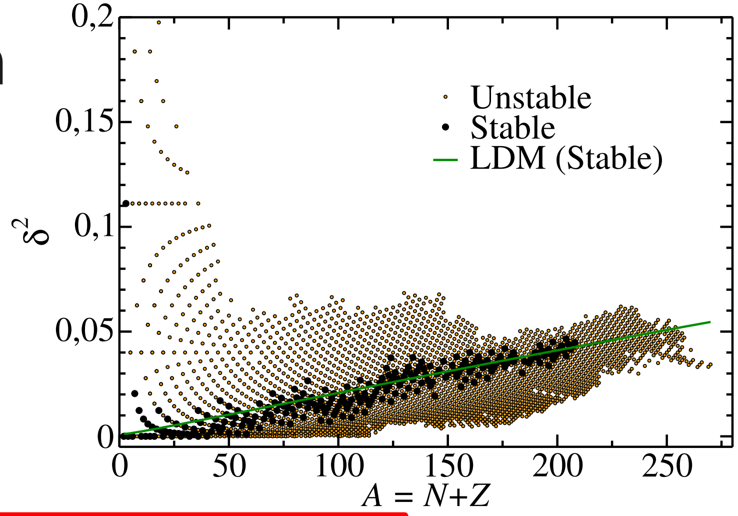
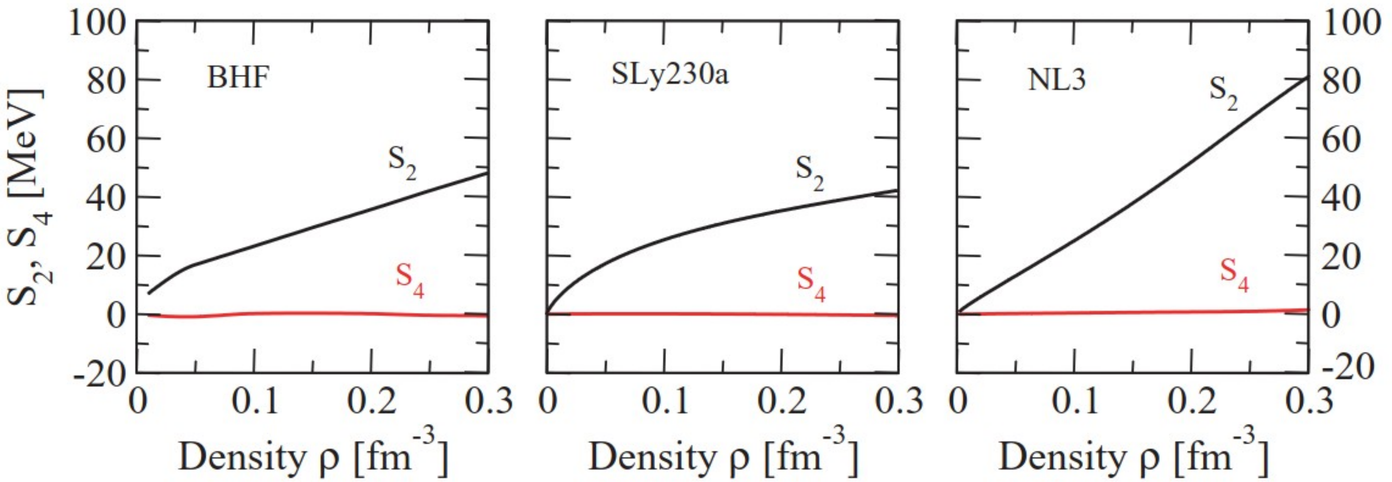
Nuclear Equation of State (EoS)

Energy per nucleon (***e***) as a **function** of the **total density** **$\rho=\rho_n+\rho_p$** and the **relative difference** **$\delta=(\rho_n-\rho_p)/\rho$** for **unpolarized uniform matter at T=0 assuming isospin symmetry** (even powers of δ). For **δ → 0**:



$$e(\rho,\delta) = e(\rho,0) + S_2(\rho)\delta^2 + S_4(\rho)\delta^4 + \mathcal{O}[\delta^6]$$

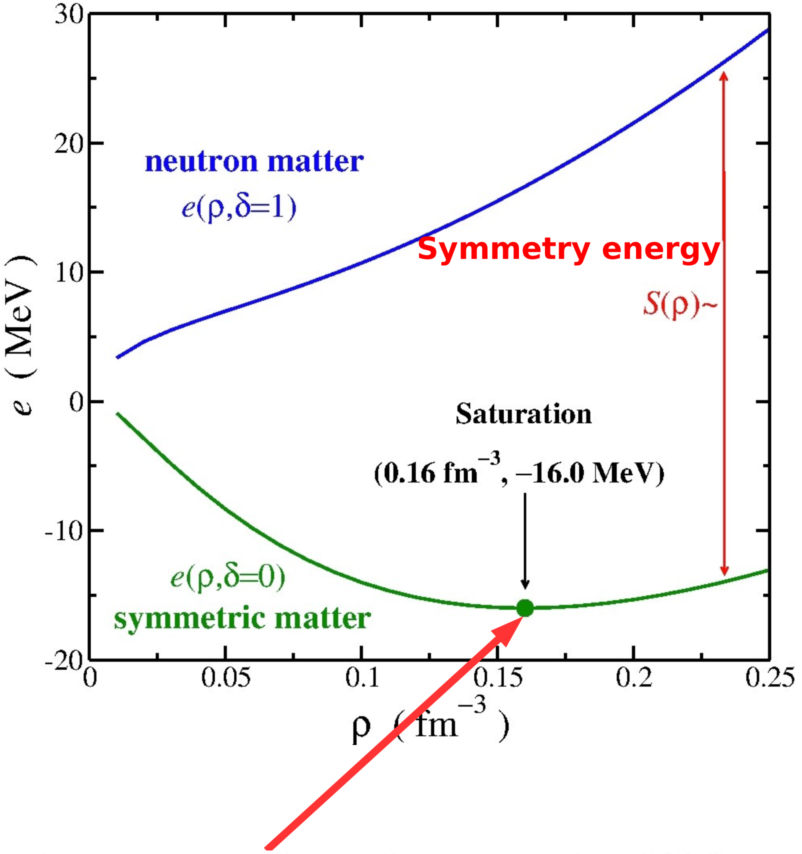


Isaac Vidaña, Constança Providência, Artur Polls, and Arnau Rios
Phys. Rev. C **80**, 045806 – Published 23 October 2009

Nuclear Equation of State (EoS)

Unpolarized, uniform nuclear matter at T=0 assuming isospin symmetry

$$e(\rho,\delta) = e(\rho,0) + S_2(\rho)\delta^2$$



$$P(\rho=\rho_0,\delta=0) = 0 \ \text{MeV fm}^{-3}$$

It is customary to also **expand** e(ρ,0) and S(ρ) around nuclear **saturation density** $\rho_0 \sim 0.16\ \text{fm}^{-3}$

$$e(\rho,0) = e(\rho_0,0) + \frac{1}{2}K_0 x^2 + \mathcal{O}[\rho^3] \text{ where } x = \frac{\rho-\rho_0}{3\rho_0}$$

$$S(\rho) = J + Lx + \frac{1}{2}K_{sym}x^2 + \mathcal{O}[\rho^3,\delta^4]$$

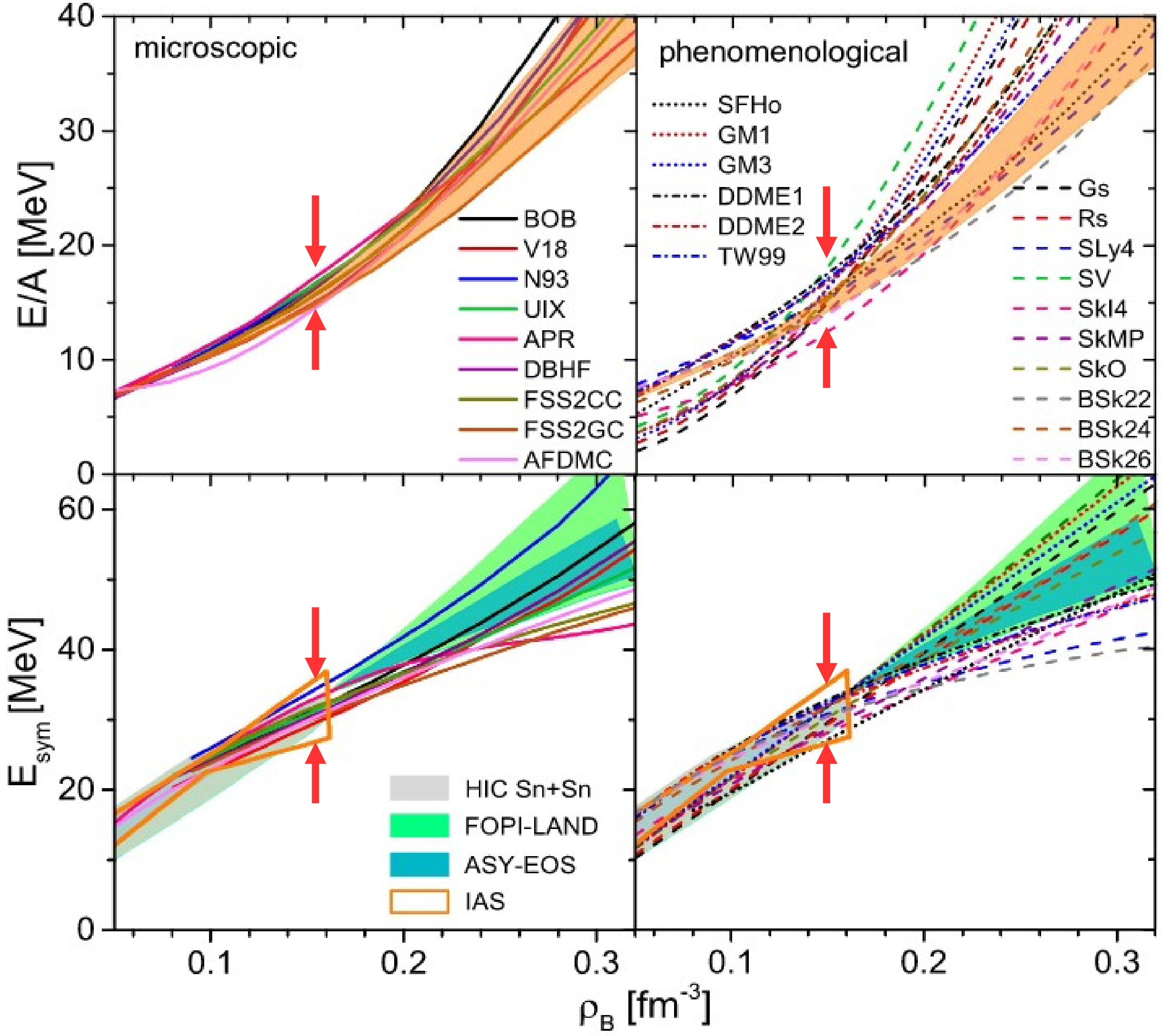
K_0 → how **compressible** is symmetric matter at ρ_0

J → **penalty energy** for converting all **protons into neutrons** in symmetric matter at ρ_0

L → **neutron pressure** in neutron matter at ρ_0

EoS from current nuclear models

Micorscopic and phenomenological models constrainted by different data display similar discrepances on the EoS



Many-body methods have been shown to agree

Main source of uncertainty in the nuclear Hamiltonian

Progress in Particle and Nuclear Physics 120 (2021) 103879

Light orange bands from χEFT from Drischler C., et al. Phys. Rev. Lett., 125 202702 (2020).

How one can connect finite nuclear properties with the symmerty energy:

Example: Neutron skin thickness ($\Delta r_{np}=r_n-r_p$) is a good proxy to L

B. Alex Brown Phys. Rev. Lett. 85, 5296 (2000)

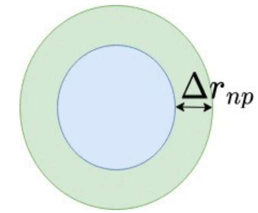
$\boldsymbol{\Delta r_{np}}$ in a **heavy** neutron rich **nucleus** is related to the **neutron pressure** ($\delta=1$) around ρ_0 (***L***).

→ EoS:

$$P(\rho_0,\delta)=\rho_0^2\frac{\partial e(\rho,\delta)}{\partial\rho}\bigg|_{\rho=\rho_0}=\frac{1}{3}\rho_0\delta^2L$$

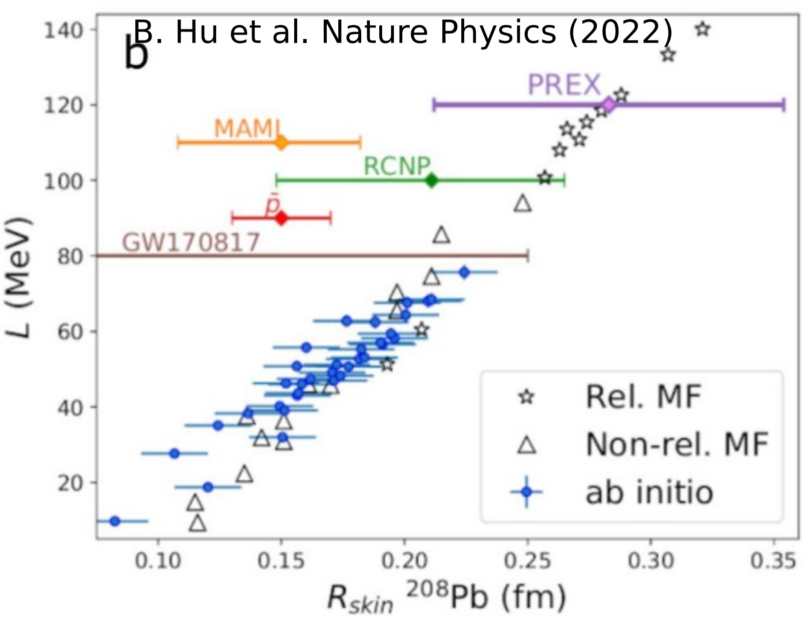
→ Macro Model:

$$\Delta r_{np}\approx\frac{1}{12}\frac{N-Z}{A}\frac{R}{J}L$$



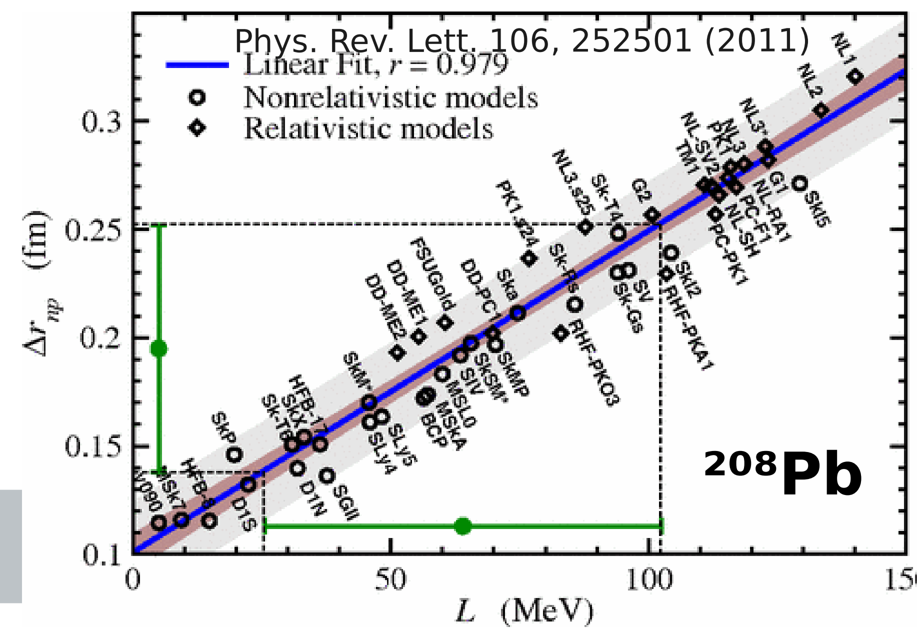
→ Micro & Pheno models:

B. Hu et al. Nature Physics (2022)



→ More Pheno models:

Phys. Rev. Lett. 106, 252501 (2011)



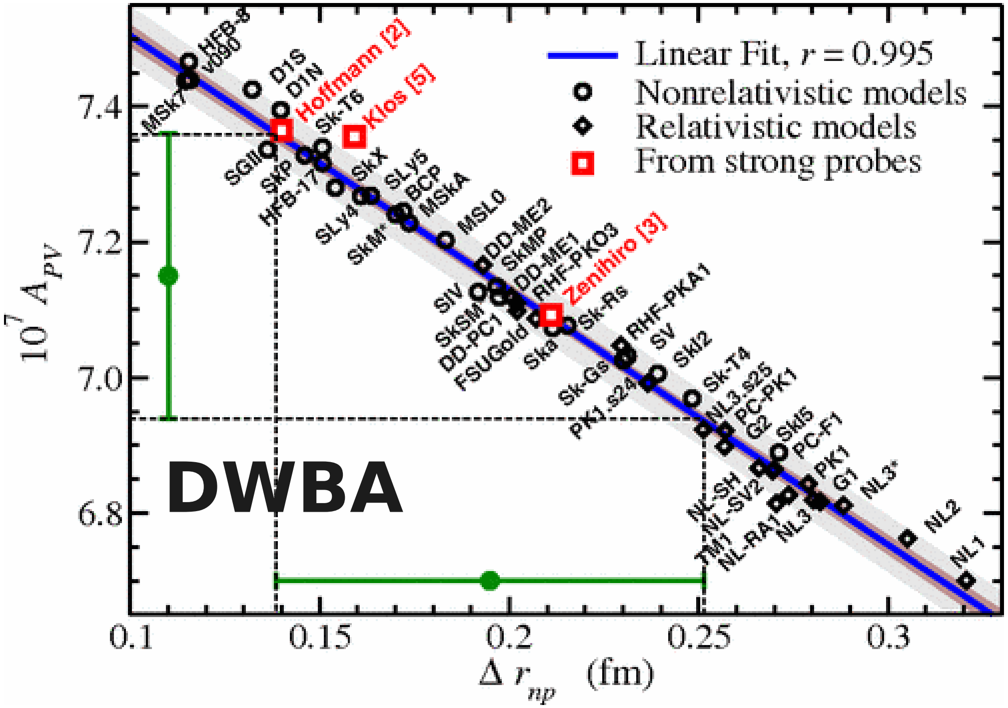
How to measure Δr_{np}? (just two examples)

Parity violating and parity conserving elastic electron scattering

Polarized electron-Nucleus scattering:

→ In good approximation, the weak interaction probes the neutron distribution in nuclei while Coulomb interaction probes the proton distribution

$$A_{pv} = \frac{d\sigma_+/d\Omega - d\sigma_-/d\Omega}{d\sigma_+/d\Omega + d\sigma_-/d\Omega} \sim \frac{\text{Weak}}{\text{Coulomb}}$$

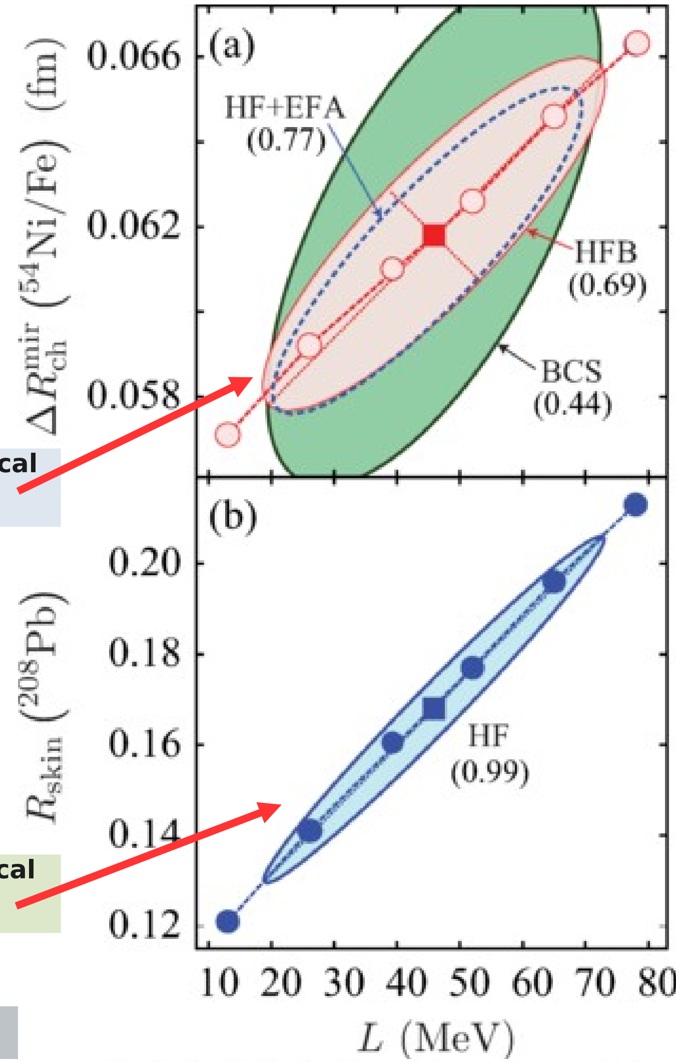


Neutron Skin of 208Pb, Nuclear Symmetry Energy, and the Parity Radius Experiment
X. Roca-Maza, M. Centelles, X. Viñas, and M. Warda Phys. Rev. Lett. 106, 252501 (2011)

Isospin symmetry →
$\Delta r_{ch} = r_{ch}(^{54}Ni) - r_{ch}(^{54}Fe) = \Delta r_{np}(^{54}Fe)$

Large theoretical uncertainties

Small theoretical uncertainties



Paul-Gerhard Reinhard and Witold Nazarewicz
Phys. Rev. C **105**, L021301 – Published 3 February 2022

Skyy V. Pineda, Kristian König, Dominic M. Rossi, B. Alex Brown, Anthony Incorvati, Jeremy Lantis, Kei Minamisono, Wilfried Nörtershäuser, Jorge Piekarewicz, Robert Powel, and Felix Sommer
Phys. Rev. Lett. **127**, 182503 – Published 29 October 2021

Other Observables?
Dipole polarizability, J and Δr_{np}

The electric dipole **polarizability** measures the **tendency** of the nuclear **charge distribution** to be **distorted** by an **external electric field**.

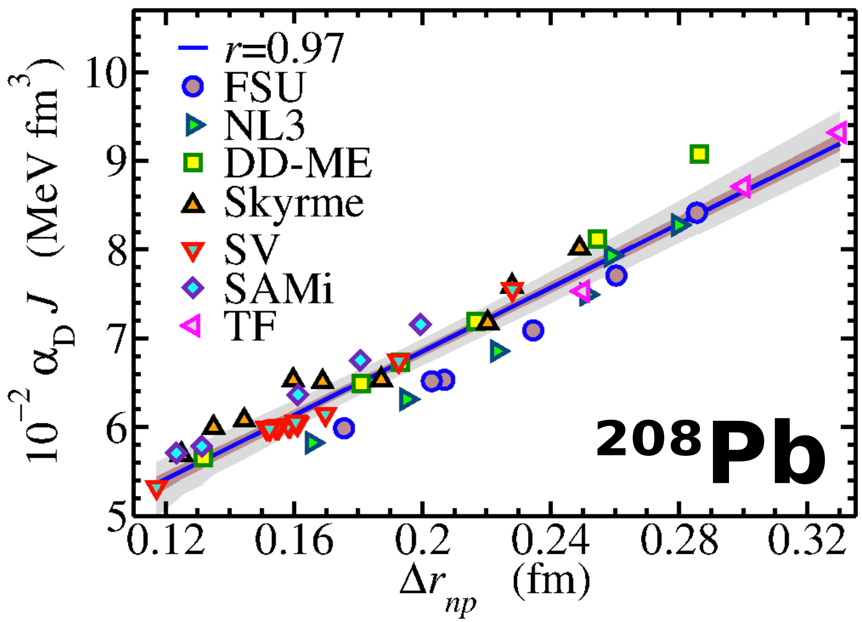
From a macroscopic point of view **α ~ (electric dipole moment)/($E_{external}$)**

→ For guidance, using the **dielectric theorem**, the polarizability can be calculated assuming the **Droplet model**:

$$\alpha_D = \frac{8\pi e^2}{9} m_{-1}(E1)$$

Meyer et al. NPA385 (1982) 269-284

$$\alpha_D \approx \frac{\pi e^2}{54} \frac{\langle r^2 \rangle}{J} A \left(1 + \frac{5}{2} \frac{\Delta r_{np} - \Delta r_{np}^{\text{surf}} - \Delta r_{np}^{\text{Coul}}}{\langle r^2 \rangle^{1/2} (I - I_{\text{Coul}})} \right)$$



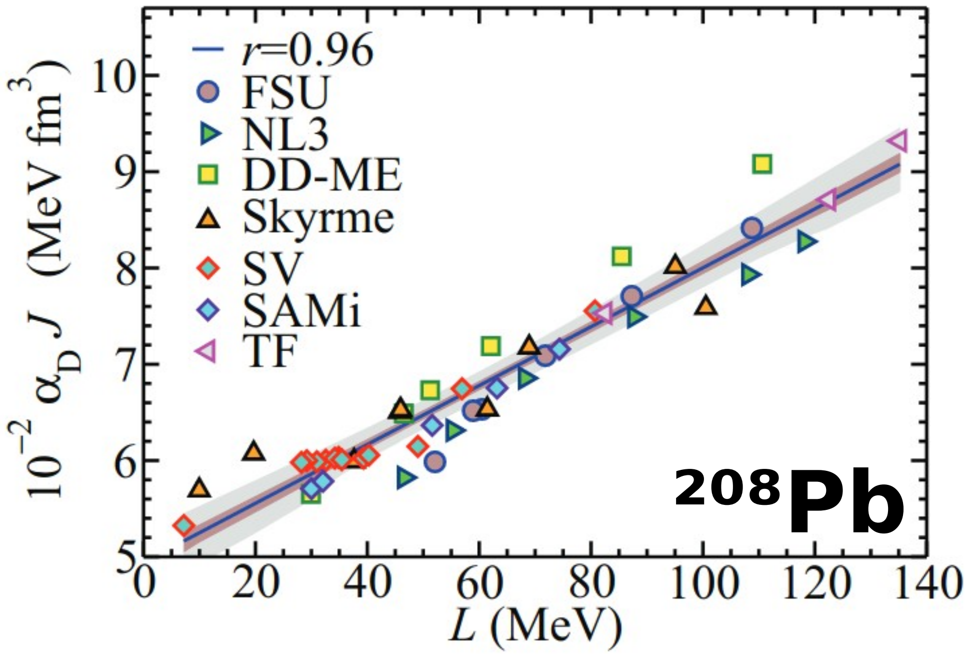
Electric dipole polarizability in 208Pb: Insights from the droplet model - X. Roca-Maza, M. Brenna, G. Colò, M. Centelles, X. Viñas, B. K. Agrawal, N. Paar, D. Vretenar, and J. Piekarewicz

Phys. Rev. C 88, 024316 (2013)

Polarizability increases with the mass (for the dipole $A^{5/3}$, for the quadrupole $A^{7/3}$ and so on …) and it **sets a relation between the EoS parameters J and L**

Dipole polarizability, J and L

Determination of the J vs L relation from experimental data according to EDFs



$$J = 25.0(2) + 0.19(2)L \quad \text{for } ^{68}\text{Ni},$$
$$J = 25.4(1.1) + 0.17(1)L \quad \text{for } ^{120}\text{Sn},$$
$$J = 24.5(8) + 0.17(1)L \quad \text{for } ^{208}\text{Pb}.$$

Assuming Taylor expansion around ρ_0:

$$S(\langle\rho\rangle) \approx J - L\frac{\rho_0 - \langle\rho\rangle}{3\rho_0} \rightarrow J \approx S(\langle\rho\rangle) + \frac{\rho_0 - \langle\rho\rangle}{3\rho_0}L$$

one can qualitatively understand the result!!

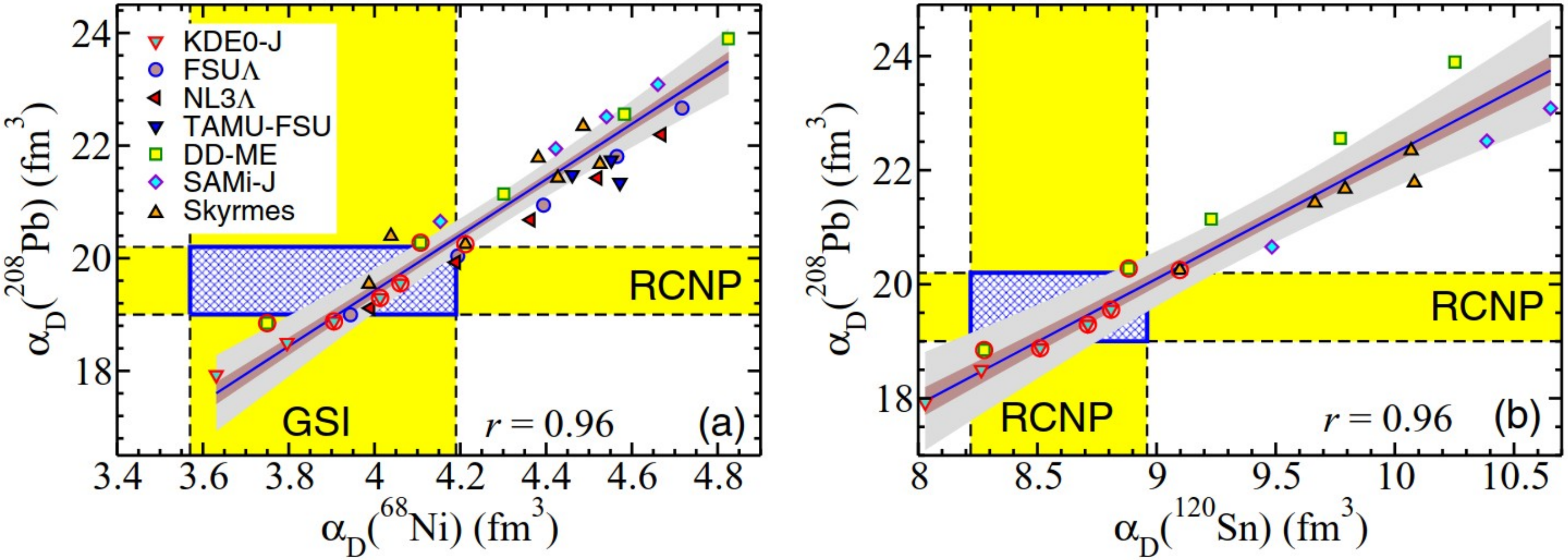
$$S(\langle\rho\rangle \approx 0.08\ \text{fm}^{-3}) \approx 25\ \text{MeV}$$

X. Roca-Maza, M. Brenna, G. Colò, M. Centelles, X. Viñas, B. K. Agrawal, N. Paar, D. Vretenar, and J. Piekarewicz
Phys. Rev. C **88**, 024316 – Published 20 August 2013

X. Roca-Maza, X. Viñas, M. Centelles, B. K. Agrawal, G. Colò, N. Paar, J. Piekarewicz, and D. Vretenar
Phys. Rev. C **92**, 064304 – Published 8 December 2015

α_D: linear correlation versus EDFs that reproduce the data

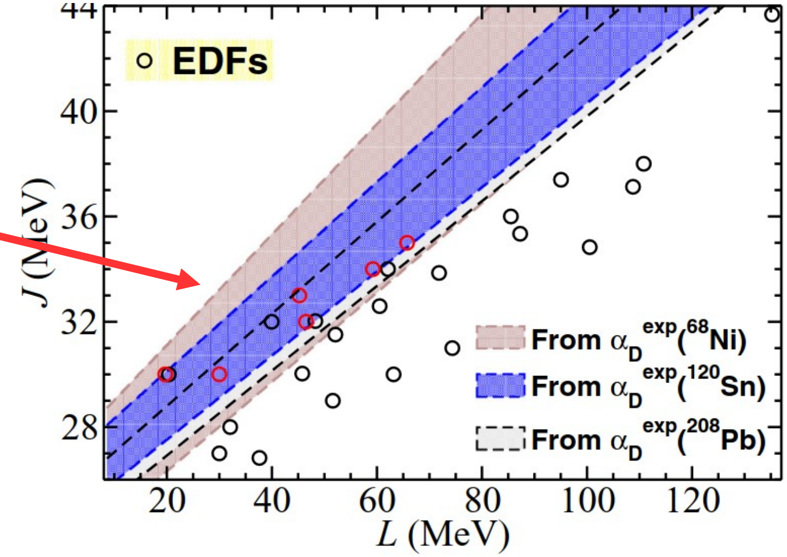
Selection of EDFs (red circles) compatible with experimental data



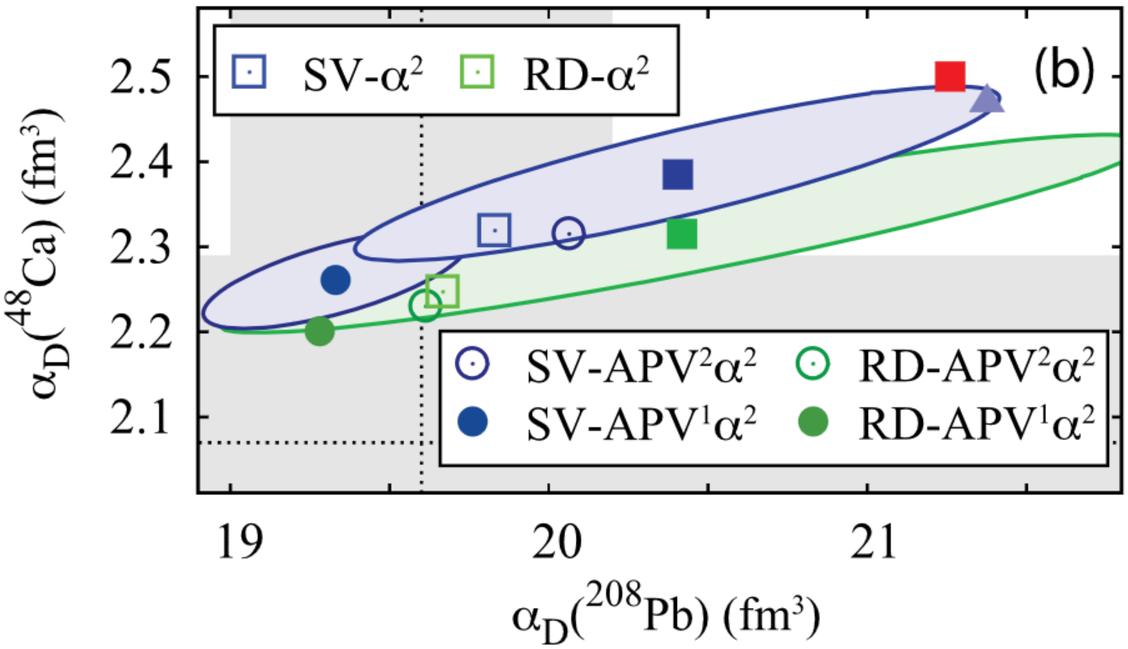
X. Roca-Maza, M. Brenna, G. Colò, M. Centelles, X. Viñas, B. K. Agrawal, N. Paar, D. Vretenar, and J. Piekarewicz
Phys. Rev. C **88**, 024316 – Published 20 August 2013
X. Roca-Maza, X. Viñas, M. Centelles, B. K. Agrawal, G. Colò, N. Paar, J. Piekarewicz, and D. Vretenar
Phys. Rev. C **92**, 064304 – Published 8 December 2015

Warning: Some EDFs not compatible with experiment (3 black circles) are within bands, but most of them outside!

This fact points to a reliable analysis but with some uncertainties not fully under control.



How models perform for A_{PV} (sensitive to Δr_{np}) and α_D (sensitive to J and Δr_{np}) in ^{48}Ca and ^{208}Pb?



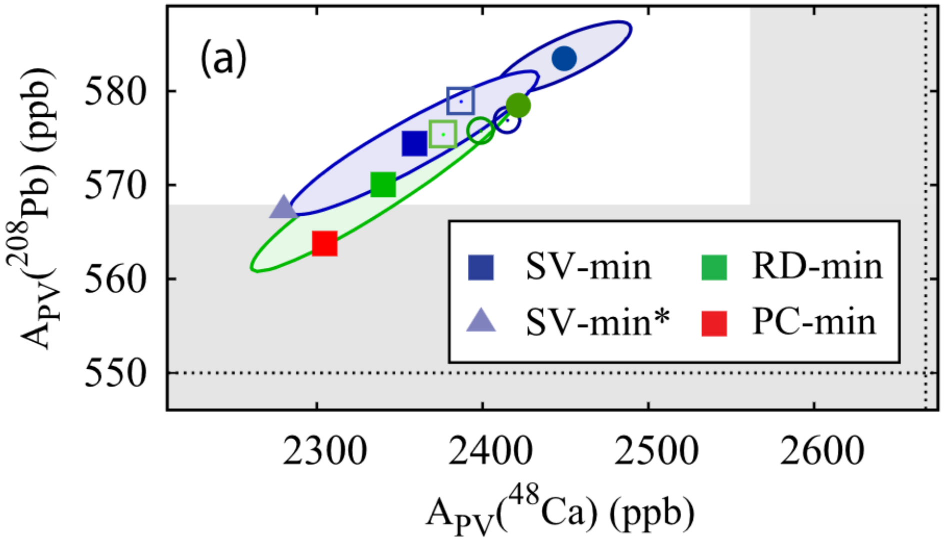
Simultaneous description of **dipole polarizabilities** → point to a **good understanding** of **symmetry energy** and **neutron skins**

Ab-initio (B. Hu) Nature Physics (2022)

$\alpha_D(^{48}\mathrm{Ca})$ $2.30^{+0.31}_{-0.26}$

$\alpha_D(^{208}\mathrm{Pb})$ $22.6^{+2.1}_{-1.8}$

No simultaneous description of **parity violating asymmetries** (ground state observable) → point to a **deficient understanding** of **neutron skins** (actually of A_{pv})

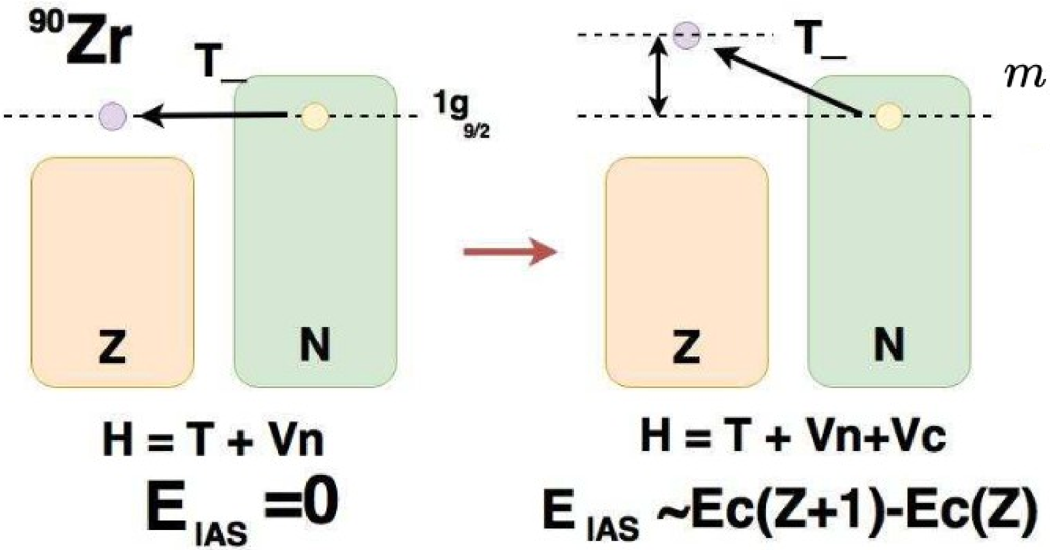


Paul-Gerhard Reinhard, Xavier Roca-Maza, and Witold Nazarewicz, PRL 127 232501 (2021) and PRL 129 232501 (2022)

Magenta dashed lines from Δr_{np} in G. Hagen et al. Nature Physics 12, 186–190 (2016) and H. Bu et al. Nature Physics (2022)

Fermi or Isobaric Analog Resonance

$$F = T_{\pm} = \sum_{i}^{A} t_{\pm}(i)$$



→ energy weighted sum rule:

$$m_1 = \sum_{\nu} (E_\nu - E_0) |\langle \nu | F | 0 \rangle|^2 = \langle 0 | T_+ [\mathcal{H}, T_-] | 0 \rangle$$

[H,T₋] different from zero only if **H** contains terms that **breaks isospin invariance**

→ **Coulomb** leading ISB**:**

$$E_{\text{IAS}} = \frac{\langle 0 | T_+ [\mathcal{H}, T_-] | 0 \rangle}{\langle 0 | T_+ T_- | 0 \rangle}$$

$$\approx \frac{6}{5} \frac{Ze^2}{r_0 A^{1/3}} \left(1 - \sqrt{\frac{5}{12}} \frac{N}{N-Z} \frac{\Delta R_{\text{np}}}{r_0 A^{1/3}} \right)$$

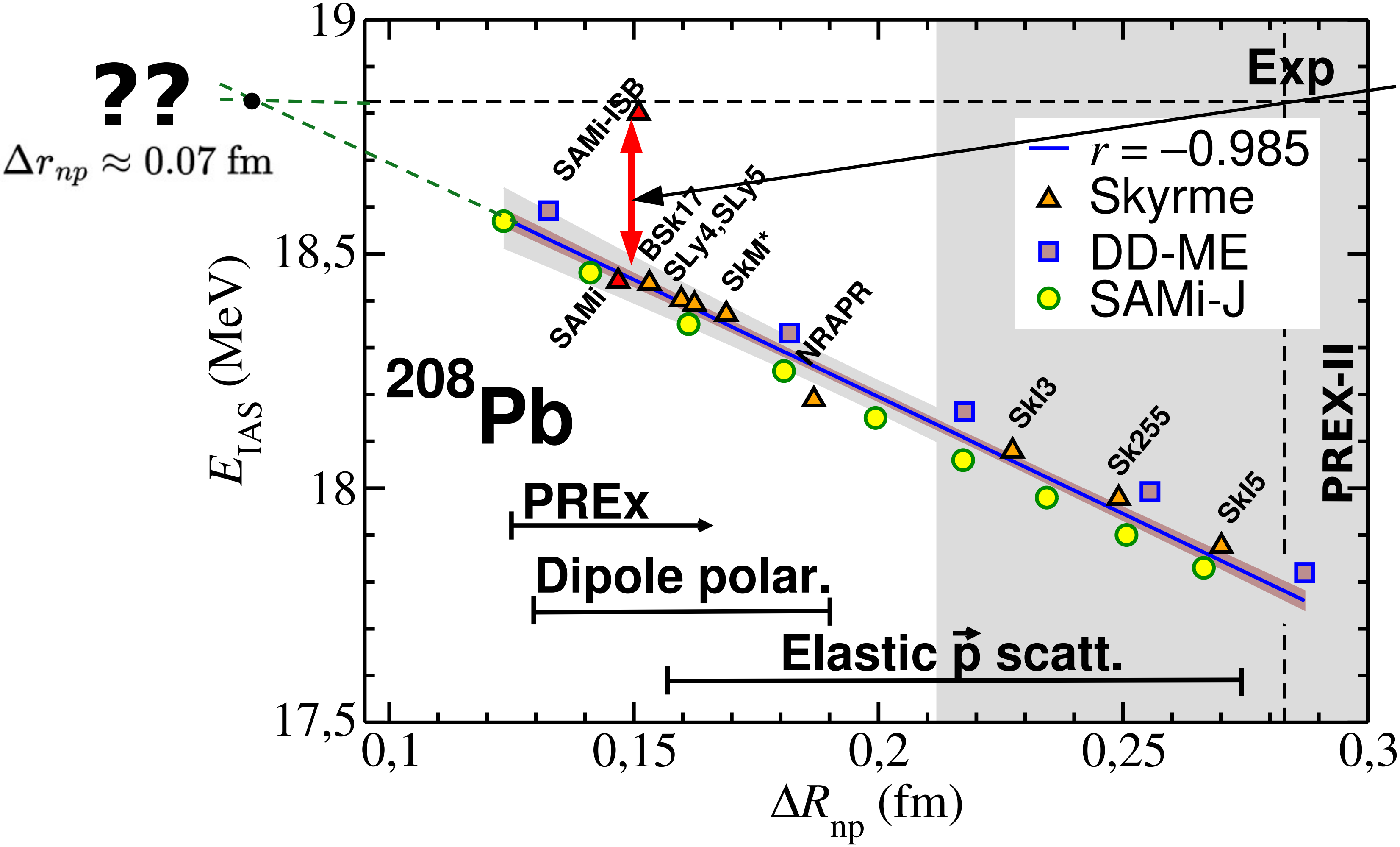
→ non-energy weighted sum rule:

$$\begin{aligned} m_0^- - m_0^+ &= \langle 0 | T_+ T_- | 0 \rangle - \langle 0 | T_- T_+ | 0 \rangle \\ &= \langle 0 | [T_+, T_-] | 0 \rangle = \langle 0 | 2T_z | 0 \rangle \\ &= N - Z \end{aligned}$$

Note: If not isospin-mixing it would be zero!!

Isobaric Analog State, ISB and Δr_{np}

$$F = T_{\pm} = \sum_{i}^{A} t_{\pm}(i)$$



Isospin symmetry breaking (ISB) missing effects:

1) **Nuclear strong int.**
2) **Coulomb corrections**

The larger the Δr_{np}, the larger the ISB contributions to IAS in ^{208}Pb

X. Roca-Maza, G. Colò, and H. Sagawa
Phys. Rev. Lett. **120**, 202501 – Published 18 May 2018

QCD-Based Charge Symmetry Breaking Interaction

arXiv:2305.17481 [pdf, other] nucl-th hep-lat hep-ph nucl-ex

QCD-Based Charge Symmetry Breaking Interaction and Okamoto-Nolen-Schiffer Anomaly

Authors: Hiroyuki Sagawa, Tomoya Naito, Xavier Roca-Maza, Tetsuo Hatsuda

Class	Method or Name	$\tilde{s}_0$ (MeV fm^3)	$\tilde{s}_1$ (MeV fm^5)	$\tilde{s}_2$ (MeV fm^5)
Pheno	SAMi-ISB	-52.6 ± 1.4	—	—
Pheno	SLy4-ISB (leading order)	-22.4 ± 4.4	—	—
Pheno	SkM*-ISB (leading order)	-22.4 ± 5.6	—	—
Pheno	SV$_\mathrm{T}$-ISB (leading order)	-29.6 ± 7.6	—	—
Pheno	SV$_\mathrm{T}$-ISB (next-leading order)	$+44 \pm 8$	-56 ± 16	-31.2 ± 3.2
Pheno	Estimation by isovector density	-17.6 ± 32.0	—	—
Theor	ΔE_tot (N^{2}LO$_\mathrm{GO}$ (394) & CC)	-4.2 ± 6.5	—	—
Theor	ΔE_tot (N^{2}LO$_\mathrm{GO}$ (450) & CC)	-5.1 ± 28.5	—	—
Theor	ΔE (AV18-UX & GFMC)	-6.413 ± 0.173	—	—
Theor	QCD sum rule (Case I)	$-15.5^{+8.8}_{-12.5}$	$+0.52^{+0.42}_{-0.29}$	—
Theor	QCD sum rule (Case II)	$-15.5^{+8.8}_{-12.5}$	—	$+0.18^{+0.14}_{-0.10}$

Estimated from IAS or mirror mass differences

Estimated from Novario, et al. PRL130, 032501 (2023) & Wiringa private comm.

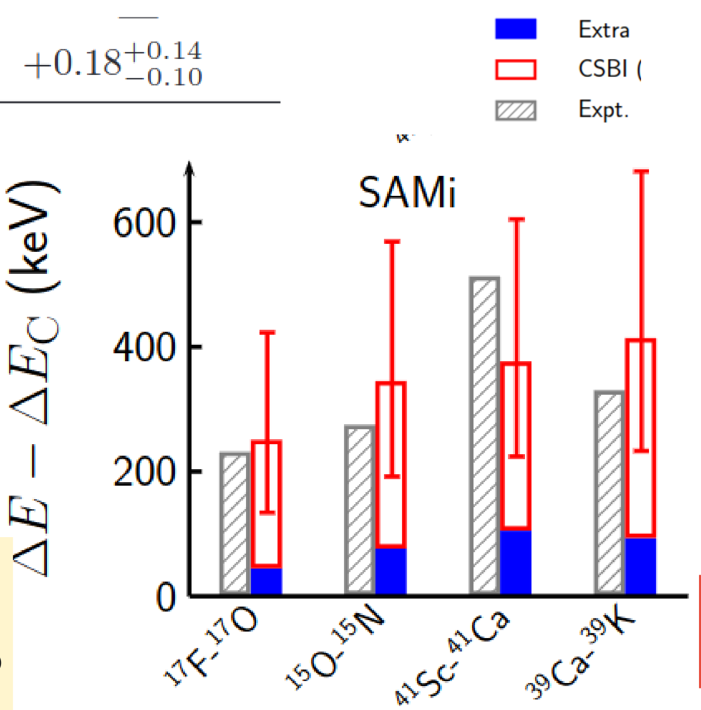
Universal form in-medium χ condensate allow

$$\delta_\mathrm{chiral} \equiv \Delta_{np}(0) - \Delta_{np}(\rho) \propto k_1 \frac{\rho}{\rho_0} + k_2 \left(\frac{\rho}{\rho_0}\right)^{5/3}$$

$$k_1 = -\frac{\sigma_{\pi N}\rho_0}{f_\pi^2 m_\pi^2} < 0, \qquad k_2 = -k_1 \frac{3k_{\mathrm{F}0}^2}{10 m_N^2}$$

$$\sigma_{\pi N} = \frac{1}{2M_N}\langle N|\hat{m}(\bar{u}u + \bar{d}d)|N\rangle,$$

Largest uncertainty on the light quark scalar current



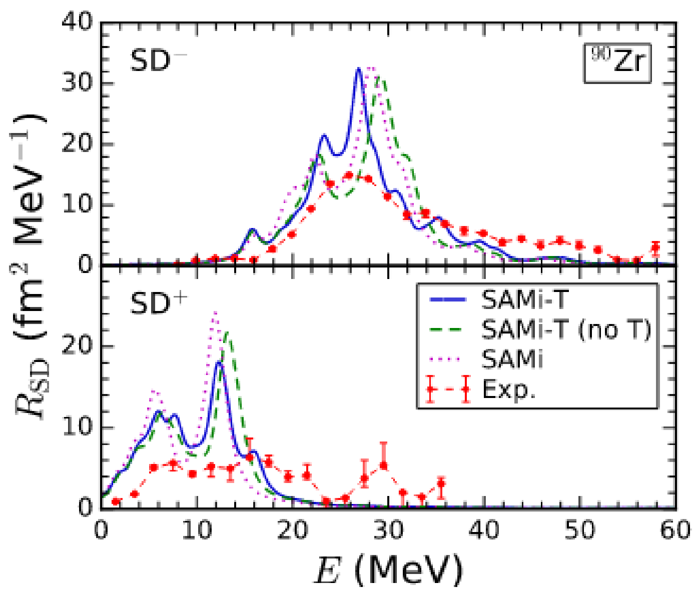
→ Theoretical results on ISB + Exp. in IAS ^{208}Pb would imply a very small Δr_{np}

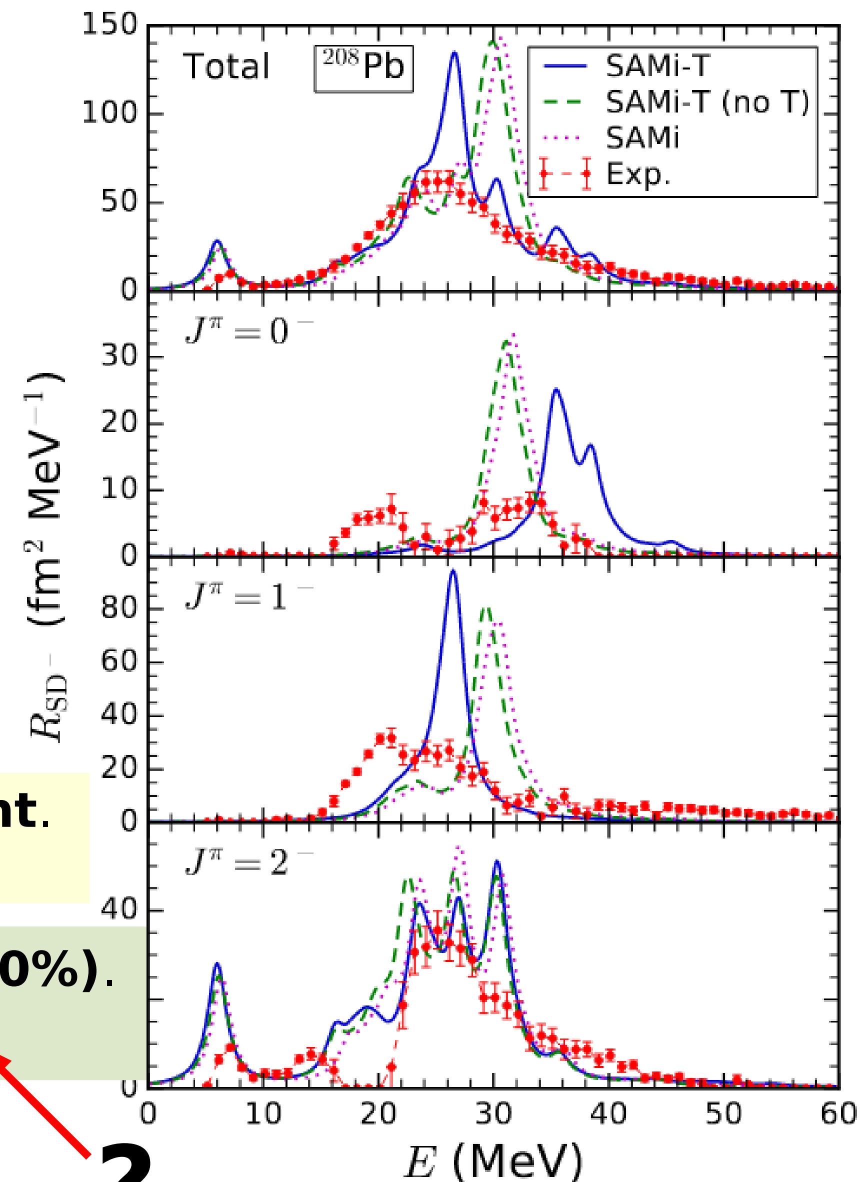
Spin Dipole Resonance and Δr_{np}

Difficult to measure and analyze?

$$\sum_{i=1}^{A} \sum_{M} \tau_{\pm}(i) r_i^L \left[Y_L(\hat{r}_i) \otimes \boldsymbol{\sigma}(i) \right]_{JM}$$

$$m_0(t_-) - m_0(t_+) = \frac{9}{4\pi}\left(N\langle r_n^2\rangle - Z\langle r_p^2\rangle\right)$$





→ ^{90}Zr **exp and theo sum rules in agreement**.
From exp. sum rule: $\Delta r_{np} = 0.07 \pm 0.04$ fm

→ ^{208}Pb **exp sum rule < theo sum rule (~20%)**.
From exp. sum rule: $\Delta r_{np} = 0.07 \pm 0.03$ fm

?

Shihang Shen (申时行), Gianluca Colò, and Xavier Roca-Maza
Phys. Rev. C **99**, 034322 – Published 20 March 2019

Conclusions

Different ways to investigate properties of the **symmetry energy** ($\rho<\rho_0$) using **different observables** provide **different answers.**

Theory:

→ An effort to understand the **parity violating asymmetry** and the **beam normal spin asymmetry** in **^{208}Pb** is needed.

→ An effort to better understand the **systematics** on the **dipole polarizability** is needed (e.g. along the Sn isotopic chain).

→ An effort to better understand **Isospin Symmetry Breaking** in nuclei in connection to the experimental knowledge of the **IAS** could provide robust insights into the symmerty energy

→ Study of **charge-exchange resonances** that naturally isolate differences between protons and neutrons could be useful to propose new experiments sensitive to the symmetry energy.

Experiment:

→ An effort to improve the accuracy in the **parity violating asymmetry in ^{208}Pb** (and/or measure other Q values) and confirm the measured values for the beam normal spin asymmetry is needed.

→ An effort to measure the **dipole polarizability** in neutron-rich Sn isotopes (N>74) will help understanding structure effects as well as provide information on the symmetry energy.

→ Measure **charge-exchange resonances** like Spin-Dipole Resonance? (any medium/heavy nucleus)

Collaborators

→ Gianluca **Colò** (University of Milan)

→ Hiroyuki **Sagawa** (University of Aizu & RIKEN)

→ Tomoya **Naito** (University of Tokyo & RIKEN)

→ Shihang **Shen** (Forschungszentrum Jülich)

→ Xavier **Vinyes** & Mario **Centelles** (University of Barcelona)

→ Jorge **Piekarewicz** (Florida State University)

→ Nils **Paar** & Dario **Vretenar** (University of Zagreb)

→ Bijay K. **Agrawal** (Saha Institute of Nuclear Physics)

→ P.-G. **Reinhard** (University of Erlangen-Nürnberg)

→ Witold **Nazarewicz** (FRIB and Michigan State University)